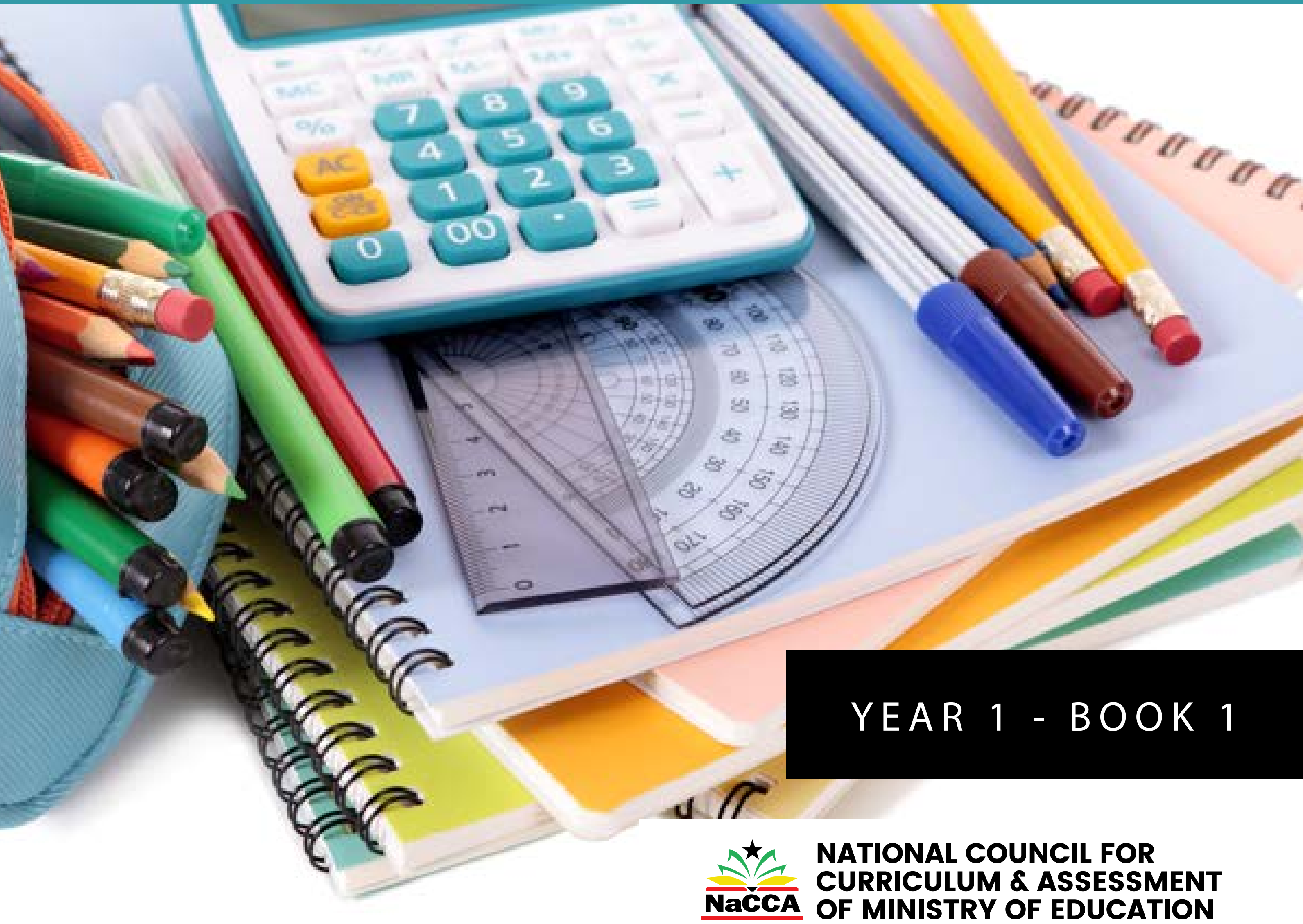
MINISTRY OF EDUCATION

Mathematics

TEACHER MANUAL



YEAR 1 - BOOK 1

NATIONAL COUNCIL FOR CURRICULUM & ASSESSMENT OF MINISTRY OF EDUCATION

MINISTRY OF EDUCATION

REPUBLIC OF GHANA

Mathematics

Teacher Manual

Year One - Book One

NaCCA
NATIONAL COUNCIL FOR
CURRICULUM & ASSESSMENT
OF MINISTRY OF EDUCATION

MATHEMATICS TEACHERS MANUAL

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INTRODUCTION

The National Council for Curriculum and Assessment (NaCCA) has developed a new Senior High School (SHS), Senior High Technical School (SHTS) and Science, Technology, Engineering and Mathematics (STEM) Curriculum. It aims to ensure that all learners achieve their potential by equipping them with 21st Century skills, competencies, character qualities and shared Ghanaian values. This will prepare learners to live a responsible adult life, further their education and enter the world of work.

This is the first time that Ghana has developed an SHS Curriculum which focuses on national values, attempting to educate a generation of Ghanaian youth who are proud of our country and can contribute effectively to its development.

This Teacher Manual for Mathematics covers all aspects of the content, pedagogy, teaching and learning resources and assessment required to effectively teach Year One of the new curriculum. It contains this information for the first 13 weeks of Year One, with the remaining 11 weeks contained within Book Two. Teachers are therefore to use this Teacher Manual to develop their weekly Learning Plans as required by Ghana Education Service.

Some of the key features of the new curriculum are set out below.

Learner-Centred Curriculum

The SHS, SHTS, and STEM curriculum places the learner at the center of teaching and learning by building on their existing life experiences, knowledge and understanding. Learners are actively involved in the knowledge-creation process, with the teacher acting as a facilitator. This involves using interactive and practical teaching and learning methods, as well as the learner's environment to make learning exciting and relatable. As an example, the new curriculum focuses on Ghanaian culture, Ghanaian history, and Ghanaian geography so that learners first understand their home and surroundings before extending their knowledge globally.

Promoting Ghanaian Values

Shared Ghanaian values have been integrated into the curriculum to ensure that all young people understand what it means to be a responsible Ghanaian citizen. These values include truth, integrity, diversity, equity, self-directed learning, self-confidence, adaptability and resourcefulness, leadership and responsible citizenship.

Integrating 21st Century Skills and Competencies

The SHS, SHTS, and STEM curriculum integrates 21st Century skills and competencies. These are:

- **Foundational Knowledge:** Literacy, Numeracy, Scientific Literacy, Information Communication and Digital Literacy, Financial Literacy and Entrepreneurship, Cultural Identity, Civic Literacy and Global Citizenship
- **Competencies:** Critical Thinking and Problem Solving, Innovation and Creativity, Collaboration and Communication
- **Character Qualities:** Discipline and Integrity, Self-Directed Learning, Self-Confidence, Adaptability and Resourcefulness, Leadership and Responsible Citizenship

Balanced Approach to Assessment - not just Final External Examinations

The SHS, SHTS, and STEM curriculum promotes a balanced approach to assessment. It encourages varied and differentiated assessments such as project work, practical demonstration, performance assessment, skills-based assessment, class exercises, portfolios as well as end-of-term examinations and final external assessment examinations. Two levels of assessment are used. These are:

o Internal Assessment (30%) – Comprises formative (portfolios, performance and project work) and summative (end-of-term examinations) which will be recorded in a school-based transcript.

o External Assessment (70%) – Comprehensive summative assessment will be conducted by the West African Examinations Council (WAEC) through the WASSCE. The questions posed by WAEC will test critical thinking, communication and problem solving as well as knowledge, understanding and factual recall.

The split of external and internal assessment will remain at 70/30 as is currently the case. However, there will be far greater transparency and quality assurance of the 30% of marks which are school-based. This will be achieved through the introduction of a school-based transcript, setting out all marks which learners achieve from SHS 1 to SHS 3. This transcript will be presented to universities alongside the WASSCE certificate for tertiary admissions.

An Inclusive and Responsive Curriculum

The SHS, SHTS, and STEM curriculum ensures no learner is left behind, and this is achieved through the following:

- Addressing the needs of all learners, including those requiring additional support or with special needs. The SHS, SHTS, and STEM curriculum includes learners with disabilities by adapting teaching and learning materials into accessible formats through technology and other measures to meet the needs of learners with disabilities.
- Incorporating strategies and measures, such as differentiation and adaptative pedagogies ensuring equitable access to resources and opportunities for all learners.
- Challenging traditional gender, cultural, or social stereotypes and encouraging all learners to achieve their true potential.
- Making provision for the needs of gifted and talented learners in schools.

Social and Emotional Learning

Social and emotional learning skills have also been integrated into the curriculum to help learners to develop and acquire skills, attitudes, and knowledge essential for understanding and managing their emotions, building healthy relationships and making responsible decisions.

Philosophy and vision for each subject

Each subject now has its own philosophy and vision, which sets out why the subject is being taught and how it will contribute to national development. The Philosophy and Vision for Mathematics is:

Philosophy: Every learner can develop their potential in mathematics through creative and innovative ways to become lifelong learners, apply mathematical skills and competencies to solve everyday problems, further their education and/or proceed to the world of work.

Vision: Trained mathematically enthusiastic learners who are highly interested in the subject and are capable of reasoning, modelling, representing and making use of mathematical tools and technology to solve problems in real life, further their studies and/or proceed to the world of work.

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SCOPE AND SEQUENCE

Mathematics Summary

S/N	STRAND	SUB-STRAND	SHS1			SHS2			SHS3		
			CS	LO	LI	CS	LO	LI	CS	LO	LI
1	Numbers for everyday life	Real number and Numeration system	3	3	8	2	2	5	-	-	-
		Proportional reasoning	2	2	4	2	2	4	2	2	4
2	Algebraic Thinking	Applications of expressions, equations and inequalities	2	3	6	1	2	4	-	-	-
		Patterns and relationships	2	2	4	1	1	5	1	1	3
3	Geometry around us	Spatial sense	1	1	5	1	1	4	2	2	7
		Measurement	3	3	8	3	3	8	1	1	2
4	Making sense of and using data	Statistical reasoning and its application in real life	3	3	8	3	3	7	2	2	4
		Chance	1	1	3	1	1	2	1	1	3
Total			**17**	**18**	**46**	**14**	**15**	**39**	**9**	**9**	**22**

Overall Totals (SHS 1 – 3)

Content Standards	**40**
Learning Outcomes	**42**
Learning Indicators	**107**

SECTION 1: NUMBER SETS

Strand: **Number for Everyday life**

Sub-Strand: Real Number and Numeration System (RNNS)

Learning Outcomes:

1. *Apply the relationship and differences between the set of rational and irrational numbers and use these to solve problems.*
2. *Analyse and solve real world problems involving union, intersection and complements and their applications to three sets problems using simple surveys.*

Content Standards:

1. Demonstrate knowledge and understanding of the real number system and the operations of the various subsets.
2. Demonstrate knowledge and understanding of the real number system with respect to the concepts and vocabulary of sets, establish their relationships and carry out simple surveys using the properties of sets.

INTRODUCTION AND SECTION SUMMARY

Welcome to the SHS1 Teachers' Manual for Mathematics, which introduces teachers and students to foundational mathematical concepts. In this section, we shall be empowered to effectively audit the entry behaviours of SHS1 students in mathematics and, thereby, employ interactive strategies to link their background to the real number system, properties and relationships among its subsets and their applications to advanced mathematical explorations for lifelong learning.

The section comprises three weeks. Week 1 deals with sets of real numbers (natural, whole, integers, rational, and irrational) and their applications to real life. In Week 2, students will explore the properties of operations, such as closure, commutative, associative, distributive, identity and inverse, among others. Finally, in Week 3, students will be given opportunities to establish relationships among sets of numbers and link these to three sets problems and their applications in their daily lives.

This concise summary seeks to audit and empower teachers with the core concepts of each week's lesson, providing a clear progression from the fundamentals of numbers and sets to more complex set relationships and problem-solving techniques. These foundational insights will serve as a strong basis for their further exploration in algebra, geometry, statistics, and beyond, necessary for mathematical proficiency and human development.

The following weeks are considered in section 1:

Week 1: Sets of Real Numbers

1. Review of and representation of the real number system (natural, whole, integers, rational, irrational).
2. Visual representation and ordering real numbers on the number line.

Week 2: Properties of Operation and Subsets

1. Closure, commutative, associative, distributive
2. Identity and inverse

Week 3: Relationship among Three Sets of Items:

1. Representing 3-set relationships with Venn diagrams
2. Set operations with three sets (union, intersection)
3. De Morgan's laws for three sets

SUMMARY OF Pedagogical Exemplars

Effective teaching of the Mathematics necessitates a dynamic approach that caters for diverse learning needs, fostering understanding, application, and extension of mathematical concepts. On teaching strategies and differentiation, teachers should employ a combination of instructional methods, including interactive approaches, collaborative activities, and hands-on exercises, to engage learners actively. This section, therefore, requires hands-on activities where learners are encouraged to adopt varied instructional strategies, including differentiation, to accommodate the diverse learning styles and abilities within the classroom. Learners should be offered varied and appropriate opportunities to work in teams to find solutions to assigned tasks. Hence, a mixture of inquiry-based learning, concept mapping, number line representations, group discussions, technology integrations, real-world connections, cooperative learning, and scaffolded instruction, will be appropriate.

Note: for the gifted and talented students, include strategies for mathematical induction where learners prove properties of natural numbers including transitivity and proofs of irrationality of numbers like $\sqrt{2}$, π,etc.

ASSESSMENT SUMMARY

The concepts under this section require learners to demonstrate conceptual understanding, including their real-life applications. Hence, the assessments should largely cover levels 1–4 of the DOK. Again, teachers should employ a variety of formative assessment (assessment for- and as-learning) strategies such as oral/written presentations, pair-tasks, reports, home tasks, etc. to gather information about learners' progress and give prompt feedback to them. Specifically, teachers should conduct the following assessments and record the performances of learners for continuous assessment records.

1. Class exercises (including individual worksheets) after each lesson.
2. Homework
3. Scores on practical group activities

Assessment strategies and how to use them during teaching and learning.

Consider the following assessment strategies and use the results to provide targeted feedback from students you consider AP, P and HP and identify areas where additional support is needed.

L2: Think-pair-share: Guide learners to reflect individually on a question or concept (classify $\left\{4, -3, 0.5, \sqrt{3}, \frac{2}{3}, 75\%\right\}$ under the real number system), then discuss it with the peer, and share their thoughts with the entire class. This strategy promotes peer-to-peer learning and allows for the assessment of individual understanding.

L3: **Homework and assignments:** Give learners tasks on the following to take away and submit after a few days or the next lesson. Review learners' completed homework or assignments and provide immediate feedback for improvement-

a. Use a number line to distinguish rational from irrational numbers.
b. Justify why $\sqrt{2}$ is not a rational number using number line.
c. Establish a set of numbers that includes 5 members each belonging to the various subsets of the real number system.

L2: **Quizzes and tests**: Teachers can give short quizzes that evaluate learners' understanding of specific content or skills covered in the teaching and learning processes. E.g. "Identify the property of operation in $3 \times (4 + 5) = (3 \times 4) + (3 \times 5)$ and explain what it means in mathematics.

L4: **Projects**: Teachers to group learners for a meaningful and comprehensive task or assignment that allows them to apply their knowledge and skills, fostering critical thinking and creativity while providing opportunities for feedback. E.g. Each group to carry out a simple survey in their community/ school to establish areas where real numbers are mostly applied in their everyday interactions and present your results in class using tables. For example, consider utility bills, telephone numbers, salaries, etc.

Real number	***Area of application***	***Number of persons contacted***
Natural	*Phone and index numbers*	
Whole	*Water bill, School fees, …*	
Integers	*Thermometer values*	
Rational	*Sharing of items*	
Irrational	*Speedometer of cars*	

WEEK 1: THE REAL NUMBER SYSTEM

Learning Indicators:

1. *Develop the real number system using the closure property.*
2. *Distinguish between rational and irrational numbers using conversion of common to decimal fractions and solve related problems.*

Theme or Focal Area 1: Establishing the Set of Real Numbers Using the Closure Property

Definition of key Concepts:

Guide learners to establish the meaning of key concepts in the section. These include

1. **Natural Numbers:** Natural numbers are known as counting numbers that contain the positive numbers from 1 to infinity. The set of natural numbers is denoted as **"N"** with the set $N = \{1, 2, 3, 4, 5, \ldots\}$
2. **Whole Numbers:** $W = \{0,1, 2, 3, 4, 5, \ldots\}$
3. **Integers:** $Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
4. **Rational Numbers:** denoted by **"Q"** with the set $\{\ldots, \frac{2}{5}, 0.4, 40\%, 0.3333.., -\frac{3}{7}, 7, -123, \ldots\}$
5. **Irrational Numbers:** it is represented by the symbol $Q^1 = \{\ldots, \sqrt{2}, \sqrt{3}, \sqrt{8}, \pi, \ldots\}$
6. **Real Numbers:** The combination of rational numbers and irrational numbers

Learning Task for Practice 1:

1) Investigate real life application of real numbers
2) Classify the subsets of real numbers using diagrams, flow charts, number lines, etc.
3) Differentiate rational from irrational numbers using recurring and non-recurring decimals
4) Investigate subsets of the real number system using visual aids

1. **Teachers guide learners to investigate real life application of real numbers**

Investigate real life situation(s) where each of the subsets can be used. i.e. Integers are used in temperature reading, medicine, agriculture, computing, business, food and cosmetic industry, energy sector, agriculture, automobile, garment production, medicine, etc.

Example 1. The set of natural numbers is always less than the set of whole numbers. **True/False**

Example 2. Real numbers can be used to investigate everyday life activities. These include

1. For measuring airspeed, rainfall, wind speed and distance.
2. In insurance policies.
3. In Medical instruments and for checking heartbeat rate.
4. To check fuel amount and vehicle driving instruments.
5. In ticket number, house number, index numbers of students, mobile phone number, etc.
6. Computer programming and usage.
7. Utility bills: power, water, telephone, TV, etc.

8. etc.

Learning Task for Practice 2:

1. Students to classify the subsets of real numbers using diagrams, flow charts, number lines, etc.:

   a. Limit content expectation to enumerating the various subsets of the real number system and its inter connections.

   b. Extend content expectation to include classifying the various subsets under rational and irrational number and the applications to their daily lives.

   c. Finally, extend content expectation to include classifying the various subsets under rational and irrational numbers (using visual aids) and the applications to their daily lives,

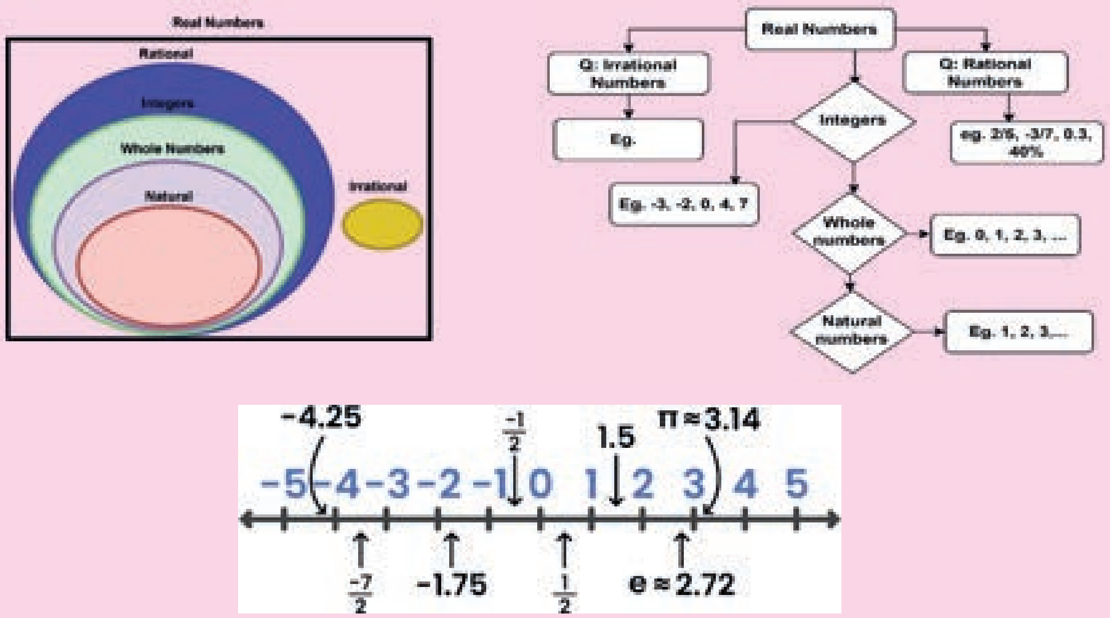


Figure 1: **visuals classification of the real number system**

2. Student to establish the subsets of the real number system using visual aids and investigate them through the closure property.

Study the diagrams above, and

1. enumerate the various subsets of the set of real numbers,
2. state their relationship to each other.
3. investigate the closure property of the subsets using the four operations.

Closure:

Investigating the closure property of whole numbers:

Example 1:

$5 + 8 = 13 \in N$;

$5 - 8 = -3 \in N$;

$5 \times 8 = 40 \in N$;

$4 \div 8 = \frac{1}{2} \notin N$

Closure property of whole numbers	
W + W = Whole numbers	Closed
W × **W = Whole numbers**	Closed
W − **W = Not always a whole number**	Not closed
W ÷ **W = Not always a whole number**	Not closed

Investigating the closure property of integers:

Example 2:

$5 + 8 = 13 \in N$;

$5 - 8 = -3 \in N$;

$5 \times 8 = 40 \in N$;

$4 \div 8 = \frac{1}{2} \notin N$

Closure property of integers	
Integer + Integer = Integer	Closed
Integer - Integer = Integer	Closed
Integer × **Integer = Integer**	Closed
Integer ÷ **Integer = not always an Integer**	Not closed

Investigating the closure property of real numbers:

Closure Property of Real Numbers	
Real number + Real number = Real number	Closed
Real number − Real number = Real number	Closed
Real number × **Real number = Real number**	Closed
Real number÷ Real number = Not always a real number	Closed only under non-zero division

Pedagogical Exemplars

A blend of the following pedagogical exemplars will be considered.

1. **Problem-based learning;** Talk for Learning; Experiential learning; and Group work/ collaborative learning.

   In mixed ability or convenience grouping, students use verbal discussions to establish **the various subsets of the real number system** through the closure property,

   Note: Accept, and redirect appropriately, related concepts that are not mentioned here. e.g. even and odd, prime and composite, triangular numbers, etc.

2. **Problem-based learning:** Here, students establish the relationship between and among the subsets of real numbers and make connections to real life problems.

3. **Using think-pair-share:** Initiate activities to help learners, individually, in pairs and in groups, establish **whether or not the various subsets of the real numbers are closed** with respect to ***addition*** (+), ***subtraction*** (−), ***multiplication*** (×), **and *division*** (÷).

   Encourage self-confidence, diversity and leadership in achieving the conceptual understanding of real numbers.

Reflective practice: Initiate activities that will lead the learner to reflect on the key concepts in the lesson.

Theme/focal area(s) 2: Distinguishing between rational and irrational numbers and their applications to the real world

Definition of key concepts

Rational numbers: Numbers of the form $\frac{a}{b}$, where $b \neq 0$. e.g. $\frac{2}{5} = 0.4 = 40\%$, 0.33…, $0.\dot{4}$, $0.\dot{2}\dot{5}$.

They mostly appear as fractions, percentages, decimals (terminating, non-terminating and recurring) decimals, or terminating and non-recurring decimals. They are mostly applied in decision making, such as tracking loan payments of debtors, calculating students' exams scores, sharing of items, etc.

Irrational numbers: Numbers that cannot be expressed as $\frac{a}{b}$, where $b \neq 0$. e.g. They include $\boldsymbol{\pi}$ **and** square roots of non-perfect numbers: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, etc. They are mostly applied in trigonometric related problems, π is also used to adjust the speedometer of vehicles. Consider the diagram below for further analysis of rational and irrational numbers.

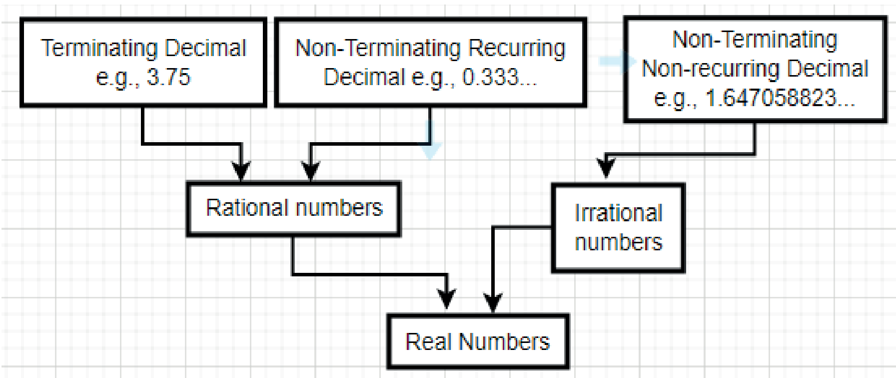


Figure 2: **Sample Diagram for comparing Rational and Irrational Numbers**

Application of the concepts with examples

After going through the simple decimal fractions, students to establish recurring and non-recurring decimals using simple fractions to help in their understanding of rational and irrational numbers.

Examples of recurring decimals:

$\frac{1}{3} = 0.333\ldots = 0.\dot{3}$, $\frac{5}{3} = 1.666\ldots = 1.\dot{6}$ $\frac{9}{11} = 0.818\ldots = 0.\dot{8}\dot{1}$

$\frac{22}{7} = 3.142857142857142857\ldots = 3.\dot{1}\dot{4}\dot{2}\dot{8}\dot{5}\dot{7}$

Examples of non-recurring non-terminating decimals:

$\sqrt{2} = 1.414213562373095\ldots$ $\frac{28}{17} = 1.647058823529412\ldots$

$2\sqrt{3} = 3.464101615137755\ldots$ $\frac{1}{\sqrt{2}} = 0.7071067811865475\ldots$

Learning Task for Practice 3

Strategy 1: using Pythagorean Spiral otherwise known as the 'Wheel of Theodorus' or 'spiral of Theodorus'.

Step 1: draw a unit right-angled triangle and find the length of the hypotenuse as: $\sqrt{1^2 + 1^2} = \sqrt{2}$

Step 2: next draw a right-angled triangle with dimensions 1 and $\sqrt{2}$, then find the length of the hypotenuse as $\sqrt{1^2 + \sqrt{2}^2} = \sqrt{3}$,

Step 3: continue with $\sqrt{1^2 + \sqrt{3}^2} = \sqrt{4} = 2$, then $\sqrt{1^2 + 2^2} = \sqrt{5}$, $\sqrt{1^2 + \sqrt{5}^2} = \sqrt{6}$, etc.

See the figure below

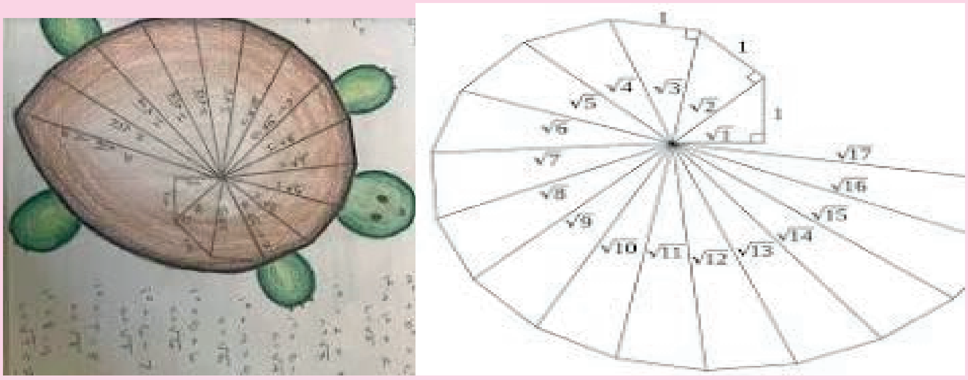


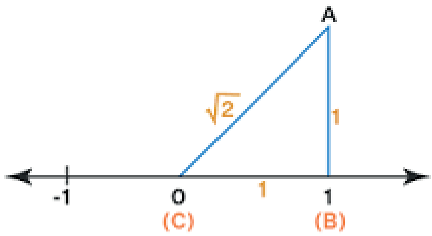
Figure 3: **Wheel of Theodorus for irrational numbers**

From the figure above, we can deduce if the radicals as either rational or irrational numbers.

1. Students to differentiate between rational and irrational numbers using various models:

   a. Strategy 1: Using number lines.
   To represent an irrational number on a number line, we break the number into two parts inside the square root. Through the Pythagoras theorem, the two parts make the sides of the triangles and the hypotenuse is the number inside the square root.

   b. Strategy 1: Using construction of right-angle triangles



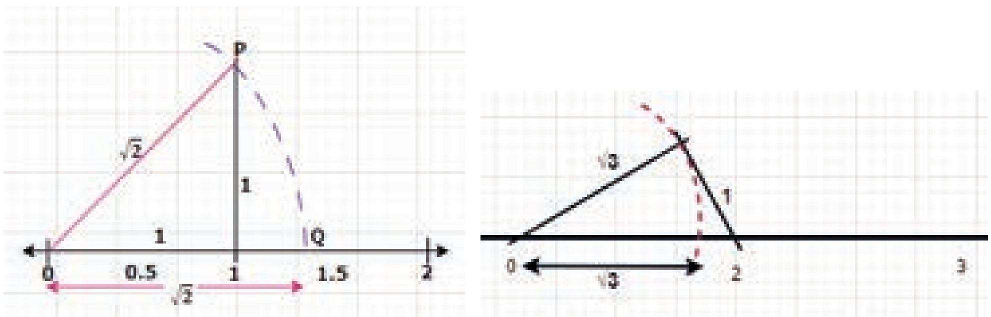


Figure 4: **Right-angle triangles**

From the figure above, we see that $\sqrt{2} < 2$ **and** $\sqrt{3} < 3$**,** etc. Note, students to investigate other radicals

Key Assessment

Answer at least one of the following questions

1. Enumerate the various subsets of the real number system using a Venn diagram/flow chart or number line. (**Assessment Level 2)**
2. Investigate the closure properties of the various subsets of the real number system, under the operations (+, −, × , ÷). (**Assessment Level 3)**
3. Investigate, where possible, application of the various subsets of the real number system in the following areas/sectors:
   a) daily transactions,
   b) computing,
   c) technology,
   d) medicine,
   e) agriculture,
   f) food and garment production
   g) energy sector,
   h) others. (**Assessment Level 3)**
4. Write down the steps by which a python can swallow a grain of corn, a chick and a hawk at different places, in a single meal. If a hunter kills the python after this and put it into his bag containing a bottle of water, use Venn diagram to illustrate the content in the hunters' bag. Relate these within the real number system. (**Assessment Level 4)**

Pedagogical Exemplars

1. **Diamond nine; Group work/collaborative learning. Using the diamond nine strategy in a collaborative group activity,** distinguish between rational and irrational numbers and establish their connections to daily activities.
2. **Group work/collaborative learning, e**stablish fractions that lead to recurring and non-recurring decimals; investigate their applications to real life problems. Promote among learners, tolerance, truth, honesty, respect for others' views, etc.
3. **Through collaborative group activity**, investigate rational and irrational numbers (i.e., $\sqrt{2}$, $\sqrt{3}$, π, etc.) from the wheel of Theodorus, number lines and other technology tools.
4. Through small group presentations, use models including construction of right-angle triangles, number lines and other technology tools to establish that for every positive integer n, $0 \leq \sqrt{\boldsymbol{n}} \leq \boldsymbol{n}$, E.g. $0 < \sqrt{1} = 1$, $0 < \sqrt{2} < 2$**, etc.**

Reflective Practice

Initiate activities that will lead the learner to reflect on the key concepts in the lesson.

Reflections: Conduct reflection on the following:

a. Real number system

b. Closure property:

c. Whole numbers are **closed** under + *and* × only

d. Integers are **not closed** under ÷ only

e. Real numbers are **closed only under non-zero division**.

f. Real numbers are used in every facet of life: business, food and garment, cosmetic industry, energy sector, agriculture, automobile, automaton, medicine, etc.

Key Assessment

Level 3: Answer at least one of the following questions.

1. Establish at least two real life applications each of rational or irrational numbers.

2. Investigate and classify each of the following as either a rational or an irrational number.

a. 75%:

b. 0.7142857142857143…

c. $\frac{125}{11050}$

d. $\sqrt{8}$

e. π (pi):

f. $2\sqrt{7}$

3. Convert the following decimals into fractions

a. $0.\dot{5}$

b. $0.\dot{6}\dot{2}$

c. $1.\dot{6}$

d. $2.\dot{8}\dot{1}$

WEEK 2: PROPERTIES OF OPERATION

Learning Indicators:

1. *Establish the properties of real numbers with respect to commutative, associative, identity, inverse, distributive, etc.*
2. *Review the properties of subsets (two and three), their vocabulary and operations and use it to solve real life problem.*

THEME OR FOCAL AREA 3: PROPERTIES OF OPERATION: COMMUTATIVE, ASSOCIATIVE, DISTRIBUTIVE, IDENTITY, INVERSE

Definition/Introduction:

1. **Commutative property**: Explains why order of terms does not matter while adding or multiplying two numerals. The positions of terms do not affect results.
2. **Associative property**: Regardless of how numbers are grouped, adding, or multiplying them together does not change the results. Here the placement of parentheses does not matter. The order of operation does not affect results.
3. **Distributive property**: This tells us that in solving expressions such as $a(b + c)$, we can find the sum of the products [[OMML-EQ–94]] and $(a \times c)$
4. **Identity property**: When a number is multiplied by 1, the product is the number itself. 1 is called multiplicative identity element. Also, when a number is added to 0, the result is the number itself. 0 is called additive identity element.
5. **Inverse property:**

   a. **Additive inverse**: This occurs when the sum of two addends gives 0.

   b. **Multiplicative inverse**: This occurs when product of two numbers gives 1.e.g. If $3 \times \Delta = 1$, then $\Delta = \frac{1}{3}$. Hence, $3 \times \frac{1}{3} = 1$. Therefore, the multiplicative inverse of 3 is $\Delta = \frac{1}{3}$.

Learning Task for Practice:

1) Establish that for a given set of numbers **a**, **b** and **c**, 1 is the multiplicative identity and 0 is the additive identity,
2) Determine identity elements, additive and multiplicative inverses of real numbers,
3) Establish the distributive property real numbers that connects (*) and (+) or (*) and (−)
4) Review subsets, unions, intersections, regions of two and three sets Venn diagrams and their properties of operations to investigate real life problems
5) Create Venn diagrams for three sets problems (intersecting, or non-intersecting) and identify the various regions.

   a. Limit content expectation to enumerating the properties of operations, commutative, associative, distributive, and their connections to real life and their inter connections.

   b. Extend content expectation to include classifying the properties of operations, commutative, associative, distributive, etc. and their existence with respect to the four operations.

   c. Extend content expectation to include applying the properties of operations, commutative, associative, distributive, etc. to real life and solving simple problems involving them.

Application of the concepts with examples:

Teacher to put students in groups and guide them to establish that for any given set of numbers *a*, *b and c*, the above properties of operations hold or otherwise.

$a * b = b * a$; *and* $a + b = b + a$.

Example 1.

2 * 7 = 7 * 2 = 14 ; 2 + 7 = 7 + 2 = 9. Thus, $a * b * c = c * b * a$; *and* $a + b + c = c + b + a$.

Example 2

. 2 * 3 * 4 = 4 * 3 * 2 = 24 ; *and* 2 + 3 + 4 = 4 + 3 + 2 = 9.

Learning task for practice 1:

Using multi-base blocks, Geodot and YouTube videos initiate activities to establish that for a given set of numbers ***a***, ***b*** *and* ***c***, 1 is the multiplicative identity and 0 is the additive identity,

Learners to extend the knowledge to identity elements, additive and multiplicative inverses as follows:

Multiplicative identity element $a * b = b * a = a$. E.g. *ie*. 4 * 1 = 1 * 4 = 4 (1 is multiplicative identity element)

Additive identity element $a + b = b + a = a$. E.g. 4 + (0) = (0) + 4 = 4 (0 is additive identity element)

Multiplicative inverse $a * b = b * a = 1$. e.g. $4 * \frac{1}{4} = \frac{1}{4} * 4 = 1$ ($\frac{1}{4}$ is multiplicative inverse of 4). In general, we say $\frac{1}{n}$ is multiplicative inverse of n

Additive inverse $a + b = b + a = 0$. E.g. $4 + (-4) = (-4) + 4 = 0$ ((-n) is additive inverse of n).

ie $a * 1 = 1 * a = a$; and $a + 0 = 0 + a = a$.

In general, we say that $-n$ is the additive inverse of n.

Example 3

Establish that for any three numbers ***a***, ***b*** *and* ***c***, the distributive property connects as follows:

(*) *and* (+) *or* (*) *and* (−) can be used as

For instance, $a*(b + c) = a * b + a * c$, [* is distributed over +)

eg . 10*(6 + 4) = (10 * 6) + (10 * 4) = 100

Also, $a*(b - c) = (a * b) - (a * c)$, [‘*’ is distributed over ‘ –‘).

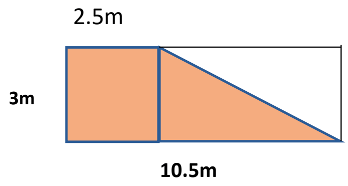
E.g. solve 108 ÷ 12 using the distributive property

Rewrite 108 ÷ 12 as (120 − 12) ÷ 12 =(120 − 12) ÷ 12 = (120 ÷ 12) − (12 ÷ 12)

This gives 10 – 1 = 9 ∴ 108 ÷ 12 = 9

Properties of operations:

Example 4: Find the area of the shaded portion in the figure below, using distributive property of operation.



Applying the distributive property, area (A), of the shaded portion (trapezium) is given by

$$A = \frac{h}{2} * \left(a + b\right) = \frac{h}{2} * a + \frac{h}{2} * b$$

$$\rightarrow \frac{3}{2} m \left(2.5m + 10.5m\right) = \frac{3}{2} m * 2.5m + \frac{3}{2} m * 4m$$

$$\rightarrow 3 \_ 2 \ m(13m) = 7.5m^2 + 12m^2$$

$$19.5m^2 = 19.5m^2$$

Example 5: Apply the distributive property of operation to divide 135 ÷ 15

Rewrite 135 ÷ 15 as (150 − 15) ÷ 15 = (150 ÷ 15) − (15 ÷ 15) = 10 − 1 = 9

Example 6:

A woman created a 2***m*** fire belt around, but within, her 200***m*** by 250***m*** rice farm. She later created a circular fishpond of diameter 6m at the middle of the farm.

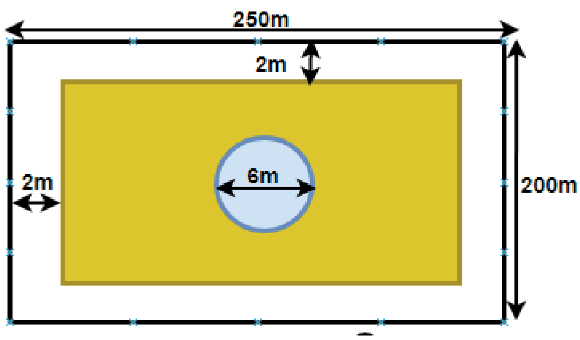
Determine the actual land area covered by the rice farm

(i). before the creation of the fishpond

(ii). after the creation of the fishpond. Give your answers to the nearest square metre.

If it cost her GHs28.00 to spray a square metre of the rice farm with weedicide, how much will it cost the farmer to spray the rice farm?

Solution:



a. Given: diameter of the pond= 6m,

Area of pond $= \pi \left(\frac{d}{2}\right)^2 = \pi \left(\frac{6}{2}\right)^2 = 9\pi = 28.23m^2 \cong 28\ m^2$

The actual land area covered by the rice farm.

(i). before the creation of the fishpond $= (250 - 4)(200 - 4) = 48{,}216\ m^2$

(ii). after the creation of the fishpond $= 48{,}216\ m^2 - 28\ m^2 = 48{,}188m^2$

b. If $GHs28.00 = 1\ m^2$, then $x = 48{,}188\ m^2$

$$x = \frac{48{,}188\ m^2}{28\ m^2} = GHs1{,}721.00$$

The farmer will need $GHs1{,}721.00$ to spray the farm.

Pedagogical Exemplars

A blend of the following pedagogical exemplars will be considered.

1. **Problem-based learning;** Talk for Learning; Experiential learning; and Group work/ collaborative learning.
2. **In mixed ability or convenience grouping**, Learners engage in group activities, make oral presentation of findings and use flowcharts/Venn diagram/number lines as well as strategies used in the class to establish the properties of operations, commutative, associative, distributive, etc.

Note: Accept, and redirect appropriately, related concepts that are not mentioned here. E.g. Accepted transitivity property of equality and inequality.

3. **Problem-based learning:** Here, Learners engage in group activities, make oral/written presentation of findings and use flowcharts/Venn diagram/number lines to determine the relationships between and among the properties of operations, commutative, associative, distributive, properties.
4. **Group presentation:** In smaller groups presentations, learners engage in group activities, make oral/written presentation of findings and use flowcharts/Venn diagram/number lines to determine the relationships between and among the properties of operations, commutative, associative, distributive, etc., and solve real life problems involving the properties.
5. **Using think-pair-share:** Initiate activities to help learners, individually, in pairs and in groups, investigate the properties of operations, commutative, associative, distributive, etc. with respect to ***addition*** $(+)$, ***subtraction*** $(-)$, ***multiplication*** $(\times)$, **and *division*** $(\div)$.

Encourage self-confidence, diversity and leadership in achieving the properties of operations, commutative, associative, distributive, etc.

6. **Talk for learning; think-pair-share; and group work/collaborative learning.**

Lead learners to establish (individually, in pairs and in groups) that for any given set of numbers *a*, *b and c* the following properties hold.

a) Enumerate the properties of operations in respect of commutative, associative, distributive, identity, inverse, etc.
b) Using strategies including think-pair-share; and group work/collaborative activities.
c) Employ differentiated content, assessment and process to ensure among learners, tolerance, truth, honesty, respect for others' views, etc.

7. **In small learning groups**, using worksheets, establish the distributive property operation and investigate its applications in other areas of knowledge.

a) Through individual work guide learners to find areas of the shaded portions, applying appropriate properties of operation.

8. **Talk for learning; think-pair-share; and group work/collaborative learning:** Use appropriate strategies teach application of properties of operation in solving daily problems involving real numbers.

Reflective practice: Initiate activities that will lead the learner to reflect on the key concepts in the lesson.

Key Assessment

Level 3: Answer at least three questions from the following:

1) Establish that for any given set of numbers *a*, *b and c* the following properties hold.
   - **a)** Commutative,
   - **b)** Associative,
   - **c)** Distributive,
2) identify the property of operation in each of the following
   - **a)** $4 * \frac{1}{4} = \frac{1}{4} * 4 = 1$
   - **b)** $4 * 3 = 3 * 4 = 12$
   - **c)** $4 * 0 = 0 * 4 = 0$

WEEK 3: PROPERTIES OF SUBSETS

Learning Indicator(s):

1. *Organise information visually to establish the relationship between and among sets of items (three sets) and apply these to conduct mini surveys in the school community and beyond.*
2. *Establish the relationship between and among three sets, including set equations and the De Morgan's law.*

Theme or Focal Area 1: Properties of Subsets (two and three), Subsets & Proper Subsets

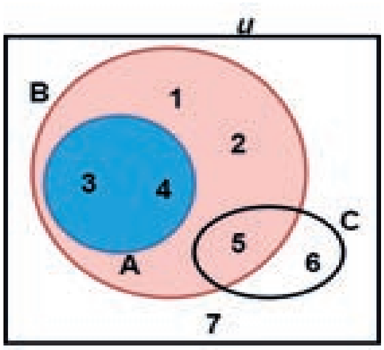
Definition/introduction

Subsets: If every member of set **A** is also a member of set **B,** then **A** is a subset of ***B***, we write this as $\boldsymbol{A \subseteq B}$. We can say **A** is contained in **B**, or $\boldsymbol{B \supseteq A}$, here, B is a superset of A, B includes A, or B contains A. If A is not a subset of B, we write $\boldsymbol{A \nsubseteq B}$.

However, if ***A*** is a subset of B ($A \subseteq B$), but A is not equal to B, then we say A is a proper subset of B, written as $A \subset B$ or $A \subsetneq B$. Example, U= {1,2, 3,4, 5,6, 7} $A = \{3,4\}$, $B = \{1,2, 3,4, 5,\}$, $C = \{5,6\}$

Venn Diagram

In mathematics, the Venn diagram is a pictorial representation of the relationship between two or more sets. It was suggested by the English mathematician and philosopher, John Venn.



- **i.** A is a proper subset of B; $A \subset B$
- **i.** A is not a subset of C;
- **ii.** B is a proper subset of U;

Pedagogical Exemplars

Determine the properties of subsets (for two sets problems), their vocabulary and operations and use it to solve real-life problems.

Employ differentiated content, assessment, and process to ensure among learners, tolerance, and respect for others' views.

Using building on what others say, managing talk for learning; diamond nine; group work/ collaborative learning, model and explain three sets problems in real life

In an interactive activity, review subsets, unions, intersections, regions of two and three sets Venn diagram and their properties of operations to investigate real life problems

Using building on what others say, managing talk for learning; diamond nine; group work/collaborative learning, model and explain three sets problems in real life

Using Diamond Nine; Group work/collaborative learning

Apply the concept of Venn diagram to solve three set problems using Group work/collaborative learning, ensuring balance in the differentiation of the various proficiency levels couple with GESI.

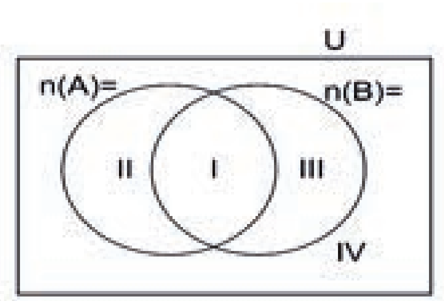
Using Think-pair-share in an interactive and learner centred classroom, model and resolve real-life problems using equations involving two and three sets and the De Morgan's law. Employ differentiated assessment and ensure values such as tolerance, truth, honesty, respect for others' views, etc. among learners.

Example: Create Venn diagrams for three sets problems (intersecting, or non-intersecting) and identify the various regions.

Theme/Focal Area (S) 2: Properties of Subsets (two and three)

Subsets & Proper Subsets

Review the Venn diagram for 2-sets problem (see Venn diagram below) and use it to introduce students to 3-sets problems.



n(A) = all elements that belong to set A,

n(B) = all elements that belong to set B,

I = elements that belong to both sets A and B

II = elements that belong sets A only

III = elements that belong to set B only

IV = elements that belong to neither set A nor B

Learning Tasks for Practice

1. Comprehend the vocabulary and operations of three sets:
2. Investigate subsets, unions, intersections, regions of three sets Venn diagram, their properties of operations and application to real life problems.

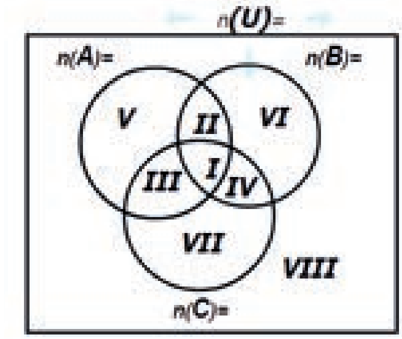
Theme/Focal Area 1: Word Problems Involving 3 Sets

Establish the relationship between subsets and indicate the regions of three sets.
Conduct activities that will lead students to enumerate all the various regions of three sets problems.

3-Sets Venn diagram

Theme/Focal Area 1:

Establish the relationship between sets of items and use the ideas to conduct mini surveys on three sets problems.



From the Venn diagram, state the regions that represent elements:

- **i.** Which are common to sets A, B, and C. i.e. $n(A \cap B \cap C)$.
- **ii.** That belong to sets A and B, but not common to C: $(A \cap B) \cap C'$.
- **iii.** That belongs to sets A and C, but not common to B: $(A \cap C) \cap B'$.
- **iv.** That belongs to sets B and C, but not common to A: $(B \cap C) \cap A'$.
- **v.** In set A that are neither in set B nor C:$A \cap (B' \cap C')$.
- **vi.** In set B that are neither in set A nor C:$B \cap (A' \cap C')$.
- **vii.** In set C that are neither in set A nor B: $C \cap (B' \cap A')$.
- **viii.** In the universal set U that are not in sets A, B and C: $(A \cup B \cup C)'$.

Application of the concept and examples:

The teacher should lead learners to establish the following properties of sets:

1. $(A \cup B)' = A' \cap B'$
2. $(A \cap B)' = A' \cup B'$
3. $(A \cup B \cup C)' = A' \cap B' \cap C'$
4. $(A \cap B \cap C)' = A' \cup B' \cup C'$

From algebraic point of view, for any three Sets A, B and C, the following properties are true.

- **i.** Commutative properties $AUB = BUA\ A\ B = B\ A$
- **ii.** Distributive properties: $A\ (B\ \ C) = (A\ \ B)\ (A\ \ C)\ ;\ A\ (B\ C) = (A\ \ B)\ (A\ \ C)$
- **iii.** Associative properties: $A \cap (B\ \ C) = (A \cap B)\ \ C\ ;\ A \cup (B\ C) = (A \cup B) \cup C)$
- **iv.** Other properties:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

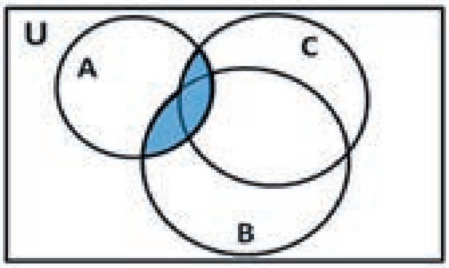
$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - [n(A \cap B) + n(A \cap C) + n(B \cap C)] + n(A \cap B \cap C)]$$

In an interactive group presentation, learners establish the identities and Venn diagrams of:

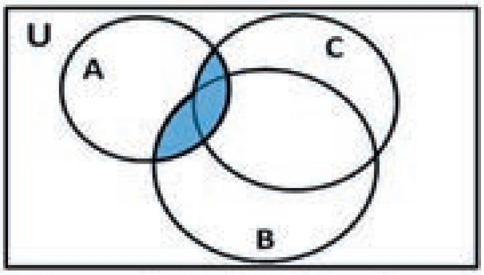
For any three sets A, B and C, the following holds:

1. AUB = BUA
2. AU (B∩ C) = (A UB) ∩(A UC)
3. A∩(BUC) = (A∩ B) U(A ∩C)

Illustration of $A\ (B\ C) = (A\ \ B)\ (A\ \ C)$ in the Venn diagrams below

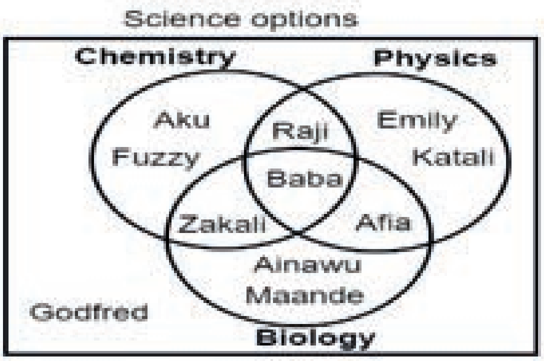


A ∩ (B U C)



(A ∩ B) U (A ∩ C)

Application of the Concept with examples:



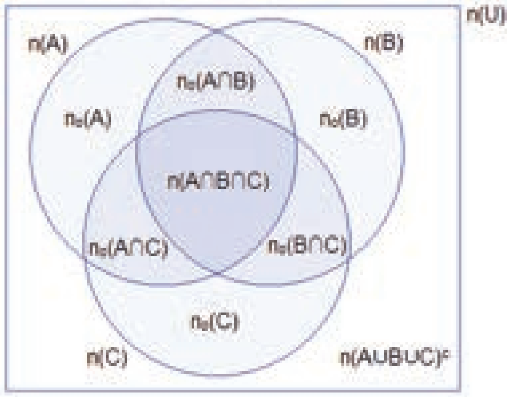
From the Venn diagram, state the regions that represent elements:

1. Which are common to sets P, B, and C. i.e. $n(P \cap B \cap C)$.
2. That belong to sets P and B, but not common to C: $(P \cap B) \cap C'$.
3. That belongs to sets P and C, but not common to B: $(A \cap C) \cap B'$.
4. That belongs to sets B and C, but not common to A: $(B \cap C) \cap P'$.
5. In set A that are neither in set B nor C:$P \cap (B' \cap C')$.
6. In set B that are neither in set P nor C:$B \cap (P' \cap C')$.
7. In set C that are neither in set P nor B: $C \cap (B' \cap P')$.
8. In the universal set U that are not in sets P, B and C: $(\text{P} \cup B \cup C)'$.

Key Assessment

Answer the questions from the following:

1. **Level 2:** Study the Venn diagram below and write down the mathematical representation of four regions using *A*, *B* and *C*



2. **Level 3:** Three committees A, B and C form a union, U in a School. If U has a total population 380 members, n(A) = 200, $n(B) = 155$, n(C) = 110, $n(A \cap B) = 50$, $n(B \cap C) = 35$, $n(A \cap C) = 44$ and $n(A \cap B \cap C) = 14$, illustrate this information in a Venn-diagram and find the following:

   1. $n(A \cap B' \cap C')$
   1. $n(A' \cap \mathrm{B} \cap C')$
   2. $n(A' \cap \mathrm{B}' \cap \mathrm{C})$
   3. $n(A \cap \mathrm{B} \cap C')$
   4. $n(A' \cap \mathrm{B} \cap C)$
   5. $n(A \cap \mathrm{B}' \cap C)$
   6. $n(A \cup B \cup C)'$

3. **Level 4:** In a survey of 100 SHS graduates who reported at the Jubilee House, on Independence Day, for employment. It was established that 55 graduates registered army, 50 graduates registered police and 30 graduates registered immigration. The entire 100 graduates registered for at least one of the forces. If 44 of the graduates registered for exactly two of the forces, how many graduates registered for all three forces?

Reflective Practice:

Initiate activities that will lead the learner to reflect on the key concepts in the lesson.

Reflections: Conduct reflection on the following:

1. vocabulary and operations of three sets:
2. subsets, unions, intersections, regions of three sets Venn diagram, their properties of operations and application to real life problems

Theme/Focal Area (S)2: Set Equations and De Morgan's Law

Definition/Introduction: Explanation of De Morgan's Law:

De Morgan's Law for Sets:

1. Establishing the relationship between sets of items (three sets)
2. Conducting mini surveys on sets
3. Set equations and De Morgan's law for two and three sets problems.
4. Applying the De Morgan's law to solve real life problems.

2-Sets Venn: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

3-Sets Venn:

$n(A \cup B \cup C)=n\ (A) + n(B) + n(C) - [n(A \cap B) + n(A \cap C) + n(B \cap C)] + n(A \cap B \cap C)]$

Examples:

Given the sets A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, and C = {5, 6, 7, 8}, verify de Morgan's laws for the following: a) (A ∪ B)' = A' ∩ B' b) (A ∩ B ∩ C)' = A' ∪ B' ∪ C'

1. **Level 2**: Let $X = \{x \mid x \text{ is an even digit}\}$, $Y = \{x \mid x \text{ is a prime number less than } 10\}$, and $Z = \{x \mid x \text{ is a perfect square less than } 10\}$.
   a) Find (X ∪ Y)'
   b) Find (X ∩ Y ∩ Z)'
   c) Verify De Morgan's laws for the sets X, Y, and Z.
2. **Level 2:** If $A = \{1, 2, 3, 4, 5\}$, $B = \{4, 5, 6, 7\}$, and $C = \{2, 4, 6, 8\}$, express the following using De Morgan's laws and set operations:
   a) The set of elements that are not in A or B but are in C.
   b) The set of elements that are in A and not in B or C.
3. **Level 2:** Suppose $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{2, 4, 6, 8, 10\}$, and $B = \{1, 3, 5, 7, 9\}$.
   a) Find (A ∪ B)'
   b) Find (A ∩ B)'
   c) Verify De Morgan's laws for the sets A and B.

Learning Tasks For Practice:

Learners to establish the de Morgan's Law and apply it to solve real life problems involving three sets.

Key Assessment

1. A survey conducted by students in Yendi Secondary School revealed that, 84 students study mathematics, 114 study ICT and 78 study Clothing. It was also realised that 48 study mathematics and ICT, 46 study mathematics and Clothing, 42 study Clothing and ICT and 34 study all the three courses.
   a) Draw a Venn diagram for this information,
   Find how many students study:
   b) One course only,
   c) Exactly two courses,
   d) At least two courses

DoK:
Level 2: up to completing the Venn diagram.
Level 3: up to completing the Venn diagram and find those who study only one course.
Level 4: answer all sub questions under question 1.

2. An advertising agency finds that, of its 170 sponsors, 115 sponsored Hearts, 110 sponsored RTU and 130 sponsored Kotoko. Also, 85 sponsored Hearts and Kotoko, 75 sponsored Kotoko and RTU, 95 sponsored Hearts and RTU, 70 sponsored all the three football clubs. Draw a Venn diagram to represent these data. Find how many sponsored.
   i. only Hearts?
   ii. only RTU?
   iii. RTU and Kotoko but not Hearts?

   DoK:
   Level 2: up to completing the Venn diagram.
   Level 3: up to completing the Venn diagram and find those who study only one course.
   Level 4: answer all sub questions under question 1.

Reflective Practice:
Initiate activities that will lead the learner to reflect on the key concepts in the lesson.

Reflections: Conduct reflection on the following:
1. Three sets problems
2. Mini surveys involving three sets
3. The Venn diagram can be used to solve real life problems.
4. De Morgan's Law

Section 1 Review

Over the last 3 weeks, we explored the basic concepts related to real numbers, sets, and mathematical operations.

On real numbers, we differentiated between natural, whole, integer, rational, irrational, and real numbers, emphasizing their unique properties like closure revealed core aspects of each number system. We also distinguished between rational and irrational numbers based on their decimal expansions.

When studying set theory, we used operations like unions, intersections, and complements to analyse relationships. Venn diagrams visually modelled operations between 2 or 3 sets.

Regarding properties of operations, we examined commutativity, associativity, distributivity, identity, and inverse elements. These qualities allow numbers to be added, subtracted, multiplied, and divided flexibly while maintaining equality. They form the basis for algebraic manipulations.

To apply these learnings, we modelled real-world scenarios with sets and Venn diagrams. We also solved problems utilizing rational and irrational numbers.

In summary, the 3 weeks provided crucial introductions to real numbers and their properties, and problem-solving skills. Students should now be familiar with these foundational math topics and equipped to build upon them in more advanced courses.

RESOURCES

- Mathematical sets.
- Geo pegs and peg boards
- Number line
- Charged particles
- Fractional bords
- Muti-base Blocks,
- Wheel of theodoras
- Manella cards
- Bundles of sticks and loose ones, abacus
- Graph board/sheets and square papers
- Cuisenaire rods
- Technology tools such as computer, mobile phone etc.
- Computer software applications like GeoGebra.
- Tape measure, carpenters square, compass, geodot,
- Algebraic Tiles etc.

SECTION 2: FRACTIONS AND PERCENTAGES

Strand: **Number for Everyday life**

Sub-Strand: Real Number and Numeration system (RNNS)

Learning Outcome(s):

1. *Make connections between fractions and decimals and use them to solve daily problems.*
2. *Create strategies for solving percent problems involving personal or household finance.*

Content Standard(s):

1. Demonstrate understanding of proportional reasoning involving fractions and its operations and use it to solve real-life problems including rounding off (decimal places and significant figures).
2. Demonstrate conceptual understanding of proportional reasoning on percentages and use it to solve everyday life problems including simple interest, discount, profit, loss, commission, etc.
3. Create strategies for solving percent problems involving personal or household finance.

INTRODUCTION AND SECTION SUMMARY

Proportional reasoning is a fundamental skill with numerous real-life applications. This section equips students with a strong conceptual foundation in fractions and percentages, essential mathematical concepts deeply rooted in proportional reasoning. The primary focus is on building problem-solving skills to tackle real-life situations effectively.

The section comprises five weeks. Week 4 deals with the concept of fractions (including benchmark, equivalent, like, unlike, mixed numbers). Week 5 will lead to operations on common fractions. Week 6 deals with decimal number and percentages while weeks 7 and 8 leads us to percentages and their applications to real life.

Students will, at the end of this section, master proportional reasoning with fractions and percentages that will enhance their problem-solving skills and empower them to make informed decisions, particularly in personal and household finance. By developing strategies to solve percent problems in financial contexts, they gain valuable insights into responsible financial management.

They will also develop a deep understanding of proportional reasoning with fractions and percentages. Key concepts include understanding proportional reasoning, performing operations with fractions, applying proportional reasoning to real-life problems, rounding off decimal places and significant figures. Additionally, students will gain a conceptual understanding of percentages, their relationship to fractions and decimals, and solve everyday life problems involving discounts, profits, losses, commissions, and more.

Finally, the section covers personal and household finance, focusing on developing strategies to solve percent problems in financial contexts, applying proportional reasoning to manage finances effectively, and exploring real-life scenarios involving budgeting, savings, investments, and other financial decisions.

The following weeks are considered in section 2

1. **Week 4**: The concept and operations of fractions
2. **Week 5**: Problem-solving on common fractions

3. **Week 6**: Decimal number and percentages.
4. **Week 7**: Application of percentages.
5. **Week 8**: Simple and compound interest.

SUMMARY OF PEDAGOGICAL EXEMPLARS

Week 4: The concept of fractions: Teachers should use visual models (fraction strips, circular representations, number lines) to introduce fractions as parts of a whole. Hands-on activities like paper folding can demonstrate equivalent fractions. Benchmark fractions (0, $\frac{1}{4}$, $\frac{1}{2}$, 1) should be related to real-life examples. Comparing and ordering fractions can be taught through interactive activities using fraction strips or number lines. For mixed numbers, visual models should be used to represent conversions to improper fractions and vice versa. Differentiation can be achieved by providing additional challenges for gifted students, such as working with higher-level fractions or more complex conversions.

Week 5: Operations on common fractions: Visual models should be used to teach addition and subtraction of fractions with like and unlike denominators. Multiplication of fractions can be introduced through real-life examples of finding a fraction of a quantity. Division of fractions should be related to the inverse operation of multiplication. Simplifying fractions after operations should be practiced through exercises and word problems. Differentiation can be achieved by providing more complex operations or multi-step word problems for advanced learners.

Week 6: Decimal number and percentages: Place value charts and base-ten blocks should be used to represent decimal numbers. Hands-on activities should be provided to convert fractions to decimals and percentages, and vice versa. Real-life examples should be used to introduce percentages and their representations. Rounding decimals and percentages should be practiced through exercises and word problems. For gifted learners, additional challenges can include working with repeating decimals or more complex conversions.

Week 7: Application of percentages: Teachers should use real-life examples (discounts, tips, taxes, gratuities) to practice finding percentages of a given quantity and finding the whole when a part and percentage are given. Word problems from various contexts should be provided to apply percentages in authentic situations. Differentiation can be achieved by providing more complex word problems or open-ended tasks for advanced learners.

Week 8: Application of percentages 2: Real-life situations (population growth, inflation rates, business scenarios, investments, loans) should be used to illustrate percentage increase/decrease, profit/loss calculations. Word problems from different contexts should be provided to apply these concepts. For gifted learners, additional challenges can include more complex multi-step problems or open-ended tasks that require higher-order thinking skills.

Throughout these sections, learners should develop a conceptual understanding of fractions, decimals, and percentages, as well as the ability to perform operations, convert between representations, and apply these concepts in real-life situations. Assessments should align with the content taught and focus on evaluating conceptual understanding, procedural fluency, problem-solving skills, and application of knowledge.

ASSESSMENT SUMMARY

The concepts under this section require learners to demonstrate conceptual understanding, including their real-life applications. Hence, the assessments should largely cover levels 1-4 of the DOK. Again, teachers should employ a variety of formative assessment (assessment for- and as-learning) strategies such as oral/written presentations, pair-tasks, reports, home tasks, etc. to gather information about learners' progress and give prompt feedback to them. Specifically, teacher should conduct the following assessments and record the performances of learners for continuous assessment records;

1. class exercises (including individual worksheets) after each lesson
2. home works
3. scores on practical group activities on measuring perimeter and area of real objects

Assessment strategies and how to use them during teaching and learning.

Consider the following assessment strategies and use the results to provide targeted feedback from students you consider AP, P and HP and identify areas where additional support is needed.

Week 4: ***The concept of fractions*****:** A formative assessment can be conducted through a hands-on activity or a short quiz. Students can be asked to represent fractions using visual models, identify benchmark fractions, generate equivalent fractions, and compare and order like and unlike fractions. The assessment should align with the Assessment Manual's guidelines for assessing conceptual understanding and representation skills.

Week 5: ***Operations on common fractions***: A summative assessment can be given at the end of this week to evaluate students' proficiency in performing operations with fractions. The assessment should include problems involving addition, subtraction, multiplication, and division of fractions, with a mix of like and unlike denominators. Word problems can be incorporated to assess their ability to apply these operations in real-life contexts, as outlined in the Assessment Manual.

Week 6: ***Decimal number and percentages***: A formative assessment can focus on students' understanding of place value in decimal numbers, converting between fractions, decimals, and percentages, and rounding decimals and percentages. The assessment should follow the guidelines in the Assessment Manual for assessing procedural fluency and conceptual understanding.

Week 7***: Application of percentages***: A project-based assessment can be given, where students are required to solve real-life problems involving percentages, such as calculating discounts, taxes, gratuities, or finding the whole when a part and percentage are given. The assessment should align with the Assessment Manual's guidelines for assessing problem-solving skills and application of knowledge in authentic contexts.

Week 8: ***Application of percentages assessment***: A summative assessment can be conducted to evaluate students' ability to solve problems related to percentage increase/decrease, profit/loss calculations. The assessment should include a variety of word problems from different contexts, as recommended in the Assessment Manual for assessing higher-order thinking skills and application of concepts.

For each assessment, detailed rubrics should be developed based on the Assessment Manual's guidelines, and students' performance should be recorded in their transcripts, highlighting their strengths and areas for improvement.

L2: Think-pair-share***:*** Guide learners to reflect individually on a question or concept (establish common fractions and convert fractious to decimals. This strategy promotes peer-to-peer learning and allows for the assessment of individual understanding.

L3: **Homework and assignments:** Give learners tasks on the following to take away and submit after a few days or the next lesson. Review learners' completed homework or assignments and provide immediate feedback for improvement-

1. Convert the following fractions to decimals and percentages

i. $\frac{1}{2}$

ii. $\frac{1}{4}$

iii. $\frac{3}{4}$

iv. $\frac{4}{5}$

2. Convert the following decimals and percentages into common fraction.

i. 0.5

ii. 0.25

iii. 75%

iv. 125%

3. Convert $12\frac{1}{2}$% into a fraction.

4. Suglo has 20 items on her shopping list. At the market, she realised from her list that she completed 40% of her shopping. Determine how many more items she has to buy?

WEEK 4 :THE CONCEPT AND OPERATIONS OF FRACTIONS 1

Learning Indicators:

1. *Establish the concept of fractions and investigate the connections between fractions and decimal numbers.*
2. *Establish basic rules for operations on fractions: addition, subtraction, multiplication and division.*

Theme/Focal Area (S)1: The Concept of Fractions

Definition/Introduction:

A fraction is a concept which represents a numerical value and defines parts of a whole/unit, a group or a ration.

Parts of Fractions

The fractions include two parts, numerator and denominator.

1. Numerator: It is the upper part of the fraction, that represents the sections of the fraction
2. Denominator: It is the lower or bottom part that represents the total parts in which the fraction is equally divided.

Example: If $\frac{2}{5}$ is a fraction, then 2 is the **numerator** and 5 is the **denominator**.

Types of Fractions

1. **Proper fractions:** The proper fractions are those where the numerator is less than the denominator. For example, $\frac{4}{5}$ will be a proper fraction since "numerator < denominator".
2. **Improper fractions:** The improper fraction is a fraction where the numerator is greater than the denominator. For example, $\frac{5}{2}$ will be an improper fraction since "numerator > denominator".
3. **Mixed fractions**: A mixed fraction is a combination of the integer part and a proper fraction. These are also called mixed numbers or mixed numerals. For example: $3\frac{1}{2}$, $4\frac{2}{3}$, $7\frac{1}{2}$, $1\frac{11}{12}$
4. **Like fractions:** Like fractions are those fractions, as the name suggests, that are alike or the same. For example, take $\frac{1}{4}$ and $\frac{2}{4}$; they are alike since they have same denominator.
5. **Unlike fractions:** Unlike fractions, are those that are dissimilar. They have different denominators. For example, $\frac{1}{2}$ and $\frac{2}{3}$ are unlike fractions.
6. **Equivalent fractions**: Two fractions are equivalent to each other if after simplification the two fractions are equal to one another. Consider the fractions

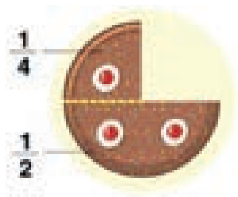
   a. $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8},$

   b. $\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}$

   c. $\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}$ are called equivalent fractions.

   a. For example, $\frac{1}{3}$ and $\frac{2}{6}$ are equivalent fractions, Since, $\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$

7. **Unit fractions:** A fraction is known as a unit fraction when the numerator is equal to 1.
   i. One half of whole = $\frac{1}{2}$
   ii. One-third of whole = $\frac{1}{3}$
   iii. One-fourth of whole = $\frac{1}{4}$
   iv. One-fifth of whole = $\frac{1}{5}$



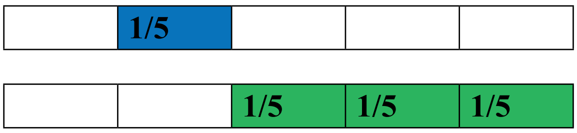
Learning Tasks for Practice

1. models the concept of fractions as part of a whole, a group or a ratio of two integers
2. reviews, parts of fractions and types of fractions.
3. initiate interactive activities that relates fractions to learners' real-life experiences
4. apply these experiences to operations on fractions.

Application of the Concept and Examples

Model simple fractions using paper strips, cuisenaire rods, or a number line. For example, if we have to represent $\frac{1}{5}$ and $\frac{3}{5}$ parts of a whole on a line, then have the figure below.

Using paper strips, we have



Guide learners to model fractions of similar nature, such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{7}$, $\frac{5}{12}$, etc. using paper strips, Cuisenaire rods, number lines, etc.

Establish benchmark fractions

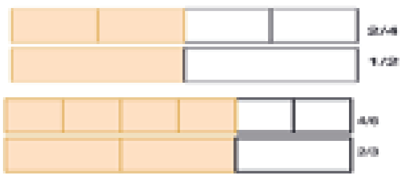
Benchmark fractions are common, easy-to-visualize fractions that are used as reference points to estimate or compare other fractions. The most used benchmark fractions are:

1. 0 (zero)
2. $\frac{1}{4}$ (One-quarter / one-fourth)
3. $\frac{1}{2}$ (one-half)
4. $\frac{3}{4}$ (Three-fourths)
5. 1 (one whole)

Establish equivalent fractions from benchmark fractions using interactive approaches.

i. $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots$

ii. $\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots$



iii. $\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots$

iv. $\frac{1}{5}, \frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \ldots$

v. $\frac{1}{6}, \frac{2}{-12}, \frac{3}{18}, \frac{4}{24}, \ldots$

Rules for operations on fractions

Guide learners to establish some basic rules necessary for operations on fractions.

Rule 1: *Addition and subtraction of fractions are possible with a common denominator.*

Rule 2: When we multiply two fractions, then the numerators are multiplied as well as the denominators are multiplied.

Rule 3: When we divide a fraction by another fraction, we have to find the reciprocal of the second fraction and then use Rule 2 above.

1: Adding Fractions

Like fractions: The addition of fractions is easy when they have a common denominator, known as the lowest common multiple (LCM).

For example, 1). $\frac{2}{5} + \frac{1}{5}$ 2). $\frac{5}{8} + \frac{2}{8}$

Solution Here, we need to just add the numerators, since the denominators are common. Thus,

1). $\frac{2}{5} + \frac{1}{5} = \frac{2+1}{5} = \frac{3}{5}$ 2). $\frac{5}{8} + \frac{2}{8} = \frac{5+2}{8} = \frac{7}{8}$,

Unlike fractions: Adding fractions with different denominators

Example, 3). $\frac{5}{8} + \frac{1}{2}$

Solution: Here, one denominator is a factor or multiple of the other and we need to adjust the denominators to be same before adding. Thus,

$\frac{5}{8} + 1 = 5 + 1 \times 4 = 5 + 4 = 5\frac{+4}{8} = 9$

Example 4). $\frac{1}{3} + \frac{3}{4} = ?$

Solution: Here, the denominators are neither factors nor multiples of the other and we need to adjust the denominators to be same before adding. Thus, multiply $\frac{1}{3}$ *by* $\frac{4}{4}$ and $\frac{3}{4} \times \frac{3}{3}$

$\frac{1}{3} + 3 = 1 \times 4 + 3 \times 3 = 4\frac{}{2} + 9\frac{}{2} = 4\frac{+9}{12} = 1\frac{3}{12}$

Using equivalent fractions to make unlike fractions common before adding them.

Example: $\frac{1}{3} + \frac{3}{4}$

Solution: Using equivalent fractions of $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = 4_{2} = 5_{5} = 6_{8} = \ldots$

Similarly, $\frac{3}{4} = \frac{6}{8} = 9_{2} = \frac{12}{16} = \frac{15}{20} = \frac{18}{24} = \ldots$

Fractions with common denominators are $\frac{4}{12}$ and $\frac{9}{12}$. Implies $\frac{1}{3} + \frac{3}{4} \equiv \frac{4}{12} + \frac{9}{12} = \frac{13}{12}$

$\therefore 1 + 3 = 1\frac{3}{12}$

Using **LCM** to solve addition of fraction problems:

Add the fractions $\frac{1}{4} + \frac{5}{6}$.

Here, we must simplify them by finding the LCM of denominators $\frac{1}{4} + \frac{5}{6}$ and then making it common for both fractions.

LCM $= \frac{1}{4} + \frac{5}{6} = \frac{3+10}{12} = \frac{13}{12} = 1\ 1_{2}$

Model similar examples for learners to discuss in class

2: Subtraction of fractions with different denominators

If the denominators of the two fractions are different, we must simplify them by finding the LCM of denominators and then making it common for both fractions.

Example: $\frac{5}{8} \frac{2}{8} = \frac{5-2}{8}$, Here, the denominators are same

Example: 5). $\frac{2}{3} - \frac{3}{4}$.

The two denominators are 3 and 4. Hence, LCM of 3 and 4 *is* 12

Therefore, multiplying $\frac{2}{3}$ by $\frac{4}{4}$ and $\frac{3}{4}$ by $\frac{3}{3}$, we get; $\frac{8}{12} - 9_{\overline{2}} = \frac{-1}{12}$

Model similar examples for learners to discuss in class

3: Multiplication of Fractions

Example 6). $\frac{2}{3} \times \frac{3}{4}$

Solution: $\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = 6_{\overline{2}} = \frac{1}{2}$

Model similar examples for learners to discuss in class

4: Division of Fractions

Example 7). $\frac{2}{3} \div \frac{3}{4}$

Solution: $\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$

Model similar examples for learners to discuss in class

Learning Tasks for Practice

1. ***Problem-Solving***: Alberta designed a pie with four slices of equal dimensions. She decided to give out two slices of the pie to her son. Represent this with a fraction.
2. ***Problem-solving***: If you owe someone $\frac{3}{4}$ of an amount of money and your friend owes $\frac{5}{7}$ of the amount, find the sum of the debt of you and your friend

Pedagogical Exemplars

A blend of the following pedagogical exemplars will be considered.

1. **Problem-based learning;** Talk for learning; experiential learning; and group work/ collaborative learning.
2. In mixed ability or convenience grouping, students use verbal discuss to establish ***concept of fractions.***

   Note: Accept, and redirect appropriately, related concepts that are not mentioned here. e.g. Explain non-examples and counter examples.
3. **Problem-based learning:** Here, students establish the relationship between and among the various types of fractions.
4. **Using think-pair-share:** Initiate activities to help learners, individually, in pairs and in groups, investigate ***concept of fractions*** *with respect to part-whole, ratio and group.*

   Encourage self-confidence, diversity and leadership in achieving the ***concept of fractions***

Reflective Practice: Initiate activities that will lead the learner to reflect on the key concepts in the lesson.

Key Assessment

1. In a school choir of 45 members, $\frac{3}{5}$ of the members are female. How many female members are in the choir?
2. A bakery has a stock of 75 loaves of bread, and $\frac{3}{5}$ of them are whole wheat bread. How many loaves of whole wheat bread are there?
3. In a company of 120 employees, $\frac{3}{5}$ of them are full-time employees. How many full-time employees are there?
4. A library has a collection of 90 books, and $\frac{3}{5}$ of them are fiction books. How many fiction books are in the collection?

WEEK 5 :THE CONCEPT AND OPERATIONS OF FRACTIONS 2

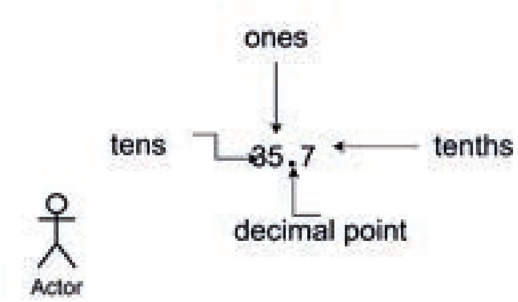
Learning Indicator(s):

1. *Investigate the connections between fractions and decimal numbers.*
2. *Establish additive and multiplicative inverses of fractions using multi-purpose model charts.*

Theme/Focal Area (S)1: Problem-Solving on Common Fractions

Definition/Introduction:

- **Decimals:** In Algebra, decimals are one of the types of numbers, which has a whole number and the fractional parts separated by a decimal point. The dot present between the whole and common fractional part.



- Percentages: it is a number or ratio that can be expressed as a fraction out of a hundred.

Learning Tasks for Practice

1. Establish decimals as fractions.
2. Teachers convert the different types of fractions into decimals and vice versa.
3. Teachers establish percentages as a fraction and/or a decimal.
4. Convert the different types of fractions into percentages and vice versa.
5. Converting the Decimal Number into Decimal Fraction.

Application of the Concept and Examples

1. Teacher establishes percentage and decimal numbers as fractions using different models such as multi-base arithmetic blocks, graph sheets/square pater, etc.

   Identify decimal names of given fractions; convert fractious to decimals (and vice versa); and convert decimals to percentages (and vice versa)

2. Convert fractions to decimals and percentages (and vice versa) and use these representations in estimations, computations, and applications. i.e.
   - If $0.5 = 5_{\overline{0}} = \frac{1}{2}$; then let learners convert 0.25 into a fraction
   - If $\frac{1}{2} = \frac{1}{2} \times \frac{5}{5} = 5_{\overline{0}} = 0.5$; then learners to convert $\frac{1}{4}$ into a decimal number

3. Establish percentage as the number of parts in every 100; use fractions and percentages to describe parts of shapes, quantities and measures. I.e.

- $\frac{1}{2} = 0.5 = \frac{50}{100} = 50\%$
- $\frac{1}{4} = 0.25 = \frac{125}{100} = 25\%$

4. Suglo has 20 items on her shopping list. At the market, she realised from her list that she completed 40% of her shopping. Determine how many more items she has to buy?

Solution: She bought 40% *of* $20 = 4\frac{0}{100} \times 20 = 8$,

She now has 60% *of* $20 = 6\frac{0}{100} \times 20 = 12$ to buy.

Alternatively, she is left with $(20 - 8) = 12$ items to buy.

3. **Convert $12\frac{1}{2}$ % into a fraction.**

Solution:

Consider the steps below:

Step 1: Convert $12\frac{1}{2}$ % into a proper fraction.

Thus, $12\frac{1}{2}\% = \frac{25}{2}\%$

Step 2: Replace the percent symbol (%) with $\frac{1}{100}$.

So, $12\frac{1}{2}\% = \frac{25}{2} \times \frac{1}{100} = \frac{25}{200}$

Step 3: Reduce it to the lowest form, to get $12\frac{1}{2}\% = \frac{1}{8}$.

Conclude that $12\frac{1}{2}\% = \frac{1}{8}$

4. Add $2\frac{2}{5}$ to $1\frac{2}{3}$,

Solution:

Step 1: Add the whole numbers and then add the fractions separately as shown below;

$2 + 1 + \left(\frac{2}{5} + \frac{2}{3}\right) = 3 + \left(\frac{6}{15} + \frac{10}{15}\right) = 3 + \frac{6 + 10}{15}$

Step II: Simplify to get $= 3 + \frac{16}{15}$

Step III: Convert the improper fraction into mix number $= 3 + 1 + \frac{1}{15} = 4 + \frac{1}{15}$

Step IV: Simplify to get $4\frac{1}{15}$.

Theme/Focal Area(S) 2: Additive and Multiplicative Inverses of Fractions Using Multi-Purpose Model Charts.

Definition/Introduction:

Additive inverses: A number that, when added to the original number, results in a sum of zero.

I . e . : $a + b = 0$, b is additive inverse of a. Eg. If $3 + (-3) = 0$, the (-3) is additive inverse of 3.

Multiplicative inverses: A number that, when multiplied by an original number, results in a product of 1.

i.e. if a × b = 1, then b is a multiplicative inverse of a.

E.g. $\frac{2}{3} \times \frac{3}{2} = 1$, hence $\frac{2}{3}$ is a multiplicative inverse of $\frac{3}{2}$

Application of the concept and examples:

Establish at least 3 pairs of numbers whose product is always 1, using the multipurpose fractional chart below.

Example:

$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$
$\frac{2}{1}$	$\frac{2}{2}$	$\frac{2}{3}$	$\frac{2}{4}$	$\frac{2}{5}$	$\frac{2}{6}$	$\frac{2}{7}$	$\frac{2}{8}$	$\frac{2}{9}$	$\frac{2}{10}$
$\frac{3}{1}$	$\frac{3}{2}$	$\frac{3}{3}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{3}{6}$	$\frac{3}{7}$	$\frac{3}{8}$	$\frac{3}{9}$	$\frac{3}{10}$
$\frac{4}{1}$	$\frac{4}{2}$	$\frac{4}{3}$	$\frac{4}{4}$	$\frac{4}{5}$	$\frac{4}{6}$	$\frac{4}{7}$	$\frac{4}{8}$	$\frac{4}{9}$	$\frac{4}{10}$
$\frac{5}{1}$	$\frac{5}{2}$	$\frac{5}{3}$	$\frac{5}{4}$	$\frac{5}{5}$	$\frac{5}{6}$	$\frac{5}{7}$	$\frac{5}{8}$	$\frac{5}{9}$	$\frac{5}{10}$
$\frac{6}{1}$	$\frac{6}{2}$	$\frac{6}{3}$	$\frac{6}{4}$	$\frac{6}{5}$	$\frac{6}{6}$	$\frac{6}{7}$	$\frac{6}{8}$	$\frac{6}{9}$	$\frac{6}{10}$
$\frac{7}{1}$	$\frac{7}{2}$	$\frac{7}{3}$	$\frac{7}{4}$	$\frac{7}{5}$	$\frac{7}{6}$	$\frac{7}{7}$	$\frac{7}{8}$	$\frac{7}{9}$	$\frac{7}{10}$
$\frac{8}{1}$	$\frac{8}{2}$	$\frac{8}{3}$	$\frac{8}{4}$	$\frac{8}{5}$	$\frac{8}{6}$	$\frac{8}{7}$	$\frac{8}{8}$	$\frac{8}{9}$	$\frac{8}{10}$
$\frac{9}{1}$	$\frac{9}{2}$	$\frac{9}{3}$	$\frac{9}{4}$	$\frac{9}{5}$	$\frac{9}{6}$	$\frac{9}{7}$	$\frac{9}{8}$	$\frac{9}{9}$	$\frac{9}{10}$
$\frac{10}{1}$	$\frac{10}{2}$	$\frac{10}{3}$	$\frac{10}{4}$	$\frac{10}{5}$	$\frac{10}{6}$	$\frac{10}{7}$	$\frac{10}{8}$	$\frac{10}{9}$	$\frac{10}{10}$

Examples,

1. $\frac{3}{4} \times \frac{4}{3} = 1$, here multiplicative inverse of $\frac{3}{4}$ is $\frac{4}{3}$ since their product is 1
2. $\frac{6}{3} \times \frac{3}{6} = 1$, similarly, multiplicative inverse of $\frac{6}{3}$ is $\frac{3}{6}$ since their product is 1
3. $\frac{5}{7} \times \frac{7}{5} = 1$, also, multiplicative inverse of $\frac{6}{3}$ is $\frac{3}{6}$ since their product is 1

Learning Tasks for Practice

Study and write down pairs of additive and multiplicative inverses of the following.

- **i.** $3 + x = 0$
- **ii.** $-5 + x = 0$
- **iii.** $x + 15 = 0$
- **iv.** $\frac{3}{5}$
- **v.** $\frac{2}{7}$
- **vi.** $\frac{7}{11}$
- **vii.** $\frac{-4}{5}$

Pedagogical Exemplars

A blend of the following pedagogical exemplars will be considered.

1. **Problem-based learning;** Talk for learning; experiential learning; and group work/ collaborative learning.
2. In mixed ability or convenience grouping, students use verbal discuss to establish *decimals and percentages as fractions.*

   Note: Accept, and redirect appropriately, related concepts that are not mentioned here. E.g. include non-examples and counter examples.
3. **Problem-based learning:** Here, students establish the relationship between and among the decimals, percentages, and fractions.
4. **Using think-pair-share:** Initiate activities to help learners, individually, in pairs and in groups, investigate *the additive and multiplicative inverses of fractions***.**

Encourage self-confidence, diversity, and leadership in achieving the *concept of fractions.*

Reflective Practice: Initiate activities that will lead the learner to reflect on the key concepts in the lesson.

Key Assessment

1. Provide tasks that will lead to application and creativity to help learners achieve strategic reasoning of operation on mixed numbers and their applications to real life.
2. Express $\frac{5}{8}$ as a decimal to two decimal places
3. A line manager is calculating the percentage of progress for a school project, and has completed 5 out of the 8 planned tasks. How will you help him to represent the completion rate as a decimal rounded to two decimal places?
4. A fraction of students in a class have access to online learning resources. If 15 out of 32 students can access these resources, convert the fraction to a decimal simplify it to show the ratio of students with online access.
5. Using the expanded form, express the following decimal numbers in the form of $\frac{a}{b}$, where $b \neq 0$

   I. 0.75 (Hint : $75 \times \frac{1}{100} = \frac{5}{100} = \frac{3}{4}$)
   II. 0.45
   III. 1.25
   IV. 45.5
6. Convert the following fractions to decimal numbers:

   i. $\frac{75}{100}$
   ii. $\frac{7}{10}$
   iii. $\frac{5}{8}$
   iv. $\frac{5}{4}$
   v. $2\frac{3}{4}$

Reflective Practice:

Initiate activities that will lead the learner to reflect on the key concepts in the lesson.

Reflections: Conduct reflection on the following:

1. Concept of fractions
2. Decimals as fractions
3. Percentages as fractions
4. Additive inverse of fractions
5. Multiplicative inverse of fractions

WEEK 6: OPERATIONS ON COMMON FRACTIONS

Learning Indicators:

1. *Review the concept of fractions and investigate the connections between fractions and decimal numbers.*
2. *Develop models to examine connections between and among fractions, percentages and decimal numbers and generalize.*

Theme/Focal Area (S): Connections Between and Among Fractions, Percentages and Decimal Numbers

Application of the Concept and Examples:

To convert a percentage into a fraction, write the figure of the percentage as "out of 100".

To turn a fraction into a percentage, we multiply the fraction by 100 and express in % form. Here, we simply multiply the numerator by 100 and divide it by 100.

i.e. $25\,\% = 25 \times \frac{1}{100} = 5\,/100$ implies $\frac{1}{4} = 25 \times \frac{1}{100} = 25\%$

Learning Tasks for Practice:

1.
   - i. Convert fractions to decimals and to percentages and use these representations in estimations, computations, and applications.
   - ii. Play mental mathematics games: Learners use simple mental strategies to perform the following:

     Addition, subtraction, multiplication and division of fractions using appropriate words and make connections.
   - iii. Models additive and multiplicative inverses of fractions using multi-purpose model charts.
2. Teachers review fractions as part of a whole, part of a group or a ratio of two integers.
3. Model additive and multiplicative inverses of fractions using multi-purpose model charts.
4. Convert fractions from one form into other forms.

Application of the Concept and Examples

Identify decimal names of given fractions; convert fractions to decimals (and vice versa); and convert decimals to percentages (and vice versa)

1. Convert fractions to decimals and percentages (and vice versa) and use these representations in estimations, computations, and applications. E.g.
   - If $0.3 = \frac{3}{10}$;
   - then learners convert 0.5 into a fraction as $0.5 = \frac{5}{10} = \frac{1}{2}$
   - If $\frac{1}{2} = \frac{1}{2} \times \frac{5}{5} = \frac{5}{10} = \frac{0.5}{1} = 0.5$; then learners to convert $\frac{1}{4}$ into a decimal number
2. Establish percentage as the number of parts in every 100; use fractions and percentages to describe parts of shapes, quantities and measures. I.e.

- $\frac{1}{2} = 0.5 =$; $0.5 \times \frac{100}{100} = 5\frac{0}{100} = 50\%$
- $\frac{1}{4} = 0.25$; $0.25 \times \frac{100}{100} = 2\frac{5}{100} = 25\%$

Trials: Convert $\frac{5}{9}$, $\frac{15}{75}$, into percentages.

a. $\frac{5}{9}$

Step 1: $\frac{5}{9}$ as a decimal is $\frac{5}{9} \approx 0.5555565$ (students may use calculators)

Step 2: Multiply by 100: $0.5556 \times 100 = 55.56\%$

So, $\frac{5}{9} = 55.56\%$

b. $\frac{15}{75}$

Step 1: $\frac{15}{75}$ as a decimal is $\frac{15}{75} = 0.2$

Step 2: Multiply by 100: $0.2 \times 100 = 20\%$

So, $\frac{15}{75}$ *s* 20 % .

3. Suglo has 20 items on her shopping list. At the market, she realised from her list that she completed 40% of her shopping. Determine how many more items she has to buy?

   Solution: She bought 40% *of* 20 $= \frac{40}{100} \times 20 = 8$,

   She now has 60% *of* 20 $= \frac{60}{100} \times 20 = 12$ times to buy.

   Alternatively, she is left with $(20 - 8) = 12$ items to buy.

Pedagogical Exemplars

A blend of the following pedagogical exemplars will be considered.

1. **Problem-based learning;** Talk for Learning; Experiential learning; and Group work/ collaborative learning.
2. In mixed ability or convenience grouping, students use verbal discuss to establish *decimals and percentages as fractions*

   Note: Accept, and redirect appropriately, related concepts that are not mentioned here. Eg. include non-examples and counter examples.
3. **Problem-based learning:** Here, students establish the relationship between and among the decimals, percentages and fractions
4. **Using think-pair-share:** Initiate activities to help learners, individually, in pairs and in groups, investigate *the additive and multiplicative inverses of fractions***.**

   Encourage self-confidence, diversity and leadership in achieving the *concept of fractions*

Reflective Practice: Initiate activities that will lead the learner to reflect on the key concepts in the lesson.

Key Assessment

Level 1 and 2 and 3

1. Express 60% as a fraction in its simplest form. $\frac{60}{100} = \frac{3}{5}$
2. Convert 0.6 to a proper fraction in its simplest form. $\frac{0.6}{1} = \frac{6}{10} = \frac{3}{5}$

3. What is 3 out of 5 expressed as a percentage? 3 *out of* 5 $= \frac{3}{5} = \frac{60}{100} = 60\%$

4. If 18 out of 30 students prefer gablee, what percentage of the students prefer gablee? $\frac{18}{30} = \frac{3}{5} = 60\%$

5. Express $\frac{5}{8}$ as a decimal (correct to two decimal places). Answer: $\frac{5}{8} = 0.625 \cong 0.63$

Level 3 and 4: Provide tasks that will lead to application and creativity to help learners achieve strategic reasoning of fraction and apply them to different scenarios in real-life.

6. A recipe calls for $\frac{5}{6}$ cup of sugar. However, you want to make only half of the recipe. How much sugar should you use in the reduced recipe? **Answer**: $\frac{1}{2}$ *of* $\frac{5}{6} = \frac{1}{2} \times \frac{5}{6} = 5_{\overline{2}}$

WEEK 7

Learning Indicator(s):

1. *Analyse daily activities/issues/businesses involving fractions, percentages and decimals.*
2. *Apply fractions, percentages and decimals to problems involving personal or household finance (such as utility bills, exchange rates, project budgeting, school sees, shopping, etc.)*

Theme or Focal Area(s)1: Application of Percentages 1

Definition/Introduction:

1. Percentage increase $= \frac{increment}{original\ price} \times 100\%$
2. Percentage decrease $= \frac{decrement}{original\ price} \times 100\%$
3. **Commission**: A fee paid to an individual or entity for facilitating a transaction or performing a service, commonly used as compensation for salespeople based on a percentage of sales.
4. **Discount**: A reduction in the listed price of a product or service, offered to encourage sales, attract customers, or clear inventory. *Can be a percentage off or a fixed amount deducted*.
5. **Profit**: The financial gain earned after deducting all expenses and costs from the total revenue generated by a business operation or investment. *A key indicator of financial success*.
6. Profit percent $= \left(\frac{profit}{cost\ price}\right)$ x 100%
7. **Loss**: The opposite of profit, occurring when expenses exceed revenue, resulting in a financial deficit or decrease in wealth/value for a business or investment.

Learning Tasks for Practice

1. Teachers review fractions, percentages and decimals and apply them to percentage increase and percentage decrease of given quantities.
2. Model real life examples of percentage increase and percentage decrease.
3. Extend the ideas to calculating other concepts such as discount, commission, utility bills, exchange rates, project budgeting, etc.

Application of the Concept and Examples:

Examples:

1. If the Public Utility and Regulatory Authority announced an increment in the price of diesel per litre from GH¢9.50 to GH¢13.20, which later declined to GH¢12.50. Find
   a. percentage increase
   b. percentage decrease.

Solution:

Percentage increase $= \frac{increment}{original\ price} \times 100\ \% = \frac{3.7}{9.50} \times 100\ \% = 38.95\ \%$.

Percentage decrease $= \frac{decrement}{original\ price} \times 100\ \% = \frac{0.7}{13.20} \times 100\ \% = 5.30\%$

2. Gladys deposited $GH¢4,000.00$ into a bank account and the annual simple interest rate is 8 % . How much interest is added to the accounts after 4 years?

Solution:

Principal = $GH¢4,000.00$, rate = $\frac{8}{100}$

Interest $= \frac{PTR}{100} = \frac{GH¢4,000.00 \times 8 \times 4}{100} = GH¢1,280$

Profit and loss;

1. Suppose a shopkeeper has bought $1kg$ of apples for $GH¢100.00$ and sold it for $GH¢120.00$ per kg. How much is the profit gained by him?

   Solution:

   Cost Price for apples is GH¢100.00,

   Selling Price for apples is GH¢120.00,

   Then profit gained by shopkeeper is; $P = SP - CP$

   $$P = 120 - 100 = \text{GH¢}20.00$$

2. For the above example calculate the percentage of the profit gained by the shopkeeper.

   Solution:

   We know, Profit percentage = $\left(\frac{profit}{cost\ price}\right) \times 100\%$

   Therefore, Profit percentage = $\left(\frac{\text{GH¢}20.00}{\text{GH¢}100.00}\right) \times 100\% = 20\ \%$.

3. If marked price = GH¢ 1700.00, selling price = GH¢1540.00 then find the discount.

   Solution:

   Given : *Marked Price* = *GH¢* 1700.00

   Selling Price = *GH¢* 1540

   By using the formula,

   Selling price = *Marked price* – *Discount*

   So, *discount* = *Marked Price* – *Selling Price*

   Discount = 1700 – 1540 = *GH¢* 160.00

   ∴ The discount is GH¢ 160.00

4. A man buys a fan for GH¢1000 and sells it at a loss of 15%. What is the selling price of the fan?

   Solution: Cost Price of the fan is GH¢1000.00

   Loss percentage is 15%

   As we know, Loss percentage = $\left(\frac{profit}{cost\ price}\right) \times 100\%$

   $15 = \left(\frac{LOSS}{1000}\right) \times 100\%$

   Therefore, Loss = GH¢150.00

   As we know,

   Loss = *Cost Price* − *Selling Price*

So, $Selling\ Price = Cost\ Price - Loss$

$= 1000 - 150$

$Selling\ Price =$ GH¢850.00

Alternatively, $Loss = 15\%\ 0f\ 1000 = 150$ therefore, $SP = 1000 - 150 = 850$

Calculating Commission and discount;

1. If marked price = GH¢ 990.00 and percentage of discount is 10%, then find the selling price.

   Solution:

   Given: Marked price = GH¢ 990.00

   discount = 10%

   Let the percentage of discount be x

   $\therefore x = 10\%$

   By using the formula,

   $$\text{Discount} = \frac{(\text{Marked price} \times \text{discount }\%)}{100\%} = \frac{990 \times 10}{100\%} = GH¢\ 99.00$$

   Now, *Selling price = Marked price – Discount* = 990 – 99 = *GH¢* 891.00

   ∴ The selling price is *GH¢* 891.00

Note: Alternatively, we can find 10% and subtract or find 90% as the amount.

2. If selling price = GH¢ 900.00 Discount is 20%, then find the marked price.

   Solution:

   Given: Selling *Price* = *GH¢* 900.00

   Discount = 20%

   Now, let us consider the marked price as GH¢ x.

   Given discount is 20% on the marked price.

   Discount = 0.2x

   By using the formula,

   Selling Price = Marked Price – Discount

   $900 = x - 0.2x$

   $900 = 0.8x$

   $x = 9\ 00\ \_\ 0.8$

   $= GH¢1125$

   ∴ The marked price is *GH¢* 1125.00

Alternatively, 80% = 900 [100 - 20% = 80%] So 100% = 900 * 100/80

Alternatively, we use proportions to find the marked price as follows

80% = 900 [100 − 20% = 80%]

So 100% = 900 * 100/80 = *GH¢* 1125.00

3. A farmer sold food grains for *GH¢*9200 through an agent. The rate of commission was 2%. How much commission did the agent get?

Solution:

Given:*Selling Price* = *GH¢* 9200.00

Commission rate = 2%

By using the formula, *Commission* = *Commission Rate* × *Selling Price*

= (2/100) × 9200

= GH¢ 184.00

∴ The agent got a commission of GH¢ 184.00

4. Kande sold flowers worth GH¢ 15,000.00 by giving 4% commission to the agent. Find

a. the commission she paid,
b. the amount received by Kande.

Solution:

Given: Selling Price = GH¢ 15000.00

Commission rate = 4%

By using the formula,

Commission = Commission Rate × Selling Price

Commission = $\frac{4}{100} \times 15000$

= GH¢ 600

Kande paid a commission of *GH¢* 600.00

Amount received by Kande = Selling Price – Commission

Amount received = 15000 – 600 = *GH¢* 14400.00

∴ The amount received by Kande is GH¢14400.00

Pedagogical Exemplars

Teachers should consider the following activities;

Experiential learning;

Apply fractions, percentages and decimals to problems involving personal or household finance (such as utility bills, exchange rates, project budgeting, school sees, shopping, etc.)

Calculate the percentage increases and decreases of a quantity using appropriate strategies. E.g.

- Percentage increase = $\frac{increment}{original\ price} \times 100\%$
- Percentage decrease = $\frac{decrement}{original\ price} \times 100\%$

Examples:

Question 1: The price of a laptop was initially GH¢4,800. After a 20% discount, the laptop's price was reduced. What is the new price of the laptop after the discount?

Solution: To calculate the percentage decrease, we use the formula:

Percentage decrease = $\frac{\text{decrease in value}}{\text{original value}} \times 100\%$

Given: - Original price of the laptop = *GH¢*4,800.00
- Discount percentage = 20%

Decrease in value = *Original value* × *Discount percentage*

Decrease in value = *GH¢*4,800.00 × (20 _ 100) = *GH¢*960.00

New price = *Original price* − *Decrease in value*

New price = *GH¢*4,800 − *GH¢*960 = *GH¢*3,840.00

Therefore, the new price of the laptop after the 20% discount is GH¢3,840.00.

Alternatively, for percentage decrease, we can subtract to get 100 % − 20 % = 80 % = 0.8

Now, by multiplying 0.8 × *GH¢*4,800.00 = *GH¢*3,840. 00

Question 2: A company experienced a 15% increase in sales this year compared to the previous year. If the sales for the previous year were GH¢8 million, what were the sales for the current year?

Solution: To calculate the percentage increase, we use the formula:

Percentage decrease = $\frac{\text{increase in value}}{\text{original value}} \times 100\%$

Given: - Previous year's sales = GH¢8 million
- Percentage increase in sales = 15%

Increase in value = Original value × Percentage increase

Increase in value = *GH¢*8,000,000 × (15 _ 100) = *GH¢*1,200,000

New value = Original value + Increase in value

New value = *GH¢*8,000,000 + *GH¢*1,200,000 = *GH¢*9,200,000

Therefore, the sales for the current year after a 15% increase are GH¢9,200,000.

Alternatively, for percentage increase, we can add to get 100 % + 15 % = 115 % = 1.15

Now, by multiplying 1.15 × *GH¢*8,000,000 = *GH¢*9,200,000

Discuss the following with students

Examples

1. A shop offers a 25% discount on all items during a sale. If a customer purchases a jacket with an original price of GH¢120, how much will they pay after the discount?

   Solution:

   Original price = GH¢120

   Discount percentage = 25%

   Discount amount = Original price × Discount % = GH¢120 × (25/100) = GH¢30

   Discounted price = Original price - Discount amount = GH¢120 - GH¢30 = GH¢90

   Therefore, the customer will pay GH¢90 for the jacket after the 25% discount.

2. A business made a profit of GH¢50,000 last year. If the business had to pay a 25% tax on its profit, how much tax did it pay?

   Solution:

   Profit = GH¢50,000

   Tax rate = 25%

   Tax amount = Profit × Tax rate = GH¢50,000 × (25/100) = GH¢12,500

   Therefore, the business paid GH¢12,500 in tax on its profit.

3. A company sold a product for GH¢800 and incurred a cost of GH¢650 to produce it. Calculate the profit or loss for the company.

   Solution:

   Selling price = GH¢800

   Cost of production = GH¢650

   Profit/Loss = Selling price - Cost of production = GH¢800 - GH¢650 = GH¢150

   Since the result is positive, the company made a profit of GH¢150 on the product.

Model similar examples with the students

Reflective Practice: Initiate activities that will lead the learner to reflect on the key concepts in the lesson.

Key Assessment

Level 3 Strategic reasoning.

1. A company's sales increased by 20% this year compared to the previous year. If the sales for the previous year were GH¢250,000, what were the sales for the current year?
2. A retail store purchases a television set for GH¢800 and plans to sell it with a mark-up of 30%. If the VAT rate is 12.5%, what will be the final selling price of the television set, including VAT?
3. A real estate agent earns a commission of 3% on the sale of a property. If the property was sold for GH¢450,000, how much commission did the agent receive?
4. A clothing store experienced a 25% decrease in sales during the previous year compared to the year before that. If the sales for the year before last were GH¢120,000, what were the sales for the previous year?
5. A household's monthly electricity bill is calculated based on the number of units consumed and the rate per unit. If the household consumed 600 units of electricity in a month and the rate per unit is GH¢0.75, calculate the total electricity bill for that month.
6. A family's monthly water bill consists of a fixed service charge of GH¢15 and a usage charge based on the number of cubic meters consumed. If the usage rate is GH¢2.50 per cubic meter and the family consumed 25 cubic meters of water in a month, calculate the total water bill for that month.
7. A small business has to pay a monthly Internet service fee of GH¢120 and a usage charge based on the amount of data consumed. If the usage rate is GH¢0.10 per gigabyte (GB) and the business consumed 250 GB of data in a month, calculate the total Internet bill for that month.

Reflective practice

Initiate activities that will lead the learner to reflect on the key concepts in the lesson.

Reflections: Conduct reflection on the following:

1. Profit and loss
2. Discount and commission.
3. Percentages increase
4. Percentages decrease
5. Applications of percentages to real life situations

WEEK 8: SIMPLE AND COMPOUND INTERESTS

Learning Indicators:

1. *Apply fractions, percentages, decimals to real life problems involving Percentage increase and percentage decrease, Profit, and loss,*
2. *Establish appropriate procedures solving problems involving simple and compound interests.*

Theme/Focal Area(s)1: Application of Percentages 2: Problem-Solving Involving Percentages

Definition/Introduction:

Principal: This is the sum of money lent or borrowed.

Interest: This is the extra money paid for taking the money as loan. This is often expressed as a percentage.

Key concepts:

Simple interest,
Compound interest.

Learning Tasks for Practice

Percentage Increase and Percentage Decrease:
Calculate the simple interest and compound interest

Simple Interest:

Task 1: Calculate the simple interest earned on a fixed deposit or savings account for a given principal amount, interest rate, and time periods.

Task 2: Solve word problems involving simple interest calculations in various contexts, such as loan repayments or rental income.

Compound Interest:

Task 3: Calculate the future value of an investment after a certain period, considering the principal amount, compound interest rate, and compounding frequency (annually, semi-annually, quarterly, etc.).

Task 4: Analyze the difference between simple interest and compound interest calculations for long-term investments and understand the impact of compounding.

Pedagogical Exemplars

Teacher should consider the following activities.

Experiential learning: teacher should review the week 7 with students and apply that to solve real life problems involving simple and compound interest.

Activity 1:

Using group collaboration, guide students to differentiate commission and discount and apply them to solve real life examples involving commission and discount**:**

Calculating Simple and Compound interest;

Review learners' background on interest and apply it to solve problems involving simple and compound interests.

Simple Interest

Simple interest is a method of interest that always applies to the original principal amount, with the same rate of interest for every time cycle. It can be calculated with the following formula: $S\,.\,I. = \frac{(P \times R \times T)}{100}$, where *P = Principal*, *R = Rate of Interest in* % *per annum*, and T = number of years. The rate of interest is in percentage is written as $\frac{R}{100}$.

Definition of key terms:

1. **Principal:** The principal is the amount that was initially borrowed (loan) from the bank or invested. The principal is denoted by P**.**
2. **Rate**: Rate is the rate of interest at which the principal amount is given to someone for a certain time, the rate of interest can be 5 % , 10 % , *or* 13 % , *etc*. The rate of interest is denoted by R.
3. **Time:** Time is the duration for which the principal amount is given to someone. Time is denoted by T.

Through change of subject, the formula $S\,.\,I. = \frac{(P \times R \times T)}{100}$ can be written as

$P = \frac{(100 \times S\,.\,I\,.)}{R \times T}$, $\quad R = \frac{(100 \times S\,.\,I\,.)}{P \times T}$ $\quad$ Or $\quad T = \frac{(100 \times S\,.\,I\,.)}{P \times R}$

Amount = *Principal* + *Simple Interest*

$\boldsymbol{A} = P + S\,.\,I.$

$\boldsymbol{A} = P + PTR.$

$\boldsymbol{A} = P(1 + TR).$

Example 1

A woman invests £250 in a building society account. At the end of the year her account is credited with 2% interest. How much interest had her £250 earned in the year?

Solution:

Interest = 2 % of £250 = $\frac{2}{100}$ × £250 = £5

Example 2

Nhyiraba invests £280 in an account that pays *r*% interest. After the first year she receives £5.60 interest. What is the value of r, the rate of interest?

Solution

After one year, the amount of interest is given by

$\frac{r}{100}$ × £280 = £5.60

$r = \frac{560}{280} = 2$

So, the interest rate, r is 2%.

Compound Interest: Is an interest calculated on the principal and the existing interest together over a given time. In compound interest, the principal amount with interest after the first unit of time becomes the principal for the next unit.

For example, when compounded annually for 2 years, the principal amount with interest accrued at the end of first year becomes the principal for the second year.

Compound Interest Formula:

Total compounded amount = $p\left(1 + \frac{r}{100}\right)^{nt}$

Where

- P is the principal amount
- r is the rate of interest
- n is the number of times the interest is compounded annually
- t is the overall tenure.

Compound Interest = $p\left(1 + \frac{r}{100}\right)^{nt} - p$

Amount per annum = $p(1 + r)^t$

Amount per semi-annual = $p\left(1 + \frac{r}{2}\right)^{2t}$

Amount per quarter-annual = $p\left(1 + \frac{r}{4}\right)^{4t}$

Amount per month == $p\left(1 + \frac{r}{12}\right)^{12t}$

Amount per day == $p\left(1 + \frac{r}{365}\right)^{365t}$

Example 3: Find the compound interest on GH¢3000 at 5% for 2 years, compounded annually.

Solution:

Amount with CI = 3000 $(1+ 5/100)^2$ **= GH¢3307.50**

Therefore, CI = 3307.5 – 3000 = GH¢307.50

Example 4: Find the compound interest on GH¢10000.00 at 12% rate of interest for 1 year, compounded half-yearly.

Solution:

Amount with CI = 10000 $[1+ (12/2 * 100)]^2$ = GH¢11236.00

Therefore, CI = 11236 – 10000 = GH¢ 1236.00

Problem 5: The difference between SI and CI compounded annually on a certain sum of money for 2 years at 8% per annum is GH¢ 12.80. Find the principal.

Solution:

Let the principal amount be x.

SI = x * 2 * 8 / 100 = 4x/25

CI = x $[1+ 8/100]^2$ **– x → 104x/625**

Therefore, 104x/625 – 4x/25 = 12.80

Solving which gives x, Principal = GH¢ 2000.

Model similar examples with the students

Key Assessment:

Mini project on proportional reasoning: Application to utility bills: water bills, light bills, telephone, etc.

Engage learners to embark on a mini-project by obtaining information on how to calculate any of the household utilities. They should obtain utility bills and apply their knowledge in percentages to calculate the bills.

Level 1: conduct a mini-project by obtaining information on how to calculate any of the household utilities

Level 2: conduct a mini-project by obtaining information on how to calculate any of the household utilities and record their findings

Level 3: conduct a mini-project by obtaining information on how to calculate any of the household utilities and record their findings and apply to solve every day problems.

Level 4: conduct a mini-project by obtaining information on how to calculate any of the household utilities and record their findings and apply to solve every day problems and design similar mini projects.

Reflective Practice: Initiate activities that will lead the learner to reflect on the key concepts in the lesson.

Applications

Simple Interest:

1. John deposited GH¢10,000 in a bank account that pays 6% simple interest per annum. How much interest will he earn after 3 years?
2. A company borrowed GH¢40,000 from a bank at a simple interest rate of 8% per year. If the loan is repaid after 4 years, how much interest will the company have to pay?

Compound Interest:

1. A sum of GH¢30,000 is deposited in a bank account that pays 6% interest compounded semi-annually. What will be the maturity value of the investment after 4 years?
2. Calculate the present value of GH¢50,000 due in 8 years, if the interest rate is 4% compounded annually.
3. A car is purchased for GH¢40,000 with a down payment of GH¢8,000. The remaining amount is financed at a simple interest rate of 7% per year for 3 years. Calculate the total amount to be paid back, including interest.
4. An investment of GH¢24,000 is expected to grow to GH¢36,000 in 6 years. Calculate the compound interest rate needed to achieve this growth.
5. If you invest GH¢10,000 today at an annual compound interest rate of 8%, how long will it take for your investment to double in value?
6. Rahul invested GH¢16,000 in a savings account that pays 5% interest compounded annually. How much will his investment be worth after 5 years?
7. A loan of GH¢20,000 is taken from a bank at a simple interest rate of 9% per year. If the loan is repaid in equal monthly instalments over 2 years, calculate the monthly instalment amount.

Section 2 Review

Over the last 5 weeks, we explored the basic concepts related to fractions, decimals and percentages and their applications to real life.

As we conclude Section two(2), it is important to reflect on the key concepts and progress achieved throughout the weeks covered in the teacher's manual. Here's a review of the topics explored:

Week 4: The focus was on understanding the concept and operations of fractions. This included essential arithmetic operations such as addition, subtraction, multiplication, and division, laying a solid foundation for further mathematical exploration.

Week 5: Problem-solving involving common fractions took center stage, providing students with opportunities to apply their understanding in practical scenarios. This fostered critical thinking and problem-solving skills.

Week 6: Decimal numbers and percentages were introduced, expanding students' numerical understanding beyond fractions. This section served as a bridge to more complex numerical systems and real-world applications.

Week 7: The focus shifted to the application of percentages in various contexts, highlighting the relevance of mathematical concepts in everyday life. Students gained insight into how percentages are used in finance, statistics, and other fields.

Week 8: The section concluded with an exploration of simple and compound interest, exposing students to fundamental financial principles. This topic provided valuable insights into economic concepts and further reinforced mathematical applications.

Resources

- Mathematical sets.
- Geo pegs and peg boards
- Number line
- Charged particles
- Fractional bords
- Muti-base Blocks,
- Wheel of theodoras
- Manella cards
- Bundles of sticks and loose ones, abacus
- Graph board/sheets and square papers
- Cuisenaire rods
- Technology tools such as computer, mobile phone etc.
- Computer software applications like GeoGebra.
- Tape measure, carpenters square, compass, geodot,
- Algebraic Tiles etc.

SECTION 3: ALGEBRAIC EXPRESSIONS & FACTORISATION

Strand: Algebraic Reasoning

Sub-Strand: Applications of Expressions, Equations and Inequalities

Content standard: Demonstrate knowledge and understanding of algebraic expressions and solve real-life problems on them.

Learning outcome: Formulate algebraic expressions using patterns to create models and solve real life problems (e.g., linear and quadratic models).

INTRODUCTION AND SECTION SUMMARY

Activities in everyday life demand that we interact with numbers. In Sections 1 and 2 of this manual, learners were taken through various concepts under the number system. This Section will introduce learners to Algebra which will assist them in solving mathematical expressions that consist of variables, constants and mathematical operations such as addition, subtraction, multiplication and division; generalize patterns and make predictions and solve equations and inequalities.

The 9^{th} and 10^{th} Weeks of the Section will assist learners to solve questions on Algebraic expressions with details on simplification and expansion which encompass monomial, binomials and trinomials; factorization which includes difference of two squares, algebraic fractions and zero/undefined algebraic fractions.

Learners are expected to develop critical thinking skills, problem solving abilities and logical reasoning which will assist learners in high-level mathematics courses and fields such as science, engineering, and economics

The weeks covered by the section are:

Week 9:

1. Use numbers, patterns, and variables to formulate mathematical expressions and apply the algebraic order of the four operations to solve problems
2. Factorisation of algebraic expressions

Week 10:

1. Perfect squares
2. Algebraic fractions

SUMMARY OF PEDAGOGICAL EXEMPLARS

This section requires hands-on activities where learners engage in practical activities involving algebraic expressions. Learners should be offered the opportunity to work in teams to develop their own real-life questions and find solutions.

Hence, experiential learning activities and Mixed-ability groupings should dominate the lessons on these concepts. All learners, irrespective of their learning abilities should be encouraged to participate fully in investigations as well as presentation of findings. However, make considerations and accommodations for the different groups. That is, offer approaching proficiency learners the opportunity to make oral presentations. Then, extend activities for the above average/highly proficient learners to using formulae and computer applications to solve problems.

ASSESSMENT SUMMARY

Many assessment strategies which vary from formative (mostly) to summative in which learners will be given various opportunities to simplify and to an extent assess themselves (assessment as learning) and also explain their thought process and reasoning behind their solutions will be used.

Assessment strategies which vary from Level 2 to Level 3 questions will be used. Teachers should conduct the following assessments and record the performances of learners for continuous assessment records;

1. class exercises (including individual worksheets) after each lesson
2. home works
3. quizzes
4. scores on practical group activities on algebraic expressions.

WEEK 9

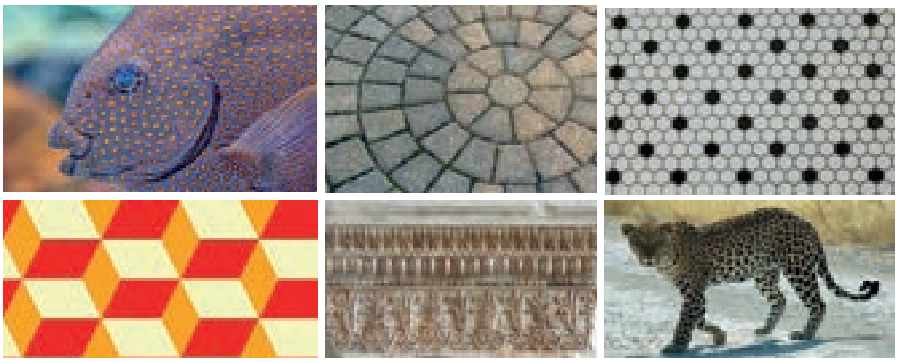
Learning Indicators:

1. *Use number patterns and variables to formulate mathematical expressions and apply the algebraic order of the four operations to solve.*
2. *Factorisation of algebraic expressions.*

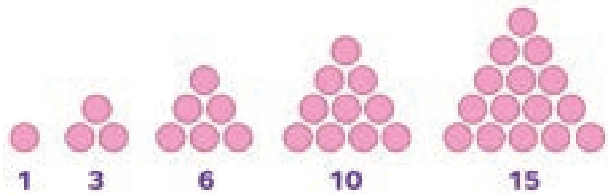
Theme or Focal Point 1: Number Patterns, Algebraic Expressions and Its Operations

Introduction

Number patterns are sequences of numbers that follow a certain rule or formula. These patterns can be found all around us, from the natural world to man-made structures. Understanding number patterns helps us make predictions, solve problems, and uncover the underlying order in seemingly random sets of numbers. Let us take a look at some examples of patterns in real life.



From the picture grid above, we realise that patterns can be found in all aspect of our lives. From our clothes, home decorations, building designs and even animals on both land and see, we see various beautiful patterns. Can you identify some patterns around you? Share with a friend.



In JHS we also learnt about patterns from shapes and numbers. Let us take a look at some examples.

Take a look at the pattern above, it is made up of circle. Can you determine how each next group of circles came about? We realise that each next pattern is generated by adding the next counting number to the preceding pattern, right?

But assuming we are to determine the number of circles in the 100^{th} term. Could that be easy? I'm sure you agree with me that it will be a challenging task. So, we will need to find a smart way to do that. This brings us to the need to find the rule or n^{th} term rule. This will help up to be able to find any term in any given pattern.

Now in this lesson our focus will be on writing **algebraic expressions** for the rules of the patterns that we will explore. Then in SHS 2 we will turn our attention to exploring patterns and sequences including arithmetic and geometric patterns.

Let's take a look at this pattern

Example 1

Pattern: 2, 5, 8, 11, 14, ...

Rule: The pattern is increasing by 3 each time.

Algebraic Expression: Let a_n represent the nth term in the pattern. The expression can be written as:

$a_n = 2 + 3(n-1)$

where n is the position of the term in the pattern. Hence, the algebraic expression is $2 + 3(n-1)$.

Example 2

The pattern below is made up of squares, made from lines, as shown below. Investigate the pattern, find the n^{th} term rule and write an algebraic expression for the number of lines required to make the pattern, if the pattern continues.

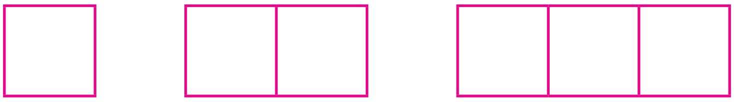


Fig. 1 **Fig. 2** **Fig. 3**

From the pattern, we realise that,

Fig 1	Fig 2	Fig 3	n^{th}
4	7	10	?

From the table let represent each term with n. Now, if we multiply n by 3 and add 1 we obtain the number of lines needed to create the shape in each term.

Therefore, our rule for the pattern is $a_n = 3(n) + 1$. Remember, $3(n) + 1$ is an algebraic expression.

Definition of key concepts

Algebraic expression in mathematics is an expression which is made up of variables, constants and arithmetic operations. (addition, subtraction, multiplication and division)

Example

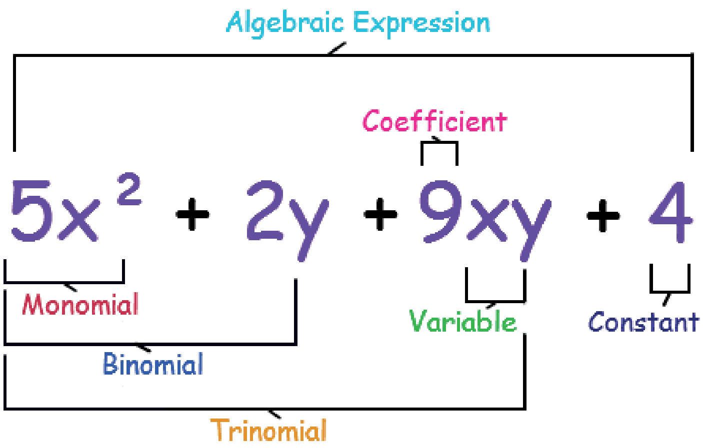
$3x - 5$, $n + 8$, x^2, $2x^2 + 3x - 5$, $2x + 5xy + 3$, etc.

1. **Term**: is the part of the expression, e.g. $x^2 + 5x - 4$ (three terms)
2. **Variable**: is a symbol (usually a letter), used to represent one or more numbers in algebraic expression (e.g. from above, x, n, are the variables)
3. **Monomials**: an algebraic expression with only one term (e.g.: y, $2x^2$, $-5a$ etc.)
4. **Binomial**: an algebraic expression with two terms (e.g.: $7x^2 + 3$, $y - 2$, $4x + 9$, $n - m$ etc.)
5. **Trinomial**: algebraic expressions with three terms (e.g: $2x^2 + 5x - 3$, $a - 5b + 2c$, $8x + 5y - z$)
6. **Coefficient**: is a number attached to a variable in an algebraic expression. Consider the expressions below and their responding coefficients

$x + 5x - 3$	$9a - 3b + 2$
1 5	9 -3

7. **Constant term**: is a number or symbol which is not attached to any variable in an expression. From the above expressions the constant terms are –3 and 2

The diagram below gives a summary of the terms explained.



Formulation of algebraic expressions

Algebraic expressions can be created using models and variables. For example, the figures shown above (model) and $2x^2 + 4x$ (variable).

Application of concepts in real world activities

a) Nine more than a certain number (x)

Solution: $x + 9$

b) Two less than one-third of a certain number (x) is

Solution: $\frac{1}{3}x - 2$Twice the square of a number (y) minus the cube of another number (n).

Solution: $2y^2 - n^3$

c) The age of Mr. Mensah is thrice his son's age (a) plus ten.

Solution: $3a + 10$

d) There are 25 oranges in a bag. Write the algebraic expression for the number of oranges in x number of bags.

Solution: $25x$

Rules for the use of the operations of algebraic expressions

1. **Addition and Subtraction**

In algebra, you can only add or subtract like terms, example $3x + 5x = 8x$, $2y - 5y = -3y$, $7a + 5b = 7a + 5b$

i. **Like terms**: Two or more terms are like terms if they have the same variables with the same exponent irrespective of their numerical coefficients. They can be added or subtracted to get a single term. For example: $3x^2$ *and* $7x^2$, $4xy$ *and* $2xy$ $5a^3b$ *and* $-7a^3b$.

ii. **Unlike terms**: are terms that cannot be added or subtracted in an expression to get a single term. Example: $3x^2$and a^2, $5y$ *and* $4xy$

2. **Multiplication and Division**

Both like terms as well as unlike terms can be multiplied and divided. In multiplying, we make sure of distributive property depending on the expression given. To divide, look for factors that are common to both numerator and denominator and this can be divided or cancelled. Factors that are common to all terms of an expression can be factored out.

Simplifying algebraic expression

Example: Simplify the following expressions.

- **a.** $x + 2y + 5x - y$
- **b.** $5p - c - 9c$
- **c.** $4x \times 2y$
- **d.** $x^2(x^3 - 3y)$
- **e.** $10b \div 2b$

Solutions

- **a.** $6x-y$
- **b.** $5p–10c$
- **c.** $8xy$
- **d.** $x^5–3x^2y$
- **e.** $5b$

Learning Tasks For Practice

Teachers engage and guide learners to expand and simplify given algebraic expressions. Ensure to include simple, intermediate as well as advanced tasks to cater for all groups of learners.

Theme/ Focal Area: Factorisation of Algebraic Expressions

In factorising algebraic expressions, we look for like terms, group them and find common factors. Factorising is the reverse process of expanding brackets in algebraic expressions. Algebraic expressions could be factorised in many ways depending on the given expression(s) and it includes the common factor approach, algebraic tiles, regrouping of terms approach, standard identity approach, splitting-the-middle-term approach and using the quadratic formula. For instance;

Worked Examples

Example: Factorise the following

1. $4x + 4by$
2. $ac + bc + ad + bd$

Solution

1. Look for the common factor(s)

   Factor the common term out (Highest common factor)

   $4x + 4by = 4(x + by)$

2. **First put the four terms in the expression into two groups of two and find the common factors from each. i.e., (*ac* + *bc*) + (*ad* + *bd*)**

   $c(a + b) + d(a + b)$ and add the outside terms and take one of the common terms to be the final answer. $ac + bc + ad + bd = (c + d)(a + b)$

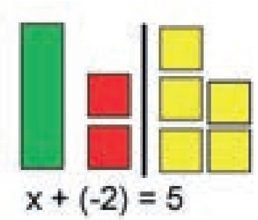
Factorising using algebraic tiles

Example 1

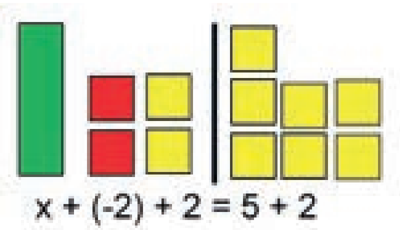
Solve x + (–2) = 5 using the algebraic tiles

Solution

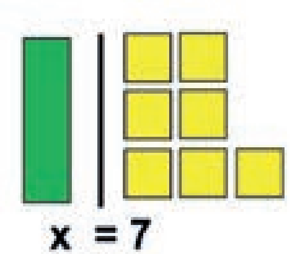
Step 1: Let's represent the equation with our algebraic tiles



Step 2: Transpose –2 by adding 2 to both sides (yellow squares are positive while red squares are negative numbers)



Step 3: Simplify



Factorisation by regrouping of terms approach

In some algebraic expressions, not every term may have a common factor. For instance, consider the algebraic expression, 12a + n − na – 12. The terms of this expression do not have a particular factor in common but the first and last term has a common factor of '12'. Similarly, second and third term has n as a common factor. So, the terms can be regrouped as:

⇒ 12a + n – na – 12 = 12a – 12 + n – an

⇒ 12a – 12 – an + n = 12(a −1) –n(a −1)

After regrouping, it can be seen that (a–1) is a common factor in each term,

⇒ 12a + n −na – 12 = (a − 1) (12 – n)

Thus, by regrouping terms we can factorise algebraic expressions.

Factorising expressions using standard identities

An equality relation which holds true for all the values of variables in mathematics is known as an identity. Consider the following identities:

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$a^2 - b^2 = (a+b)(a-b)$$

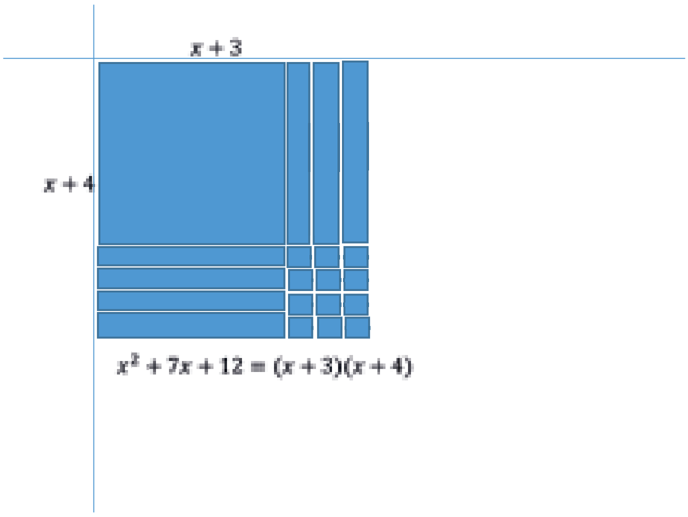
On substituting any value of *a and* *b*, both sides of the given equations remain the same. Therefore, these equations are called identities.

Factorise quadratic trinomials using algebraic tiles

Example: Factorise $x^2 + 7x + 12$ using algebraic tiles

Solution

Use algebraic tiles



Factorise quadratic trinomials using the standard identities

Example 1:

Solve $x^2 + 6x + 9 = 0$

$x^2 + 6x + 9 = 0$

This can be written as:

LHS is of the form $a^2 + 2ab + b^2$,

$\Rightarrow (x + 3)^2 = 0$

Or

$(x + 3)(x + 3) = 0$

Factors are $(x + 3)$ and $(x + 3)$.

Factoring quadratic equation using formula approach

This method is almost similar to the method of splitting the middle term.

Step 1: Consider the quadratic equation ax2 + bx + c = 0

Step 2: Now, find two numbers such that their product is equal to ac and sum equals to b.

(number 1) (number 2) = ac

(number 1) + (number 2) = b

Step 3: Substitute these two numbers in the formula given below:

(1/a) [ax + (number 1)] [ax + (number 2)] = 0

Step 4: Finally simplify the equation.

Example: Solve $3x^2$ **+ 7x + 4 = 0**

Solution:

$3x^2 + 7x + 4 = 0$

Here, a = 3, b = 7, c = 4

ac = (3)(4) = 12

Let's identify two numbers such that their sum is 7 and the product is 12.

Factors of 12: 1, 2, 3, 4, 6, 12

Sum of two factors = 7

Product of those two factors = 12

Number 1 = 3 and number 2 = 4

Now, substitute these two numbers in the formula (1/a) [*ax* + (number 1)] [*ax* + (number 2)] = 0.

$(\frac{1}{3})(3x + 3)(3x + 4) = 0$

$(3x + 3)(3x + 4) = 0$

$\frac{1}{3} \times 3(x + 1)(3x + 4) = 0$

$(x + 1)(3x + 4) = 0$

Thus, $(x + 1)$ and $(3x + 4)$ are the factors of the given quadratic equation.

Factorization of quadratic equation by splitting the middle term

Step 1: Consider the quadratic equation $ax^2 + bx + c = 0$

Step 2: Now, find two numbers such that their product is equal to *ac* and sum equals to b.

(number 1) (number 2) = *ac*

(number 1) + (number 2) = *b*

Step 3: Now, split the middle term using these two numbers,

ax^2 + (number 1) x + (number 2) x + c = 0

Step 4: Take the common factors out and simplify.

Let's have a look at the example problem given below:

Example: Solve the quadratic equation $x^2 + 7x + 10 = 0$ by splitting the middle term.

Solution:

Given,

$x^2 + 7x + 10 = 0$

Here, a = 1, b = 7, c = 10

ac = (1) (10) = 10

Factors of 10: 1, 2, 5, 10
Let's identify two factors such that their sum is 7 and the product is 10.
Sum of two factors = 7 = 2 + 5
Product of these two factors = (2)(5) = 10
Now, split the middle term.
$x^2 + 2x + 5x + 10 = 0$
Take the common terms and simplify.
$x(x + 2) + 5(x + 2) = 0$
$(x + 5)(x + 2) = 0$
Thus, $(x + 2)$ and $(x + 5)$ are the factors of the given quadratic equation.
$x+ 2=0$ $x=0–2$ $x= –2$
$x+ 5=0$ $x=0–5$ $x= –5$
Solving these two linear factors, we get x = –2, –5 as roots.

Factoring quadratic equation using quadratic formula

In quadratic formula, to get the roots of a quadratic equation $ax^2 + bx + c = 0$ is given by

$x = [-b \pm \sqrt{(b^2 - 4ac)}]/ 2a$

Substituting the values of a, b, c and simplifying the expression, we get the values of roots.

For example;
Given,
$x^2 + 4x – 21 = 0$
Here, a = 1, b = 4, c = –21
$b^2 – 4ac = (4)^2 – 4(1) (–21) = 16 + 84 = 100$
Substituting these values in the quadratic formula, we get;

$x = [–4 \pm \sqrt{100}]/ 2(1)]$
$= (–4 \pm 10)/2$
$x = (–4 + 10)/2, x = (–4 – 10)/2$
$x = 6/2, x = –14/2$
$x = 3, x = –7$
Therefore, the factors of the given quadratic equation are (x – 3) and (x + 7).

Learning Task for Practice

Learners to perform tasks on the following areas. Please ensure to include a variety of tasks that are of different difficulty levels to help all groups of learners.

1. Factorise expressions using algebraic tiles
2. Factorise expressions by regrouping of terms approach
3. Factorise quadratic trinomials using algebraic tiles
4. Factorise quadratic trinomials using the standard identities
5. Factorise quadratic equation using formula approach
6. Factorise quadratic equation by splitting the middle term
7. Factorise quadratic equation using quadratic formula

Pedagogical Exemplars

Review Previous concepts: Learners have been taught patterns and finding rules for patterns both in words and algebraic expressions. Therefore, take time to review these concepts and note down any misconceptions that learners may carry and address them immediately or as and when appropriate.

Collaborative learning: Make use of group and pair activities to engage learners to explore simple patterns to generate rules and write them in algebraic form. Ensure to include a wide range of tasks that appeal to the various groups of learners. Particularly, make accommodations for learners who may be struggling by using advance learners to support them.

Problem-based & experiential learning: Using mixed-ability groups, learners formulate mathematical expressions by investigating patterns and its applications in everyday life. For example, house numbers in the street, seating in the café and church.

Problem-based learning: In convenient mixed-ability groups, guide learners to simplify the addition, subtraction, multiplication and division of algebraic expressions. Please ensure to include tasks for beginning learners, intermediate learners as well as advance learners.

Individual tasks: Present learners with individual worksheets to complete. Also, ask learners to write some word expressions for their friends to write their algebraic expressions.

Learners in mixed ability/gender groups, investigate the concept of factorisation by reviewing expansion of algebraic expressions and apply the idea to factorise expressions.

Problem-based learning: In mixed-gender groups, engage learners to explore how to factorise algebraic expressions using the common factor approach. Then, review with learners the concepts of finding the area of a rectangles and use the idea to factorize the trinomial expressions using algebraic tiles**.**

Pair activities: using think pair shares, learners factorize direct and indirect expressions using algebraic tiles**.** Then task learners to factorise algebraic expressions using the regrouping-of-like-terms and standard identities approaches.

Problem-based learning: In mixed gender groups, factorise quadratic equations using the splitting-the-middle-term and the quadratic formula approaches and apply in their real world.

Key Assessment

Level 3

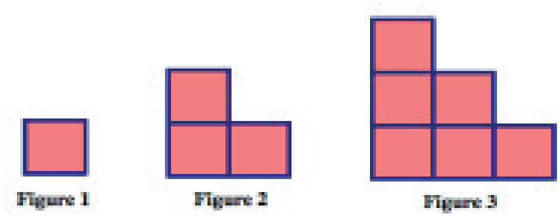
Answer the following questions

1. Identify patterns from the following sets of numbers and create a mathematical expression for each sequence.
   a. 1, 3, 5, 7, 9…
   b. –1, 2, 5, 8, 11…
2. Ama is collecting seashells on the beach. On her first day, she collected 1 seashell. Each day after that she collected 2 more seashells than the previous day. How many seashells will she have on the fifth day? Generate a pattern and use it to formulate mathematical expression.
3. In a math competition, participants are asked to solve a series of problems. In the first round, each contestant solves 1 problem. In each subsequent round, they solve 2 more problems than in the previous round. How many problems will they need to solve in the 7th round?

Level 2

1. Expand the algebraic expressions.
   a. $(x + 5)(x + 3)$
   b. $(x + 2)\ (x - 4)$
   c. $(y + 7)\ (y + 1)$
   d. $(z + 2)\ (z - 5)(x + 1)$

2. How many squares will be in the 5^{th} pattern?



3. Factorize completely the following
   a. $xy + 2xz$
   b. $4(4n - 12x)$
   c. $3x^2 - 9xy$
   d. $6ab - 4pb - 2pq + 3aq$

4. Factorize completely the following algebraic expressions
   a. $x^2 + 2x - 3$
   b. $6 - 5x - x^2$
   c. $3x^2 - 17x + 10$
   d. $p^2 - 1$

5. The length of a rectangular field is 5 more than its width. Write down an expression for the perimeter of the field.

6. The length of a rectangular garden is 4 meters longer than its width. Write an expression to find the area of the garden.

WEEK 10

Learning Indicators:

i. *Recognize perfect squares and apply the idea to solve problems including the difference of two squares of binomials.*

ii. *Analyse and apply operations on simple algebraic fractions including monomial and binomial denominators and determine the conditions under which algebraic fraction is zero or undefined.*

Theme/Focal Area: Perfect Squares

A perfect square is an integer that can be expressed as the square of another number. E.g. 4 is a perfect of 2, because it can be expressed as $2^2 = 2 \times 2$

Example: Add consecutive odd numbers to generate some perfect squares

1, 2, **3**, 4, **5**, 6, **7**, 8, **9**, 10…

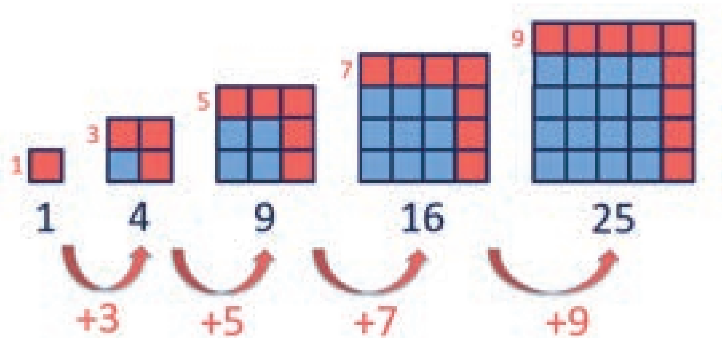
$1 = 1 = 1^2$

$1 + 3 = 4 = 2^2$

$1 + 3 + 5 = 9 = 3^2$

$1 + 3 + 5 + 7 = 16 = 4^2$

$1 + 3 + 5 + 7 + 9 = 25 = 5^2$



Perfect square and difference of two squares (square with expression as the sides and some taken out, the area of the rest

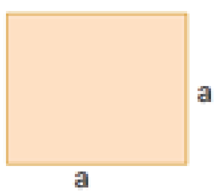


Fig. 1

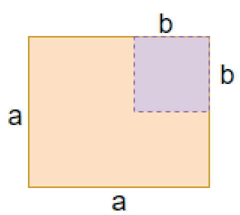


Fig. 2

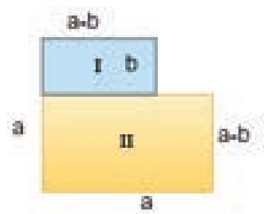


Fig. 3

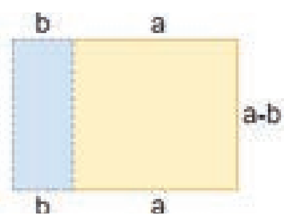


Fig. 4

1. Fig I above shows a square of **a** side and its Area= $\boldsymbol{axa} = \boldsymbol{a}^2$

   $\mathbf{a^2}$ is the perfect square of **a**

2. Fig. 2 shows the result of taking a square of side **b** from one of the four corners of Fig. 1. The result from Fig. 2 gives Fig. 3.

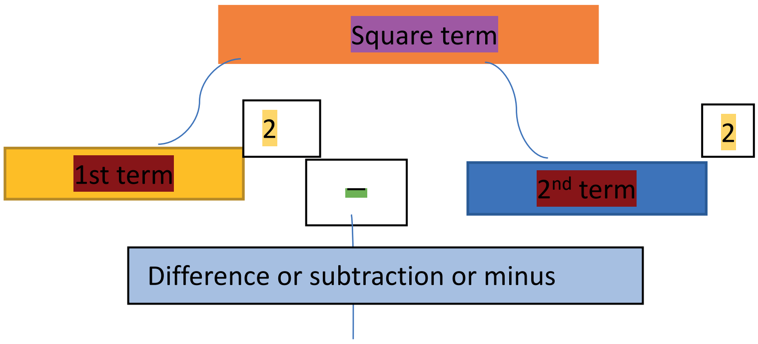
   The Area of Fig. 3 = Area of region I + Area of region II

   ((a -b)×b)+(a×(a-b)) $=a^2 - b^2$

   Area of Fig. 4 (a+b) (a-b) = Length x breadth

   Area of Fig. 3 = area of Fig. 4

   (a + b) (a - b) = $a^2 - b^2$



For example, $x^2 - 4$, $y^2 - 1$, $a^2 - b^2$ etc

$x^2 - 4 = (x + 2)(x - 2)$, $y^2 - 1 = (y + 1)(y + 1)$, $a^2 - b^2 = (a-b)(a+b)$

NOTE: *Anytime there is a binomial with each term being squared (i.e. having an exponent of 2) and subtraction as the middle sign then you are guaranteed to have the case of difference of two square.*

Learning Task for Practice

Engage learners to solve problems on perfect squares.

Theme/Focal Area: Algebraic Fractions

Definition of key concepts:

Algebraic fraction is a fraction that contains at least one variable. For example,

$\frac{x}{2}$, *x is the numerator.* $\frac{3}{x+1}$, *the denominator is an expression in terms of x*

$\frac{x+5}{2x}$, *both the numerator and the denominator contain an x term*

Just as ordinary fractions, you can add, subtract, multiply and divide them.

Application of concepts

To simplify algebraic fractions, we do it in the same way as numerical fractions. We cancel common factors from the numerator and denominator until no common factors remain.

Note the following:

1. Algebraic fractions can be added or subtracted if they have the same denominators. (a common denominator). $\frac{4}{3x} + \frac{1}{x}$ *or* $\frac{2x}{5} - \frac{3y}{5}$ *or* $\frac{x}{x-1} + \frac{5}{x-1}$ and use the Lowest Common Multiple (LCM) approach when the denominators are not the same.

2. When the fraction is multiplying, such as
$\frac{m}{n} \times a = m\frac{a}{nb}$ where there are common factors, then cancel out if not, multiply the numerators and the denominators separately.

3. When a fraction is divided by another, multiply the first fraction by the reciprocal of the second fraction. Thus $\frac{m}{n} \div a = m \times b = m\frac{b}{na}$ etc.

Operations on algebraic fractions including monomial and binomial denominators

Examples

Simplify the following.

i. $\frac{10}{x-4} + 2\frac{}{+1}$

Solution

$$\frac{10(x+1)+2(x-4)}{(x-4)(x+1)}$$

$$\frac{10x+10+2x-8}{(x-4)(x+1)}$$

$$\frac{12x+2}{(x-4)(x+1)}$$

ii. $\frac{x}{x+1} - 2\frac{x}{x+2}$

Solution

$$\frac{x(x+2)-2x(x+1)}{(x+1)(x+2)}$$

$$\frac{x^2+2x-2\,x^2-2x}{(x+1)(x+2)}$$

$$\frac{-x^2}{(x+1)(x+2)}$$

iii. $\frac{2x}{1} \times \frac{2y}{1}$

Solution

$4xy$

iv. $\frac{6x+8}{4} \div \frac{x^2+3}{5\,x^2}$

Solution

Simplify the numerator and denominator of the fraction $\frac{6x+8}{4}$.

$$\frac{6x+8}{4} = 2\frac{(3x+4)}{4} = 2\frac{(3x+4)}{2 \times 2} = 3\frac{x+4}{2}$$

v. Simplify the fraction $\frac{x^2+3}{5\,x^2}$:

Since the numerator and denominator cannot be simplified further, we leave it as is.

Now, the expression becomes:

$$\frac{3x+4}{2} \div \frac{x^2+3}{5\,x^2}$$

To divide fractions, we multiply by the reciprocal of the second fraction:

$$\frac{3x+4}{2} \times \frac{5x^2}{x^2+3}$$

Multiplying the numerators and denominators:

$$\frac{(3x + 4) \times 5x^2}{2 \times (x^2 + 3)}$$
$$\frac{15x^3 + 20x^2}{2x^2 + 6}$$

Learning Task for Practice

Engage learners with tasks on operations on algebraic fractions including monomial and binomial denominators. The tasks should cover all the four operations and must include tasks of varying difficulty to cater for the different learning needs.

Conditions for algebraic fraction to be undefined or zero

Algebraic fraction is said to be undefined or have no meaning if the denominator is equal to zero.

Note:

$\frac{\text{Dividend}}{\text{Divisor}}$ = Quotient⇒ Quotient × Divisor = Dividend

Example 1:

$\frac{16}{2} = 8$. This implies $8 \times 2=16$

$\frac{0}{13} = 0$. This implies $13 \times 0 = 0$

Now;

$\frac{5}{0} \neq 0$, This implies $0 \times 0 \neq 5$ (It cannot be defined)

Example 2: Condition under which $\frac{3a}{a-4}$ is undefined and determine the value of a

Condition: a – 4 = 0

a = 4

Condition: under which the fraction is zero, put the whole fraction to zero

$\frac{3a}{a-4} = 0$, 3a = 0, a=0 Therefore when a is 0 the fraction is zero

Example 3:

Find the value of x which makes the expressions undefined:

i. $\frac{3}{x-1}$

Solution

For the expression $\frac{3}{x-1}$ to be undefined, the denominator $x-1$ must be equal to zero: $x-1=0$ Solving for x:

$x = 1$

Therefore, the expression is undefined at $x=1$.

ii. $\frac{(2x-1)(x-4)}{4x^2-1}$

Solution

For the expression $\frac{(2x-1)(x-4)}{4x^2-1}$ to be undefined, the denominator $4x^2-1$ must be equal to zero:

$4x^2-1= 0$

This can be factored as a difference of squares:

$(2x+ 1)(2x-1)=0$

Setting each factor to zero:

$2x + 1 = 0$ or $2x - 1 = 0$

Solving for x in each case:

$x = - 1 \text{ or } x = 1 -$

Therefore, the expression is undefined at $x = -\frac{1}{2}$ *and* $x = \frac{1}{2}$.

Pedagogical Exemplars

1. **Revision of concepts:** Review previous concepts on algebraic expressions and factorisations from previous week.
2. **In convenient groups activity**, learners explore the concept of perfect squares using whole numbers.
3. **Using problem-based learning approach:** Learners in a mixed/gender group, investigate the concept of perfect square using paper cut /graph sheet and stickers out.
4. **Using talk for learning:** Learners work collaboratively to brainstorm the meaning of perfect squares, identify differences of two squares, and use problem-solving strategies to apply the concepts of perfect squares.
5. **Using talk for learning strategy**, the class review normal fractions, discuss and explain what an algebraic fraction is and apply operations on simple algebraic fractions.
6. **Collaborative learning:** using think pair share in mixed ability groups, learners investigate, identify and explain the condition under which an algebraic fraction is zero or undefined. In a mixed gender/ability groups, engage learners to simplify algebraic fractions which involves monomial and binomial denominators.
7. **Experiential learning:** In mixed ability/gender/cultural groups, discuss and solve questions on the conditions under which algebraic fractions are referred to as undefined or zero and present the findings to the larger class.

Key Assessment

Assessment Level 2:

Simplify the following

a. $\frac{x}{x^2 - 5x + 6} + 1\frac{}{-2} + 3\frac{}{-3}$

b. $\frac{2x - 1}{3} - x\frac{+3}{2}$

c. $\frac{x - 2}{3} + x\frac{+3}{5}$

d. $\frac{5x}{3} \times x\frac{+3}{2}$

e. $\frac{2x - 1}{2x} \div 4\frac{x^2 - 1}{6}$

Assessment Level 3

Solve the following questions

1. Identify with reasons which of the following fractions have monomial or binomial denominators. $\frac{2}{5x}, \frac{y}{x + 1}, \frac{x - 3}{x + 6}, \frac{7y - 1}{x^2 - 4}, \frac{3z + 4}{z^2}$

2. At a youth club $\frac{2}{5}$ of those present were playing football and $\frac{1}{4}$ were playing other games.

i. what fraction was playing games?
ii. what fraction was not playing games?

Assessment Level 2

1. Given that $p = 2\frac{x}{1 - x^2}$

And $q = 2\frac{x}{1 + x}$

simplify 3p – 2q

2. Simplify the following and state the value of the variable that makes the expression undefined or zero.

i. $\frac{4m^2 - 9}{(m - 1)^2} \div 2\frac{m + 3}{m^2 - m}$
i. $\frac{6x^2 + 12x}{x^2 + x} \times \frac{x^2 - 1}{x^2 + 10x + 16}$
ii. $\frac{y^2 - 7y + 12}{y^3 - 9y}$

3. Explain how to factorize completely the following

i. $4x^2 - 9$
ii. $16y^2 - 25a^2$
iii. $a^2b^2 - 121$
iv. $4m^2 - 100$

4. Write the following numbers as perfect squares

a. 121

b. 81

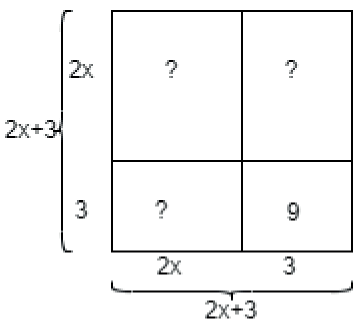
c. 64

d. 169

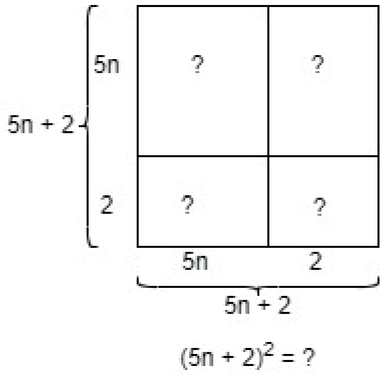
e. 100

Assessment Level 3

1. Copy and complete the following diagrams by filling in the area of each region and the write, in simplified form, an expression for the entire square.



$(2x + 3)^2 = ?$



2. Find the values for the following unknown using the idea of perfect squares

a. $3z + 132 = 162$

b. $3.462 - 1.542 = 10p$

Section 3 Review

This section reviews all the lessons taught for weeks 9 and 10. A summary of what the learner should have learnt.

REFLECTION

1. **Review of algebraic expression**
   a. Algebra is used to express thoughts, ideas etc.
   b. Algebraic expressions are used in various real life scenarios, including physics, engineering, economics and more
   c. Algebraic expressions can be created using models and variables
   d. Variables (letters) are used to represent the unknown
   e. We can only add or subtract like terms
   f. To factorise we look for the factors and the highest common factor.

2. **Review of factorisation**
   In factorising algebraic expressions, learners look for like terms, group them and find common factors. There are different ways/methods of factorizing depending on the given expression. It includes
   a. Factorisation by common factor approach
   b. Factorisation by regrouping terms
   c. Factorisation of quadratic trinomials using algebraic tiles
   d. Factorisation of quadratic trinomials using identities
   e. Factorisation of quadratic equations by splitting the middle term
   f. Factorisation of quadratic equations using the quadratic formula

3. **Review of algebraic fractions**
   a. Algebraic fractions are fractions that can be expressed as a _ b where $b \neq 0$
   b. Algebraic fractions need to have a common denominator when performing addition and subtraction.

c. The value of the fraction does not change if the numerator and denominator are multiplied by the same number or expression

4. **Review of perfect squares**
   a. Perfect squares are also called square numbers
   b. *Perfect square is a value / integer that can be expressed as the square of another number.* Perfect square values are always positive or equal to zero.
   c. Perfect numbers cannot be negative.
   d. The difference of squares may be positive, negative or zero depending on the order of the numbers taken.
   e. Difference of two squares is expressed as $a^2 - b^2$ and be factorized as $(a + b)(a - b)$.
   f. Difference of two squares can be used to factorize expressions, simplify calculations and simplify expressions.

Teaching/Learning Resources

Algebraic tiles, cut out shapes of different colours, A4 sheets, coloured pencils/pens, paper grids etc.

References

1. Foundation, C. K. (2017): Creating algebraic expressions from fractions. Retrieved from Creating Algebraic Expressions from Patterns | CK-12 Foundation (ck12.org)

2. J. Hacker (2024). Revision notes on algebraic expressions and identities. Retrieved from Revision Notes for Maths Chapter 9 - Algebraic expressions and identities (Class 8th) | askIITians

SECTION 4: LINEAR EQUATIONS, RELATIONS AND FUNCTIONS

Strand: **Algebraic Reasoning**

Sub strand: Patterns & Relationships (PR)

Content Standard: Demonstrate understanding of mapping, relations, and functions and the ability to interpret the graph of a function and its applications in real-life problems in a different context.

Learning outcome: *Distinguish between relations and functions including determining the rules for functions, then draw graphs of functions and interpret them.*

INTRODUCTION AND SECTION SUMMARY

In the first part of Section 4 of this manual, facilitators are expected to take learners through Linear equations and inequalities. Aspects of linear equations and inequalities to be taught includes change of subject of a relation; Linear equations in One and Two variable(s), Equations with brackets and fractions; Linear inequalities and number lines; and real world situations involving linear equations and inequalities.

The second part of the section will also espouse on Relations and Functions. Emphasis will be placed on the concept of relations and functions and their types; rules of mapping; gradient and equation of a straight line; magnitude of a line segment; and graphs of linear equations.

These concepts are used in the fields of physics, engineering, and economics and many others to model and solve real-life problems. It is expected that understanding these concepts will assist learners in making informed decisions and solve practical problems efficiently.

The weeks covered by the section are:

Week 11:

1. Construct and interpret formulae for a given task and apply to problems involving a change of subjects.
2. Solve linear equations involving one and two variable(s); brackets and fractions and relate to real world problems.
3. Find solution set or truth set of linear inequalities and illustrate this on a number line.

Week 12:

1. Identify relations from functions and differentiate between the types of relations and functions.
2. Determine the rule of a given mapping.

Week 13:

1. Find the distance (magnitude) between two points.
2. Find the gradient and the equation of a straight line between two points on the line.
3. Draw line graphs for given relations and read values from the graphs.

SUMMARY OF PEDAGOGICAL EXEMPLARS

This section requires hands-on activities where learners engage in practical activities of linear equations and inequalities; and relations and functions. Learners should be given the platform to work in teams to develop their own real life questions and find answers.

Therefore, Experiential learning activities and Mixed-ability groupings should dominate the lessons on these concepts. All learners, irrespective of their learning abilities should be assisted to take part fully in investigations as well as presentation of findings. However, make considerations and accommodations for the different groups. That is, offer approaching proficiency learners the opportunity to make oral presentations. Then, extend activities for the above average/highly proficient learners to using formulae and computer applications to solve problems.

ASSESSMENT SUMMARY

Assessment methods which range from quizzes, tests, and homework assignments can be used to evaluate learners understanding of concepts and their ability to solve problems. Performance tasks like solving real-world problems involving linear equations and inequalities will also be used to assess learner's application of these mathematical skills.

Also, relations and functions will make use of various visual aids such as diagrams of types of mappings; and charts on graphs and interactive mapping tools will also be incorporated to engage learners in hands-on learning experiences.

Assessment strategies which vary from Level 1 to Level 4 questions of the DoK will be used. Teacher should record the performances of learners for continuous assessment records.

WEEK 11

Learning indicators:

1. *Construct and interpret formulae for a given task and apply to problems involving a change of subjects.*
2. *Solve linear equations in one and two variable(s); and brackets and fractions for given problems and relate it to real life situations.*
3. *Find solution set or truth set of linear inequalities and illustrate on the number line.*

Theme/Focal Area (S) 1: Change of Subjects

Definition of key concepts

A **formula** is a rule which gives the relationship between things or quantities. The letters in the formula always stand for something specific, like cost, speed, number of books. There is always more than one unknown in a formula. E.g. C=4v + 5. The perimeter of a rectangle P=2l + 2b, the area of a triangle$A = \frac{1}{2}bh$.

Application of concepts and examples

Change of subjects and its application of relation

Examples

1. Make c the subject of the relation $y = mx + c$

   Solution

   To make c the subject, subtract mx from both sides of the equation.

   $\boldsymbol{mx - mx} + c = y - mx$

   $\therefore \boldsymbol{c = y - mx}$

2. From the equation $3c + 2r = md + k$, make r the subject.

   Solution

   Given $3c + 2r = md + k$,

   Make 2r the subject:

   $\therefore 2r = md + k - 3c$

   Divide both sides by 2.

   $\therefore \ r = m\frac{d + k - 3c}{2}$

3. The relation between energy E, mass m, and velocity of light v, is given by $E = mv^2$.

   Find the value v, when $E = 20$ and $m = 5$

   Solution

   Given that $E = mv^2$

   $\therefore \ v^2 = \frac{E}{m}$

   $\therefore v = \sqrt{\frac{E}{m}}$

   $v = \sqrt{\frac{20}{5}} \ \therefore v = 2$

Theme/Focal Area (S) 2: Linear Equations

Definition of key concepts

An *equation* of the form ax + b=c, where a, b and c real numbers, and a≠0.Linear equation in one variable has exactly one solution.

Linear equations involving one and two variable(s)

Linear equations in one variable have exactly one solution.

Example 1

Solve for the variable indicated in the following equations.

a) $3x - 12 = 21$

b) $5 - 3\boldsymbol{y} = 3\boldsymbol{y} + 7$

Solution

(a) To solve $3x - 12 = 21$**,**

Add 12 to both sides of the equation, $3x = 33$

Then divide both sides of the equation by 3, to make x the subject

$x = 3\frac{3}{3} = 11$

(b) To solve $5 - 3\boldsymbol{y} = 3\boldsymbol{y} + 7$

First group the like terms $5 - 7 = 3y + 3y$

Simplify the terms $-2 = 6y$

Make y the subject by dividing both sides by coefficient of y.

Therefore $y = -\frac{1}{3}$

Linear equations involving brackets

Learners are expected to apply the concept of distributive property in the expansion of brackets.

Example

Solve the equation $7(x - 6\ ) = 3(x + 9)$

Answer

To solve the equation$7(x - 6\ ) = 3(x + 9)$,

First multiply the brackets (expansion) on both sides of the equation,

$7x - 42 = 3x + 27$

Then subtract $3x$ from both sides $4x - 42 = 27$

Then add 42 to both sides of the equation $4x = 69$

Divide both sides of the equation by 4.

$x = 6\frac{9}{4} = 17\frac{1}{4}$

Linear equations involving fractions

This concept was treated in the JHS Curriculum and as such teachers are expected to give the learners a few examples.

Example: Solve $\frac{3x}{2} - 2 = \frac{1}{2}$

Solution

Solve $\frac{3x}{2} - 2 = \frac{1}{2}$

1. Eliminate the fraction by multiplying both sides of the equation by the reciprocal of the fraction coefficient $= 2 \times \frac{3x}{2} - 2 \times 2 = \frac{1}{2} \times 2$
2. Group the like terms and simplify the equation

   $3x - 4 = 1$

   $3x = 1 + 4$

   Dividing both sides by 3, $x = \frac{5}{3}$

Linear Inequalities

Linear inequality in one variable is of the form $ax + b < c$, $ax + b \leq c$, $ax + b > c$, $ax + b \geq c$.

Folding the human right arm resembles the idea of greater than (>) symbol and that of the left arm also resembles the idea of less than (<) symbol.

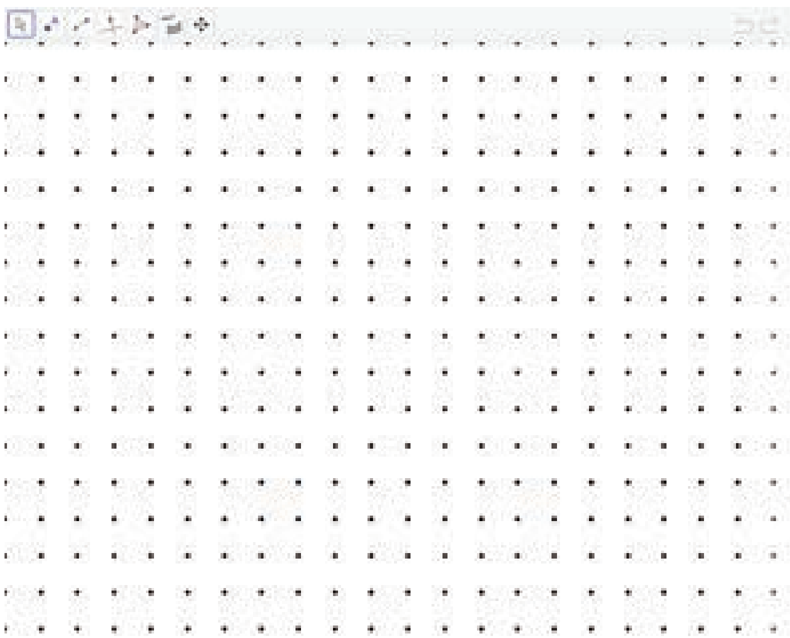
$4 < 5$(4 is less than 5)

$6 > 4$(6 is greater than 4)

$x \leq 4$(Depending on the values of x, which is an unknown variable)

Suppose we are given an inequality of the form $x > -5$, the solution set for an inequality (as it is for an equation) is the set of all values for the variable that make the inequality a true statement.

An appropriate way to picture the solution set is by a graph on a **number line** or the use of **Geodot** to generate conjectures. A sample of the geodot is shown here.



Discuss graphs of inequalities and graph the set; $\{x: x < 4\}$

Explanations: We want to include all real numbers less than 4, that is, to the left of 4 on the number line. An open circle is used to indicate that the point corresponding to 4 is not included in the graph. It is called **an open half line**; it extends to the left and not including4.

Two other symbols as shown in the introduction, ≤ and≥, are also used in writing inequalities. In each case, they combine the inequality symbols for less than or greater than with the symbol for equality.

The following explains the use of these symbols. The expression $\boldsymbol{a} \leq \boldsymbol{b}$ is read as "***a*** is less than or equal to ***b***"

Note that this combines the symbol '< 'and '=' and means that either

$\boldsymbol{a} < \boldsymbol{b}$ or $\boldsymbol{a}$ = b. Similarly, a ≥ b reads "a is greater than or equal to b". Implying, either $\boldsymbol{a} > \boldsymbol{b}$ or $\boldsymbol{a}$ = ***b***. etc.

Example

Solve $\frac{1}{2}x - \frac{1}{3}(x + 4) > 4x + \frac{2}{3}$

Multiply through by LCM: 6

$6 \times \frac{1}{2}x - \frac{1}{3}(x + 4) \times 6 > 4x \times 6 + \frac{2}{3} \times 6$

Cancelling and dividing through, we have $3x - 2x - 8 > 24x + 4$

Grouping like terms $3x - 2x - 24x > 4 + 8$

Simplifying,$-23x > 12$

Dividing both sides by 23 yields$x < \frac{12}{23}$

Application of concept

Translation of real life problems into Mathematical statements.

Example 1

If Kofi's age now is 30 years, what is his age in 5 years' time?

Explore: what facts are you given?

* The fact already given is 30 years.

* The fact yet to be found is 5 years' time.

What fact do you need to find?

* Thus, how many years together would Kofi in the coming 5 years(future)?

Plan; Write an equation

Let **m** be Kofi's age now

His age in 5 years' time

$\Rightarrow m = 30 + 5$

$m = 35$ years, Kofi will be 35years

Example 2

If Ama is 40 years now, what was her age 4 years ago?

Let n be her age now = 40 years. (given fact)

Her age 4 years ago (past years)

$\Rightarrow (n - 4)$

$\Rightarrow 40 - 4$

- n=**36 years.**
- Ama was 36years ago

Learners to;

1. Review and model the concepts of solving linear equations and constructing and interpreting formulae for a given task and apply them so finding solutions involving change of subjects.
2. Solve linear equations involving one and two variable(s), brackets and fractions.
3. Develop and solve real-world situations involving linear equations in One variable.
4. Find the solution or truth set of linear inequalities and illustrate on number lines.

Pedagogical Exemplars

The aim of the lessons for the week is for all learners be able generate their equations and change the subjects, solve questions on linear equations and inequalities and solve real-world applications questions on linear equations. These are suggested activities for facilitators to take learners through.

1. **Review previous knowledge:** Review learners' previous knowledge on how to write equations. Take note to make corrections and support learners who have little or no idea about the concepts.
2. **Using talk for learning,** engage and guide learners to construct and interpret formulae for a given equation and apply to problems involving a change of subjects. Make sure that enough examples of varying difficulty are provided to cater for the different group of learners to grasp the concept well.
3. **Using think-pair- share activity and mixed ability/gender groups** engage learners through demonstrations and one-on-one discussions to formulate and solve linear equations in one and two variables and those involving brackets and fractions. Give learners the opportunity to make group presentations (oral and written) with each member of the pair/group taking their turn to help in solving assigned task.
4. **In mixed ability groups:** Learners work in harmonised mixed-abilities groups to extend the idea of linear equation to explain linear inequality including real-life activities to develop conceptual understanding. Give learners the platform to work out enough examples whilst you also address challenges faced by learners through one-on-one discussions.
5. **Using think-pair-share in mixed-ability groups**: In a well-controlled discussion, guide learners to solve word problems involving linear equations and inequalities and translate them into mathematical statements and solve. Encourage learners to share solutions to the whole class whiles you provide feedback which promotes further explanations on the strategies they used and how they arrived at their answers.
6. **In a mixed ability/gender group**: Guide and engage learners through a well-supervised grouping to find solution set or truth set of linear inequalities and illustrate them on the number line and solve real-world/life problems involving linear equations in one variable. Equal

opportunities should be given to all groups to present their findings/solutions to the class whiles those struggling to grasp the concepts are supported.

7. **In a well-regulated class discussion**, summarise the lesson for the week and give learners tasks to solve. Such tasks in the form of assignment or take home tasks could be given to learners.

Key Assessment

Assessment Level 2: Solve for y in the following equations;

1. $6 + y - 2 = 12$
2. $14y - 5 = 2y$
3. $10(y + 2) = 14$
4. $-4(2 - y) = 9(y - 4)$
5. $4 - (y - 8) = -12$
6. $\frac{2}{3}y = \frac{4}{5}(4 + y)$

Assessment Level 3:

1. Find the solution set of the following equations and illustrate your answer on the number line;
   i. $3x - 2 \geq 12 - x$
   ii. $x - \frac{2}{3}(x + 1) \leq \frac{1}{2}(4 - x) - 5$
2. Answer the following questions;
   i. The sum of four consecutive even numbers is 36. Find the numbers.
   ii. The perimeter of a football field in a rectangular form of a certain school is 296m. If the breadth is $\frac{2}{3}$ of the length, find the length.
   iii. Find the number N, such that when $\frac{1}{3}$ of it is added to 8, the result is the same as 18 from $\frac{1}{2}$ of it.

WEEK 12

Learning indicators

1. *Identify relations from functions and differentiate between the types of relations and functions using models such as graphs.*
2. *Investigate relationships between two number sets and determine the rules of given mappings or functions.*

Theme/Focal Area (S)1: Relations and Functions

Definition of key concepts

Relations

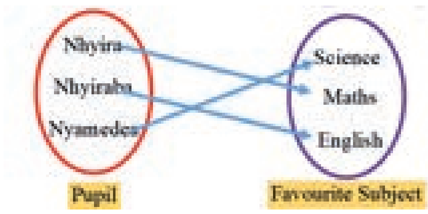
A **relation** is a rule which connects one set to another. We can express relations as an ordered pair (2, 6), in a diagram form, a rule form or graphical form. Relations could be a connection between the first set known as the domain and the second set called the co-domain.

Types of Relations

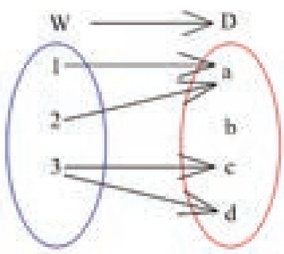
There are **four types of relations** and they are;

1. **One -to-one relation:** This is a relation in which each element in the domain (first set) has exactly one image in the co-domain (second set).

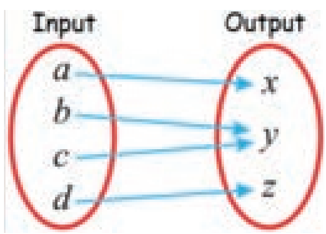
Example:



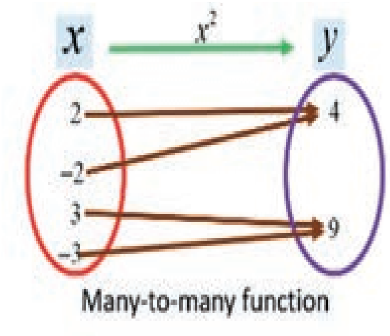
2. **One-to-many relation**: This is a relation in which one element in the domain has more than one(many) in the co-domain



3. **Many- to-one relation:** This is a relation in which more than one element in the domain has only one element in the co-domain.



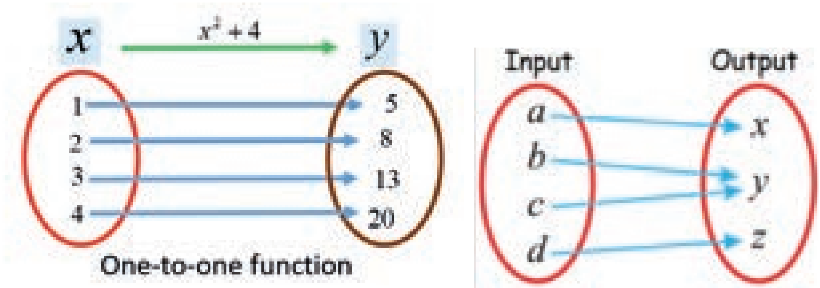
4. **Many-to-many relation**: This is a relation in which many elements in the domain have many images in the co-domain.



Many-to-many function

Functions

Functions are relations where each element in domain has only one image in the co-domain. **One -to-one and many-to-one relations are functions** because each element in the domain has exactly one image in the co-domain. Thus, every member of the domain has only one image in the co-domain.

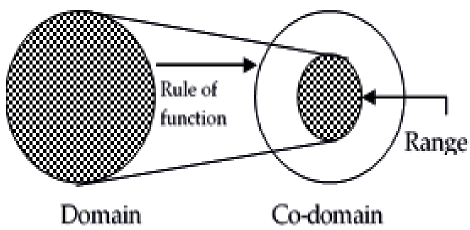


Mapping

Mapping is a relation such that each element in the domain is associated with an element in the co-domain**.**

A subset of the ***co-domain*** which is actually used by the function is called the ***range*** of the function. This is illustrated in figure below.

A subset of the ***co-domain***, the ***range*** of the function as shown

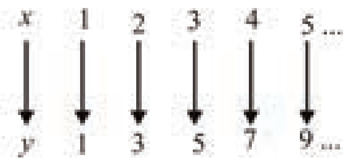


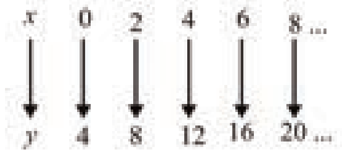
Rules of Mappings

There are two rules for mappings

1. Linear mapping

A **mapping is said to be linear** if the difference between the consecutive elements in both the domain and the co-domain is constant.



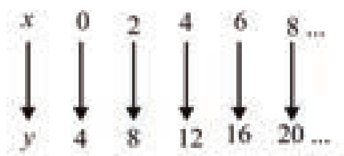


The rule for linear mapping is of the form$y = mx + c$

Where

$$m/a = \frac{\text{constant difference of the co-domain}}{\text{constant difference of the domain}}$$

E.g. What is the rule of the mapping?



Solution

The rule of the mapping is of the form$y = ax + b$

$$a = \frac{8-4}{2-0}$$

$$a = 2$$

Put the value of a into the equation $y = 2x + b \ldots (1)$

Now take any coordinate say (0, 4) and put it into equation 1(i.e. $x = 4$
And $y = 4$),

$4 = 2(0) + b$

$b = 4$

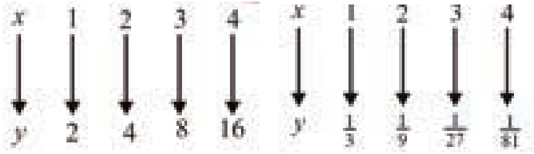
Put b back into equation 1 to give the rule for the mapping above.

$\therefore y = 2x + 4$

2. **Exponential mapping**

A **mapping is said to be exponential mapping** if the ratio between the consecutive elements in the co-domain is constant. The rule for exponential mapping is given as; $y = arx - b$

Example:



Put the values of a, b and c into the form $y = ax^2 + bx + c$

$$\therefore y = 2x^2 + 3x + 1$$

Application of concepts in real-world

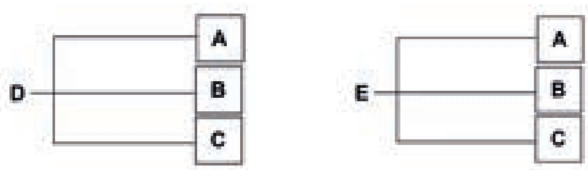
Example1

Adowa has three jumpers and two skirts, combine these in six possible ways using arrow diagram.

Solution

Let the jumpers be A, B, and C

Let the skirts be D and E.



Theme/Focal Area (S) 2: Graphs of Linear Functions

Draw graphs of linear functions and interpret them.

A **straight line graph** is a visual representation of a linear function.

A straight line has a general equation of

$$y = mx + c$$

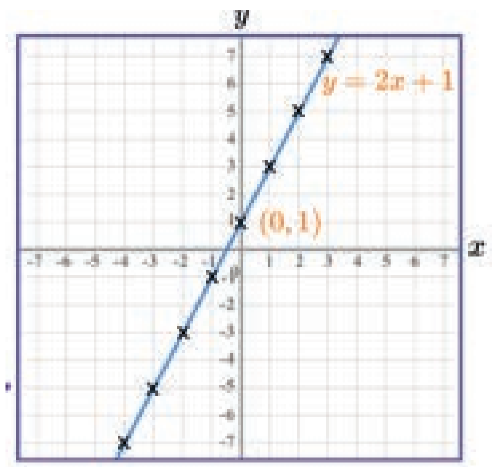
gradient (m) y-intercept (c)

Example

y = 2x + 1

$m = 2$, and $c = 1$

The graph of this equation looks like this: →



Identifying linear graphs

Example:

Mr. Benyah asks Yakubu to identify whether the given equation $3x - 7y = 16$ form a linear graph or not without plotting its values.

Solution

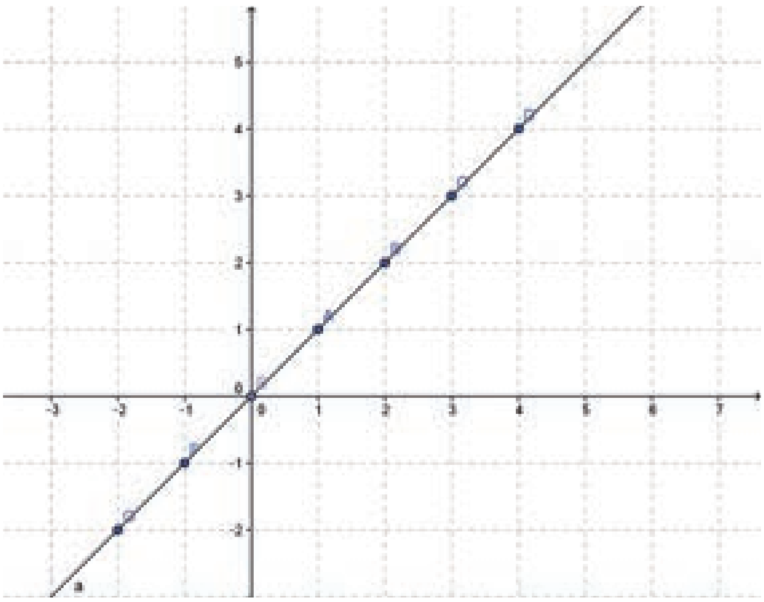
First, Yakubu needs to identify the type of equation. Next, he needs to remember that any linear equation in two variables always represent a straight line. Therefore, the above equation represents a straight line.

Drawing Linear Graphs

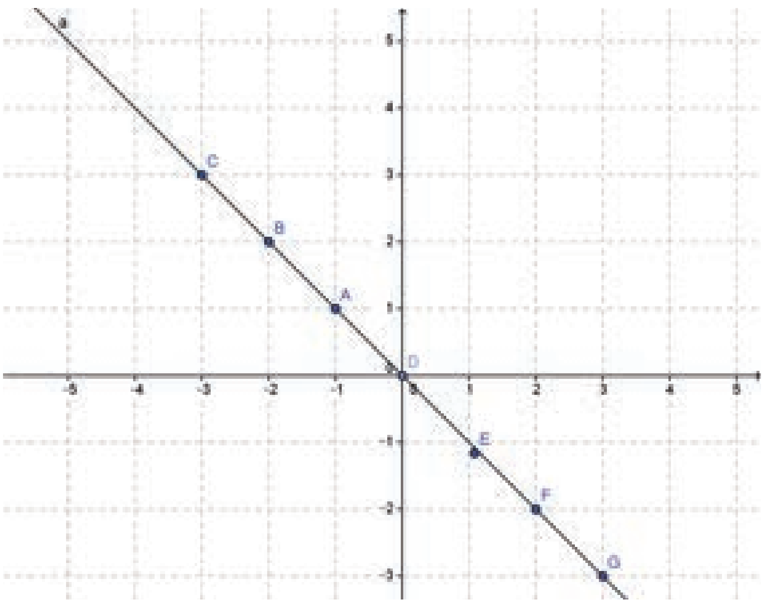
Example:

Draw the graph of a straight line with the following gradient and explain your answer.

a) 1 b) –1



The graph with the gradient 1 passes through the origin and slopes from left to right.

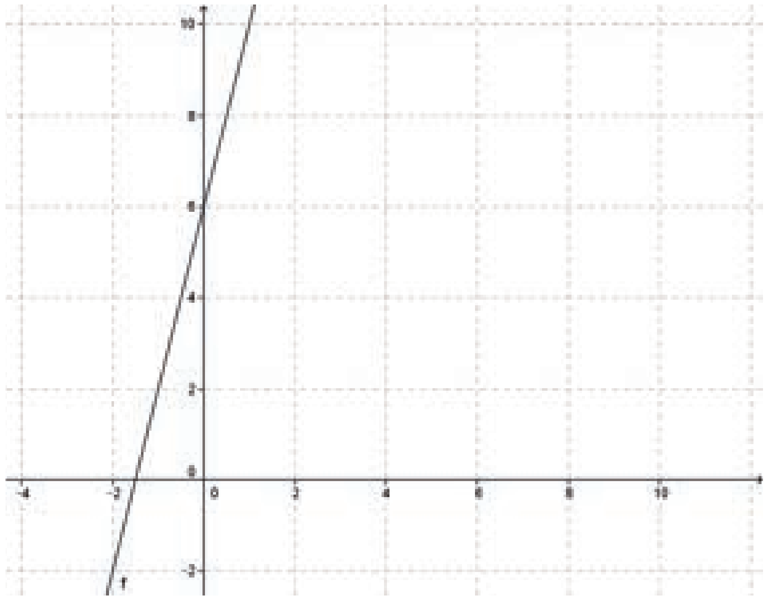


a) When the gradient is –1

The graph slopes downwards and it passes through the origin.

Example:
Draw the graph of $y = 4x + 6$ and explain what happens if the constant 6 is changed to 1 .

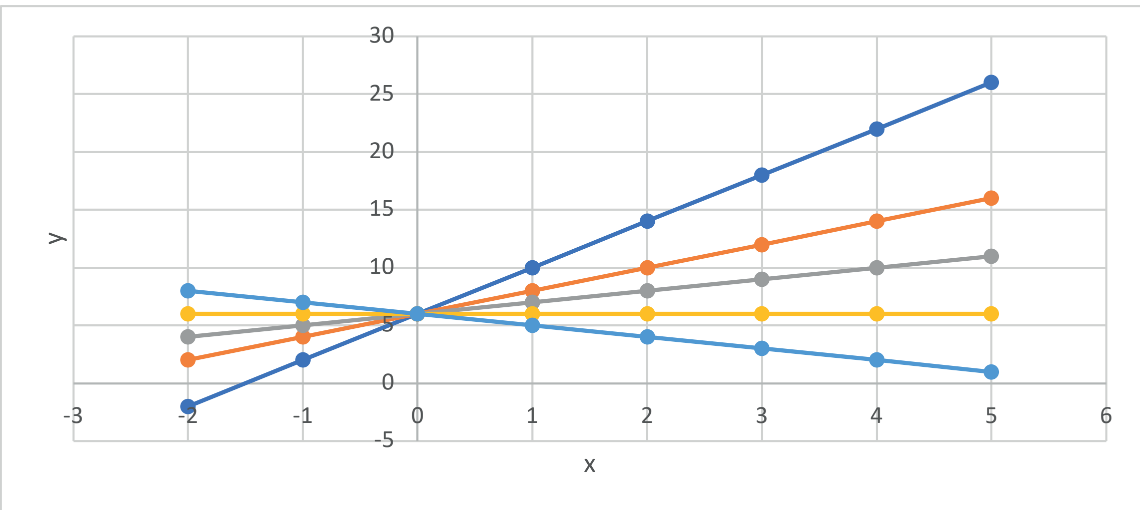
Solution



Explanation: The line moves "down" and cuts y-axis at (0, 1).

Example:
Draw the graph of $y = 4x + 6$ and explain what happens if the coefficient 4 is changed to 2, 1 , 0 and –1 respectively.

Solution:
As the co-efficient decreases, the gradient changes as the line is rotated clockwise about (0, 6)



Learners to;

1. Review and model the concepts and types of relations and functions.
2. Develop the concept of mapping and make investigations of the relationships between two number sets and determine the rules of given mappings or functions.

3. Develop and solve real-world situations involving the concepts of relations, functions and mappings.
4. Draw graphs of linear functions and interpret them.

Pedagogical Exemplars

The objective of the lessons for the week is for all learners to develop the ability to perform tasks on relations and functions and its real-world applications. Take into consideration the following proposed pedagogical strategies in the curriculum.

Review previous knowledge: Review learners' previous knowledge on how to meaning of relations and functions since it was treated in the Junior High School Curriculum. Attention should be given to learners to make correction of their output.

Using talk for learning strategy in mixed-ability/gender groups, learners discuss the meaning of relations and use models to explain examples and types of relations. Ensure that the scenarios provided are in connection with real-world applications.

Using talk for learning in a small mixed ability/gender groups, learners identify functions out of the relations and establish the meaning of a function as a relation which derives one output for each given input and give the types of function as one-to-one and one-to-many function.

In collaborative and well-supervised groups, learners brainstorm and establish that mapping is the same as function which maps element in one set to a unique element in another set. In a well-rehearsed approach, engage learners in groups to model at least one type of relation and represent these relations on mapping diagrams.

Talk for learning: In controlled and convenient groups, learners should discuss the exponential mapping and establish the rule of exponential mapping. Engage learners to solve questions involving experiential mapping and make oral and written presentation to the whole class. Ensure that you attend to learners with low and medium learning abilities as well as high ability learners.

Talk for learning: In mixed-gender/ability groups, learners extend their idea of mapping to discuss linear mapping and establish the rule for linear mapping. Engage learners to solve questions involving experiential mapping and make oral and written presentation to the whole class. Ensure that you attend to learners with low and medium learning abilities as well as high ability learners.

Experiential and problem-based learning approaches: Learners work in well-developed mixed abilities groups to develop and solve real-world situations involving the concepts of relations, functions and mappings. Make oral and written presentations to the class for discussion and also ensuring that every learner in the group makes a presentation. As part of activities to perform at home, task learners each to develop and solve at least two real-world situations involving relations, functions and mapping and submit for scoring and continuous assessment.

Initiating talk for learning in a whole class discussion, review the form of a linear function as y = mx + c where m and c are constant (include the form ax + by + c = 0).

Experiential learning: In small groups, learners research using any of the available IT tools, to explain why the graph of a linear function is a straight line. In pairs, task learners to draw a straight line given a gradient and justify their answer.

Collaborative learning: In pairs, task learners to brainstorm on how to draw a straight line on a graph sheet using the slope intercept form (i.e., $y = mx + c$) and investigate what happens if the constant c (i.e., y-intercept) keeps on changing in a particular equation.

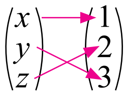
Collaborative learning and review: In pairs, task learners to brainstorm on how to draw on a graph sheet a straight line, using equations in the form $y = mx + c$ and investigate what happens if the coefficient of x keeps on changing in a particular equation.

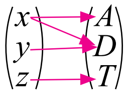
Review: Using a whole class discussion, review on how to draw a linear function with a given interval and assign individual task to learners which they can take home and submit later for scoring and continuous assessment.

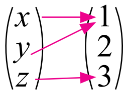
Key Assessment

Assessment Level 2:

1. Which of the following mappings is/are function?

i. 

ii. 

iii. 

2. Which of the following relations defined on the set of real numbers are functions?

   A = $(x, y): y = 3x + 1$

   B = (x, y): $y = 2x^2$

   C = (x, y): $x = y^2$

Assessment Level 3:

1. Draw mapping diagrams to represent the following

   i. $y = 2x^2 - 1$
   ii. $y = 3x + 2$

2. Find the rule of the following mapping

P	**5**	**6**	**7**	**8**
↓	↓	↓	↓	↓
q	**17**	**21**	**25**	**29**

3. Draw the following linear graphs and interpret them

   i. $y = 2x + 1$
   ii. $y = -3x + 1$
   iii. $y = -5x - 1$

WEEK 13

Learning Indicators

1. *Extend the knowledge of coordinates of two points to find the gradient and equation of a straight line*
2. *Recognize and interpret two points on a straight line and use it to find the distance between them.*

Theme/Focal Area(S) 1: Gradient and Equation of Straight Line

Facts about gradient

1 The gradient of a line is the measure of the steepness of a straight line.

2 The gradient of a line can be either positive or negative or 0 and does not need to be a whole number.

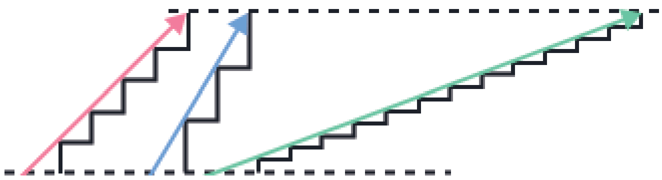
3 The gradient of a line can either be in an uphill (positive value) or downhill direction (negative value).

4 The gradient of a line is the measure of the steepness of a straight line.

How to understand the gradient of a line

Imagine walking up a set of stairs. Each step has the same height and you can only take one step forward each time you move. If the steps are taller, you will reach the top of the stairs quicker, if each step is shorter, you will reach the top of the stairs more slowly.

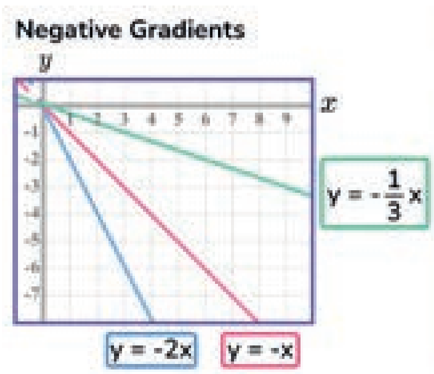
Let’s look at sets of stairs,

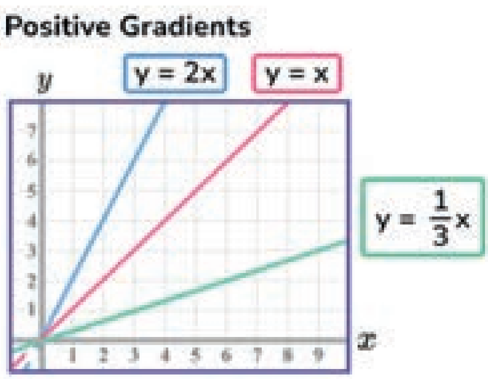


The blue steps are taller than the red steps and so the gradient is steeper (notice the blue arrow is steeper than the red arrow).

The green steps are not as tall as the red steps so the gradient is shallower (the green arrow is shallower than the red arrow).

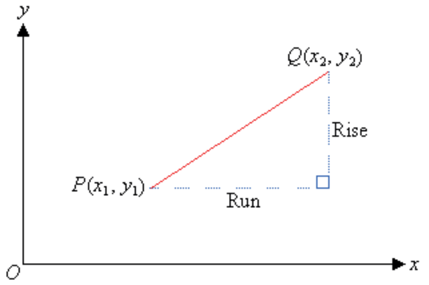
Gradients can be positive or negative but are always observed from left to right.





Finding gradient: In placing a ladder against a wall or tree, a change in the position of the top of the ladder will be as a result of change in position of the foot of the same ladder.

$$\text{Gradient} = \frac{\text{Rise}}{\text{Run}} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$



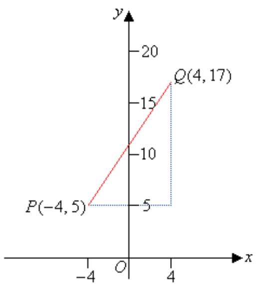
The gradient of a straight line is denoted by m where: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Example

Find the gradient of the straight line joining the points P(– 4, 5) and Q(4, 17).

Solution

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{17 - 5}{4 - (-4)} = \frac{12}{8} = 1.5$$



So, the gradient of the line PQ is 1.5.

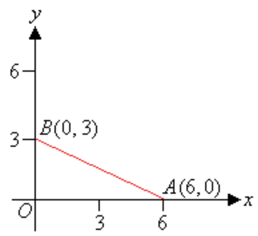
Note: If the gradient of a line is **positive**, then the line **slopes upward** as the value of x increases.

Example 2

Find the gradient of the straight line joining the points A(6, 0) and B(0, 3).

Solution:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 0}{0 - 6} = \frac{3}{-6} = -\frac{1}{2}$$



So, the gradient of the line AB is $-\frac{1}{2}$

Note: When the gradient of a line is **negative**, it indicates that the line **slopes downward from the left to right.** As the value of (x) increases, the corresponding (y) values decrease, resulting in a reduction along the line.

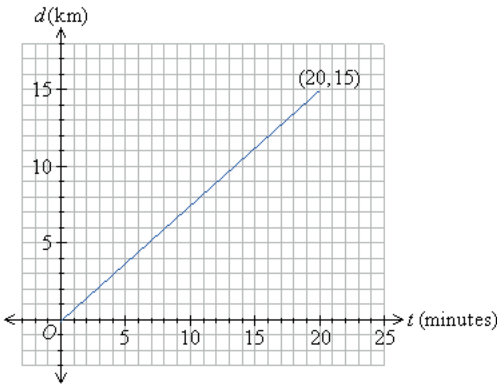
Applications of Gradients

Gradients are an important part of life. The roof of a house is built with a gradient to enable rain water to run down the roof. An aeroplane ascends at a particular gradient after take-off, flies at a different gradient and descends at another gradient to safely land. Tennis courts, roads, football and cricket grounds are made with a gradient to assist drainage.

Example 1

A horse gallops for 20 minutes and covers a distance of 15 km, as shown in the diagram.

Find the gradient of the line and describe its meaning.



Solution

Let $(t_1, d_1) = (0, 0)$ and $(t_2, d_2) = (20,15)$

Now, $m = \frac{d_2 - d_1}{t_2 - t_1}$

$= \frac{15 - 0}{20 - 0}$

$= \frac{15}{20}$

$= \frac{3}{4}$

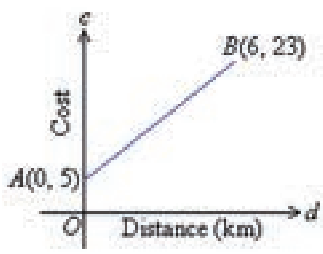
So, the gradient of the line is 3/4 km/min

In the above example, we notice that the gradient of the distance-time graph gives the speed (in kilometres per minute); and the distance covered by the horse can be represented by the equation:

$d = \frac{3}{4}t$ ($\because$ Distance = Speed × Time)

Example 2

The cost of transporting documents by courier is given by the line segment drawn in the diagram. Find the gradient of the line segment; and describe its meaning.



Solution:

Let $(d_1, c_1) = (0, 5)$ and $(d_2, c_2) = (6, 23)$

Now, $m = \frac{c_2 - c_1}{d_2 - d_1}$

$= \frac{23-5}{6-0}$

$= \frac{18}{6}$

$= 3$

So, the gradient of the line is 3. This means that the cost of transporting documents is GHc 3 per km plus a fixed charge of GHc 5, i.e. it costs GHc 5 for the courier to arrive and GHc 3 for every kilometre travelled to deliver the documents.

Finding gradient of a straight line given the equation

We can determine the gradient from a given equation of a straight line.

For example,

Given the equation $2y - 6x = 12$, first rewrite the equation in the general form $y = mx + c$, where m is the gradient.

Therefore, $2y - 6x = 12$ can be written as $2y = 6x + 12$

Now, making y the subject we have $y = 3x + 6$.

Since our new equation is in the general form, we compare and identify the gradient. Hence, the gradient of the equation $2y - 6x = 12$ is **3.**

Finding the equation of a straight line

Find the equation of the line with gradient −2 that passes through the point (3, −4).

Solution

Put $m = -2$, $x_1 = 3$ and $y_1 = -4$ into the formula $y - y_1 = m(x - x_1)$

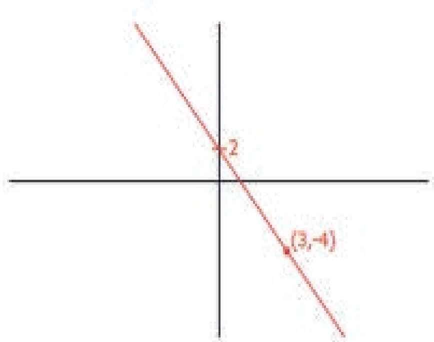
$y - y_1 = m(x - x_1)$

$y + 4 = -2(x - 3)$

Expand the brackets and simplify.

$y + 4 = -2x + 6$

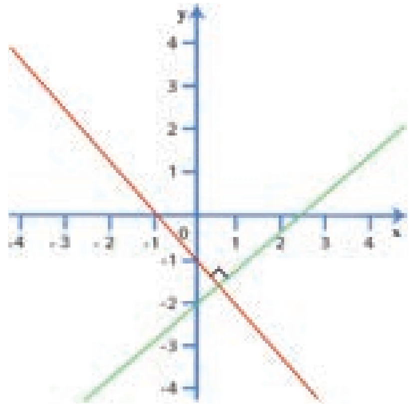
$y = -2x + 2$



Theme/Focal Area(S) 2: Magnitude of A Line Segment (Including Parallel, Perpendicular Lines And Midpoint)

Perpendicular Lines:

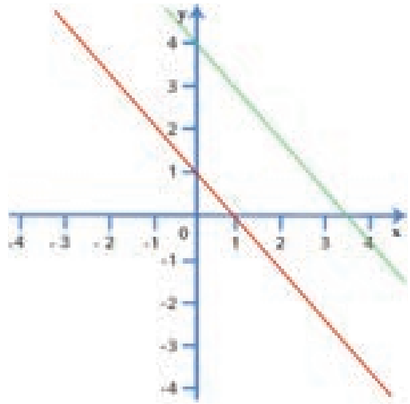
Definition: Perpendicular lines are two lines that intersect at a 90^o angle, forming right angles.



Real-Life Example: Consider a door frame and the floor. The door frame (vertical line) and the floor (horizontal line) intersect to form a right angle, making them perpendicular.

Parallel Lines:

Definition: Parallel lines are two or more lines that never intersect. They have the same slope and are equidistant from each other.



Real-Life Example: Look at railroad tracks. The two tracks run alongside each other, never converging or intersecting. This demonstrates parallel lines in real life.

Worked Examples

Example 1

Identify which of the lines are parallel and which perpendicular**.**

i. $y = 3x + 1$

ii. $y = 3x + 12$

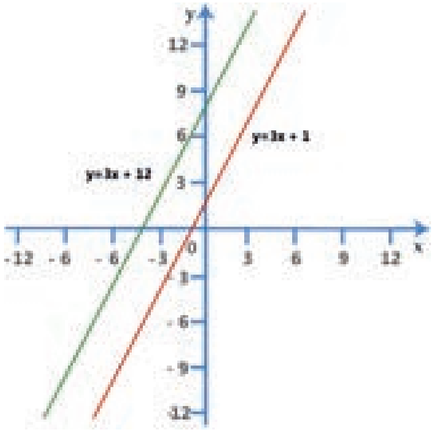
iii. $y = \frac{1}{4}x - 5$

iv. $y = -4x + 1$

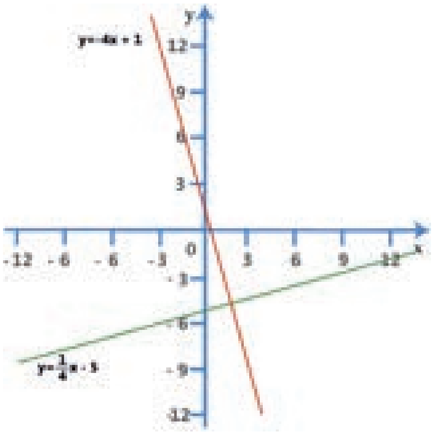
Solution

Parallel lines have the same slope. Since the functions $y = 3x + 1$

$y = 3x + 12$ each have the same slope 3, they represent parallel lines.



Perpendicular lines have negative reciprocal slopes. Since –4 and 1/4 are negative reciprocals the equations $\boldsymbol{y = \frac{1}{4}x - 5}$ **and** $\boldsymbol{y = -4x + 1}$**,** they represent perpendicular lines.



Example 2

Find the equation of a line that is perpendicular to the line $y = 2x - 2$ and goes through the point (1,3).

Solution

We know that the general equation of a straight line is $y = mx + c$.

Firstly, we need to find the gradient of the line $y = 2x – 2$.

If we label the gradient of our line as m_1 and the gradient of the line that is perpendicular with our line m_2 then we know that the product of those two gradients should be –1.

The gradient of our line is $m_1 = 2$

Meanwhile the gradient of the line perpendicular to our line is: $m_2 = -\frac{1}{m_1}$

$m_2 = -\frac{1}{2}$

After finding the gradient the equation of the line we want to find takes the form

$y = -\frac{1}{2}x + c.$

To find the value of c, we substitute the point (1,3) on our equation, since the graph of this line passes through this point.

$y = -\frac{1}{2}x + c$

$3 = -\frac{1}{2} \times 1 + c$

$3 = -\frac{1}{2} + c$

$c = 3 + \frac{1}{2}$

$c = \frac{6}{2} + \frac{1}{2} = \frac{7}{2}$

The final form of our line that is perpendicular with the given line is:

$y = -\frac{1}{2}x + \frac{7}{2}$

Example 3

Find the equation of a line that is parallel to the line $x + y - 1=0$ and goes through the point (–1,1).

Solution

Firstly, we rewrite our line $\boldsymbol{x} + \boldsymbol{y} - 1 = 0$ in the correct form $\boldsymbol{y} = -\boldsymbol{x} + 1$

Then we find the gradient of our line that is $m_1 = -1$

We know that parallel lines have the same gradient so the gradient of the line we are going to find is $m_1 = m_2 = -1$

The line takes the form $y = -x + c$

To find the value of c, we substitute the point (–1,1) on our equation, since the graph of this line passes through this point.

$y = -x + c$

$1 = -(-1) + c$

$1 = 1 + c$

$c = 1 - 1 = 0$

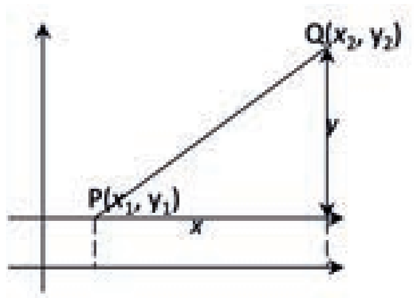
The final form of our line that is parallel with the given line is: $y = -x$

Finding the magnitude of line segment

The magnitude of a line also known as "length", "distance" or "modulus" of a line describe how long a line links to two points.

Relating length of objects discussed from activities, if P and Q have coordinates (x_1, y_1) and (x_2, y_2). From the figure below

Example 1



$\Delta x = x_2 - x_1$

$\Delta y = y_2 - y_1$

Where Δ means a change

By Pythagoras theorem,

$|PQ|^2 = \Delta x^2 + \Delta y^2$

$|PQ| = \sqrt{\Delta x^2 + \Delta y^2}$

$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

So, if P (x_1, y_1) and Q(x_2, y_2) are two points in the oxy plane, then the distance between P and Q is $|PQ|= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

***Note**: The symbol* **(Δ) called delta** *as used here implies change in* $\boldsymbol{x_2}$, $\boldsymbol{x_1}$ *and* y_2, y_1 *or the differences in their values.*

Example 2:

Determine the distance between the points

(a) P(2, 1) and Q(5, 5)(b) A(7, –3) and B(–1, 5)(c) D(4, 1) and E(–3, –5)

Solution

The distance between two points is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(a) P(2, 1) and Q(5, 5)

$|PQ|= \sqrt{(5 - 2)^2 + (5 - 1)^2} = \sqrt{3^2 + 4^2}$

. $|PQ| = \sqrt{25} = 5$units.

(b) A(7, –3) and B(–1, 5)

$\Rightarrow |AB| = \sqrt{(-1 - 7)^2 + (5 + 3)^2}$

$|AB| = \sqrt{(-8)^2 + (8)^2} = \sqrt{128}$units

(c) D(4, 1) and E(–3, –5)

$\Rightarrow |DE| = \sqrt{(-3 - 4)^2 + (-5 - 1)^2}$

$\Rightarrow |DE| = \sqrt{(-7)^2 + (-6)^2} = \sqrt{49 + 36}$

. $|DE| = \sqrt{85}$ units

Determine the mid-point of a line

What is a midpoint?

In geometry, **the midpoint** is the middle point of a line segment. It is equidistant from both endpoints, and it is the midpoint of both the segment and the endpoints. It bisects the segment.

For example;

Consider the following line segment and the points A (3, 4) and B (5, 10). We can determine the mid-point.

HOW?

If you add both x co-ordinates and then divide by two you get

$$\frac{(3 + 5)}{2} = 8 - = 4$$

If you add both y co-ordinates and then divide by two you get

$$\frac{(4 + 10)}{2} = \frac{14}{2} = 7$$

This gives a new point with co-ordinates (4, 7). This point is exactly half way between A and B. Learners to;

1. Review and draw linear graphs using linear functions.
2. Find solutions to equations involving gradient of the straight line.
3. Find solutions to equations involving equations of the straight line.
4. Solve equations and gradient of the straight line involving real-world applications.
5. Determine the equation of perpendicular and parallel lines.
6. Determine the magnitude of a straight line.
7. Determine the midpoint of a straight line.

Pedagogical Exemplars

The aim of the lesson is for all learners to solve equations involving gradient and equations of the straight line. Also, learners should be able to answer questions pertaining to the real-world situations and be able to determine the equation on perpendicular and parallel lines. Finally, learners must be able to determine the magnitude and midpoint of straight lines.

Facilitators can consider these pedagogical strategies when facilitating these lessons.

Initiating talk for learning in a whole class discussion, review the form of a linear function as $y = mx + c$ where m and c are constant (include the form $ax + by + c = 0$).

Experiential learning: In convenient small groups, learners research, using any of the available IT tools to come out with an explanation why the graph of a linear function is a straight line.

Collaborative learning/talk for learning approaches*:* Engage learners to draw a straight line for given gradient by ensuring that the lesson is tailored-measured for learners taking into consideration the different ability learners in the class.

Collaborative learning: In pairs, and using talk for learning, engage learners to brainstorm on how to draw a straight line, using the slope intercept form (I.e., $y = mx + c$) and investigate what happens if the constant c (i.e., y-intercept) keeps on changing in a particular equation and guide learners to review how to draw a linear function with a given interval through demonstrations.

Problem-based learning approach: Learners investigate parallel and perpendicular lines through demonstrations and discussions to establish their relationships through problem solving.

Using talk for learning and experiential learning: Engage in small groups with at most five learners in a group, on how to find the magnitude of a line segment and determine the distance/magnitude between two points using everyday activity while you employ think-pair share in discussing and determining midpoint of line segment using real-world situations.

Using the mixed ability/gender groups: Engage learners to discuss and summarize the lessons and give them group task which may be taken home and presented later for scoring and recorded for continuous assessment.

Key Assessment

Assessment Level 2:

1. Suppose A is the point (3, 4) and B is the point (8, 14). What is the gradient/straight line joining these points?
2. Calculate the gradient of the straight line given the coordinates A (2,6) and B (8,24)

Assessment Level 3:

1. Find the midpoint of the line joining the points or with end points of (1,3) and (5,6).
2. Show that the line segment joining the points (1, 4) and (3, 10) is parallel to the line segment joining the points (−5, −10) and (−2, −1).
3. Are the lines L_1 through (2, 3) and (4, 6) and L_2 through (–4, 2) and (0, 8) parallel, or do they intersect? Give reason for your answer.
4. Determine the equation of the line perpendicular to $2y + 3x = 6$ through the point (5, 2)

Section Review

REFLECTION

1. **Review of Change of subjects and Linear equations and Inequalities**
   a. Develop and interpret linear equations.
   b. Solve change of subject involving one variable.
   c. Solve change of subjects involving two variables.
   d. Solve change of subject involving brackets.
   e. Solve change of subject involving fractions.
   f. Solve inequalities and illustrate it on number lines.
   g. Solve equations which involves real-life applications involving linear equations and inequalities.

2. **Review of Relations and Functions**
   a. Develop a model to explain relations and functions and their types.
   b. Use models to explain the concepts of mapping.
   c. Solve questions involving exponential and linear mapping.
   d. Make investigations of the relationships between two number sets and determine the rules of given mappings or functions.
   e. Solve real-life applications involving mapping.
   f. Draw graphs of linear functions using the general equation $y = mx + c$ and interpret them.

3. **Review of Relations and Functions**
   a. Review and draw linear graphs using linear functions.
   b. Solve equations involving gradient of the straight line.
   c. Solve equations involving equations of the straight line.
   d. Solve equations and gradient of the straight line involving real-world applications.
   e. Solve the equation of perpendicular and parallel lines.
   f. Determine the magnitude of a straight line.
   g. Determine the midpoint of a straight line.

Teaching/Learning Resources

Mathematics posters, white board pan balance, videos, mini whiteboards or laminated white paper, dry erase markers and erasers, algebraic tiles, patterns, calculator, paper grids, technological tools such as computer, mobile phone, you tube videos.