SECTION

1 Number Sets

NUMBERS FOR EVERYDAY LIFE

Real Number and Numeration System

INTRODUCTION

This section introduces you to the real number system, its properties and the relationships among its subsets and their applications to advanced mathematical explorations for lifelong learning. The first part deals with sets of real numbers (natural, whole, integers, rational, and irrational) and their applications to real life. You will then explore the properties of operations, such as closure, commutative, associative, distributive, identity and inverse, and then finally, you will be given opportunities to establish relationships among sets of numbers and link these to three-set problems and their applications in daily life.

Understanding subsets and their properties is fundamental in various branches of mathematics, including set theory and combinatorics. Subsets play a crucial role in analysing relationships between elements within sets and are essential for solving problems in fields such as probability, algebra, and computer science. This section delves into the properties of subsets, focusing on both two-subset and three-subset scenarios and distinguishing between subsets and proper subsets.

A good appreciation of the Real Number System will not only help you understand other mathematical concepts but will also enable you to perform daily activities and solve real-life problems without difficulty.

At the end of the section, you should be able to:

1. Develop the real number system using the closure property.
2. Distinguish between rational and irrational numbers using the conversion of common to decimal fractions and solve related problems.
3. Establish the properties of real numbers with respect to Commutative, Associative, Distributive, Identity, and Inverse.
4. Review the properties of subsets (two and three), their vocabularies and operations and use it to solve real life problems.

5. Organise information visually to establish the relationship between and among sets of items (three sets) and apply these to conduct mini-surveys in the school community and beyond.
6. Establish the relationship between and among three sets, including set equations and the De Morgan's law.

Key Ideas

- Real numbers are used in everyday life. It starts with the use of natural numbers (used in counting) and gradually fuses integers, rational numbers and irrational numbers.
- Closure property will be applied under real numbers using the basic arithmetic operations.
- Properties of Real Numbers include Commutative Property, Associative Property, Distributive Property, Identity Property, and Inverse Property.
- The properties of real numbers are fundamental in mathematics and have numerous real-world applications.
- Similarly, understanding the concepts of subsets, proper subsets, union, intersection and universal sets will enable you to visually represent real life situations using Venn diagrams.

These concepts are applied in the following real-life situations:

- Finance: Calculating profits and losses involving multiple financial assets (stocks, bonds, etc.) using operations on subsets.
- Data Analysis: Using subsets to categorise and analyse data sets based on various criteria.
- Inventory Management: Determining inventory levels based on the intersection of demand from multiple markets and current stock levels.
- Properties of subsets explore the fundamental concepts of set theory, including subsets, union, intersection, and complements, and how to represent these relationships using Venn diagrams visually.
- Algebraic properties of sets, such as set equations and De Morgan's Law help to simplify complex set expressions and establish the relationships between sets.

ESTABLISHING THE SET OF REAL NUMBERS USING THE CLOSURE PROPERTY

Numbers play significant roles in everyday life — in business for buying and selling; for measuring of a patient's vitals (weight, temperature, rate of heartbeat etc.) in the hospital; for checking the speed limit of a moving vehicle when driving; to represent the quantities of food substances that should be combined to constitute a balanced diet to ensure good nutrition, etc.

As a class or in small groups discuss the following scenarios:

1. Imagine a world without numbers. What challenges would this create? Do we consider numbers essential?
2. How do we use numbers in our everyday lives? How did you use them in coming to school? Have you already used them in your lessons?
3. How do different professions use them? Can you think of a profession where numbers are not required?
4. Looking back to JHS, can we remember different types of numbers that we came across? E.g., odd numbers, even numbers, what other types? Below is a breakdown of some of the different types of numbers.

Key Concepts in the Real Number System

- **Natural Numbers**: These are counting numbers that contain positive numbers from 1 to infinity. The set of natural numbers is denoted as $\mathbb{N}$, where $\mathbb{N} = \{1, 2, 3, 4, 5, 6, 7, 8,...\}$
- **Whole Numbers**: These numbers start from 0 to infinity. Set of whole numbers is denoted as $\mathbb{W}$, where $\mathbb{W} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \ldots\}$.
- **Integers**: These contain infinite positive and negative whole numbers. A set of integers is usually denoted as $\mathbb{Z}$, where $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$
- **Rational Numbers**: These are numbers that can be expressed as a common fraction (i.e. $\frac{a}{b}$, where a and b are integers but $b \neq 0$). The set of rational numbers is usually denoted as $\mathbb{Q}$. Examples of rational numbers are $\frac{2}{5}$, 0.4, 40%, 0.3333…., $\frac{-3}{7}$, 7, -123. These numbers are either terminating or recurring decimals.
- **Irrational Numbers**: These numbers cannot be expressed in the form $\frac{a}{b}$, where a and b are integers but $b \neq 0$. The set of irrational numbers is usually denoted as $\mathbb{Q}'$. Examples of irrational numbers are $\sqrt{2}$, $\sqrt{3}$, $2\sqrt{2}$, $2\sqrt{5}$, $\sqrt{8}$, π. These numbers are non-terminating and non-recurring decimals.

- **Real Numbers**: These are the combination of rational numbers and irrational numbers.
- **Terminating Decimals**: They are decimal numbers that end (i.e. Terminate) after a finite number of digits.
- **Non-terminating Decimals**: They are decimal numbers that continues endlessly with or without repeating digits. These are numbers that do not end or terminate.
- **Recurring Decimals**: They are decimal representations of rational numbers where one or more digits repeat infinitely. For example, $0.33\dot{3}$, $5.2525\dot{2}\dot{5}$, $0.153\dot{1}5\dot{3}$
- **Closure Property**: A set is said to be closed under an operation if performing that operation on any two elements from the set always produces a result that is also in the set.

Which of the sets above contains the most elements and which the fewest? Can you explain why this is the case?

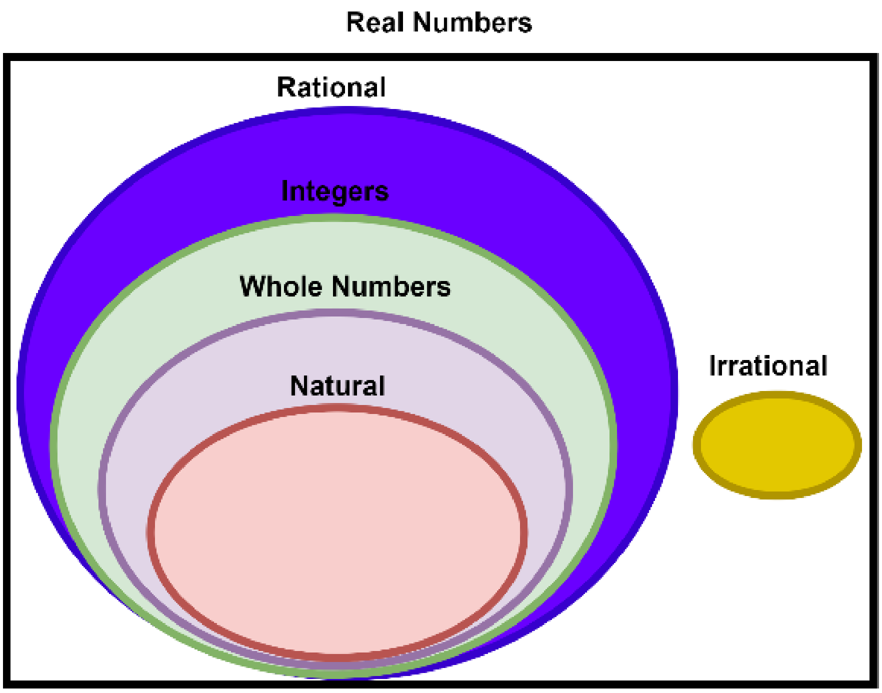


Fig. 1.1: Classification of the subsets of real numbers using Venn diagram

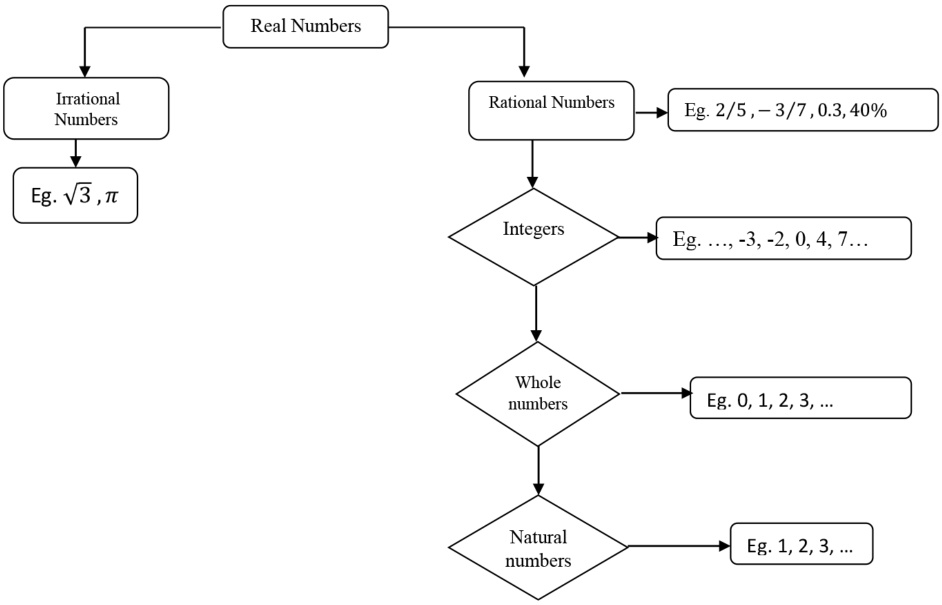


Fig. 1.2: Classification of the subsets of real numbers using a flowchart

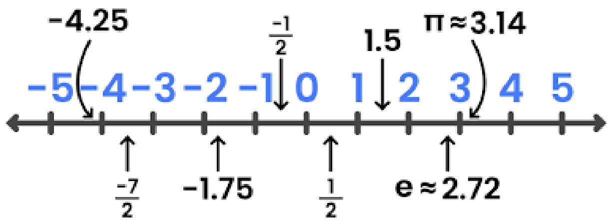


Fig. 1.3: Classification of the subsets of real numbers using a Number line

CLOSURE PROPERTY

Now, think about whole numbers. When you add or multiply two whole numbers is the answer always another whole number? This means that a set of whole numbers is closed under addition and multiplication. When you subtract or divide two whole numbers is the result always a whole number? This shows that the set of wholes is not closed under subtraction and division.

If the answers remain as whole numbers always, then this property is said to be 'closed.

Activity 1.1

Now investigate integers and then real numbers in the same way. What can you say about the closure property with these different types of numbers? Let the tables below guide your investigations.

Table 1.1: Closure properties of whole numbers

Example	Closure property of Whole Numbers	
$5 + 8 = 13 \in \mathbb{W}$	Whole number + Whole number = Whole number	Closed
$5 \times 8 = 40 \in \mathbb{W}$	Whole number × Whole number = Whole number	Closed
$5 - 8 = -3 \notin \mathbb{W}$	Whole number – Whole number = Not always a whole number	Not closed
$4 \div 8 = \frac{1}{2} \notin \mathbb{W}$	Whole number ÷ Whole number = Not always a whole number	Not closed

Table 1.2: Closure properties of integers

Examples	Closure Property of Integers	
$5 + 8 = 13 \in \mathbb{Z}$	Integer + Integer = Integer	Closed
$5 - 8 = -3 \in \mathbb{Z}$	Integer – Integers = Integer	Closed
$5 \times 8 = 40 \in \mathbb{Z}$	Integer × Integer = Integer	Closed
$4 \div 8 = \frac{1}{2} \notin \mathbb{Z}$	Integer ÷ Integer = Not always an Integer	Not closed

Table 1.3: Closure properties of real numbers

Closure Property Of Real Numbers	
Real number +Real number = Real number	Closed
Real number – Real number = Real number	Closed
Real number × Real number = Real number	Closed
Real number ÷ Real number = Not always a real number	Closed only under non-zero division

RATIONAL AND IRRATIONAL NUMBERS AND THEIR APPLICATIONS IN THE REAL WORLD

Rational Numbers

Numbers which can be written in the form $\frac{a}{b}$, where $b \neq 0$, are called rational numbers.

E.g. $\frac{2}{5}$, 0.4, 40%, 0.33…, $0.\dot{4}$, $0.\dot{2}\dot{5}$

Rational numbers mostly appear as fractions, percentages, and decimals that are either terminating or non-terminating but recurring.

Examples of recurring decimals:

(i) $\frac{1}{3} = 0.333, \ldots = 0.\dot{3}$

ii) $\frac{5}{3} = 1.666, \ldots = 1.\dot{6}$

iii) $\frac{9}{11} = 0.818, \ldots = 0.\dot{8}\dot{1}$

iv) $\frac{22}{7} = 3.142857\ 142857\ 142857\ldots = 3.\dot{1}4285\dot{7}$

Examples of non–recurring and non–terminating decimals:

i) $\sqrt{2} = 1.414213562373095, \ldots$

ii) $2\sqrt{3} = 3.464101615137755, \ldots$

iii) $\frac{1}{\sqrt{2}} = 0.7071067811865475, \ldots$

Rational numbers are used everywhere, for example, in decision-making, such as tracking debtors' loan payments, calculating students' exam scores, sharing items, etc.

Expressing recurring decimals as common fractions

Recurring decimals can also be expressed as common fractions. In pairs, work through the examples below so you can see how it is being done.

Worked Example 1.1

1. Express $0.\dot{6}$ as a common fraction.

Solution

Let $x = 0.\dot{6}$......................equation (1)

Multiplying equation (1) by 10

Then $10x = 6.\dot{6}$.................equation (2)

Equation (2) – Equation (1)

$$9x = 6$$

$$x = \frac{9}{6}$$

$$x = \frac{3}{2} = 1\frac{1}{2}$$

Therefore $0.\dot{6} = 2\frac{=}{3}$

2. Express $0.\dot{8}\dot{1}$ as a common fraction.

Solution:

Let $y = 0.\dot{8}\dot{1}$....................*equation* (1)

Multiplying equation (1) by 100

Then $100x = 81.\dot{8}\dot{1}$......*equation* (2)

Equation (2) – Equation (1)

$$99x = 81$$

$$x = \frac{81}{99} = \frac{9}{11}$$

Therefore $0.\dot{8}\dot{1} = \frac{9}{11}$

Irrational Numbers

Numbers that cannot be expressed as $\frac{a}{b}$, where a and b are integers, but $b \neq 0$, are irrational numbers. They include π and square roots of non-perfect numbers: $\sqrt{2}, \sqrt{3}, \sqrt{5}$, etc.

Irrational numbers are applied in trigonometric-related problems. π is also used in measurements involving circles and to adjust speedometers in vehicles.

Consider the diagram below for further analysis of rational and irrational numbers

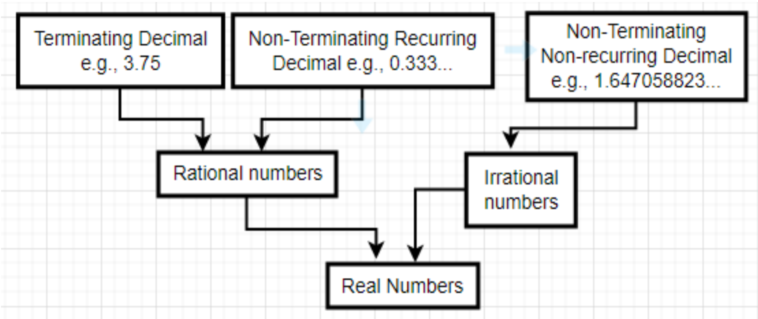


Fig. 1.4: Rational and irrational numbers

The table below further summarises the differences between rational and irrational numbers.

Table 1.4: Differences between rational and irrational numbers

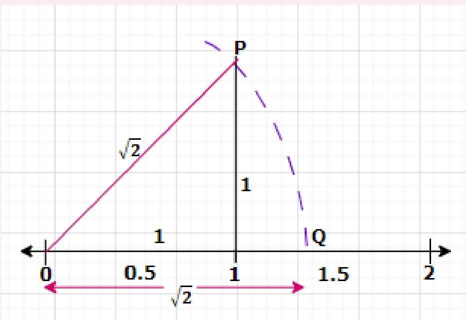
Rational Numbers	Irrational Numbers
Numbers that can be written as fractions $(\frac{a}{b})$ are rational numbers. Both the numerator (a) and denominator (b) are integers in their simplest form, and the denominator (b) is not equal to zero.	Numbers that cannot be expressed as fractions are irrational numbers.
The decimal representation of rational numbers is either terminating or recurring.	The decimal representation of irrational numbers is non-terminating and non-recurring.
Rational numbers are applicable in many areas, including banking, sharing items, and calculating students' scores.	Irrational numbers are applicable in areas such as architecture, engineering, physics and so on.

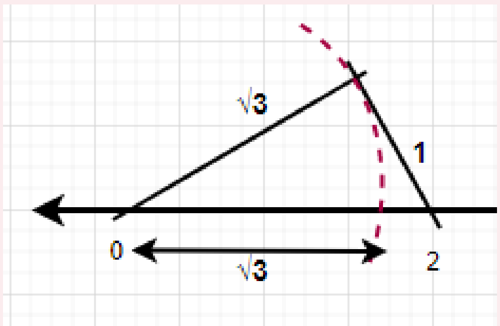
Rational Numbers	**Irrational Numbers**
Rational numbers include perfect squares such as 4, 9, 16, 25, and so on.	Irrational numbers include surds such as $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$, and so on.
Examples: $\frac{3}{2} = 1.5$, $\frac{364}{99} = 3.67676767....$, $\frac{1}{3} = 0.33333...$, etc	Examples: $\sqrt{5} = 2.236067977...$, $\sqrt{11} = 3.31662479...$, etc

Activity 1.2

Let us investigate the differences between Rational and Irrational numbers.

1. Using the construction of right-angle triangles





Watch this https://youtu.be/g5vfSPAlrVM

From the above, we can see that $\sqrt{2} < 2$ and $\sqrt{3} < 3$, etc.

Therefore for every positive integer $\boldsymbol{n}$, $0 < \sqrt{\boldsymbol{n}} < n$

Example: $0 < \sqrt{1} = 1$, $0 < \sqrt{2} < 2$, etc.

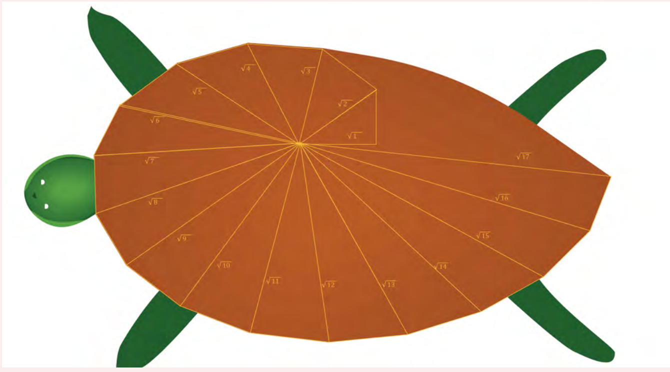
2. Using the Pythagorean Spiral (Wheel of Theodorus)

i. Step 1: Draw a unit right-angled triangle and find the hypotenuse as $\sqrt{1^2 + 1^2} = \sqrt{2}$

ii. Step 2: Draw a right-angled triangle with dimensions 1 and $\sqrt{2}$, then find the length of the hypotenuse as $\sqrt{1^2 + \sqrt{2}^2} = \sqrt{3}$,

iii. Step 3: continue with $\sqrt{1^2 + \sqrt{3}^2} = \sqrt{4} = 2$, then $\sqrt{1^2 + 2^2} = \sqrt{5}$, $\sqrt{1^2 + \sqrt{5}^2} = \sqrt{6}$

The images below show the Wheel of Theodorus for Irrational Numbers.



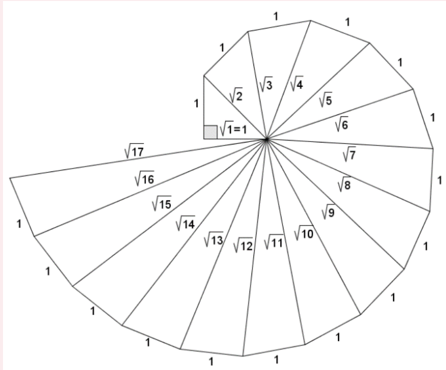


Fig. 1.5: The wheel of Theodorus for Irrational Numbers

From these, can you deduce if the radicals are either rational or irrational numbers?

REAL NUMBERS AND THEIR PROPERTIES

Real numbers are the numbers we use in everyday life, including integers, fractions, decimals, and irrational numbers. The properties of real numbers, including commutative, associative, distributive, identity, and inverse, dictate how we can manipulate and operate on these numbers. These properties provide a framework for performing arithmetic operations efficiently and accurately. Whether it is adding up expenses, calculating dimensions in construction, or analysing data in science, a solid grasp of these properties is essential.

Subsets are collections of elements that are entirely contained within another set. Understanding subsets, their relationships, and operations is crucial in various fields. With two subsets, you can perform operations like union (combining elements from both sets), intersection (finding common elements), and set difference (finding elements unique to each set).

Extending this to three subsets allows for more complex analyses and problem-solving. Real-life scenarios often involve categorising, comparing, and combining sets of data or objects, making knowledge of subset properties invaluable. Whether organising customer data in marketing, managing inventory in logistics, or analysing survey responses in social sciences, the ability to manipulate subsets is a powerful tool. Understanding the properties of operations will enable you to manipulate algebraic expressions with ease.

PROPERTIES OF OPERATIONS

In pairs or small groups, investigate these properties of operations. Can you make general conclusions from this?

1. Commutative

Is it possible to rearrange the numbers in a calculation and still get the same results?

For example, is 3×2 the same as 2×3? Is $3 - 2$ the same as $2 - 3$? Check what happens with addition and division. If it does remain true, then the operation is said to be commutative.

2. Associative

If we group numbers differently does it affect the outcome?

For example, if we substitute different numbers for a, b and c, would it be true that $(a + b) + c = a + (b + c)$?

Can this also work for multiplication, subtraction or division? If it is true, then the operation is said to be associative.

Remember always to do the calculation within the bracket first.

3. Distributive

This is specific to multiplication, alongside addition, and subtraction. Substitute in numbers for a, b and c to see if it is true that $a(b + c) = ab + ac$. What about $a(b - c) = ab - ac$? Can you work out something similar for division?

4. Identity

Can you think of a number that when you add another number to it (addition), you always have the number you started with? Can you do the same for multiplication?

5. Inverse

Can you find a number that when added to another number, results in the same number you found in (4) above? Can you do the same for multiplication?

Below is a summary of the conclusions you should have made.

Commutative Property

This property states that when we add or multiply two numbers, the order in which we do it does not change the answer.

- Commutative Property of Addition: $a + b = b + a$ (where $a, b \in R$).

Example, $5 + 3 = 3 + 5 = 8$

- Commutative Property of Multiplication: $a \times b = b \times a$ (where $a, b \in R$).

  Example, $2 \times 6 = 6 \times 2 = 12$

Associative Property

This property of numbers states that no matter how we group the real numbers when adding or multiplying them, the sum or product will always be the same.

- Associative Property of Addition: $(a + b) + c = a + (b + c)$ where a, b and $c \in R$. Example, $(2 + 3) + 4 = 2 + (3 + 4) = 9$
- Associative Property of Multiplication: $(a \times b) \times c = a \times (b \times c)$, where a, b and $c \in R$. for example $(2 \times 3) \times 4 = 2 \times (3 \times 4) = 24$. This Property involves three values or variables with the same sign.

NOTE: Subtraction and Division do not work for Commutative and Associative properties. That is, for these two operations, the order matters. This implies that the change in order for a set of two numbers rarely gives the same answer.

For example, $5 - 2 \neq 2 - 5$ or $\frac{2}{5} \neq \frac{5}{2}$, $(5 - 2) - 7 \neq 2 - (5 - 7)$ or $(2 \div 6) \div 4 \neq 2 \div (6 \div 4)$

Distributive Property

The distributive property is a fundamental property that defines how the multiplication operation is distributed over addition and subtraction. It is a very useful property that helps us simplify expressions in which we multiply a number by the sum or difference of two other numbers.

- Distributive property of multiplication over addition: This property of multiplication over addition is used when we need to multiply a number by a sum. If a, b and $c \in R$, then $a(b + c) = ab + ac$.

  Example: $7(3 + 5) = 7(3) + 7(5)$

  $7(8) = 21 + 35$

  $56 = 56$

- Distributive Property of Multiplication over Subtraction: The distributive property of multiplication over subtraction states that the multiplication of a number by the difference of two other numbers is equal to the difference of the products of the distributed number. If a, b and $c \in R$, then $a(b - c) = ab - ac$.

  Example: $7(9\text{-}5) = 7(9) - 7(5)$

  $7(4) = 63 - 35$

  $28 = 28$

Identity Property

It states that when a specific operation is performed on an element with its identity element, the result is the original element itself.

Additive identity

It states that adding zero to any number does not change the value of the number. **Zero** is the additive identity. For example, $6 + 0 = 0 + 6 = 6$.

Multiplicative identity

It states that multiplying a number by 1 does not change the value of the number. Therefore **1** is a multiplicative identity. For example, $10 \times 1 = 10$

Inverse Property

The inverse property under the operations of addition or multiplication will return the identity value of that operation. For addition, the result is zero while for multiplication, the result is one.

Additive inverse property

This states that when a number is added to its additive inverse, the sum of the 2 numbers will be zero (additive identity). Thus, $a + (-a) = -a + a = 0$

For example, $5 + (-5) = 0$.

Multiplicative inverse property

This states that when a number is multiplied by its multiplicative inverse, the result is one (multiplicative identity).

Thus, $a \times \frac{1}{a} = \frac{1}{a} \times a = 1$ or $a \times a^{-1} = a^{-1} \times a = 1$

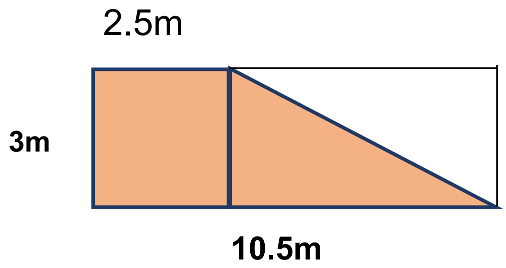
For example, $4 \times \frac{1}{4} = \frac{1}{4} \times 4 = 1$

APPLICATIONS OF THE PROPERTIES OF OPERATIONS

This area looks at applications of subsets in solving real-life problems depicted in worked examples.

Worked Example 1.2

1. Find the area of the shaded portion in the figure below using the distributive property of operations.



Solution

1. Applying the distributive property, area (A), of the shaded portion (trapezium) is given by

$$A = \frac{h}{2}(a + b) = \frac{h}{2}a + \frac{h}{2}b$$

$$\frac{3}{2}m(2.5m + 10.5m) = \frac{3}{2}m \times 2.5m + \frac{3}{2}m \times 4m$$

$$\frac{3}{2}m(13m) = 7.5\,m^2 + 12m^2$$

$$19.5\,m^2 = 19.5\,m^2$$

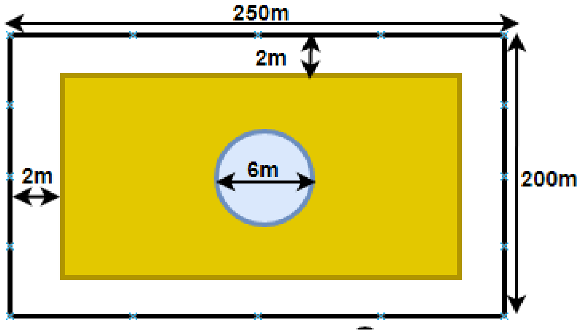
Worked Example 1.3

a) A woman created a 2m fire belt around but within her 200m by 250m rice farm. She later created a circular fishpond with a diameter of 6m in the middle of the farm. Determine the actual land area created by the rice farm,

- **i)** Before the creation of the fishpond.
- **ii)** After the creation of the fishpond. Give your answers to the nearest square metre.

b) If it costs her GH¢28.00 to spray a square metre of the rice farm with weedicide, how much will it cost the farmer to spray the rice farm?

Solution



(This image is NOT drawn to scale)

Given: diameter of the pond = 6m

Area of pond = $\pi\left(\frac{6}{2}\right)^2 = \pi\ (3)^2 = 9\pi = 28.23\,m^2 = 28\,m^2$

The actual land area covered by the rice farm:

(a) **i.** before the creation of the fishpond = (250 – 4)(200 – 4) =48,216m^2

ii. after the creation of the fishpond = 48,216m^2 – 28.23m^2

= 48187.77m^2

= 48,188m^2

(b) If 1m^2 = GH¢28.00

Then, 48,188m^2 = GH¢28.00 × 48,187.77m^2 = GH¢1,349,257.56

Lets perform the following activities to investigate the properties of operations

Activity 1.3

1. Take a piece of paper. Cut it into six boxes.
2. Take another piece of paper the same size as the first paper and cut it into seven boxes.
3. Place one set of the six boxes to your left and one set of the seven boxes to your right. Count the boxes altogether.
4. Place the second set of seven boxes to your left and the second set of six boxes to your right. Count the boxes altogether.
5. Compare your answers in (iii) and (iv). What property is that?

Activity 1.4

1. Pick any three different numbers and write each on separate pieces of paper.
2. Use the numbers to create different addition equations.
3. Record the sum of each equation on separate sheets of paper.
4. Compare the answers. What can you say about the results?

Activity 1.5

Kwame and Ato were given some ingredients to bake biscuits. Each was given 2 cups of flour, 3 eggs and a cup of sugar to make 5 biscuits. Kwame first mixed the sugar and flour, followed by the eggs, to make the dough for the five biscuits. Ato first mixed the flour and the eggs, and then mixed it with sugar to also make the same dough for five biscuits. Which property is being illustrated?

Activity 1.6

1. Write 3 different numbers and group them in two different ways.

   Example: $(5 - 2) - 7$ vs. $5 - (2 - 7)$
2. Find the answer for each of the groups, compare the answer and make conclusions.

Activity 1.7

1. Write down 3 different numbers and group them in two different ways.

   Example: $(6 \div 3) \div 4$ vs. $6 \div (3 \div 4)$
2. Find the answer for each of the groups and compare the answers. Write down what you have observed.

Activity 1.8

1. Take two sheets of plain paper that are of the same size.
2. Give one sheet of paper to your friend. Divide the sheets into two equal parts and shade one-half.
3. Cut out the shaded parts and join them to form a whole.
4. What conclusion can be drawn based on the figure formed?

PROPERTIES OF SUBSETS (TWO AND THREE), SUBSETS AND PROPER SUBSETS

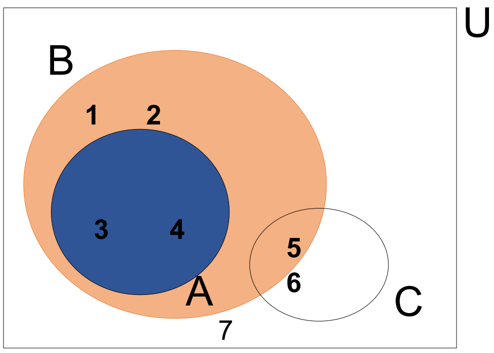
Subsets

If every member of a set $\boldsymbol{A}$ is also a member of set $\boldsymbol{B}$**,** then $\boldsymbol{A}$ is a subset of $\boldsymbol{B}$, we write this as $\boldsymbol{A} \subseteq \boldsymbol{B}$**.** We can say $\boldsymbol{A}$ is contained in $\boldsymbol{B}$**,** or $\boldsymbol{B} \supseteq \boldsymbol{A}$, here, B is a superset of A, B includes A, or B contains A. If A is not a subset of B, we write $\boldsymbol{A} \not\subseteq \boldsymbol{B}$.

However, if $\boldsymbol{A}$ is a subset of **B** ($A \subseteq B$), but A is not equal to B, then we say A is a **proper subset** of B, written as $A \subset B$ *or* $A \subsetneq B$. Example, $U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{3,4\}$, $B = \{1, 2, 3, 4, 5\}$, $C = \{5, 6\}$

The Venn Diagram

In mathematics, the Venn diagram is a pictorial representation of the relationship between two or more sets. It was suggested by the English mathematician and philosopher, John Venn.



i. A is a proper subset of B; $A \subset B$

ii. A is not a subset of C

iii. B is a proper subset of U

Worked Example 1.4

Given that $A = \{1, 3, 5\}$, $B = \{1, 2, 3, 4, 5\}$ *and* $C = \{1, 2, 3, 4, 5\}$. Using the concepts of subsets, show the relationship between A, B and C.

Solution

A is a subset of B, $A \subseteq B$. because every element of A is also in B.

A is also a proper subset of B, $A \subset B$. because every element in A is also in B and $A \neq B$.

C is a subset of B, $C \subseteq B$, but is not a proper subset of B because $C = B$.

Worked Example 1.5

If $X = \{1, 3, 5\}$, $Y = \{2, 3, 4, 5, 6\}$, determine whether X is a subset of Y

Solution

X is not a subset of Y, $X \not\subseteq Y$, because element 1 is in X.

The Number of Subsets in a Given Set

The table below shows the relationship between the number of elements in a set and the number of subsets of that set.

Table 1.5

Sets	Subsets	No. Of Elements	No. Of Subsets
$\{\ \}$	$\{\ \}$	**0**	$\mathbf{1} = 2^0$
$\{x\}$	$\{\ \}, \{x\}$	1	$\mathbf{2 = 2^1}$
$\{x, y\}$	$\{\ \}, \{x\}, \{y\}, \{x, y\}$	2	$\mathbf{4 = 2^2}$
		:	:
		n	$\mathbf{2^n}$

Worked Example 1.6

Find all the subsets of the set $A = \{1,2,3\}$

Solution

The subsets are; $\{\ \},\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},$ *and* $\{1,2,3\}$. There are 3 elements in set A, so there are 2^3 subsets, which is equal to 8.

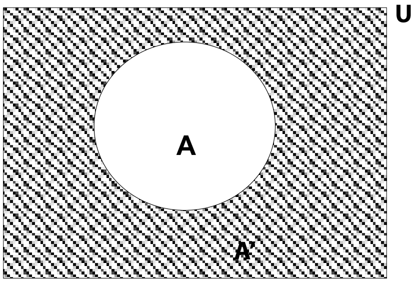
To find all the subsets of a given set, note that the empty set is a subset of any set. Also, any set is a subset of itself.

The Complement of a Set

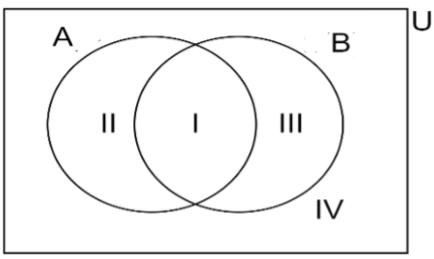
When we talk about the complement of set A in relation to the universal set U, we are referring to all the elements in U that are not in set A. The complement of set A is written as A' and read as "A complement".

For example, if $U = \{a, b, c, d, e, f\}$ and $A = \{a, b, e\}$ then $A' = \{c, d, f\}$.

The shaded portion in the Venn diagram below shows "A complement".



Properties of a Venn Diagram for a Two–Set Problem



Below is a detailed description of the various regions in the diagram

- n(A) = all elements that belong to set A
- n(B) = all elements that belong to set B
- I = elements that belong to both sets A and B

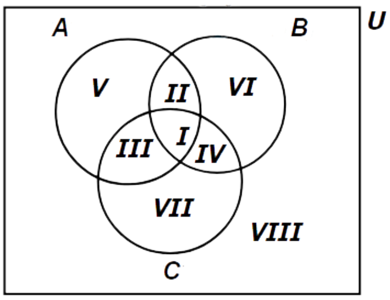
- II = elements that belong to set A only
- III = elements that belong to set B only
- IV = elements that belong to neither set A nor B

The set notations for the various regions of the diagram are given as:

1. I = $A \cap B$
2. II = $A \cap B'$
3. III = $A' \cap B$
4. IV = $(A \cup B)'$

Can you see why it works to describe the different regions? In pairs, come up with your own examples and task your partner to define the regions.

Properties of a Venn Diagram for a Three–Set Problem



From the above Venn diagram, identify the regions that represent:

I Elements common to sets A, B and C: $(A \cap B \cap C)$

II Elements that belong to sets A and B, but not common to C: $(A \cap B) \cap C'$

III Elements that belong to sets A and C, but not common to B: $(A \cap C) \cap B'$

IV Elements that belong to sets B and C, but not common to A: $(B \cap C) \cap A'$

V Elements in set A that are neither in set B nor C: $A \cap (B' \cap C')$

VI Elements in set B that are neither in set A nor C: $B \cap (A' \cap C')$

VII Elements in set C that are neither in set A nor B: $C \cap (B' \cap A')$

VIII Elements in the universal set U that are not in sets A, B and C: $(A \cup B \cup C)'$

Activity 1.9

Look at the examples below and establish whether the following properties of sets are always true?

1. $(A \cup B)' = A' \cap B'$
2. $(A \cap B)' = A' \cup B'$
3. $(A \cup B \cup C)' = A' \cap B' \cap C'$
4. $(A \cap B \cap C)' = A' \cup B' \cup C'$

Again, working in pairs, work out other properties that are true for sets.

Did you conclude that, for any three sets A, B and C, the following properties are true?

i. Commutative properties $A \cup B = B \cup A$ *and* $A \cap B = B \cap A$

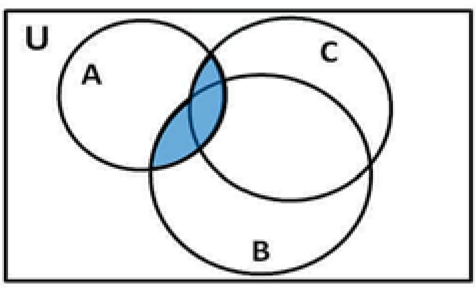
ii. Distributive properties: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$;
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

iii. Associative properties: $A \cap (B \cap C) = (A \cap B) \cap C$; $A \cup (B \cup C) = (A \cup B) \cup C$

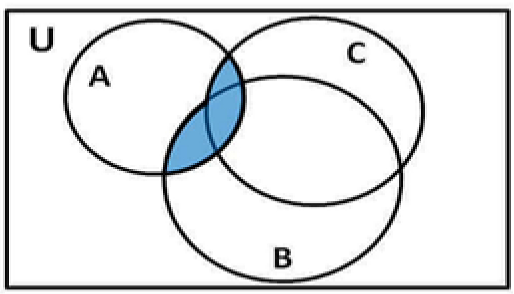
iv. Other properties:

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - [n((A \cap B) + n(A \cap C) + n(B \cap C)] + n(A \cap B \cap C)$

The Venn diagrams below illustrate $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$



A ∩ (B U C)



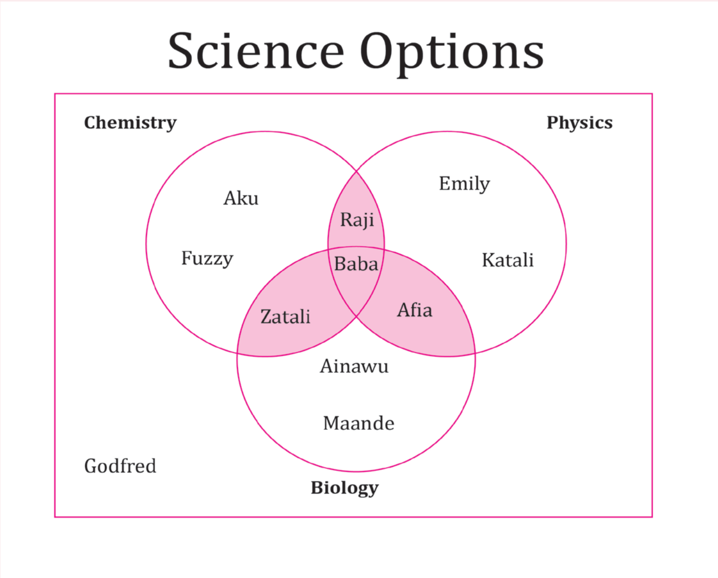
(A ∩ B) U (A ∩ C)

Activities Including Self-Assessment 1.10

1.

- Write down the first 10 natural numbers
- Label it as the universal set "U".
- From the Universal Set "U", identify the even numbers:
- Write down these even numbers as a separate set, which you will call "E".
- Compare the sets "U" and "E".
- What is the relationship between the sets "U" and "E"

2.



From the Venn diagram, identify the regions that represent:

i. Elements common to sets P, B, and C., i.e. $n(P \cap B \cap C)$.

ii. Elements that belong to sets P and B, but not common to C: $(P \cap B) \cap C'$

iii. Elements that belong to sets P and C, but not common to B: $(P \cap C) \cap B'$

iv. Elements that belong to sets B and C, but not common to P: $(B \cap C) \cap P'$

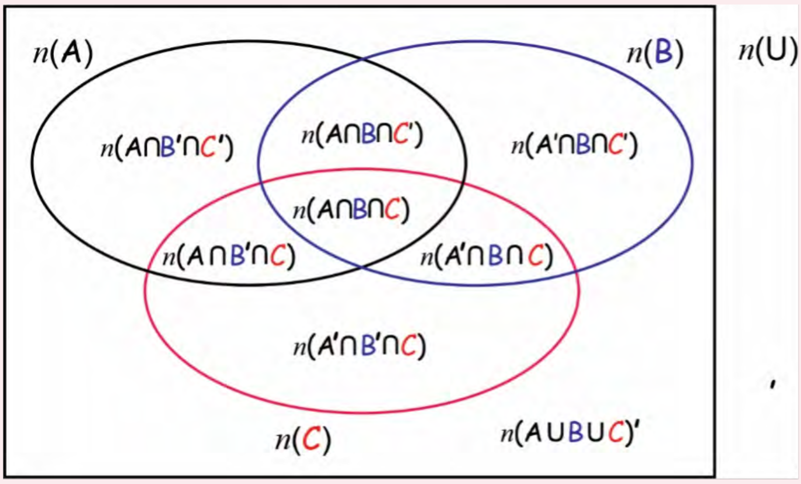
v. Elements in set P that are neither in set B nor C: $P \cap (B' \cap C')$

vi. Elements in set B that are neither in set P nor C: $B \cap (P' \cap C')$

vii. Elements in set C that are neither in set P nor B: $C \cap (B' \cap P')$

viii. Elements in the universal set U that are not in sets P, B and C: $(P \cup B \cup C)'$

3. Study the Venn diagram below and write down the mathematical representation of four regions using A, B and C.



ESTABLISH THE RELATIONSHIP BETWEEN AND AMONG THREE SETS, INCLUDING SET EQUATIONS AND THE DE MORGAN'S LAW

De Morgan's Laws are fundamental principles in set theory and logic named after the British mathematician and logician Augustus De Morgan. There are two primary laws, often referred to as De Morgan's Laws:

The First Law (De Morgan's Law for Union): $(A \cup B)' = A' \cap B'$

This law states that the complement of the union of two sets is equal to the intersection of their complements. In simpler terms, it says that "not *A* or *B*" is equivalent to "not *A* and not *B*".

The Second Law (De Morgan's Law for Intersection): $(A \cap B)' = A' \cup B'$

This law states that the complement of the intersection of two sets is equal to the union of their complements. In simpler terms, it says that "not *A* and not *B*" is equivalent to "not *A or B*"

Two-set problem equation: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Three-set problem equation: $n(A \cup B \cup C) = n(A) + n(B) + n(C) - [n(A \cap B) + n(A \cap C) + n(B \cap C)] + n(A \cap B \cap C)$

Activities Including Self-Assessment 1.11

1. Your school library has two collections of books:

A = {books written by female authors}

B = {books published after 2010}

Using De Morgan's Law, find the set of books that are NOT written by female authors AND NOT published after 2010. Simplify your answer using set notation.

2. Given the sets $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, and $C = \{5, 6, 7, 8\}$, verify de Morgan's laws for the following:

a) $(A \cup B)' = A' \cap B'$

b) $(A \cap B \cap C)' = A' \cup B' \cup C'$

Review Questions

Review Questions 1.1

1. Classify the following under Rational ($\mathbb{Q}$), Irrational ($\mathbb{Q}'$), Integer ($\mathbb{Z}$), Whole ($\mathbb{W}$) and Natural numbers ($\mathbb{N}$). Some will be in more than one classification. Write down all of those that apply.

   i. $\frac{2}{3}$
   ii. -4.75
   iii. 0
   iv. π
   v. $2.435435435\dot{4}3\dot{5}$
   vi. -0.236574876967674......
   vii. 100
   viii. -8
   ix. 1.120361203612036...
   x. 0.5
   xi. $\frac{22}{7}$

2. Find out which of the following elements of the set $M = \left\{ -3, 2.3, \sqrt{16}, -\pi, 0, \frac{5}{6}, 0.\dot{2}\dot{3}, 6 \right\}$ is/are

   (a) Natural numbers
   (b) Real numbers
   (c) Integers
   (d) Irrational numbers
   (e) Rational numbers

3. Abena wants to categorise the following numbers into their respective subsets of the real number system: -3, 0, $\sqrt{2}$, $\frac{1}{2}$ and 7. Help her classify each number as a natural number, whole number, integer, rational number or irrational number.

4. A chick swallowed a grain of corn. A hawk then ate the chicken. A python then swallowed the hawk. A hunter then killed the python and put it into his bag containing a bottle of water, illustrate the content in the hunter's bag using a Venn diagram. Relate these within the real number system.

5. Convert the following decimals into fractions.

   (a) $0.\dot{5}$
   (b) $0.\dot{6}\dot{2}$
   (c) $1.\dot{6}$
   (d) $2.\dot{8}\dot{1}$

Review Questions 1.2

1. Name the property depicted in $(7 \times 15) \times 9 = 7 \times (15 \times 9)$
2. Regroup $7 + (3 + 2)$ and compare the answers. What conclusion can you draw?
3. Use the distributive property to find the area of a rectangular cocoa farm that is 43m long and 8m wide.
4. A group of 5 students was asked to bring 6 pebbles each, while another group of 6 students was asked to bring 5 pebbles. How many pebbles does each group have? Draw a valid conclusion on the results.
5. Simplify the equation $2x + 3 = 5$. Identify the properties needed to solve this equation.
6. Senyo buys three packs of pineapples, each priced at Gh¢ 12.00 and three packs of oranges, each priced at Gh¢ 6.00. Use the distributive property to write an equation that represents the total cost of fruits Senyo purchased.
7. Mr Asiedu has a monthly budget and allocates 20% of his income to savings and 30% to rent. If his monthly income is Gh¢ 800.00. Use the distributive property to write an equation that represents his total monthly expenses?
8. Amina bought 11 pens, and then returned 5 of them because they were not working. If each pen costs Gh¢ 3.00, use the distributive property to write an equation that represents the total cost of the pens Amina kept.
9. Imagine you are at a grocery store with your friend, and you both are buying some items. You have 3 apples, and your friend has 2 oranges. You decide to combine your fruits and put them in a single bag. Later, you meet another friend who has 4 bananas. You put all the fruits together.

   Using the associative property, how do you add the fruits in different orders?

Review Questions 1.3

1. Three committees A, B, and C form a union, U in school. If the U has a total population 380 members. $n(A) = 200$, $n(B) = 155$, $n(C) = 110$, $n(A \cap B) = 50$, $n(B \cap C) = 35$, $n(A \cap C) = 44$ *and* $n(A \cap B \cap C) = 0$, Illustrate this information in a Venn diagram and find the following:

   (a) $n(A \cap B' \cap C')$ (b) $n(A' \cap B \cap C')$

   (c) $n(A' \cap B' \cap C)$ (d) $n(A \cap B \cap C')$

   (e) $n(A' \cap B \cap C)$ (f) $n(A \cap B' \cap C)$

   (g) $n(A \cup B \cup C)'$

2. In a survey of 100 SHS graduates looking for employment. It was established that 55 graduates chose army, 50 graduates chose police and 30 graduates chose immigration. The entire 100 graduates chose at least one of the forces. If 44 of the graduates chose exactly two of the forces, how many graduates chose all three forces?

3. Draw Venn diagrams to illustrate the following relationship among sets M, A and U, where M is the set of students studying Mathematics in Nchumuruman SHS, A is the set of students studying Additional Mathematics in the same school, and U is the set of all students in that school.

   i. All the students who study Additional Mathematics also study Mathematics, but some students who study Mathematics do not study Additional Mathematics.

   ii. Not all students study Additional Mathematics, but every student studying Mathematics studies Additional Mathematics

4. A group of 22 travellers were each required to obtain a passport, health certificate and convertible currency. 12 had passports, 14 had health certificates, and 11 had currency. 6 had passports and health certificates, 6 had passports and currency, and 7 had health certificates and currency. Each traveller had at least one of the required items.

   (a) Draw a Venn diagram to illustrate the given information.

   (b) Find the number of travellers who had

      i. all three required items

      ii. exactly two of the three required items.

5. In a group of 59 traders, 26 sell gari, 8 sell only rice and 15 sell only maize. 10 sell gari and rice, 16 sell rice and maize and 42 sell maize. Each trader sells at least one of the three items. Find the number of traders who sell

(a) gari or maize.

(b) gari and maize.

(c) exactly two items.

6. Let $X = \{x \mid x \text{ is an even digit}\}$, $Y = \{x \mid x \text{ is a prime number less than } 10\}$, and $Z = \{x \mid x \text{ is a perfect square less than } 10\}$ and the universal set, $U = \{x : x \text{ is an integer and } 0 < x < 11\}$

(a) Find $(X \cup Y)'$

(b) Find $(X \cap Y \cap Z)'$

(c) Verify De Morgan's laws for the sets X, Y, and Z.

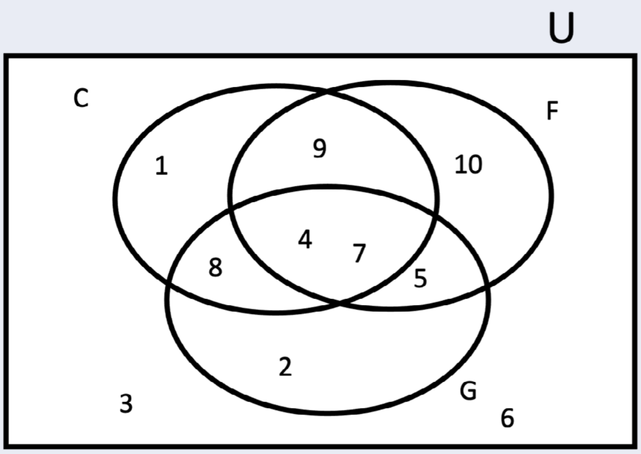
7. Suppose $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{2, 4, 6, 8, 10\}$, and $B = \{1, 3, 5, 7, 9\}$.

(a) Find $(A \cup B)'$

(b) Find $(A \cap B)'$

(c) Verify de Morgan's laws for the sets A and B.

8. The sets C, F and G are subsets of the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ as shown in the diagram below.



List the elements of:

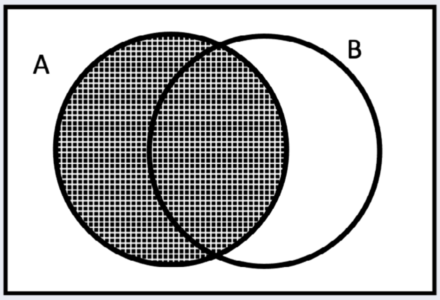
(a) C

(b) $C \cap F$

(c) $C \cap F \cap G'$

(d) $C \cap F' \cap G'$

9. Name the shaded region in the Venn diagram below.



A. A

B. B

C. $A \cap B'$

D. $A' \cap B$

EXTENDED READING

1. Watch this video for further learning on rational and irrational numbers in link provided: https://www.youtube.com/watch?app=desktop&v=aDKek9X4jC4
2. Watch this video to learn more about irrational numbers using the link provided:

   https://youtu.be/g5vfSPAlrVM
3. Baffour Asamoah, B. A. (2015). **Baffour BA series: Core mathematics**. Accra: Mega Heights.
4. Watch these videos using the links provided for further explanations on real numbers, and the closure property:
   a. https://www.youtube.com/watch?v=0OwvN-957aE&pp=ygUScmVhbCBudW1iZXIgc3lzdGVt
   b. Real Numbers (youtube.com)
   c. The Closure Property (youtube.com)
5. Akrong Series: Core Mathematics for Senior High Schools New International Edition
6. Aki –Ola Series: Core Mathematics for Senior High Schools in West Africa, Millennium edition 5 (Pages 1-21)

REFERENCES

1. Stewart, J. (2008). *Calculus: Early Transcendentals.* Cengage Learning.
2. Rudin, W. (1976). *Principles of Mathematical Analysis.* McGraw-Hill Education.
3. Abbott, S. (2012). *Understanding Analysis.* Springer Science & Business Media
4. Baffour BA Series: Core Mathematics for schools and Colleges
5. Asiedu, P. (2016). *Mathematics for Senior High Schools 1*. S. Coleman, K. A. Benson, H. A. Baah – Yeboah (Eds.). Aki – Ola Publications.
6. Trybulec, A.(1990). Enumerated sets. *Formalized Mathematics*, 17. Asiedu, P. (Millennium Edition 5). Core Mathematics for Senior High Schools in West Africa. Aki – Ola Publications.

ACKNOWLEDGEMENTS

Ghana Education Service (GES)

List of Contributors

NAME	INSTITUTION
Janet Waponde	OLA SHS
Yamusah Issahaku	Kumbungu SHS
Joseph Bakpil Nagbija	Nchumuruman SHS, Chinderi
Modzaka Godfred	Keta SHTS, Keta