SECTION 2

GEOMETRICAL FIGURES

GRAPHIC COMMUNICATION

Plane Geometry

INTRODUCTION

In this unit, we will delve into the fundamental aspects of plane geometry, exploring various geometrical figures that lie flat on a single plane. Understanding these figures not only involves recognising their properties and applications but also mastering techniques such as using ratios for scaling, blending circles and lines with arcs, constructing ellipses, and creating Archimedean spirals. Through this exploration, you will gain practical insights into how these geometrical concepts are applied in design, architecture, engineering, and other creative disciplines. Let's dive into the fascinating world of plane geometry and its role in graphic communication.

At the end of this section, you should be able to:

- Explain plane geometrical figures, state their properties, applications and give examples.
- Use ratio to enlarge, reduce, or divide plane geometrical figures.
- Blend circles and lines with arcs.
- Construct an ellipse as a plane geometrical figure.
- Construct an Archimedean spiral as a plane geometrical figure.

Key Ideas

- Plane Geometrical Figures refer to shapes that lie flat on a plane and have two dimensions (length and width), such as squares, circles, and triangles.
- Properties are characteristics or attributes of geometrical figures, such as size, shape, and angles.
- Line Segments are straight parts of a line with two endpoints.
- Curves are smooth, flowing lines that are not straight.
- Circles refer to round shapes where all points are equidistant from the centre.
- Triangles are three-sided polygons with three angles.
- Quadrilaterals are four-sided polygons with four angles, such as squares and rectangles.

- Polygons are closed-plane figures with three or more straight sides and angles, such as triangles, quadrilaterals, pentagons, and hexagons.
- Ellipse refers to a special type of oval shape resembling a squashed or stretched circle.
- Elliptical shapes are utilised in creating structures in architecture, woodwork objects, and metal fabrications.
- An Archimedean spiral is a curve that starts at a central point and moves outward, getting further away from the centre at a constant rate as it turns.
- An Archimedean spiral finds diverse applications in the following areas: **maritime engineering**, **hydrometers**, **hydraulic systems**, **repellents**, **and architectural design.**

PLANE GEOMETRICAL FIGURES

There are different types of plane geometrical figures. This unit will help you explore different types of plane geometrical figures, their properties, examples, and their applications in everyday life.

Plane geometrical figures are shapes created using points, line segments, circles, or curves. They are also known as two-dimensional shapes because they are flat and have only two dimensions: length and width (or breadth). These shapes can have any number of sides, and their sides can be straight, curved, or a combination of both. Such shapes are commonly seen in various designs around us. Examples include triangles, squares, rectangles, circles, ovals, and more. Some common examples along with their properties are seen in **Table 2.1**.

Characteristics of Plane Geometrical Figures

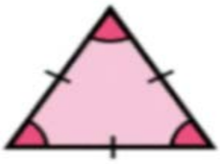
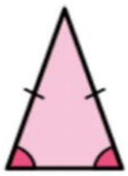
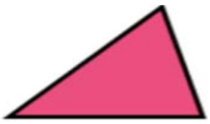
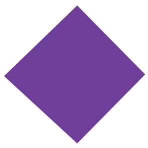
As stated earlier, plane geometrical figures are fundamental shapes in geometry that are defined by their flat surfaces and lack of thickness. These figures, commonly seen in various designs and everyday objects, are essential for understanding basic geometrical concepts. Below are some key characteristics of plane geometrical figures:

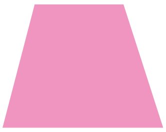
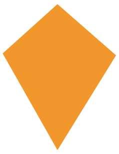
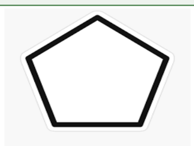
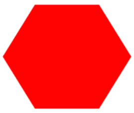
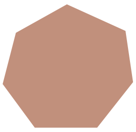
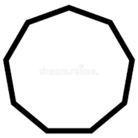
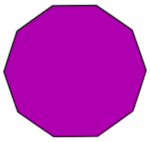
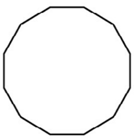
1. Plane geometrical figures have flat surfaces.
2. They have two dimensions.
3. They do not have thickness.
4. All two-dimensional shapes are known as polygons.

Below are two tables: **Table 2.1.1** shows types of polygons whereas **Table 2.1.2** shows other plane geometrical shapes.

Type of Polygons, Examples and Their Properties

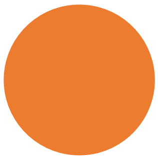
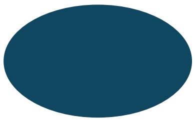
Table 2.1.1 Type of polygons, examples and their properties

Types of polygons	Examples	Properties
Triangles (Trigon)		➢ **Bounded by three straight lines** ➢ **Three internal angles**
	Equilateral	➢ All sides are equal in length ➢ All angles are equal in size 60^{0}
	Right-angled	➢ One right angle ➢ The side opposite the right angle is called the hypotenuse
	Isosceles	➢ Two sides are equal in length ➢ Two equal angles
	Scalene	➢ Three unequal sides ➢ Three unequal angles
Quadrilaterals		➢ **Bounded by four striaght lines** ➢ **Four intenal angles**
	Parallelogram	➢ The opposite sides are parallel. ➢ Opposite sides are equal. ➢ Opposite angles are equal.
	Square	➢ All sides are equal. ➢ All angles are equal and measure 90^{o}.
	Rectangle	➢ The opposite sides are parallel. ➢ Opposite sides are equal. ➢ All angles are equal and measure 90^{o}.
	Rhombus	➢ All sides are equal. ➢ Opposite angles are equal.

	Trapezoid	➢ The opposite sides are parallel. ➢ Adjacent angles add up to 180^o.
	Kite	➢ Adjacent sides are equal. ➢ One pair of opposite angles are equal.
Polygons with more than four sides.		
	Pentagon	➢ Has 5 sides ➢ The sum of interior angles is 540^0
	Hexagon	➢ Has 6 sides ➢ The sum of interior angles is 720^0
	Heptagon	➢ Has 7 sides ➢ The sum of interior angles is 900^0
	Octagon	➢ Has 8 sides ➢ The sum of interior angles is 1080^0
	Nonagon	➢ Has 9 sides ➢ The sum of interior angles is 1260^0
	Decagon	➢ Has 10 sides ➢ The sum of interior angles is1440^0
	Undecagon	➢ Has 11 sides ➢ The sum of interior angles is 1620^0
	Dodecagon	➢ Has 12 sides ➢ The sum of interior angles is 1800^0

Other Geometrical Shapes

Table 2.1.2 Other plane geometrical shapes

Types of shape	Examples	Properties
	Circle	Circles are plane shapes with a curved boundary with all points equidistant from a fixed point
	Ovals (ellipses)	An oval is a plane shape that resembles the shape of an egg or a squashed ball.

Application of Plane Geometry

Plane geometry plays a significant role in various aspects of our daily lives, often without us even realising it. The principles of plane geometry are applied in numerous practical situations, making it an essential area of study. Here are some common applications of plane geometry:

1. Folders/envelopes
2. Tabletops
3. Walls
4. Paper
5. Chalkboard/whiteboard
6. TV screens

In addition, plane geometrical shapes find extensive applications across various fields, demonstrating their versatility and importance. Here are some examples:

- **Artwork:** Geometrical shapes are integral to creating patterns, logos, road signs, and mosaics. Artists and designers use these shapes to create visually appealing and structured designs, often combining them to produce complex and intricate artworks.
- **Engineering:** Plane geometry is crucial in designing and constructing various engineering structures such as gears, roof trusses, and bridges. Engineers rely on geometrical principles to ensure stability, functionality, and aesthetic appeal in their designs.
- **Transportation:** Geometrical shapes are foundational in the design of transportation vehicles and components. Bicycle frames, boat hulls, and wheels are all designed using geometric principles to ensure strength, efficiency, and optimal performance.

- **Architecture:** In architecture, plane geometrical shapes are employed in designing walls, floors, arches, and roof structures. Architects use these shapes to create functional and visually pleasing buildings and structures, ensuring they are both stable and aesthetically pleasing.
- **Sports:** Many sports equipment and fields are designed using plane geometrical shapes. Footballs, volleyballs, tennis balls, and other sports equipment are manufactured based on geometric principles to ensure uniformity, balance, and optimal performance.
- **Nature:** Geometrical shapes are abundant in nature, exemplified by the structure of leaves, honeycombs, shells, and celestial bodies like the sun and star crescent. These natural geometrical shapes inspire many human-made designs and structures.
- **Everyday Objects:** Numerous everyday objects incorporate plane geometrical shapes in their design. TV screens, paper, tabletops, and envelopes all rely on geometric shapes to provide functionality and practicality in daily use.

ACTIVITY 2.1.1

Group Project work

In your groups, do activities 1 (a) and (b) and any other one.

1. (a) Create a 'warning sign' using any modeling material, based on the idea of plane geometrical shapes, and place it on the school grounds. The design should have at least a combination of three different shapes in it.

   (b) Make drawings of the shapes, their names, and their properties in table or chart form.

2. (a) A tiler in an architecture firm wants a pattern to use in laying the tiles for a conference hall. Make a design of a pattern to be recommended for the tiler on an A4 or A3, sheet using at least three different geometrical figures. Give two reasons for the choice of shapes.

   (b) Make templates of the geometrical figures used and put them in a self-made envelope for storage. Write the names of the shapes on the templates.

3. An upper primary school learner claims that a square and a rhombus are the same because they each have sides of equal length and the same number of sides. Explain to the child why their view is right or wrong, using diagrams to illustrate your explanation.
4. A primary school teacher in your community needs teaching and learning material (TLM) to facilitate a lesson on plane geometrical shapes for her Basic 1 class. Create a TLM that will help her class understand that no matter the orientation of the shape, it remains the same. Include at least five different shapes.

Extended Reading

- Morling, K. (2021), Geometric and Engineering Drawing (4TH Edition), Edward Arnold, London. Chapter 3; pages 39 - 50

USING RATIO TO ENLARGE AND REDUCE PLANE GEOMETRICAL FIGURES.

In this lesson, you will be able to use ratio to enlarge and reduce regular and irregular plane geometrical figures using ratio and proportions.

A plane geometrical figure can be enlarged or reduced when all its dimensions are changed using the same ratio without changing the actual shape of the figure. In instances where various sizes of objects are required, for example, small size, medium size, and large size, the principles of enlarging and reducing plane geometrical figures are required.

Enlargement and Reduction

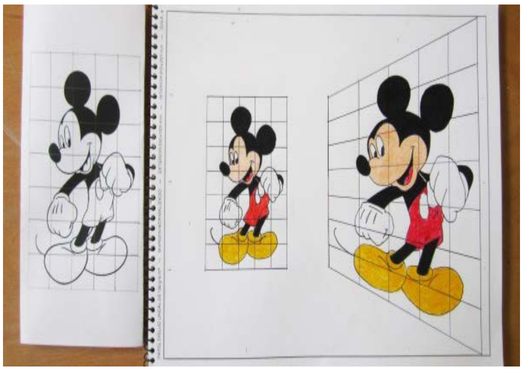


Fig. 2.1.1: A

Fig. 2.1.2: B

In small groups, observe the pictures (A and B) above and give your comments.

Using Ratio to Reduce the Size of a Four-Sided Polygon

Reducing the size of a four-sided polygon using a ratio involves a systematic approach that ensures accuracy and precision. This technique is fundamental in geometry and is often used in various fields, such as architecture and design, to create scaled-down versions of shapes while maintaining their proportions. By following the outlined procedures, you will learn how to effectively use ratios to achieve the desired reduction of a polygon.

Procedures:

1. Draw the given plane figure.
2. Choose a polar point **O** outside the given figure.
3. Draw lines from the polar point **O** to touch the vertices of the plane figure.
4. Construct the given ratio (using division of lines) on one of the radial lines.
5. Draw lines parallel to the given figure based on the ratio to obtain the reduction or enlargement of the four-sided polygon.

Examples

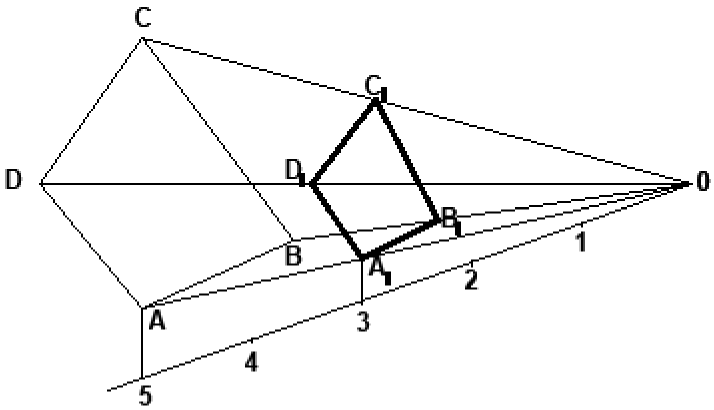


Fig 2.1.3: Reduction of plane figures

Using Ratio to Enlarge a Four-Sided Polygon

Procedures:

1. Draw the given plane figure.
2. Choose a polar point **O** outside the given figure.
3. Draw lines from the polar point **O** to touch the vertices of the plane figure.
4. Construct the given ratio (using division of lines) on one of the radial lines.
5. Draw lines parallel to the given figure based on the ratio to obtain the reduction or enlargement of the four-sided polygon.

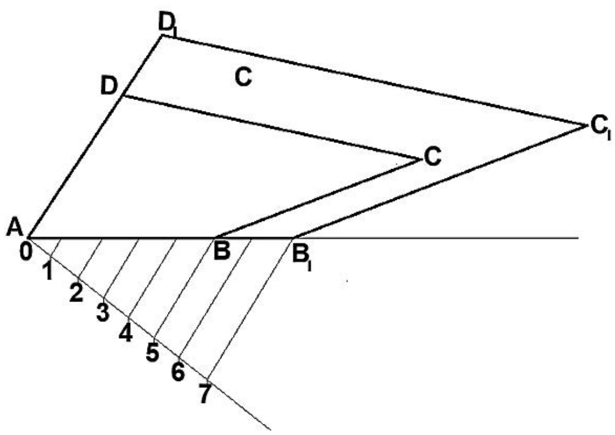


Fig 2.1.4: Enlargement of plane figures

Using Ratio to Enlarge a Three-Sided Plane Figure

Procedures:

1. Draw the given plane figure.
2. Choose a polar point **O** outside the given figure.
3. Draw lines from the polar point **O** to touch the vertices of the plane figure.
4. Construct the given ratio (using division of lines) on one of the radial lines.
5. Draw lines parallel to the given figure based on the ratio to obtain the reduction or enlargement of the four-sided polygon.

<u>Examples</u>

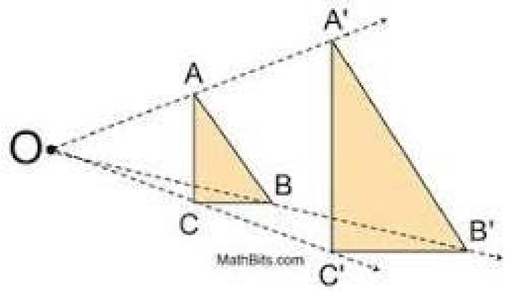


Fig. 2.1.5: Enlargement of a three-sided figure

Construction of Plane Figures of Equal Areas

A. Rectangle Equal in Area to a Given Triangle.

Example 1:

Draw a rectangle equal in area to a triangle with sides AB = 50, AC = 70 and CB = 60

Steps:

1. Draw the given triangle ABC.
2. Draw a perpendicular from B (the apex) to intersect the base line AB at D.
3. Bisect the line BD.
4. From points A and C, project perpendicular lines to intersect the bisected line at E and F respectively.
5. Firm in lines AB, BQ, QP, and PA to obtain the required rectangle.

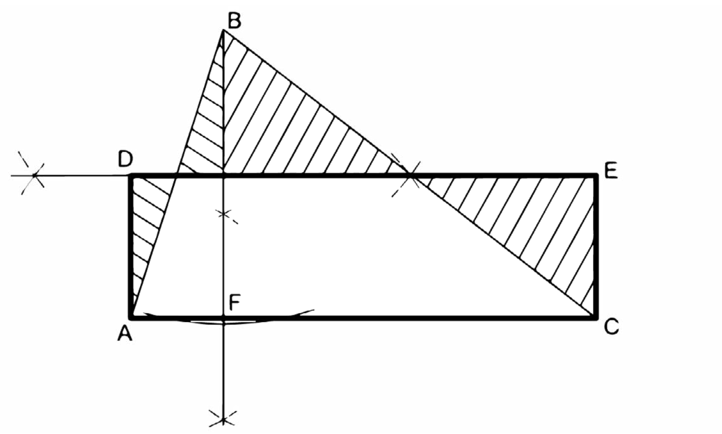


Fig. 2.1.6: Rectangle equal in area to a given triangle

B. Square Equal in Area to a Given Rectangle

Example 2:

A rectangular swimming pool measures 70 mm by 40 mm. Redesign the swimming pool into a square with the same area.

Steps:

1. Draw the given rectangle, ABCD.
2. With point D as the centre and radius DC, draw an arc to intersect the line AD produced at E.
3. Bisect line AE to obtain point F.
4. With point F as the centre and radius FE or AF, draw a semi-circle.
5. Produce a line DC to intersect the semi-circle at point G. Line DG is one side of the square.
6. With radius DG and D as the centre, draw an arc to intersect the line AE produced at L.
7. With the same radius as DG and points G and L as they enter, draw arcs to intersect at H.
8. Firm in lines DG, GH, HL, and LD to obtain the required square.

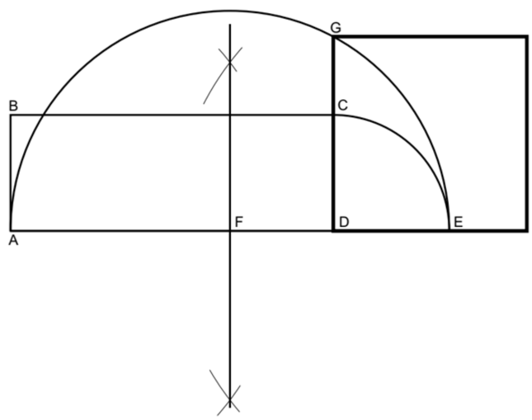


Fig. 2.1.7: Square equal in area to a given rectangle.

C. Triangle to Square of Equal Area

Example 3:

Draw a square equal in area to a triangle with sides AB = 70, AC = 60 and CB = 40

Construction refers to figure 2.1.8

Steps:

1. Draw the given triangle ABC.
2. Convert the triangle to a rectangle of equal area.
3. Convert the rectangle obtained in 2 above to a square of equal area.

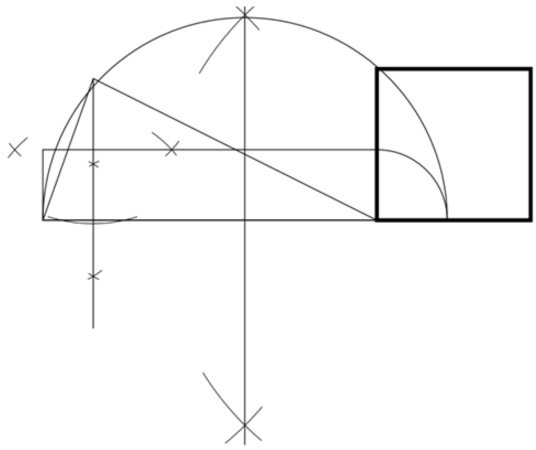


Fig. 2.1.8: Triangle to square of equal area.

D. Triangle Equal in Area to a Given Polygon with More than Four Sides.

Example 4:

Fig. 2.1.9 shows a pentagon. Let's convert the pentagon to a triangle of equal area. Read and follow the steps in **Table 2.1.3** below.

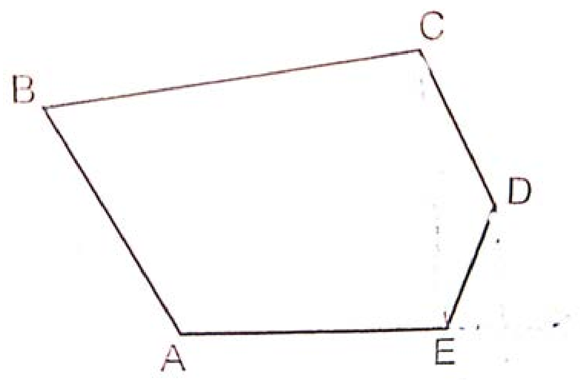
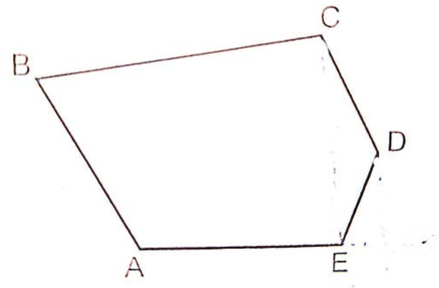
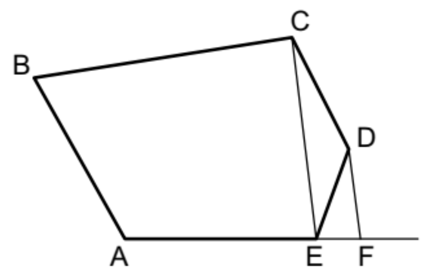
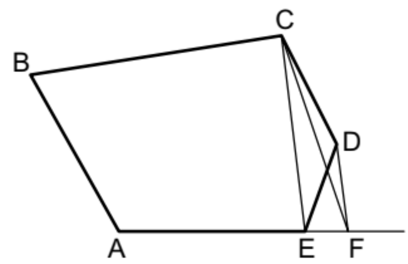
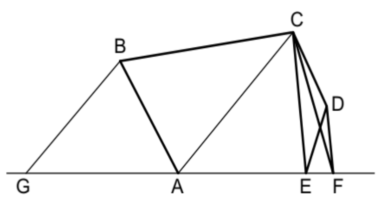
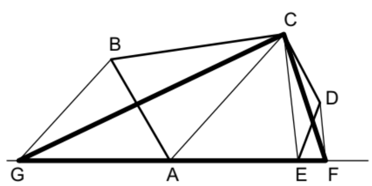


Fig. 2.1.9

Table 2.1.3: Steps to convert a pentagon to a triangle of equal area

PROCEDURE	SKETCH
STEP 1 1. Draw the given polygon **ABCDE**	
STEP 2 1. Extend the base line AE. 2. Join CE. 3. From D, draw a line parallel to CE to meet the extended line at F	
STEP 3 1. Join CF	
STEP 4 1. Join CA 2. From B, draw a line parallel to CA to meet the extended line at G	
STEP 5 1. Join G to C. 2. Figure **CGF** is the required **triangle.**	

Application of Reduction and Enlargement of Plane Geometrical Figures

The principles of reducing and enlarging plane geometrical figures are widely applied in various industries to achieve practical and aesthetic purposes. By scaling shapes accurately, designers and engineers can create efficient and appealing products. Here are a few examples of how these techniques are used:

1. **Designing of Packages:** Ensuring products are securely and attractively packaged by adjusting the dimensions of packaging materials.
2. **Electrical Gadgets:** Developing components and devices in different sizes to fit various specifications and user needs.
3. **Varied Sizes of Canned Food:** Producing cans in different dimensions to offer consumers multiple portion options while maintaining the same shape and design.

ACTIVITY 2.1.2

1. **Draw a rectangle equal in area to an equilateral triangle of side 60.**

Steps:

i. Follow the steps in **Example 1** above to obtain **Fig. 2.1.10**.

ii. Shade the part of the triangle outside the rectangle XYZ. (*Refer to Fig. 2.7 below*).

iii. Cut the shaded portions XYC and XCZ.

iv. Match XYC with AEX and XCZ with BEZ.

v. Write your observations in the space provided below and share them with a friend.

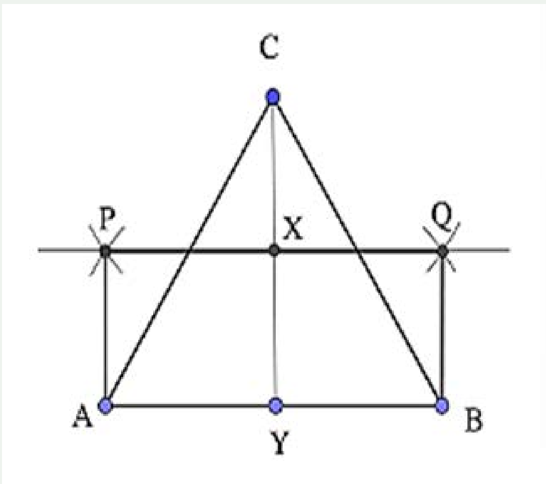


Fig. 2.1.10: A rectangle equal in area to a given triangle.

2. Designing three different sizes of Pizza box.

Imagine you are a packaging designer tasked with enlarging the base of a four-sided pizza box. The current base has a length of 40 mm, and you need to enlarge it using the ratio 2:4:6. By following the steps for Fig. 2.1.10, you can achieve this transformation. Use the hints below to guide you through the process:

i. **Draw the Original Figure**: Start by sketching the original four-sided base of the pizza box with a length of 40 mm.

ii. **Choose a Polar Point:** Select a polar **point O** outside the figure from which you will draw radial lines.

iii. **Connect Vertices:** Draw lines from **point O** to touch each vertex of the original figure.

iv. **Construct the Given Ratio (using division of lines):** Use the ratio 2:4:6 to divide one of the radial lines, ensuring each segment reflects the specified proportion.

v. **Draw Parallel Lines:** Draw lines parallel to the sides of the given figure based on the constructed ratio, to obtain a reduced or an enlarged version of the pizza box base.

Extended Reading

- https://www.youtube.com/watch?v=NokTEaoGFE8
- Morling, K. (2021), *Geometric and Engineering Drawing (4TH Edition)*, Edward Arnold, *London. Pages 75 – 84.*
- https://www.youtube.com/watch?v=NokTEaoGFE8

BLENDING CIRCLES AND LINES WITH ARCS

In this lesson, you will learn about the fusion of circles, lines, and arcs—a vital aspect of plane geometry. Mastering this blend deepens your understanding and ignites creativity where ideas are conveyed through design. You will understand how circles, lines, and arcs blend, enhance geometric skills, and cultivate visual literacy.

You will also explore techniques and principles, carry out hands-on activities, and see real-world examples.

Blending of Arcs

When constructing an outline that contains blending curves, be concerned with the positions of the centres of the curves. A curve will blend exactly with another curve or line when the centre of the curve is correctly found.

Below are examples of objects we commonly use that are made based on the blending of circles and lines with arcs.

Fig. 2.1.11: Items that are based on the blending of circles and lines with arcs.

Reasons for Blending Circles and Lines with Arcs

Curves are included in the outlines of components for several reasons. These include:

1. To remove sharp edges to make it safe for handling.
2. To avoid extra machining costs.
3. To enhance appearance or aesthetics.

Principles for Blending Circles

For a curve or an arc to blend exactly, the centre of the curve should be found. When the actual centre is located, the curve will blend perfectly.

Conditions for Finding Centres of Arcs

1. Find the centre of an arc with radius *r*, which blends with two straight lines meeting at right angles.

Principle 1

The blending of two lines meeting at right angle

- Let **r** be the given radius and **AB**, and **AC** the given straight lines
- With **A** as centre and radius equal to **r**, draw arc intersecting **AB** at **P**
- With **A** as centre and radius equal to **r**, draw arc intersecting **AC** at **Q**
- With **P** as centre and radius equal to **r**, draw an arc
- With **Q** as centre and radius equal to **r**, draw an arc to intersect the previous arc at **O**
- With **O** as centre and radius equal to **r**, draw the required arc to blend the two straight lines at **P** and **Q**.

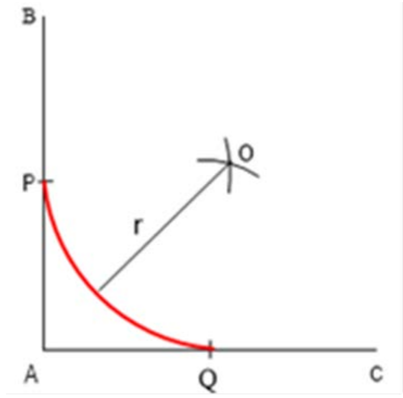


Fig. 2.2.12

Principle 2

The blending of two lines meeting at angles other than 90^0

- Let **AB** and **DC** be the two straight lines and r the radius
- Draw a line **RQ** parallel to **AB** at a distance **r** from **AB**
- Draw a line **ST** parallel to **DC** at a distance **r** from **DC**
- Extend the lines **ST** and **DC** to meet at **O**
- With **O** as centre and radius equal to **r** draw the arc to blend the two straight lines **AB** and **DC**

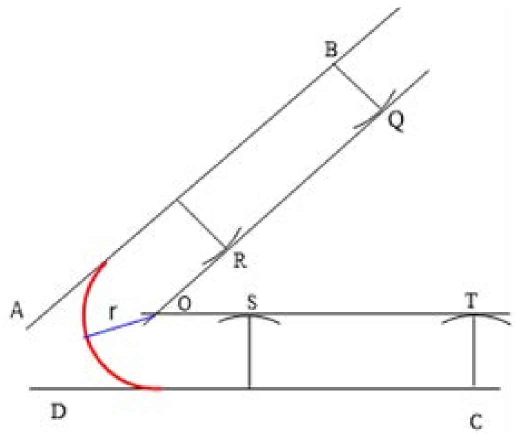


Fig. 2.2.13

Principle 3

Blending a line and a point,

- Construct a line DC, parallel to the given line AB, distance r away
- Position a point S at a convenient place between the two lines
- With centre S, radius r, draw an arc to cut line DC at O
- With O as centre and radius r, draw an arc to pass through S and meet the given line AB

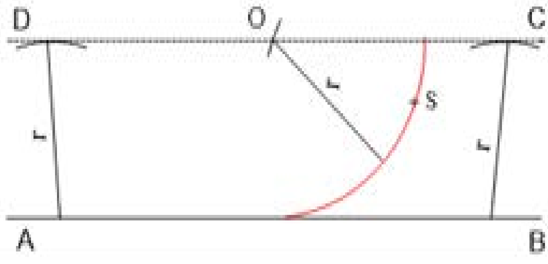


Fig. 2.1.14

Principle 4

- Mark points P and Q, 100 mm apart
- Draw a circle with centre P and radius 20 mm
- Draw another circle with centre Q and radius 25 mm
- Draw an arc with centre P and radius 70 mm (20 + 50 = 70).
- Draw another arc with centre Q and radius 75 mm (25 + 50 = 75) to intersect the previous arc at point O.
- Draw an arc AB with centre O and radius 50 mm.
- This arc AB blends internal the two circles

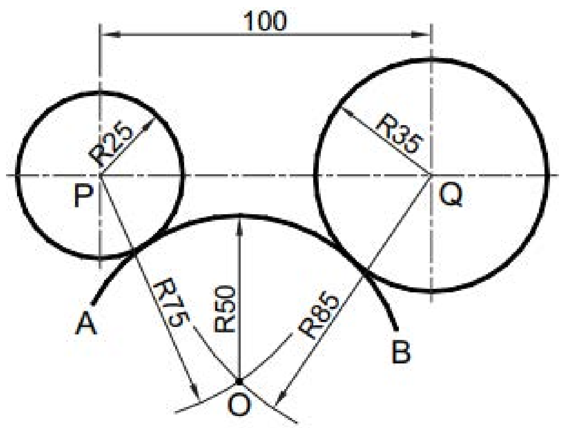


Fig. 2.1.15

Principle 5

- Mark points **P** and **Q**, 100 mm apart.
- Draw a circle with centre **P** and radius 25 mm
- Draw another circle with centre **Q** and radius 35 mm.
- Draw an arc with centre **P** and radius 75 mm (100 – 25 = 75)
- Draw another arc with centre **Q** and radius 65 mm (100 – 35 = 65) to intersect the previous arc at point **O**
- Draw an arc **AB** with centre **O** and radius 100 mm.
- This arc **AB** is external to both the circles and touches them tangentially.

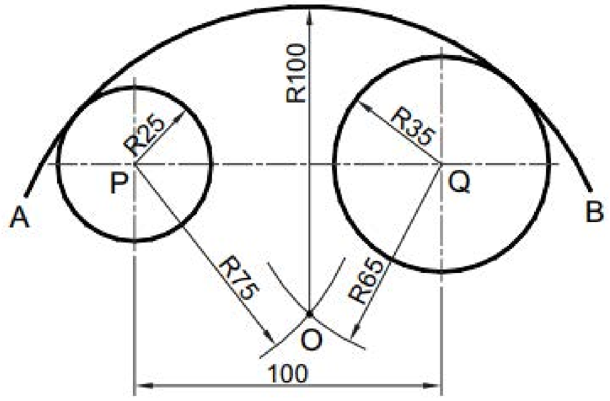


Fig. 2.1.16

Principle 6

Blending two circles of different radii internally and externally

- Mark points P and Q, 100 mm apart
- Draw a circle with centre P and radius 25 mm
- Draw another circle with centre Q and radius 35 mm.
- Draw an arc with centre P and radius 125 mm (100 + 25 = 125)
- Draw another arc with centre Q and radius 65 mm (100 – 35 = 65) to intersect the previous arc at point O
- Draw an arc AB with centre O and radius 100 mm
- The arc AB blends internal to the circle with centre P and blends external to the circle with centre Q

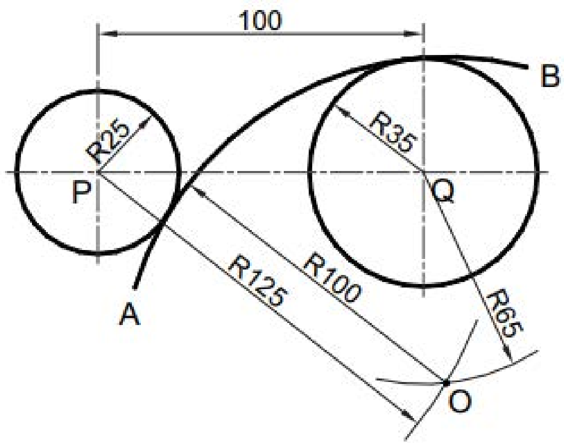


Fig. 2.1.17

Activity 2.1.3

1. In groups, discuss and complete the table below

	Object	Name	Description of object
1.		....	....
2.		....	....
3.		....	....
4.	....	Baby's feeding bottle	It has two main parts: an irregular curved-shape teat and a cylindrical body.

2. The step leading from the veranda to the classroom floor of the 2 Art 1 class is so sharp with jagged edges that it causes students to fall, resulting in bruised knees. Draw a diagram to show how the edge of the riser and tread can be redesigned to make it safe for students to use.
   - Let **r** be the given radius and **AB**, and **BC** the given straight lines
   - With **B** as centre and radius equal to **r**, draw arc cutting **AB** at **Q** and **BC** at P
   - With **Q** as centre and radius equal to **r** draw an arc
   - With **P** as centre and radius equal to **r**, draw another arc to intersect the previous arc at **O**
   - With **O** as centre and radius equal to **r**, draw the required arc to blend the two straight lines at **P** and **Q.**

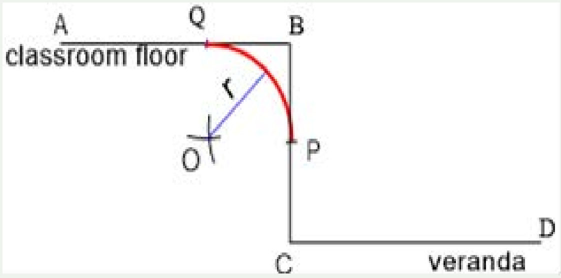


Fig. 2.1.18: Step from veranda to classroom floor

3. Based on the principles and concept of blending lines and circles with arcs construct a one end open spanner.

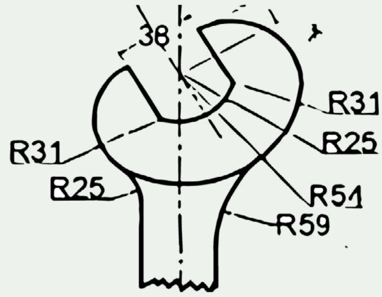


Fig. 2.1.19: Spanner construction

4. To blend two circles, U and V, of different radii with an arc of radius Y externally:

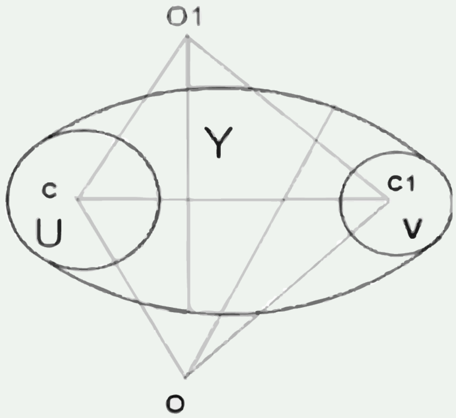


Fig. 2.1.20: Blending of two circles of different radii externally

DATA

Ur = 35, Vr = 25, Y = arc, radius = 80 and cc1= 60

Steps

a. Draw two circles with the given radii, placing their centres *P* units apart.
b. Find the centre of the arc by subtracting the radius of circle U from the radius of the arc *Y*, that is, *Y*-radius of circle *U*.
c. With *U* as the centre and a radius of *Y*-radius of circle *U*, draw an arc.
d. Subtract the radius of circle *V* from the radius of the arc *Y*.
e. With *V* as the centre and a radius of *Y*-radius of circle *V*, draw another arc to intersect the first arc at point *O*. *O* is the centre of the arc to blend the circles.
f. With *O* as the centre and a radius *Y*, draw the arc to blend the two circles *U* and *V*.
g. Repeat the process to obtain the centre *O1*.
h. With *O1* as the centre and a radius *Y*, draw the arc below to blend the two circles.

5. To blend two circles, U and V, of different radii with an arc of radius R externally and internally.

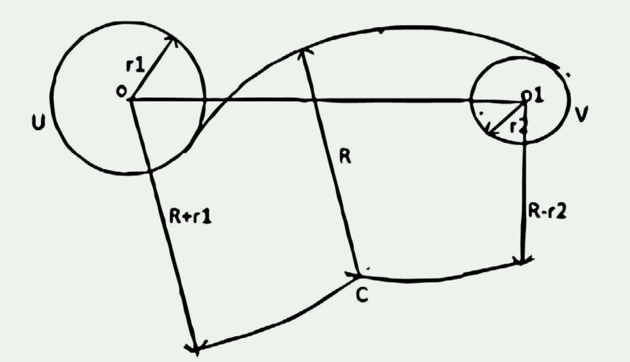


Fig. 2.1.21: Blending of two circles of different radii externally and internally

Steps

a. Draw two circles with the given radii r1 and r2, placing their centres P units apart.
b. Find the centre of the arc by adding the radius of circle U to the radius of the arc R. That is, R + r1. With centre O and radius R + r1, draw an arc.
c. Subtract the radius of circle V from the radius of the arc R. That is, R - r2. Then, with centre O1 and radius R - r2, draw another arc to intersect the first arc at point C.
d. From centre C and with radius R, draw the arc to blend the two circles U and V externally and internally.

CONSTRUCTING AN ELLIPSE AS A PLANE GEOMETRICAL FIGURE

Geometrical constructions provide a practical understanding of various shapes and curves. Two interesting figures in this area are the ellipse and the Archimedean spiral, each with unique properties and applications.

In this lesson, you will explore the steps to construct an ellipse and an Archimedean spiral, enhancing your geometric drawing skills and visualisation.

Ellipse

An ellipse is a geometric shape formed when a cone is sliced at an angle that is not perpendicular to its base. The resulting figure has two axes of symmetry, with the longer axis being the major axis and the shorter one the minor axis. See the example illustrated in **Fig. 2.1.22** below. Ellipses are widely found in both natural and man-made objects, from planetary orbits to architectural designs and various forms of artistic expression.

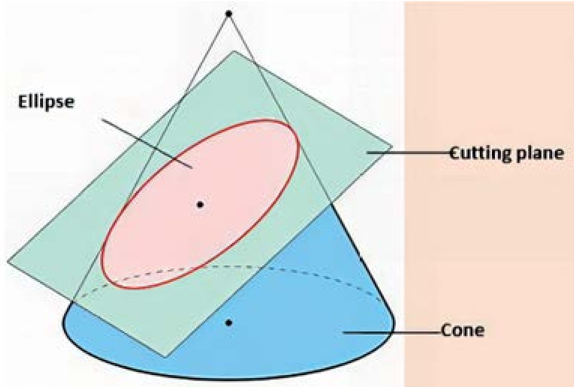


Fig.2.1.22: Ellipse

An ellipse can be defined as the locus of a point that moves in such a way that its distance from a fixed point (known as the focus) maintains a constant ratio, which is always less than one (1), to its perpendicular distance from a fixed straight line (called the directrix).

It can also be defined as the path of a point that moves in such a way that its distance from a fixed point (the focus) always has a constant ratio, less than one (1), to its perpendicular distance from a straight line (the directrix).

Fig. 2.1.23: An elliptical faced clock

Properties of an Ellipse

The following are key properties of an ellipse:

1. **The major axis**: This is the maximum distance measured between its two vertices, i.e., vertex A and vertex B, on the horizontal axis.
2. **The minor axis**: This is the maximum distance measured on the vertical axis.
3. **Vertex:** Each of the two points where the ellipse intersects its major axis. These points represent the turning points of the path traced by the locus of points that form the ellipse.
4. **Directrix:** A straight line perpendicular to the major axis of the ellipse. The perpendicular distance from any point on the ellipse to the directrix is essential in defining the ellipse.

5. **Focal Points (Foci):** The two fixed points within the ellipse such that the sum of the distances from any point on the ellipse to these two points remains constant. The position of the focal points determines the shape of the ellipse.
6. **Eccentricity:** A measure of how much the ellipse deviates from being circular. It is defined as the ratio of the distance from any point on the ellipse to one of the foci, to the perpendicular distance from that point to the nearest directrix.

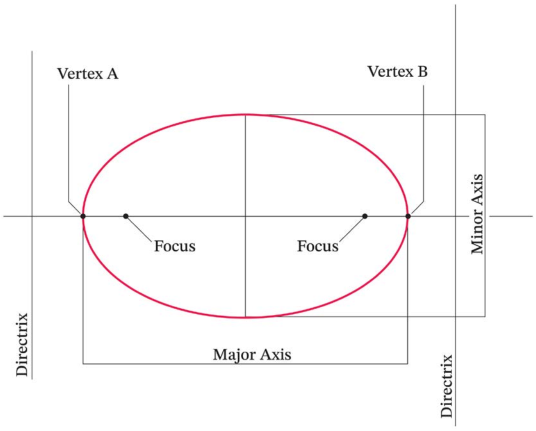


Fig. 2.1.24: Properties of an ellipse

Methods of Constructing an Ellipse

The common methods of constructing an ellipse with the major and minor axes are:

1. **Concentric circles method:** a technique that uses two circles with a common centre to help you draw an ellipse.
2. **The rectangular method** involves using a rectangle to guide the drawing of an ellipse.
3. **Trammel method:** A mechanical method for drawing an accurate ellipse.

1. Concentric Circles Method

Below is an example showing how to construct an ellipse using concentric circles when the length of the major axis is 120 mm and the minor axis 80 mm.

Steps:

a. Draw the major and minor axes AB and CD and then locate the centre O.

b. With center O and major axis and minor axis as diameters, draw two concentric circles.

c. Divide both the circles into an equal number of parts, say 12 and draw the radial lines.

d. Considering the radial line 0-1^1-1, draw a horizontal line from 1^1 to meet the vertical line from 1 at P_1.

e. Repeat the fourth point and obtain other points P_2, P_3, etc.

f. Join the points by a smooth curve to obtain the required ellipse.

See **Figure 2.1.25** below to view what the results look like.

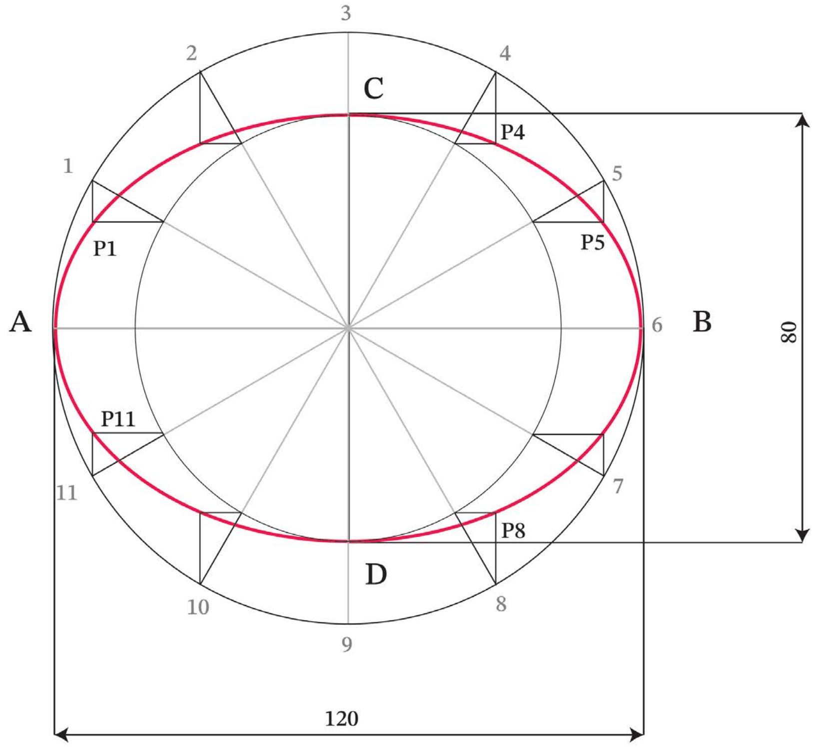


Fig. 2.1.25: Concentric circles method

2. Rectangular Method

Steps

a. Draw the major and minor axes AB = 120 and CD = 80 and locate the centre O.

b. Draw the rectangle KLMN passing through A, D, B, and C.

c. Divide AO and AN into the same number of equal parts, say 4.

d. Join C with the points 1′, 2′, and 3′.

e. Join D with points 1, 2, and 3 and extend till they meet the lines C-1′, C-2′, and C-3′ respectively at P1′, P2′ and P3′.

f. Repeat the third and fifth points to obtain the points in the remaining three quadrants.

g. Join the points with a smooth curve to form the required ellipse.

See **Figure 2.1.26** below to view what the results look like.

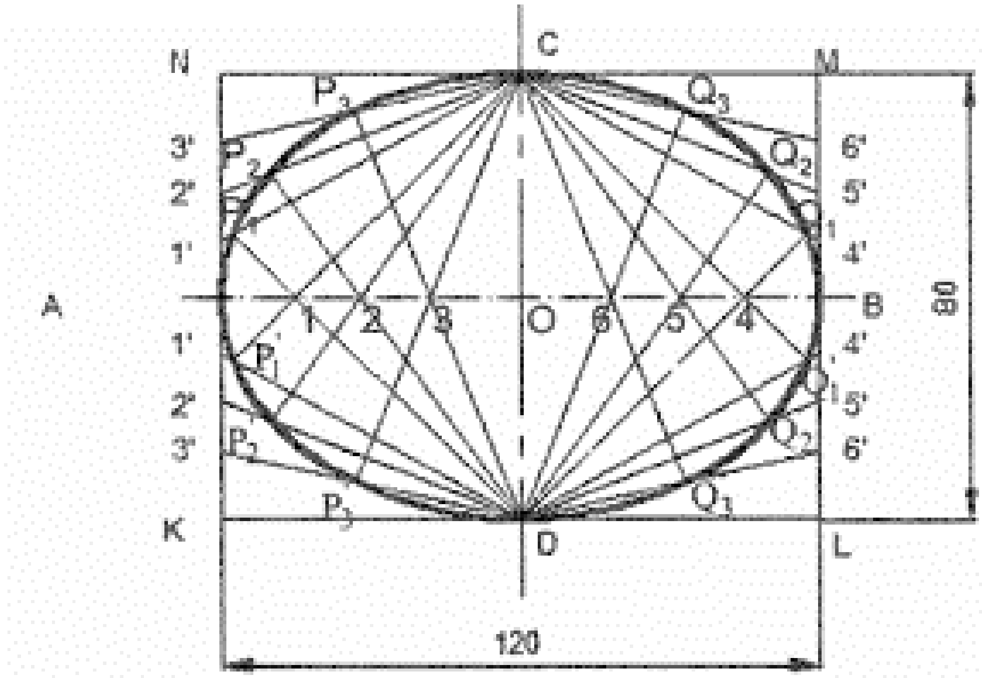


Fig. 2.1.26: Rectangular method

3. Trammel Method

Steps:

a. Take a strip of paper (Trammel) and mark PQ = half the minor axis and PR = half the major axis, as shown.

b. Draw AB = 120 mm to represent the major axis, and bisect it at 0. Through 0 draw a vertical CD = 80 mm to represent the minor axis.

c. Keep the trammel such that Q is lying on the major axis and R on the minor axis. Now the position of the point P is one of the points on the ellipse.

d. Then change the position of the trammel so that Q and R always lie on AB and CD, respectively. Now the new position of the point P is another point to construct the ellipse.

e. Repeat the above and rotate the trammel for 360°, always keeping Q along AB and R along CD.

f. For different positions of Q and R, locate the positions of point P and draw a smooth ellipse.

See **Figure 2.1.27** below to view what the results look like.

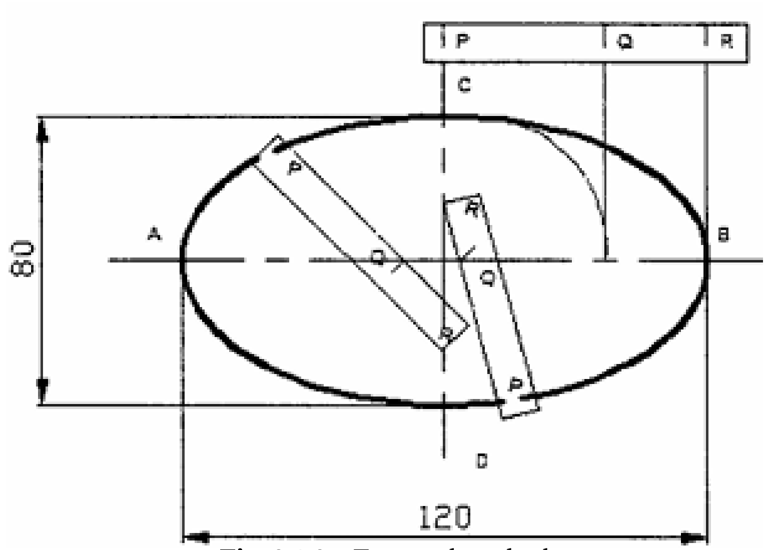


Fig. 2.1.27: Trammel method

Applications of an Ellipse

The geometric properties of an ellipse have numerous practical applications across various fields. From architecture and engineering to astronomy and art, ellipses play a crucial role in designing structures, understanding planetary orbits, and even in acoustics. Below are some areas where ellipses are applied, demonstrating their significance in both theoretical and practical contexts.

a. **Architecture:** Ellipses are used in the design of buildings to create aesthetically pleasing curves and arches.

Fig. 2.1.28: Elliptical Building in Architecture

b. **Woodworking:** Elliptical shapes are used in designing and constructing tables, shelves, and other furniture pieces for a distinctive look.

Fig. 2.1.29: Elliptical table in Woodwork

c. **Metal Fabrication:** In metalwork and automotive design, ellipses are utilised to create various components and parts that require smooth, curved surfaces. For example, elliptical shapes are often employed in the design of exhaust pipes, body panels, and decorative elements to achieve both functionality and aesthetic appeal.

Fig. 2.1.30: Elliptical mirror frame in Metal Fabrication

Archimedean Spiral

The Archimedean spiral is a curve named after the ancient Greek mathematician Archimedes. It is characterised by the property that the distance between consecutive turns of the spiral is constant. Unlike other spirals where the gap between coils widens or narrows, in an Archimedean spiral, the spacing remains uniform. This type of spiral has practical applications in fields such as engineering, art, and physics, where consistent spacing is essential.

The Archimedean Spiral (as a Locus)

It is the locus of a point that moves away from a fixed point at a constant speed along a line which simultaneously rotates with a constant angular velocity. This consistent motion results in a spiral with evenly spaced turns, making the Archimedean spiral unique among spirals and widely applicable in fields like gear design, antenna engineering, and artistic patterns.

Archimedean spirals can be observed in nature, for instance, in the shells of snails and the coils of millipedes.


Fig. 2.1.31: Coiled millipedes


Fig. 2.1.32: Snail shell

They can also be seen in artificially designed objects such as mosquito coils and spiral stairs.


Fig. 2.1.33: Mosquito coil


Fig. 2.1.34: Spiral stairs

Activity 2.1.4

Below are the steps used to construct an Archimedean spiral. Follow the steps and construct the spiral in your notebooks.

1. **Steps** Start from the point of origin (centre O) and draw a circle using the radius given.
2. Divide the circle into 12 equal parts.
3. Divide the radius into 12 equal parts.
4. With centre O and radius 1, 2, 3,…12, draw arcs to intersect the corresponding radii.
5. Join the intersecting points with a smooth curve to obtain an Archimedean spiral.

In the end, your Archimedean spiral should look like **Fig. 2.1.35** below

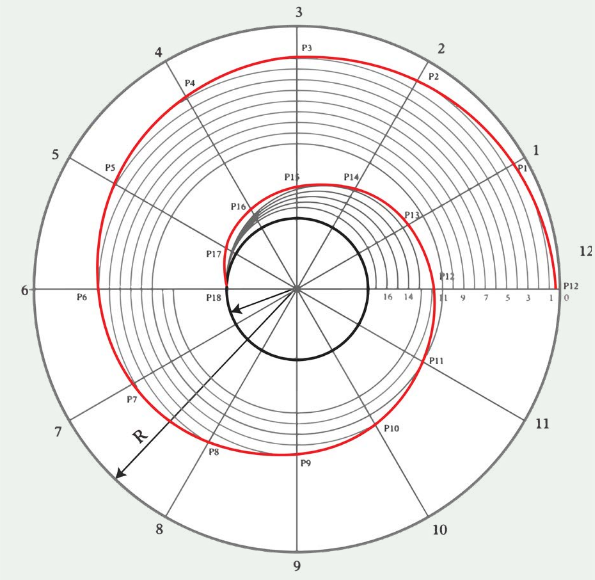


Fig. 2.1.35: Construction of Archimedean spiral

Applications of an Archimedean Spiral

Archimedean spirals are not only fascinating mathematical constructs but also have practical applications across various fields. Their unique properties make them useful in both natural and man-made designs. Here are some key areas where Archimedean spirals are applied:

1. **Maritime Engineering:** Used in the design of ships and submarines to optimise hull shapes for efficiency and stability.
2. **Hydrometry** is employed in hydrometers to accurately measure the density of fluids based on the depth of immersion.
3. **Hydraulic Systems:** Utilised in hydraulic lifts to control the movement of fluids and ensure smooth operation.
4. **Insect Repellents** are round in the design of mosquito coils to create an even and effective burning pattern.
5. **Architectural Design:** Used in the layout of spiral stairs to achieve a visually pleasing and structurally sound staircase design.

Extended Reading

- Agrawal, C. M. and Agrawal, B. (2015). Engineering Drawing, McGraw Hill Education (India) Private Limited. Pages 5.16, 6.21.
- Visit YouTube and watch videos on Blending Arcs and Lines
- https/eccotradingdesginlondon.com
- Lawrence, J. D. (1972), A Catalog of Special Plane Curves, New York Dover, pp. 186-187

REVIEW QUESTIONS, UNIT 1

1. Explain the concept of plane geometry and give three examples.

2. (a) Within your immediate environment, there are different geometrical shapes in use. Identify four of such plane geometrical figures with the help of the table below:

S/No.	Name	Number sides	Properties
1			
2			
3			
4			

(b) What might have informed the choice of the use of those figures, in your opinion? (**Hint:** Explain by using two properties for each of the geometrical figures).

3. In what real-life applications will a circle be preferred to an oval in a design?

4. In your groups, discuss how you would enlarge a triangular plot of land with a side length of 60mm using a ratio of 8:5.

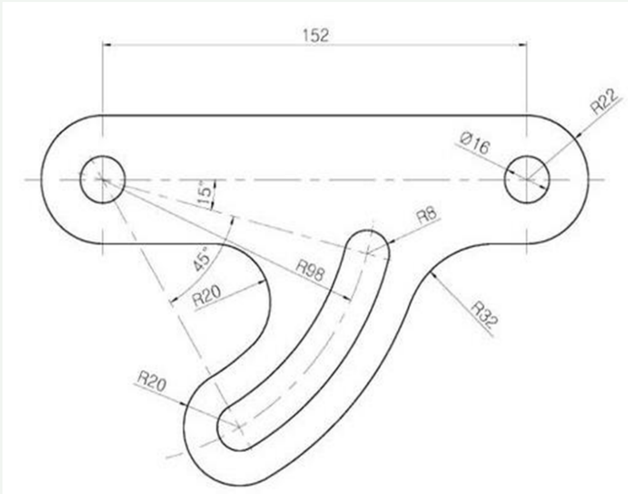
5. As a design technology student, you have been contacted to draw a regular pentagonal tabletop with a side of 600mm, which is far bigger than A3 drawing paper. How will you draw the figure when given the ratio of 5:8?

Project work

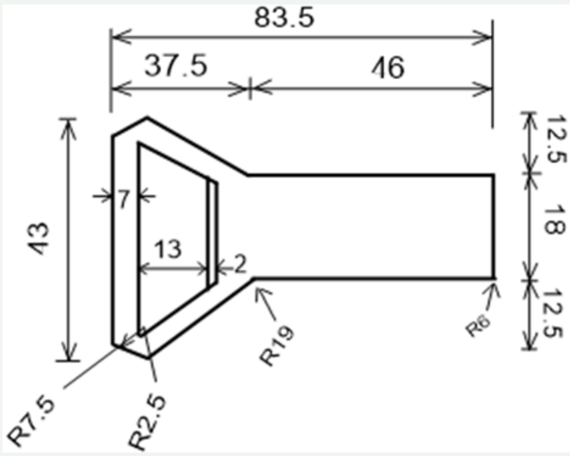
In groups, select any one of the following and solve the problem:

6. Your school library lacks units to display the recently donated books. Design a unit suitable for displaying the library books, using the concept of blending circles and straight lines with arcs. Create a cut-out of your design and paste it in your classroom.

7. An engine part has been designed to replace a damaged one. Discuss the part by identifying five (5) sides blended with arcs, based on your understanding of blending circles and lines with arcs. Illustrate the figure on an A3 sheet and label the identified sides.



8. Observe the figure below and identify any five (5) faults that make it difficult to use. Redesign this item and correct the identified faults to make it safe and user-friendly.



9. Imagine you are walking through a park. You notice many objects; identify any three everyday objects around you that feature elliptical shapes.

10. You are tasked with preparing a road sign in an elliptical shape on a piece of paper. **Draw the ellipse with a major axis of 100 mm and a minor axis of 75 mm using any of the three methods available.**

11. At the workshop, you are tasked with crafting a spiral. **Construct an Archimedean spiral with a radius of 75 mm.**

12. As an interior designer, draw an elliptical tabletop with a major axis of 1000mm and a minor axis of 750mm for an elegant yet functional dining table.

13. A building contractor came to you, as an architect planner, to design a design compact spiral staircase that can save space as well as enhance the aesthetic appeal of a multi-story office building.

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- Hu, Z.H. Ding, Y.S. Zhang, W.B. et al., An interactive co-evolutionary CAD system for garment pattern design. Computer. Aided Des. 40(12), 1094–1104 (2008)
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Graphic Communication

Solid Geometry

INTRODUCTION

This unit introduces you to the basics of solid geometrical figures—three-dimensional (3D) objects that take up space in the real world. Unlike plane geometry, which focuses on flat, two-dimensional (2D) shapes like squares, triangles and circles, solid geometry deals with shapes that have height, width, and depth. These three dimensions give objects their volume and allow them to exist in real life and occupy physical space. Gaining knowledge of and practical skills in solid geometry will help you in visualising and constructing shapes used in various important fields like engineering, architecture, design, and art, where accurate working with three-dimension (3D) objects is essential.

At the end of this section, you will be able to:

- Explain the types of solid geometrical figures and give examples
- Construct objects in isometric and oblique
- Construct objects in perspective
- Construct the surface development of prisms

Key Ideas

SOLID GEOMETRICAL FIGURES

- A **Cylinder** can be known as a three-dimensional geometric shape with two parallel circular bases connected by a curved surface.
- A **Cone** refers to a three-dimensional geometric shape with a circular base and a pointed top called the apex or vertex.
- A **Prism** is a three-dimensional geometric shape with two parallel, congruent bases connected by rectangular (or parallelogram) faces.
- A **Pyramid** is a three-dimensional geometric shape with a polygonal base and triangular faces that converge to a single point called the apex or vertex.
- A **Sphere** refers to a perfectly round, three-dimensional geometric shape where every point on the surface is equidistant from the centre.
- A **Cube** is a three-dimensional geometric shape with six equal square faces, twelve equal edges, and eight vertices

CONSTRUCTING OBJECTS IN ISOMETRIC AND OBLIQUE

- **Isometric drawing** is a simple and effective way to show a three-dimensional object on a two-dimensional surface without losing accuracy.

- In isometric drawings, vertical lines are straight up and down, and horizontal lines are at a 30-degree angle.
- **Oblique drawing** is a three-dimensional drawing where the side of the object that shows the depth (how far it goes back), is drawn at an angle, usually 45 degrees.
- In oblique drawings, vertical lines stay upright, one horizontal line is straight and at 90 degrees to the vertical, while the other horizontal line is tilted at an angle less than 90 degrees.

PERSPECTIVE DRAWING

- **Perspective drawings** refer to the technique that creates a sense of depth and realism, making the drawn scene look more lifelike.
- **Vanishing Points** are points on the horizon line where parallel lines seem to converge.
- **Horizon Line** shows the level at which the sky and ground appear to meet.
- **Orthogonal Lines** are lines that lead to the vanishing point, helping to create the illusion of depth.
- **One-Point Perspective** means using a single vanishing point; often used for compositions where the object is directly facing the viewer.
- **Two-Point Perspective** is when using two vanishing points; commonly used for drawing objects at an angle.

SURFACE DEVELOPMENT OF PRISMS

- A prism is a three-dimensional solid with two identical and parallel polygonal bases connected by flat faces.
- Prism development refers to the process of creating a two-dimensional layout, or net, that can be folded to form a three-dimensional prism.
- Unfold refers to the process of laying out the surfaces of a three-dimensional (3D) object into a two-dimensional (2D)
- A truncated prism is a prism that has had the top part cut off at an angle, making the top face smaller and slanted compared to the base.

SOLID GEOMETRICAL FIGURES

Solid geometry is a branch of geometry that deals with three-dimensional (3D) space. Solid geometry includes the length, breath, and height of objects.

Everyday objects such as cupboards, footballs, cardboard tubes, and gas cylinders are examples of solid geometrical shapes

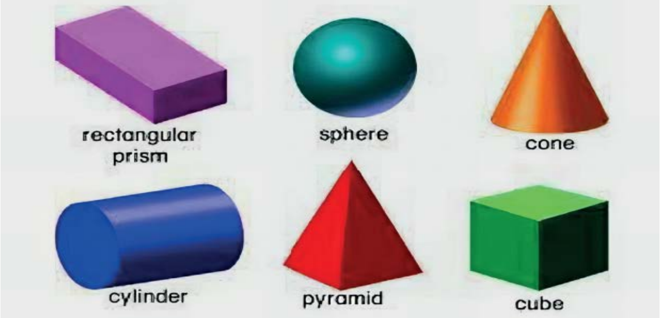


Fig 2.2.1: Common solid geometrical shapes

Classifications of Solid Geometry

Solid geometrical figures are classified into several categories. The two most common ones are:

1. **Prisms:**

   A Prism is a geometric shape with two parallel, congruent bases connected by rectangular (or parallelogram) faces

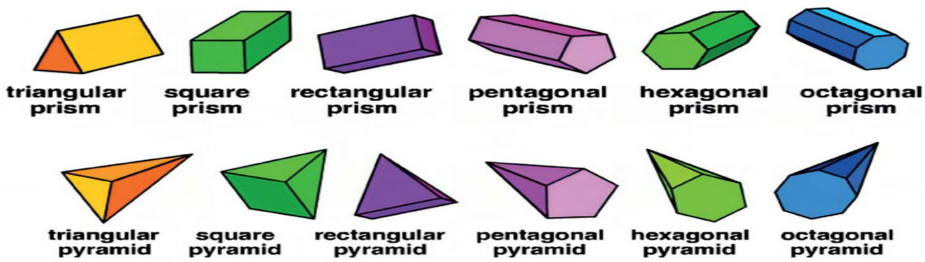


Fig 2.2.2: Prisms

2. **Pyramids:**

   A Pyramid is a geometric shape with a polygonal base and triangular faces that converge to a single point called the apex or vertex.

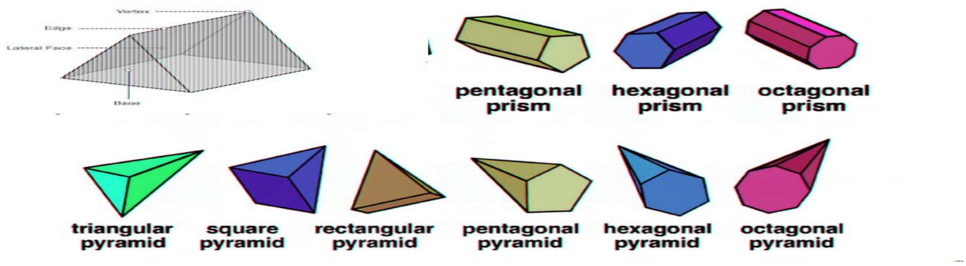


Fig 2.2.3: Pyramids

Properties of Solid Geometrical Figures

Solid geometrical figures have the following features:

Edges: An edge is the line segment that joins one vertex to another. It helps in forming the outline of 3D shapes. It means it joins one corner point to another.

Faces: It is defined as the flat surface enclosed by edges that geometric shapes are made up of. It is a 2D figure for all 3D figures.

Vertices: A vertex is a point where the edges of the solid figure meet each other. It can be referred to as a point where the adjacent sides of a polygon meet. The vertex is the corner where edges meet.

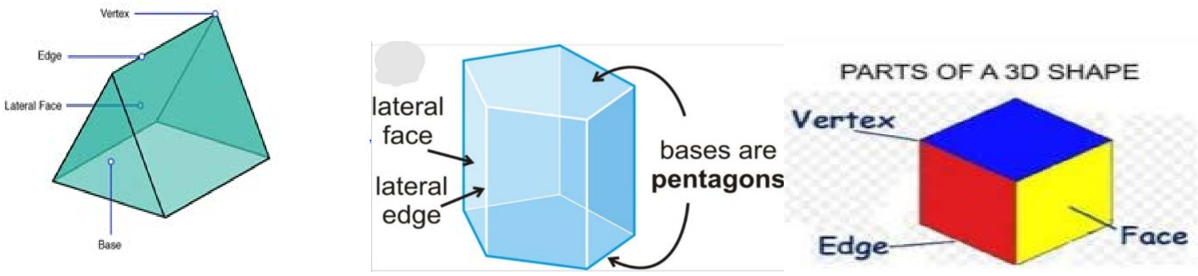


Fig 2.2.4: Features of solid geometrical figures

Table 2.2.1: Properties of prisms and pyramids

PRISMS	PYRAMIDS
Have a constant cross-section along their height.	Have a tapering shape with a base and sloping sides that meet at the apex.
Have flat faces which are polygons.	Have a polygon base.
Have straight edges that connect to vertices of the faces.	Have triangular faces that meet at the apex.
Have equal height along the faces.	Have a single apex (vertex).

Application of Solid Geometry in Real Life

Solid geometry has numerous real-life applications. Some are seen in the following areas:

1. **Construction of structures**
   Used in construction of structures such as buildings, bridges, and other structures
2. **Art works**
   Sculpturing, modelling, and visual arts can be made from solid geometry.
3. **Interior designing**
   Solid geometries are used in the halls, kitchens, conference rooms, and other related areas
4. **Ship building**
   They can be used in ship building, creating machines, mechanisms and other products
5. **Vehicle manufacture**
   Modern vehicles are designed in these shapes
6. **Most Computer Aided Designs (CAD)** such as gaming, and simulations are in the form of solid geometry.

Activity 2.2.1

1. A group of children were arguing that a Hip Roof design they saw on a building is either a prism or a pyramid. With your knowledge in solid geometry, explain to the children why it is a pyramid or prism. Present your response in a table by stating the two types of solid geometry, listing three examples of each, and stating two properties of each. Support your work with drawings or pictures of prisms and pyramids, and that of a Hip Roof design.

Solid Geometry	Three Examples each	Two Properties of each	Pictures/drawings
Prism	i. ii. iii.	i. ii.	
Pyramid	i. ii. iii.	i. ii.	

2. Sometimes drawings will get damaged or lost. Having now gained some knowledge of solid geometry, and using neat sketches, design an envelope to store the drawing sheets.

CONSTRUCTING OBJECTS IN ISOMETRIC AND OBLIQUE

Isometric Drawings

These drawings are ways to show three-dimensional objects on a flat surface. They clearly represent the shape and dimensions of an object without distorting its proportions.

Principles of Isometric Drawing

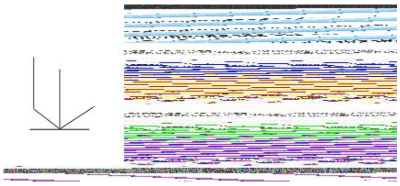


Fig 2.2.5: An isometric axis.

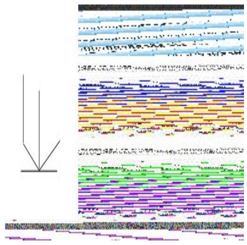


Fig 2.2.6: An isometric prism

1. **Vertical lines remain vertical:** the edges of objects such as a box, poles, or any structure, remain in straight lines that go up and down, parallel to the sides of the drawing paper or screen.
2. **Horizontal lines are inclined at 30 degrees:** objects are horizontally inclined at a 30-degree angle and applied to both the left and right horizontal lines, which represent the width and depth of the object.

Activity 2.2.2

Follow the guidelines below to draw the object shown in Fig 2.2.7, in isometric drawing on your sketch pad.

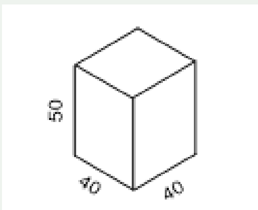


Fig 2.2.7: A square prism

Guidelines (Steps for Isometric Drawing)

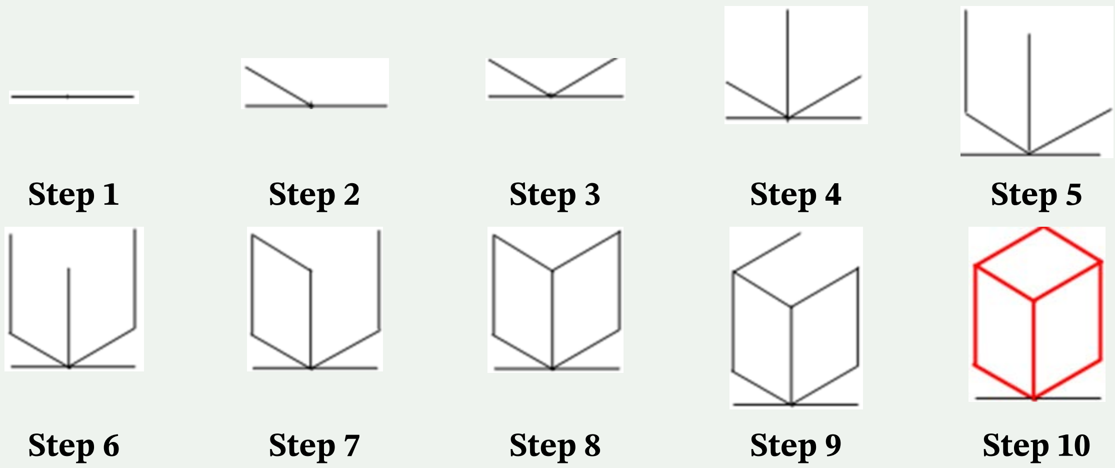
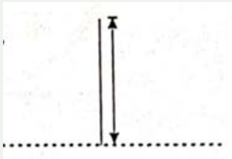
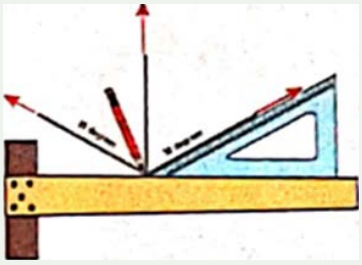
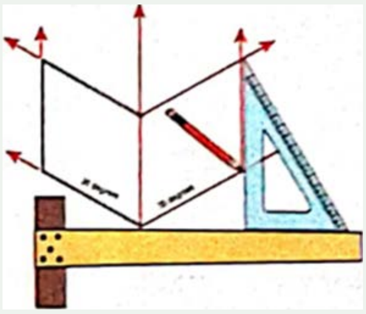
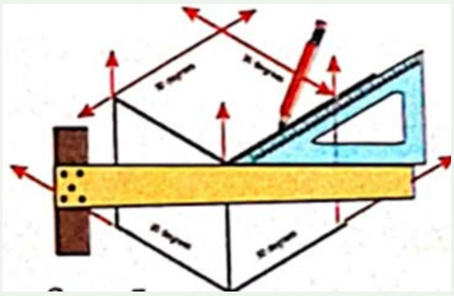
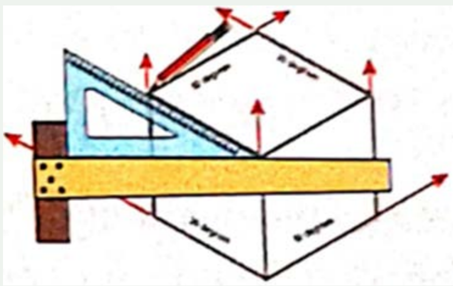


Fig 2.2.8: Steps for drawing isometric views.

Activity 2.2.3

Drawing a box in Isometric using instruments.

Steps	Procedure	Object
1	Draw a straight horizontal line, using a set square, close to the bottom edge of the drawing surface.	
2	Draw the front vertical edge of the cube (the first of the three axes). Use a set square and a T-square.	
3	Construct the two inclined axes, one to the left and one to the right. They should be at 30^0 to the horizontal line.	
4	Draw in the left and right sides of the cube. The vertical line should be parallel to the vertical axis, and the horizontal lines parallel to the horizontal axis. Use a set square to ensure the lines are parallel.	
5	Draw the top of the cube with all drawn 30^0 to the horizontal. Use the set square to draw one edge of the top and then flip it over to draw the other edge.	

Activity 2.2.4

1. Use the isometric drawing principles acquired to draw the uncompleted urinal shown in Fig 2.2.9 in your sketch pad.

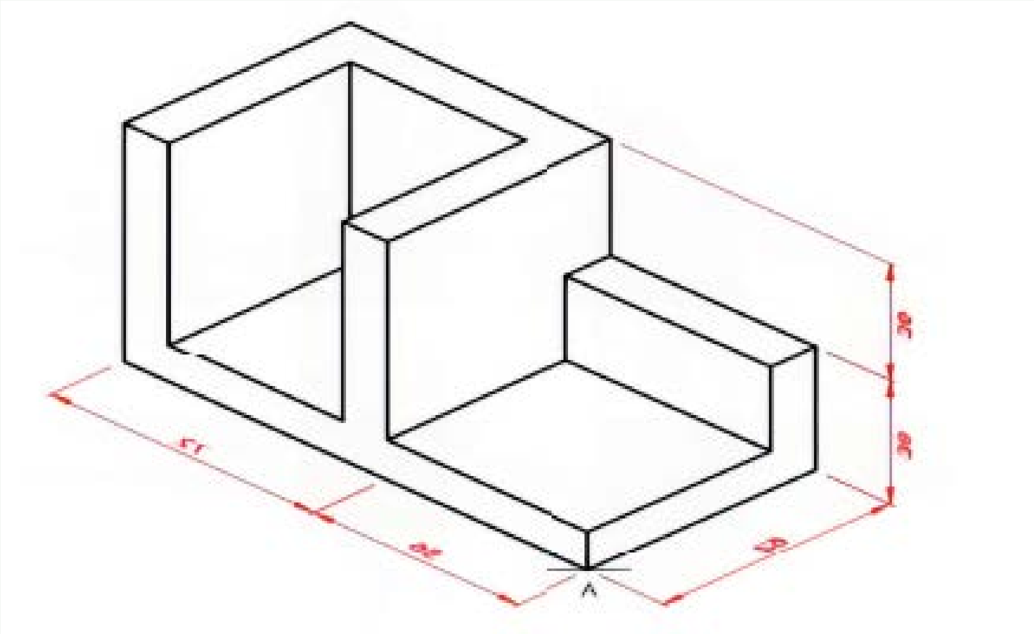


Fig 2.2.9: An uncompleted urinal

Oblique Drawings

Oblique drawing is a way of showing three-dimensional objects on a flat surface, like isometric drawings, but with a key difference in how the depth of the object is represented. In oblique vertical lines remain vertical, one horizontal line remains horizontal and 90° to the vertical. The other horizontal line should be inclined at an angle less than 90°

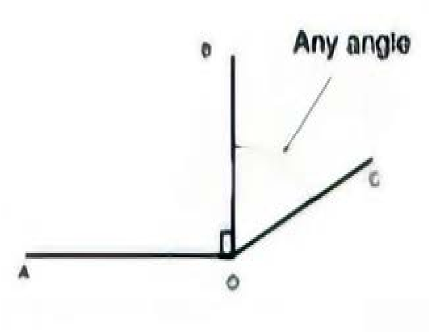


Fig 2.2.10: Oblique Axes

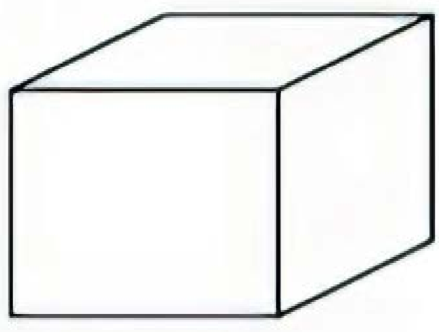
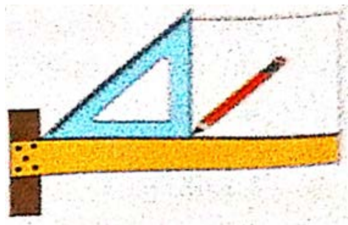
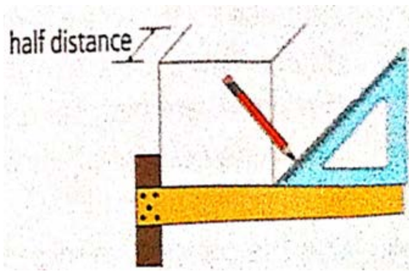
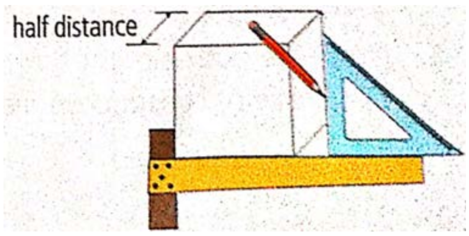


Figure 2.2.11: A box in oblique view

How to Draw a Cube Accurately in Oblique View, Using an Instrument.

Table 2.2.2: Activity table on Oblique drawing.

Step	Procedure	Object
1	Draw the front view. Use a T-square and 45^0 set square.	
2	Draw a line at an angle of 45° from each corner of the square. The distance of any line drawn back at 45° should be halved. For example, a cube may have sides of 100 mm in length. This will be drawn at 50mm to make the cube look more realistic and proportional.	
3	Draw the last vertical line and the last horizontal line to complete the cube.	

Activity 2.2.5

1. Follow the steps in Table 2.2.2 above, Construct the object in **Fig 2.2.12** below in an Oblique in your drawing sketch pad.

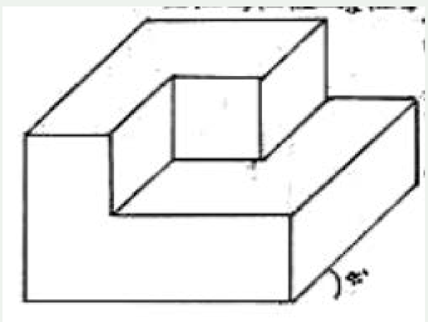
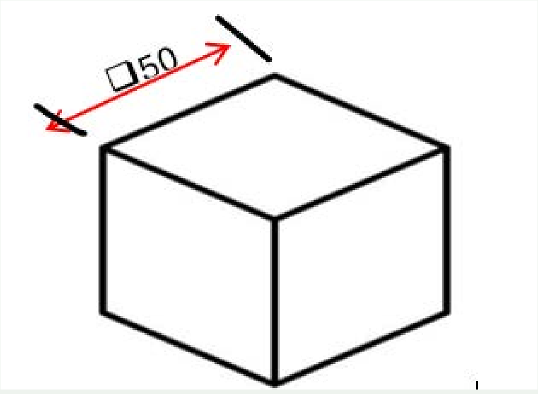


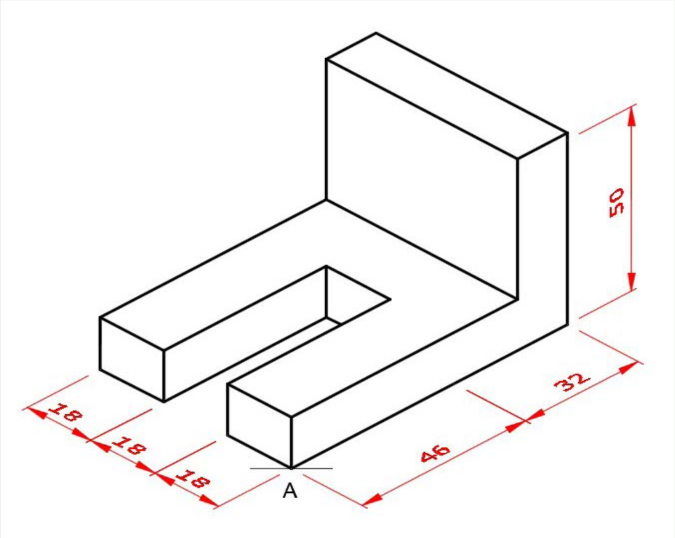
Fig. 2.2.12: A step in a swimming pool

2. Construct the blocks below in Oblique in your drawing sketch pad.

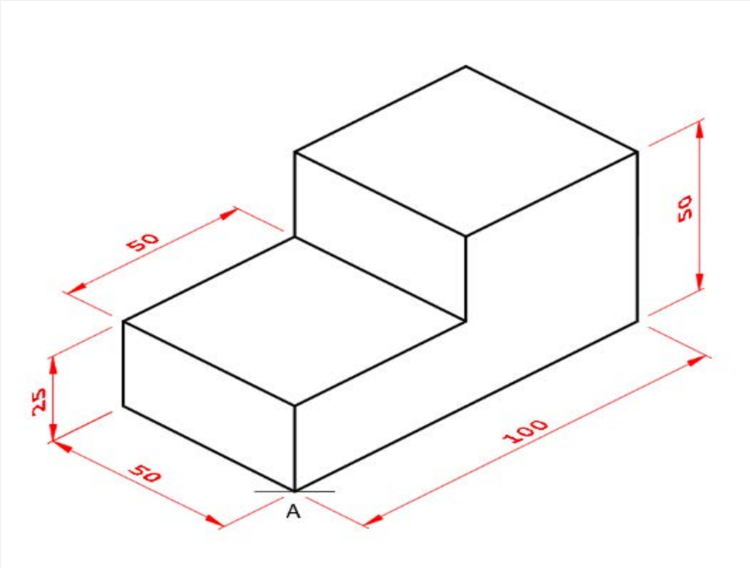
a.



b.



c.



Extended Reading

- Aidoo, F. F. & Co. (2022). Essential Career Technology, Cambridge University Press. United Kingdom. (Pages 151-153).
- Morling, K. (2021), Geometric and Engineering Drawing (4TH Edition), Edward Arnold, London. (Pages 121 – 133)
- Tufnell, R. (1987), Introducing Design and Communication, Scotprint Ltd, Great Britain. Page 46.

PERSPECTIVE DRAWING

Perspective drawing helps to understand how to interpret 3D images that appear on a flat surface picture. As you have learnt from the previous section, many drawings for designing and making products have been discovered because of technological development. This sub-strand will cover the concept of how to construct solid geometrical figures in different perspectives. There are several perspective styles to consider and two of them will be covered in this section, namely: One-point perspective and Two-point perspective

You will cover the perspective principles step by step and acquire the skills to present different figures in perspective.

One-Point Perspective

One-point perspective is a technique to create **solid geometrical drawings** on a flat surface using a **single vanishing point** on the horizon line. **A vanishing point (VP)**, or point of convergence, in a linear perspective drawing, is the spot on the horizon line to which the receding parallel lines diminish or disappear. This can be seen in the figures below.

Fig 2.2.13: A train and railway line.

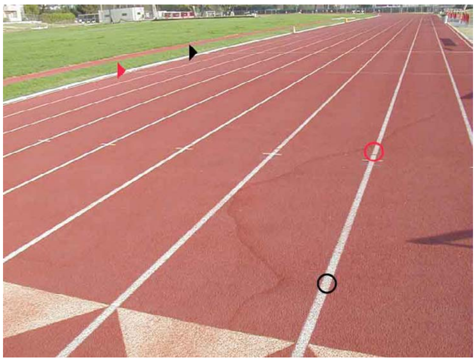


Fig 2.2.14: A 100-meter track

Fig 2.2.15: A seedling nursery

Two-point Perspective

Two-point perspective is a technique to create **solid geometrical drawings** on a flat surface using **two vanishing points** on the horizon line. This can be seen in the figures below.

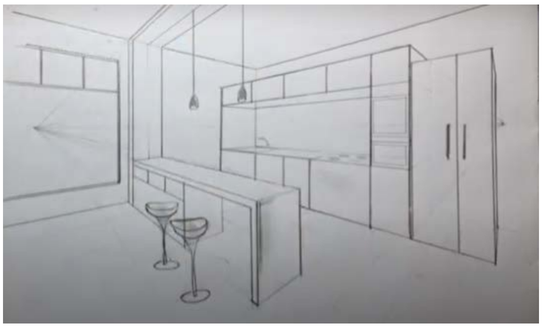


Fig 2.2.16: A modern kitchen design

Fig 2.2.17: A city street

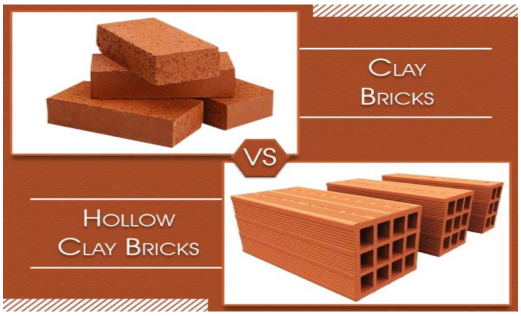


Fig 2.2.18: Burnt hollow bricks

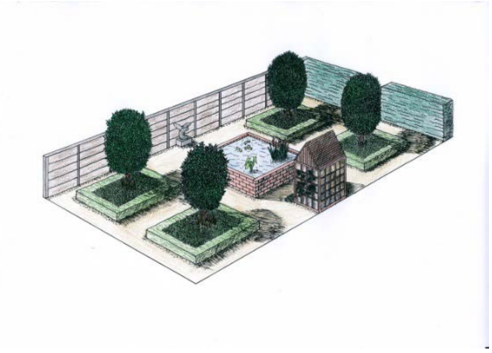


Fig 2.2.19: A garden design

Terminologies Used in Perspective Drawing

Eye levels/ horizon

1. **Human eye level (at eye level)**

Example:

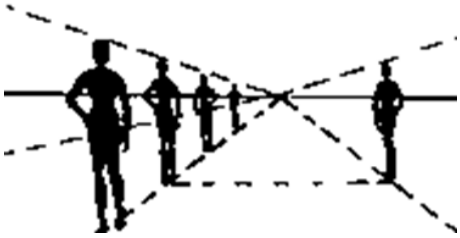


Fig 2.2.20: A queue of people

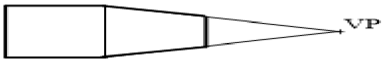


Fig 2.2.21: Human eye level in one-point perspective

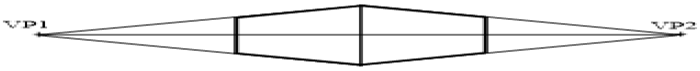


Fig 2.2.22: Human eye level in two-point perspective

2. **Worm's eye level (above eye level)**
   Examples:

Fig 2.2.23: A human queue.

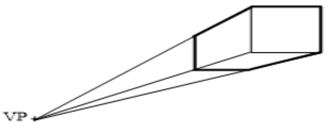


Fig 2.2.24: Worm's eye level in one-point perspective.

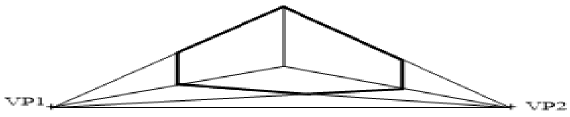


Fig 2.2.25: Worm's eye level in 2-point perspective

3. Bird's eye level (below eye level)

Examples:

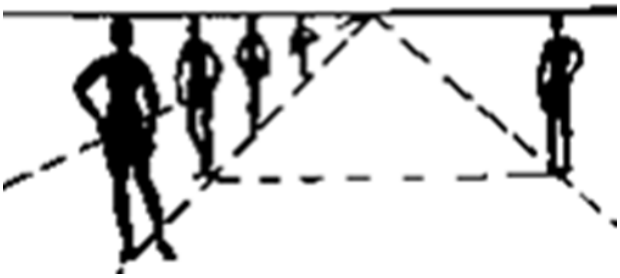


Fig 2.2.26: A human queue.

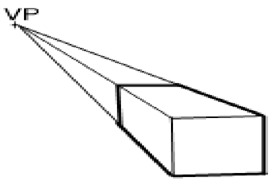


Fig 2.2.27: Bird's eye level in one-point Perspective.

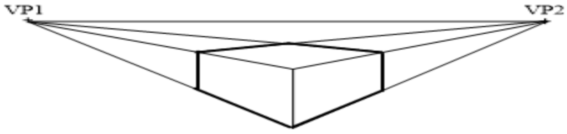
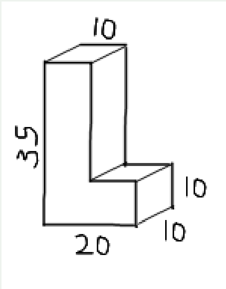


Fig 2.2.28: Bird's eye level in two-point perspective.

Activity 2.2.6

Follow the following perspective drawing principles and produce other objects individually. Use freehand sketches.



1. Step for drawing One-point Perspective:

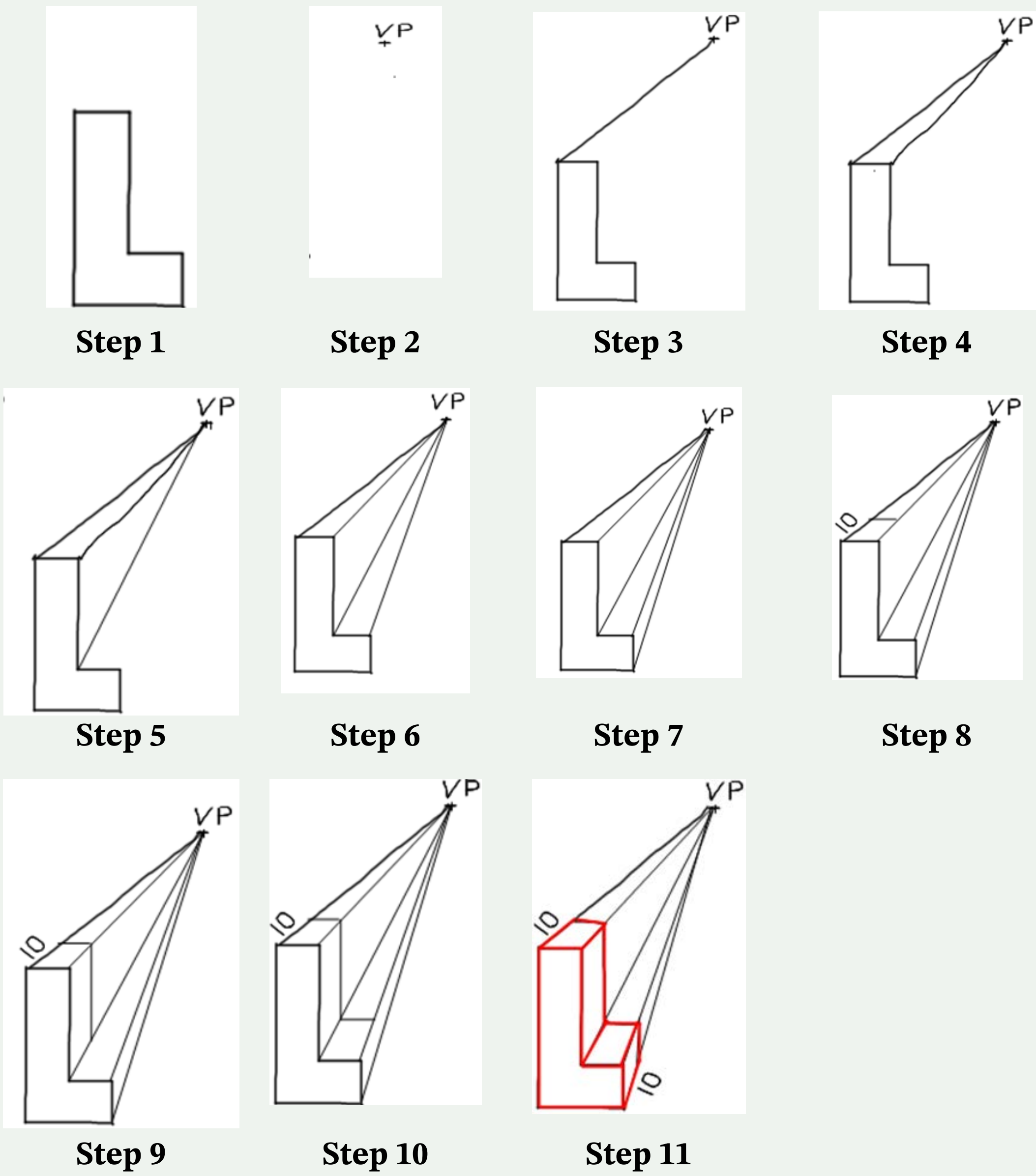
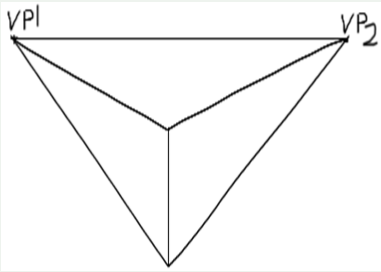


Fig 2.2.29: Step for drawing One-point perspective

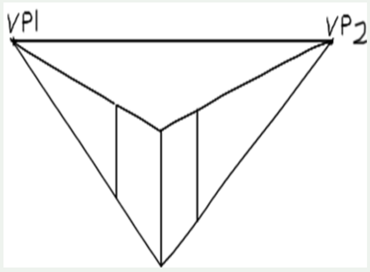
2. Steps for drawing a two-point Perspective:

Step 1. Position horizon and VP1, VP2.

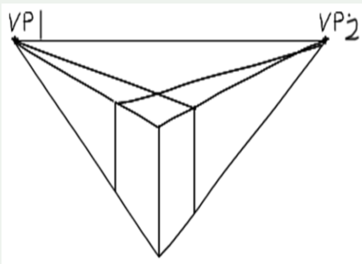
Step 2. Draw the nearest vertical corner and the full height of the object.



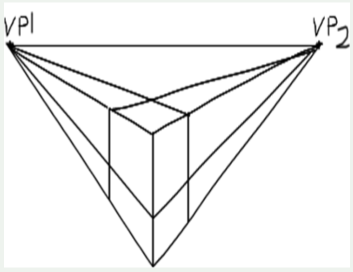
Step 3. Join end points to VP1 and VP2



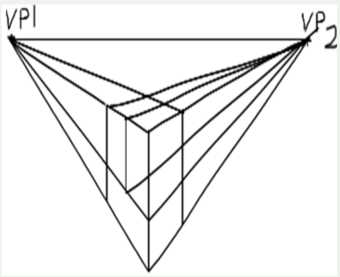
Step 4. Drop the verticals.



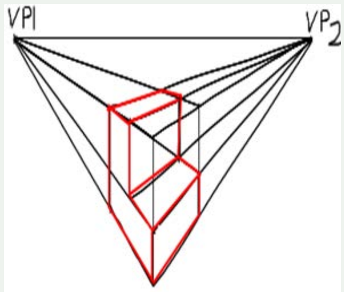
Step 5. Join vertical ends to VP1 and VP2.



Step 6. Mark out base thickness and join to VP1 and VP2.



Step 7. Mark out top thickness and join to VP1, VP2



Step 8. Join base breadth to VP 1

Fig 2.2.30: Steps for drawing a Two-point perspective

Extended Reading

- Tufnell, R. (1987), Introducing Design and Communication, Scotprint Ltd, Great Britain. Page 44, 45.
- Morling, K. (2021), Geometric and Engineering Drawing (4TH Edition), Edward Arnold, London. (Pages 118 – 120)

SURFACE DEVELOPMENT OF PRISMS

The surface development of an object means the **unfolding of all surfaces that form the object on a plane**. To explain further, we can identify the development of the solid cube, or box (see fig 2.2.31) when it is spread out flat various square shapes are formed, this is a typical representation of surface development.

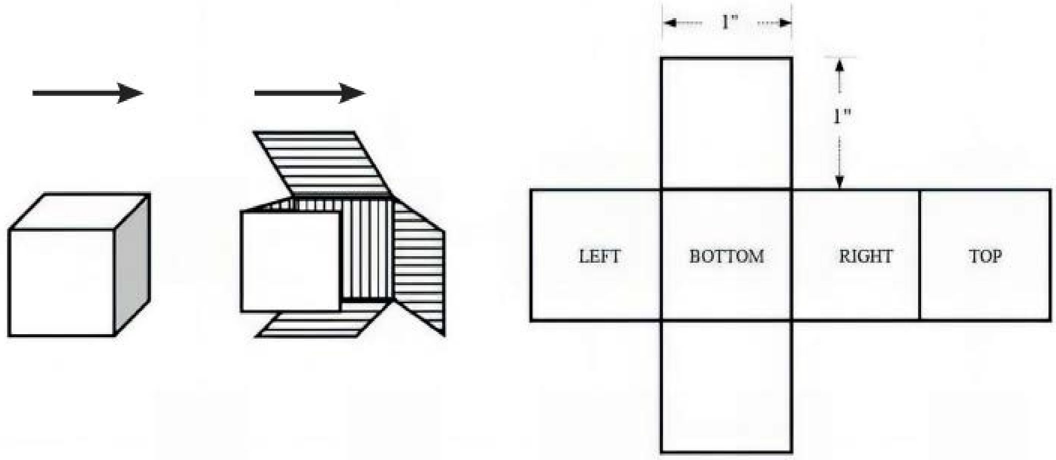


Fig 2.2.31: Surface development of a cube

Principles of Surface Development

There are three constructional procedures used in developing surfaces of prisms, two of these are:

1. Parallel lines method
2. Radial lines method

NB. The Parallel lines method is used to develop prisms and cylinders. The Radial lines method is used for the development of pyramids and cones.

Surface Development of a Hexagonal Prism

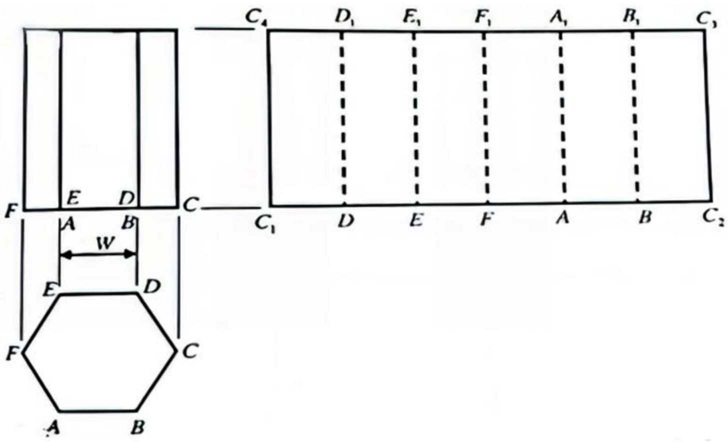


Fig 2.2.32: Hexagonal prism

Procedure:

1. Draw the plan of the hexagonal prism and letter the corners as A, B, C, D, E, F.
2. Project vertical lines to produce the front elevation and label as shown in Fig 2.2.32.
3. Draw horizontal lines from points C and C' to a convenient distance.
4. Draw vertical lines to intersect the horizontal lines at points C1 and C4.
5. With radius AB = [one side of the prism] step off six times on the horizontal line.
6. Join the last point C2 to C 3 and project broken lines through the points.
7. Firm in the rectangular shape as the development of the hexagon.

Activity 2.2.7

1. Construct the surface development of a rectangular prism of sides 30 by 50 and height 80 in Figure 2.2.33 in your sketch pad

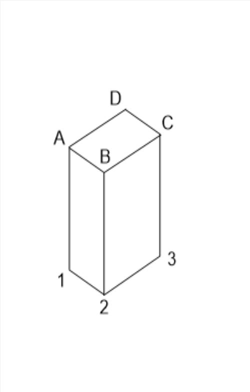


Fig 2.2.33: A rectangular prism

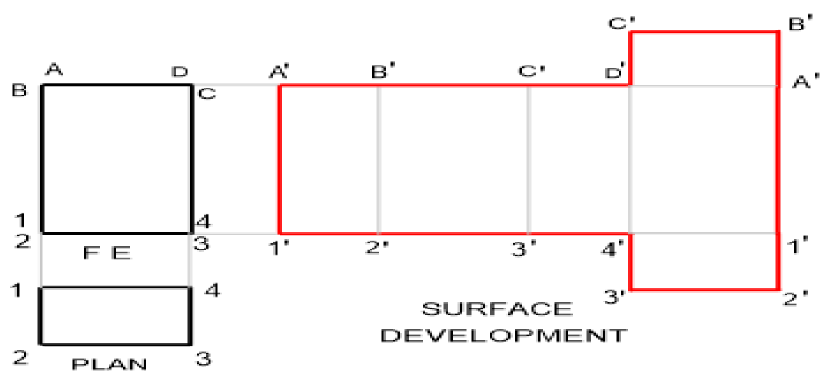


Fig 3.3.34: Development of a rectangular prism

Procedure:

1. Draw a rectangle 1234, to represent the top view, (PLAN).
2. Project all the points to obtain rectangle A, D, 3, 2 as the front view, (FE).
3. Stretch out line 1'–1' and A'– A' from the front view. Equal to the perimeter of the base (160mm).
4. Divide 1'-1' and A'-A' in four equal parts and name their intermediate points as 2', 3' 4' at the base and B', C', D' at the top respectively.
5. Join vertical edges 1'A', 2'B', 3'C', 4'D', and 1'A in the development.
6. Attach rectangle 1234 and ABCD to D'A' and 4'1' respectively as the base and top of the prism to obtain a complete development of the prism.

Extended Reading

- Agrawal, B. and Agrawal, C.M, (2015). Engineering Drawing, McGraw Hill Education (India) Private Limited, New Delhi. (Chapter 13.4, Page 13.2-13.3).
- Morling, K. (2021), Geometric and Engineering Drawing (4TH Edition), Edward Arnold, London. (Pages 216-225)

SURFACE DEVELOPMENT OF TRUNCATED PRISMS

This shows how the lateral faces of the truncated prism unfold when flattened out. It helps visualise the shape and layout of the lateral faces. Truncation occurs when a cut is made across the height of a prism at an angle to the base.

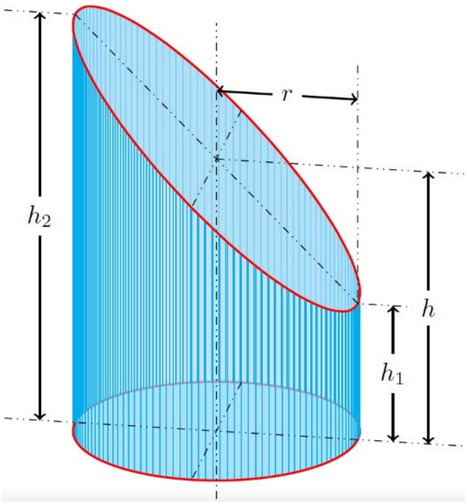


Fig 2.2.35 : Example of a solid truncated cylinder prism. Source: TikZ.net

The steps to construct the surface development of a truncated prism (as shown in Fig 2.2.36, below) are:

1. Draw the views:

a. Draw the front view of the truncated prism, showing both the original base and the truncated top.

b. Label the vertices and edges of the base and the truncated top.

2. Draw parallel lines:

a. Draw horizontal lines parallel to the front elevation from the plan of the prism. This line represents the **stretch-out line**.

b. The length of this line should be equal to the **perimeter** of the base of the prism.

3. Fold lines:

a. Along the stretch-out line, locate points corresponding to the lengths of the sides of the prism.

b. These points represent the **fold lines** where the lateral faces will be connected.

4. Surface development:

a. From each fold line, draw vertical lines perpendicular to the stretch-out line.

b. These vertical lines represent the lateral faces of the truncated prism.

c. Connect the corresponding points on adjacent vertical lines to complete the surface development.

Example

Development of a truncated cylinder.

Procedure

a. Draw the front and the plan views of the cylinder.

b. Divide the plan of the cylinder into several equal parts e.g. 12 divisions.

c. Project these lines to intersect line a1 and g1.

d. Extend the base line a1 and g1 of the elevation so a reasonable distance.

e. With a pair of compasses measures 1-12 on the plan.

f. With the same distance step off 12 divisions on the extended line.

g. Draw horizontal lines through the point on line a1 and g1 on the elevation to intersect the projected lines.

h. Draw a smooth line through the intersection to obtain the curve.

i. Firm in the outlines to obtain the development.

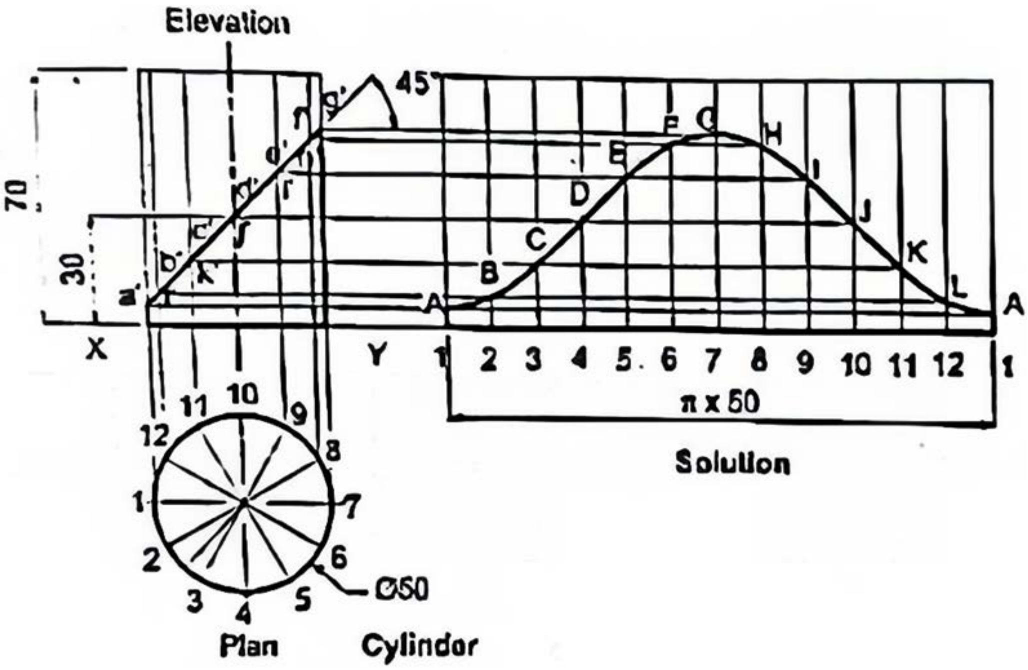


Fig 2.2.36: Development of a truncated cylinder

Activity 2.2.7

The figure below is a front view of a rectangular prism of height 60mm. Construct the surface development of the prism.

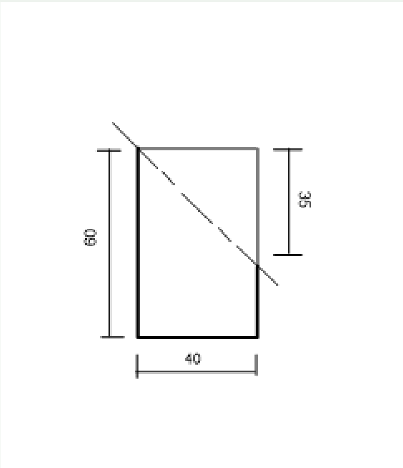


Fig 2.2.37: Truncated rectangular prism

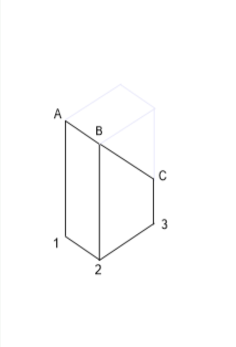


Fig 2.2.38: Truncated rectangular prism in isometric view

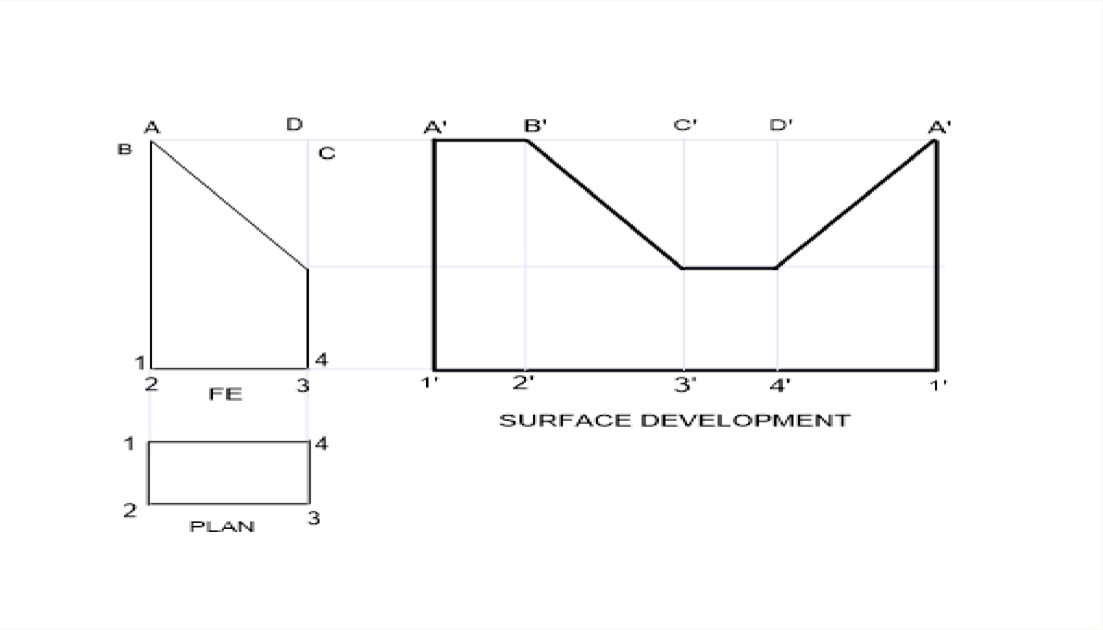


Fig 2.2.39: Surface Development of a Truncated Rectangular Prism

Procedure:

1. Draw the given rectangle A, 2, 3, D, (Front elevation).
2. Project all the points to obtain rectangle 1, 2, 3, 4. (Plan).
3. Extend out line 4, C and A to obtain 1'–1' and A'– A' area. (Development)
4. Extend another line from (FE) the cut point online C, 4 or B,3 to the last edge 1', A'.
5. Transfer sides 1234 from the Plan unto 1' – 1' and name their intermittent points as 2', 3' 4' at the base and B', C', D' at the top respectively.
6. Join vertical edges 1'A', 2'B', 3'C', 4'D', and 1'A in the development.
7. Join with straight lines A' to B', B' to middle of C', 3, continue to middle of D', 4', and continue to A'.
8. Firm the outline to obtain a complete development of a prism.

REVIEW QUESTIONS 2.2

SOLID GEOMETRICAL FIGURES

1.

a. List at least four differences between 2D figures and 3D figures.

b. Using some or the entire list in (a) above, explain what is meant by 'solid geometry' in your own words.

CONSTRUCTING OBJECTS IN ISOMETRIC AND OBLIQUE

1. You are working on a project in your technical drawing class where you need to design a toolbox. The toolbox has a simple rectangular shape with a handle on top. Design an isometric drawing of the toolbox, showing all three dimensions length (45cm), width (30cm), and height (20cm) clearly.

2. You have been tasked with designing a basic dustbin suitable for your classroom. The dustbin will feature a body with a flat, rectangular lid. Your design is to be in an oblique drawing to depict the dustbin's shape and structure.

PERSPECTIVE

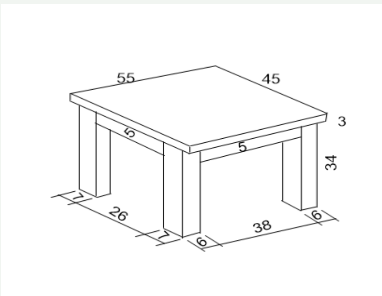
Group Project Work: In your groups, select two of the objects below. Using an A3 sheet, construct each object in:

a. One-point perspective at bird's-eye level

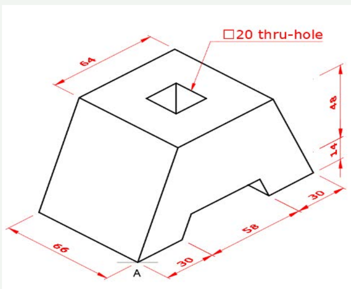
b. Two-point perspective at human eye level.

c. Cut out the drawings and paste them on cardboard.

1



2



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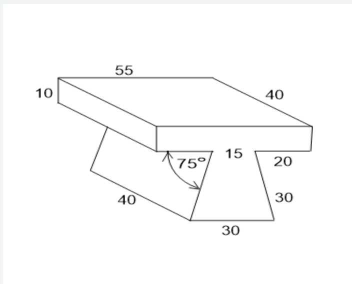

SURFACE DEVELOPMENT OF PRISMS

Individual exercise

1. You are working as a packaging designer for a renowned food company in Ghana. The company is launching a new snack product and needs an innovative packaging design that is both functional and attractive. The packaging should not only protect the product but also stand out on the shelves to attract customers.

Your task is to design and construct the surface development of this product package. The package should be a rectangular box with the following dimensions:

a. **Height**: 150 mm

b. **Width**: 50 mm

c. **Length**: 80 mm

SURFACE DEVELOPMENT OF TRUNCATED PRISM

Individual exercise

1. You are an apprentice in an engineering workshop, where your supervisor has tasked you with designing a component that will be fabricated using sheet metal. The component has a pentagonal cross-section and needs to be constructed as a prism. However, due to specific design requirements, the prism is truncated at a certain height and angle.

   Develop the surface layout of a pentagonal prism for fabrication. The prism has a side length of 30 mm and a height of 70 mm. The unique challenge is that the prism is truncated at a height of 35 mm at an angle of 45° to the base.

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1. Acquaye, E. A (2022) Technical Drawing, Yetoda Publishing.
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Graphic Communication

Fractal Geometry

INTRODUCTION

This section introduces you to the world of fractal geometry and its uses. You will learn about fractal patterns through practical demonstrations and hands-on projects. Fractal geometry appears in nature, such as in flowers, tree leaves, and the scales of some fish and snakes. It can also be created artificially using principles from solid and plane geometry.

Fractal geometry expresses complex, self-repeating patterns, found in a unique and compelling application in design. By incorporating the principles of fractal geometry, designers can create both visually stunning and intricately detailed patterns known as fractal designs. These patterns, which repeat at various scales, add depth and sophistication to a wide range of design projects. Whether in architecture, art, or digital media, the use of fractal geometry enhances creativity and innovation, transforming simple elements into mesmerising and harmonious designs.

At the end of this section, you will be able to:

- Explain the basic concept of fractal geometry
- Use the concept of fractal geometry in fractal designs

Key Ideas

- Fractal is a complex pattern that repeats itself at different sizes, creating intricate and detailed designs.
- Pattern means a design element that repeats to create visual interest and texture, like the designs on wallpaper or fabric.
- Creating refers to the process of coming up with new design ideas and making them real, like sketching a new product or making a prototype.
- Fabric is a material used in design projects, especially textiles for clothing, upholstery, and other soft goods.
- Design involves the process of planning and creating something that is both functional and visually appealing, like designing a chair or a website.

INTRODUCTION TO FRACTAL GEOMETRY

Explanation of Fractal Geometry

Fractal geometry involves creating intricate designs by continuously repeating a geometric pattern. Fractals are complex shapes that display self-similarity at different scales. This means that when you zoom in or out on a fractal, you will see smaller versions of the overall structure repeating themselves.

These patterns can be found both in nature and in artificial designs.

Examples of Artificial Pattern Design

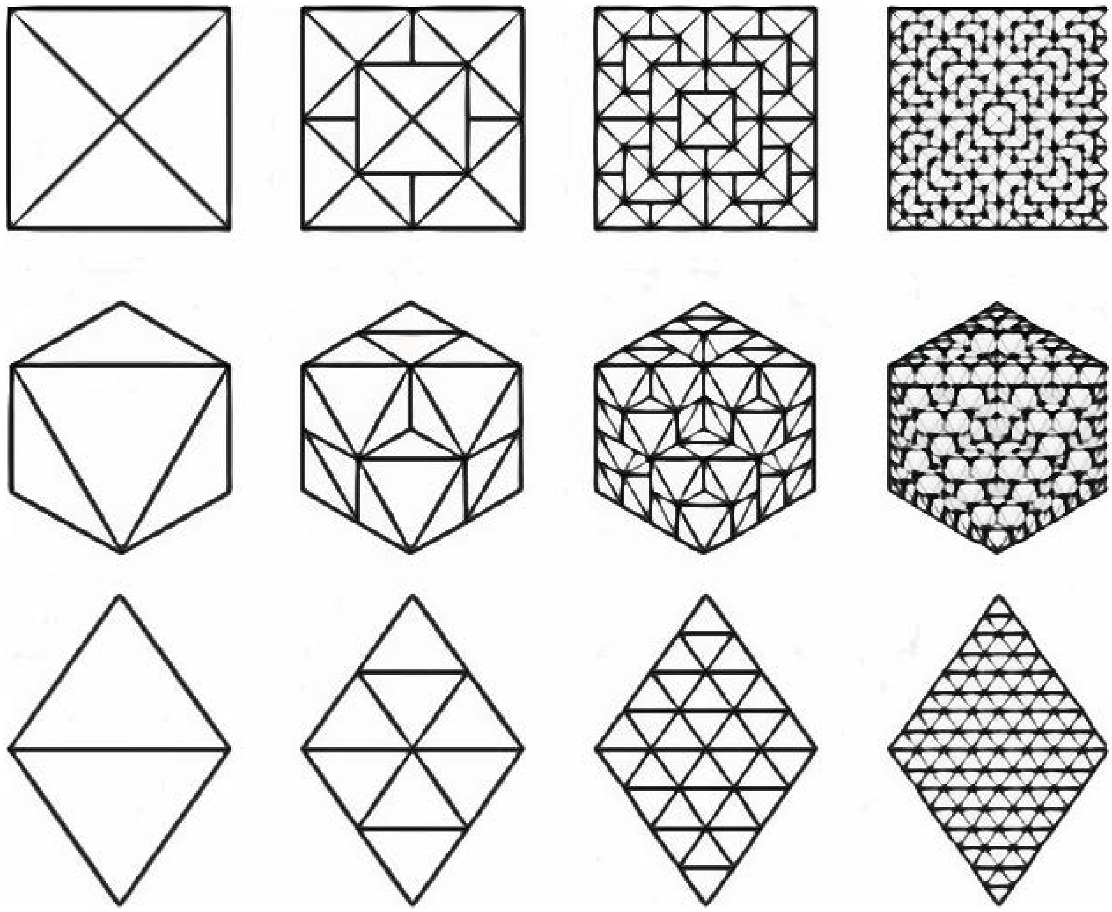


Fig 2.3.1: Kepler's fractals

Types of Fractal Geometry

Common types of fractals include:

1. **Iterated Function Systems (IFS):** These fractals are generated by repeatedly applying a set of similar shapes to an initial geometric shape. Examples include the Barnsley fern and the Sierpinski triangle.

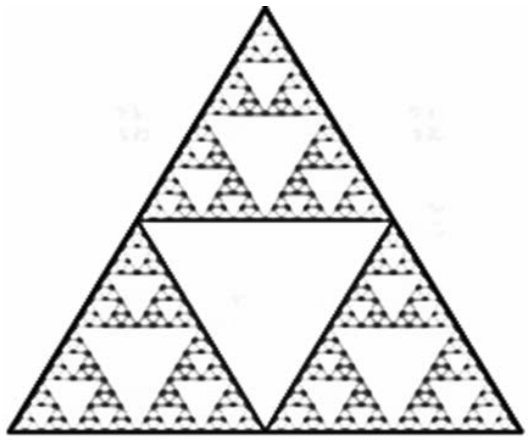


Fig 2.3.2: Sierpinski triangle

Fig 2.3.3: Barnsley fern

2. **Lindenmayer systems (L-systems):** are a type of formal grammar used to model the growth of biological structures such as plants. They generate fractal-like patterns through the iterative application of production rules.

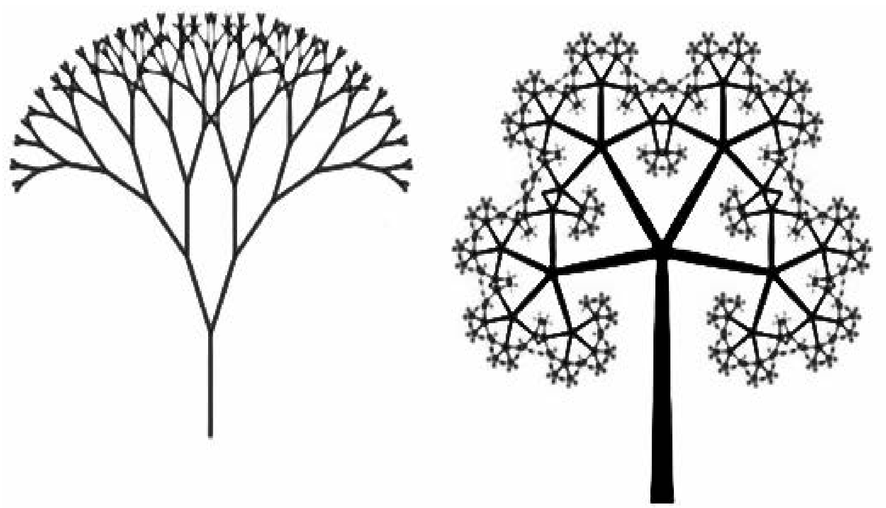


Fig 2.3.4: Fractals in nature (fractal trees)

3. **Cantor Set:** The Cantor set is an example of a self-similar fractal constructed by iteratively removing middle thirds from a line segment. It has a fractal dimension between 0 and 1, indicating its fractional nature.

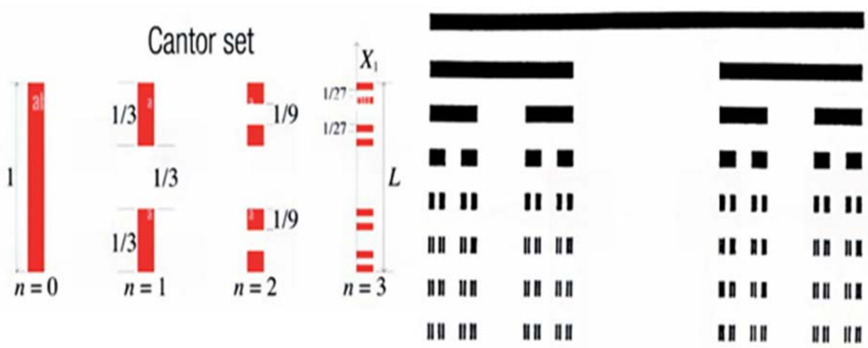


Fig 2.3.5: Cantor set

More Examples of Fractal in Nature.

a. b. c.

Fig 2.3.6: Flower in fractals

Fig 2.3.7: Tree in fractals

Fig 2.3.8: Water in fractals

Characteristics of Fractal Geometry

a. **Self-similarity**: Parts of the fractal look like the whole structure, no matter how much you zoom in or out.

b. **Infinite complexity**: Fractals have intricate patterns that remain detailed at every scale.

c. **Fractal dimensions**: Unlike traditional geometric shapes like lines, squares, or cubes, fractals can have dimensions that are not whole numbers, often called fractal dimensions.

d. **Iteration:** Fractals are often formed through iterative processes, where a basic geometric pattern is repeated and altered with each iteration.

Application of Fractal Geometry

1. **Creation of visual art**: **Visual Art Creation:** Artists use fractal geometry to craft complex and visually captivating artworks. Using mathematical algorithms and software such as Mandelbrot Set or Julia Set, artists explore a vast array of unique shapes and patterns.

2. **Digital art and graphics:** Fractal geometry has transformed digital art and graphics by facilitating the development of intricate and sophisticated images. Software based on fractals enables artists to produce impressive visual effects, textures, and patterns that replicate natural occurrences or delve into abstract ideas.
3. **Fractal music and sound**: Fractal geometry extends beyond visual art into the realm of music and soundscapes. Compositions inspired by fractal patterns feature self-similar structures and recursive motifs, offering listeners a dynamic and immersive auditory journey.
4. **Design and architecture**: Fractal geometry has impacted architectural design by introducing innovative approaches to creating structures with organic, self-repeating forms. Architects apply fractal principles to design buildings, facades, and urban landscapes that blend harmoniously with nature and showcase visually appealing patterns.
5. **Textile and fashion design**: Fractal geometry is utilised in textile and fashion design, where designers integrate fractal patterns into fabrics, clothing, and accessories. These designs enhance fashion collections by introducing depth, texture, and captivating visual elements, resulting in distinctive and memorable pieces
6. **Generative art and procedural generation:** Fractal geometry serves as a foundational tool in generative art, enabling artists to autonomously generate artworks using algorithms. By leveraging fractal patterns and procedural generation techniques, artists can produce limitless variations of images, animations, and interactive experiences.
7. **Science**: Fractal geometry has contributed to understanding:
    a. The folding of brains of mammals during growth.
    b. The fragmentation of landscapes during earthquakes.
    c. Model of human lungs, blood vessels, and neurological systems.
    d. Insights into bacterial growth patterns.

Activity 2.3.1

Explain the concept of fractal geometry.

Group work

Your group is to work on a special project to design a new children's park in your village. The planners want the park to be visually stunning and inspired by nature. They have heard about the beauty and efficiency of fractal patterns found in nature and want to include these designs in various parts of the park, such as pathways, flower arrangements, and sculptures.

Use the following steps to explain how you will accomplish the project

1. Discuss how fractals show self-similarity and complexity across different scales (nature, art, architecture, and science).
2. Identify patterns within the fractal shapes.
3. Draw a simple fractal pattern, such as a Sierpinski Triangle or a Koch Snowflake, on paper.
4. Start with a basic shape and repeat a transformation or pattern at each iteration, consider how this could inform the layout of the children's playground.
5. Are there items that you might expect to find in a children's park that could utilise fractal patterns?
6. Present your findings to the class for other groups to discuss.

Activity 2.3.2

Individual activity

Identify three types of fractal geometry in the environment and explain their real-life applications.

Copy and complete the table below.

NO	TYPE OF FRACTAL GEOMETRY	REAL-LIFE APPLICATION
1.		
2.		
3.		

Activity 2.3.3

A

B

C

Individual activity

Use the steps below to explain three applications of fractal geometry.

1. Research from reference books or the internet to help identify three types of fractal geometry.
2. Explain how each type can be applied in real-life situations.
3. Provide examples and visual aids to illustrate these applications.

Extended Reading

- https://www.pardesco.com *(Open with Hyperlink)*
- https://www.re-thinkingthefuture.com *(Open with Hyperlink)*
- Ball, P. (2001). The self-made tapestry: pattern formation in nature. Oxford, Oxford University Press. Page 5-9
- Pohl, G. and Nachtigall, W. (2015). Biomimetics for Architecture & Design Nature, Analogies Technology 1st Edition. (Pages 10-18)

Application of Fractal Geometry in Designing

Exploring Fractal Patterns

Fractal patterns exist in numerous forms which include geometrical patterns. For example, circle and polygon.

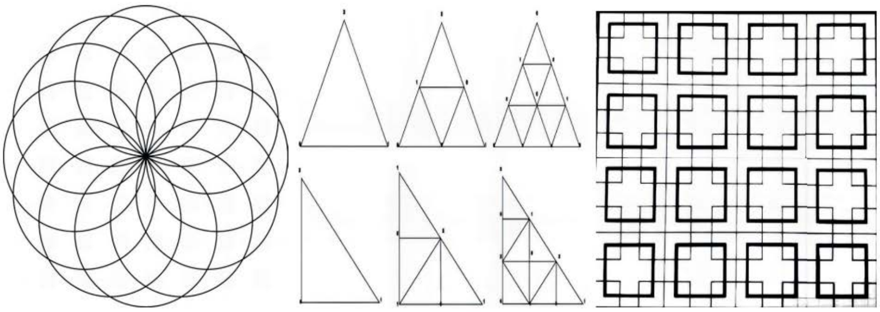
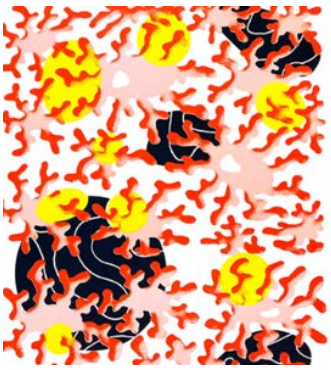
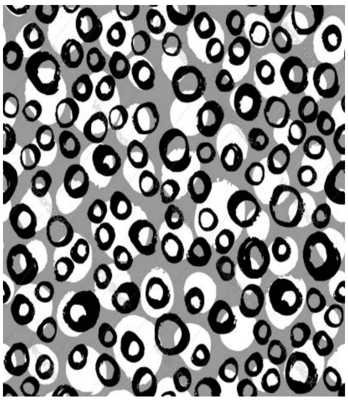


Fig 2.3.9: Geometrical fractals

Apophysis design.



Bryce design.



Chaotica design.

Fig 2.3.10: Computer fractals

Generating Fractal Patterns Geometrically

The beauty of fractals lies in their simplicity at each level, yet they are complex when viewed as a whole. Fractal patterns can be generated using various drawing instruments or freehand. Computer applications such as Apophysis, Bryce, Chaotica and Fractal now can also be used to generate fascinating fractal designs.

Creating a fractal pattern involves repeating a simple process repeatedly, at smaller scales. Fractals are infinitely complex shapes that exhibit self-similarity across different sizes.

Steps to Develop your Own Fractal

1. **Start with a simple shape:**

   Review **Fig 2.3.9**, above. Begin with a basic geometric shape or pattern of your choice. For instance, you can start with a line segment and then branch off smaller segments from one end.

2. **Iteration and repetition:**

   a. Divide the line segment into smaller segments.

   b. At each iteration, create new branches by repeating the same process.

   c. Adjust angles and lengths to achieve the desired fractal structure.

Examples of Fractals

1. **Sierpinski Triangle:**
   Start with an equilateral triangle and recursively remove smaller triangles from its centre.

2. **Mandelbrot Set:**
   Explore the fascinating world of complex numbers and their iterations to create intricate patterns.

3. **Space-Filling Curves**:
   Construct curves that fill space densely, such as the Hilbert Curve or Peano Curve.

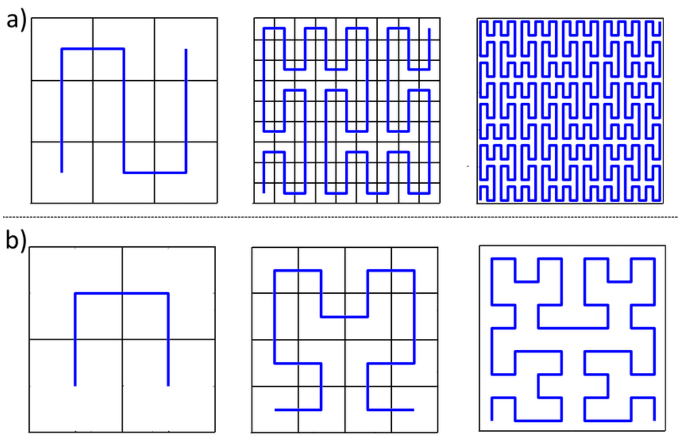


Fig 2.3.11: (a) Peano curve (b) Hilbert curve Source: researchgate.net

Application of Fractal Patterns in Design

1. Architecture:

Fractal patterns have been used in architecture to design buildings of self-similarity in the centuries and in recent urban development. Elements like staircases, railings, and decorative mouldings can incorporate fractal-inspired designs. These details create a cohesive and visually engaging environment, enhancing the overall aesthetics. Examples are found in **Fig 2.3.12** below.

Columns **Staircase** **Mouldings**

Figure 2.3.12: Architecture

2. Textile design:

Patterns utilised by textile design through printing or embroidery are very important in the art world due to the impact they can have on making beautiful designs, colours, and infinite details. Fractal patterns can be woven into textiles, creating visually appealing fabrics. These fabrics can be used for clothing, scarves or home decor.

Fig 2.3.13: Textile design

3. Interior designs:

Fractals play a fascinating role in interior design, infusing spaces with complexity, visual interest, and a sense of natural harmony. Let us explore how fractals enhance interior design

Fig 2.3.14: Fractal Interior Design

Activity 2.3.4

In your groups, study the animation and follow the outlined steps to design a fabric pattern for a clothing company in Ghana.

1. Identify and gather inspiration from the animation.
2. Sketch initial ideas and create mood boards to visualise the design.
3. Select a colour palette that resonates with Ghanaian culture and the company's image.

4. Use hand or design software to create hand or digital versions of the chosen patterns.
5. Test the pattern on fabric samples to see how it translates to the actual material.
6. Prepare the pattern for production, ensuring it meets the technical specifications.
7. Display your work and make a gallery walk to appreciate other groups' work.

Extended Reading

1. Shier, J. (2019) Fractalize That: A Visual Essay on Statistical Geometry (Vol. 3), U.S.A. World Scientific Publishing Co. Pte Ltd (Page 24).
2. https://www.freepik.com (Open with Hyperlink)

REVIEW QUESTIONS 2.3

Fractal geometry

1. Briefly explain how fractal geometry can boost creativity and innovation in designing different objects or structures.
2. Describe the following types of fractal geometry and give one (1) example each in a real-life application.
   a. Mandelbrot set,
   b. Sierpinski triangle

Application of Fractal Geometry in Designing

1. Design two fractal patterns using basic geometric shapes for home decor products, such as rugs and wallpapers that aim to blend contemporary style with elegance.
2. You have been assigned to a project where you aim to enhance the visual appeal of your classroom. Identify three fractal patterns, to be applied for ceiling, walls and floor.
3. A clothing company in Ghana seeks to incorporate traditional Ghanaian motifs and modern fashion design trends into their new fabric designs. Design a fractal pattern in cooperating with Adinkra symbols design for the company.

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Acknowledgements

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