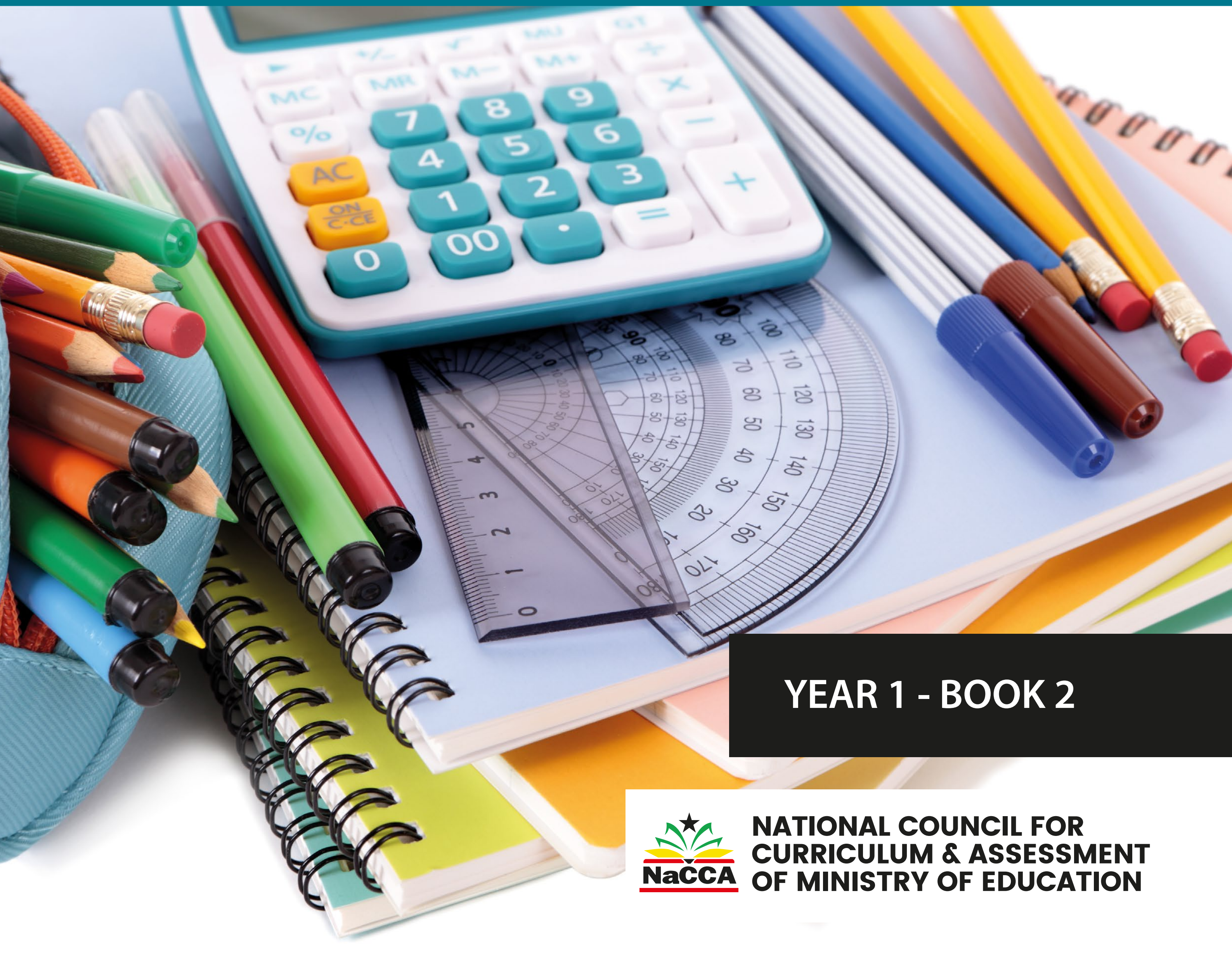
MINISTRY OF EDUCATION

MATHEMATICS

For Senior High Schools

TEACHER MANUAL



YEAR 1 - BOOK 2

NaCCA
NATIONAL COUNCIL FOR CURRICULUM & ASSESSMENT OF MINISTRY OF EDUCATION

MINISTRY OF EDUCATION

REPUBLIC OF GHANA

Mathematics

For Senior High Schools

Teacher Manual

Year One – Book Two

MATHEMATICS TEACHER MANUAL

Enquiries and comments on this manual should be addressed to:
The Director-General
National Council for Curriculum and Assessment (NaCCA)
Ministry of Education
P.O. Box CT PMB 77
Cantonments Accra

Telephone: 0302909071, 0302909862
Email: info@nacca.gov.gh
website: www.nacca.gov.gh

CONTENTS

INTRODUCTION

The National Council for Curriculum and Assessment (NaCCA) has developed a new Senior High School (SHS), Senior High Technical School (SHTS) and Science, Technology, Engineering and Mathematics (STEM) Curriculum. It aims to ensure that all learners achieve their potential by equipping them with 21st Century skills, competencies, character qualities and shared Ghanaian values. This will prepare learners to live a responsible adult life, further their education and enter the world of work.

This is the first time that Ghana has developed an SHS Curriculum which focuses on national values, attempting to educate a generation of Ghanaian youth who are proud of our country and can contribute effectively to its development.

This Book Two of the Teacher Manual for Mathematics covers all aspects of the content, pedagogy, teaching and learning resources and assessment required to effectively teach Year One of the new curriculum. It contains information for the second 11 weeks of Year One. Teachers are therefore to use this Teacher Manual to develop their weekly Learning Plans as required by Ghana Education Service.

Some of the key features of the new curriculum are set out below.

Learner-Centred Curriculum

The SHS, SHTS, and STEM curriculum places the learner at the center of teaching and learning by building on their existing life experiences, knowledge and understanding. Learners are actively involved in the knowledge-creation process, with the teacher acting as a facilitator. This involves using interactive and practical teaching and learning methods, as well as the learner's environment to make learning exciting and relatable. As an example, the new curriculum focuses on Ghanaian culture, Ghanaian history, and Ghanaian geography so that learners first understand their home and surroundings before extending their knowledge globally.

Promoting Ghanaian Values

Shared Ghanaian values have been integrated into the curriculum to ensure that all young people understand what it means to be a responsible Ghanaian citizen. These values include truth, integrity, diversity, equity, self-directed learning, self-confidence, adaptability and resourcefulness, leadership and responsible citizenship.

Integrating 21st Century Skills and Competencies

The SHS, SHTS, and STEM curriculum integrates 21st Century skills and competencies. These are:

- **Foundational Knowledge:** Literacy, Numeracy, Scientific Literacy, Information Communication and Digital Literacy, Financial Literacy and Entrepreneurship, Cultural Identity, Civic Literacy and Global Citizenship
- **Competencies:** Critical Thinking and Problem Solving, Innovation and Creativity, Collaboration and Communication
- **Character Qualities:** Discipline and Integrity, Self-Directed Learning, Self-Confidence, Adaptability and Resourcefulness, Leadership and Responsible Citizenship

Balanced Approach to Assessment - not just Final External Examinations

The SHS, SHTS, and STEM curriculum promotes a balanced approach to assessment. It encourages varied and differentiated assessments such as project work, practical demonstration, performance assessment, skills-based assessment, class exercises, portfolios as well as end-of-term examinations and final external assessment examinations. Two levels of assessment are used. These are:

- Internal Assessment (30%) – Comprises formative (portfolios, performance and project work) and summative (end-of-term examinations) which will be recorded in a school-based transcript.

- External Assessment (70%) – Comprehensive summative assessment will be conducted by the West African Examinations Council (WAEC) through the WASSCE. The questions posed by WAEC will test critical thinking, communication and problem solving as well as knowledge, understanding and factual recall.

The split of external and internal assessment will remain at 70/30 as is currently the case. However, there will be far greater transparency and quality assurance of the 30% of marks which are school-based. This will be achieved through the introduction of a school-based transcript, setting out all marks which learners achieve from SHS 1 to SHS 3. This transcript will be presented to universities alongside the WASSCE certificate for tertiary admissions.

An Inclusive and Responsive Curriculum

The SHS, SHTS, and STEM curriculum ensures no learner is left behind, and this is achieved through the following:

- Addressing the needs of all learners, including those requiring additional support or with special needs. The SHS, SHTS, and STEM curriculum includes learners with disabilities by adapting teaching and learning materials into accessible formats through technology and other measures to meet the needs of learners with disabilities.
- Incorporating strategies and measures, such as differentiation and adaptative pedagogies ensuring equitable access to resources and opportunities for all learners.
- Challenging traditional gender, cultural, or social stereotypes and encouraging all learners to achieve their true potential.
- Making provision for the needs of gifted and talented learners in schools.

Social and Emotional Learning

Social and emotional learning skills have also been integrated into the curriculum to help learners to develop and acquire skills, attitudes, and knowledge essential for understanding and managing their emotions, building healthy relationships and making responsible decisions.

Philosophy and vision for each subject

Each subject now has its own philosophy and vision, which sets out why the subject is being taught and how it will contribute to national development. The Philosophy and Vision for Mathematics is:

Philosophy: Every learner can develop their potential in mathematics through creative and innovative ways to become lifelong learners, apply mathematical skills and competencies to solve everyday problems, further their education and/or proceed to the world of work.

Vision: Trained mathematically enthusiastic learners who are highly interested in the subject and are capable of reasoning, modelling, representing and making use of mathematical tools and technology to solve problems in real life, further their studies and/or proceed to the world of work.

SUMMARY SCOPE AND SEQUENCE

S/N	STRAND	SUB-STRAND	SHS1			SHS2			SHS3		
			CS	LO	LI	CS	LO	LI	CS	LO	LI
1.	Numbers for everyday life	Real number and Numeration system	3	3	8	2	2	5	-	-	-
		Proportional reasoning	2	2	4	2	2	4	2	2	4
2.	Algebraic Thinking	Applications of expressions, equations and inequalities	2	3	6	1	2	4	-	-	-
		Patterns and relationships	2	2	4	1	1	5	1	1	3
3.	Geometry around us	Spatial sense	1	1	5	1	1	4	2	2	7
		Measurement	3	3	8	3	3	8	1	1	2
4.	Making sense of and using data	Statistical reasoning and its application in real life	3	3	8	3	3	7	2	2	4
		Chance	1	1	3	1	1	2	1	1	3
Total			**17**	**18**	**46**	**14**	**15**	**39**	**9**	**9**	**22**

Overall Totals (SHS 1 – 3)

Content Standards	**40**
Learning Outcomes	**42**
Learning Indicators	**107**

SECTION 5: ANGLE AND THE PYTHAGOREAN THEOREM

Strand: **Geometry Around Us**

Sub-Strand: Spatial Sense

Learning Outcome: *Draw and describe angles of various measures; solve problems on Pythagorean theorem, parallel lines, perpendicular lines and transversal; use the exterior angle theorem of a triangle and calculate the sums of interior and exterior angles of polygons.*

Content Standard: Demonstrate conceptual understanding of spatial sense with respect to angles, parallel lines, transversal and polygons, and apply their properties to solve everyday life problems.

INTRODUCTION AND SECTION SUMMARY

Angles and polygons are fundamental concepts in geometry that provide the foundation for understanding shapes and spatial relationships. Angles are formed when two rays are joined together at a common point. They are measured in degrees and play a crucial role in defining the orientation and position of geometric figures. Polygons, on the other hand, are closed two-dimensional shapes made up of straight line segments. They include familiar shapes like triangles, quadrilaterals, pentagons, and so on. Understanding the properties of angles within polygons is essential for determining the interior and exterior angles of these shapes. Learning about angles and polygons helps students develop spatial reasoning skills and understand the properties of shapes in both two and three dimensions. These concepts are foundational for further studies in geometry and trigonometry. Additionally, they are interrelated with other areas such as algebra, physics, and engineering, where geometric concepts are applied to solve real-world problems.

The weeks covered by the section are:

Week 14:

1. Referents for angles
2. Replicating and Bisecting Angles
3. Types of Angles
4. Real-life problems involving Angles
5. Parallel lines, Perpendicular lines and Transversal
6. Complementary and Supplementary Angles
7. Pairs of angles formed by parallel lines and a transversal
8. Solve problems on parallel lines, perpendicular lines and transversal

Week 15:

1. Exterior Angle Theorem of a triangle
2. Sum of the interior angles of polygons

Week 16:

1. The Pythagorean theorem
2. Real-life uses of Pythagorean theorem

3. Applications of Pythagoras's Theorem
4. Use the Pythagorean theorem to determine if a given triangle is a right triangle

SUMMARY OF PEDAGOGICAL EXEMPLARS

This section focuses on hands-on, **experiential learning activities** for teaching angles and the Pythagorean theorem. Learners will engage in **practical measurements, collaborate in teams**, and work in **mixed-groupings** to enhance their comprehension. Activities are tailored to different learning abilities and styles to ensure active participation. For instance, learners will identify and measure angles in their surroundings to understand referents for angles. They will use protractors and paper folding to replicate and bisect angles, categorizing them based on measurements. Real-life angle problems will be tackled collaboratively to develop critical thinking. Learners will also explore parallel lines, perpendicular lines, and transversals by creating and measuring angles in geometric shapes. They will engage in **hands-on tasks** to grasp complementary and supplementary angles, and analyse angles formed by parallel lines and a transversal. Furthermore, students will solve real-world problems involving these concepts. The section also covers the exterior angle theorem of a triangle and the sum of interior angles of polygons, using manipulatives and visual aids for better understanding. The Pythagorean theorem will be taught through **practical activities**, showcasing its applications in architecture, engineering, and navigation. Students will apply the theorem to identify right triangles, reinforcing their understanding through practical applications.

ASSESSMENT SUMMARY

The concepts covered in this section require learners to demonstrate a conceptual understanding, including their real-life applications. Therefore, assessments should primarily focus on levels 2 and 3 of the Depth of Knowledge (DOK) framework. Specifically, the following assessments should be conducted and recorded for continuous assessment records:

1. Class exercises, including individual and group worksheets, after each lesson to assess understanding of concepts related to referents for angles, types of angles, and the Pythagorean theorem.
2. Homework assignments to reinforce learning and assess understanding of concepts such as replicating and bisecting angles, complementary and supplementary angles, and the exterior angle theorem of a triangle.
3. Scores on practical group activities involving measuring perimeter, area, and volume in real-life contexts, to assess understanding of parallel lines, perpendicular lines, transversals, and the sum of the interior angles of polygons.
4. Real-life problem-solving tasks to assess application of concepts, including solving problems involving angles, parallel lines, perpendicular lines, and the Pythagorean theorem.
5. Assessments focusing on the application of the Pythagorean theorem to determine if a given triangle is a right triangle, as well as real-life uses and applications of the theorem.

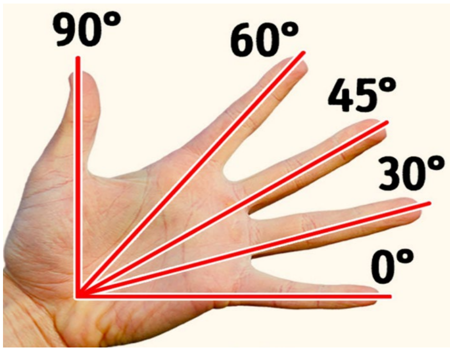
Week 14

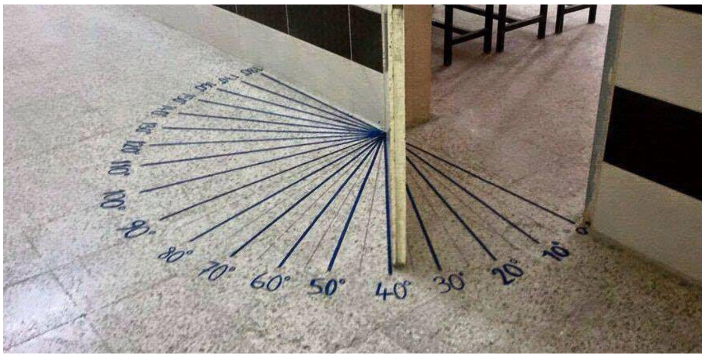
Learning Indicators:

1. *Draw and describe angles with various measures, including acute, right, straight, obtuse and reflex angles.*
2. *Solve problems that involve parallel lines, perpendicular lines and transversal, and pairs of angles formed between them.*

Theme or Focal Area: Referents for Angles

A referent is an object/item that can be used to help understand/represent a concept. Some referents of angles are corners of rooms and doors, the human palm, tree branches, adjustable chairs, etc.





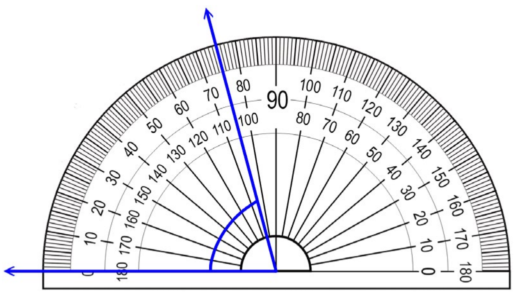
Sketching common angles around us

We can make free hand sketch of angles such as acute, right, straight, obtuse and reflex angles and verify the closeness of their sketch with the actual angle.

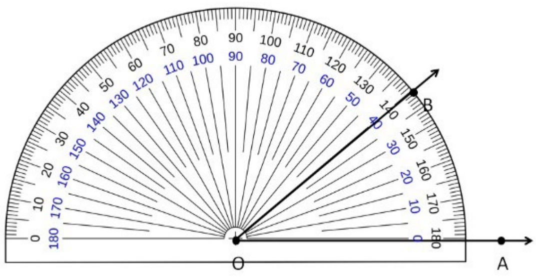
Measuring Angles such as 30°, 45°, 60°, 75º, 90° and 180°

We can measure angle sizes using a protractor.

Examples



75°



40°

Replicating Angles

Angles can be replicated in a variety of ways; examples include, protractor, compass and straightedge, carpenter's square, geometry software (e.g., Geiger).

Constructing Angles

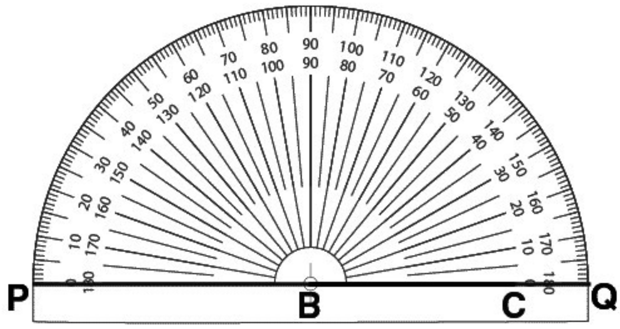
We can construct various angles using the protractor as well as a pair of compass just as you learnt in JHS. Let us try an example of each process.

Method 1: Constructing Angles Using a Protractor

For constructing angles of any given measure, be it an acute, an obtuse or a right-angle, the simplest method is by using a protractor. Let us say, you are asked to construct an angle of 120 degrees. The required steps are:

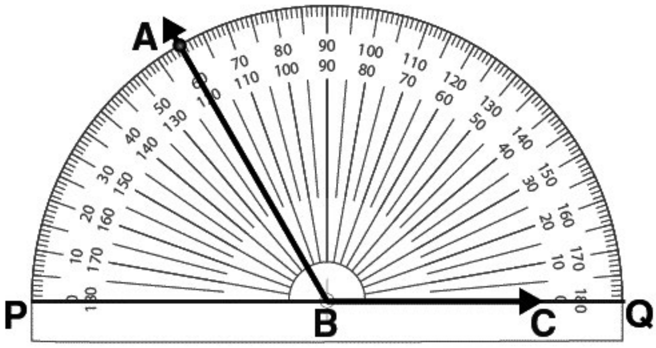
Step 1: Draw a line segment BC, which is one of the arms of the angle that is to be constructed.

Step 2: Place the protractor with its point O on point B of the line segment BC.

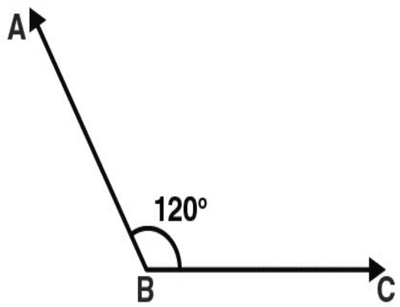


Step 3: Align OQ along the edge BC.

Step 4: The protractor has two-way markings. We consider the scale which has 0 degrees near point C for construction. Mark point A next to the 120 degrees' mark on the scale.

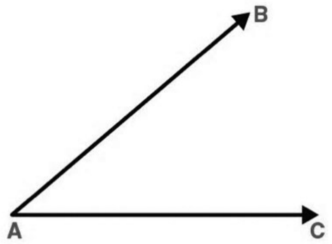


Step 5: Join points A and B. ∠ABC = 120 degrees is the required angle.



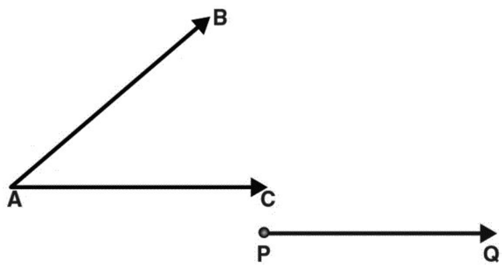
Method 2: Constructing Angles Using Compass

Constructing angles of unknown measure is basically copying a given angle whose measure is not known. We accomplish this task using compasses. Let us say that you are given ∠BAC that you are supposed to copy.

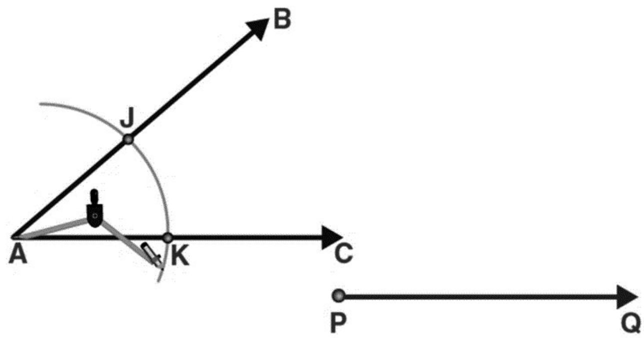


The steps to construct angles using compass are given below:

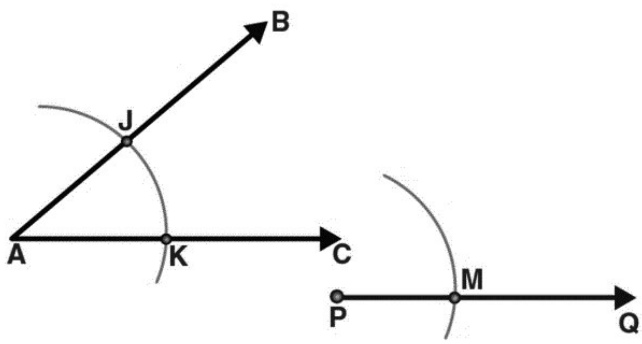
Step 1: Draw a line PQ. Point P is the vertex of the copied angle.



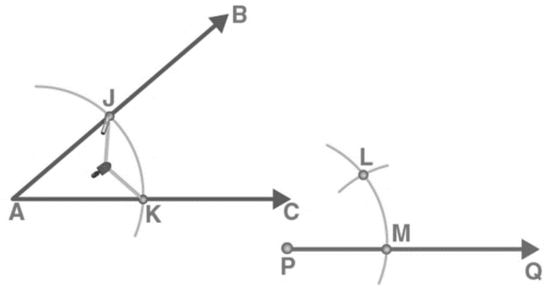
Step 2: Place the compass pointer at point A and make an arc that cuts arms AC and AB at points K and J respectively.



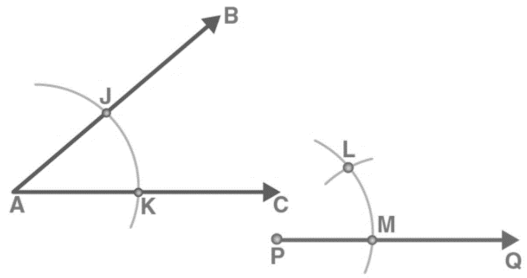
Step 3: Without changing the radius of the compass, cut an arc on PQ at point M.



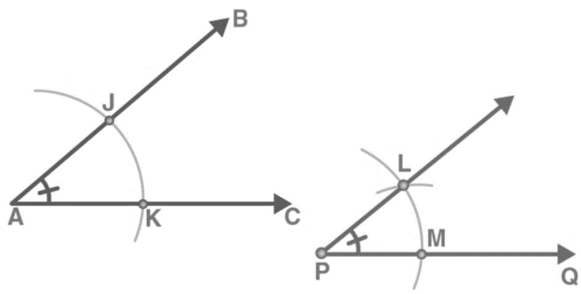
Step 4: Adjust the compass such that the pointer is placed at K and the pencil head at J.



Step 5: With the same radius, draw an arc on the first arc with the compass pointer at M. Mark the intersecting point as L.



Step 6: Join the points P and L using a ruler. Extend the line up to R.



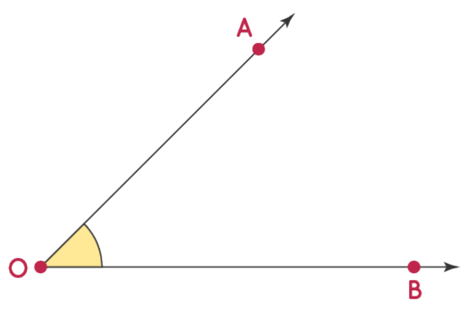
Step 7: ∠RPQ is the required angle.

N.B. *Take learners through the construction of various angle using both methods demonstrated above.*

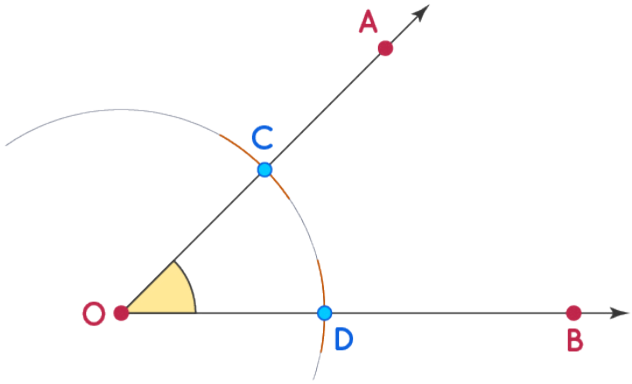
Bisecting Angles

Angles can be bisected to generate other angles that are required to carry out a particular activity. In real life situations, we bisect angles in order to be able to make models such as building construction, carpentry works, etc.

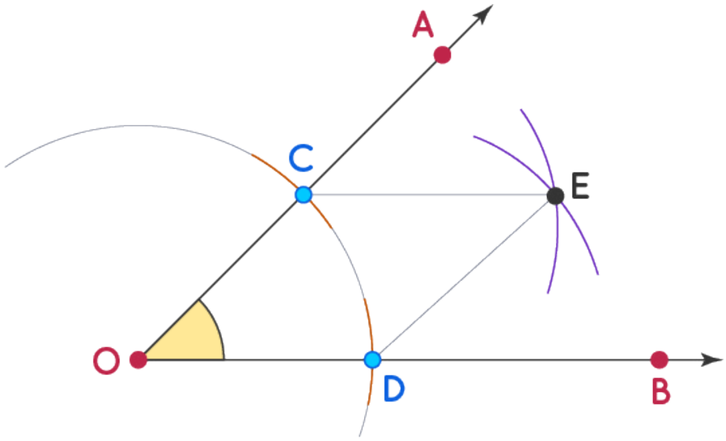
Step 1: Given Angle AOB



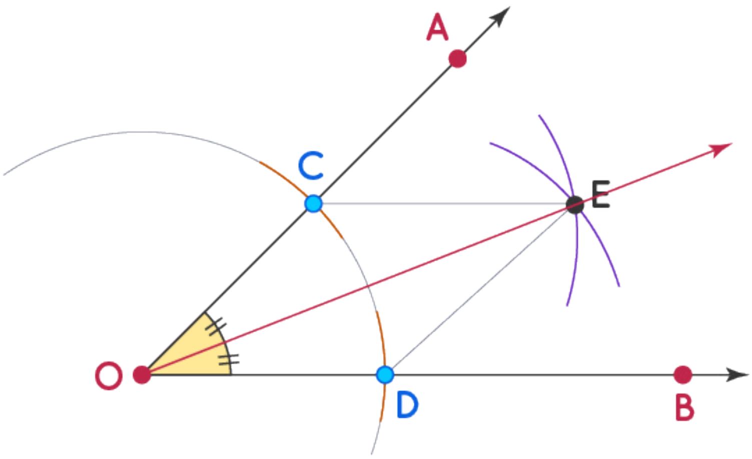
Step 2: Span any width of radius in a compass and with O as the centre, draw two arcs such that it cut the rays OA and OB at points C and D respectively.



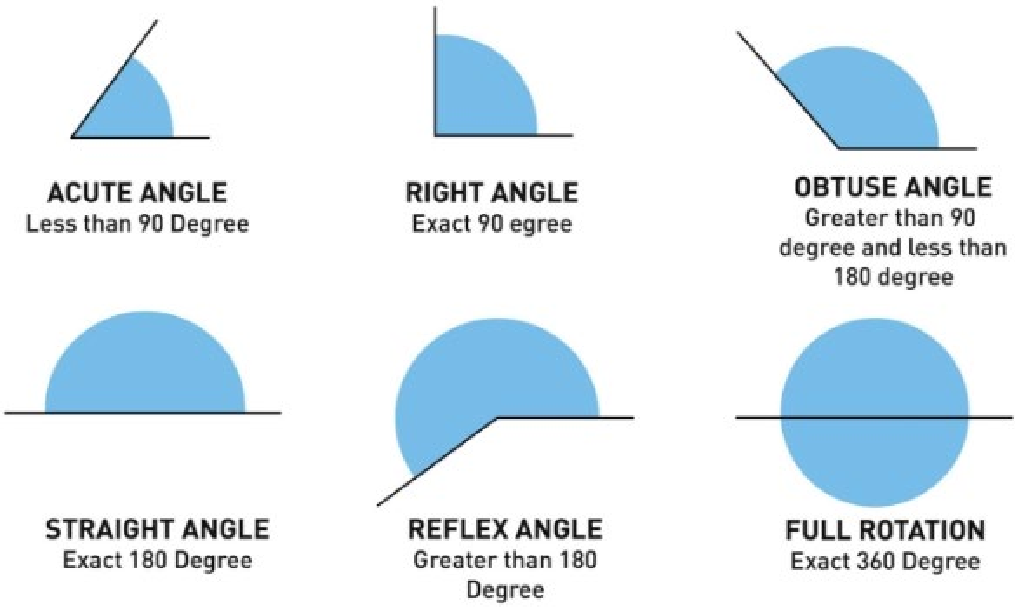
Step 3: Without changing the distance between the legs of the compass, draw two arcs with C and D as centres, such that these two arcs intersect at a point named E



Step 4: Join the ray OE. This is the required angle bisector of angle AOB.



Types of Angles



Pair of Angles

When two angles are paired, then there exist different angles, such as;

- Complementary angles
- Supplementary angles
- Linear Pair
- Adjacent angles
- Vertically Opposite angles

Learning Tasks for Practice

Learners construct angles such as;

a. 45^0

b. 75^0

c. 60^0

d. 135^0

Parallel lines, Perpendicular lines and Transversal

Perpendicular Lines:

Definition: Perpendicular lines are two lines that intersect at a 90-degree angle, forming right angles.

Real-Life Example: Consider a door frame and the floor. The door frame (vertical line) and the floor (horizontal line) intersect to form a right angle, making them perpendicular.

Parallel Lines:

Definition: Parallel lines are two or more lines that never intersect. They have the same slope and are equidistant from each other.

Real-Life Example: Look at railroad tracks. The two tracks run alongside each other, never diverging or intersecting. This demonstrates parallel lines in real life.

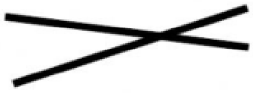
Transversal:
Definition: A transversal is a line that intersects with two or more other lines at distinct points.

Real-Life Example: Imagine a set of power lines suspended on poles at either side of a road. The road acts as the transversal line that intersects the power lines at multiple points.

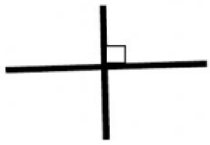
Determine whether these lines as perpendicular, parallel or neither, and justify it.

Example:

Parallel lines



Neither

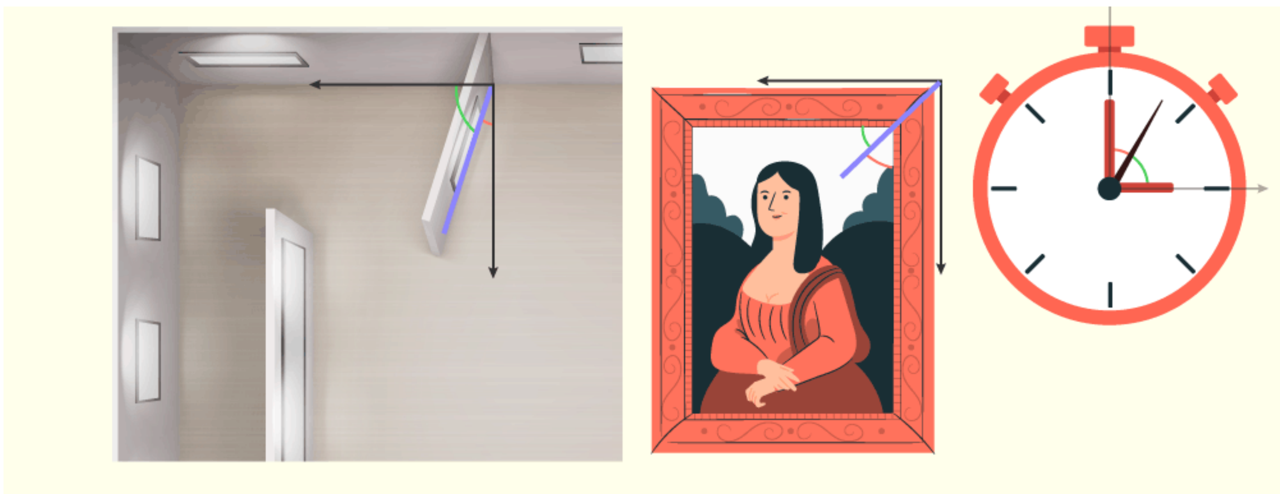


Perpendicular lines

Complementary and Supplementary Angles

Complementary Angles:
Definition: Complementary angles are two angles that add up to 90 degrees.

Real-Life Example:

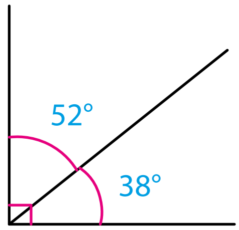
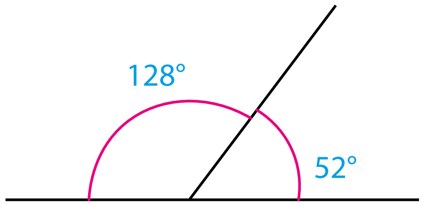


Consider the door opening. The two angles made by the door will always add up to 90, as the door swings open.

Supplementary Angles:
Definition: Supplementary angles are two angles that add up to 180 degrees.

Real-Life Example: Think about a straight road with a street sign or a lamppost. If you stand at the base of the sign and measure the angle to the left and the angle to the right of the road, those two angles will be supplementary because they add up to a straight angle of 180 degrees.

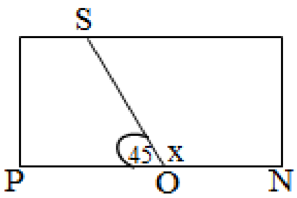
Let's take a look at these examples.

Type of Angles	Description	Example
Complementary Angles	Angles that add up to 90°	
Supplementary Angles	Angles that add up to 180°	

Real-life problems involving Angles

Example 1: Kwamena cut the straight edge of a piece of wood to make a 45^0 angle. At what angle was the other piece cut?

Solution



Think:

We need to find ∠SON

Since PON is a straight line, its angle is 180^0.

Therefore:∠PON = ∠POS + ∠ SON

∠PON − ∠POS = ∠SON

180º – 45º = 135º

Therefore, angle was the other piece cut is 135^0.

Example 2: Two angles are complementary. If one of the angles is double the other angle, find the two angles.

Solution:

Let x be one of the angles

Then the other angle is $2x$.

Because x and $2x$ are complementary angles, we have;

$x + 2x = 90$

$3x = 90$

Divide each side by 3.

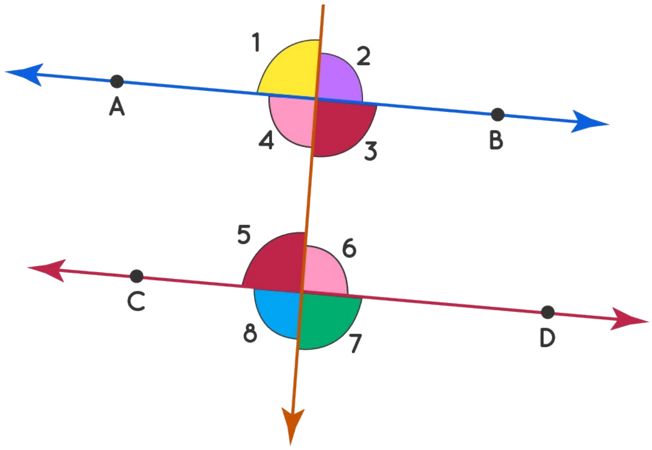
$x = 30$

$2x = 2(30) = 60$

So, the two angles are 30° and 60°.

Pairs of angles formed by parallel lines and a transversal

Study the diagram carefully,



A) In the figure given above, the **corresponding angles** formed by the intersection of the transversal are:

- $\angle 1$ and $\angle 5$
- $\angle 2$ and $\angle 6$
- $\angle 3$ and $\angle 7$
- $\angle 4$ and $\angle 8$

It should be noted that the pair of corresponding angles are equal in measure, that is, $\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 3 = \angle 7$, and $\angle 4 = \angle 8$

B) In the figure given above, there are two pairs of **alternate interior angles.**

- $\angle 3$ and $\angle 6$
- $\angle 4$ and $\angle 5$

It should be noted that the pair of alternate interior angles are equal in measure, that is, $\angle 3 = \angle 6$, and $\angle 4 = \angle 5$

C) In the figure given above, there are two pairs of **alternate exterior angles**.

- $\angle 1$ and $\angle 8$
- $\angle 2$ and $\angle 7$

It should be noted that the pair of alternate exterior angles are equal in measure, that is, $\angle 1 = \angle 8$, and $\angle 2 = \angle 7$

D) In the given figure, there are two pairs of **consecutive interior angles.**

- $\angle 4$ and $\angle 6$
- $\angle 3$ and $\angle 5$

It should be noted that unlike the other pairs given above, the pair of consecutive interior angles are supplementary, that is, $\angle 4 + \angle 6 = 180°$, and $\angle 3 + \angle 5 = 180°$.

E) In the figure given above, there are four pairs of **vertically opposite angles**.

- $\angle 1$ and $\angle 4$
- $\angle 2$ and $\angle 3$
- $\angle 6$ and $\angle 7$

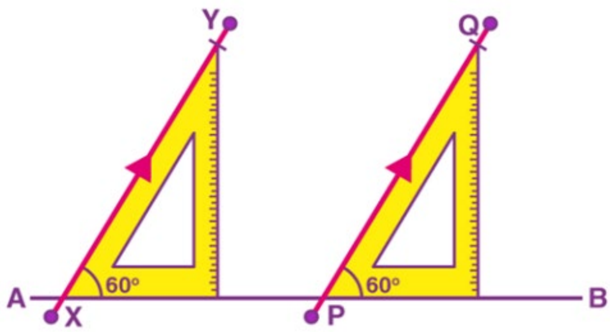
- $\angle 5$ and $\angle 8$

It should be noted that the pair of vertically opposite angles are equal in measure, that is, $\angle 1 = \angle 4$, $\angle 2 = \angle 3$, $\angle 6 = \angle 7$ and $\angle 5 = \angle 8$.

Solve problems on parallel lines, perpendicular lines and transversal

Example

A teacher asked Kofi to draw two parallel lines. With the help of his set squares and ruler, he drew a straight line segment AB and then placed the set square on this line and drew two line segments XY and PQ, by changing the position of the set squares as shown.

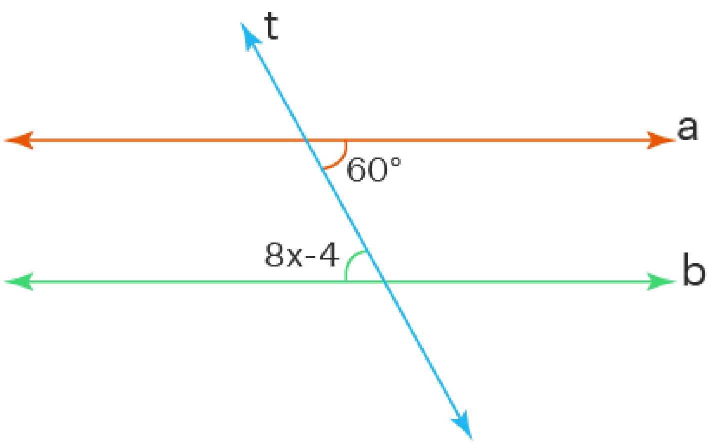


Kofi claimed that XY and PQ are parallel. Can you tell how?

Determine the measures of angles involving parallel lines and a transversal, using angle relationships.

Example

Find the value of x in the given parallel lines ‘a’ and ‘b’, cut by a transversal ‘t’.



Solution: The given parallel lines are cut by a transversal, therefore, the marked angles in the figure are the alternate interior angles which are equal in measure. This means,

$8x - 4^\circ = 60^\circ$, and $8x = 64^\circ$, $x = 8^\circ$.

Therefore, the value of $x = 8^\circ$.

Learning Task for Practice

Learners determine missing angles in angles formed by parallel lines and cut by transversal.

Possible Misconceptions

Some common misconceptions include:

1. **All angles are the same size:** Some learners may think that all angles are the same size or that they all look the same. This misconception can be addressed by showing them different angles and explaining how the size and shape of an angle can vary.
2. **All angles are measured the same way:** Learners may think that all angles are measured in the same direction (e.g., clockwise) or that the direction of measurement does not matter. It is important to clarify that angles are measured in a specific direction (usually counterclockwise) from one side of the angle to the other.
3. **Complementary and supplementary angles always have a common vertex:** Some learners may think that complementary and supplementary angles always have a common vertex. While complementary and supplementary angles can be adjacent (share a common vertex and side), they do not have to be. They can be separate angles that add up to the specified total measure.
4. **Straight angles are the same as reflex angles:** Learners may confuse straight angles (exactly 180 degrees) with reflex angles (more than 180 degrees). It is important to clarify the difference between these two types of angles and provide examples to help them understand.

Pedagogical Exemplars

Teachers should consider the following activities;

Reviewing previous knowledge: Review learners' previous knowledge on angles from JHS. Take the opportunity to discuss some referents for angles.

Experiential learning: In mixed-gender/ability groups, engage learners to explore the immediate school and classroom environment to investigate referents for angles.

Task learners to free-hand sketch some angles they know from JHS. Now, present learners with a worksheet containing various angles and ask them to estimate them, then measure the angles with protractor to compare the results with their estimates.

Problem-based learning: In small groups, engage learners to bisect given angles. Then discuss in groups the types of angles.

Group & pair activities: Using think-pair- share, task learners to solve problems on pairs of angles.

Whole Class discussions and demonstrations: In a whole class, discuss the meaning of Parallel lines, Perpendicular lines and Transversal with real life examples. Then discuss the various angles formed by parallel lines and a transversal.

Problem-based group learning: Using mixed-ability groups, present learners with task sheets on angles formed by parallel lines and a transversal to solve.

Take time to deal with common misconceptions that learners are likely to have.

Individual Task

Present learners with individual worksheets to complete. You may also ask learners to pose some questions for their friends to solve.

Key Assessment

Level 2

1. Give 4 examples of referents for angles.
2. Make a free hand sketch of the following angles: 30°, 45°, 60°, 75°, 90° and 180°.
3. Using a protractor or a pair of compasses, and a straight edge construct the following angles; 30°, 45°, 60°, 75°, 90° and 135°.

Level 3

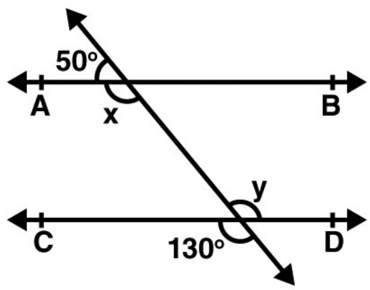
1. Using a ruler, a pair of compasses and a protractor, construct angles 60, 90, and 120 and bisect each of them.
2. Solve the following word problems on angles measurement;
   a. Find the measure of angle that is 10° more than its complement.
   b. Find the measure of an angle that is 30° less than its supplement
3. Write two word problems on angles and share with other students to solve.

Level 2

1. Differentiate between perpendicular and parallel lines and support your answer with a sketch for each type of lines.
2. State one real-life application, each, of perpendicular and parallel lines.
3. Define Complementary and Supplementary angles and give one real-life example for each.
4. Draw, analyse and name pairs of angles formed by parallel lines and a transversal by identifying corresponding angles, alternate interior angles, alternate exterior angles, and vertically opposite angles.

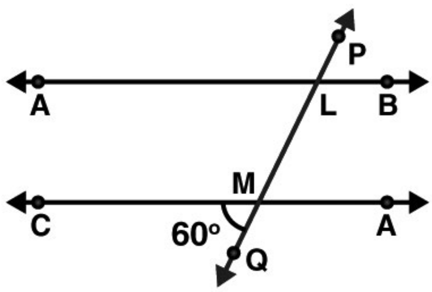
Level 2

I. Observe the figure given below: Find the values of x and y.



[ANS: $x = 130°$; $y = 130°$]

II. In Figure, AB and CD are parallel lines intersected by a transversal PQ at L and M respectively, if ∠CMQ = 60, find all other angles in the figure.



[ANS: M = 120°; P = 60° and L = 120°]

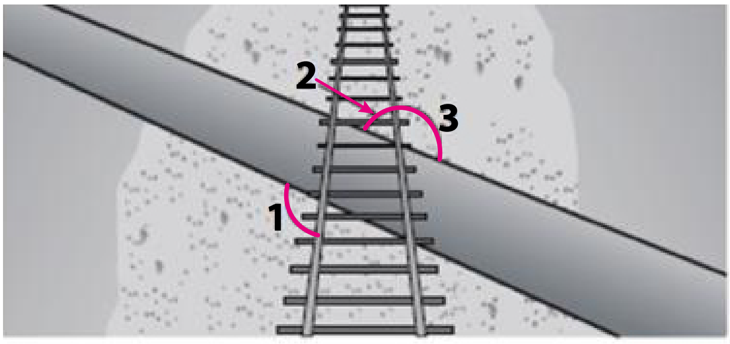
III. If x and y are pair of interior angles on the same side of transversal and x is equal to y, then what are the angles?

[ANS: x and y are supplementary angles. This means that the sum of the measures of x and y is 180 degrees.]

Level 3

Solve the following real life problems on angles formed by parallel lines and transversal.

1. A hiking trail crosses a set of train tracks as shown in the diagram. The path of the hiking trail forms angles 1, 2, and 3 with the parallel tracks.



a) If the angle at 1 is 1350, what is the angle at 3?

b) If the angle at 1 is 1350, what is the angle at 2?

2. If one of the angles is 45 degrees, then its corresponding angle will be?

3. If one of the angles is 108 degrees, then its vertically opposite angle is?

Week 15

Learning Indicators:

1. *State and use the exterior angle theorem of a triangle and identify various properties of special triangles.*
2. *State and use the properties of quadrilaterals and calculate the sums of interior angles and exterior angles of a polygon.*

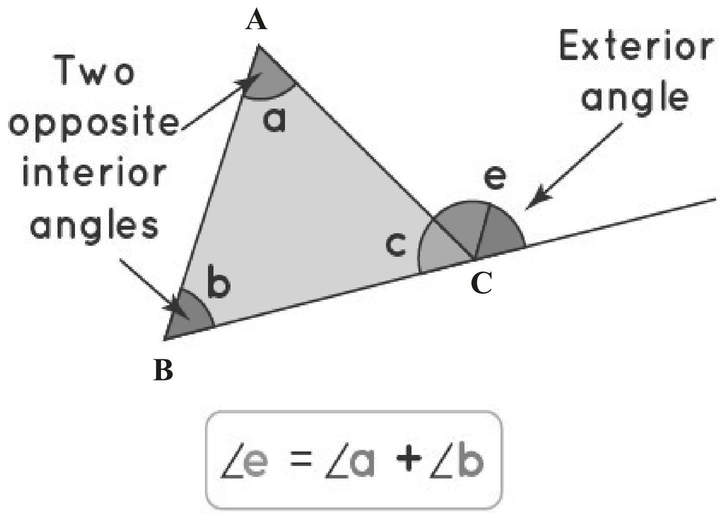
Theme or Focal Area: Exterior Angle Theorem of a triangle

Exterior Angle Theorem

According to the exterior angle theorem, the exterior angle that results from stretching a triangle's side is equal to the sum of the dimensions of the triangle's two opposed interior angles. The theorem can be used to find the measure of an unknown angle in a triangle.

Example:

Three internal angles in a triangle always add up to 180°. This theorem is applied to each of the outer angles, which total six. As they constitute a linear pair of angles, take note that an exterior angle is supplementary to the neighbouring interior angle. Exterior angles are those that are created between a polygon's side and its extended neighbouring side.



Proof of the exterior angle theorem

We can verify the exterior angle theorem with the known properties of a triangle. Consider a Δ ABC.

The three angles a + b + c = 180° (angle sum property of a triangle) ----- Equation 1

c = 180° - (a + b) ----- Equation 2 (rewriting equation 1)

e = 180° - c----- Equation 3 (linear pair of angles)

Substituting the value of c in equation 3, we get

e = 180° – [180° - (a + b)]

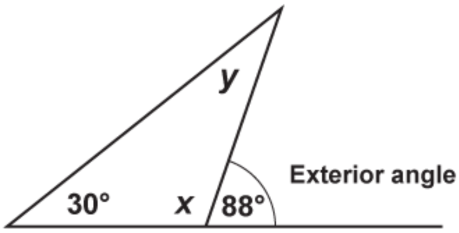
e = 180° – 180° + (a + b)

e = a + b

Hence verified.

Worked Examples

Use the properties of exterior angles of a triangle to determine the measure of the angles represented by x and y.



Solution

$x + 88^o = 180^o$

$x + 88^o - \mathbf{88^o} = 180^o - \mathbf{88^o}$

$x = 92$

$x + y + 30^o = 180^o$

$92 + y + 30^o = 180^o$

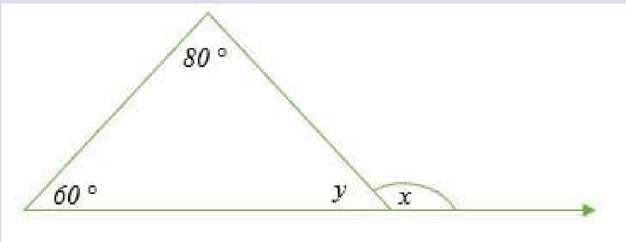
$y + 122^o = 180^o$

$y = 180^o - \mathbf{122^o}$

$y = 58^o$

Learning Tasks for Practice

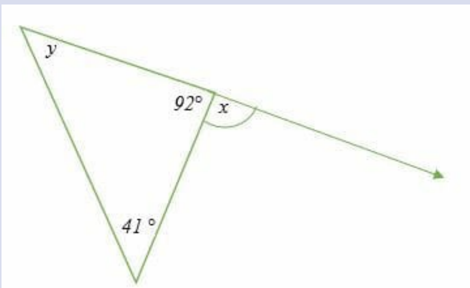
1. Calculate the values of x and y in the following triangle.



[$y = 40^0$; $x = 140^0$]

2. The exterior angle of a triangle is 120°. Find the value of x if the opposite non-adjacent interior angles are $(4x + 40)$ ° and 60°. [$x = 5^0$]

3. Determine the value of x and y in the figure below.



[$y = 47^0$; $x = 88^0$]

Sum of the interior angles of polygons

Generalisation/formula for determining the sum of the interior angles of polygons

Interior angles of a polygon are the angles formed inside a closed plane figure with straight sides, known as a polygon. The interior angles are formed at each vertex or corner of the polygon.

Key points about the interior angles of a polygon:

Sum of Interior Angles: The sum of the interior angles of a polygon with ***n*** sides can be calculated using the formula (n-2) *x* 180º. This formula holds true for all polygons, regardless of their size or shape.

Regular Polygons: In a regular polygon, all interior angles have the same measure. For example, in a regular hexagon, each interior angle measures 120 degrees because $\frac{(6-2) \times 180°}{6} = 120$.

Irregular Polygons: In an irregular polygon, the interior angles can have different measures. The sizes of the interior angles depend on the specific lengths of the polygon's sides and the arrangement of its vertices.

Relationship between Number of Sides and Interior Angles: As the number of sides in a polygon increases, the sum of the interior angles also increases. However, the measure of individual interior angles increases at a slower rate in larger polygons.

The interior angle of a pentagon

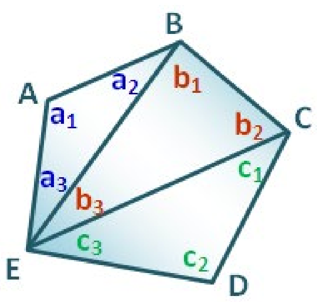
To determine the sum of interior angle of a polygon, we will go through the following activities. Let's begin with an example and work backwards.

Determine the sum of the interior angle of a pentagon.

Solution

Calculating the angle sum of pentagon ABCDE we have;

$$\text{Pentagon Angle Sum} = \overset{A}{a_1} + \overbrace{(a_2 + b_1)}^{B} + \overbrace{(b_2 + c_1)}^{C} + \overset{D}{c_2} + \overbrace{(c_3 + a_3 + b_3)}^{E}.$$



$$= (a_1 + a_2 + a_3) + (b_1 + b_2 + b_3) + (c_1 + c_2 + c_3)$$
$$= 180° + 180° + 180°$$
$$= 540°$$

Explanation:

You realise that the angle measured in the first line of our equation is just a rearrangement of the measures of the interior angles of the three triangles. Hence, the sum of the interior angles of the pentagon is equal to the angle sum of the three triangles. Therefore, we can conclude that the sum of the interior angles of a polygon is equal to the angle sum of the number of triangles that can be formed by dividing it using the method described above.

Using this conclusion, we will now relate the number of sides of a polygon, the number of triangles that can be formed by drawing diagonals and the polygon's angle sum.

Polygon	Number of Vertices (n)	Number of triangles	Sum of Angles (m^o)
Triangle	3	1	1(180) = 180
Quadrilateral	4	2	2(180) = 360
Pentagon	5	3	3(180) = 540
Hexagon	6	4	4(180) = 720
Heptagon	7	5	5(180) = 900
…	…	…	…
Decagon	10	8	8(180) = 1440
100-gon	100	?	?
n-gon	n	n-2	(n-2)180

From the table, we observe that the number of triangles formed is 2 less than the number of sides of the polygon. This is true, because n – 2 triangles can be formed by drawing diagonals from one of the vertices to n – 3 non-adjacent vertices. Therefore, the angle sum m of a polygon with n sides is given by the formula $m = 180°(n - 2)$.

Learning Task for Practice

Find missing angles in a given polygon.

Possible Misconceptions

1. **Exterior angles are always larger than interior angles in triangles:** Learners may mistakenly believe that the exterior angle of a triangle is always larger than any of its interior angles. While this is true for one specific exterior angle of a triangle (the one adjacent to the third side), it is not true for all exterior angles of the triangle.
2. **The exterior angle theorem for triangles states that the exterior angle is equal to the sum of the two remote interior angles:** Some learners may incorrectly believe that the exterior angle theorem of a triangle states that the exterior angle is equal to the sum of **any** two interior angles. In reality, the theorem states that the exterior angle is equal to the sum of the two **opposite** interior angles.
3. **Incorrect use of the sum of the interior angles of a polygon is always (n-2) × 180 degrees:** Whilst this is always true, that the sum of the interior angles of any polygon can be found by multiplying the number of sides (n) minus two by 180 degrees, for all polygons, including concave ones, learners may apply it incorrectly or without understanding its derivation.
4. **Forgetting that exterior angles are always supplementary to the interior angles:** Learners easily forget that the exterior angles of a polygon are always supplementary (add up to 180 degrees) to the interior angles.

Pedagogical Exemplars

Teacher should Consider the following activities;

Reviewing Previous concepts: Review learners' previous knowledge on angles in a triangle as well calculating unknown angles in a triangle from JHS.

Whole Class Discussions: In a whole class, discuss the exterior angle theorem and demonstrate how the theorem can be verified.

Group & pair activities: Present groups of learners with some problems on finding the exterior angle of a triangle.

Whole Class discussions and demonstrations: Lead a whole class discussion on the formula for determining the sum of the interior angles of polygons.

Problem-based group learning: Present groups of learners with some problems on finding the sum of the interior angles of polygons.

Individual Task

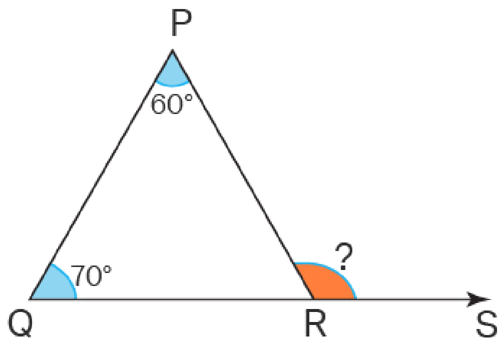
Present learners with individual worksheets to complete. Alternatively, allow learners to take home the tasks for later submission.

Key Assessment

Level 2 & 3

Solve the following problems using the exterior angle theorem of a triangle.

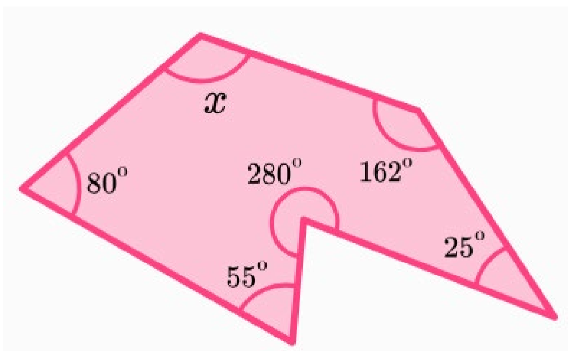
1. Find the value of the exterior angle in the triangle. **AP**



1. If one interior angle of a triangle is 56°, find the measure of its corresponding exterior angle. **HP**
2. If one interior angle of a triangle is 70°, find the measure of its corresponding exterior angle. **HP**

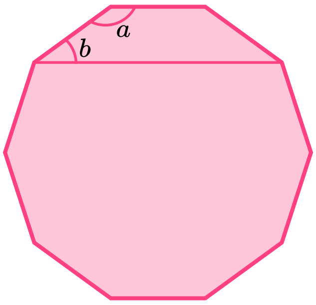
Level 2 & 3

1. Use an example to develop a formula for determining the sum of the interior angles of a polygon. **HP**
2. Work out the size of the angle labelled *x*. **HP**



3. A regular polygon's interior and exterior angles are in the ratio 9:1. How many sides does the polygon have?

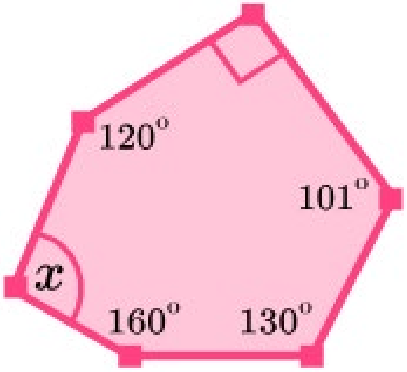
4. The diagram below shows a regular decagon.



(a) Work out the size of angle a.

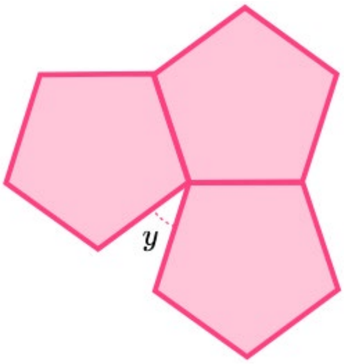
(b) Work out the size of angle b.

5. The diagram shows a polygon. Find the size of angle x. [ANS: 119^0]



6. Each of the interior angles of a regular polygon is 140°. How many sides does the polygon have? [ANS: 9 sides]

7. Shown below are three congruent regular pentagons. Find angle y. [ANS: $y = 36^0$]



8. Use the generalisation to come out with general statements on the sum of the interior angles for the various polygons.

Week 16

Learning Indicator: *Solve problems on Pythagorean theorem by identifying situations that involve right triangles, verify the formula and apply it.*

Theme or Focal Area: The Pythagorean theorem

The Pythagorean theorem states that in a right-angle triangle, the square of the length of the hypotenuse (the side opposite the right angle and the longest side in a right-angle triangle) is equal to the sum of the squares of the lengths of the other two sides.

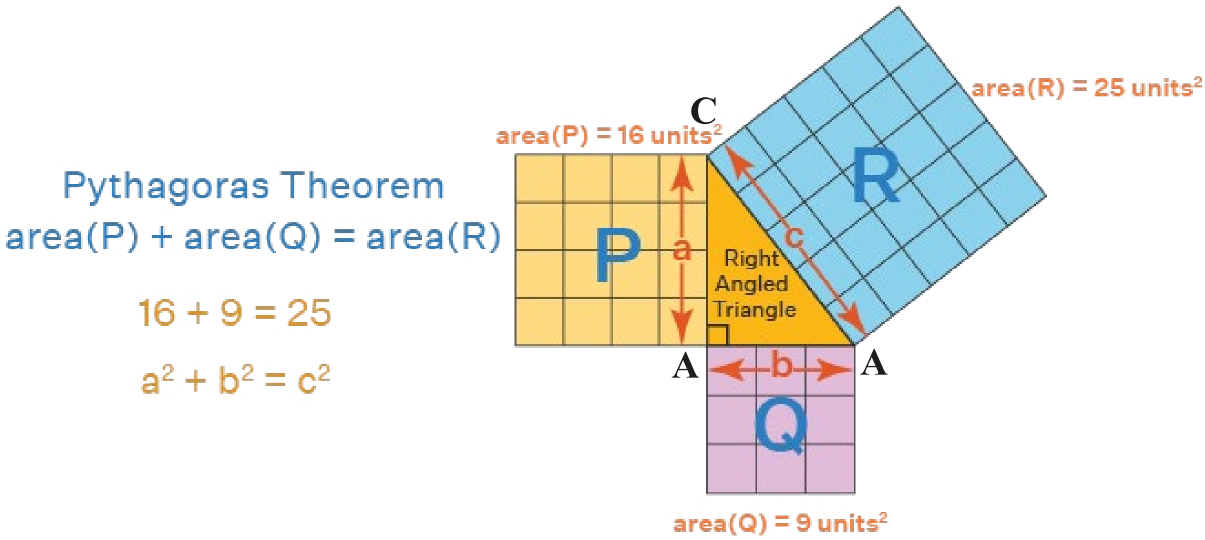
The reason why the Pythagorean theorem only applies to right triangles is rooted in the geometric properties and relationships within such triangles. In a right-angle triangle, one of the angles is always a right angle, which measures exactly 90 degrees. This characteristic allows for a specific set of relationships among the triangle's sides and angles, which the Pythagorean theorem captures.

If we consider a triangle that is not a right-angle triangle, the sum of the squares of the lengths of any two sides will not equal the square of the length of the remaining side. This is because the angle opposite the longest side, known as the hypotenuse in a right-angle triangle, will not be a right-angle, and the triangle's proportions will differ.

Verify the Pythagorean theorem

Example 1:

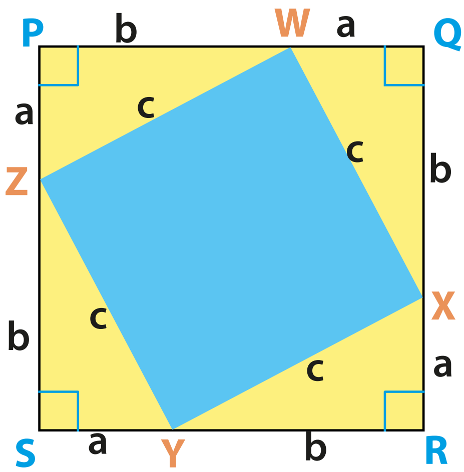
Pythagoras' theorem using the following figure shows that the area of the square formed by the longest side of the right triangle (the hypotenuse) is equal to the sum of the area of the squares formed by the other two sides of the right triangle.



Thus, if AB and AC are the shorter sides and BC is the hypotenuse of the triangle, then:

$BC^2 = AB^2 + AC^2$. In this case, AB is the base, AC is the altitude or the height, and BC is the hypotenuse.

Example 2: Use the values a, b, and c as shown in the following figure and follow the steps given below:



Step 1: Arrange four congruent right triangles in the given square PQRS, whose side is a + b. The four right triangles have 'b' as the base, 'a' as the height and, 'c' as the hypotenuse.

Step 2: The 4 triangles form the inner square WXYZ as shown, with 'c' as the four sides.

Step 3: The area of the square WXYZ by arranging the four triangles is c^2.

Step 4: The area of the square PQRS with side (a + b) = Area of 4 triangles + Area of the square WXYZ with side 'c'. This means $(a + b)^2 = [4 \times \frac{1}{2} \times (a \times b)] + c^2$.

This leads to $a^2 + b^2 + 2ab = 2ab + c^2$. Therefore, $a^2 + b^2 = c^2$. **Hence proved.**

Real-life uses of Pythagorean theorem

- The Pythagorean Theorem is useful for two-dimensional navigation.
- Painting on a Wall: To paint tall structures, painters make use of ladders, and they frequently employ Pythagoras' theorem to carefully position the ladder's base away from the wall, so it won't topple over.
- What size of TV should you buy? The size of a television is always specified in terms of its diagonal. If a television is specified as 43 inches in size, its true size is the diagonal's or hypotenuse's measurement.

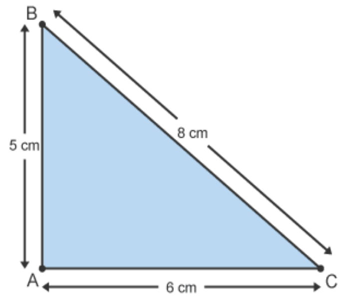
Applications of Pythagoras Theorem

- To know if the triangle is a right-angled triangle or not.
- In a right-angled triangle, we can calculate the length of any side if the other two sides are given.
- To find the diagonal length of a square.

Use the Pythagorean theorem to determine if a given triangle is a right triangle

Pythagoras' theorem can be used to determine whether a triangle has a right-angle. The triangle contains a right-angle if the sum of the squares of the two shorter sides equals the square of the hypotenuse.

Example 1: Does the triangle ABC contain a right angle?



$/AB/^2 + /AC/^2 = 5^2 + 6^2 = 25 + 36 = 61$

The side of the triangle /BC/ is 8units.

$8^2 = 64$

$61 \neq 64$

Thus, 61 does not equal 64. Therefore, the triangle does **not** contain a right angle.

Example 2: Provide an explanation on why a triangle with the side length ratio of 3:4:5 is a right-angle triangle.

Solution

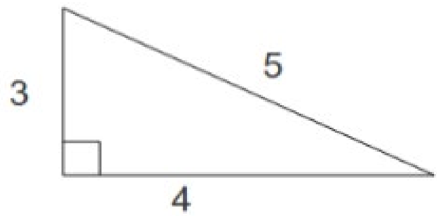
We can prove this by using the Pythagorean Theorem as follows:

Longest side $= 5^2 = 25$

$3^2 + 4^2 = 9 + 16 = 25$

$25 = 25$

Therefore, right angled.



Solve problems using Pythagorean theorem.

Example:

i. A rectangular playing field is 20 metres long. A straight path is cut across the field along one of its diagonals. If the length of the path in metres is 25m; how wide is the playing field?

Solution

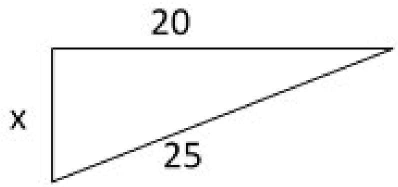
This is a right-angled triangle.

Let x = width

$20^2 + x^2 = 25^2$

$x^2 = 625 – 400 \Rightarrow x^2 = 125$

$x = 15$m



Learning Task for Practice

Learners determine if a given triangle is a right-angled triangle and give reasons for their answer.

Possible Misconceptions

1. **The Pythagorean theorem can be applied to any triangles:** Some learners may mistakenly believe that the Pythagorean theorem can be applied to any triangle, regardless of whether it is right-angled or not. In reality, the theorem only applies to right-angled triangles, where the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.
2. **The Pythagorean theorem applicable to all polygons:** Some learners may mistakenly believe that the Pythagorean theorem can be used for any polygons. This can only happen when polygons can be broken down into right-angled triangles.
3. **The Pythagorean theorem is limited to two dimensions:** Learners may mistakenly believe that the Pythagorean theorem can only be applied in two-dimensional space. In reality, the theorem can be extended to three-dimensional space, where the square of the length of the hypotenuse of a right-angled triangle in three dimensions is equal to the sum of the squares of the lengths of the other two sides.
4. **The Pythagorean theorem is only useful in geometry:** Learners may mistakenly believe that the Pythagorean theorem is only useful in geometry and has no applications in real life or other areas of mathematics. In reality, the theorem has numerous applications in various fields, including physics, engineering, and computer science, where it is used to solve problems involving distances, lengths, and areas.

Pedagogical Exemplars

Teachers should consider the following activities;

Reviewing Previous concepts: Review learners' previous knowledge on the right-angled triangle. Also, engage learners to understand what they know about the Pythagoras' theorem from JHS. Correct any misconception that learners may have.

Group & pair activities: In mixed-gender/ability groups, engage learners to discuss the theorem of Pythagoras and use any model of the choice to proof the theory. Offer learners the opportunity to make presentations and summarise the main ideas as well as deal with what lacked.

Group & pair activities: Task groups to discuss and make presentations on the real-life uses of the Pythagoras theorem.

Problem-based group learning: Using mixed-ability groups, present learners with task sheets on using the Pythagorean theorem to determine if a given triangle is a right triangle.

Engage learners to solve problems using Pythagorean theorem.

Individual Task

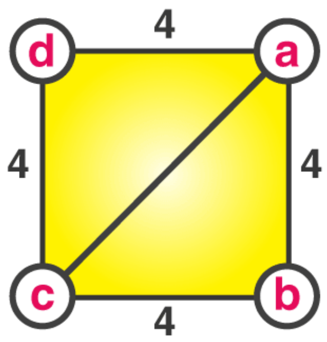
Present learners with individual worksheets to complete. Also, task learners to pose some problems for their friends to solve.

Key Assessment

Level 3

1. Explain the Theorem of Pythagoras and give reasons why it only applies to the right-angled triangle.
2. Use a model to verify the Pythagoras' theorem.

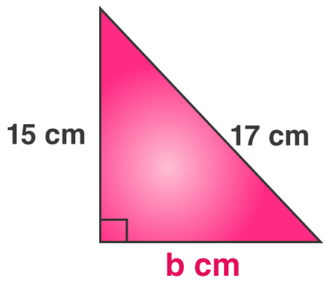
3. Research and provide at least 4 examples of real-life uses of Pythagorean Theorem.
4. Given the side of a square to be 4 cm. Find the length of the diagonal.



5. In a right-angle triangle ABC, right-angled at B, the lengths of AB and BC are 7 units and 24 units, respectively. Find AC.
6. A triangle is given whose sides are of length 11 cm, 60 cm, and 61 cm. Check whether these are the sides of a right-angled triangle.

Level 2

1. Suppose a triangle with sides 10cm, 24cm, and 26cm are given. Determine if this is a right-angled triangle. Give a reason for your answer.
2. The sides of a triangle are 5, 12 & 13 units. Check if it is right-angled.
3. The two sides of a right-angled triangle are given as shown in the figure. Find the third side.



4. An anchor line for a tower needs to be replaced. The tower is 96ft tall. The anchor line is 105ft long. How far from the tower must it be placed, to the nearest foot?

Level 3

Solve by giving real-life problems on Pythagorean theorem and pose problems for others to solve.

i. An animal shed with a pent roof need to have some new roof beams fitted. The width of the shed is 10m and the height of the pent roof is 1.3m. Work out the length of the roof beams needed.

ii. An army captain is on a hunt for a criminal. Her GPS tells her that she is 50m away from the criminal. She walks 34m due west. The GPS compass now tells her that the criminal is due south from where she is standing. How far south does she need to go to find the criminal?

Section Review

This section is for review of all the lessons taught for the last 14, 15 & 16 weeks. A summary of what the learner should have learnt.

Reflection and key ideas

- Angles are found in everyday activities such as opening doors, bending the elbow, walking, etc.
- The concept of angle is applied in many fields of life such as carpentry, building construction, etc.
- Angles such as 30°, 45°, 60°, 75°, 90° and 180° can be measured, constructed and bisected using a ruler, pair of compass and a protractor.
- There are various types of angles such as acute, right, obtuse, straight, reflex, etc.
- There are pairs of angles such as; complementary, supplementary, linear pair, adjacent angles, vertically opposite angles, etc.
- Pairs of angles formed by parallel lines and a transversal produce;
  - Corresponding angle,
  - Alternate interior and exterior angles
  - Consecutive interior angles,
  - Vertically opposite angles, etc.
- There are real-life applications of these angle concepts in the area of construction, etc.
- The exterior angle states that the results from stretching a triangle's side is equal to the sum of the dimensions of the triangle's two opposed interior angles.
- The theorem can be applied in finding missing angles in a triangle.
- The Pythagoras theorem can be useful in many contexts. Examples include;
  - Painting on a Wall
  - What size of TV should you buy?

Resources

- Mathematical sets.
- Technology tools such as computer, mobile phone etc.
- Computer software applications like GeoGebra.
- Tape measure, carpenters square, compass, clock face, geodot, etc.

References

1. What is Tranversal | Angles formed between Transversal and Parallel Lines (byjus.com)
2. Constructing Angle Bisectors - Construction using a compass, proof of angle bisector, examples. (cuemath.com)

SECTION 6: VECTORS AND TRIGONOMETRY

Strand: **Geometry Around Us**

Sub-Strand: Measurement

Learning Outcomes:

1. *Interpret information about real-world applications of vectors and recognise vectors with the same magnitude and direction but different positions as equal vectors.*
2. *Investigate and determine the trigonometric functions of special angles and solve problems using the three primary trigonometric ratios.*

Content Standards:

1. Demonstrate knowledge and understanding of the concept of measurement with respect to bearings and vectors.
2. Demonstrate conceptual understanding of the primary trigonometric ratios and apply it to solve problems that involve right triangles.

INTRODUCTION AND SECTION SUMMARY

Vectors and trigonometry are interconnected concepts that play a crucial role in mathematics, physics, engineering, and various other fields. Vectors represent quantities that have both magnitude and direction and are essential for describing motion, forces, and other physical quantities. Understanding vectors involves applying trigonometric ratios and functions, which relate the angles and sides of right-angle triangles. Trigonometry helps us calculate unknown angles and side lengths, serving as a tool for solving real-world problems involving distances, velocities, and forces. Learning about vectors and trigonometry enhances students' spatial reasoning skills and prepares them for advanced studies in mathematics. These concepts are also closely linked to other subjects such as geography, where bearings are used to navigate, and physics, where vectors are used to describe motion. Mastery of these concepts opens doors to a wide range of career paths in science, engineering, and technology.

The weeks covered by the section are:

Week 17:

1. Explanation of Vectors
2. Real life Applications of vectors
3. Types of vectors
4. Vectors in 2-D
5. Equal Vectors

Week 18:

1. Explanation of Trigonometry
2. Basic Trigonometric Ratios
3. Trigonometric functions of special angles 300, 450 and 600
4. Application of the three primary trigonometric ratios to solving Real-life Problems

SUMMARY OF PEDAGOGICAL EXEMPLARS

Throughout the section, hands-on activities and real-life examples will be used to enhance understanding and application. Learners should be engaged in a variety of teaching and learning strategies as the section aims to equip learners with a solid foundation in vectors and trigonometry, preparing them for advanced topics and real-world problem-solving.

The pedagogies will include:

Hands-on Activities: Use physical objects or models to demonstrate vector quantities and their operations. For trigonometry, use manipulatives like triangles or trigonometric circles to visualize angles and ratios.

Visual Representations: Use diagrams, graphs, and animations to illustrate vector operations and trigonometric concepts.

Collaborative Learning: Encourage group work and discussions where learners can share their understanding, solve problems together, and learn from each other's perspectives.

Problem-based Learning: Present learners with real-life problems that require the application of vectors and trigonometry.

Differentiated Instruction: Provide different learning materials and tasks based on learners' abilities and preferences. Offer additional challenges for advanced learners and additional support for those who need it.

Scaffolded Learning: Break down complex concepts into smaller, more manageable parts. Start with simpler concepts and gradually introduce more complex ideas as learners' progress.

ASSESSMENT SUMMARY

The concepts covered in this section require learners to demonstrate both conceptual understanding and practical application, especially in real-life contexts. Assessments should target levels 2 and 3 of the Depth of Knowledge (DOK) framework to ensure that learners grasp the concepts deeply. A variety of assessment strategies should be used, including but not limited to:

- Class exercises and quizzes to assess understanding of concepts such as vectors, trigonometry, and trigonometric ratios.
- Practical assessments involving real-life applications of vectors, such as velocity and force problems, to assess application skills.
- Homework assignments to reinforce learning and assess understanding of types of vectors, vectors in 2-D and equal vectors.
- Group activities where learners work together to solve problems involving vectors and trigonometry, fostering collaboration and communication skills.
- Projects where learners research and present on real-life applications of vectors and trigonometry, demonstrating their understanding and creativity.
- Problem-solving tasks that require the application of trigonometric ratios to solve real-life problems, such as finding heights or distances.

Week 17

Learning Indicators:

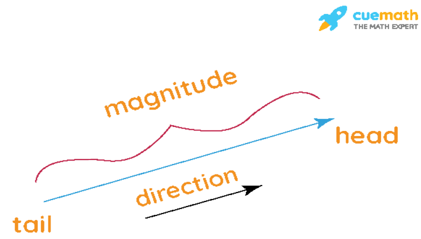
1. *Recognise a vector as a quantity with both magnitude and direction, and identify, gather, and interpret information about real-world applications of vectors.*
2. *Represent a vector in two-space geometrically as a directed line segment, with directions expressed in different ways (e.g., 320°, N40°W) and algebraically; then recognise vectors with the same magnitude and direction but different positions as equal vectors.*

Review Bearings

Review Learners' previous ideas on bearings from JHS. Include ideas such as true bearing, forward and back bearing, bearings with vectors, etc.

Theme or Focal Area: What are Vectors?

Vectors define the movement of objects from one point to another. Vectors carry a point A to point B. The length of the line between the two points A and B is called the magnitude of the vector and the direction of the displacement of point A to point B is called the direction of the vector AB.



Real life Applications of vectors

Vectors play an important role in physics. For instance, velocity, displacement, acceleration, forces are all vector quantities that have a magnitude as well as a direction.

Real-life uses of vectors

- Vectors can be used in finding the direction in which the force is applied to move an object.
- The concept of vectors aids in understanding how gravity uses a force of attraction on an object to work.
- Vectors can be used in obtaining the motion of a body which is confined to a plane.
- Vectors help in defining the force applied on a body simultaneously in the three dimensions.
- In the field of Engineering, for a structure not to collapse, vectors are used where the force is much stronger than the structure will sustain.
- Vectors are used in various oscillators.

Types of vectors

Zero Vectors	Negative Vector
Unit Vectors	Parallel Vectors
Position Vectors	Orthogonal Vectors
Equal Vectors	Co-initial Vectors

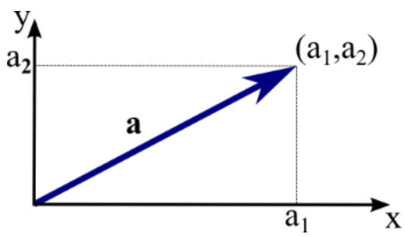
Learning Tasks for Practice

1. Learners explain parallel lines and state why parallel vectors are also considered collinear vectors.
2. Learners explain some basic concepts about vectors

Vectors in 2-D

Description: By this stage, learners are familiar with the standard (x, y) Cartesian coordinate system in the plane. That is, each point P in the plane is identified with its x and y components: P (p_1, p_2).

To determine the coordinates of a vector, **a,** in the plane, the first step is to translate the vector so that its tail is at the origin of the coordinate system. Then, the head of the vector will be at some point (a_1, a_2) in the plane. We call (a_1, a_2) the coordinates or the components of the vector **a**. We often write $a \in R^2$ to denote that it can be described by two real coordinates.



Using the Pythagorean Theorem, we can obtain an expression for the magnitude of a vector in terms of its components. Given a vector $a = (a_1, a_2)$, the vector is the hypotenuse of a right triangle whose legs are length a_1 and a_2. Hence, the length of the vector a is $|a| = \sqrt{a_1^{\,2} + a_2^{\,2}}$

Example

Consider the vector ***a*** represented by the line segment which goes from the point (1, 2) to the point (4, 6). Calculate the components and the length of this vector?

Solution

To find the components, translate the line segment one unit left and two units down. The line segment begins at the origin and ends at $(4-1, 6-2) = (3, 4)$.

Therefore, $a = (3, 4)$. The length of a is $|a| = \sqrt{3^2 + 4^2} = 5$

Equal Vectors

Equal vectors in mathematics refer to vectors that have the same magnitude and direction. Here are some key points about equal vectors:

Definition: Two vectors are equal if they have the same magnitude and the same direction.

Magnitude: The magnitude of a vector refers to its length or size. If the magnitudes of two vectors are equal, they have the same length.

Direction: The direction of a vector refers to the line along which it points. If two vectors have the same direction, they are parallel and point in the same line or path.

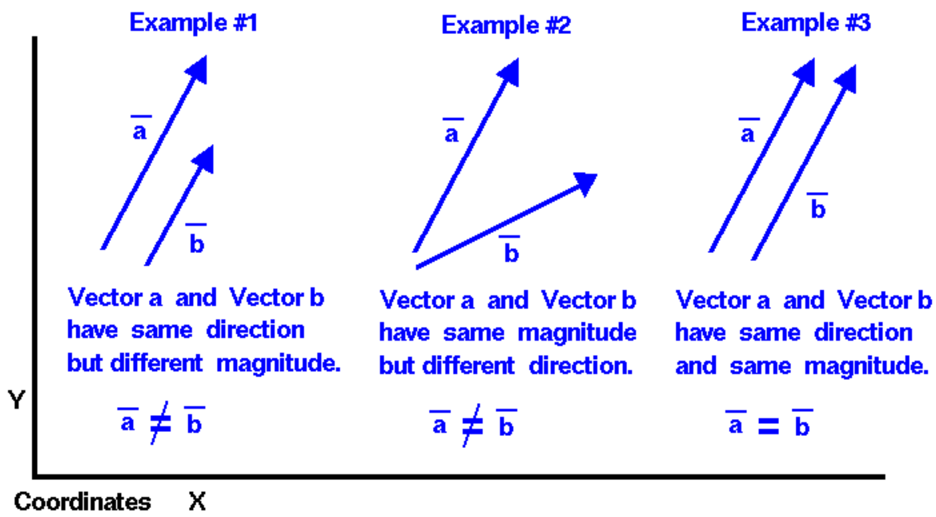
Notation: In mathematical notation, equal vectors are typically denoted by placing an arrow on top of the vector symbols. For example, if vector **a** and vector **b** are equal, it is written as $\vec{a} = \vec{b}$.

Examples

Example 1: Consider two displacement vectors in a coordinate system. If vector A represents a displacement of 5 meters to the east, and vector B represents a displacement of 5 meters to the east, then vector A and vector B are equal. They have the same magnitude (5 meters) and the same direction (east).

Example 2: Imagine two velocity vectors of moving objects. If vector C represents a velocity of 30 kilometres per hour north, and vector D represents a velocity of 30 kilometres per hour north, then vector C and vector D are equal. They have the same magnitude (30 kilometres per hour) and the same direction (north).

Equal vectors are important in various mathematical operations, such as vector addition, subtraction, and comparison. When vectors have the same magnitude and direction, they exhibit similar characteristics and can be treated as equivalent in many mathematical contexts.



Learning Task for Practice

Learners solve problems on determining the length of a vector as well as solve problems on equal vectors.

Pedagogical Exemplars

Teachers should consider the following activities;

Reviewing previous knowledge: Review learners' previous knowledge on bearings and vectors from JHS. Task groups of learners to investigate some of the everyday life uses of vectors that they learnt in JHS or they know in their everyday activities.

Group & pair activities: Using mixed ability groups, task learners to discuss and present on types of vectors with supported examples.

Problem-based learning: In small groups, engage learners to discuss and represent graphically 2D vectors. They should also show graphically when two vectors are considered equal.

Problem-based group learning: Using mixed-ability groups, present learners with task sheets on equal vectors to solve. Take note of learners' misconceptions and address them.

You may consider the following activities to differentiate the lessons;

- Provide concrete examples and visuals to explain the concept of vectors, emphasising basic properties like magnitude and direction. Use simple, relatable examples such as displacement in a straight line or speed and direction of an object. Vectors in 2-D: Begin with examples in two dimensions, allowing students to visualize vectors in a plane. Equal Vectors: Explain equal vectors using graphical examples where two vectors have the same magnitude and direction.
- Provide more abstract explanations, including mathematical notation and the concept of components. Types of Vectors: Introduce additional types like position vectors, unit vectors, and free vectors, discussing their properties and uses.
- Vectors in 2-D: Move on to more challenging problems involving vectors in two dimensions. Discuss the concept of collinear and parallel vectors, emphasizing that equal vectors have the same magnitude and direction. Equal Vectors: Discuss equivalence relations between vectors, emphasizing that equal vectors have the same length and direction, regardless of their initial points.

Whole Class discussions and demonstrations: Lead the class to discuss the main ideas of the lesson and take the opportunity to demonstrate [or learners volunteer to demonstrate] challenging concepts, including resolving all misconceptions.

Individual Task

Present learners with individual worksheets to complete.

Possible Misconceptions

Vectors are only used in mathematics: Learners may mistakenly believe that vectors are only used in mathematics and have no real-life applications. In reality, vectors are used in various fields such as physics, engineering, and computer graphics to represent quantities with both magnitude and direction.

Key Assessment

Level 3

1. Explain vectors, including using models, to demonstrate its meaning and state at least 3 real-life applications of vectors.
2. Explain the following concepts as used in vectors
   - o Why might a Parallel Vector also be considered Collinear Vectors?
   - o What is the magnitude and direction of a Zero Vector?
   - o Vectors that have the same initial point are known as…?
   - o What is the start and endpoint of a vector called?
3. Find the length of the vector a = $\binom{3}{4}$. [ANS: 5 units]
4. Using the magnitude formula, find the magnitude of the vector with u = $\binom{6}{8}$. [ANS: 10 units]

5. Determine if the vectors $\vec{a} = (3, -2)$ and $\vec{b} = (-6, 4)$ are equal. [ANS: since $3 \neq -6$ and $-2 \neq 4$, $\vec{a}$ and $\vec{b}$ are not equal]

4. Given the vectors $\vec{u} = (2,5)$ and $\vec{v} = (4,10)$, find a scalar k such that $u = kv$. [the scalar $k = \frac{1}{2}$ makes the vectors $\vec{u}$ and $\vec{v}$ equal]

Level 2

1. The vector u has an initial point at $(-2, 1)$ and an endpoint at $(4, -2)$. What is the vector's length?

2. Determine whether given pairs of vectors are equal or not. Justify your answer.

3. Determine if the vectors $\vec{a} = (3, -4)$ and $\vec{b} = (-6, 8)$ are equal.

4. Determine whether the two vectors are equal.

- Vector P: Magnitude = 5, Direction = 30 degrees above the positive x-axis.
- Vector Q: Magnitude = 5, Direction = 60 degrees above the positive x-axis.

Week 18

Learning Indicators:

1. *Investigate the three basic trigonometric ratios (tangent, sine and cosine) of an acute angle in degrees.*
2. *Find the trigonometric functions of special angles 30^0, 45^0 and 60^0, including using the calculator to determine the values of sine, cosine and tangent of angles up to 360^0.*
3. *Solve problems, using the three primary trigonometric ratios for angles from 0° to 360° in standard position.*

Review of previous knowledge

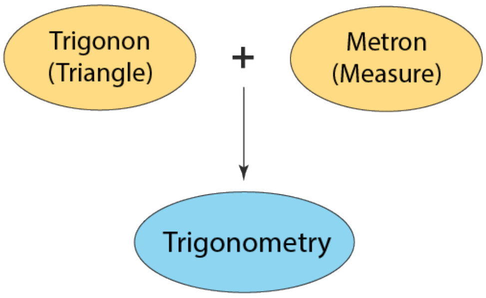
Engage learners on the following pre-requisite concepts.

- Recall learners' familiarity with Pythagoras' theorem.
- Review learners' basic knowledge of congruence and similarity of triangles.
- Knowledge of the basic properties of triangles, squares and rectangles.
- Familiarity with the use of a calculator.
- Bearings and vectors

Them or Focal Area: Trigonometry and Its Applications

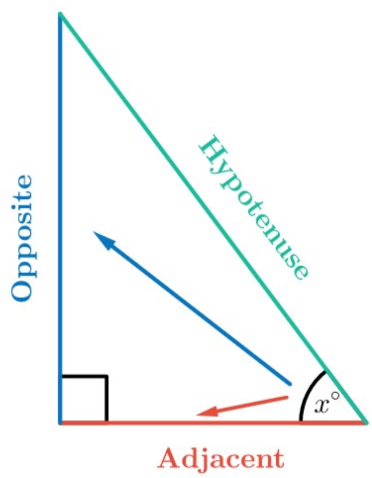
What is Trigonometry?

Trigonometry is the study of the relation between the sides and angles of triangles, particularly right-angle triangles. It thus helps in finding the measure of unknown dimensions of a triangle using formulas and identities based on this relationship.

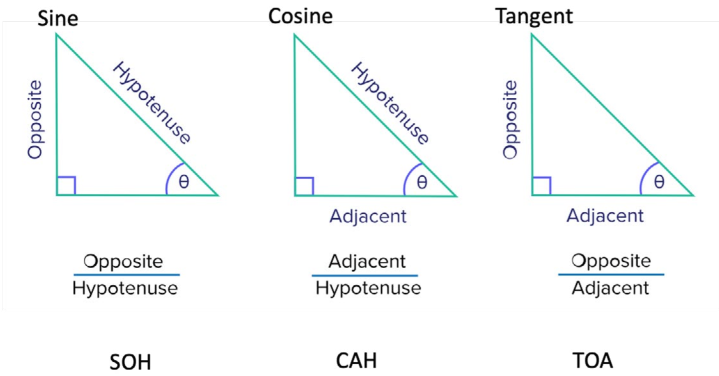


Trigonometric Ratios

A ratio is a statement of a mathematical relationship between two objects, often represented as a fraction. If we consider and two sides of a right-angled triangle with respect to a given internal angle, the ratio of the two sides has a special relationship to the angle. Take a look at the three sides of the triangle.



In relation to the angle (x) using ideas from the concept of Pythagoras' theorem, we can deduce the basic six ratios in trigonometry that help in establishing a relationship between the ratios of sides of a right-angled triangle with the angle.



The three common trigonometric ratios we see above are Sine, Cosine and Tangent, shortened to become sin, cos and tan respectively. The trigonometric ratios enable us to determine the ratio of two sides of a right-angled triangle given an internal angle or find an angle given the ratio of two sides of a right-angled triangle. The three trigonometric ratios are defined as follows:

For example,

If θ is the angle in a right-angled triangle formed between the adjacent and hypotenuse, then

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

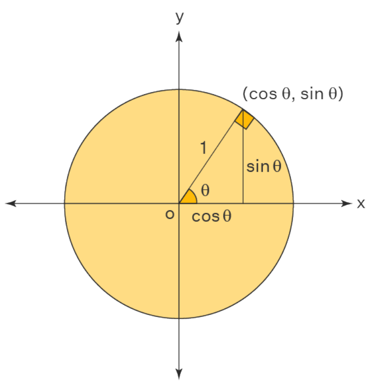
$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

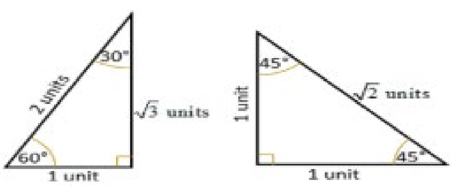
Trigonometric functions of special angles 30^0, 45^0 and 60^0.

Use a unit circle to calculate the values of basic trigonometric functions- sine, cosine, and tangent.

The following diagram shows how trigonometric ratios sine and cosine can be represented in a unit circle.



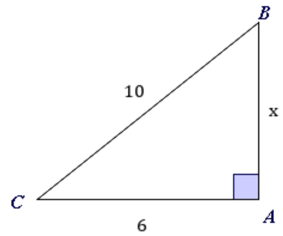
Derive the trig ratios of 30°, 45° and 60° from the 30-60-90 and 45-45-90 special triangles.



	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Worked Example

Consider the triangle in the figure.



If Cos θ = 6/10

- **a** Which angle is represented by θ? How do you know?
- **b** Find the numerical value of sin θ. Express your answer as a simplified fraction.
- **c** Find the numerical value of tan θ. Express your answer as a simplified fraction.

Solution

a. Since Cos $\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$, then the angle is *BCA*.

b. Sin $\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{x}{10}$

Find the length of the adjacent side, x, by using the Pythagoras' theorem, $C^2 = A^2 + B^2$.

Substitute $C = 10$ and $A = 6$ into the Pythagoras' theorem, then square root the expression to find the value of x.

Sin $\theta = \frac{\sqrt{10^2 - 6^2}}{10} = \frac{\sqrt{100 - 36}}{10} = \frac{\sqrt{64}}{10} = \frac{8}{10}$

Sin $\theta = \frac{4}{5}$

c. In right-angled triangles, recall that the tan of an angle is equal to the ratio of the side length opposite that angle and the adjacent length.

That is, tan $\theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{x}{6}$

We previously found that $x = 8$. Therefore, substitute the value into the trigonometric identity.

tan θ = 8–

Simplify the fraction.

tan θ = 4–

Learning Task for Practice

Learners determine the sine, cosine and tangent ratios for a given triangle.

Application of the three primary trigonometric ratios to solve Real-life Problems

The practical applications of these ratios are vast and diverse. Architects and engineers employ trigonometry to design structures, surveyors rely on it to measure land, astronomers utilise it to calculate distances between celestial bodies, and pilots use it for navigation purposes. Trigonometry can be used to measure the height of a building or mountains. Trigonometry truly lies at the heart of many scientific and technical fields.

Trigonometry in video games: Have you ever played the game, Mario? When you see him so smoothly glide over the road blocks. Trigonometry helps Mario jump over these obstacles.

Trigonometry in flight engineering: Flight engineers have to take into account their speed, distance, and direction along with the speed and direction of the wind. The wind plays an important role in how and when a plane will arrive wherever needed this is solved using vectors to create a triangle using trigonometry to solve. For example, if a plane is travelling at 234 mph, 45 degrees N of E, and there is a wind blowing due south at 20 mph. Trigonometry will help to solve for that third side of your triangle which will lead the plane in the right direction, the plane will travel with the force of wind added on to its course.

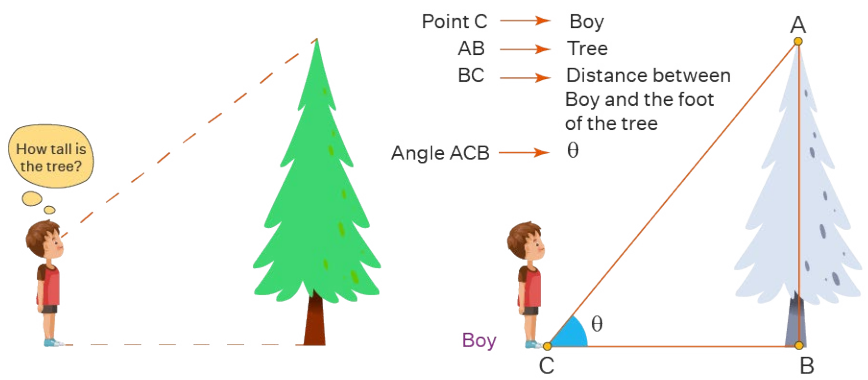
Trigonometry in physics: In physics, trigonometry is used to find the components of vectors, model the mechanics of waves (both physical and electromagnetic) and oscillations, sum the strength of fields, and use dot and cross products. Even in projectile motion you have a lot of application of trigonometry.

Other real-life Applications of Trigonometry are;

- archaeology
- criminology
- marine biology
- marine engineering
- navigation

Example

A boy is standing near a tree. He looks up at the tree and wonders "How tall is the tree?"



Solution

The height of the tree can be found without measuring it. What we have here is a right-angled triangle, i.e., a triangle with one of the angles equal to 90 degrees.

It is determined using the tangent function, such as tan of angle is equal to the ratio of the height of the tree and the distance. Let us say the angle is θ, then tan θ = Height/Distance from object & base of the tree.

Let us assume that distance is 30m and the angle formed is 45 degrees, then

Height = distance × tan θ

Height = 30 × tan 45°

Since, tan 45° = 1

So, Height = 30 m

Learning Task for Practice

Learners calculate for the values of trigonometric ratios for given angles.

Possible Misconceptions

1. **Trigonometry is only for advanced learners:** Learners may feel intimidated by trigonometry, thinking that it is only for advanced learners. In reality, trigonometry concepts can be introduced at the basic level and gradually built upon to develop a deeper understanding.
2. **Trigonometric functions have no real-world applications:** Learners may underestimate the real-world applications of trigonometric functions. These functions are used in various fields such as architecture, navigation, and engineering to solve problems involving angles and distances.
3. **Confusing sine, cosine, and tangent:** Learners may confuse the definitions of sine, cosine and tangent, leading to errors in applying these functions in different contexts.
4. **Misunderstanding angle measurement:** Learners may incorrectly measure or interpret angles, leading to errors in calculating trigonometric ratios.
5. **Forgetting the definitions of trigonometric ratios:** Learners may forget the definitions of sine, cosine, and tangent, especially in relation to the sides of a right triangle, leading to errors in calculations.
6. **Misconceptions about the unit circle:** Learners may have misconceptions about the unit circle and how it relates to trigonometric functions, leading to errors in graphing trigonometric functions.
7. **Confusing acute and obtuse angles:** Learners may confuse acute and obtuse angles, leading to errors in determining the appropriate trigonometric ratio to use in a given situation.

Pedagogical Exemplars

Teacher should Consider the following activities;

Reviewing Previous concepts: Review learners' previous knowledge on simple trig ratios from JHS. Take time to identify their common misconceptions and address them in the lesson.

Whole Class discussions and demonstrations: In a whole class, discuss the opposite, adjacent and hypotenuse of a right-angled triangle with respect to a given angle.

Group & pair activities: Put learners into mixed-ability groups and task them to investigate and discuss the three basic trig ratios and use the knowledge to solve problems.

Group & pair activities: In pairs, task learners to discuss and demonstrate the application of the three primary trigonometric ratios to solving real-life problems.

Problem-based group learning: Using mixed-ability groups, present learners with task sheets on trig ratios, including word/real-life problems to solve.

Whole Class discussions and demonstrations: Lead the class to discuss the main ideas of the lesson and take the opportunity to demonstrate [or learners volunteer to demonstrate] challenging concepts, including resolving all misconceptions.

Individual Task

Present learners with individual worksheets to complete. Alternatively, allow learners to take home the tasks for later submission.

You may consider the following activities to differentiate your lessons:

Use concrete examples and visuals to demonstrate how to calculate sine, cosine and tangent for simple angles. Use manipulatives or drawings to help visualise triangles. Start with straightforward problems involving angles from 0° to 90°. Provide guided practice and use real-life examples to make the concepts more relatable.

Focus on memorisation of the values for 30°, 45°, and 60°. Provide step-by-step guidance on using a calculator to find trigonometric ratios. Introduce more complex examples involving angles beyond 90°. Encourage students to use the unit circle to find trigonometric ratios for any angle.

Introduce problems with angles in standard position up to 360°. Encourage students to use trigonometric identities and apply the ratios to real-world scenarios. Emphasise understanding the patterns and relationships between the trigonometric ratios of special angles.

Key Assessment

Level 2

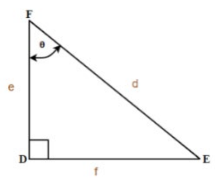
1. Given a right-angle triangle ABC with angle A as the right-angle, and side lengths as follows: AB = 5 cm, BC = 12 cm. Calculate the values of the trigonometric ratios for angle B (sine, cosine, and tangent). Round your answers to two decimal places.

   [ANS: Sine of angle B (sin B) = 0.38 Cosine of angle B (cos B) = 0.92 Tangent of angle B (tan B) = 0.42]

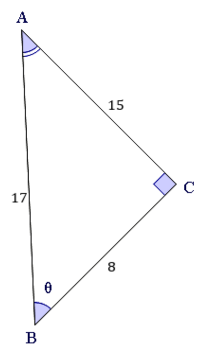
2. State the exact values of the trigonometric functions (sine, cosine, and tangent) for the special angles 30 degrees, 45 degrees, and 60 degrees. [ANS: Refer to notes above]

Level 2

1. For the given triangle find the sine, cosine and tangent ratio.



   [ANS: Sin a = $\frac{f}{d}$; Cos a = $\frac{e}{d}$ and tan a = $\frac{f}{e}$]

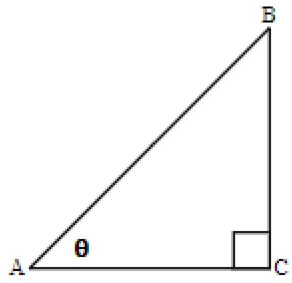


2. For the given triangle above find the sine, cosine and tangent ratios in relation to angle θ.

   [Sin θ = $\frac{15}{17}$; Cos θ = $\frac{8}{17}$; tan θ = $\frac{15}{8}$]

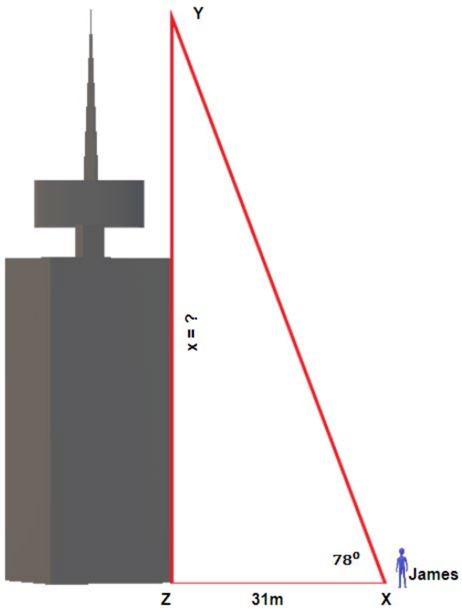
Level 1

1. Label the sides of the triangle and find the hypotenuse, opposite, and adjacent with regards to θ.

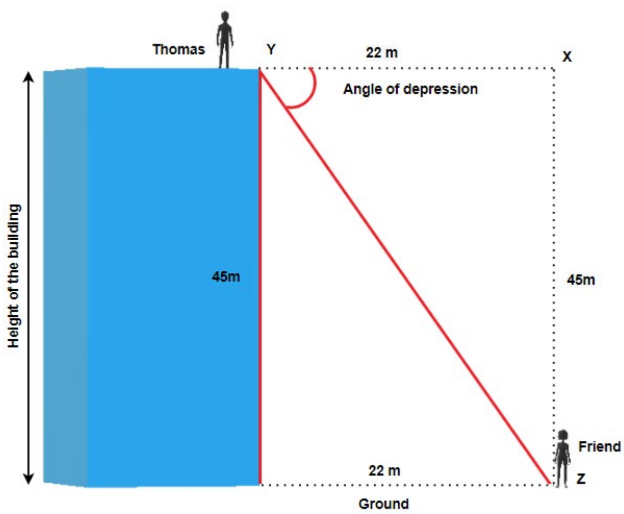


Level 3

1. James is standing 31 metres away from the base of a Harbour Centre. He looks up to the top of the building at a 78° angle. How tall is the Harbour Centre?



2. Thomas is standing at the top of the building that is 45 metres high and looks at her friend who is standing on the ground, 22 metres from the base of the building. What is the angle of depression?



Level 3

Solve a given real-life.

1. Sakumonor Skating Club has a beginner ski slope which makes a 150 angle with the ground. How far does a skier travel over a horizontal distance of 120 meters?
2. A ramp is to be built from the ground to the back of a semi-truck. The bed of the truck is 1.3 meters above the ground. The ramp makes a 22^0 angle with the ground. How long will the ramp be?
3. The angle of elevation from a boat to the top of a 48-meter lighthouse is 25^0. How far is the boat from the base of the lighthouse?
4. A prince is standing 100 meters from the base of a tower where a beautiful princess is being held captive. The princess sees the prince at a 30^0 angle of depression. How high will the prince have to climb to rescue the princess?

Section Review

This section is for the review of all the lessons taught for the last 17 & 18 weeks. A summary of what the learner should have learnt.

Reflection and key ideas

- Trigonometry is the study of the relation between the sides and angles of triangles, particularly right triangles.
- If θ is the angle in a right-angled triangle formed between the adjacent and hypotenuse, then

  $\sin \theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$

  $\cos \theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$

  $\tan \theta = \dfrac{\text{Opposite}}{\text{Adjacent}}$
- Trigonometry can be applied in various real-life situations such as;
  - Physics
  - Video games
  - archaeology
  - criminology
  - marine biology
  - marine engineering
  - navigation

Resources

- Mathematical sets
- Cut out shapes
- Magnetic demonstration set
- SAFE-T Rulers
- Centimetre grid board

- Folded geometric shapes
- Technology tools such as computer, mobile phone etc.
- Computer software applications like GeoGebra.
- Tape measure, compass, geodot, etc.

References

1. Trigonometry Questions | Trigonometry Questions with Solutions (byjus.com)
2. Sides and angles of right-angled triangles | Year 12 Maths | Australian Curriculum 12 Essential Mathematics - 2020 Edition | Mathspace
3. Solve Applications: Sine, Cosine and Tangent Ratios. – Business/Technical Mathematics (opentextbc.ca)

SECTION 7: PERIMETER, AREA AND VOLUME

Strand: **Geometry Around Us**

Sub-Strand: Measurement

Learning Outcomes:

1. *Identify and compare referents for SI and imperial area measurements, estimate perimeter and area of 2-D shapes and solve problems that involve of a given regular, composite or irregular 2-D shapes.*
2. *Determine the volume and capacity of solid shapes and solve problems that involve SI and imperial units.*

Content Standards:

1. Demonstrate conceptual understanding of the measurement of perimeter and area of and quadrilaterals.
2. Demonstrate conceptual understanding of the measurement of surface area, volume and capacity of solid shapes.

INTRODUCTION AND SECTION SUMMARY

Perimeter, area and volume are essential concepts in geometry that help us quantify and compare the sizes of two-dimensional and three-dimensional shapes. Understanding these concepts is essential for solving real-world problems involving measurement and spatial reasoning. Mastery of perimeter, area and volume allows us to perform real-life activities such as fencing required for a garden, the amount of paint needed to cover a wall, or the amount of liquid in a container. These concepts are interrelated and form the foundation for more advanced topics in geometry and calculus. Additionally, they are closely linked to subjects such as physics and engineering, where measurements of shapes and volumes are essential for design and analysis.

The weeks covered by the section are:

Week 19:

1. Referents for measuring
2. Perimeter of 2-D shapes
3. Area of 2-D shapes
4. Real-life problems on perimeter and area measurement

Week 20:

1. Volume of prisms
2. Real world problems volume of prisms

SUMMARY OF PEDAGOGICAL EXEMPLARS

This section requires hands-on activities where learners engage in practical measurements of the perimeter and area of 2-D shapes, as well as volume of prisms. Learners should be offered the opportunity to work in teams to find solutions to assigned tasks. Hence, **Experiential learning activities and mixed-groupings [ability, gender, etc.]** should dominate the lessons on these concepts. All learners, irrespective of their learning abilities should be encouraged to participate fully in investigations as well as presentation of findings. However, make considerations and accommodations for the different groups. That is, offer approaching proficiency learners the opportunity to make oral presentations and use graphical methods in determining the area and perimeter of shapes and volume of prisms. Then, extend activities for the above average/highly proficient learners to using formulae and computer applications to solve problems.

ASSESSMENT SUMMARY

The concepts under this section require learners to demonstrate conceptual understanding, including their real-life applications. Hence, the assessments should largely cover levels 2 and 3 of the DOK. Again, teachers should employ a variety of formative assessment strategies such as oral/written presentations, pair-tasks, reports, home tasks, etc. to gather information about learners' progress and give prompt feedback to them. Specifically, teachers should conduct the following assessments and record the performances of learners for continuous assessment records;

- class exercises (including individual/group worksheets) after each lesson
- home works
- scores on practical group activities on measuring perimeter, area and volume in *real life context.*

Week 19

Learning Indicators:

1. *Solve problems that involve identifying and comparing referents for SI and imperial area measurements of regular, composite and irregular 2-D shapes including decimal and fractional measurements and verify the solutions.*
2. *Estimate the perimeter and area of a given regular, composite or irregular 2-D shapes.*
3. *Solve a contextual problem that involves the perimeter and area of a regular, a composite or an irregular 2-D shape.*

Theme or Focal Area: Referents for measuring

Measurement is an essential aspect of understanding our world, and referents play a vital role in providing reference points for accurate quantification. Referents are the benchmarks or standards against which other quantities are compared, establishing a common language for measurement. They enable us to make objective assessments, draw meaningful conclusions and ensure consistency across different contexts.

We will discuss the significance of referents in measurement, covering natural, artificial and relative referents. We will also discuss the implications of selecting or modifying referents and how they shape our perception of reality. By understanding referents, we gain a deeper understanding of the foundations of quantification and the interplay between measurement and the referential framework.

Example

Investigate to validate the following referents in the tables.

Table 1: *Referents for Imperial Linear Measurement*

Imperial Measurement	Referent
Inch	Thumb length thickness of a hockey puck
Foot	Standard floor tile in classroom
Yard	Arm span from tip of nose, yard stick, length of a guitar
Mile	Distance walked in 20 minutes

Table 2: *Referents for SI Linear Measurement*

SI Measurement	Referent
Millimetres	Thickness of a dime, or fingernail
Centimetres	Width of a fingernail, black keys on a piano, crayon, paper clip, or AA battery
Meter	Distance from a doorknob to the floor, width of a volleyball net, meter stick, waist height
Kilometre	Distance walked in 15 minutes

Table 3: *Referents for Area*

Referent	Measurement
Area of a floor tile	≈1 ft^2
Area of a postage stamp	≈1 in^2
Area of a fingernail	≈1 cm^2
Area of an exterior house door	≈ 2 m^2
Area of exercise notebook	≈ 93.5 in^2 or 600 cm^2
Area of an ice rink surface	≈ 1500 m^2 or 17 000 ft^2
Area of a sheet of plywood	≈ 32 ft^2 or 3 m^2

Learning Task for Practice

Estimate the area for the following referents

Referent	Measurement
Area of your television set	
Area of the marker board in the classroom	
Area of the classroom	
Area of the school field	
Area of the head teacher's office	

Perimeter and Area of 2-D shapes

Reviewing Concepts

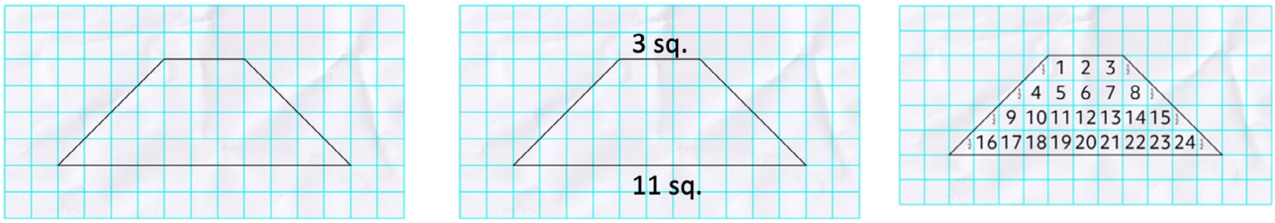
Two dimensional shapes, also known as plane shapes define the surface of solid objects They do not exist on themselves except on a solid. For instance, we cannot touch a rectangle except observing its properties on an object. In Primary school and Junior High school, learners were taken through the concept of determining perimeter and area of basic shapes such as squares, rectangles, triangles and circles. Please review these concepts with learners, including their real-life applications, before starting the new concepts on kites, rhombuses, parallelograms, trapezoids, etc.

Perimeter and area of shapes

Perimeter is the total distance around a shape. To find the perimeter, you add up the lengths of all sides. Area is the measure of the space enclosed by a 2D shape. Area is measured in square units.

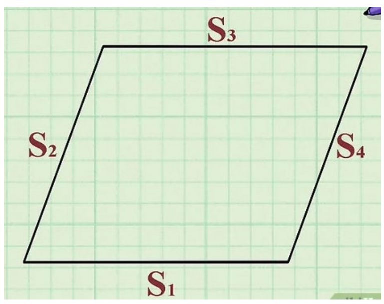
Investigating the area and perimeter of shapes using graphs/geodots

We can use graph sheets to investigate the perimeter and area of shapes. Look at the picture. A trapezium has been drawn on a graph sheet to help determine its area.



We know that area is a measure of how many square units will fit inside a shape. So, how many squares are inside our trapezoid? There are 24 full squares plus eight half squares, which means the area of the trapezoid is 28 square units.

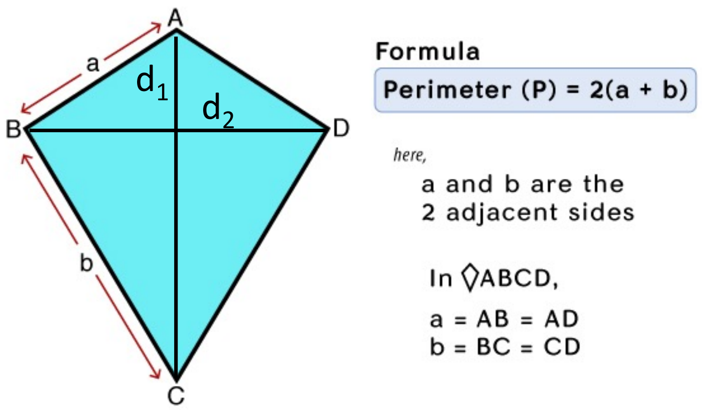
Take a look at the rhombus drawn on the graph sheet below;



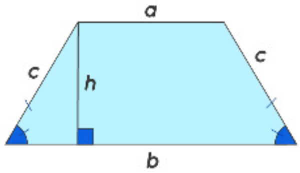
We can determine the perimeter of the shape by counting the number of squares covered by the line at S1 or S3. Each of these two sides cover approximately 12 squares. Now, since all the sides of a rhombus are equal, we can say that S2 and S4 are 12 squares as well. Assuming the side of a square on the graph is 1cm, then we can calculate the perimeter as S1 + S2 + S3 + S4 = 12cm + 12cm + 12cm + 12cm = 48cm.

Note: *Please explore the perimeter and area of kites, parallelogram and rhombus using the graph sheet of geodot.*

Calculating the perimeter and area of shapes using simple formulae. Take a look at the formulae for the following shapes;

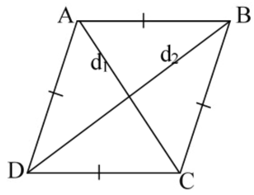


Area of a kite = ½ ×(d_1)×(d_2)square units



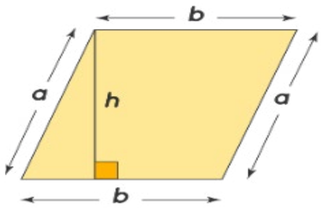
Perimeter = c+c +a + b

Area of an isosceles trapezoid = 1/2 (a+b)h square units



Perimeter = AB+AD+BC+CD

Area of a rhombus = $\frac{1}{2}\times(d_1)\times(d_2)$ square units



Perimeter = 2(a + b) or (a + a + b + b)

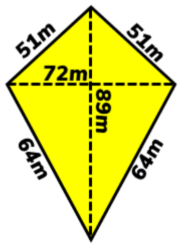
Area of a parallelogram = base × perpendicular height = b × h square units

Note: For trapezoids, discuss the types. These are;

- Isosceles Trapezium
- Scalene Trapezium
- Right Trapezium

Worked Examples

The perimeter and area for the following shapes can be determined as;

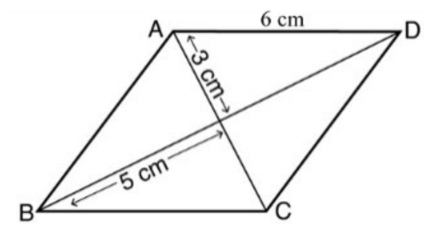


Perimeter (P) = 2(a + b),

∴P = 2(51 + 64) = 230 m

Area = $\frac{1}{2}\times 72\times 89$

= 3204 m^2

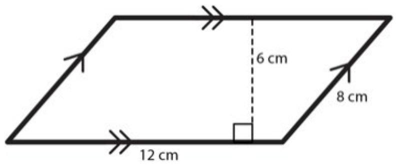


Perimeter = A + BC + CD + DA

= 6+ 6 + 6 + 6 = 24 cm

Area of one triangle = $\frac{1}{2}\times 3\times 5 = 7.5\ cm^2$

So area of rhombus = 4 × 7.5 = 30 cm^2

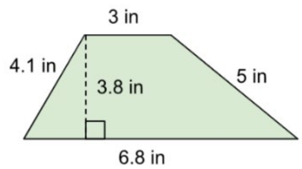


Perimeter = 8cm + 8cm + 12 cm + 12 cm

∴P = 40 cm

Area = 12 x 6 = 72cm^2

Area = 12 × 6 = 72 cm^2



Perimeter = 4.1 + 3 + 5 + 6.8 = 18.9 cm

Area = $\frac{1}{2}$ (3 + 6.8) × 3.8 = 18.62 in^2

Learning Task for practice

Learners solve problems on finding the perimeter and area of kites, rhombus, parallelogram and trapezium.

Solve real-life problems on perimeter and area of 2-D shapes

Example 1: The perimeter of a parallelogram is equal to 48cm. Two of its sides are 16cm each. How long is each of the other side?

Solution: Given: Perimeter (P) = 48 cm, and side, a = 16 cm. To find the length of the other side b of the parallelogram, we will use the formula of the perimeter of a parallelogram P = 2(a + b).

$P = 2(a + b)$

$\Rightarrow 48 = 2\ (16 + b)$

$\Rightarrow 16 + b = \frac{48}{2}$

$\Rightarrow b = 24 - 16$

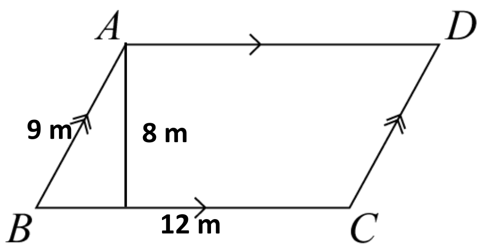
$\Rightarrow b = 8$ cm

Answer: The other side of the parallelogram is 8 cm.

Example 2: A garden in the shape of a parallelogram has dimensions 12 meters and 9 meters and a perpendicular height of 8 meters. If the owner wants to put a fence around the garden, how much fencing material is needed? Also, what is the area of the garden?

Solution:

The shape of the garden can be depicted as show in the diagram;



To find the perimeter of the parallelogram, we need to add the lengths of all four sides. Since a parallelogram has opposite sides of equal length, the perimeter is:

Perimeter = 2(12 + 9) = 2(21) = 42 meters

To find the area of the parallelogram, we use the formula:

Area = base × perpendicular height = 12 × 8 = 96 square meters

Therefore, the owner needs 42 meters of fencing material, and the area of the garden is 96 square meters.

Example 3 [challenge]

Gina is designing a rectangular banner to promote her school's art exhibition. She wants to include a large rhombus-shaped logo in the middle of the banner. The diagonal dimensions of the logo rhombus are 96 cm and 60 cm. The banner dimensions are 1.5 m *x* 1 m.

Gina has a few design questions:

- **i.** What is the height and the width of the rhombus logo?
- **ii.** What is the area of the rhombus logo?
- **iii.** Will her logo rhombus fit inside the 1.5 m *x* 1 m banner dimensions if she aligns the edges parallel?
- **iv.** How much blank space will there be at the top/bottom or sides once the rhombus logo is cantered aligned in the banner?

Solution

i. Height = Shorter diagonal = 60 cm Width = Longer diagonal = 96 cm

ii. **Area** $= \frac{1}{2} \times \text{Height} \times \text{Width} = \frac{1}{2} \times 60 \times 96 = 2{,}880\ \text{cm}^2$

iii. Yes, aligned vertically or horizontally, the rhombus height 60 cm and width 96 cm fit within the 150 cm *x* 100 cm banner with room to spare.

iv. Top/Bottom blank space $= \frac{(\text{Banner height} - \text{Rhombus height})}{2} = \frac{(150 - 60)}{2} = 45\ \text{cm}$

Side blank space $= \frac{(\text{Banner width} - \text{Rhombus width})}{2} = \frac{(100 - 96)}{2} = 2\ \text{cm}.$

So when cantered, there would be 45 cm blank space above and below the rhombus logo in the banner and 2 cm on either side.

Pedagogical Exemplars

Teachers should consider the following activities;

Reviewing previous knowledge: Review learners' previous knowledge on perimeter and area of the four basic shapes [square, rectangle, triangle and circles]. In doing so, identify the misconceptions that they may have and address them.

Experiential learning: In mixed-gender/ability groups, engage learners to explore the immediate school environment to investigate referents for the measuring various items. Examples could include referent for a centimetre, metre, kilometre, etc.

Use their experiences from their investigations on referents for linear and area measurements to estimate the perimeter or area of a given object.

Note: This activity should focus on learners' ability to estimate and not necessarily getting absolute figures. Encourage shy learners to share their findings from the investigations.

Problem-based learning: In small groups, engage learners to investigate the perimeter and area of kites, parallelogram, rhombus and trapezoids using geodot/graphs.

Alternatively, this activity, including that of the investigation of referents, could be given to learners ahead of the lesson schedule. This is to help free some time for teaching the rest of the concepts.

Group & pair activities: Using think-pair- share, task learners to solve problems, using formulae for determining the perimeter/area of kites, parallelogram, rhombus and trapezoids. Learners may use digital mathematics tools where applicable and available.

Problem-based group learning: Using mixed-ability groups, present learners with task sheets on perimeter and area, including word/real-life problems to solve.

Whole Class discussions and demonstrations: Lead the class to discuss the main ideas of the lesson and take the opportunity to demonstrate [or learners volunteer to demonstrate] challenging concepts, including resolving all misconceptions.

Individual Task

Present learners with individual worksheets to complete. Alternatively, give them some answers, e.g. 60cm, or 56m^2 and they must come up with the questions.

Possible Misconceptions

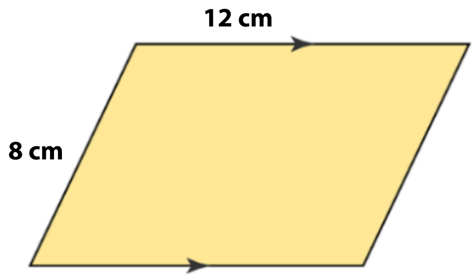
There are several misconceptions that learners may have regarding the concepts of perimeter and area for kites, parallelograms, rhombuses, and trapeziums. Some common misconceptions include:

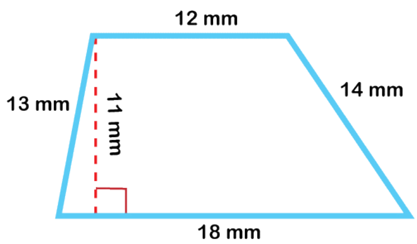
1. **Misunderstanding of diagonals:** Learners may incorrectly believe that the diagonals of a parallelogram are always equal in length.
2. **Confusion between perimeter and area:** Learners might confuse the concepts of perimeter (the distance around the shape) and area (the amount of space inside the shape), leading to incorrect calculations.
3. **Incorrect application of formulae:** Learners might apply the wrong formula for calculating the area or perimeter of these shapes, leading to inaccurate results.
4. **Misconceptions about angles:** Learners may have misconceptions about the properties of angles in these shapes, such as believing that all angles in a rhombus are right angles.

Key Assessment

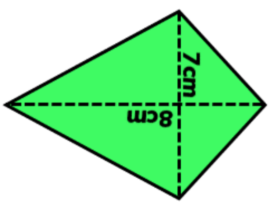
Level 2

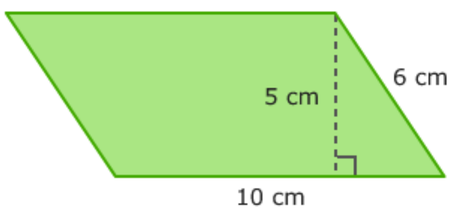
1. Find the perimeter of the following shapes.

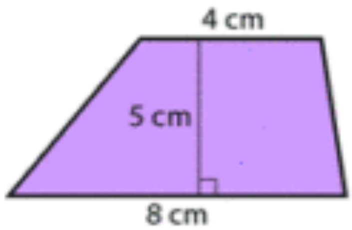




2. Calculate the area of the following shapes

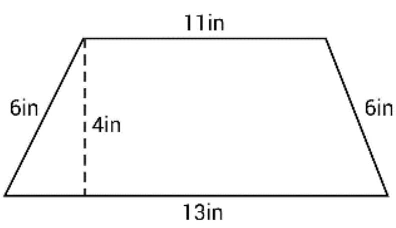


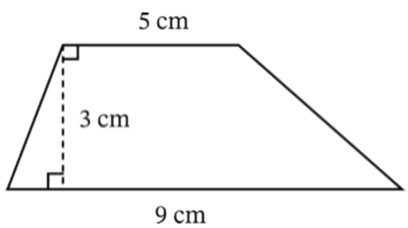


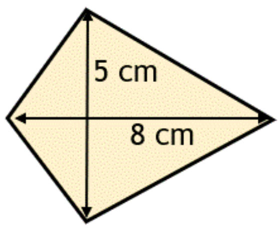


Level 2:

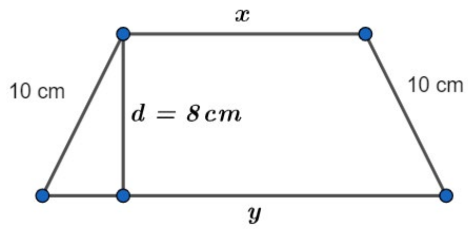
1. Find the area for the shapes below.







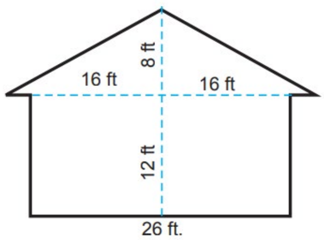
2. The perimeter of the given trapezium is 104 cm. Find its area. [ANS. 336cm^2]



Level 3

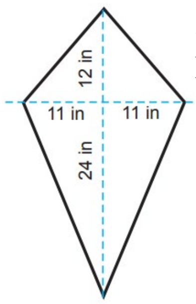
1. Solve the following problems

i.



Miss. Tackie wants to paint her house. To buy paint, she must know the area. What's the area of one side of Miss. Tackie's house? [ANS 440 ft^2]

ii.



Charles needs paper to cover his kite.
Find the area of the kite. [ANS. 396 in^2]

2. A field in the shape of a rhombus has an area of 60 square meters. If one of its diagonals is 12 meters long, find the length of the other diagonal.

3. Jamila is building a deck outside her house with a rectangular area that has outer dimensions of 8m *x* 6m. She wants to have decorative floor tile patterns in a parallelogram shape inside the deck. Each parallelogram tile has one side measuring 1.2m and a perpendicular height of 0.8m against that side.

a) What is the area of each parallelogram tile? [ANS. 0.96 m^2]

b) If Jamila covers the entire inner deck area with these tiles, how many full tiles would she need? [ANS. 50 tiles]

Week 20

Learning Indicators:

1. *Solve problems that involve SI and imperial units in volume of prisms.*
2. *Solve real world problems that involves the volume of prisms.*

Theme or Focal Area: Volume of prisms

Understanding volume helps quantify three-dimensional spaces which allows for practical applications like determining storage capacities, amounts that fit inside containers, calculating dosages and more.

Volume is a measure of the amount of three-dimensional space that an object occupies. Some key points about volume:

1. Volume measures the space taken up by a 3D object. It quantifies how much a substance or solid material would fill a three-dimensional container.
2. Volume is measured in cubic units such as cubic meters (m^3), cubic centimeters (cm^3) and cubic inches (in^3).
3. The volume of a 3D shape depends on the dimensions of height, width and depth - unlike area which uses just height and width. The standard mathematical formula used is:

   $$\text{Volume} = \text{Depth} \times \text{Width} \times \text{Height}$$

   Some examples of volumes we encounter:

   - Volume of liquid a bottle can contain
   - Volume of concrete needed to lay a house's foundation
   - Volume of medicines and dosage instructions based on that.

4. Understanding and calculating volumes has many practical real-world applications in fields from shipping and storage to calculating health or building metrics to determining carrying capacity of containers.
5. The basic volume formulas work for standard 3D geometrical shapes. However, for irregular shapes, volumes are measured by instruments or by estimation using simpler building blocks and their volume formulae.

The week will cover;

i. Define prisms and identify the elements: bases, height/length, lateral faces

ii. Understand the appropriate formulae for calculating volume of prisms

iii. Calculate volume by plugging dimensions into formula

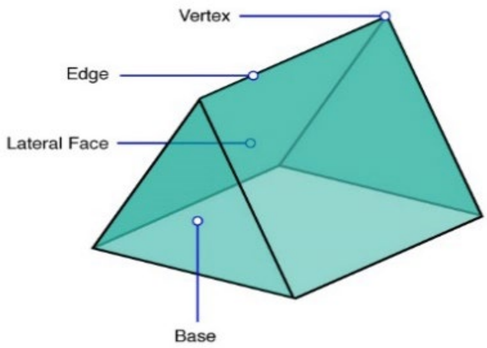
iv. Solve problems involving prism volumes contextually

Define and identify the elements: bases, height/length, lateral faces

A prism is a three-dimensional solid object having two identical and parallel shapes facing each other. Thus, a prism has a constant cross-section. The identical shapes are called the bases. The bases can have any shape of a polygon such as triangles, square, rectangle, or a pentagon. The diagram below shows a triangular prism

Parts

A prism has bases, lateral faces, edges, and vertices.



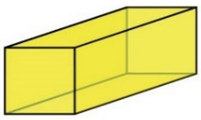
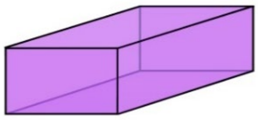
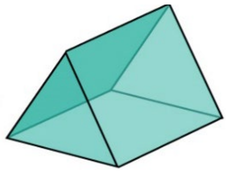
a. **Base** – The **parallel faces** which makes the 2 ends of any prism. They are congruent. The base determines the cross-section of any prism and it remains uniform throughout the shape.

b. **Lateral faces** – The **non-parallel faces** which connects the 2 bases.

c. **Vertices** – The corners.

d. **Edges** – Where any 2 faces meet.

Examples of other prisms include;

		
Square-faced cuboid • 6 faces (2 squares and 4 rectangular) • 12 edges • 8 vertices	**Rectangular-faced cuboid** • 6 faces (all rectangular) • 12 edges • 8 vertices	**Triangular** • 5 faces (2 triangular and 3 rectangular) • 9 edge • 6 vertices

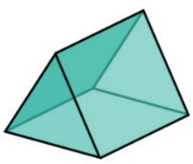
Learning Task for practice

Identify other prisms and indicate the type of face, number of faces, edges and vertices.

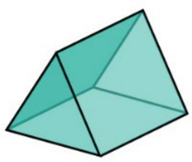
A prism can also be classified into **regular** or **irregular** based on the uniformity of its cross-section. It can be **right** or **oblique**, depending on the alignment of its bases.

Regular and Irregular Prisms

The diagram shows the difference between a regular and irregular triangular prism.



Regular



Irregular

1. **Regular Prism** – It has a base which is a regular polygon with equal side lengths. Regular prisms have identical bases and identical lateral faces. Thus, all the above examples of prisms are regular such as triangular, rectangular, and pentagonal.
2. **Irregular Prism** – It has a base which is an irregular polygon with unequal side lengths. Irregular prisms have identical bases. However, the lateral faces are not identical.

Right and Oblique Prisms

The diagram shows the difference between a right and an oblique pentagonal prism.

Right   Oblique

1. **Right Prism** – Its lateral faces are perpendicular to its bases. The 2 bases of a right prism is aligned perfectly over one another.
2. **Oblique Prism** – It is a slanted prism. So its lateral faces are not perpendicular to its bases. The 2 bases are not aligned perfectly over one another.

Understand the appropriate volume formula

Study the table

Shape	Base	Volume of Prism = Base area × height
Triangular Prism	Triangular	Volume of triangular prism = Area of triangle × height of the prism
Square Prism	Square	Volume of square prism = Area of square × height of the prism
Rectangular Prism	Rectangular	Volume of rectangular prism = Area of rectangle × height of the prism

Worked Examples

Example 1: Find the volume of a triangular prism whose base area is 64 cm^2 and height is 7 cm.

Solution:

As we know,

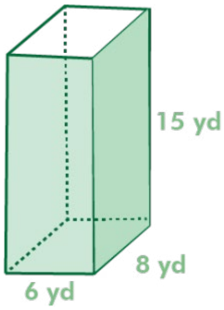
Volume (V) = Base Area × Height

$\therefore$ V = B × h, here B = 64 cm^2, h = 7 cm

$= 64 \times 7$

$= 448\ cm^3$

Example 2: Find the volume of the prism shown in the figure.

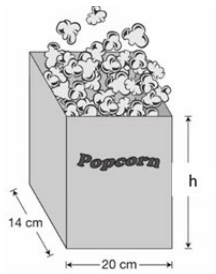


Solution:

V = B × h [Volume of a prism formula]
= l × b × h [Area of rectangle formula]
= 6 × 8 × 15 [Substitute values of l and b]
= 720 [Simplify]

The volume of the prism is 720 cubic yards.

Example 3: A box of popcorn holds 7000 cubic centimetres of popcorn. The length and width of the base of the box are 14cm and 20cm, respectively. Find the height of this box of popcorn**.**



Solution:

V = B × h [Formula for the volume of a prism]

7000 = l × b × h [Formula for the area of a rectangle; replace V with 7000]

7000 = 14 × 20 × h [Substitute values of l and b]

7000 = 280 × h [Simplify]

25 = h [Divide each side by 280]

The height of the popcorn box is 25cm.

Example 4: What is the base area of the prism if the volume of the prism is 324 cubic units and the height of the prism is 9 units.

Solution:

The given dimensions are the volume of the prism = 324 cubic units and the height of the prism = 9 units. Let the base area of the prism be "B".

Substituting the values in the volume of the prism formula, Volume of prism = V = B × H = 324 cubic units

⇒ 9B = 324

⇒ B = 36 square units

Therefore, the base area of the prism is 36 square units.

Learning Task for practice

Learners determine the volume of given prisms.

Possible Misconceptions

There are several misconceptions that learners may have regarding the concept of prisms. Some common misconceptions include:

1. **Misconceptions about cross-sectional shapes:** Learners may not understand that the cross-sectional shape of a prism remains the same throughout its length.
2. **Confusion between prisms and pyramids:** Learners may confuse prisms (which have two parallel and congruent bases connected by rectangular or parallelogram sides) with pyramids (which have a polygonal base and triangular sides that meet at a common vertex).
3. **Misunderstanding of volume and surface area:** Learners might incorrectly believe that the volume and surface area of a prism are always the same.
4. **Incorrect identification of faces, edges, and vertices:** Learners may have difficulty identifying the faces, edges, and vertices of a prism, especially with complex shapes.
5. **Confusion with formulae:** Learners might struggle with applying the correct formulae for calculating the volume and surface area of prisms, leading to errors in calculations.

Pedagogical Exemplars

Teachers should consider the following activities;

Reviewing Previous concepts: Review learners' previous knowledge on surface area and volume of solid shapes from JHS. In doing so, take notice of various misconceptions that they may bring and address them later.

Experiential learning: In mixed-gender/ability groups, engage learners to explore the immediate school environment to investigate items/objects that are considered prisms. They should further talk about the properties as well as the nets.

Use their experiences from their investigations to estimate the volume of the objects they found applying their experiences form JHS.

Group & pair activities: Present groups of learners with the formulae of the three prisms [triangular, square and rectangular prism] and task them to discuss and provide justification for each formulae.

Using think-pair- share, task learners to solve problems, using formulae for determining the volume of the prisms with given dimensions. Learners may use digital mathematics tools where applicable and available.

Problem-based group learning: Using mixed-ability groups, present learners with task sheets on volume, including word/real-life problems to solve.

Whole Class discussions and demonstrations: Lead the class to discuss the main ideas of the lesson and take the opportunity to demonstrate [or learners volunteer to demonstrate] challenging concepts, including resolving all misconceptions.

Individual Task

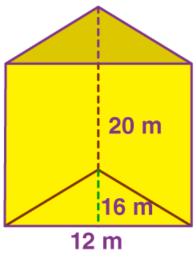
Present learners with individual worksheets to complete. Alternatively, allow learners to take home the tasks for later submission.

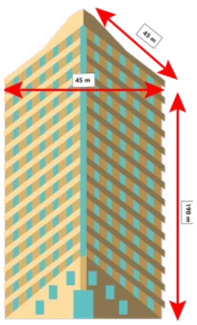
Key Assessment

Level 2

1. Find the height of the prism if the volume of the prism is 729 cubic units and the base area is 27 square units. **[ANS. 27 units]**
2. What is the base area of a prism if the volume of the prism is 300 cubic feet and the height of the prism is 6 feet? **[ANS.** 50 square feet**]**
3. The base area of a prism is 123 square yards. The height is 9 yards. Find the volume. **[ANS.** 1107 cubic yards**]**

Level 3

1. 

What is the volume of a triangular prism with dimensions of 12 m, 16 m and 20 m as given in figure [**ANS**. 1920 cubic meters]
2. 
A building is in the shape of a triangular prism. The height of the building is 190 meters. The base of the building is in the form of an equilateral triangle whose side is 45 meters. Find the lateral surface area of the building. [**ANS**. 25,650 square meters] .
3. A swimming pool in the shape of a cuboid has a length of 10 meters, a width of 5 meters, and a depth of 2 meters. If the pool is filled with water, how much water does it hold? [ANS. 100 cubic meters of water]
4. The height of a square prism is 11 yards. The volume is 99 cubic yards. What is the length of the side of the square at the base of the prism? [3 yards]

Section Review

This section is for review of all the lessons taught for the last 18 & 19 weeks. A summary of what the learner should have learnt.

Reflection and key ideas

1. Referents are materials that we can use as a reference to estimate other measurements.
2. Perimeter is the total distance around a shape. To find the perimeter, you add up the lengths of all sides.

3. Area is the total space taken up by a flat (2-D) surface or shape of an object. The space enclosed by the boundary of a plane figure is called its area. The area of a figure is the number of unit squares that cover the surface of a closed figure. Area is measured in square units like cm^2 and m^2.
4. Volume is a measure of the amount of three-dimensional space that an object occupies. Volume is measured in cubic units such as cubic meters (m^3), cubic centimetres (cm^3) and cubic inches (in^3).
5. Volume of prisms can be obtained using; Volume = Length *x* Width *x* Height for cuboids, or Volume = Area of base *x* Height.

Resources

- Mathematical sets
- Geo pegs and peg boards
- Cut out shapes
- Magnetic demonstration set
- SAFE-T Rulers
- Centimetre grid board
- Folded geometric shapes
- Technology tools such as computer, mobile phone etc.
- Computer software applications like GeoGebra.
- Tape measure, compass, geodot, etc.

References

1. Perimeter of Rhombus (math-salamanders.com)

2. Prism - Definition, Shape, Types, Formulas, Examples & Diagrams (mathmonks.com)

3. Volume of Prism - Formula, Derivation, Definition, Examples (cuemath.com)

SECTION 8: DATA ORGANISATION, ANALYSIS AND PRESENTATION

Strand: **Making Sense of and Using Data**

Sub-Strand: Statistical reasoning and its application in real life

Learning Outcomes:

1. *Decide whether or not a selected data collection method is appropriate given a particular data, justify responses, and collect both qualitative and quantitative data with the appropriate methods.*
2. *Apply the knowledge of organising and presenting data (grouped/ungrouped) using frequency tables, line graphs, pie charts, multiple bar graphs, info graphics, etc.; generate 3D graphs/charts with appropriate digital technology (where available) and solve problems on them.*
3. *Organise and present data (grouped/ungrouped) using frequency tables, line graphs, pie charts, multiple bar graphs, info graphics, etc.; generate 3D graphs/charts with appropriate digital technology (where available) and solve problems on them.*
4. *Design and execute a project by posing and refining questions to collect, analyse and interpret quantitative and/or qualitative data directly from the school community and beyond, draw useful conclusions and make recommendations.*

Content Standards:

1. Demonstrate conceptual understanding of the appropriateness of data collection methods to collect everyday life data.
2. Demonstrate conceptual understanding of data organisation and presentation for grouped and ungrouped data, including 3D graphs/charts with appropriate digital technology.
3. Demonstrate the ability to embark on a project involving the collection, analysis and interpretation of quantitative and qualitative data within the school environment.

INTRODUCTION AND SECTION SUMMARY

Understanding data is crucial in today's information-driven world. Data comes in various types, including numerical, categorical and ordinal, and can be collected, organised, and analysed in different ways. Data presentation involves representing data visually, such as through graphs, charts and tables, to facilitate understanding and interpretation. Measures of central tendency, such as mean, median, and mode, help summarise a dataset by providing a single value that represents the "centre" of the data. These measures are useful for making comparisons and drawing conclusions from the data. Real-life projects involving data collection, analysis and presentation are valuable for students as they provide hands-on experience and demonstrate the practical applications of data concepts. Such projects can be interdisciplinary, involving subjects like science, history, geography and economics, where data analysis plays a significant role. Overall, proficiency in these areas enables students to effectively collect, analyse and present data, making informed decisions and solving real-world problems. It also prepares them for further studies in fields like statistics, economics and data science.

The weeks covered by the section are:

Week 21:

1. Types of data
2. Identify and validate quantitative & qualitative data collection methods

Week 22:

1. Data organisation & presentation
2. Measures of central tendencies

Week 23: Real-life projects on data collection, analysis and presentation

SUMMARY OF PEDAGOGICAL EXEMPLARS

This section requires hands-on activities and the use of real-life examples to enhance understanding and application. Learners should therefore be engaged in a variety of teaching and learning strategies as the section aims to equip learners with a solid foundation data organisation and presentations. **The pedagogies will include:**

- **Hands-on Data Collection:** Engage students in collecting real data through surveys, observations, or experiments. This provides them with a practical understanding of data collection methods and the types of data they might encounter.
- **Visual Representations:** Use charts, graphs, and diagrams to visually represent data. This helps students see patterns and trends in data more easily.
- **Real-life Context:** Use real-life examples and scenarios to illustrate the concepts of data. This helps students see the relevance of data analysis in their everyday lives.
- **Collaborative Learning:** Encourage collaborative learning by having students work in groups to analyse and interpret data. This can help them develop communication and teamwork skills.
- **Problem-based Learning:** Present students with real-world problems that require them to collect, analyse, and present data. This helps them see the practical applications of data analysis.

ASSESSMENT SUMMARY

The concepts covered in this section require learners to demonstrate a comprehensive understanding of different types of data, data collection methods, data organisation, presentation, measures of central tendencies, and real-life applications. Assessments should target levels 2, 3 & 4 of the Depth of Knowledge (DOK) framework to ensure learners grasp the concepts deeply and can apply them effectively. A variety of assessment strategies should be used, including:

- Quizzes and tests to assess understanding of types of data, data collection methods, and measures of central tendencies.
- Practical assessments involving **real-life projects** on data collection, analysis, and presentation, allowing learners to apply their knowledge in authentic scenarios.
- Homework assignments to reinforce learning and assess understanding of data organisation and presentation techniques.
- Group activities where learners work together to analyse and interpret data, fostering collaboration and critical thinking skills.
- Presentations where learners communicate their findings from data analysis, demonstrating their ability to present data effectively.

Week 21

Learning Indicators:

1. *Classify data (primary and secondary) as quantitative (discrete and continuous), qualitative (nominal and ordinal), numerical, categorical, grouped, ungrouped, etc.*
2. *Identify and validate quantitative data collection methods (Survey/Questionnaire, Interviews, Observation, Existing Data, and Probability) and use it to collect everyday-life data.*
3. *Identify and validate qualitative data collection methods (interviews, observations, focus groups, oral histories, online tracking, social media monitoring, etc.) and use it to collect everyday-life data.*

Theme or Focal Area: Types of Data

Primary and secondary data

Primary and secondary data are two fundamental types of data used in research and analysis. They serve different purposes and have distinct characteristics, each contributing to the understanding of a given subject.

Primary data refers to information collected first-hand, specifically for a particular research study or investigation. This data is original and directly obtained from the source. Primary data collection methods may include surveys, interviews, experiments, observations, or direct measurements.

On the other hand, secondary data refers to existing data that has been collected by someone else for a different purpose but can be repurposed for new research or analysis. Secondary data sources include published literature, reports, databases, government records and data collected by other researchers.

Activity to reinforce the concept

Using data hunting game, write a variety of data types on different cards and task learners to sort the cards under given headings and justify their reason for the sorting.

Grouped and Ungrouped Data:

When working with data, it can be classified as either grouped or ungrouped, depending on how the values are organised and presented. Ungrouped data is a raw set of individual observations or values without any categorisation or grouping. Each data point is unique and stands alone. For instance, if you record the heights of a group of individuals without categorising them into ranges, you would have ungrouped data. Grouped data involves categorising values into intervals or classes, creating a summary of data within each category. Grouping data helps simplify large data sets and provides a broader overview of the distribution.

For example:

Height (in inches): 62, 65, 67, 62, 70, 65, 66, 64, 67, 71 -

Ungrouped

Height (in inches)	Frequency
62	2
64	1
65	2
66	1

Height (in inches)	Frequency
67	2
70	1
71	1

In the above frequency table, each data point is unique and stands alone.

Grouped

Height Range (in inches)	Frequency
60 - 64	3
65 - 69	4
70 - 74	3

In the example above, the heights have been grouped into three categories: 60-64 inches, 65-69 inches, and 70-74 inches. The frequency column represents the number of observations falling within each category.

Discrete and continuous data

Discuss and draw out the differences, with examples, between discrete and continuous data.

Hint:

1. Discrete data includes discrete variables that are finite, numeric, countable and non-negative integers and rarely involves decimals.

   E.g.:

   i. The number of students who have attended the class.

   ii. The number of customers who have bought different products.

   iii. The number of groceries people are purchasing every day.

   iv. Population of a country

N.B. Discrete data typically consists of whole numbers or counts of items and does not include decimal values. However, in certain cases, discrete data may be represented as decimal values for practical purposes.

For example, consider a situation where you are measuring the number of defective items produced in a factory per hour. While the number of defects is a discrete variable (you can't have half a defect), it might be represented as a decimal to indicate the average number of defects per hour over time. In such cases, the decimal values are used for measurement or calculation purposes, but the actual data points remain discrete. Another example is that in the UK, shoe sizes come in half sizes. But nothing in between. E.g. 5, 5.5, 6, 6.5 etc. - and this is discrete.

2. Continuous data is the unspecified number of possible measurements between two presumed points.
   - The weather temperature.
   - The wind speed.
   - The weight of the children.

Learning Tasks for Practice

1. Learners differentiate between the following concepts
   i. Discrete and continuous data
   ii. Primary and secondary data
   iii. Grouped and ungrouped Data
2. Learners organise data into frequency distribution tables by grouping them.

Quantitative and qualitative methods

Quantitative data is anything that can be counted or measured; it refers to numerical data. Qualitative data is descriptive, referring to things that can be observed but not measured—such as colours or emotions. Data collected about a numeric variable will always be quantitative and data collected about a categorical variable will always be qualitative.

Examples of data that can be considered as quantitative and qualitative

Quantitative data:

- My best friend is 5 feet and 7 inches tall
- They have size 6 feet
- They weigh 63 kilograms
- My best friend has one older sibling and two younger siblings
- They have two cats
- My best friend lives twenty miles away from me
- They go swimming four times a week

Qualitative data:

- My best friend has curly brown hair
- They have green eyes
- My best friend is funny, loud, and a good listener
- They can also be quite impatient and impulsive at times
- My best friend drives a red car
- They have a very friendly face and a contagious laugh

Here are examples of qualitative and quantitative data collection methods:

Quantitative Data Collection Methods:

1. Surveys: Using close-ended questions with fixed response options to gather numerical data from a large number of respondents.
2. Experiments: Conducting controlled experiments to collect quantitative data on the effects of variables on outcomes.
3. Tests and Assessments: Administering standardised tests or assessments to measure specific characteristics or abilities quantitatively.

4. Observational Studies: Systematically observing and recording behaviors or events to gather numerical data.
5. Sensor Data Collection: Using sensors and devices to collect quantitative data on physical phenomena, such as temperature, pressure, or motion.

Qualitative Data Collection Methods:

1. Interviews: Conducting one-on-one or group interviews to gather in-depth information and insights from participants.
2. Focus Groups: Bringing together a small group of people to discuss specific topics and gather opinions and perceptions.
3. Observations: Systematically observing and recording behaviors, interactions, and phenomena in natural or controlled settings.
4. Surveys: Using open-ended questions to gather detailed responses and opinions from participants.
5. Document Analysis: Analyzing written or visual materials such as documents, photographs, or videos to understand a specific topic or phenomenon.

Note: take time to explain how some of these methods (survey and observations) can both be used as qualitative and quantitative, thus depending on how the tool is constructed and used.

Application of quantitative data collection methods

Project Work

i. Research and select an existing survey questionnaire, observation guide, etc., then discuss its features, validate its usefulness and collect quantitative data with it in the classroom or within the school community.

ii. Design a mini project where you choose a data collection method of choice and collect real quantitative data with it.

Application of qualitative data collection methods

Project work

i. Students research and select an existing interview guide, observation guide, etc., then discuss its features, validate it usefulness and collect qualitative data with it.

ii. Students design a mini project where they select a data collection method of choice and collect real qualitative data with it.

Note: Offer learners who are capable the opportunity to construct their own tool and use it to collect the data.

Pedagogical Exemplars

Teacher should Consider the following activities;

Reviewing previous knowledge: Review learners' previous knowledge on data types from JHS.

Group & pair activities: Put learners into convenient groups and task them to discuss the various types of data and support their discussions with relevant examples.

Offer groups the opportunity to make presentations of their work and give others the opportunity to contribute to the presentations.

Encourage learners to be circumspect in their contributions and commentary in order not to hurt others.

Problem-based learning: Demonstrate (call a learner to volunteer) how data can be organised into a grouped data. Then assign learners tasks to work on in pairs or groups.

Problem-based learning: Present samples of qualitative and quantitative data collection tools to the class and lead the class to discuss the features and justifications on why the tool is valid or otherwise for collecting a given data. You may consider these activities to differentiate the lesson.

1. **Below Average Learners**:
   - **Presentation**: Provide simplified samples of data collection tools with clear explanations of their features and purposes.
   - **Discussion**: Lead a guided discussion focusing on identifying basic features of the tools and discussing simple justifications for their validity.
2. **Average Learners**:
   - **Presentation**: Present a variety of samples of data collection tools with moderate complexity, including examples from different fields or contexts.
   - **Discussion**: Facilitate a discussion where students analyse the tools' features, compare their strengths and weaknesses, and provide basic justifications for their validity.
3. **Above Average Learners**:
   - **Presentation**: Provide more complex samples of data collection tools, including advanced tools used in research or specialised fields.
   - **Discussion**: Engage students in a detailed analysis of the tools, requiring them to identify specific features. Evaluate their effectiveness for collecting different types of data, and provide thorough justifications for their validity.
4. **Group Activity (All Levels)**:
   - Divide the class into groups with mixed-ability levels.
   - Assign each group a different data collection tool to analyse and discuss.
   - Have groups present their findings to the class, with each member contributing based on their proficiency level.
5. **Extension Activity**:
   - For above average learners, provide additional samples of data collection tools and ask them to research and present advanced topics related to data collection methods, such as bias reduction techniques or innovative tools used in specific industries.

Experiential learning;

i. In mixed-gender/ability groups, engage learners to embark on a mini-project within the school environment by researching and selecting an existing survey questionnaire, interview guide, observation guide, etc., then discussing its features, validating its usefulness and collecting quantitative/qualitative data with it in the classroom or within the school community.

Note: You may have to support learners in researching and obtaining the research tool. Where learners are capable, provide support for them to construct the tool themselves and help them validate it.

Whole Class discussions and demonstrations

Lead the class to discuss learners' successes and challenges with the project and lead them to offer solutions to these challenges in order not to affect them in similar future projects.

Note: This project does not go beyond collecting the data. Other weeks will be used to continue with analysis and presentation of the data, and including publishing the findings.

Possible Misconceptions

1. **Data can only be quantitative:** Learners may mistakenly believe that data can only be numerical (quantitative) and overlook the possibility of categorical (qualitative) data. Understanding the distinction between the two types of data is crucial for proper analysis and interpretation.
2. **All data collection methods are equally valid:** Learners may assume that all data collection methods are equally valid for all situations. In reality, the choice of data collection method should be based on the research question, the nature of the data, and other contextual factors.
3. **Data organisation is solely about formatting:** Learners may misunderstand data organisation as simply formatting data in tables or charts. However, it also involves structuring data in a way that facilitates analysis and interpretation, such as using appropriate categories and variables.
4. **Real-life projects must involve complex data:** Learners may feel intimidated by the idea of real-life projects involving data collection, analysis, and presentation, assuming that they require complex datasets. In reality, real-life projects can involve simple datasets and focus on the process of data collection and analysis rather than the complexity of the data itself.

Key Assessment

Level 2

1. Describe each of the following data types as primary or secondary data;
    - interviews
    - reports
    - experiments
    - observations
    - published literature
    - databases

Level 3

1.
    i. Select an existing interview guide and discuss its features.
    ii. Validate the guide and collect a qualitative data with it.
2.
    i. Select an existing survey questionnaire and discuss its features.
    ii. Validate the questionnaire and collect a quantitative data with it.
3. The data below are the scores of 21 learners in a creative arts test. Organise it into a frequency distribution table by grouping them 59, 65, 61, 62, 53, 55, 60, 70, 64, 56, 58, 58, 62, 62, 68, 65, 56, 59, 68, 61, 67

Week 22

Learning Indicators:

1. *Organise and present data (grouped/ungrouped) using frequency tables, line graphs, pie charts, multiple bar graphs, info graphics, etc., including generating 3D graphs/ charts with appropriate digital technology (where available) and solve problems on them.*
2. *Analyse (include using appropriate computer applications) and interpret data using descriptive statistics (i.e., measures of central tendency/location and minimum & maximum values) and justify which of the averages best represent the data.*

Theme or Focal Area: Data Presentation methods: Frequency Distribution Table

Example 1: Ungrouped data

A jar containing beads of different colours- red, green, blue, black, red, green, blue, yellow, red, red, green, green, green, yellow, red, green, yellow. Organise the data in a frequency table.

Categories	Tally Marks	Frequency
Red	~~IIII~~	5
Green	~~IIII~~ I	6
Blue	II	2
Black	I	1
Yellow	III	3

Example 2: Grouped data

The data below represents the number of trees that survived in a survey by 100 schools that decided to plant 100 tree saplings in their gardens on world environment day.

95, 67, 28, 32, 65, 65, 69, 33, 98, 96, 76, 42, 32, 38, 42, 40, 40, 69, 95, 92, 75, 83, 76, 83, 85, 62, 37, 65, 63, 42, 89, 65, 73, 81, 49, 52, 64, 76, 83, 92, 93, 68, 52, 79, 81, 83, 59, 82, 75, 82, 86, 90, 44, 62, 31, 36, 38, 42, 39, 83, 87, 56, 58, 23, 35, 76, 83, 85, 30, 68, 69, 83, 86, 43, 45, 39, 83, 75, 66, 83, 92, 75, 89, 66, 91, 27, 88, 89, 93, 42, 53, 69, 90, 55, 66, 49, 52, 83, 34, 36

Represent the given data in the form of frequency distribution.

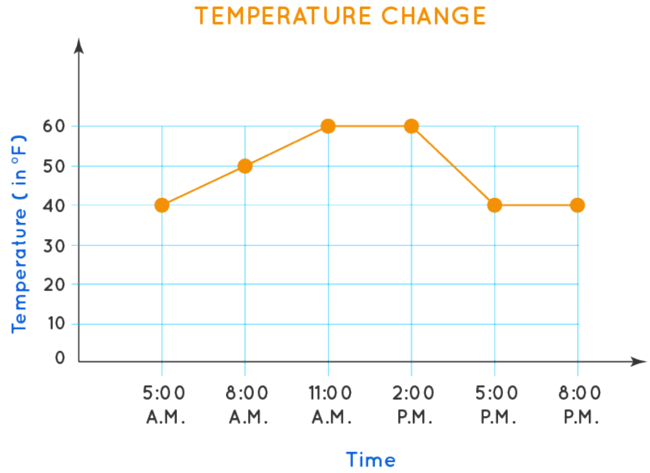
Solution

Number of plants survived	Tally Marks	Number of schools (frequency)
20 - 29	III	3
30 - 39	~~IIII~~ ~~IIII~~ IIII	14
40 - 49	~~IIII~~ ~~IIII~~ II	12
50 - 59	~~IIII~~ III	8
60 - 69	~~IIII~~ ~~IIII~~ ~~IIII~~ III	18
70 - 79	~~IIII~~ ~~IIII~~	10
80 - 89	~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ III	23
90 - 99	~~IIII~~ ~~IIII~~ II	12
Total		100

Line Graphs

Example

The temperature recorded in a city from 5 a.m. to 8 p.m. on a day was recorded in the form of a line graph as shown below. Study the graph and answer the following questions.



a) At which time(s) of the day was the temperature 40° F?

b) What was the maximum recorded temperature?

Solution

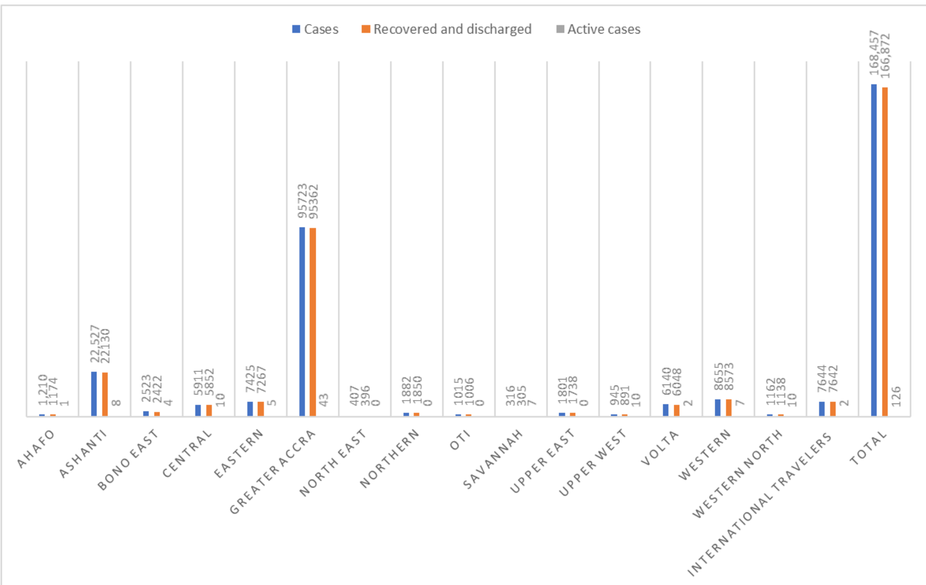
a. 5:00 a.m. and 5-6 pm.

b. 60° F

Bar Graphs

Example

The chart below is the summary of cases and recoveries of corona virus pandemic recorded in various regions in Ghana from March, 2020 to August, 2022.



From the graph:

1. Which region recorded the least cases over the period?
2. What is the percentage difference between cases recorded in Greater Accra and the total cases?

Solution

1. Savannah

2. Approximately, 43%

Pie Charts

The data below shows the amount of money that Akosua spent on buying some items:

- pepper - GH4
- onions - GH4
- salt - GH2
- fish - GH10

Draw a pie chart for the data.

Solution

Steps to draw a pie chart for a given data

1. Find the sum of the frequencies i.e. 4+4+2+10=20. Since a circle contains a total of 360°, we need to express each fraction for a sector as an angle per 360°.

   To calculate the angle for a data value of each sport:

   i. Find the angle for a data value, $\frac{360°}{20} = 18°$.

   ii. Next, we find the angle for each sector by multiplying the number of pupils (frequency) for that sector by 18 as shown in the table below.

   Alternatively, express each frequency over the total frequency and multiply by 360°.

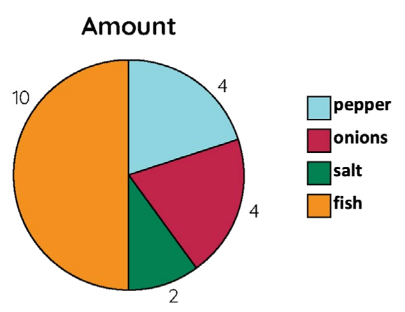
Items	Amount Spent (GHc)	Angles of sector	Alternatively Angle of sector
pepper	4	$4 \times 18 = 72°$	$\frac{4}{20} \times 360° = 72°$
onions	4	$4 \times 18 = 72°$	$\frac{4}{20} \times 360° = 72°$
salt	2	$2 \times 18 = 36°$	$\frac{2}{20} \times 360° = 36°$
fish	10	$10 \times 18 = 180°$	$\frac{10}{20} \times 360° = 180°$
Total	**20**	**360°**	

2. Use a pair of compasses to draw a circle with a radius of at least 5cm.

3. Use a protractor to draw (measure) the angles for each sector in the circle.

   i. Write and label for each sector horizontally.

   ii. It is usually appropriate to write the angles (magnitudes) in the sectors.



Learning Task for Practice

Learners construct bar charts, pie charts and line graphs for given data

Measures of Central Tendency

Mean, Median and Mode

As data analysts, statisticians, or curious learners, understanding the central tendencies of a dataset is crucial in gaining a meaningful understanding of its characteristics. These statistical measures provide us with a concise summary of the central or typical values within a dataset. They help us identify the centre around which the data clusters and give us insights into the overall distribution.

We will look at three primary measures of central tendency: the mean, the median, and the mode.

The mode refers to the value or values that appear most frequently in a dataset. It helps identify the most common or popular value(s) within a dataset and is especially useful in categorical or discrete data analysis.

The median, on the other hand, represents the middle value of a dataset when arranged in ascending or descending order. It is robust to extreme values and is particularly useful when dealing with skewed distributions or datasets with outliers.

The mean, often referred to as the average, is computed by summing all the values in the dataset and dividing by the total number of observations. It represents the arithmetic centre and provides a measure of the typical value in the dataset.

Note: Review learners' ideas on calculating the mode, median and mean for raw unorganised data as well as ungrouped data from JHS.

Calculating mean, median and mode for raw data

The marketing department of a company has collected the following sales figures (in thousands of Ghana cedis) for its top 10 sales representatives for the previous quarter:

11, 18, 15, 22, 19, 14, 17, 20, 16, 22

Calculate the following measures of central tendency for the given sales figures:

1. Mode
2. Mean
3. Median

Solution:

1. Calculating the Mode: The mode is the value that appears most frequently in the data set. In the given data, 22 appears twice, hence the mode in the data set.
2. Calculating the Mean: The mean is the sum of all the values divided by the total number of values. Sum of all values = 11 + 18 + 15 + 22 + 19 + 14 + 17 + 20 + 16 + 22 = 174

   Total number of values = 10

   Mean = $\frac{174}{10} = 17.4$
3. Calculating the Median: The median is the middle value when the data is arranged in ascending or descending order. Arranging the data in ascending order: 11, 14, 15, 16, 17, 18, 19, 20, 22, 22 Since there are 10 values, the median is the average of the 5th and 6th values.

   Median = (17 + 18) / 2 = 17.5

Therefore, the measures of central tendency for the given sales figures are:

- Mode: 22
- Mean: 17.4
- Median: 17.5

Calculating mean, median and mode from frequency distribution table

Mode and Median

Example 1: The data below shows the marks obtained in a science test. Organise the data in a frequency distribution table and find the mode and median.

2	2	4	3	1	4	1	0
1	2	3	1	2	1	3	0
4	2	3	3	3	4	3	1
1	0	1	4	0	0	1	4

Solution

Mode

To obtain the mode, look for the mark with the highest frequency. From our frequency table, below, "mark 1" had the highest frequency of 9, hence it is the mode of the data set.

Median

To obtain the median from the frequency table, follow the following steps:

1. Find the total frequency. To do this, sum up all the frequencies.
2. Divide the total frequency by 2.
3. Find the item that correspond to the frequency from your answer in point two above.

In the example, total frequency = 32 ∴ 32/2 =16

Marks	Tally	Frequency
0	𝍸	**5**
1	𝍸 ////	9
2	𝍸	**5** **16th & 17th**
3	𝍸 //	7
4	𝍸 /	**6**

From the frequency table the,

The 16th and 17th frequencies fall on 5.

Therefore, the median is **2.**

N.B.: When the total frequency is an even number, it means there are two numbers in the middle. It means the median will be the 16th and the 17th number.

Mean

The mean formula for grouped data. Which is expressed as:

Estimated mean = $\bar{X} = \sum \frac{fx}{\sum f}$

Where:

$\sum$ = is the summation sign

$\overline{X}$ = the mean value of the set of given data

f = frequency of the individual data

Example

The data below are marks obtained in a social studies test in JHS 1. Draw a frequency distribution table for the data.

3	2	2	3	4
2	5	4	2	2
2	2	1	4	2
4	5	3	2	7
2	3	2	6	3

To obtain the mean from the frequency table, follow the following steps:

1. Organise the data into a frequency distribution table.
2. Find the total frequency (f). To do this, sum all the frequencies.
3. Sum all the values under the (fx) column.
4. Divide the summation of fx by the summation of f.

Frequency distribution table

Marks (x)	Tally	Frequency (f)	fx
1	/	**1**	**1**
2	~~////~~ ~~////~~ /	**11**	**22**
3	~~////~~	**5**	**15**
4	////	**4**	**16**
5	//	**2**	**10**
6	/	**1**	**6**
7	/	**1**	**7**

$\sum f = 25$ $\sum fx = 77$

Mean - $\frac{\sum fx}{\sum f} = \frac{77}{25} = 3.08$

$\therefore$ the mean score is 3.08

Mean median and mode for grouped data

Worked Example

Example: You grew fifty carrots using special soil. You dig them up and measure their lengths (to the nearest mm) and group the results:

Length (mm)	Frequency
150 - 154	5
155 - 159	2
160 - 164	6
165 - 169	8
170 - 174	9
175 - 179	11
180 - 184	6
185 - 189	3

Mean

Length (mm)	Midpoint (x)	Frequency (f)	fx
150 - 154	152	5	760
155 - 159	157	2	314
160 - 164	162	6	972
165 - 169	167	8	1336
170 - 174	172	9	1548
175 - 179	177	11	1947
180 - 184	182	6	1092
185 - 189	187	3	561
	Totals:	**50**	**8530**

Estimated Mean $= \frac{\sum fx}{\sum f} = \frac{8530}{50} = 170.6$ mm

Median

The Median is the mean of the 25th and the 26th length, so is in the **170 - 174** group.

Mode

The Modal group is the one with the highest frequency, which is **175 - 179**:

Establish that the mean is a form of average

The term average is frequently used in everyday life to denote a value that is typical for a group of quantities. Average rainfall in a month or the average age of employees of an organisation is a typical example.

- Average is the value that indicates what is most likely to be expected.
- They help to summarise large data into a single value.

An average tends to lie centrally within the values of the observations arranged in ascending order of magnitude. So, we call an average a measure of the central tendency of the data. Averages are of different types. What we refer to as mean i.e., the arithmetic mean is one of the averages. Mean is called the mathematical average whereas median and mode are positional averages.

Differences between mean and median

Mean	Median
The mean, also known as the average, is calculated by summing all the values in a dataset and dividing by the total number of observations.	The median represents the middle value in a dataset when arranged in ascending or descending order.
It is influenced by extreme values or outliers in the dataset.	It is not affected by extreme values or outliers in the dataset.
The mean represents the arithmetic center or the typical value of the dataset.	The median is particularly useful when dealing with skewed distributions or datasets with outliers.
It is widely used and provides a measure of the central tendency that takes into account all values in the dataset.	It provides a measure of the central value that is robust to extreme values.

Justifying which of the averages best represent a given data

Example

A department of an organisation has 5 employees which include a supervisor and four executives. The executives draw a salary of GH 10,000 per month while the supervisor gets GH 40,000. Calculate the mean and median for the data. Which do you think better represents the data set?

Solution: Mean $= \frac{(10000 + 10000 + 10000 + 10000 + 40000)}{5} = \frac{80000}{5} = 16000$

Thus, the mean salary is GH 16,000.

To find the median, we consider the ascending order: 10000, 10000, 10000, 10000, 40000.

$n = 5$, so, $\frac{(n + 1)}{2} = 3$

Thus, the median is the 3rd observation.

Median = GH10,000

Thus, the median is GH10,000 per month.

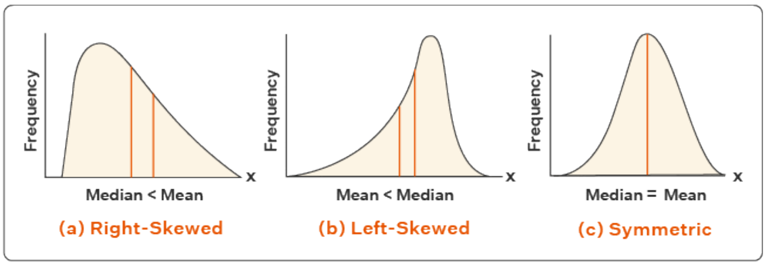
Now let us compare the two measures of central tendencies. We can observe that the mean salary of GH 16,000 does not give even an estimated salary of any of the employees whereas the median salary represents the data more effectively. One of the weaknesses of mean is that it gets affected by extreme values.

Effect of an extreme value on the mean

Example

The following graph show how extreme values affect mean and median:

- **Symmetric Data**
  - Data sets whose values are evenly spread around the centre
- **Skewed Data**
  - Data sets that are not symmetric



So, mean is to be used when we don't have extremes in the data. If we have extreme points, then the median gives a better estimation.

Learning Task for Practice

Learners determine the mode, median and mean for grouped data.

Pedagogical Exemplars

Teacher should Consider the following activities;

Reviewing Previous concepts: Review learners' previous knowledge on data presentation methods such as bar charts, frequency tables, pie charts, line graphs, etc. from JHS. Note any misconceptions that they have and address them immediately or later where appropriate.

Group & pair activities: In mixed-ability groups, assign the groups variety of data to organise them into, frequency tables, pie charts, line graphs, bar charts, etc.

You may have to take time to demonstrate how to organise date into grouped frequency tables.

Problem-based group learning: Assign some task on determining mean, median and mode for ungrouped data to groups of learners to complete and make presentations on them.

Now, demonstrate how to estimate the mean, median and mode for grouped data. Then assign learners some tasks on these concepts to solve.

You may consider the following activities to differentiate the group as well as the problem-based group activities

Below average learners

- Provide step-by-step guidance on how to organise data into frequency tables and create basic graphs (e.g., bar graphs, line graphs).
- Offer simplified explanations of descriptive statistics like mean, median, and mode.
- Use concrete examples and visual aids to help students understand the concepts.

- Allow students to work in small groups or pairs to create simple frequency tables and graphs using provided data sets.
- Provide templates and tools (such as graphing software or apps) to assist with creating visual representations of data.

Average learners

- Encourage students to organise data into more complex tables and graphs, such as pie charts or multiple bar graphs.
- Teach students how to calculate and interpret measures of central tendency (mean, median, mode).

Above average learners

- Introduce the use of technology (e.g., spreadsheet software) for creating and analysing data.
- Challenge students to create advanced visualisations, such as 3D graphs, using specialised software or programming languages.

Whole Class discussions and demonstrations: In a whole class, discuss the differences between mean and median and how to make justifications for which of the averages best represent a given data, as well as the effect of an extreme value on the mean.

Assign learners some tasks on these concepts to complete and address any misconceptions that they may have.

Individual Task

Present learners with individual worksheets to complete at their own time.

Key Assessment

Level 2

1. The table below shows the temperature, in degrees Celsius, recorded on 22 February, 2023. Construct a line graph for the data in the table.

Table: Temperature 22 Feb			
8:00	**10:00**	**12:00**	**14:00**
24°	32°	36°	29°

2. The table below shows the ice cream sales for a particular week. Construct a bar chart of the data.

	Table: Ice Cream Sales						
Day	**Mon**	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**	**Sun**
Amount (GH¢)	410.00	440.00	550.00	420.00	610.00	790.00	770.00

3. The table below shows the favourite type of movie from some movie enthusiasts. Construct a pie chart for the data.

Table: Favourite Type of Movie				
Comedy	**Action**	**Romance**	**Drama**	**Sci-Fi**
6	7	8	4	5

Level 2

1. Suppose we conduct a survey in which we ask 15 households how many pets they have in their home. The results are as follows:

   1, 1, 1, 1, 2, 2, 2, 3, 3, 4, 5, 5, 6, 7, 8

   Construct both a grouped and an ungrouped frequency tables for the data.

2. The table shows Russel's height at 3-year intervals. Make a line graph to display the data.

Age	Height(ft)
3	1.9
6	2.4
9	2.7
12	3.0
15	4.6
18	5.5
21	6.1
24	6.2

3. A survey of 145 people asked them "Which is the nicest fruit?" Construct a bar chart for the information.

Fruit	People
Apple	35
Orange	30
Banana	10
Kiwifruit	25
Blueberry	40
Grapes	5

4. The table below shows the number of boys and girls in all JHS classes at Winneba Don Bosco B JHS.

SEX	JHS1	JHS2	JHS3
BOYS	14	19	23
GIRLS	20	18	21

Construct a bar chart for the data.

Level 2

1. Find the mode of the data {14, 16, 16, 16, 17, 16, 18}.
2. The ages of the members of a community centre have been listed below: {42, 38, 29, 37, 40, 33, 41}. Calculate the median of the given data.
3. Find the mean, median and mode for the following list of values:13, 18, 13, 14, 13, 16, 14, 21, 13

Level 3

1. Estimate the mean, median and mode for the data set. **AP**

Seconds	Frequency
51 - 55	2
56 - 60	7
61 - 65	8
- 70	4

2. The ages of the 112 people who live on a tropical island are grouped as follows: **P**

Age	Number
0 - 9	20
10 - 19	21
20 - 29	23
30 - 39	16
40 - 49	11
50 - 59	10
60 - 69	7
70 - 79	3
80 - 89	1

   i. Estimate the mean, median and mode for the data.

   ii. Analyse the three central scores (mean, median and mode) calculated and justify why a particular one best represents the data

3. Estimate the mean, median and mode for the data set.

Length (mm)	Frequency
150 - 154	5
155 - 159	2
160 - 164	6
165 - 169	8
170 - 174	9
175 - 179	11
180 - 184	6
185 - 189	3

   Analyse the three central scores (mean, median and mode) calculated and justify why a particular one best represent the data.

4. State and explain, with practical examples two differences between mean and median.

5. Explain, with practical examples the effect of an extreme value on the mean.

Week 23

Learning Indicators:

1. *Use mathematical arguments to support personal choices as well as incorporate the views and perspectives of others to assess and make inferences of data presented in everyday life (including class discussions, school debates, textbooks, school clubs, etc.)*
2. *Develop and execute a project with a team by collecting and analysing data within the school environment and give useful conclusions and recommendations (including the use of appropriate computer applications, e.g., excel).*
3. *Present a project report to your class or at a school forum. Include the use of presentation software such as power point, infographics, etc., and publish the report in a school magazine, school notice board, school social media platforms, etc.*

Theme or Focal Area: Use of mathematical language in making inferences of data

Example:

The table below is the summary of cases and recoveries of coronavirus recorded in various regions in Ghana from March 2020 to 11th August 2022. Use the information from the table to make recommendations to the government on resource allocation to the various regions.

Regions	Cases	Recovered and discharged	Active cases
Ahafo	1,210	1174	1
Ashanti	22,527	22130	8
Bono east	2523	2422	4
Central	5911	5852	17
Eastern	7425	7267	5
Greater Accra	95723	95362	43
North East	407	396	0
Northern	1882	1850	0
Oti	1015	1006	0
Savannah	316	305	7
Upper east	1801	1738	0
Upper west	945	891	10
Volta	6140	6048	2
Western	8655	8573	7
Western north	1162	1138	10
International Travellers	7644	7642	2
Total	**168,457**	**166,872**	**126**

Examples of recommendations and their justifications

Recommendation 1: More resources should be allocated to Greater Accra region and Ashanti region since they recorded the highest cases, i.e., Greater Accra 95723 (56.8%) and Ashanti 22,527 (13.4%).

Recommendation 2: More resources should be allocated to Greater Accra region and Central region since they have the highest number of active cases, i.e., Greater Accra 43 (34%) and Central 17 (13.5%).

The two recommendations are based on the data. Each of the recommendation has a good basis per the data in the table. Students must learn to make such recommendations based on data and accept alternative views of others since it can also be useful.

Real-life Project on Data collection, analysis and presentation

Project 1

- In convenient groups, obtain the West Africa Secondary School Certificate Exams (WASSCE) result of your school for the past five years and analyse it by looking at the overall differences in the performance by years, by programme/courses/subject.
- As part of the analysis, obtain the frequencies and percentages, then draw charts (line, pie, bar etc.) for the data.
- Also, from the analysis make conclusions and give recommendations to the school.

Project 1

- Design Student Course Evaluation Questionnaire and use it to collect data from students in your school.
- Analyse the data and make conclusions and recommendations based of the results.

Publishing Data Findings

Example:

i. Make summaries of your results, conclusions and recommendations of your project and present them using a power point, infographics design, Microsoft word or handwritten to the class or at a mini forum (including school clubs) in the school.

ii. Make oral presentations on the project by explaining the choice of project topic and its relevance, choice of data collection method(s) and the analysis and talk about the challenges faced.

iii. Publish the summary of the findings and recommendations of the project on school notice board or magazines, school social media platforms, etc.

Pedagogical Exemplars

Teacher should Consider the following activities;

Reviewing previous knowledge: Review learners' previous knowledge on data analysis including finding the mode, median and mean.

Problem-based learning: Demonstrate (call a learner to volunteer) how to interpret a given data and make recommendations on the data. Then assign some tasks for learners to do.

Experiential learning: In mixed-gender/ability groups, engage learners to embark on a mini-project within the school environment by obtaining say WASSCE result of the school for the past five years and analyse it by looking at the overall differences in the performance either by years, by programme/courses/subject.

Again, as part of the analysis, they should obtain the frequencies and/or percentages, then draw charts (line, pie, bar etc.) for the data. They should also, from the analysis, make conclusions and give recommendations to the school.

Experiential learning: Learners should publish the findings of their work by making summaries of their results, conclusions and recommendations of the project and present them using a power point, infographics design, Microsoft word or handwritten to the class or at a mini forum (including school clubs) in the school. They may also publish it in a school magazine, school social media platform, etc.

You may consider the following activities to differentiate the lessons:

For below average learners: Provide guided practice in using basic mathematical reasoning to support personal choices and assess data. Offer structured activities where students work in pairs or small groups to analyse simple data sets. Scaffold discussions to help students incorporate the views of others in their assessments.

Assign tasks where students write short explanations or make simple presentations using basic mathematical arguments. Have students work in small groups to collect and analyse data on a familiar topic within the school environment.

For average learners: Teach students to use more advanced mathematical reasoning, including statistical analysis, to support their choices and assess data. Provide opportunities for students to collaborate in teams to plan and execute a project, including collecting and analysing data using computer applications.

Require students to write a detailed project report that includes a thorough analysis of the data collected, conclusions drawn, and recommendations made. Have students present their project findings to their class or at a school forum using presentation software.

For above average learners: Challenge students to use sophisticated mathematical arguments and statistical methods to support their choices and assess complex data sets. Encourage students to think critically and creatively, incorporating diverse perspectives in their assessments.

Assign a project that requires students to design and execute a comprehensive research study, including collecting and analysing data, and presenting their findings in a professional report. Have students present their project findings in a compelling and engaging manner to a wider audience, such as through a school magazine or social media platform.

Key Assessment

Level 4 [All – AP, P & HP]

1. Collect, analyse and present a real-life data (WASCE Result, Sports Records, Enrollment Records, Awards Records, Teacher Employment Records, etc.) from within the school environment.
2. From the findings, make recommendations and give reasons for such recommendations.
3. Work with colleague learners to develop a mini-research project by;
   - i. Developing a questionnaire
   - ii. Collecting data
   - iii. Analysing the data and
   - iv. Presenting the data

   Use an appropriate IT tool to execute any of the activities where necessary.
4. Make a publication of the findings of the project on the school notice, social media platform, etc.

Section Review

This section is for review of all the lessons taught for the last 21, 22 & 23 Weeks. A summary of what the learner should have learnt.

Reflection and key ideas

Types of data

a) Discrete data includes discrete variables that are finite, numeric, countable, and non-negative integers.

b) Continuous data is the unspecified number of possible measurements between two presumed points.

c) Primary data refers to information collected first hand.

d) Secondary data refers to existing data that has been collected by someone else for a different purpose but can be repurposed for new research or analysis.

e) Ungrouped data refers to a raw set of individual observations or values without any categorisation or grouping. Grouped data involves categorising values into intervals or classes.

When dealing with data, know that;

a) Different data sets require different data collection methods if the finding/results of the data are to be accepted by the consumers of the data.

b) We can organise and present data (grouped/ungrouped) using frequency tables, line graphs, pie charts, multiple bar graphs, info graphics, etc., including generating 3D. each of these data presentation methods has its own advantages and disadvantages depending on the audience and the nature of the survey.

Measures of central tendency

a) The mode refers to the value or values that appear most frequently in a dataset.

b) The median, on the other hand, represents the middle value of a dataset when arranged in ascending or descending order.

c) The mean, often referred to as the average, is computed by summing all the values in the dataset and dividing by the total number of observations

d) An extreme data affects the mean

Benefits of embarking on projects on data

a) Acquisition of practical application of concepts learned on data collection, analyses and interpretations.

b) Acquisition of skills in embarking on mini-surveys to collect information on pertinent issues in the school environment and providing solutions which will lead learners into becoming critical thinkers and problem solvers in their latter stages of life.

c) Acquisition of communication/presentation skills, summarising ideas and publishing findings of a study.

Resources

Graph sheets, mathematical sets, computer with data organising software like excel, power point, A4, A3 papers, flip charts, markers, colour pens, Reports from analysed data, manila cards, worksheets, posters, teaching presentations, enquiry project-template, etc.

Sample Rubric to assess leaners project in Week 21

Criteria	Exemplary (4)	Proficient (3)	Basic (2)	Needs Improvement (1)
Research and Selection	Thoroughly researched and selected a highly relevant and effective data collection tool, demonstrating a deep understanding of its features and applicability.	Researched and selected a relevant data collection tool, demonstrating an understanding of its features and applicability.	Researched and selected a data collection tool, but the choice may be somewhat limited or not fully explained.	Limited research or selection of a data collection tool, with little explanation or relevance to the project.
Discussion of Features	Provided a comprehensive discussion of the features of the selected tool, including its design and relevance to the research topic.	Discussed the features of the selected tool, including its design and relevance to the research topic.	Provided a basic discussion of the features of the selected tool, but lacked depth or detail.	Provided a superficial or incomplete discussion of the features of the selected tool.
Validation of Usefulness	Conducted a rigorous validation process, including pilot testing and analysis of results, demonstrating a clear understanding of how to assess the tool's usefulness.	Conducted a validation process, including pilot testing, and discussed the results, showing an understanding of how to assess the tool's usefulness.	Conducted a validation process, but the analysis of results was limited or unclear.	Limited or no validation process conducted, or the analysis of results was missing or incorrect.
Data Collection	Collected quantitative data using the selected tool in a classroom or school community setting, demonstrating excellent organisation and attention to detail.	Collected quantitative data using the selected tool in a classroom or school community setting, with good organisation and attention to detail.	Collected quantitative data using the selected tool, but with some organisational or detail issues.	Collected quantitative data, but the use of the selected tool was unclear or ineffective.

Criteria	Exemplary (4)	Proficient (3)	Basic (2)	Needs Improvement (1)
Presentation of Findings	Presented findings clearly and effectively, including an analysis of the data collected and its implications for the research topic.	Presented findings clearly, with an analysis of the data collected and some discussion of its implications for the research topic.	Presented findings, but the analysis was limited or the implications for the research topic were unclear.	Presented findings, but the analysis was missing or incorrect, and the implications for the research topic were not discussed.
Overall Quality and Creativity	Demonstrated exceptional creativity, originality, and effort in the project, exceeding expectations.	Demonstrated creativity, originality, and effort in the project, meeting expectations.	Demonstrated some creativity and effort in the project, but lacked originality or depth.	Demonstrated little creativity, originality, or effort in the project.

Sample Rubric to assess learners project in Week 23

Criteria	Approaching Proficiency	Proficiency	Highly Proficient
Data Collection and Analysis	Collects WASSCE results for past five years.	Collects and organises WASSCE results for analysis.	Collects and organises detailed WASSCE results for in-depth analysis.
Graphical Representation	Attempts to create basic charts (e.g., bar, pie) for data.	Creates clear and appropriate charts for data analysis.	Creates sophisticated charts, selecting best representations for different data sets.
Analysis and Interpretation	Provides basic analysis of overall performance trends.	Analyses performance trends by program/ course/subject.	Conducts comprehensive analysis, identifying patterns and trends, and provides insightful conclusions.
Statistical Measures	Uses basic statistical measures (e.g., frequencies, percentages).	Utilises statistical measures effectively for analysis.	Applies advanced statistical measures to enhance analysis.
Conclusions and Recommendations	Provides basic conclusions and recommendations.	Presents clear and logical conclusions with relevant recommendations.	Offers insightful conclusions and actionable recommendations based on the analysis.
Presentation	Presents findings in a basic format.	Presents findings in a clear and organised manner.	Presents findings professionally, using visual aids effectively.
Collaboration and Teamwork	Participates minimally in group work.	Collaborates effectively with team members.	Leads and collaborates effectively, ensuring all team members contribute.
Use of Technology	Uses technology for basic data analysis and presentation.	Utilises technology proficiently for analysis and presentation.	Integrates technology innovatively for advanced analysis and presentation.
Overall Quality and Creativity	Demonstrates basic understanding of the project.	Shows proficiency in executing the project.	Demonstrates exceptional quality, creativity, and depth in project execution.

References

1. Perimeter of Rhombus (math-salamanders.com)
2. Prism - Definition, Shape, Types, Formulas, Examples & Diagrams (mathmonks.com)
3. Volume of Prism - Formula, Derivation, Definition, Examples (cuemath.com)

SECTION 9: PROBABILITY OF INDEPENDENT EVENTS

Strand: **Making Sense of and Using Data**

Sub-Strand: Probability/Chance

Learning Outcome: *Determine the sample space for simple and compound probability experiments involving independent events; express the probabilities of given events as fractions, decimals, percentages and solve problems everyday life problems.*

Content Standard: Demonstrate conceptual understanding of simple and compound probability experiments involving two independent events.

INTRODUCTION AND SECTION SUMMARY

Probability is the mathematical study of uncertainty and chance. It plays a crucial role in various fields such as statistics, economics, and finance. Simple probability experiments involve outcomes that are equally likely, such as flipping a coin or rolling a fair die. Compound probability experiments involve multiple stages or events, such as drawing two cards from a deck without replacement. Understanding the probability of independent events is essential for calculating the likelihood of multiple events occurring simultaneously. Independent events are events where the outcome of one event does not affect the outcome of another event. Proficiency in these areas enables students to make informed decisions in situations involving uncertainty. It also provides a foundation for further studies in statistics and data analysis. Probability concepts are also linked to other subjects such as physics and biology, where probability is used to model and analyse random processes. Overall, learning about probability experiments and independent events enhances students' problem-solving skills and critical thinking abilities.

The weeks covered by the section are:

Week 24

1. Simple and compound probability experiments
2. Probability of independent events

SUMMARY OF PEDAGOGICAL EXEMPLARS

To teach the concepts of simple and compound probability experiments, as well as the probability of independent events, the following pedagogical strategies can be employed:

1. **Hands-on Probability Experiments:** Engage students in hands-on activities that involve probability, such as rolling dice, flipping coins, or drawing cards. This can help them understand the concept of probability through concrete examples.
2. **Real-life Scenarios:** Present students with real-life scenarios where probability is involved, such as weather forecasting or sports statistics. This can help them see the relevance of probability in everyday situations.
3. **Problem-solving Activities:** Provide students with problem-solving activities that require them to calculate probabilities. This can help them develop critical thinking and analytical skills.
4. **Collaborative Learning:** Encourage collaborative learning by having students work in groups to solve probability problems. This can help them learn from each other and develop teamwork skills.

5. **Use of Technology:** Utilise technology tools, such as probability calculators or software, to help students calculate probabilities more efficiently and accurately.

6. **Visual Representations:** Use visual representations, such as diagrams or charts, to illustrate probability concepts. This can help students better understand the relationships between different outcomes.

ASSESSMENT SUMMARY

The concepts covered in this section require learners to demonstrate a solid understanding of simple and compound probability experiments, as well as the probability of independent events. Assessments should target levels 2 and 3 of the Depth of Knowledge (DOK) framework to ensure learners have a deep understanding of the concepts and can apply them effectively. A variety of assessment strategies should be used, including:

- Written assessments, such as quizzes and tests, to assess understanding of probability concepts and calculations.
- Experiential learning through practical assessments involving simple and compound probability experiments, allowing learners to apply their knowledge in real-world scenarios.
- Problem-solving tasks that require learners to calculate the probability of independent events, demonstrating their ability to apply probability rules.
- Collaborative & Group activities where learners work together to solve complex probability problems, fostering collaboration and communication skills.
- Real-life scenarios where learners must apply probability concepts to make predictions or decisions, demonstrating their understanding of probability in practical situations.

Week 24

Learning Indicators:

1. *List the elements of the sample space from a simple or compound experiment involving two independent events.*
2. *Determine the probabilities of independent events and express the results as fractions, decimals, percentages and/or ratios.*
3. *Solve everyday life problems involving the probability of two-independent events.*

Theme or Focal Area: Probability of Independent Events (Sample Space)

Probability is a branch of mathematics that deals with the likelihood or chance of an event occurring. It is used to quantify uncertainty and predict the likelihood of different outcomes in situations where the outcome is uncertain. Probability is expressed as a number between 0 and 1, where 0 indicates that the event will not occur, and 1 indicates that the event will definitely occur.

Terminologies relating to the concept of probability

Example

- **i.** Experiment
- **ii.** Random Experiment
- **iii.** Trial
- **iv.** Sample space
- **v.** Event
- **vi.** Equally Likely Events
- **vii.** Exhaustive Events
- **viii.** Favourable Events
- **ix.** Additive Law of Probability

A *random experiment* is a mechanism that produces a definite outcome that cannot be predicted with certainty. The sample space associated with a random experiment is the set of all possible outcomes. An event is a subset of the sample space.

Independent experiments and list the sample space

Example 1.

Two events, A and B, are independent if $P(A \cap B) = P(A) \times P(B)$.

Consider tossing a coin three times in a row. Since each of the throws is independent of the other two, we consider all 8 ($= 2^3$) possible outcomes as equiprobable and assign each the probability of 1/8. Here is the sample space of a sequence of three tosses:

{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}.

Example 2.

A fair die is rolled twice. List the sample space for the experiment.

Solution: Sample space for two dice (outcomes):

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Learning Tasks for Practice

1. Learners determine whether given experiments are independent events or not.
2. Learners list the sample space of a given probability event. Ensure to include simple scenarios as well as intermediate and complex ones to help all the different groups of learners.

Probabilities of independent events

Description:

Probability serves as a powerful tool in understanding and quantifying uncertainty. It enables us to make informed decisions, assess risks, and analyse the likelihood of various outcomes. We will delve into independent events, where the occurrence or outcome of one event does not affect the occurrence or outcome of another event.

Example 1:

Let's suppose there are ten balls in a box. Four balls are Green (G), and six balls are Red (R). If we draw two balls, one at a time, with replacement, find the probability of the following events:

1. Both balls are green.
2. The first ball is red and the second is green.
3. At least one ball is red.

Solution

Let G1 and R1 be the events that the first ball is Green/Red respectively. Similarly, let G2 and R2 be the events that the second ball is Green/Red. Since we are dealing with sampling with replacement so

$P(G1) = P(G2) = \frac{4}{10} = \frac{2}{5}$ and $P(R1) = P(R2) = \frac{6}{10} = \frac{3}{5}$

1. P(both balls are green) = P(G1 and G2) = $P(G1 \cap G2)$. Since the trials are independent, so $P(G1 \cap G2) = P(G1) \times P(G2) = \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$.
2. P(First Red and Second Green) = P(R1 and G2) = $P(R1 \cap G2)$. since the trials are independent, so $P(R1 \cap G2) = P(R1) \times P(G2) = \frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$.
3. We use the fact that P(at least one ball is Red) = 1 – P(both balls are Green). Hence, P(at least one ball is Red) $= 1 - \frac{4}{25} = \frac{21}{25}$.

Example 2:

A poll finds that 72% of Kumasi indigenes consider themselves football fans. If you randomly pick two people from the population, what is the probability the first person is a football fan and the second is as well? That the first one is and the second one isn't?

Solution:

One person being a football fan doesn't have an effect on whether the second randomly selected person is. Therefore, the events are independent, and the probability can be found by multiplying the probabilities together:

First one and second are football fans: $P(A \cap B) = P(A) \cdot P(B) = 0.72 \times 0.72 = 0.5184$. First one is a football fan, the second one isn't: $P(A \cap B) = P(A) \cdot P(B) = 0.72 \times (1 - 0.72) = 0.202$.

In the second part, we multiplied by the complement. As the probability of being a fan is 0.72, then the probability of not being a fan is 1 – 0.72, or 0.28.

Events A and B are independent if the equation $P(A \cap B) = P(A) \times P(B)$ holds true. You can use the equation to check if events are independent; multiply the probabilities of the two events together to see if they equal the probability of them both happening together.

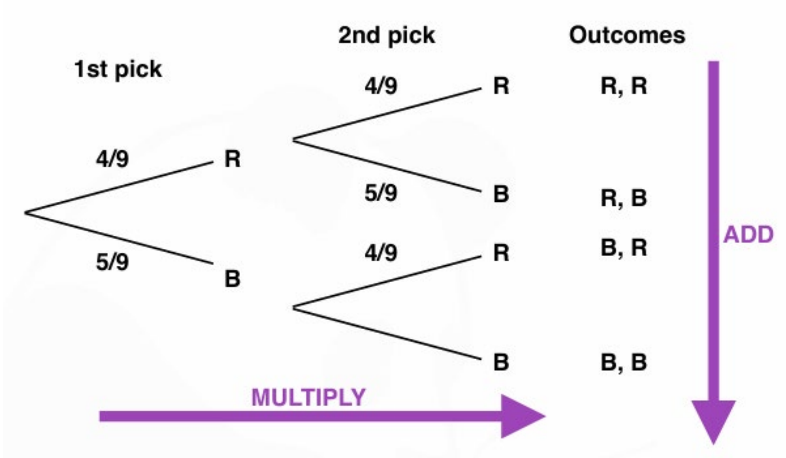
Tree diagrams

A tree diagram is a useful tool to solve probability problems and would make it easier.

Example

A bag has 9 discs — 4 red and 5 blue. A disc is chosen at random, its colour noted and then replaced in the bag. Another disc is then chosen.

Study the tree diagram carefully.



Key ideas

- Probabilities go on the branches and outcomes at the end of each branch.
- The sum of the probabilities on the branches must be 1.
- When going across the tree diagram, we multiply probabilities.

  E.g. $P\ (2\ red\ discs) = P(R, R)$

  $$= P(R) \times P(R)$$
  $$= \frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$$

- When going down the tree diagram, we add the probabilities.

  E.g. $P\ (same\ colour) = P(R, R) + P\ (B, B)$

  $$= P(R) \times P(R) + P(B) \times P(B)$$
  $$= \frac{4}{9} \times \frac{4}{9} + \frac{5}{9} \times \frac{5}{9} = \frac{16}{81} + \frac{25}{81} = \frac{41}{81}$$

Two common questions

1. What is the probability of getting ***only one*** red disc?

   Working: To get one red disc, we could choose a red disc first and then a blue disc or could a blue one and then a red one.

   $= P(\text{only 1 red disc}) = P(R, B) + P(R ,B)$

   $= P(R)\times P(B) + P(B)\times P(R)$

   $= \frac{4}{9} \times \frac{5}{9} + \frac{5}{9} \times \frac{4}{9} = \frac{20}{81} + \frac{20}{81} = \frac{40}{81}$

N.B. This is the same as finding the probability of getting different colours.

2. What is the probability of getting at least one blue disc?

   Working: "At least one blue disc" is the complementary event of "no blue discs".

   "No blue discs" means that two red discs have been chosen

   $$\begin{aligned} P(\text{at least 1 blue disc}) &= 1 - P(\text{no blue discs}) \\ &= 1 - P(\text{2 red discs}) \\ &= 1 - P(R, R) \\ &= 1 - \frac{4}{9} \times \frac{4}{9} \\ &= 1 - \frac{16}{81} \\ &= \frac{65}{81} \end{aligned}$$

N.B. You could also do P(at least 1 blue disc) = P(1 blue disc) + P(2 blue discs)

Learning Tasks for Practice

1. Learners determine the probability of independent events and express the results as fractions, decimals, percentages and/or ratios.
2. Learners use tree diagrams to solve probability of independent events.

Real-life Problems of Probabilities of independent events

Example 1:

A message is transmitted from Node-A to Node-B through three intermediate nodes. The message will be successfully transmitted only if all the intermediate nodes are working. The probability that an intermediate node will fail is 1%. All nodes are independent of each other. What is the probability that you will not successfully transmit the message?

Solution

We first find the probability that you will successfully transmit the message. For successful transmission, we need all nodes to be working. The probability that a node will not fail is

P(Node does not fail) = 1– P(Node fails) = 1 – 0.01 = 0.99.

Since all the nodes are independent so the probability that node 1 AND node 2 AND node 3 do not fail = 0.99 × 0.99 × 0.99 = 0.97

Accordingly, P(message is not successful) = 1 – P(message is successful) = 1-0.97 = 0.03 = 3%.

Example 2:
A retail store sells two types of products: electronics and clothing. The store manager wants to analyse the sales data to understand the relationship between the sales of these two product categories.

The store's sales data for the last month shows the following:

- The probability of a customer buying an electronic product is 0.3.
- The probability of a customer buying a clothing product is 0.4.

Assume that the purchases of electronic and clothing products are independent events.

Question: What is the probability that a randomly selected customer will buy both an electronic product and a clothing product?

Solution:
To solve this problem, we need to use the concept of the probability of independent events.

The probability of two independent events occurring together is the product of their individual probabilities.

Let's define the events:

- E: A customer buys an electronic product
- C: A customer buys a clothing product

Given information:

- P(E) = 0.3 (the probability of buying an electronic product)
- P(C) = 0.4 (the probability of buying a clothing product)

We want to find the probability of a customer buying both an electronic product and a clothing product, which is P(E and C).

Since the purchases of electronic and clothing products are independent events, we can use the multiplication rule for independent events:

$P(E \text{ and } C) = P(E) \times P(C)$ $P(E \text{ and } C) = 0.3 \times 0.4$ $P(E \text{ and } C) = 0.12$

Therefore, the probability that a randomly selected customer will buy both an electronic product and a clothing product is 0.12 or 12%.

Learning Task for Practice

Learners solve real-life problems involving the probability of two-independent events.

Pedagogical Exemplars

Teachers should consider the following activities;

Reviewing previous knowledge: Review learners' previous knowledge on probability. Note the misconceptions that learners may have about the concept and address them immediately or as and when appropriate.

Group & pair activities: In mixed-gender/ability groups, engage learners to discuss and present on the various terminologies relating to the concept of probability.

Whole Class discussions and demonstrations: Have a whole class discussion on conditional probability as a pre-requisite to understanding probability with independent events, and discuss independent experiments including listing the sample space.

Problem-based learning: In small groups, engage learners to solve some problems on determining which given experiments are considered independent events.

Group & pair activities: Using think-pair- share, task learners to solve problems on probability of independent events including real-life problems. Please make accommodations for the various ability groups in the class as you carryout this activity.

For instance; for some of the learners give simple scenarios for them to identify the outcomes of two independent events and list them systematically. Then, other learners should analyse real-life scenarios to identify two independent events and construct the sample space for these events. Then for some other learners, they should evaluate complex scenarios involving multiple independent events and create sample spaces for these scenarios.

Whole Class discussions and demonstrations: Lead the class to discuss the main ideas of the lesson and take the opportunity to demonstrate [or learners volunteer to demonstrate] challenging areas, including resolving all misconceptions.

Individual Task

Present learners with individual worksheets to complete. Alternatively, task learners to create some questions for their colleagues to solve.

Please make some considerations for the various ability groups of learners as they embark on individual tasks: thus include simple tasks where learners calculate simple probabilities for independent events and express the results in simplified fractions or basic percentages. Then include tasks where learners solve practical problems involving the probability of two independent events, expressing the answers as fractions, decimals, and percentages. Then finally, include tasks where learners solve intricate problems involving the probability of two independent events, presenting solutions in various formats, including ratios.

Possible Misconceptions

There are several misconceptions that learners may have regarding the concepts of perimeter and area for kites, parallelograms, rhombuses, and trapeziums. Some common misconceptions include:

1. **More trials lead to more accurate probabilities:** Learners may mistakenly believe that conducting more trials in a probability experiment will always lead to more accurate probabilities. While this can be true in some cases, the accuracy of probabilities also depends on the randomness and representativeness of the trials.
2. **Compound events are always more likely than simple events:** Learners may mistakenly believe that compound events (events that consist of two or more simple events) are always more likely to occur than simple events. However, this is not always the case, as the probability of a compound event depends on the probabilities of its individual components.
3. **Independence implies lack of influence:** Learners may misunderstand the concept of independence in probability and think that if two events are independent, they cannot influence each other. In reality, independence in probability simply means that the outcome of one event does not affect the outcome of the other.

4. **Probability is always expressed as a fraction:** Learners may incorrectly believe that probabilities are always expressed as fractions. While probabilities can be expressed as fractions, they can also be expressed as percentages or decimals.

Key Assessment

Level 2

1. Determine which of the following are examples of independent events.
   a. Rolling a 5 on one die and rolling a 5 on a second die. [ANS: independent]
   b. Randomly picking a cookie from the cookie jar and picking a jack from a deck of cards. [ANS: independent]
   c. Winning a hockey game and scoring a goal. [ANS: not independent or dependent]

2. Determine which of the following are examples of independent events.
   a. Choosing an 8 from a deck of cards, replacing it, and choosing a face card. [ANS: independent]
   b. Going to the beach and bringing an umbrella. [ANS: not independent or dependent]
   c. Getting gasoline for your car and getting diesel fuel for your car. [ANS: independent]

Level 2

1. A fair die is rolled once. List the sample space.
2. Two coins are tossed once. List the sample space for the experiment.
3. A dice and a coin are tossed once. List the sample space for the experiment.
4. Explain, with relevant examples, the meaning of probability of independent events.

Level 3

1. Two (2) cards are chosen from a deck of cards. The first card is replaced before choosing the second card. What is the probability that they both will be face cards?
2. If the probability of receiving at least 1 piece of mail on any particular day is 22%, what is the probability of not receiving any mail for 3 days in a row?
3. Johnathan is rolling 2 dice and needs to roll an 11 to win the game he is playing. What is the probability that Johnathan wins the game?
4. While driving to work, Sarah passes through two sets of traffic lights. The probability the first set is green is 0.3 and the probability the second is green is 0.4. Draw a tree diagram to represent this.

   Find the probability that:
   i. neither set of lights is green
   ii. only one set of traffic lights is green
   iii. at least one set of traffic lights is green

5. A dollar-bill changer on a snack machine was tested with 100 $1 bills. Twenty-five of the bills were found to be counterfeit, but only one was accepted by the machine. However, six of the legal bills were rejected. Draw a chart to show the number of legal and counterfeit bills that were accepted or rejected.
   i. What is the probability that a bill will be rejected given that it is legal?
   ii. What is the probability that the counterfeit bill is accepted?

6. Joshua and his friend bought movie tickets and ate a pizza. Can you tell us whether it is a dependent event or an independent event?
7. Alexander tossed a coin and rolled a single 6-sided die. Find the probability of landing head side of the coin and rolling a 3 on the die.
8. Researchers collected a random sample of people and documented their eye colour.

Eye colour	Probability
Blue	0.33
Brown	0.39
Hazel	0.16
Green	0.12

If the researchers randomly select two people from their sample, what is the probability that both will have hazel eyes?

8. A coin and a die are tossed. Calculate the probability of getting tails and a 5.
9. In Tania's homeroom class, 9% of the students were born in March and 40% of the students have a blood type of O+. What is the probability of a student chosen at random from Tania's homeroom class being born in March and having a blood type of O+ if the two events are independent events? [ANS:0.036 or 3.6%]
10. What is the probability of tossing 2 coins in any order one after the other and getting 1 head and 1 tail? [ANS: 1/2 or 50%]

Level 3

1. Joseph and David are playing with cards in a pack, there are 52 cards. Joseph drew a card at random Then replaced it, and drew another card. Then he asked David what is the probability of drawing a queen followed by a king? [ANS: 1/169]
2. A juggler has seven red, five green, and four blue balls. During his stunt, he accidentally drops a ball and then picks it up. As he continues, another ball falls. What is the probability that the first ball that was dropped is blue, and the second ball is green? [ANS: 5/64]
3. In a survey, a company found that 6 out of 10 people eat pizza. If three people are chosen at random with replacement, what is the probability that all 3 people eat pizza? [ANS: 0.216]

Section Review

This section is for review of all the lessons taught for week 24. A summary of what the learner should have learnt.

Reflection and key ideas

A random experiment is a mechanism that produces a definite outcome that cannot be predicted with certainty. The sample space associated with a random experiment is the set of all possible outcomes. An event is a subset of the sample space.

Two events, A and B, are independent if $P(A \cap B) = P(A)\ x\ P(B)$.

Events A and B are independent if the equation $P(A \cap B) = P(A)\ x\ P(B)$ holds true. You can use the equation to check if events are independent; multiply the probabilities of the two events together to see if they equal the probability of them both happening together.

Independent probability refers to the occurrence of one event not affecting the probability of another event. Examples;

i. Riding a bike and watching your favourite movie on a laptop.

ii. Studying hard for the exam and hitting six in cricket.

Resources

1. Memory cards, etc.
2. Graph sheets
3. Mathematical sets
4. Computer with data organising software like excel, power point, etc.,
5. A4 and A3 papers
6. Flip charts
7. Markers
8. Colour pens, etc.
9. Technology tools such as computer, mobile phone etc.
10. Sample questionnaire, interview guides, observation guide, computer-assisted telephone interview guide, mail survey, computer-assisted personal interview guide.
11. Manila cards

References

1. Perimeter of Rhombus (math-salamanders.com)
2. Prism - Definition, Shape, Types, Formulas, Examples & Diagrams (mathmonks.com)
3. Volume of Prism - Formula, Derivation, Definition, Examples (cuemath.com)
4. Independent_Events_and_Tree_Diagrams_LESSON.pdf (mathspanda.com)

ACKNOWLEDGEMENTS

Special thanks to Professor Edward Appiah, Director-General of the National Council for Curriculum and Assessment (NaCCA) and all who contributed to the successful writing of the Teacher Manuals for the new Senior High School (SHS), Senior High Technical School (SHTS) and Science Technology, Engineering and Mathematics (STEM) curriculum.

The writing team was made up of the following members:

NaCCA Team	
Name of Staff	**Designation**
Matthew Owusu	Deputy Director-General, Technical Services
Reginald Quartey	Ag. Director, Curriculum Development Directorate
Anita Cordei Collison	Ag. Director, Standards, Assessment and Quality Assurance Directorate
Rebecca Abu Gariba	Ag. Director, Corporate Affairs
Anthony Sarpong	Director, Standards, Assessment and Quality Assurance Directorate
Uriah Kofi Otoo	Senior Curriculum Development Officer (Art and Design Foundation & Studio)
Nii Boye Tagoe	Senior Curriculum Development Officer (History)
Juliet Owusu-Ansah	Senior Curriculum Development Officer (Social Studies)
Eric Amoah	Senior Curriculum Development Officer (General Science)
Ayuuba Sullivan Akudago	Senior Curriculum Development Officer (Physical Education & Health)
Godfred Asiedu Mireku	Senior Curriculum Development Officer (Mathematics)
Samuel Owusu Ansah	Senior Curriculum Development Officer (Mathematics)
Thomas Kumah Osei	Senior Curriculum Development Officer (English)
Godwin Mawunyo Kofi Senanu	Assistant Curriculum Development Officer (Economics)
Joachim Kwame Honu	Principal Standards, Assessment and Quality Assurance Officer
Jephtar Adu Mensah	Senior Standards, Assessment and Quality Assurance Officer
Richard Teye	Senior Standards, Assessment and Quality Assurance Officer
Nancy Asieduwaa Gyapong	Assistant Standards, Assessment and Quality Assurance Officer
Francis Agbalenyo	Senior Research, Planning, Monitoring and Evaluation Officer
Abigail Birago Owusu	Senior Research, Planning, Monitoring and Evaluation Officer
Ebenezer Nkuah Ankamah	Senior Research, Planning, Monitoring and Evaluation Officer

NaCCA Team	
Name of Staff	**Designation**
Joseph Barwuah	Senior Instructional Resource Officer
Sharon Antwi-Baah	Assistant Instructional Resource Officer
Dennis Adjasi	Instructional Resource Officer
Samuel Amankwa Ogyampo	Corporate Affairs Officer
Seth Nii Nartey	Corporate Affairs Officer
Alice Abbew Donkor	National Service Person

Subject	Writer	Designation/Institution
Home Economics	Grace Annagmeng Mwini	Tumu College of Education
	Imoro Miftaw	Gambaga Girls' SHS
	Jusinta Kwakyewaa (Rev. Sr.)	St. Francis SHTS
Religious Studies	Dr. Richardson Addai-Mununkum	University of Education Winneba
	Dr. Francis Opoku	Valley View University College
	Aransa Bawa Abdul Razak	Uthmaniya SHS
	Godfred Bonsu	Prempeh College
RME	Anthony Mensah	Abetifi College of Education
	Joseph Bless Darkwa	Volo Community SHS
	Clement Nsorwineh Atigah	Tamale SHS
Arabic	Dr. Murtada Mahmoud Muaz	AAMUSTED
	Dr. Abas Umar Mohammed	University of Ghana
	Mahey Ibrahim Mohammed	Tijjaniya Senior High School
French	Osmanu Ibrahim	Mount Mary College of Education
	Mawufemor Kwame Agorgli	Akim Asafo SHS
Performing Arts	Dr. Latipher Osei Appiah-Agyei	University of Education Winneba
	Desmond Ali Gasanga	Ghana Education Service
	Chris Ampomah Mensah	Bolgatanga SHS, Winkogo
Art and Design Studio and Foundation	Dr. Ebenezer Acquah	University for Education Winneba
	Seyram Kojo Adipah	Ghana Education Service
	Dr. Jectey Nyarko Mantey	Kwame Nkrumah University of Science and Technology
	Yaw Boateng Ampadu	Prempeh College
	Kwame Opoku Bonsu	Kwame Nkrumah University of Science and Technology
	Dzorka Etonam Justice	Kpando Senior High Sschool

Subject	**Writer**	**Designation/Institution**
Applied Technology	Dr. Sherry Kwabla Amedorme	AAMUSTED
	Dr. Prosper Mensah	AAMUSTED
	Esther Pokuah	Mampong Technical College of Education
	Wisdom Dzidzienyo Adzraku	AAMUSTED
	Kunkyuuri Philip	Kumasi SHTS
	Antwi Samuel	Kibi Senior High School
	Josiah Bawagigah Kandwe	Walewale Technical Institute
	Emmanuel Korletey	Benso Senior High Technical School
	Isaac Buckman	Armed Forces Senior High Technical School
	Tetteh Moses	Dagbon State Senior High School
	Awane Adongo Martin	Dabokpa Technical Institute
Design and Communication Technology	Gabriel Boafo	Kwabeng Anglican SHTS
	Henry Agmor Mensah	KASS
	Joseph Asomani	AAMUSTED
	Kwame Opoku Bonsu	Kwame Nkrumah University of Science and Technology
	Dr. Jectey Nyarko Mantey	Kwame Nkrumah University of Science and Technology
	Dr. Ebenezer Acquah	University for Education Winneba
Business Studies	Emmanuel Kodwo Arthur	ICAG
	Dr. Emmanuel Caesar Ayamba	Bolgatanga Technical University
	Ansbert Baba Avole	Bolgatanga Senior High School, Winkogo
	Faustina Graham	Ghana Education Service, HQ
	Victoria Osei Nimako	SDA Senior High School, Akyem Sekyere
Agriculture	Dr. Esther Fobi Donkoh	University of Energy and Natural Resources
	Prof. Frederick Adzitey	University for Development Studies
	Eric Morgan Asante	St. Peter's Senior High School
Agricultural Science	David Esela Zigah	Achimota School
	Prof. J.V.K. Afun	Kwame Nkrumah University of Science and Technology
	Mrs. Benedicta Carbiliba Foli	Retired, Koforidua Senior High Technical School

Subject	Writer	Designation/Institution
Government	Josephine Akosua Gbagbo	Ngleshie Amanfro SHS
	Augustine Arko Blay	University of Education Winneba
	Samuel Kofi Adu	Fettehman Senior High School
Economics	Dr. Peter Anti Partey	University of Cape Coast
	Charlotte Kpogli	Ho Technical University
	Benjamin Agyekum	Mangoase Senior High School
Geography	Raymond Nsiah Asare	Methodist Girls' High School
	Prof. Ebenezer Owusu Sekyere	University for Development Studies
	Samuel Sakyi Addo	Achimota School
History	Kofi Adjei Akrasi	Opoku Ware School
	Dr. Anitha Oforiwah Adu-Boahen	University of Education Winneba
	Prince Essiaw	Enchi College of Education
Ghanaian Language	David Sarpei Nunoo	University of Education Winneba, Ajumako
	Catherine Ekua Mensah	University of Cape Coast
	Ebenezer Agyemang	Opoku Ware School
Physical Education and Health	Paul Dadzie	Accra Academy
	Sekor Gaveh	Kwabeng Anglican Senior High Technical School
	Anthonia Afosah Kwaaso	Junkwa Senior High School
	Mary Aku Ogum	University of Cape Coast
Social Studies	Mohammed Adam	University of Education Winneba
	Simon Tengan	Wa Senior High Technical School
	Jemima Ayensu	Holy Child School
Computing and Information Communication Technology (ICT)	Victor King Anyanful	OLA College of Education
	Raphael Dordoe Senyo	Ziavi Senior High Technical School
	Kwasi Abankwa Anokye	Ghana Education Service, SEU
	Millicent Heduvor	STEM Senior High School, Awaso
	Dr. Ephriam Kwaa Aidoo	University for Education Winneba
	Dr. Gaddafi Abdul-Salaam	Kwame Nkrumah University of Science and Technology
English Language	Esther O. Armah	Mangoase Senior High School
	Kukua Andoh Robertson	Achimota School
	Alfred Quaittoo	Kaneshie Senior High Technical School
	Benjamin Orrison Akrono	Islamic Girls' Senior High School
	Fuseini Hamza	Tamale Girls' Senior High School

Subject	Writer	Designation/Institution
Intervention English	Roberta Emma Amos-Abanyie	Ingit Education Consult
	Perfect Quarshie	Mawuko Girls Senior High School
	Sampson Dedey Baidoo	Benso Senior High Technical School
Literature-in-English	Blessington Dzah	Ziavi Senior High Technical School
	Angela Aninakwah	West African Senior High School
	Juliana Akomea	Mangoase Senior High School
General Science	Dr. Comfort Korkor Sam	University for Development Studies
	Saddik Mohammed	Ghana Education Service
	Robert Arhin	SDA SHS, Akyem Sekyere
Chemistry	Ambrose Ayikue	St. Francis College of Education
	Awumbire Patrick Nsobila	Bolgatanga SHS, Winkogo
	Bismark Tunu	Opoku Ware School
	Gbeddy Nereus Anthony	Ghanata Senior High School
Physics	Dr. Linus Labik	Kwame Nkrumah University of Science and Technology
	Henry Benyah	Wesley Girls High School
	Sylvester Affram	Kwabeng Anglican SHS
Biology	Paul Beeton Damoah	Prempeh College
	Maxwell Bunu	Ada College of Education
	Ebenezer Delali Kpelly	Wesley Girls' SHS
	Doris Osei-Antwi	Ghana National College
Mathematics	Edward Dadson Mills	University of Education Winneba
	Zacharia Abubakari Sadiq	Tamale College of Education
	Collins Kofi Annan	Mando SHS
Additional Mathematics	Dr. Nana Akosua Owusu-Ansah	University of Education Winneba
	Gershon Mantey	University of Education Winneba
	Innocent Duncan	KNUST SHS
Intervention Mathematics	Florence Yeboah	Assin Manso SHS
	Mawufemor Adukpo	Ghanata SHS
	Jemima Saah	Winneba SHS
Robotics	Dr. Eliel Keelson	Kwame Nkrumah University of Science and Technology
	Dr. Nii Longdon Sowah	University of Ghana
	Isaac Nzoley	Wesley Girls High School

Subject	Writer	Designation/Institution
Engineering	Daniel K. Agbogbo	Kwabeng Anglican SHTS
	Prof. Abdul-Rahman Ahmed	Kwame Nkrumah University of Science and Technology
	Valentina Osei-Himah	Atebubu College of Education
Aviation and Aerospace Engineering	Opoku Joel Mintah	Altair Unmanned Technologies
	Sam Ferdinand	Afua Kobi Ampem Girls' SHS
Biomedical Science	Dr. Dorothy Yakoba Agyapong	Kwame Nkrumah University of Science and Technology
	Jennifer Fafa Adzraku	Université Libre de Bruxelles
	Dr. Eric Worlawoe Gaba	Br. Tarcisius Prosthetics and Orthotics Training College
Manufacturing Engineering	Benjamin Atribawuni Asaaga	Kwame Nkrumah University of Science and Technology
	Dr. Samuel Boahene	Kwame Nkrumah University of Science and Technology
	Prof Charles Oppon	Cape Coast Technical University
Spanish	Setor Donne Novieto	University of Ghana
	Franklina Kabio Danlebo	University of Ghana
	Mishael Annoh Acheampong	University of Media, Art and Communication
Assessment	Benjamin Sundeme	St. Ambrose College of Education
	Dr. Isaac Amoako	Atebubu College of Education
Curriculum Writing Guide Technical Team	Paul Michael Cudjoe	Prempeh College
	Evans Odei	Achimota School