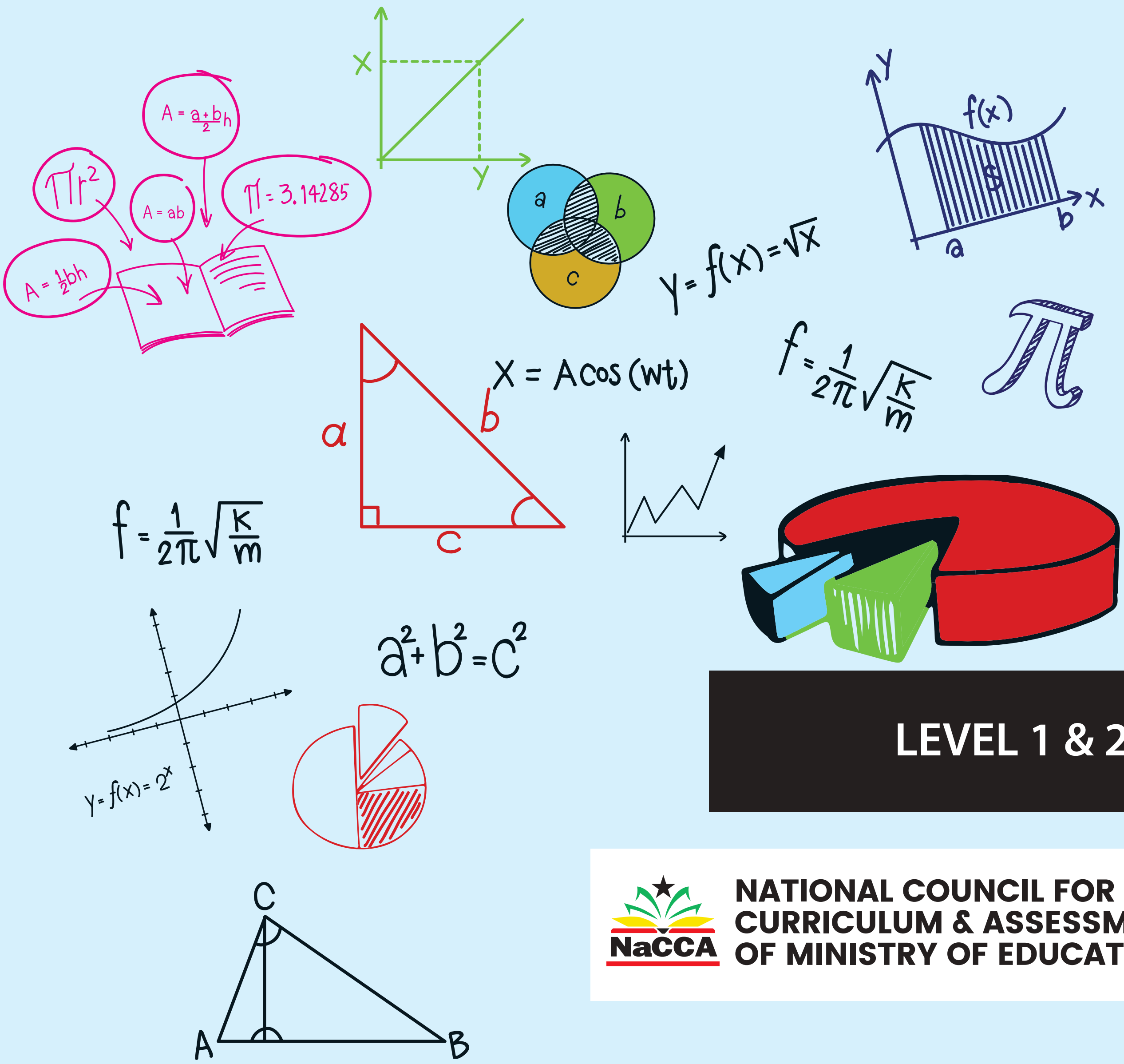
MINISTRY OF EDUCATION

Intervention Mathematics

For Senior High Schools

TEACHER MANUAL



LEVEL 1 & 2

NATIONAL COUNCIL FOR CURRICULUM & ASSESSMENT OF MINISTRY OF EDUCATION

MINISTRY OF EDUCATION

REPUBLIC OF GHANA

Intervention Mathematics

For Senior High Schools

Teacher Manual

Level One & Two

INTERVENTION MATHEMATICS TEACHER MANUAL

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CONTENTS

MODULE 1

SECTION 1: MAKING SENSE WITH NUMBERS

Strand: **Numbers for Everyday Life**

Sub-Strand: Real Number and Numeration System

Content Standards

1. Demonstrate an understanding of quantities and place value for multi-digit numerals up to 1 000 000.
2. Demonstrate an understanding of factors, multiples of numbers including composite, even, odd and prime numbers from 1 to 100.

INTRODUCTION AND SECTION SUMMARY

The concepts of reading, writing, and comparing number quantities up to 1 000 000, rounding whole numbers, identifying even and odd numbers, and understanding factors and multiples are foundational in developing mathematical literacy. Using graph sheets and multi-base blocks, students can visualise and manipulate large numbers, enhancing their comprehension. Rounding numbers to the nearest tens, hundreds, thousands, and tens of thousands helps students simplify and estimate values in real-world contexts. Identifying even and odd numbers by arranging them in arrays provides a concrete understanding of numerical properties. Recognising factors and multiples and applying this knowledge to problem-solving enables students to tackle various mathematical challenges with confidence.

The section will cover the following focal areas:

1. *Reading, writing and comparing modelled number quantities up to 1 000 000 using graph sheets and multi-base block.*
2. *Rounding (off, up, down) whole numbers up to 100 000 to the nearest tens, hundreds, thousands and tens of thousands.*
3. *Identifying even and odd numbers between 1 and 100 as numbers that can be arrayed in twos array and those which cannot.*
4. *Identifying factors and multiples of numbers and use the knowledge to solve problems.*

SUMMARY OF PEDAGOGICAL EXEMPLARS

Learners will benefit from a variety of teaching strategies designed to solidify their understanding of these concepts.

1. **Visualisation and Manipulatives:** Use graph sheets and multi-base blocks to help students read, write, and compare large numbers up to 1 000 000. This hands-on approach allows students to see and interact with numbers, fostering deeper understanding.
2. **Rounding Activities:** Engage students in exercises that involve rounding numbers up to *100000*. Use number lines and practical examples, such as estimating costs or distances, to make rounding relevant and meaningful.
3. **Array Models:** Teach even and odd numbers by having students arrange numbers in twos arrays. This visual and tactile method helps them grasp the concept of numerical parity and its practical implications.

4. **Factors and Multiples:** Use interactive activities to help students identify factors and multiples. Apply this knowledge to solve problems, such as finding common factors or multiples in real-world scenarios, to enhance their problem-solving skills.
5. **Collaborative Learning:** Encourage group activities where students work together to compare large numbers, round values, and solve problems involving factors and multiples. This fosters teamwork and critical thinking.

ASSESSMENT SUMMARY

Assessments for these concepts should be diverse and address different levels of cognitive demand.

1. **Class Exercises and Tests:** Assess students' ability to read, write, and compare large numbers up to 1 000 000 using graph sheets and multi-base blocks. Include exercises on rounding numbers to the nearest tens, hundreds, thousands, and tens of thousands.
2. **Practical Applications:** Present real-life problems that require rounding numbers, such as estimating costs or distances, to test students' understanding and application of rounding concepts.
3. **Array Models:** Use exercises where students identify and categorise even and odd numbers between 1 and 100 by arranging them in arrays. Assess their ability to explain the reasoning behind their categorisation.
4. **Factors and Multiples Problems:** Provide problems that require identifying factors and multiples of numbers. Include tasks that involve finding common factors or multiples, solving word problems, and applying these concepts in practical scenarios.
5. **Group Activities:** Engage students in collaborative tasks where they compare large numbers, round values, and solve problems involving factors and multiples. Assess their teamwork, communication, and problem-solving skills.
6. **Presentations:** Have students present their solutions to problems involving factors and multiples, explaining their thought process and the strategies they used. This assesses their understanding and ability to communicate mathematical concepts effectively.

Week 1: Modelling and Comparing Number Quantities

Learning Indicator: *Read, write and compare modelled number quantities up to 1000 000 using graph sheets and multi-base block.*

Focal Area: Modelling Number Quantities

Representing numbers using graph sheets and base 10 blocks (up to 6 digits)

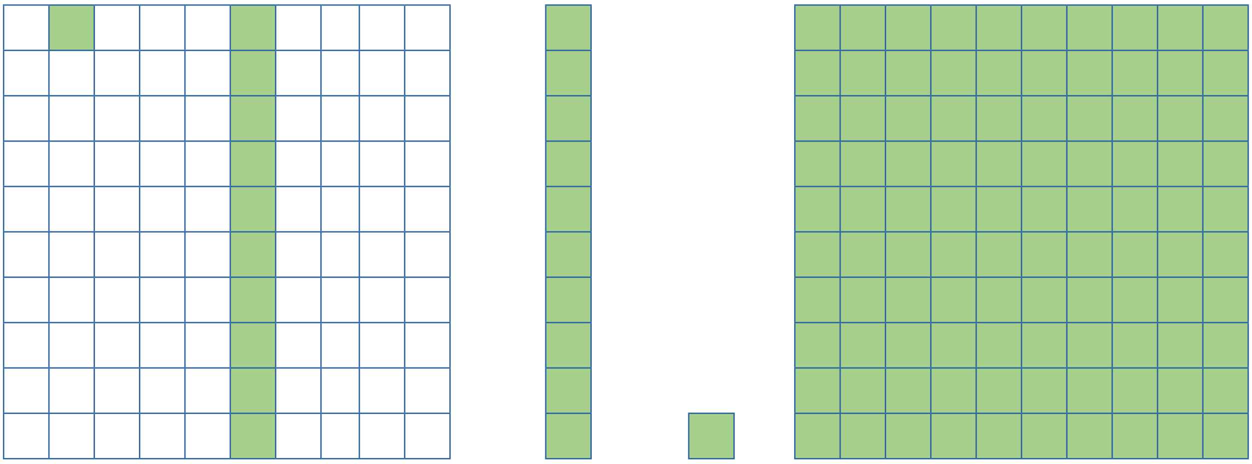
Graph sheets

Graph sheets (or grid paper) help visualise numbers in a structured format, making it easier to understand place value and compare numbers.

- **Plotting Numbers**: Writing numbers on a graph sheet to visualise their magnitude.
- **Place Value Columns**: Using columns on graph paper to separate digits according to their place value (ones, tens, hundreds, etc.).

Example:

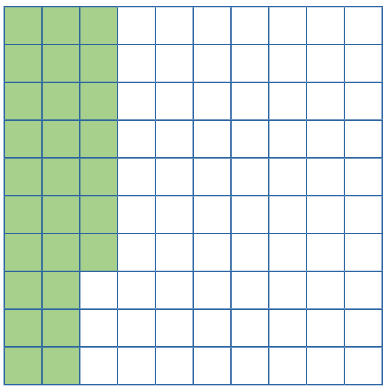
Let a small square represent one unit of area. This means that a shaded column or row represents a ten as it consists of ten squares and the whole grid shaded represents one hundred as it consists of one hundred squares.

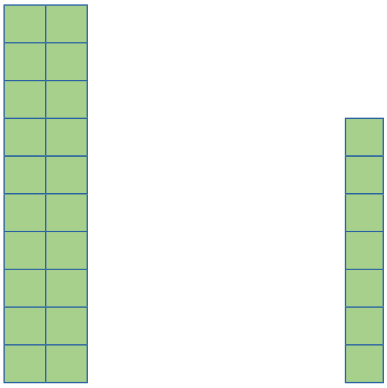


Ten One Hundred

Example:

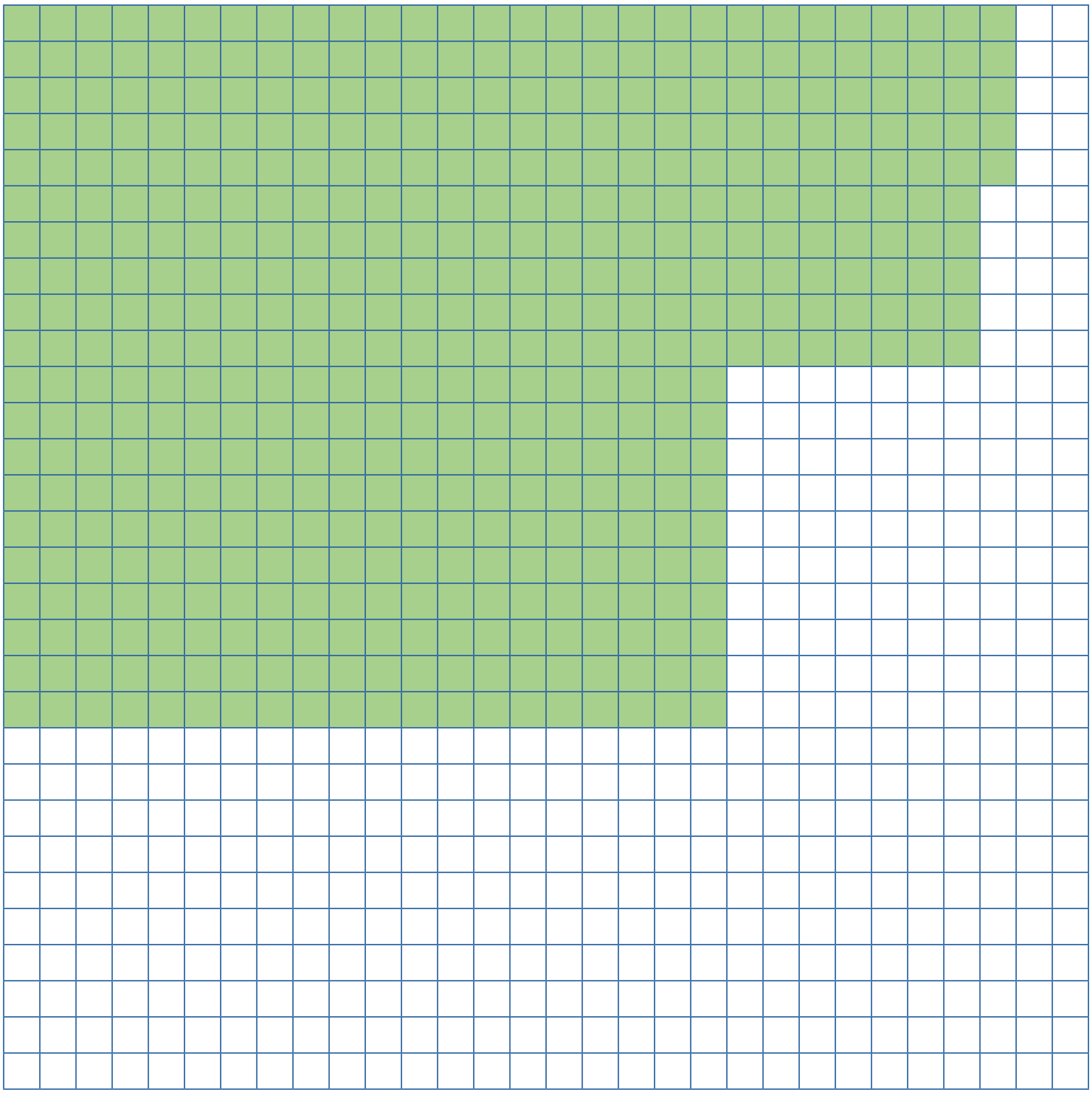
1. 27 = 2 Tens and 7 ones

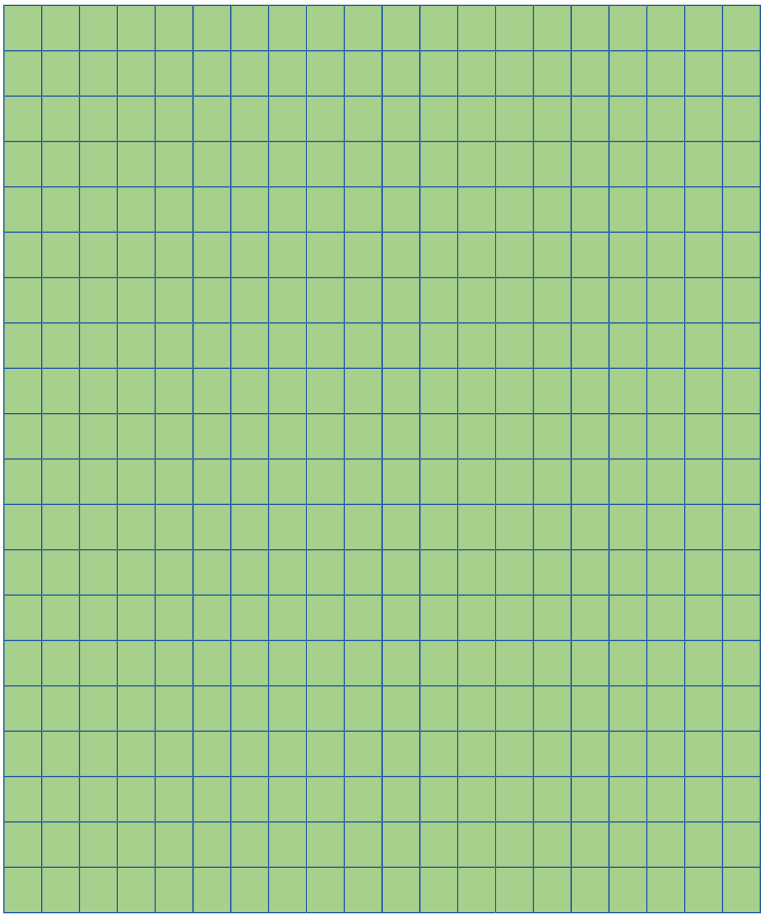




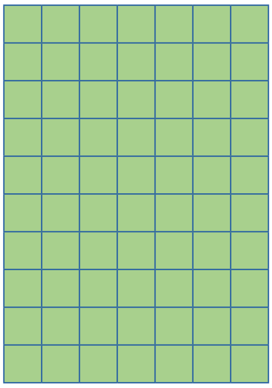
2 Tens and 7 ones

2. 475





4 Hundreds = 4 by Hundred = 400



7 tens = 7 by Ten = 70

5 ones = 5 by 1 = 5

This implies that 4 Hundreds and 7 Tens and 5 ones will be written in expanded form as;

Four Hundred and Seventy-Five.

400 + 70 + 5 = 475

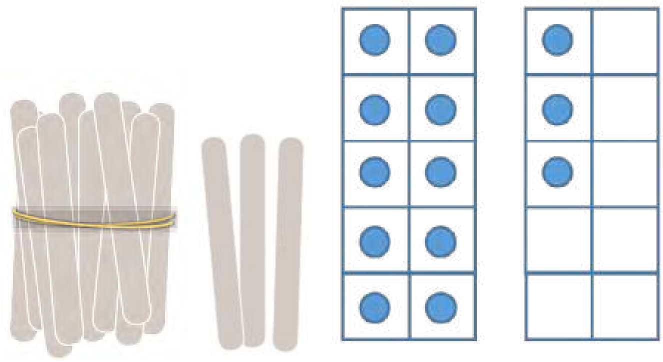
Concept of Place Value

Place value is a fundamental concept in mathematics that refers to the value of a digit in a number based on its position or place. It is the idea that each digit in a number has a specific value depending on its location, starting from the right: The value of each place increases by a factor of 10 as you move left.

The order of place value of digits of a number of right to left is units (ones), tens, hundreds, thousands, ten thousands, hundred thousands, and so on.

Understanding place value is crucial for performing arithmetic operations, such as addition, subtraction, multiplication, and division, as well as for rounding numbers, estimating quantities, and solving problems involving decimals and fractions. Place value is a building block of mathematics and is essential for developing a strong foundation in numeracy.

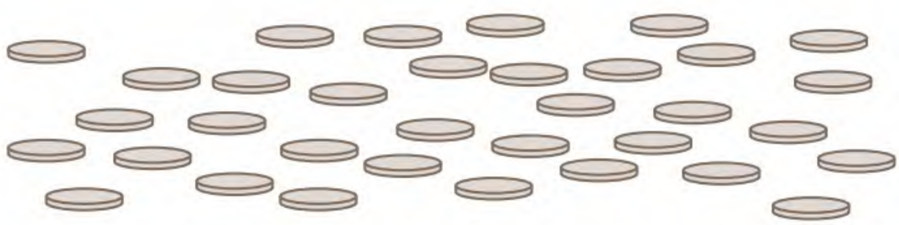
To ensure that students develop the concepts related to 'ten more than' learners are encouraged to practice modelling numbers using materials such as sticks and counters.



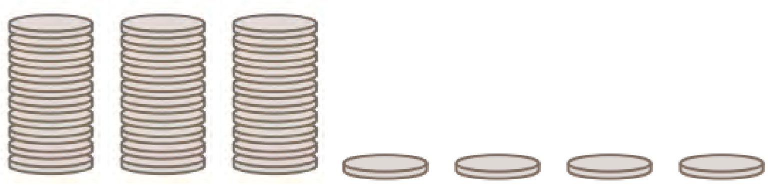
Two Tens and Beyond

Students should experience counting using manipulatives such as icypole sticks and counters with numbers that are large enough for them to see the need to make and use 'tens'. We want them to become accustomed to organising the items they are counting into 'tens' as an efficient means of finding the total of the count.

A collection such as this,

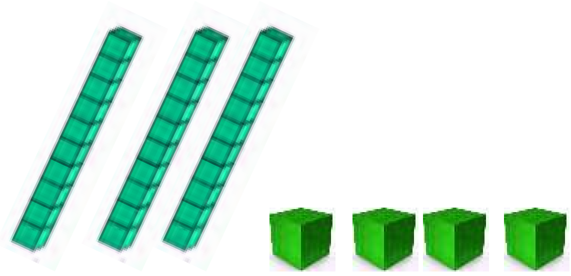


Can be organised in tens for easy counting and the tens are counted first; Ten, Twenty, Thirty, Thirty-one, Thirty-two, Thirty-three, Thirty-four counters.

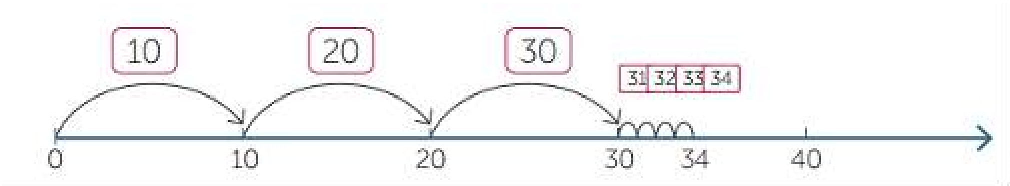


This can also be demonstrated using base-ten blocks, the number line and tens frames.

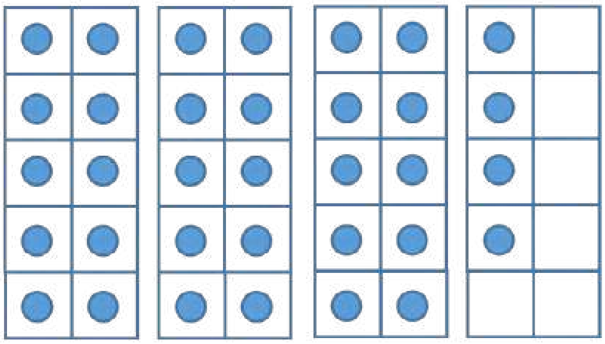
Using Base Ten Blocks



Using the Number line

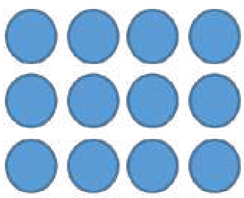


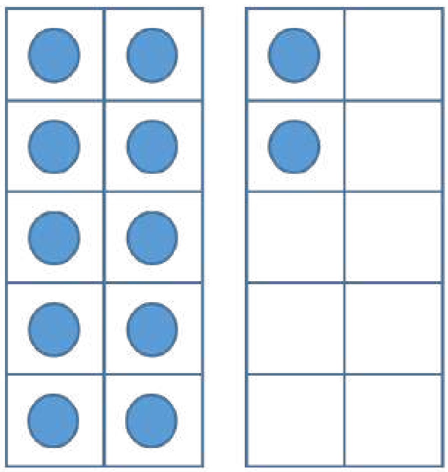
Using Coloured Blocks



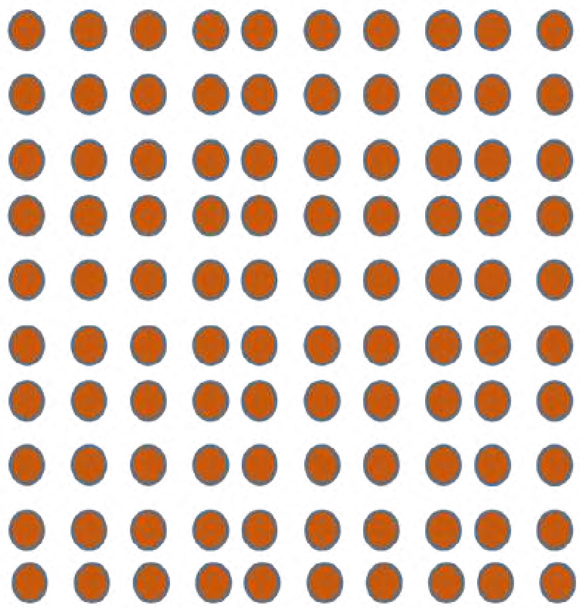
Example

Provide blocks or counters and ask learners to arrange the number 12 in different ways (counters, base 10 blocks or the number line)





Hundreds and beyond

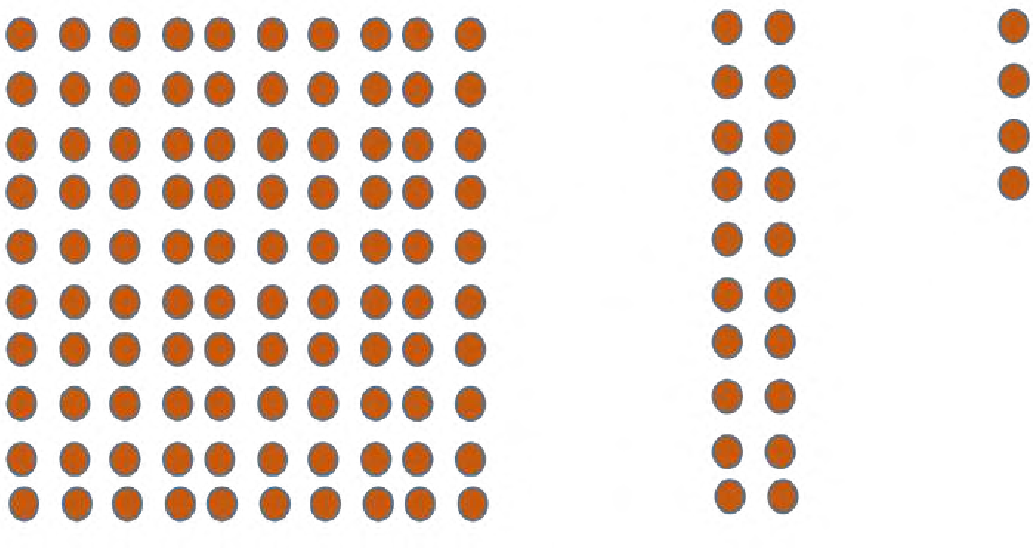


Learners can count to 100 by tens. The activity above shows coloured discs arranged 10 by 10 (10 on the vertical row and 10 on the horizontal).

Hundreds = $100 = 10 \times 10 = 10^2$

Example

Represent 124 using the concept of place of value.



Example

Using dummy currencies

1. How much money is there in total?

Writing in Terms of Place Value:

2000 (thousands place): 10 × 200 cedi notes

200 + 200 + 200 + 200 + 200 + 200 + 200 + 200 + 200 + 200 = 2 000 (2 Thousands)

200 (hundreds place): 1 × 200cedi note

20 (tens place): 1 × 20 cedi note

2 (units place): 1 × 2 cedi note

This indicates

Breakdown of 2222 cedis:

Thousands place: 2000 cedis (10 × 200 cedi notes)

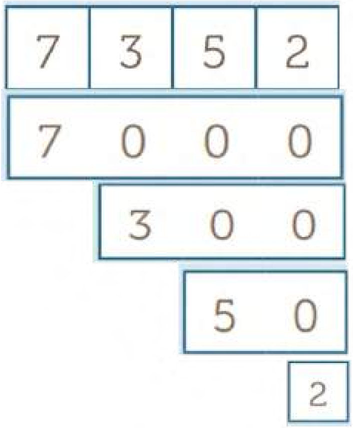
Hundreds place: 200 cedis (1 × 200 cedi note)

Tens place: 20 cedis (1 × 20 cedi note)

Units place: 2 cedis (1 × 2 cedi note)

Example

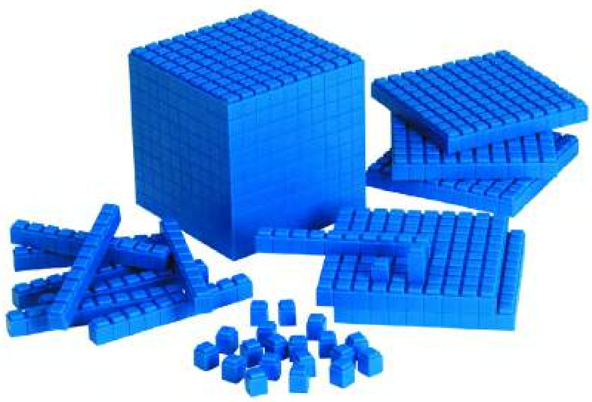
Model the number 7352 using place value chart.

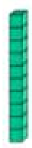
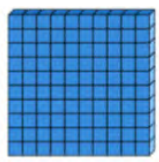
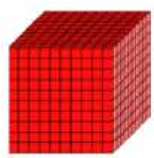


This shows that 7352 = 7000 + 300 + 50 + 2

Using Multi-base blocks

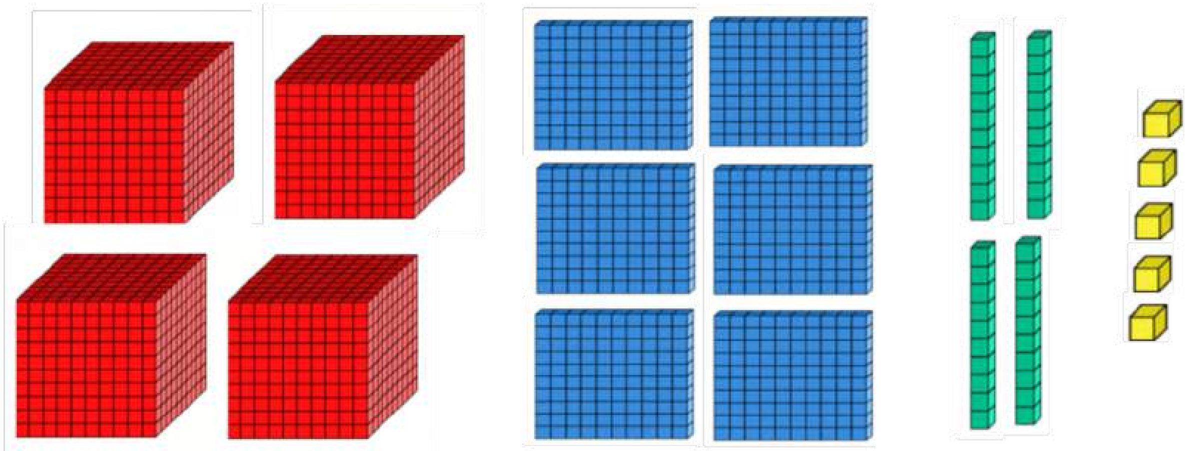
Multi-base blocks (such as base-10 blocks) are physical tools that represent different place values. They help in visualising and manipulating numbers.



• **Units**: Represent the ones place.	
• **Rods**: Represent the tens place.	
• **Flats**: Represent the hundreds place.	
• **Cubes**: Represent the thousands place and beyond.	

Example:

Write the number that is represented by the base 10 blocks



Four Thousands and Six Hundreds and Four Tens and Five Ones

This implies that,

Four Thousands = 4000

Six Hundreds = 600

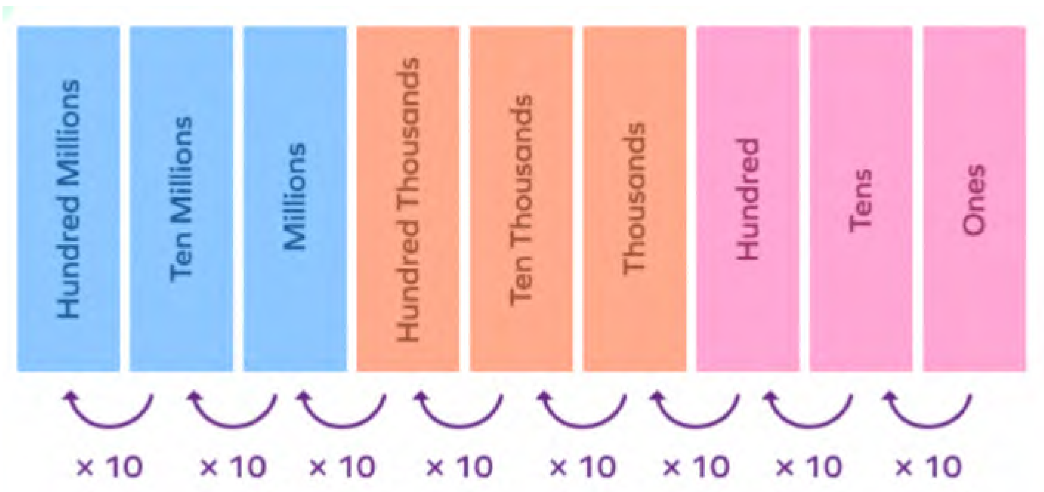
Four Tens = 40

Five Ones = 5

4000 + 600 + 40 + 5 = 4645

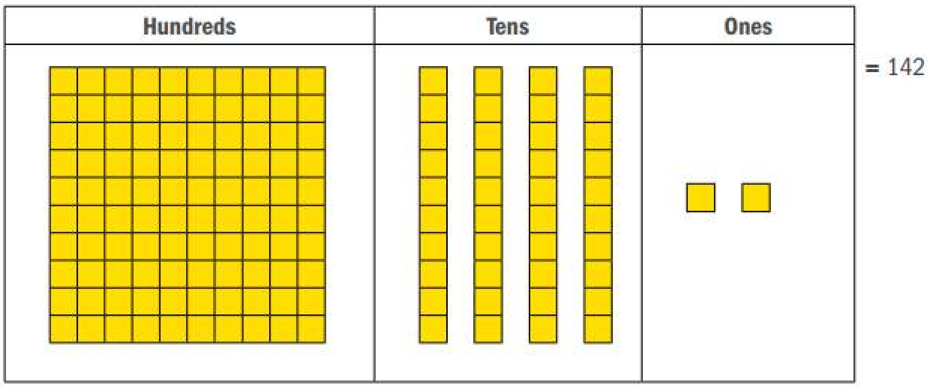
This can be read as Four Thousand, Six Hundred and Forty- Five

Place Value Chart

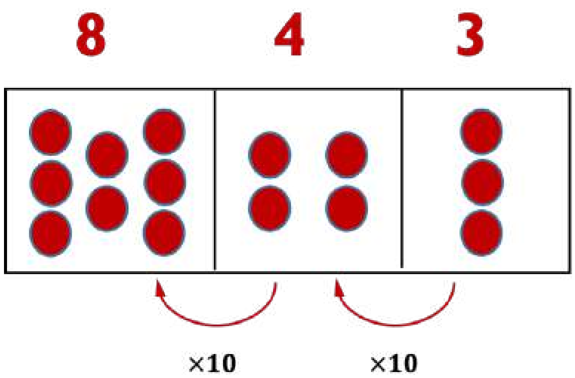


Example:

1. Illustrate the use of a place-value chart with the number **142.**



2. Illustrate the use of a place-value chart with the number **843.**



The ones digit is 3, so 3 discs are placed in the rightmost column, representing 3 ones.

The tens digit is 4, so 4 discs are placed in the next column, representing 4 tens.

The hundreds digit is 8, so 8 discs are placed in the next column, representing 8 hundred.

Writing and Naming Larger Numbers

1. 1 000 000

Millions			Thousands			Hundreds	Tens	Ones
Hundred Million	**Ten million**	**Million**	**Hundred Thousand**	**Ten Thousand**	**Thousand**	**Hundred**	**Tens**	**Ones**
		1	**0**	**0**	**0**	**0**	**0**	**0**

2. 345 678

Millions			Thousands			Hundreds	Tens	Ones
Hundred Million	**Ten million**	**Million**	**Hundred Thousand**	**Ten Thousand**	**Thousand**	**Hundred**	**Tens**	**Ones**
			3	**4**	**5**	**6**	**7**	**8**

Encourage learners to work in pairs or in small groups to break down the quantity of each digit represented based on its place value.

3 in the hundred thousand place = 3 × 100 000 = 300 000

4 in the ten thousands place = 4 × 10 000 = 40 000

5 in the thousands place = 5 × 1 000 = 5 000

6 in the hundreds place = 6 × 100 = 600

7 in the tens place = 7 × 10 = 70

8 in the ones place = 8 × 1 = 8

To further illustrate the place value, assist learners to write the number 345678 in its expanded form.

345,678 = 300,000 + 40,000 + 5,000 + 600 + 70 + 8

To represent this visually,

Place Value	Ten million	Million	Hundred Thousand	Ten Thousand	Thousand	Hundred	Tens	Ones
Digit			**3**	**4**	**5**	**6**	**7**	**8**
Value			300 000	40 000	5 000	600	70	**8**

Example:

1. Break down the number 789 012

   7 in the hundred thousands place: 7 × 100 000 = 700 000

   8 in the ten thousands place: 8 × 10 000 = 80 000

   9 in the thousands place: 9 × 1 000 = 9 000

   0 in the hundreds place: 0 × 100 = 0

   1 in the tens place: 1 × 10 = 10

   2 in the ones place: 2 × 1 = 2

   Expanded form: 789 012

   = 700 000 + 80 000 + 9 000 + 0 + 10 + 2

   = 7 Hundred Thousands + 8 Ten Thousands + 9 Thousands + 0 hundreds + 10 tens + 2 ones

2. What is the value of the digit 4 in the number 254 831?

   The digit 4 is in the thousands place.

   Value: 4 × 1,000 = 4 000

Reading Numbers

Pronouncing numbers correctly (e.g., 345 678 as "three hundred forty-five thousand, six hundred seventy-eight").

Writing Numbers

Normal Form: Writing numbers in digits (e.g., 345 678).

Expanded Form: Breaking down numbers according to their place value

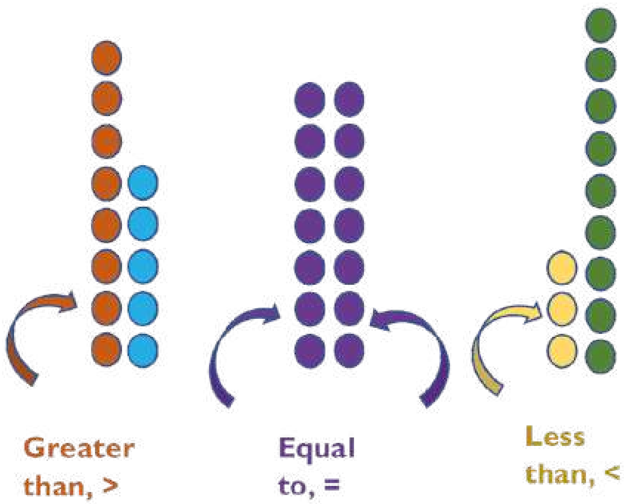
(e.g., 345 678 = 300 000 + 40 000 + 5 000 + 600 + 70 + 8).

Word Form: Writing numbers in words (e.g., three hundred forty-five thousand, six hundred seventy-eight).

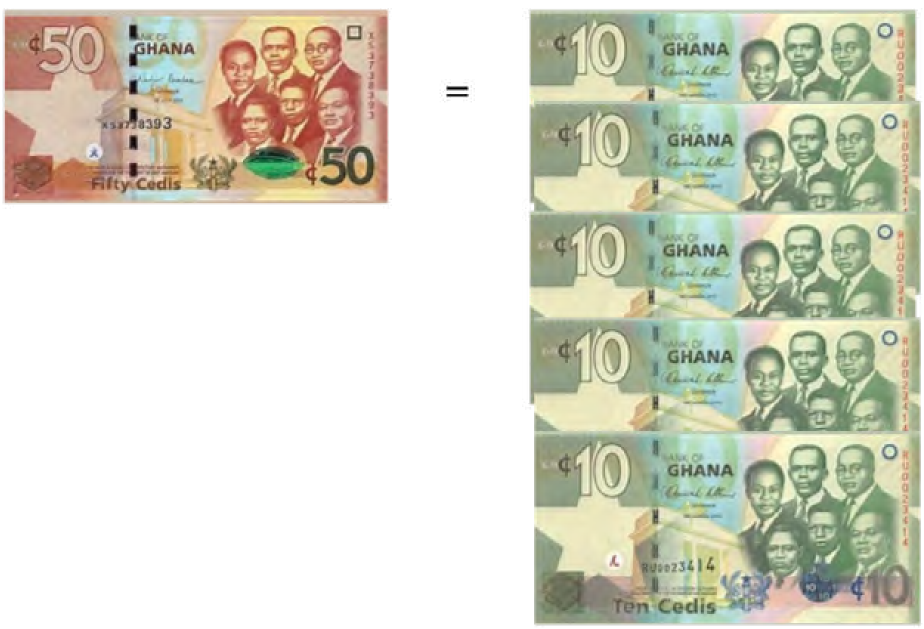
Comparing Numbers

Example:

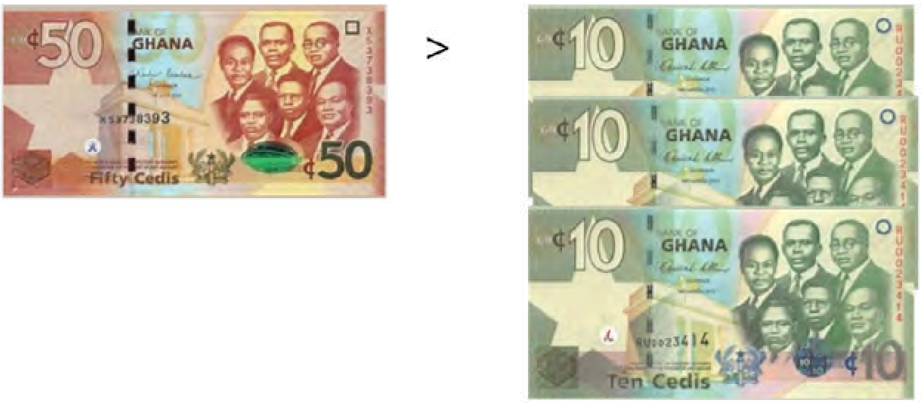
1.



2.



3.



4. Let's compare the amounts 3450 cedis and 3290 cedis.
   - Thousands place: 3 vs 3 (they are equal, move to the next digit)
   - Hundreds place: 4 vs 2 (4 is greater than 2, so 3450 cedis is greater than (>) 3290 cedis)

Therefore, **3450 cedis is greater than 3290 cedis, or 3450 > 3290**.

Learners Task for Practice

1. Learners design and complete place value charts using multi-base blocks and graph sheets to represent various numbers up to 1 000 000.
2. Learners use graph sheets to draw visual representations of large numbers and label each place value accurately.

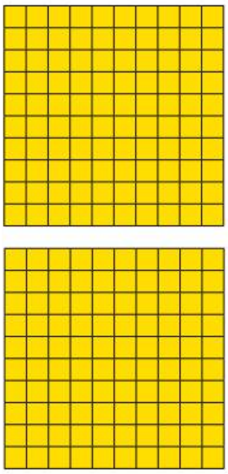
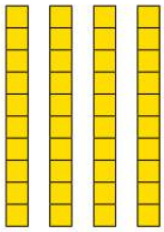
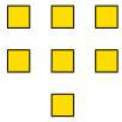
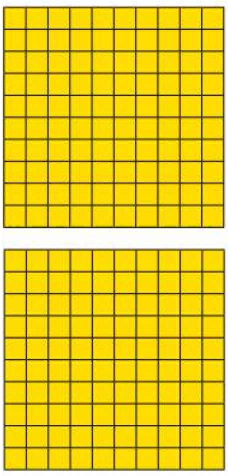
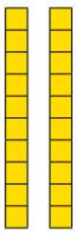
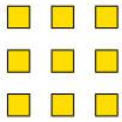
Pedagogical Exemplars

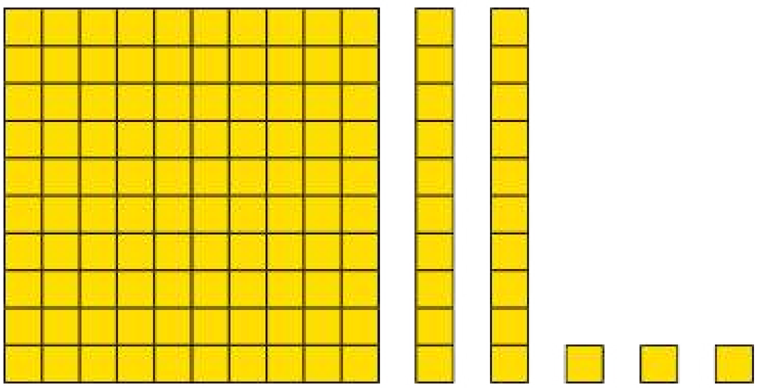
1. **Collaborative/ Experiential learning**: In mixed-gender/ability groups, engage learners read, write, and compare numbers up to 1 000 000 using graph sheets, multi-base blocks and other improvised materials to explore and enhance their understanding of place value and large numbers.
2. **Think-Pair-Share**: Engage learners to individually model a given number (e.g. 345 678) using multi-base blocks or graph sheets and pair up to discuss and compare their models. In their pairs let learners present their models to the class with explanations.

Note: Learners should be encouraged to use other means for modelling for quantities, such as dummy monies etc.

Key Assessment

1. Write the number shown using base-10 blocks below.

Hundreds	Tens	Ones	Number
 ________	 ________	 ________	
 ________	 ________	 ________	

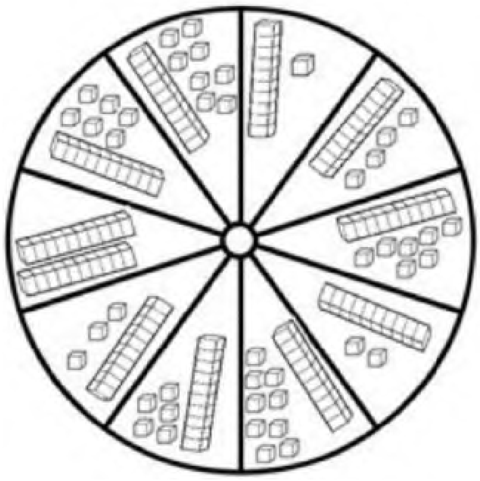


2. Write the number "three hundred fifty-two thousand, six hundred eighteen" in digit form on your graph sheet.

3. Observe the place value wheel below and answer the following questions:

i. Write down each of the numbers shown on each section of the wheel.

ii. Using the base 10 blocks, create the largest number possible on the place value wheel. What is that number?



4. City A has a population of 789 012. Write this number in words.

City B has a population of 654 321.

Which city is larger?

Week 2: Approximations & Odd and Even Numbers

Learning Indicators

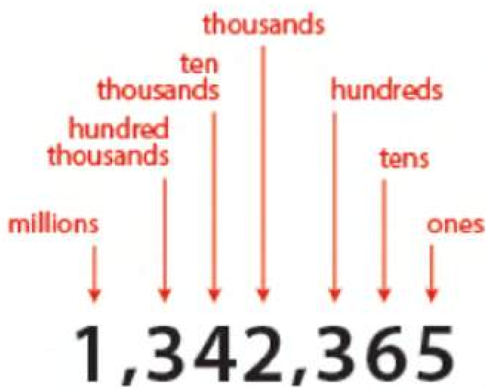
1. *Round (off, up, down) whole numbers up to 100 000 to the nearest tens, hundreds, thousands and tens of thousands*
2. *Identify even and odd numbers between 1 and 100 as numbers that can be arrayed in twos array and those which cannot*

Focal Area: Approximating and Rounding Numbers

Measures of length, mass, time, area, population, money etc., should always be given to a reasonable degree of approximation, especially when they cannot be stated with exactness or precision. There are three main ways by which approximations may be done. That is by rounding off to a nearest appropriate unit, rounding to a given number of decimal places, or rounding numbers to a given number of significant figures.

We can use the idea of rounding numbers to get a rough estimate which might be a little more or less than the actual amount.

Approximating and rounding numbers are methods used to simplify numbers to make them easier to work with, especially when exact precision is not necessary. Here's a detailed look at these concepts:



Example:
Emefa bought 11 pens at 0.95p each. In order to find out approximately how much she must pay, we can round 11 pens down to 10 pens and 0.95 up to GH₵1.

Rounding to the Nearest Ten

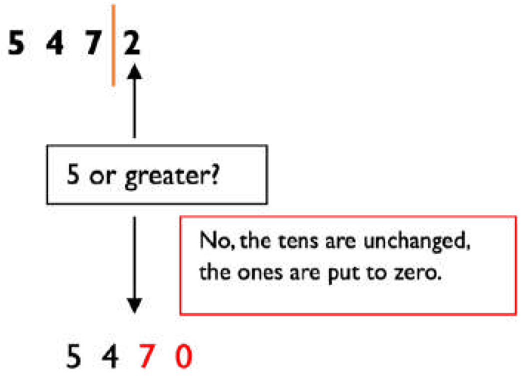
Rounding numbers to the nearest ten involves looking at the ones place and determining if the tens number should be rounded up or if it should remain the same. To round a number to the nearest ten, if the number ends with a 0, 1, 2, 3 or 4 the tens number is unchanged. If the number ends with a 5, 6, 7, 8 or 9 the number is rounded up. The basic rules are:

- If the digit in the ones place is 5 or greater, round up the tens.
- If the digit in the ones place is less than 5, the tens remain the same.

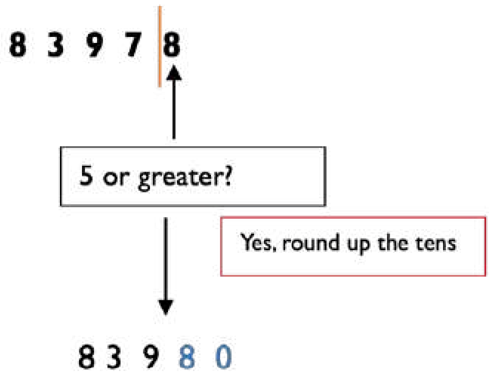
This can be expanded to rounding to the nearest hundred, thousand etc. Here you change to zeros all the digits after the place you are rounding to. If rounding up, the digit in the place you are rounding to is increased by 1.

Example:

1. Round 5 472 to the nearest ten.



2. Round 83 978 to the nearest ten.



3. Round 24 693 to the nearest ten.

When rounding 24 693 to the nearest ten, the number in the tens digit is considered; which is 9 in this instance, 3 is the number on the right of 9, which is smaller than 5, so the 9 digit is kept as it is, therefore there is no need to change the tens.

The final answer is 24, 690

4. Round 22 to the nearest ten.

The number in the ones digit is 2, which is less than 5, therefore we leave the tens alone and replace the ones with 0.

22 is rounded down to 20.

5. Use the number line to round 750 to the nearest ten.

Draw a number line and locate 750.

The number close to the tens digit is 0, which is less than 5, hence we round down by maintaining the digit as it is.

Therefore, 750 to the nearest ten is still 750.

Rounding to the Nearest Hundred

Rounding numbers to the nearest hundred involves looking at the tens place and determining if the number should be rounded up or remain the same. To round a number to the nearest hundred, if the number in the tens column is 5 or greater, we round the hundreds up by 1, if the tens number is less than 5 the hundreds number remains the same.

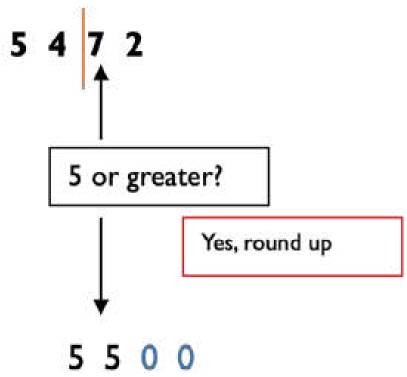
The basic rules are:

- If the digit in the tens place is 5 or greater, round the hundreds number up by 1.
- If the digit in the tens place is less than 5, the hundreds number remains the same.

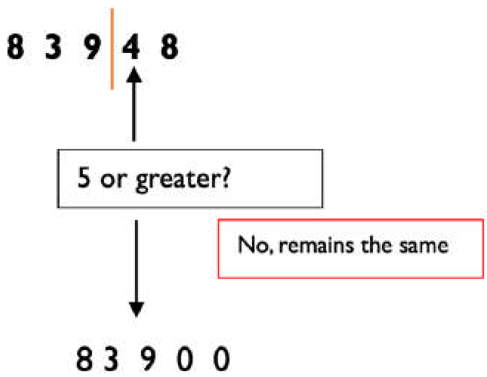
More generally, change to zeros all the digits after the place you are rounding to. If rounding up, the digit in the place you are rounding to is increased by 1

Example

1. Round 5 472 to the nearest hundred.



2. Round 83 948 to the nearest hundred.

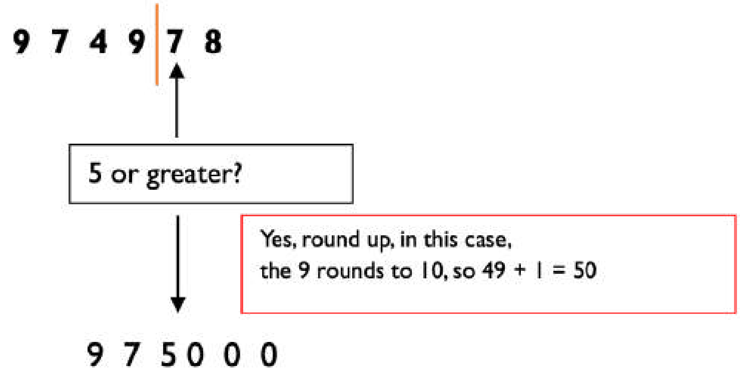


3. Round 24 693 to the nearest hundred.

   When rounding 24 693 to the nearest hundred, the number in the tens column is considered. In this case, it is 9, which is greater than 5, so we round up by increasing the hundreds digit by 1 (from 6 to 7);

   The final answer is 24 700.

4. Round 974 978 to the nearest hundred.



5. Use the number line to round 750 to the nearest hundred.
   Draw a number line and locate 750.

The number in the tens digit of 750 is greater than or equal to 5 therefore we round up by increasing the number in the hundreds digit by 1; so from 7 to 8 and replace the tens with 0.

Therefore, 750 to the nearest hundreds is 800.

Rounding to the Nearest Thousand

Use the number line to round the following to the nearest thousand.

1. 3 518

   The number in the hundreds digit of 3 518 is greater or equal to 5 therefore we round up by increasing the number in the thousands column from 3 to 4 and replace the following digits with 0.

   Therefore, 3518 to the nearest hundreds in 4 000.

2. 6 344

   The number in the hundreds digit of 6 344 is less than 5 therefore we maintain the number in the thousands digit; and replace the remaining numbers with 0.

   Therefore, 6 344 to the nearest hundreds is rounded down to 6 000

Learning Task for Practice

1. Learners round given numbers to the nearest tens, hundreds, thousands, and tens of thousands using number lines and visual aids.
2. Learners use real-life examples, such as rounding prices or distances, to understand the practical applications of rounding.
3. Learners work in pairs to create word problems involving rounding, then swap with another pair to solve.

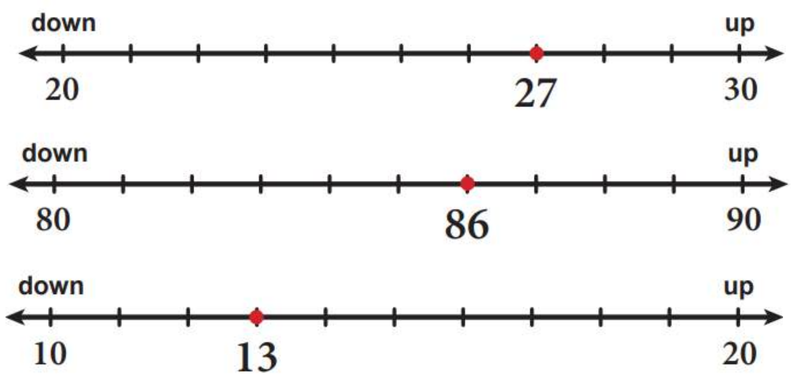
Pedagogical Exemplars

1. **Collaborative learning**: In small groups, learners round a set of given numbers to the nearest tens, hundreds, thousands, and tens of thousands using worksheets.
2. **Think Pair-Share**: In pairs, or mixed ability groupings, use base ten blocks or other manipulatives to represent numbers and visually understand rounding to the nearest tens, hundreds, thousands, and tens of thousands.

   Example: Using base ten blocks, learners represent the number 34 572. They physically adjust the blocks to round to the nearest ten, hundred, thousand, and ten thousand, then record the results: 34 570 (nearest ten), 34 600 (nearest hundred), 35 000 (nearest thousand), and 30 000 (nearest ten thousand).

Key Assessment

1. Use the number lines provided below to round to the nearest ten.



2. The table lists some tall buildings and their heights.

Building	Height	Height (rounded)
Supreme Court	2, 717 ft	
Parliament House	2, 073 ft	
Axim Forte	1, 667 ft	
Elmina Castle	1, 483 ft	
Manhyia Palace	1, 250 ft	

Round the height of each building to the nearest hundred feet.

4. Jake's yearly earnings are GH₵ 47 807.
   That means GH₵____________ to the nearest thousand.
   So, he earns about _______ monthly.

Focal Area: Understanding Even and Odd Numbers

Numbers that can be divided by 2 without a remainder are called even numbers, meaning that they can be divided into two equal parts. Odd numbers are those numbers that cannot be divided into two equal parts.

Examples of odd numbers are 1, 3, 5, 7, 9, 11, 13, 15,…

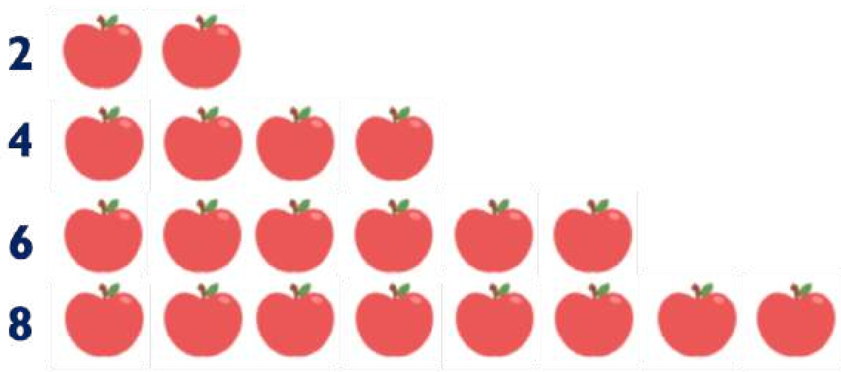
Examples of even numbers are 2, 4, 6, 8, 10, 12, 14,…

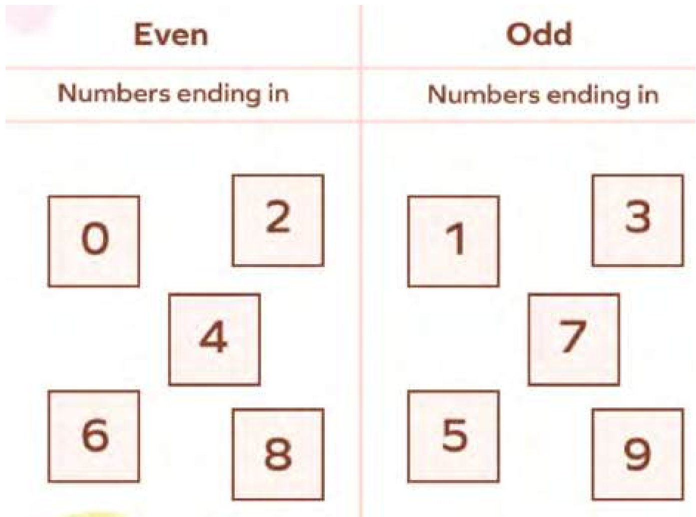
Let learners explore some special features of odd and even numbers by arranging counters/ colored discs etc. in pairs.

Even Numbers

Even numbers end with 0, 2, 4, 6 and 8.

Example:
Count the number of children below and divide into two teams



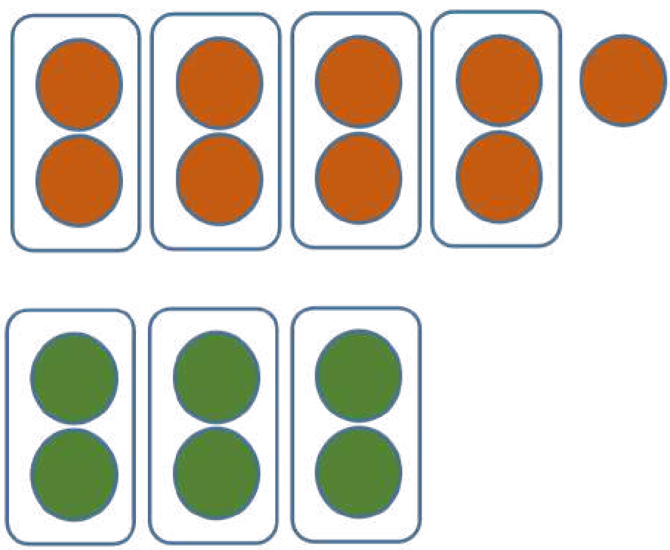


Array representation

- Even numbers can be arranged into two equal groups.
- Odd numbers cannot be evenly divided into two groups; there will always be one left over.

Ask learners what makes a number even? Odd? There are two ways to identify odd/even numbers: if at the end there is a remainder (of 1) the number is odd; otherwise it is even.

9 is odd: 6 is even:



Examples

Even Number:

1. **6**: Can be divided into 3 groups of 2 (6 ÷ 2 = 3).
2. **14**: Can be divided into 7 groups of 2 (14 ÷ 2 = 7).

Odd Numbers:

1. **5**: Can be divided into 2 groups of 2 with 1 left over (5 ÷ 2 = 2 remainder 1).
2. **13**: Can be divided into 6 groups of 2 with 1 left over (13 ÷ 2 = 6 remainder 1).

Odd and Even Number Definition

Even numbers are numbers that are divisible by 2. Even numbers always end with 0, 2, 4, 6 or 8.

Odd numbers, when divided by 2, leave a remainder of 1. Odd numbers always end with 1, 3, 5, 7 or 9.

[Note: what is 0? It is not odd, there is no remainder after sharing 0 into pairs.]

How are even and odd numbers identified?

1. By comprehending the number at the "ones" place

In this approach, we analyse the number in the "ones" place in an integer to check if the number is even or odd.

To identify even numbers, we observe the last digit or the ones digit of the number. If it ends in one of the digits **0, 2, 4, 6, or 8**, then it is an even number. Otherwise, it is an odd number.

Example:

Determine whether the following numbers are even or odd using the concept of "ones" digit in a place value

1. 248

Hundreds	Tens	Ones
2	**4**	**8**
Even Number		

The ones number is even, so the whole number is even.

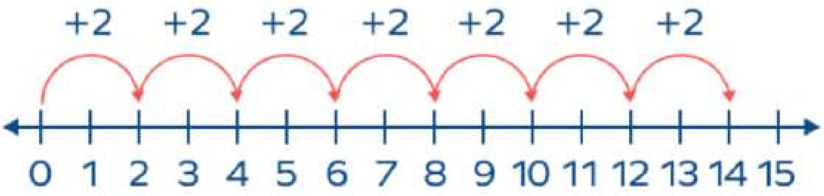
2. 103

Hundreds	Tens	Ones
1	**0**	**3**
Odd number		

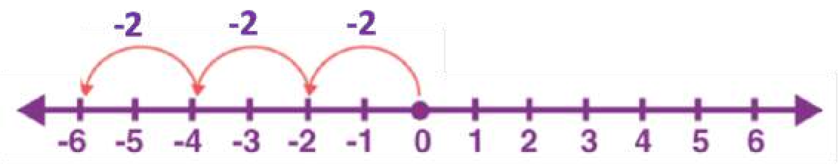
The ones number is odd, so the whole number is odd.

Identifying Even and Odd Numbers on a Number Line

Use a number line and have learners make jumps of 2 starting from 0. They will land on even numbers. Explain that numbers they land on are even, and the numbers in between are odd.



If you make the jumps of 2 to the right of 0, you will meet the positive integers that are even. Notice that between two even numbers, there's an odd number!



If you make the jumps of 2 to the left of 0, you will meet the negative integers that are even. Notice that between two even numbers, there's an odd number!

Properties of even and odd numbers

Take a look at the properties of even and odd numbers with addition, multiplication, and subtraction.

Property of Addition

- Even number + Odd number = Odd number
  An even number plus an odd number equals an odd number.
  For example, 8 + 5 = 13; 6 + 7 = 13
- Even number + Even number = Even number
  Adding two even numbers results in an even number.
  For example, 8 + 4 = 12; 12 + 8 = 20
- Odd number + Odd number = Even number
  When adding two odd numbers, the result is an even number.
  For example, 3 + 5 = 8; 15 + 11 = 26

Operation (+)	Example
Even + Even = Even	**6 + 6 =12**
Even + Odd = Odd	**6 + 3 = 9**
Odd + Even = Odd	**3 + 6 = 9**
Odd + Odd = Even	**3 + 3 = 6**

Property of Subtraction

- Even number – Odd number = Odd number
  When you subtract an odd number from an even number, the result is an odd number.
  For example, 8 – 5 = 3; 32 – 7 = 25
- Even number – Even number = Even number
  Subtracting two even numbers results in an even number.
  For example, 16 – 10 = 6; 38 – 4 = 34

- Odd number – Odd number = Even number

  Subtracting two odd numbers results in an even number.

  For example, 13 – 5 = 8; 63 – 17 = 46

Operation (−)	Example
Even – Even = Even	6 – 4 = 2
Even – Odd = Odd	6 – 3 = 3
Odd – Even = Odd	7 – 2 = 5
Odd – Odd = Even	7 – 5 = 2

Property of Multiplication

- Multiplying an even number and an odd number (and vice versa) always results in an even number.
  For example, 5 × 6 = 30.
- Multiplying an even number with an even number always results in an even number.
  For example, 6 × 10 = 60.
- Multiplying odd and odd always results in an odd number.
  For example, 13 × 5 = 65.

Operation (×)	Example
Even × Even = Even	4 × 2 = 8
Even × Odd = Even	4 × 3 = 12
Odd × Even = Even	3 × 4 = 12
Odd × Odd = Odd	3 × 3 = 9

Even Numbers up to 100				
2	4	6	8	10
12	14	16	18	20
22	24	26	28	30
32	34	36	38	40
42	44	46	48	50
52	54	56	58	60
62	64	66	68	70
72	74	76	78	80
82	84	86	88	90
92	94	96	98	100

Odd Numbers up to 100				
1	3	5	7	9
11	13	15	17	19

Odd Numbers up to 100				
21	23	25	27	29
31	33	35	37	39
41	43	45	47	49
51	53	55	57	59
61	63	65	67	69
71	73	75	77	79
81	83	85	87	89
91	93	95	97	99

Facts about even and odd numbers

- 0 is an even number.
- 1 is an odd number.
- The sum of two or more even numbers is always even.
- The product of two or more even numbers is always even.
- If you can form two equal groups of the given number, or form a "doubles fact," it is an even number

Examples

Is 29 510 an even number?

Solution:

The ones place of the given number is 0, which is an even number. Thus, the number 29,510 is an even number.

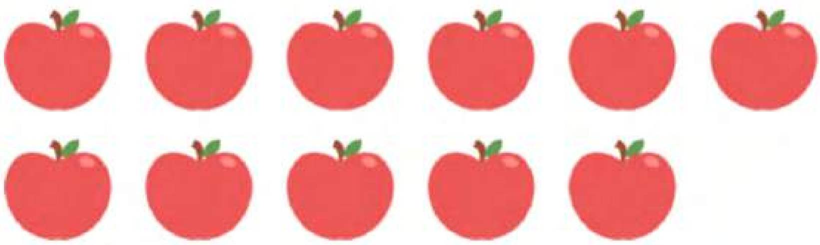
What is the sum of the first and last even numbers between 1 and 99?

Solution:

Between 1 and 99, the largest even number is 98, and the smallest even number is 2.

So, the required sum is 98 + 2 = 100.

Check whether the number of apples in the image are even or odd.



Solution

There are 11 apples.

11 apples cannot be divided into two equal groups. If we divide 11 by 2, the remainder is 1. One apple will be left out.

Thus, 11 is an odd number.

Learning Tasks for Practice

Learners practice problems on even and odd numbers.

Pedagogical Exemplars

1. **Problem Based Learning**: Create contextual problems or present scenarios involving even and odd numbers where learners need to group items in pairs.

   Example - If you have 15 apples and you want to pack them in pairs, will any be left without a pair?
2. **Experiential Learning**: Let learners make a chart listing numbers from 1 to 100, where learners can colour-code even numbers in one colour and odd numbers in another. Each learner picks a number, checks if it can be paired, and colours it accordingly on the chart.
3. **Think-Pair-Share**: Prepare flashcards with numbers from 1 to 100. In pairs, task learners to take turns to draw a card and share whether the number is even or odd and why.

Key Assessment

1. Is 350 an even number? Explain your answer.
2. Determine which of the groups of numbers below are even numbers.
   - **i.** 5, 23, 147
   - **ii.** 2, 16, 234
   - **iii.** 89, 573, 1257
   - **iv.** 123, 567, 897
3. Is the number 0 an even or an odd number?
4. How many even numbers are there between 20 and 50 (excluding 20 and 50)

Week 3: Factors and Multiples

Learning Indicator: *Identify factors and multiples of numbers and use the knowledge to solve problems*

Focal Area: Factors, Multiples and Highest Common Factor

Factors

A factor is a number that goes exactly into a given number, or divides into a particular number with no remainder (fraction or decimal). If two numbers multiply to give another number, those two multipliers will both be factors of the number. A factor is always a positive integer.

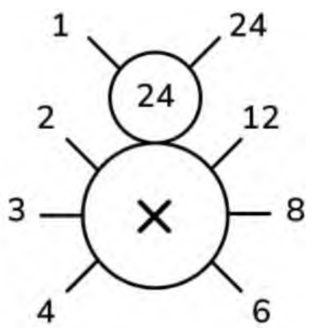
To find all the factors of a given number, start with 1 and systematically work through each number to see if it has a factor pair that will multiply to make the given number, until the factors end up repeating themselves. For example, to find all the factors of 24:

For example, 4 is a factor of 8, 12, 16, 20, etc. because 8 ÷ 4 = 2, 12 ÷ 4 = 3, 16 ÷ 4 = 4, 20 ÷ 4 = 5, etc.; therefore, all the numbers in the 4 times tables have 4 as a factor.

Examples:

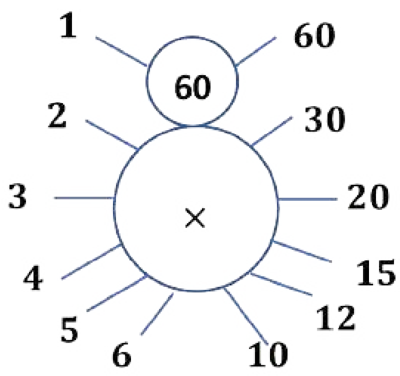
1. Find the factors of 24 .

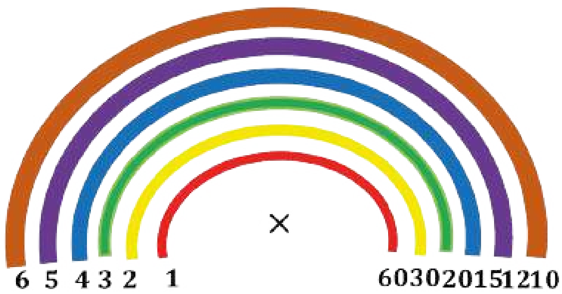
   Using factor bugs, arrays or rainbows to model the factors of 24. This involves finding factor pairs that multiply to make the given number, in this case, 24.



Therefore, 1, 2, 3, 4, 6, 8, 12 and 24 are all the factors of 24.

Find the factors of 60





Therefore, 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30 and 60 are all the factors of 60.

Factors of a number are the integers that can be multiplied together to produce that number.

The factors of 12 are 1, 2, 3, 4, 6, and 12.

- $1 \times 12 = 12$
- $2 \times 6 = 12$
- $3 \times 4 = 12$

Common factor

Common factors are factors shared between two or more given numbers. For example, 5 is a common factor of 5, 10 and 25 as it is a factor of all three numbers (note that 5, 10 and 25 are therefore all multiples of 5).

7 is a common factor of 14 and 21, as it fits exactly into both. Another example would be the common factors of 8 and 12 which are 1, 2 and 4.

Highest Common Factor

Highest Common Factor (HCF) of two or more numbers is the greatest number that divides into the given numbers exactly. HCF is the acronym of Highest Common Factor. It is calculated for two or more numbers.

Finding the Highest Common Factor (HCF)

In order to find the Highest Common Factor of two or more numbers we look for the common factors in their prime factorisations and multiply these together.

Example: find the HCF of 24 and 60

We worked out above that $24 = 2 \times 2 \times 2 \times 3$ and $60 = 2 \times 2 \times 3 \times 5$.

To find out the HCF we look for the common prime factors (the prime numbers that are in both) and multiply them together. In this case we have 2 × 2 × 3 = 12. Therefore, the HCF of 24 and 60 is 24.

Multiples

A multiple of a number is the result when that number is multiplied by an integer. Compared to the factor examples above, whereas 4 is a factor of 8 and 12, 8 and 12 are multiples of 4.

Other examples of multiples of 4 could be 4 (4 × 1), 36 (4 × 9) or 400 (4 × 100). Multiples can also be described as numbers in a given times table – for example, 4, 8, 12 and 16 are all in the 4 times table, therefore, they are also all multiples of 4.

Multiples of 2: 2, 4, 6, 10, 12, 14, 16, 18, 20, 22, 24 …

Multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24 …

Multiples of 6: 6, 12, 18, 24 …

Multiples of 8: 8, 16, 24 …

Common multiples

Common multiples are multiples shared between two or more given numbers. For example, 24 is a common multiple of 2, 3, 6 and 8 as it is a multiple of all four numbers (note that 2, 3, 6 and 8 are therefore all factors of 24).

Lowest Common Multiple (LCM)

The lowest common multiple (LCM), also referred to as the least common multiple, is the smallest multiple shared between two or more given numbers. For example, the lowest common multiple of 10 and 15 is 30 as it is the smallest multiple that both given numbers share (10 × 3 = 30 and 15 × 2 = 30).

Let us take two numbers, 3 and 4.

Multiples of 3 = 3, 6, 9, 12, 15, 18, 21, 24…….

Multiples of 4: 4, 8, 12, 16, 20, 24…..

Looking at the lists, the common multiples of 3 and 4 are 12, 24,……

The lowest common multiple is 12, because it is the smallest number.

Hence the L.C.M of 3 and 4 is 12.

Example: Find the LCM of 8, 12 and 24

Multiples of 8 : 8, 16, 24, 32, 40, 48….

Multiples of 12 : 12, 24, 36, 48, ….

Multiples of 24 : 24, 48, 72, 96, …..

Looking at the lists, the common multiples are 24, 48…

Therefore, the LCM is 24.

Learning Tasks for practice

1. Learners list the factors of two given numbers and identify the common factors.
2. Learners create a list of multiples for two numbers and find the common multiples.
3. Learners use Venn diagrams to find common factors of two numbers.

4. Learners perform prime factorisation of two numbers and use it to find the HCF and create factor trees for given numbers and identify the common prime factors.

Pedagogical Exemplars

1. **Think-Pair-Share:** Learners individually think about the multiples of 6 and 8. They then pair up to discuss their lists and identify the common multiples, followed by sharing their results with the class.
2. **Experiential Learning:** Create Bingo cards with numbers. Call out criteria such as “a factor of 36” or “a multiple of 7.” Learners mark the numbers that meet the criteria.
3. **Collaborative learning:** Learners work in small groups to use Venn diagrams to visually represent the factors of two numbers and identify the common factors.

Key Assessment

1. What are the factors of 24?
2. List the first five multiples of 6.
3. What is the HCF of 15 and 25?
4. Identify the common factors of 18 and 27.
5. Find the common multiples of 4 and 10 up to 50.
6. Explain how you can find the HCF of 40 and 60 using prime factorisation.
7. Find the common factors and the HCF of 36 and 54.
8. You have 30 red balls and 45 blue balls. How can you divide them into the largest possible equal groups without any leftovers?
9. Analyse the factors of 42 and 56 to determine their HCF. Explain your reasoning

Section Review

In this section, we delved into fundamental number concepts and operations, focusing on understanding large numbers, rounding, and identifying number properties. Here is an overview of the key concepts and skills covered:

1. **Reading, Writing, and Comparing Number Quantities:**
   - **Modeling Large Numbers:** We practiced reading, writing, and comparing numbers up to 1 000 000 using graph sheets and multi-base blocks. This approach helps in visualising and understanding large quantities, making it easier to work with and compare different numbers.
2. **Rounding Whole Numbers:**
   - **Rounding Techniques:** We explored how to round whole numbers up to 100 000 to the nearest tens, hundreds, thousands, and tens of thousands. This skill is essential for simplifying numbers and making them more manageable in various mathematical contexts.
3. **Identifying Even and Odd Numbers:**
   - **Classification of Numbers:** We identified even and odd numbers between 1 and 100, understanding that even numbers can be arrayed in twos and odd numbers cannot. This classification helps in recognising patterns and properties of numbers.

4. **Identifying Factors and Multiples:**
   - **Factors and Multiples:** We learned to identify factors and multiples of numbers and used this knowledge to solve problems. Understanding factors and multiples is crucial for solving problems involving divisibility, prime numbers, and lowest common multiples.

This section provided a comprehensive foundation in working with large numbers, rounding, and understanding number properties. These skills are fundamental for more advanced mathematical concepts and problem-solving.

SECTION 2: NUMBER AND OPERATIONS ON NUMBER

Strand: **Numbers for Everyday Life**

Sub-Strand: Number Operations

Content Standards

1. Describe and apply mental mathematics strategies and number properties involving the four basic operations to solve problems
2. Demonstrate conceptual understating Interpret negative and positive numbers in context
3. Develop knowledge and understanding of the concept of fractions and its application in real life

INTRODUCTION AND SECTION SUMMARY

Mastering multi-step word problems involving the four basic operations, understanding and applying positive and negative numbers in real-life situations, and modeling fractions are essential skills in mathematics. These concepts help learners develop critical thinking and problem-solving abilities. Solving word problems using mental strategies enhances their arithmetic proficiency and boosts confidence. Describing real-life situations with positive and negative numbers, including performing operations on integers, provides a practical understanding of numerical relationships. Modeling and representing fractions, including naming fractions and representing quantities as fractions, ensures a solid foundation in understanding parts of a whole and their applications.

The concepts covered by the section are:

1. *Solving multi-step word problems involving the four basic operations using mental strategies.*
2. *Describing real-life situations using positive and negative numbers including operations on integers*
3. *Modelling and representing given fractions, naming fractions, representing quantities as a fraction*

SUMMARY OF PEDAGOGICAL EXEMPLARS

Learners will benefit from a variety of instructional strategies designed to deepen their understanding of these concepts.

1. **Multi-Step Word Problems:** Use real-life scenarios and practical examples to teach students how to solve multi-step word problems involving addition, subtraction, multiplication, and division. Encourage the use of mental strategies and estimation to find solutions.
2. **Positive and Negative Numbers:** Integrate activities that involve describing real-life situations using positive and negative numbers. Use number lines and real-world contexts, such as temperature changes or financial transactions, to help students grasp the concept of integers and their operations.
3. **Modeling Fractions:** Utilise visual aids, such as fraction strips, pie charts, and number lines, to teach students how to model and represent fractions. Provide hands-on activities where students can name fractions and represent quantities as fractions, reinforcing their understanding of parts of a whole.

4. **Collaborative Learning:** Encourage group activities and discussions where students solve multi-step word problems, describe situations with integers, and model fractions together. This promotes teamwork and enhances problem-solving skills.

ASSESSMENT SUMMARY

Assessments for these concepts should be varied and address different cognitive levels.

1. **Class Exercises and Tests:** Evaluate students' ability to solve multi-step word problems involving the four basic operations. Include tasks that require the use of mental strategies and estimation.
2. **Real-Life Application Problems:** Present real-life scenarios that involve positive and negative numbers. Assess students' ability to describe these situations and perform operations on integers accurately.
3. **Modeling Fractions:** Use exercises that require students to name fractions, represent quantities as fractions, and model fractions using visual aids. Assess their ability to accurately model and interpret fractions.
4. **Collaborative Projects:** Engage students in group tasks where they work together to solve complex word problems, describe real-life situations with integers, and model fractions. Assess their teamwork, communication, and problem-solving skills.
5. **Presentations:** Have students present their solutions to word problems, real-life integer applications, and fraction models. This assesses their understanding and ability to communicate mathematical concepts effectively.

Week 4: Mental Mathematics Strategies

Learning Indicator: *Solve multi step word problems involving the four basic operations using mental strategies.*

Focal Area: Operation of Numbers Up to 100 000 Using Mental Strategies.

Mental strategies for addition and subtraction of large numbers (up to 100 000) involve breaking down the numbers into more manageable parts. Here are some effective techniques:

Addition

Breaking Down into Place Values:
Example:

Add 47,653 and 23,457.

Break down each number by place value:

47 653 = 40 000 + 7 000 + 600 + 50 + 3

23 457 = 20 000 + 3 000 + 400 + 50 + 7

Add corresponding place values:

40 000 + 20 000 = 60 000

7 000 + 3 000 = 10 000

600 + 400 = 1 000

50 + 50 = 100

3 + 7 = 10

Combine the results:
60 000 + 10 000 + 1 000 + 100 + 10 = 71 110

Subtraction

Breaking Down into Place Values:
Example:
Subtract 12 345 from 78 654.

Break down each number by place value:
78 654 = 70 000 + 8 000 + 600 + 50 + 4
12 345 = 10 000 + 2 000 + 300 + 40 + 5

Subtract corresponding place values:
70 000 – 10 000 = 60 000
8 000 – 2 000 = 6 000
600 – 300 = 300
50 – 40 = 10
4 – 5 = –1 (borrow from 10s place, so it becomes 9 in the next step)

Combine the results:

60 000 + 6 000 + 300 + 10 - 1 = 66 309

Addition Example, using the vertical method:

$$\begin{array}{r} {}^{1\,1} \\ 173 \\ +\ 67 \\ \hline 240 \end{array}$$

7 + 3 = 10, remember 0, carry 1

1 + 7 + 6 = 14, remember 4, carry 1

1 + 1 = 2.

Remember, if you are doing this in your head, the numbers come out in the order 042 as we start with the units and then work up. This means we must then reverse them to assign the correct place value and we have 240.

Addition Example, using place value:

$$\begin{array}{r} 46 \\ +\ 38 \\ \hline \end{array}$$

Using place value add from left to right,

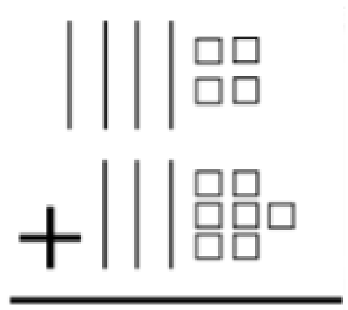
6 + 8 = 14

40 + 30 = 70

70 + 14 = 84

Addition Example, using base- 10 blocks:

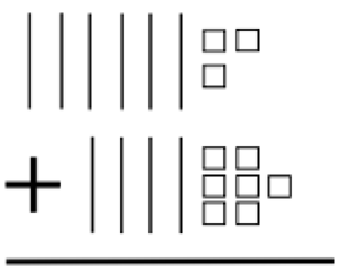
1. Add 44 and 37.



Show 44 + 37 with Base-10 blocks.

- show adding from left to right by adding groups of tens first, then regrouping units into tens if possible, and finally counting the remaining units
  40 + 30 + 10 + 1 = 81
- compare this to trying to add the total, starting with the units first
  4 + 7 = 11
  11 + 70 = 81

2. Add 63 and 47



Show 63 + 47 with Base-10 blocks.

Reflect: How many different ways can the total be found?

63 + 7 = 70

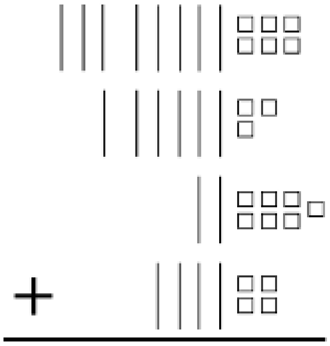
70 + 40 = 110

60 + 40 = 100

100 + 3 + 7 = 110

3 + 7+60 + 40 = 110

3. Add 86, 63, 27 and 44



Model 86 + 63 + 27 + 44, using Base-10 blocks.

Reflect: What is the most efficient method of finding the total?

- Adding all of the units and all of the tens separately and then adding them both together (similar to the algorithm)
- Counting all
- Adding tens and counting up
- Regrouping in some other method.

Addition Example, using a number line:

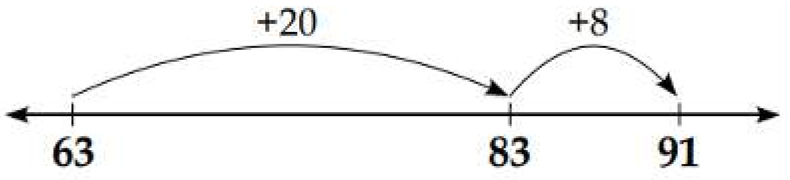
1. Add 63 and 28

$$\begin{array}{r} 63 \\ +\ 28 \\ \hline \end{array}$$

Break the numbers up and add the parts in the order that works best for you.

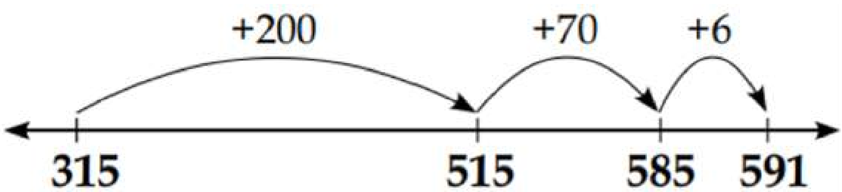
63 + 20 = 83

83 + 8 = 91



2. Add 315 and 276

315 + 200 + 70 + 6 = 591



Subtraction Example, using a number line:

Subtract 59 from 132

132 – 50 = 82

82 – 9 = 73

Multiplication Example, using the vertical method

1.

$$\begin{array}{r} 635 \\ \times \quad 4 \\ \hline \end{array}$$

600 × 4 = 2 400

30 × 4 = 120

5 × 4 = 20

2 400 + 120 + 20 = 2 540

2.

$$\begin{array}{r} 6000 \\ \times \quad 30 \\ \hline \end{array}$$

6 000 × 30 = (6 ×1 000) × (3 ×10)

= 6 × 3 ×1 000 ×10

= 18 × 10 000 = 180 000

Note: In multiplication and division, annexing zeros allows for quick mental calculations of whole numbers that are multiples of powers of ten.

Annexing zeros algorithm for multiplication:

1. Cut all the trailing zeros for numbers being multiplied.
2. Multiply the remaining numbers.
3. Paste all the zeros back.

For example, 6 000 × 30 = 6 × 3 × 10 000 = 180 000

Example 2

$$\begin{array}{r} ₵91 \\ \times \; 13 \\ \hline 273 \\ 910 \\ \hline ₵1183 \end{array}$$

1 × 3 = 3, remember 3
9 × 3 = 27, remember 27
Rearrange mentally into 273.

Add a place-holder zero for when we multiply by the tens.
1 × 1 = 1, remember 1
9 × 1 = 9, remember 9 Rearrange mentally into 910.
3 + 0 = 3, remember 3
7 + 1 = 8, remember 8
2 + 9 = 11, remember 11

Rearrange mentally and reassign place value and units to 1183.

Multiplication using the expanded method

The expanded form of multiplication is useful for splitting the higher-order digits and representing them in the form of units (ones), 10s, 100s, and 1000s. A number expansion is the separation of numbers by their place values

Examples

1. 4 537 is written in expanded form as,

$= 4000 + 500 + 30 + 7$

$= 4 \times 1\,000 + 5 \times 100 + 3 \times 10 + 7 = 4\,537$

2. Multiply 76 by 34

	70	6	
30	30×70 = 2100	30×6 = 180	2100 + 180 = 2280
	4×70 = 280	4×6 = 24	280 + 24 = 304

304 + 2280 = 2584

Therefore, 76 × 34 = 2584

3. Multiply 12 by 344

	300	40	4	
10	10×30 = 3000	10×40 = 400	10×4 = 40	3000 + 400 + 40
2	2×300 = 600	2×40 = 80	2×4 = 8	600 + 80 + 8

3 440 + 688 = 4 128

4. Multiply 902 by 7

	900	00	2	
7	7×900 = 6300	7×00 = 00	7×2 = 14	6300 + 00 + 14 = 6314

Multiplication using the lattice method

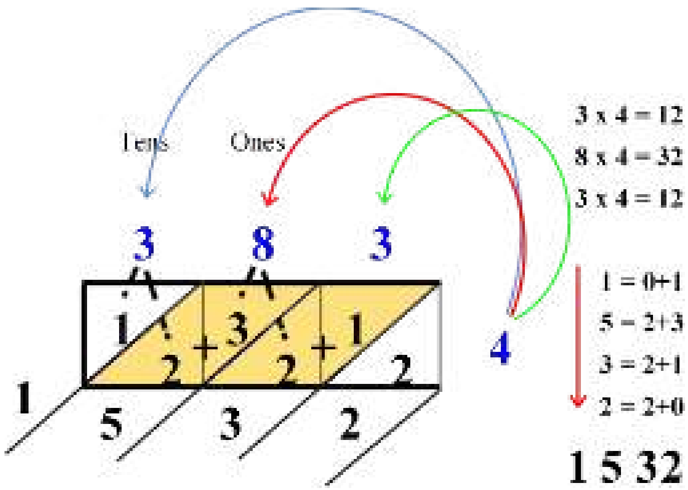
The lattice method is a visual way to multiply numbers. Here's how to multiply 127 by 65 using the lattice method:

Step-by-Step Solution:

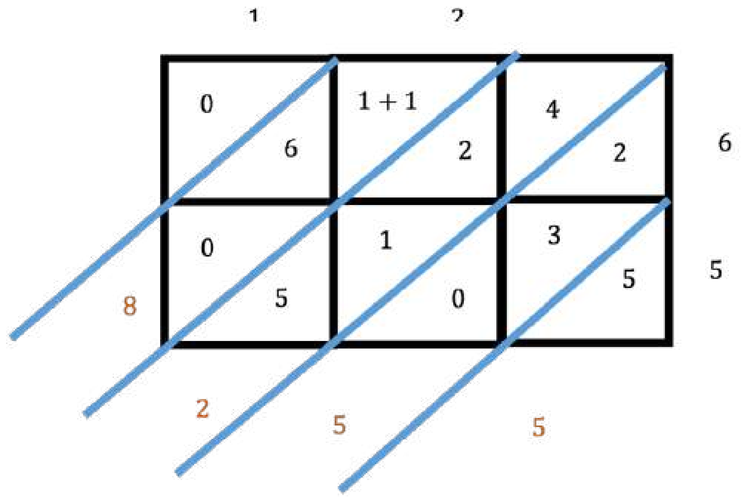
1. **Set Up the Grid**:
   - Draw a grid with 3 columns (since 127 has 3 digits) and 2 rows (since 65 has 2 digits).
   - Write the digits of 127 across the top of the grid.
   - Write the digits of 65 along the right side of the grid.
2. **Draw Diagonals**:
   - Draw diagonal lines from the top right to the bottom left of each cell in the grid, extending outside the grid.
3. **Fill in the Products**:
   - Multiply each digit of 127 by each digit of 65, and write the products in the corresponding cells, with the tens digit above the diagonal and the ones digit below the diagonal.
4. **Sum Along the Diagonals**:
   - Add the numbers along each diagonal, starting from the bottom right.
5. **Write the Final Answer**:
   - Read the sum of the diagonals from left to right to get the final product.

Examples

1. 383 ×4 = 1,532



2. 127 × 65



Set up the grid and fill in the Products:

- Multiply 6 by 1, 2, and 7:
  - 6 × 1 = 06
  - 6 × 2 = 12
  - 6 × 7 = 42
- Multiply 5 by 1, 2, and 7:
  - 5 × 1 = 05
  - 5 × 2 = 10
  - 5 × 7 = 35

Sum Along the Diagonals

Starting from the bottom right:

- 5 (from the bottom cell)
- 3 + 2 = 5
- 4 + 2 + 1 + 5 = 12 = 2, and carry the tens to the next diagonal with a 1
- 1 + 1 + 6 = 8
- 0 (from the top cell)

Read the sum of the diagonals from left to right: The final product is 8 255

Division Strategies

Using the "Big 7 strategy" for division

Example 1: 318 ÷3

Solution

3	3	1	8	
	− 3	0	0	100
		1	8	
	−	1	8	6
			0	**106**

100 + 6 = 106

So, 318 ÷ 3 = 106

Example 2: 156 ÷12 =

Solution

12	1	5	6	
	− 1	2	0	10
		3	6	+
	−	3	6	3
			0	**13**

So, 156 ÷12 = 13

Use "repeated subtraction" to solve division problems.

Example: 144 ÷ 24 = 5

Solution

144 – 24 = 120	①
120 – 24 = 96	②
96 – 24 = 72	③
72 – 24 = 48	④
48 – 24 = 24	⑤
24 – 24 = 0	6

So,144 ÷ 24 = 6

Example 2: A class teacher shared 165 exercise books equally among 15 pupils. How many did each pupil get?

Solution

165 ÷15 =

Repeated subtraction

165 – 15 = 150	**1**
150 – 15 = 135	**2**
135 – 15 = 120	**3**
120 – 15 = 105	**4**
105 – 15 = 90	**5**
90 – 15 = 75	**6**
75 – 15 = 60	**7**
60 – 15 = 45	**8**
45 – 15 = 30	**9**
30 – 15 = 15	**10**
15 – 15 = 0	**11**

"Big 7" strategy

15	1	6	5	
	– 1	5	0	10
		1	5	+
	–	1	5	1
			0	**11**

Learning Task for practice

1. Learners solve a series of addition and subtraction problems mentally within a time limit.
2. Learners estimate the sum, difference, product, or quotient of large numbers before calculating the exact answer.
3. Create Bingo cards with various results of addition, subtraction, multiplication, and division problems. The teacher calls out a problem, and learners mark the answer if it appears on their card.

Pedagogical exemplars

1. **Think- Pair- Share**: Present learners with a problem, such as "Calculate 56 789 + 34 567 mentally." Each learner thinks about their solution individually, then pairs up with a partner to discuss their methods and solutions. Finally, they share their answers and strategies with the class.
2. **Problem-based learning:** Learners are given a hypothetical budget of GHS 100 000 and a list of expenses. They use mental math to calculate the total expenses and determine how much money is left.
3. **Group work:** Learners brainstorm different mental math strategies they use for large numbers, such as breaking down numbers, using benchmarks, or compensation methods.

Key Assessment

1. What is 23 456 + 34 567?
2. Estimate the result of 45 678 – 12 345 before calculating it exactly.

   Explain your estimation process.

3. Compare different strategies for mentally multiplying 123 by 456.
Which strategy do you find most efficient and why?

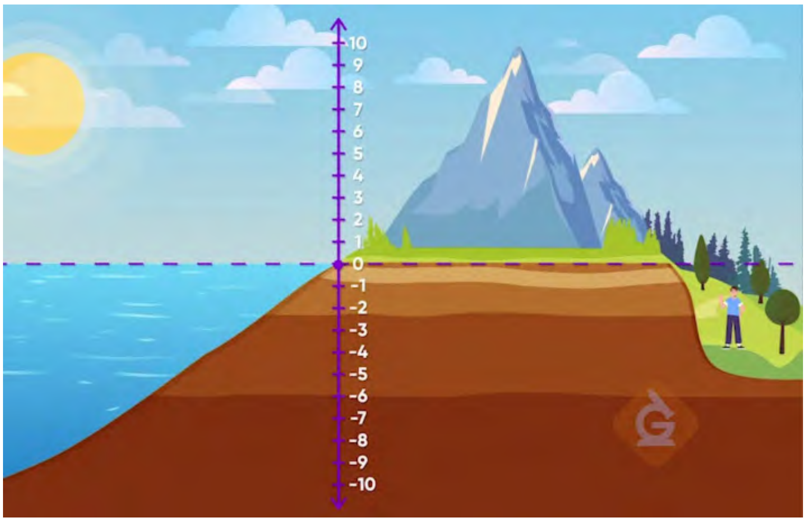
4. Create a real-world problem that requires using mental math with large numbers to solve.
Explain the steps and reasoning behind your solution.

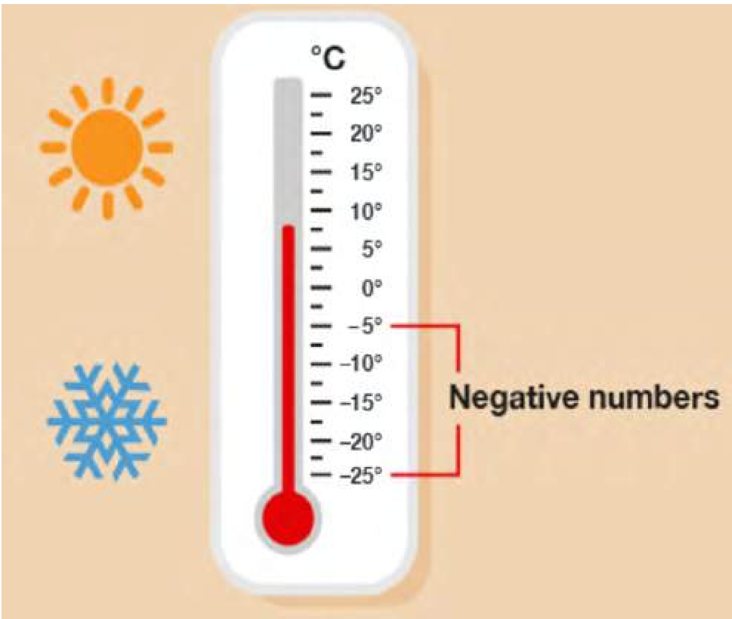
Week 5: Positive and Negative Whole Numbers

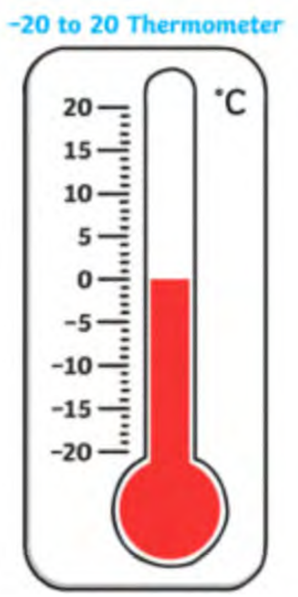
Learning Indicator: *Describe real life situations using positive and negative numbers including operations on integers*

Focal Area: Concept of Positive and Negative Whole Numbers [Integers]

Consider the image below; and ask learners what they observe



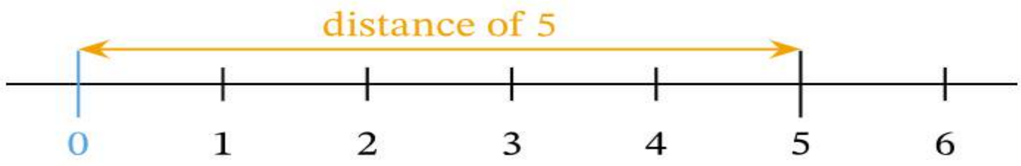




Negative and Positive numbers on the number line

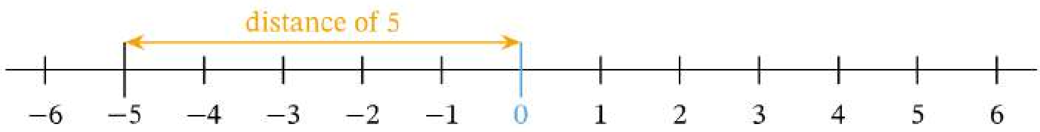
The positive and negative number line is a traditional tool used to help students learn to add and subtract positive and negative numbers. It looks like a traditional early number line, however, instead of starting at 0 or 1 it starts with a negative number on the far left of the line (the actual starting number is variable depending on the detail of the number line). The digits then increase from left to right until they reach 0. As the number line progresses to the right, the numbers rise as the traditional number line.

Learners are familiar with whole numbers, which are the counting numbers 1, 2, 3, 4, 5, and so on together with zero, can be drawn on a number line. When you do this, you can think of each number having a distance from zero. For example, 5 is 5 units to the right from zero, or a distance of 5 from zero.



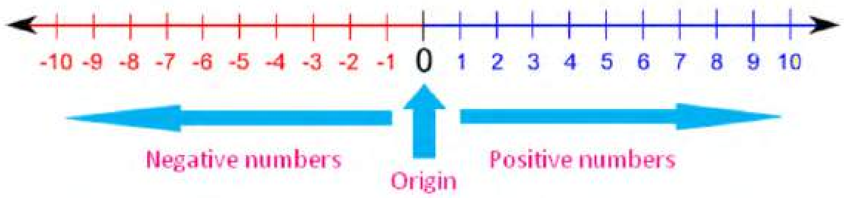
The numbers 1, 2, 3, count from zero to the right. We call these numbers the positive numbers. We can also think of counting from zero in the other direction. When we do this we get the negative numbers. Negative numbers are always written with a negative sign (−). We can write positive numbers with the positive sign (+) but we do not have to.

For example, 5 units to the left of zero we find the number negative 5, which we write −5. The positive number 5 and the negative number −5 are the same distance from zero in opposite directions.



A **positive number** is a number that is greater than zero. It falls above zero on a vertical number line or to the right of zero on a horizontal number line.

A **negative number** is a number that is less than zero. It falls below zero on a vertical number line or to the left of zero on a horizontal number line.



Real Life Situations using positive and negative numbers

1.

Positive	Negative
Above	Below
Right	Left
Win	Lose
Increase	Decrease
Fast	Slow
Over	Under
More	Less
Grow	Shrink
Higher	Lower
Ascend	Descend

2. Banking and Money
    - Positive Numbers: When you put money into your bank account.
        - Example: If you deposit GHS 50 into your bank account, you add GHS 50 to your balance.
    - Negative Numbers: When you take money out of your bank account.
        - Example: If you withdraw GHS 20, you subtract GHS 20 from your balance.
3. Temperature
    - Positive Numbers: When it's warmer than 0 degrees.
        - Example: A sunny day might be 25°C.
    - Negative Numbers: When it's colder than 0 degrees.
        - Example: A very cold day might be -5°C.
4. Elevation (Height)
    - Positive Numbers: Places that are above sea level.
        - Example: A mountain that is 1 000 metres high.
    - Negative Numbers: Places that are below the ground.
        - Example: A mine that is 50 metres underground is at -50 metres.
5. Sports Scores
    - Positive Numbers: Points that your team scores.
        - Example: If your team scores 10 points in a game, that's +10 points.
    - Negative Numbers: Points taken away because of penalties.
        - Example: If your team loses 5 points due to a penalty, that is -5 points.
6. Health and Weight
    - Positive Numbers: Gaining weight or muscle.
        - Example: If you gain 3 kilograms, that's +3 kg.
    - Negative Numbers: Losing weight.
        - Example: If you lose 2 kilograms, that's -2 kg.
7. Elevators
    - Positive Numbers: Floors above the ground floor.
        - Example: The 5th floor of a building is +5.
    - Negative Numbers: Floors below the ground floor.
        - Example: The basement level is -1.
8. Shopping and Discounts
    - Positive Numbers: Adding items to your cart.
        - Example: If you buy 5 apples, you have +5 apples.
    - Negative Numbers: Returning items.
        - Example: If you return 1 apple, you have -1 apple.

9. Owing Money (Debt)

- Positive Numbers: Money you have.
  - Example: You have GHS 10, which is +10.
- Negative Numbers: Money you owe.
  - Example: If you owe GHS 5 to someone, that's −5.

10. Driving and Directions

- Positive Numbers: Moving forward.
  - Example: Driving 10 kilometers forward is +10 km.
- Negative Numbers: Moving backward.
  - Example: Reversing 2 kilometers is −2 km.

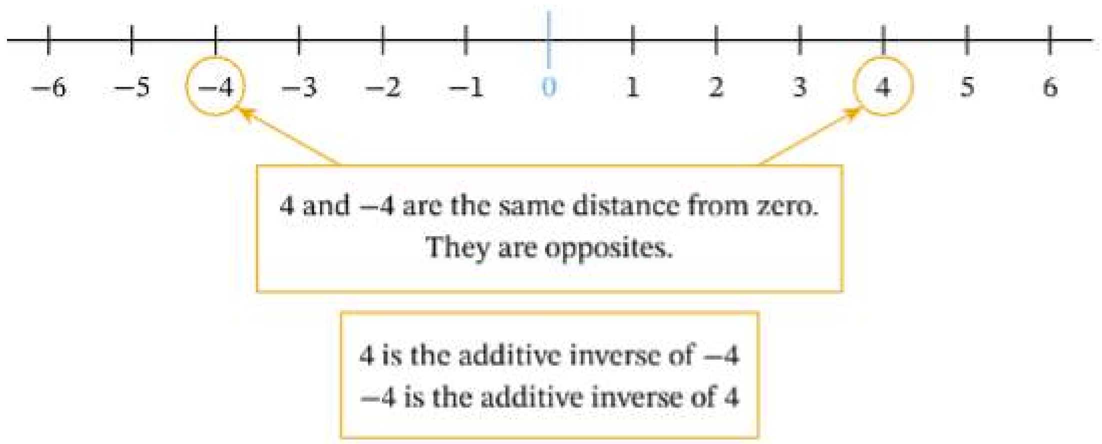
11. Test Scores

- Positive Numbers: Correct answers.
  - Example: Getting 8 questions right on a test is +8 points.
- Negative Numbers: Incorrect answers (if they reduce your score).
  - Example: If wrong answers deduct points and you get 2 wrong, you might get −2 points.

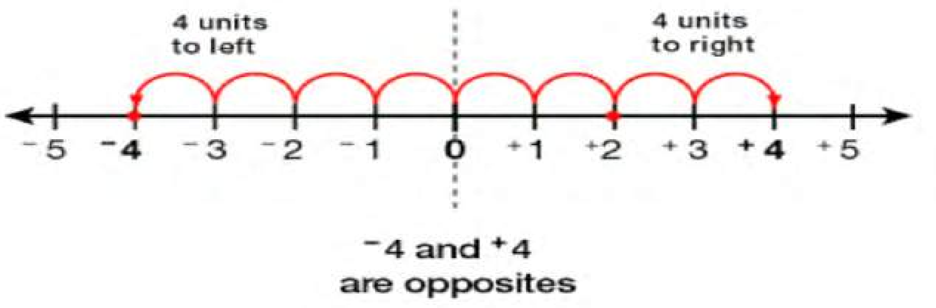
Integers

The integers are the positive counting numbers 1,2,3,4, 5…, together with their additive inverses, the negative numbers, −1, −2, −3, −4, −5…, and 0, which is neither positive nor negative.

When we plot these numbers on a number line, 0 is in the centre, the positive numbers count up to the right of zero, and the negative numbers count down to the left of zero.



Pairs of numbers like +4 and −4 are exactly the same distance from 0, but on opposite sides, so they are called "opposites"



When we measure from zero, there are always two numbers that are the same distance away but in opposite directions. We call these numbers opposites. For example,

- 1 and −1 are opposites,
- 2 and −2 are opposites, and
- 53 and −53 are opposites.

The positive integers are more than zero and the negative integers are less than zero. Therefore, integers can be used to describe real-world situations, which represent quantities that are more or less than zero.

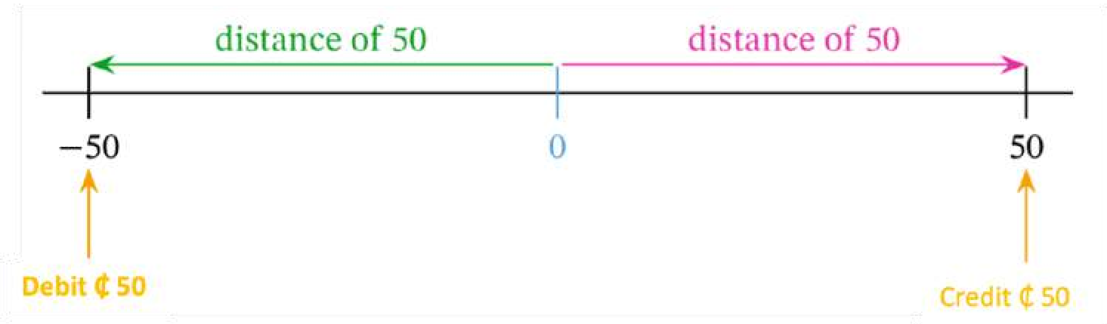
Example:

Suppose that Sarah has a bank account. If the bank account contains ₵ 50, then Sarah has ₵ 50 to spend. We can represent this amount with the positive integer 50 because she has an amount that is more than zero.

However, if her account is in debt by ₵.50, then she owes ₵.50. To represent the balance in her account, we would use the negative integer −50.

In this situation, a positive integer represents money she has or money she has gained, and a negative number represents money she has lost or money she has to pay. When we talk about the balance in a bank account, the amount can either be more than zero (having money to spend) or less than zero (owing money).

We can model this on a number line.



If the bank account is empty, the balance can be represented by 0. If she deposits ₵ 50 into the account when the balance is 0, this represents a gain of ₵ 50 that we represent with the positive integer 50. If instead she withdraws 50 from the account when the balance is zero, this represents a debt, or a loss, of ₵50 that we can represent with the negative integer −50.

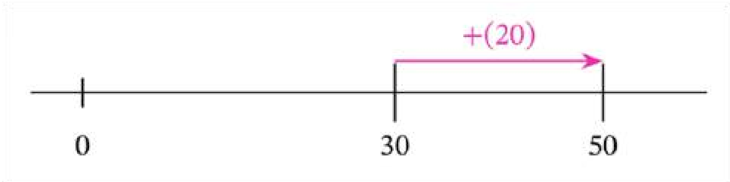
A deposit (gain) of ₵50 and a withdrawal (loss) of ₵50 are represented by integers that are the same distance from zero but in opposite directions. This is because they represent the same amount of money each time but we use either a positive or negative number to indicate whether the amount is being gained or lost.

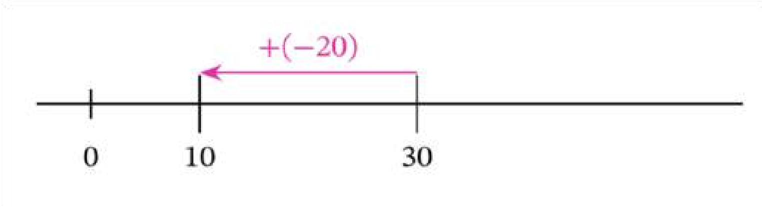
The numbers 50 and −50 are called opposites, or additive inverses, because they sum to zero. To see this, observe that a deposit of ₵50 and then a withdrawal of ₵50 takes the balance in the account back to zero. When we model this addition on a number line, we see that 50 + (−50) = 0.

Example:

Kwame is playing a game and starts with 30 points.

- If, after his first turn, his score changes by +20, what is his score after the first turn?
- If, instead, after his first turn, his score changes by −20, what is his score after the first turn?





Learning Tasks for Practice

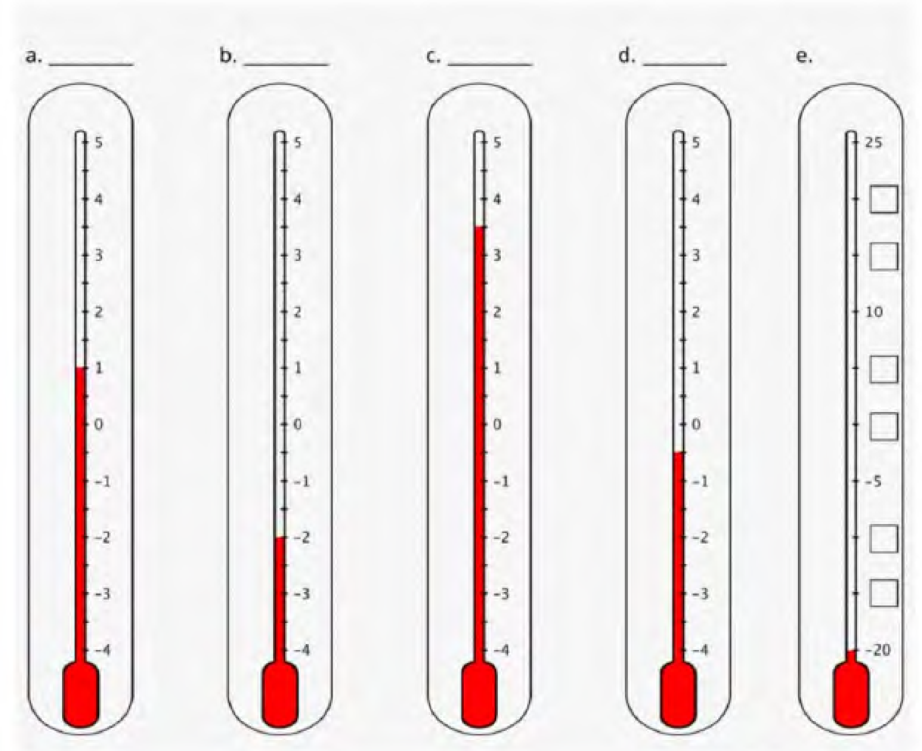
1. Learners simulate a bank account where deposits are positive numbers and withdrawals are negative numbers. They track the balance after a series of transactions.
2. Learners research different elevations, such as sea level, mountains (positive numbers), and ocean depths (negative numbers). They plot these elevations on a vertical number line.
3. Learners create scenarios where a team scores points (positive) and loses points due to penalties (negative). They determine the final score after a series of events.

Pedagogical exemplars

1. **Collaborative Learning**: Learners work in pairs to record daily temperatures over a week, then compare their results.
2. **Think-Pair-Share**: Learners individually predict temperature changes, pair up to discuss their predictions, and share with the class.
3. **Inquiry-Based Learning**: Learners explore the impact of different transactions on the balance and present their findings.
4. **Problem-based learning:** Learners create and solve problems involving real-life scenarios that use positive and negative numbers.

Key Assessment

1. What is the opposite of −5 on the number line?
2. Compare the temperatures: If one day is −5°C and the next day is 3°C, which day is warmer?
3. Calculate the new balance: If you have ₵20 and withdraw ₵5, what is your new balance?
4. Read and record the temperatures in the figure below.



Focal Area: Operations on Positive and Negative Whole Numbers (Addition and Subtraction).

In this focal area we will explore the basic arithmetic processes of addition and subtraction involving integers. We will also, delve into the rules and techniques in solving real life problems.

The result of adding two or more numbers is a **sum,** and the result of subtracting a number from another is the **difference.**

In order to add positive and negative integers, we will imagine that we are moving along that number line.

When we add a positive number to another, we start at the first number mentioned and then move to the right the amount added. Positive numbers make us move to the right side of the number line.

Examples: $4 + 3 \rightarrow$ We start at 4 and go 3 units to the right. We end up at 7, so: $4 + 3 = 7$

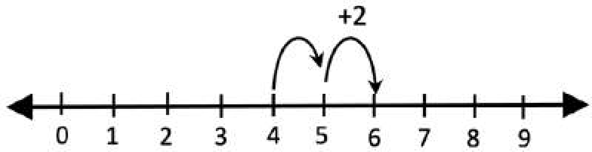
$-2 + 6 \rightarrow$ We start at -2 and go 6 units to the right. We end up at 4, so: $-2 + 6 = 4\cdot$

When we add a negative number to another, we start at the first number mentioned and then move to the left the amount added. Negative numbers make us move to the left side of the number line. Adding a negative number is the same as subtracting a positive number.

Examples:

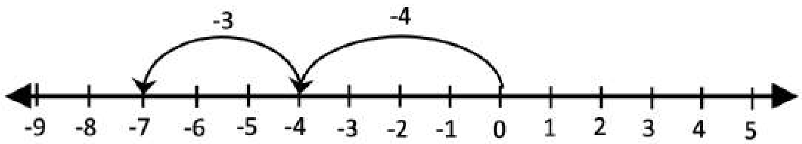
1. Kwame moved +4 steps from the starting point at 0 and then proceeded to take an additional +2 steps. What is the total number of steps he took?

$$4 + 2 = 6$$



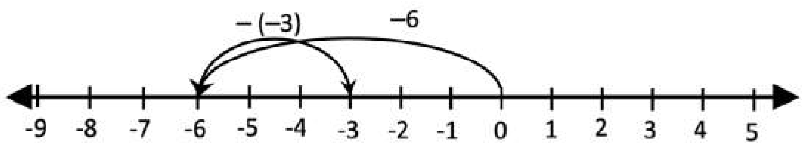
2. Alima has no money, so she borrowed *GH¢* 4.00 from her friend to buy food. She borrowed an extra *GH¢* 3.00 later, how much does she owe?

$$-4 + -3 = -7$$



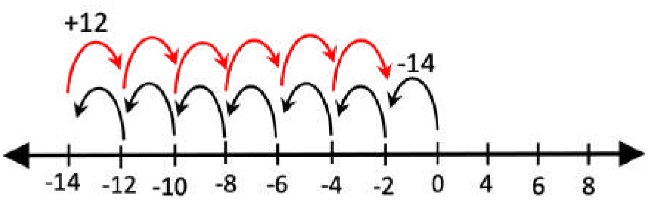
$-4 + (-3) = -7$

3. Mawuli borrowed GH¢ 6.00 from Esi, and later returned GH¢ 3.00. The expression $(-6) - (-3)$ which is $(-6) + (3)$ models this situation.



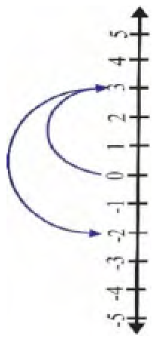
$(-6) - (-3) = -6 + 3 = -3$

4. Mensima has a jar of toffees, initially, she had 14 fewer toffees than she needed for a party. Kwesi gives her an additional 12. The expression $(-14) + 12$ models this situation.



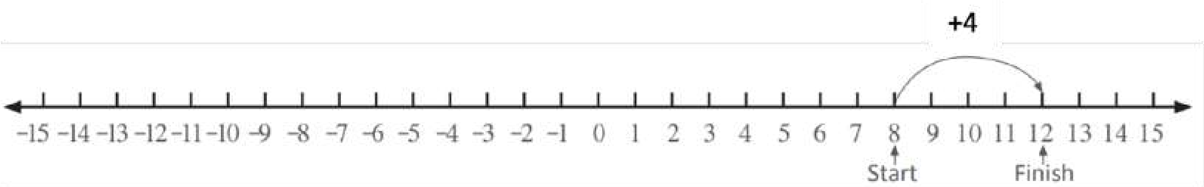
$(-14) + 12 = (-2)$

5. Find 3 +(− 5)



3 + (−5) = −2

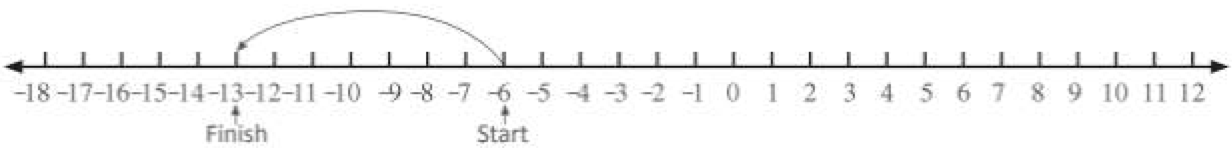
6. Find 8 + 4



Starting at +8, 4 to the right finishes at 12 right (+ 12)

8 + 4 = 12

7. Find −6 − 7



Starting at −6, then 7 to the left finishes at −13.

−6 − 7 = −13

Learning Tasks for Practice

1. Provide a series of integer addition problems for students to solve, both horizontally and vertically. Include a mix of positive and negative integers to reinforce the rules of adding integers. Limit content expectations to addition and subtraction of integers.
2. Present subtraction problems involving integers, emphasising the concept of "adding the opposite." Include scenarios where students need to subtract a negative number or subtract a positive number from a negative integer. Extend content expectations to solving word problems on addition and subtraction of integers.
3. Present word problems or real-life scenarios where students need to use integer operations to solve problems. For example, scenarios involving finances, temperatures, or distances can help students understand the practical applications of integer operations. Extend content expectations to include writing word problems on addition and subtraction on integers and solving them.

Pedagogical Exemplars

1. **Think-pair-share:** In pairs, learners engage in different problem-solving processes in numbers to perform basic operations on real numbers. Pose a question related to real number operations, such as "How would you explain adding a positive and a negative number?" Allow learners time to think individually, discuss their thoughts with a partner, and then share their ideas with the class. This encourages active engagement and provides a platform for diverse perspectives.
2. **Experiential learning**: Provide concrete manipulatives like number lines, counters, or coloured chips and ask learners to physically manipulate these objects to represent addition and subtraction of real numbers. For example, use counters to model combining positive and negative values.
3. Ask learners to perform addition by moving to the right or forward and subtraction by moving to the left or backward.
4. **Structuring Talk for learning**: Conduct structured mathematical discussions. Present a problem-solving scenario involving real numbers and guide learners in discussing their approaches, strategies, and reasoning. Encourage them to use precise mathematical language to express their thoughts.

Key Assessment

1. Using the number line and other models (coloured chips, counters), find the sum of:
   - **i.** 4 and 2
   - **ii.** 5 and – 3
2. Find the difference between:
   - **i.** 6 and – 3
   - **ii.** – 4 and – 5
3. You have *GH¢* 25.00 and spend *GH¢* 12.00. How much money do you have left?
4. The temperature was 35 degrees Celsius, and it dropped by 9 degrees. What is the new temperature?
5. The temperature was 20 degrees Celsius, and it rises by 8 degrees. What is the new temperature?

Focal Area: Operations on Positive and Negative Whole Numbers (Multiplication and Division)

Here we will explore how multiplication and division allow us to combine and separate quantities, paving the way for deeper understanding and problem-solving skills.

Multiplication is the process of grouping quantities to find their total product. We can do this by finding the total cost of items at a store, area of a table top, or determining the distance travelled by a car etc. Multiplication helps us to understand and quantify relationships between quantities.

Division, on the other hand, is the process of sharing or distributing quantities into smaller, equal groups or to find the ratio between two quantities. Division helps us in our daily activities such as sharing items equally among friends, calculating rates and proportions, and comparing quantities.

Multiplication of Integers

Example:

1. A box contains 8 pencils. If there are 5 boxes in total, how many pencils are there altogether?

8 + 8 + 8 + 8 + 8 = 40

5 groups of 8

$(8 \times 5) = 40$

This implies that, $8 + 8 + 8 + 8 + 8 = (8 \times 5) = 40$

Therefore, multiplication is a repeated addition.

2. Evaluate 14 x 3:

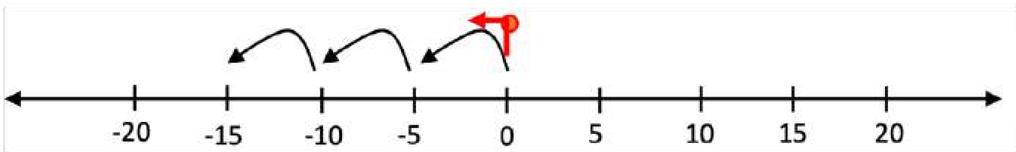
This means we have 14 + 14 +14 or 3 groups of 14 = 14 x 3 = 3 x 14 = 42

Therefore, regardless of what order the numbers are written, the product is the same.

Multiplication of negative integers using number line.

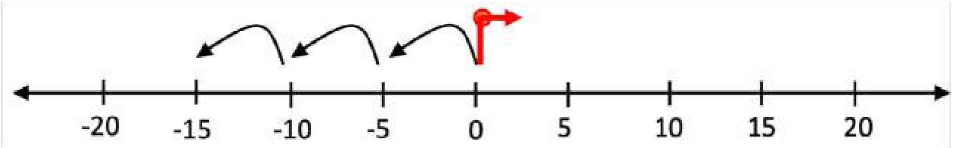
Example:

1. Evaluate -5×3



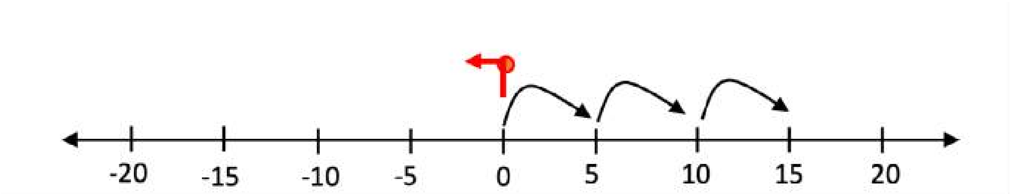
The negative attached to the number 5 indicates the direction you will face at the origin as shown in the diagram above. The 3 indicates the size of the movement from the origin in that direction. This implies that, there is a jump of each one of 5, three times, from the origin. Therefore, $-5 \times 3 = -15$.

2. Evaluate 5 × –3



In the second example, the number 5 is positive and it indicates the direction you will face at the origin as shown in the diagram above. The number 3 is now negative, and it indicates the direction of the movement from the origin. This implies that, there is a jump of each one of 5 backward (negative direction) from the origin. Therefore, 5 × –3 = –15.

3. Evaluate –5 × –3



In the third example, the number 5 is negative and it indicates the direction you will face at the origin as shown in the diagram above. The number 3 is also negative, and it indicates the direction of the movement from the origin. This implies that, there is a jump of each one of 5 backwards (negative direction) from the origin. Therefore, –5 × –3 = 15.

4. If the cost of a notebook is GH¢ 13.00. How much would you pay for, if you buy 16 of them?

Solution

13 × 16 = 208

From the above illustrations, the following rules of multiplication can be deduced.

Rules of multiplication

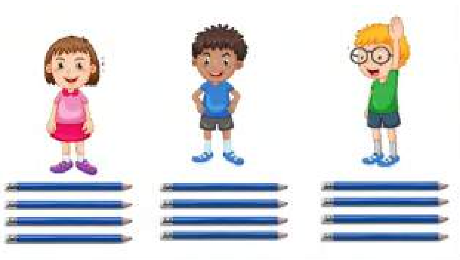
In multiplication, the sign of the product depends on the signs of the numbers being multiplied.

- Positive × Positive = Positive (e.g., 3 × 4 = 12)
- Negative × Negative = Positive (e.g., –5 × –2 = 10)
- Positive × Negative = Negative (e.g., 2 × –7 = –14)
- Negative × Positive = Negative (e.g., −2 × 7 = –14)

Division of Integers

Remember that division is the process of sharing or distributing quantities into smaller groups.

a. Kofi is to share 12 pencils among his three friends. How many pencils will each receive?



From the illustration, each friend receives 4 pencils. Therefore, 12 ÷3 = 4

b. Your class is going on a field trip and the bus can hold 36 passengers There are 108 students in your class. How many buses will be needed to transport all the students?

Solution

This is a division problem: 108 ÷36 = 3

c. A household owes electricity bill of GH¢450.00. This amount was shared among three members equally, how much will each tenant pay?

Solution

Since this is a debt, the amount owe will be written as – 450. Therefore, each member will pay an amount of – 450 ÷ 3 = − 150, which is a debt and can be written as – 150.

d. Again, you owe a friend GH¢30, and you owe another friend GH¢10. "What happens if you divide your total debt (–GH¢40) equally between these two friends?" To divide the total debt (–GH¢40) by the number of friends (–2) gives a positive result, indicating each friend receives a share of the debt.

Mathematically, – 40 ÷ – 2 = 20.

Rules of division

In division, the sign of the division depends on the signs of the numbers being divided.

- Positive ÷ Positive = Positive (e.g., 12 ÷ 4 = 3)
- Negative ÷ Negative = Positive (e.g., –20 ÷–2 = 10)
- Positive ÷ Negative = Negative (e.g., 28 ÷ –2 = –14)
- Negative ÷ Positive = Negative (e.g., −28 ÷ 2 = –14)

Learning Tasks for Practice

1. Use the number line to solve problems on multiplication and division of integers. Solve word problems involving multiplication and division of integers. Model word problems on multiplication and division of integers.
2. Create problem-solving tasks based on real-world contexts, such as budgeting, temperature changes, or distance calculations. Ensure that scenarios are relatable and culturally relevant to students from different backgrounds

Pedagogical Exemplars

1. **Initiating talk for learning:** In a whole class discussion, explore concepts of the operations of integers using multiplication and division.
2. **Collaborative learning:** In mixed ability/gender groups of five, use models to explore the multiplication of integers.

   E.g., pencils, coloured chips, number line.
3. **Managing Talk for learning:** In a whole class discussion, use the number line to investigate the rules of multiplication and division such that:

   a. Positive x Positive = Positive

   b. Negative x Negative = Positive

   c. Positive x Negative = Negative

   d. Negative x Positive = Negative
   e. Positive ÷ Positive = Positive
   f. Negative ÷ Negative = Positive
   g. Positive ÷ Negative = Negative
   h. Negative ÷ Positive = Negative

4. **Experiential learning:** teacher gives 12 pencils to a learner to share among three friends from the larger group. Learners then discuss the number of pencils each receives.
5. **Structuring Talk for learning:** in a whole class discussion create a scenario where a household owes electricity bill of GH¢450.00, and that this amount was shared among three members equally, how much will each member pay? Discuss with learners how the debt of GH¢450.0 will be negative and will be divided by positive 3
6. **Problem based learning:** Put learners in their mixed ability gender groups and create another scenario of a learner owing a friend GH¢10 (negative debt). If he/she pays the friend back GH¢10 (negative payment), The learner's debt is settled (positive outcome), think, ink, and share your ideas.

Key Assessment

1. **Assessment Level 1**
   a. A factory produces 240 toys in 6 hours, how many toys are produced in one hour?
   b. A company has 180 employees and wants to form 6 teams. How many employees will be in a team?
2. **Assessment Level 2**
   a. The cost of a book is GH¢ 3.00. A student bought 24 books. How much will the student pay for all the books.
   b. You have 72 m^2 of a wall to paint, and each bucket of paint covers 8 m^2. How many buckets of paint do you need to buy?
   c. If you are in a group of 5 students and you are given GH¢.125.00 to share equally, how much will you receive?
3. **Assessment Level 3**

   Evaluate the following operation and explain the relationship between multiplication and division.

Multiplication	Division

Section Review

In this section, you explored key mathematical concepts that are essential for solving problems in everyday life. In this section:

1. You learned how to use mental strategies and number properties to efficiently solve problems involving the four basic operations—addition, subtraction, multiplication, and division. These strategies help you make quick calculations, improving your problem-solving speed and accuracy in both academic and real-world contexts.
2. **Interpreting Negative and Positive Numbers**: You developed a deeper understanding of how to interpret and use positive and negative numbers in various contexts. This knowledge is crucial for situations involving temperature changes, financial transactions, or any scenario where numbers represent values that can go above or below zero.
3. **Understanding Fractions and Their Real-Life Applications**: You gained knowledge of fractions, including how to identify, compare, and operate with them. You also explored how fractions are used in real life, such as dividing a piece of bread among friends, measuring ingredients in a recipe, or understanding discounts during sales.

This section has equipped you with practical skills to handle numerical challenges in daily life and future mathematical learning.

SECTION 3: REASONING WITH ALGEBRA

Strands: **Algebra**

Sub-Strand: Patterns and Relations; Algebraic Expressions; Variables and Equations

Content Standards

1. Determine the pattern rule to make predictions about subsequent elements.
2. Demonstrate understanding of algebraic expressions
3. Solve problems involving single variable, one-step equations with whole number coefficients

INTRODUCTION AND SECTION SUMMARY

Understanding patterns and algebraic expressions is crucial for developing mathematical thinking. Representing and extending patterns visually, describing patterns using mathematical language, and solving problems using pattern rules enable students to recognise and predict regularities. Modeling real-life situations as mathematical statements helps students connect mathematics to their everyday experiences. Performing basic operations on algebraic expressions, including addition, subtraction, and multiplication, builds a foundation for more advanced algebraic concepts. These skills are essential for students to analyse and interpret data, solve complex problems, and understand the underlying structures in mathematics.

The section will cover the following concepts:

1. *Representing and extending a given pattern visually, and explain how each element differs from the preceding one.*
2. *Describing, orally or in writing, a given pattern (rule), using mathematical language and predict subsequent elements in the pattern.*
3. *Solving a given problem (including tables/charts) using a pattern rule to determine subsequent elements (predictions).*
4. *Modelling real-life situations as mathematical statements*
5. *Performing basic operations (add, subtract and multiplication) on algebraic expressions.*

SUMMARY OF PEDAGOGICAL EXEMPLARS

Learners will benefit from diverse instructional strategies to master these concepts effectively.

1. **Pattern Representation and Extension:** Use visual aids and manipulatives to help students represent and extend patterns. Encourage students to explain how each element in the pattern differs from the preceding one, fostering their ability to recognise and articulate patterns.
2. **Describing Patterns:** Provide opportunities for students to describe patterns orally and in writing using mathematical language. Use interactive activities where students predict subsequent elements in a pattern based on identified rules.
3. **Problem Solving with Patterns:** Engage students in problem-solving tasks that involve using pattern rules to determine subsequent elements. Incorporate tables and charts to help students organise information and make predictions.

4. **Modeling Real-Life Situations:** Use real-life examples to illustrate how mathematical statements can model situations. Encourage students to translate everyday scenarios into mathematical language, enhancing their understanding of the practical applications of mathematics.
5. **Operations on Algebraic Expressions:** Teach students to perform basic operations on algebraic expressions through step-by-step instruction and practice exercises. Use real-life contexts to show the relevance of algebraic operations.
6. **Collaborative Learning:** Promote group activities where students work together to represent patterns, describe rules, solve problems, model situations, and perform operations on algebraic expressions. This fosters teamwork, critical thinking, and communication skills.

ASSESSMENT SUMMARY

Assessments for these concepts should be varied and comprehensive, addressing different cognitive levels.

1. **Class Exercises and Tests:** Assess students' ability to represent and extend patterns visually, describe patterns using mathematical language, and predict subsequent elements. Include tasks that require performing basic operations on algebraic expressions.
2. **Pattern-Based Problems:** Present problems that involve using pattern rules to determine subsequent elements. Use tables and charts to test students' organisational skills and ability to make predictions.
3. **Real-Life Modeling:** Provide real-life scenarios and ask students to model these situations as mathematical statements. Assess their ability to connect mathematical concepts to practical applications.
4. **Collaborative Projects:** Engage students in group tasks where they represent and describe patterns, solve pattern-based problems, model real-life situations, and perform operations on algebraic expressions. Assess their teamwork, problem-solving, and communication skills.
5. **Presentations:** Have students present their solutions to pattern problems, real-life models, and algebraic operations. This assesses their understanding, ability to communicate mathematical concepts, and application of learned skills to different contexts.

Week 6: Extending Patterns Visually

Learning Indicators

1. *Represent and extend a given pattern visually, and explain how each element differs from the preceding one.*
2. *Describe, orally or in writing, a given pattern (rule), using mathematical language and predict subsequent elements in the pattern.*

Focal Area: Represent and Extend a Given Pattern Visually

Patterns are arrangements that follow a particular rule or set of rules. They can be found in numbers, shapes, and daily activities. Patterns can be visual, numerical, or symbolic. Visual patterns involve shapes or objects arranged in a particular order. Numerical patterns involve sequences of numbers that follow a specific rule, such as increasing by a certain amount each time. Symbolic patterns use letters to represent elements in the pattern. Patterns help develop critical thinking and problem-solving abilities, which are essential for academic success and everyday life.

This week, we will learn how to identify and understand these rules helping learners predict what comes next, solve problems, and apply these skills in real-life situations. Through, a combination of visual aids, hands-on activities, and clear explanations, aiming to make learning about patterns is an engaging and rewarding experience for learners.

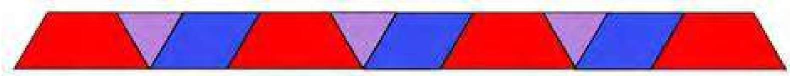
Identifying and understanding and identifying patterns

All around us, we find various real-life objects arranged in order. Below are some real-life objects, let's take a look at these and identify and discuss the meaning of patterns. Discuss the different types of patterns in each object.



A pattern is a repeated or regular arrangement of numbers, shapes, or objects. They follow a specific rule or set of rules that determine the order and structure of the elements.

Examples of patterns

1. 

   This is a pattern because it follows a repeating cycle of colors. Thus the colors "red, violet, and blue" form a cycle that starts over after every three elements.

2. 3, 6, 9, 12, 15
   This is a pattern because each number is obtained by adding 3 to the previous number.

3. Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday

   This is a pattern because it represents the continuous and repeating cycle of the days of the week.

Types of patterns.

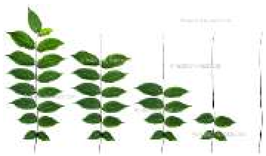
1. Repeating patterns: arrangements that follow a specific rule and repeat themselves at regular intervals.

Visual representation of repeating patterns in the environment

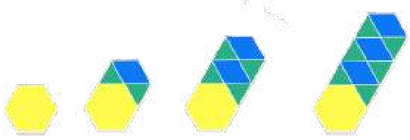



Growing patterns: arrangements where each subsequent element in the sequence builds upon or increases from the previous elements according to a specific rule.

Visual representation of growing patterns in the environment







Learning Task for Practice

1. Learners discuss and present various objects in the school and explain their patterns.
2. Learners discuss the various types of patterns and give examples.
3. Learners discuss and present various objects in the environment in terms of visual, numerical, or symbolic patterns.
4. Learners discuss and give examples of growing and repeating patterns.

Pedagogical Exemplars

1. **Experiential learning**: In collaborative and mixed-gender/ability groups, engage learners to explore and identify examples of patterns in the environment.
2. **Project/Collaborative learning:** Engage learners to use the explored patterns to discuss the types of patterns.
3. **Collaborative learning**: using mixed-gender/ability grouping, learners investigate discuss and make presentations on various objects in their immediate environment in terms of visual, numerical or symbolic patterns.
4. **Problem-based learning**: in groups/pairs, engage learners to take photos of patterns around them and create a collage or presentation.

Focal Area: Describe Patterns Orally Using Mathematical Language and Extend The Patterns

Describing patterns is important for learners to understand and communicate mathematical ideas. This activity focuses on using mathematical language to describe patterns, helping learners articulate their observations and reasoning orally or in writing.

Reinforcement activities:

- Describing patterns orally
- Extending and explaining the patterns.

Describing patterns orally

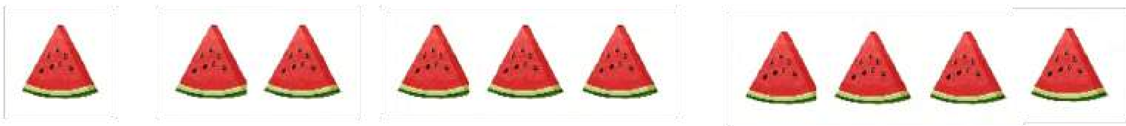
Examples

Describe the following patterns orally

1.

The pattern alternates between 2 peppers and 1 tomato, repeating every four items.

2.



The pattern starts with one slice of watermelon and adds one more slice with each step. The pattern begins with a single slice of watermelon, two slices of watermelon, three slices of watermelon, and four slices of watermelon.

3. 2, 4, 6, 8, 10

The pattern starts with 2, and each number is obtained by adding 2 to the previous one

4. 40, 20, 10, 5

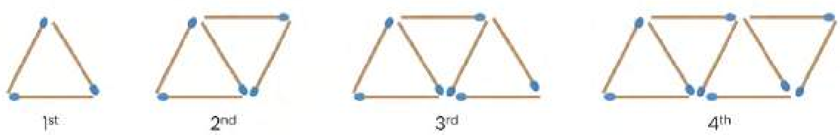
Each number in this pattern is obtained by dividing the previous number by 2.

Extend a given pattern.

Visual, numerical, or symbolic patterns can be extended based on how each number relates to the previous one.

Examples

1. Study the pattern and find the number of sticks that could be used in the 5th term.



Solution:

Look at the number of sticks in each term.

- 1st term: 3 sticks
- 2nd term: 5 sticks
- 3rd term: 7 sticks
- 4th term: 9 sticks

Observe how the number of sticks changes from one term to the next.

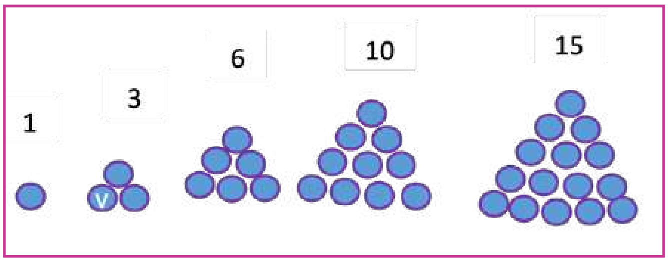
- From 3 to 5, the number of sticks increases by 2.
- From 5 to 7, the number of sticks increases by 2.
- From 7 to 9, the number of sticks increases by 2.

This shows that each term increases by 2 sticks.

To find the 5^{th} term, start with the 4^{th} term, which has 9 sticks and add 2 sticks to the 4^{th} term to get the 5^{th}

Therefore, the number of sticks used in the 5^{th} term is **11**.

2. Write the next term in the pattern below



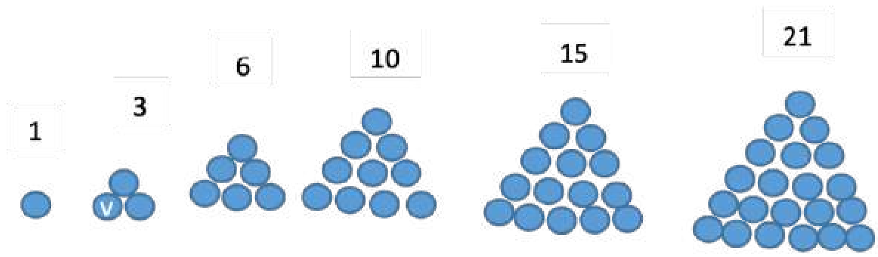
Solution:

Each number in the pattern is the sum of the first natural numbers. Therefore, we add the next natural number to the last number in the pattern to obtain the next number.

Thus,

- The first number is 1.
- The second number is $1 + 2 = 3$.
- The third number is $1 + 2 + 3 = 6$.
- The fourth number is $1 + 2 + 3 + 4 = 10$.
- The fifth number is $1 + 2 + 3 + 4 + 5 = 15$.
- The sixth number is $1+ 2+ 3+ 4+ 5+ 6 = 21$

Therefore, the pattern extends to 21



3. Write the 4th and 5th terms of the patterns 3, 9, 27, ...

Solution:

Look at the sequence: 3, 9, 27, ...

First term: 3

Second term: 9

Third term: 27

To find the next number in the pattern, we must see how we get from one number to the next.

- To go from 3 to 9, we multiply by 3: $3 \times 3 = 9$
- To go from 9 to 27, we multiply by 3 again: $9 \times 3 = 27$

So, this shows that each term is obtained by multiplying 3 by the previous term.

To find the 4th term, multiply the 3rd term, 27 by 3.

Thus, $27 \times 3 = 81$

So, the 4th term is **81**.

To find the 5th term, multiply the 4th term, 81 by 3.

Thus, $81 \times 3 = 243$

So, the 5th term is **243**.

Learning Task for Practice

1. Learners are tasked to investigate how each number relates to the previous number in a given pattern.
2. Learners are tasked to discuss and write the next term or extend a given number.

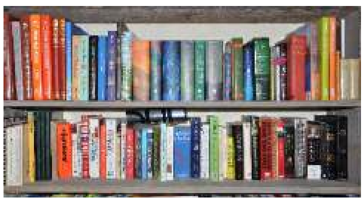
Pedagogical Exemplars

1. **Initiating talk for learning:** In a whole class discussion explain the key terms and encourage the learners to use the correct in describing the patterns orally and in writing.
2. **Inquiry-Based Learning:** In collaborative and mixed-gender/ability groupings, engage learners to investigate how each number relates to the previous one in a given pattern.
3. **Project/Collaborative learning**: Engage learners to discuss, explain and extend a given pattern based on how the numbers relate.

Key Assessment

1. Identify the growing patterns in the following real-life objects.



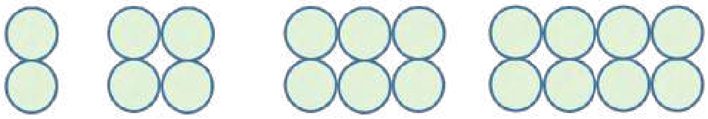



2. Describe the following patterns.

i. 55, 65, 75, 85

ii. 2, 8, 14, 20, 26,

iii.



3. Study the patterns carefully and complete them.

i. 1, 3, 6, 10, ___, ___.

ii. 10, 100, 1000, 10000, …., ……

4. Mina increases her savings by ¢5 each week:

Week 1: ¢5 Week 2: ¢10 Week 3: ¢15

The pattern continues with an increase of ¢5 each week.

How much will be saved in week 6?

6. A tile floor has a pattern of red, blue, red, blue, and so on. If the pattern is extended for the next four tiles, the sequence will be red tile, blue tile, red tile, blue tile.

a. The relationship is to alternate between red and blue tiles. True / False

b. The tile floor has a growing pattern. True / False

7. The schedule of examination timings for some candidates is given below.
What is the time of the sixth examination if the pattern continues

Examination:	1st	2nd	3rd	4th	5th
Time:	1:00 pm	1:35 pm	2: 10 pm	2:45 pm	3:20 pm

Week 7: Using Pattern Rules to Solve Problems

Learning Indicators

1. *Solve a given problem (including tables/charts) using a pattern rule to determine subsequent elements (predictions).*
2. *Model real-life situations as mathematical statements.*

Focal Area: Applying Pattern Rules to Solve Problems and Make Predictions

Patterns help us to understand how different elements relate to each other in a sequence. Identifying the rule that governs the pattern, helps predict what comes next and solve various real-life problems that involve patterns. For instance, you may use pattern rules to solve puszles, identify consistencies in real-life situations, or predict future trends in data.

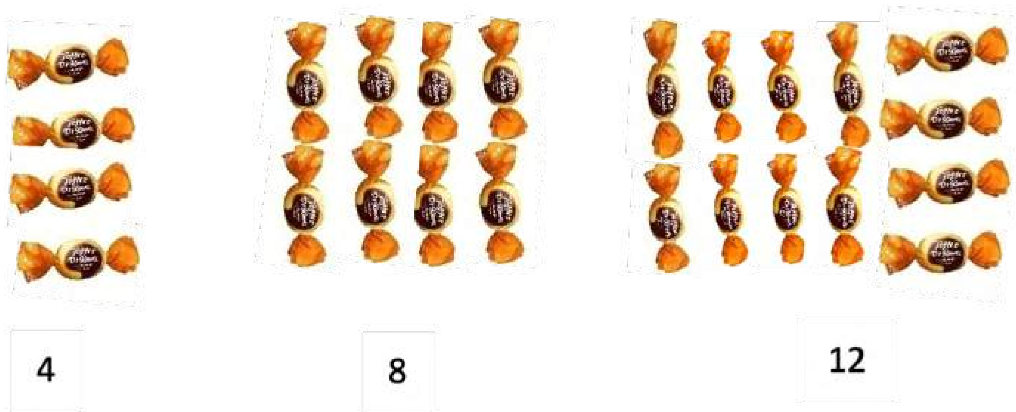
This area focuses on analysing and solving problems that involve patterns, including using tables and charts to help find and apply pattern rules.

Identify and write down the rule that describes the pattern and predicts the subsequent elements.

Identifying and writing the rule that defines a given pattern involves looking at the sequence of numbers, shapes, or symbols to notice any regularity or repetition. Find the relationship by determining how each element changes from one to the next. This could involve addition, subtraction, multiplication, or another operation. Write down the rule that describes this relationship.

Examples

1. Write the rule for the following sequences and predict the next two numbers.



Solution:

First we must find the relationship:

The difference between consecutive terms:

$8 - 4 = 4$

$12 - 8 = 4$

The difference between each consecutive pair of number of toffees is 4.

The rule: We can see that this means that the rule is to add 4 to the previous number of toffees to find the number of toffees in the next term.

Thus,

First number of toffees = 4

Second number of toffees = *first number of toffees* + 4 = 8

Third number of toffees = *second number of toffees* + 4 = 12

Therefore, the next two terms:

Fourth number of toffees = *third number of toffees* + 4 = 16

Fifth number of toffees = *fourth number of toffees* + 4 = 20

2. 1, 3, 5, 7

Solution:

Find the relationship by identifying the difference:

$3 - 1 = 2$

$5 - 3 = 2$

$7 - 5 = 2$

The difference between each consecutive pair of numbers is 2

The rule: From the above, the rule is to add 2 to the previous number to get the next number.

Therefore,

First number = 1

Second number = 1 + 2 = 3

Third number = 3 + 2 = 5

Fourth number = 5 + 2 = 7

Therefore, the next two terms are:

7 + 2 = **9**

9 + 2 = **11**

Learning Task for Practice

Learners are tasked to investigate and write the rule for a given pattern.

Pedagogical Exemplars

Inquiry-Based Learning: In collaborative and mixed-gender/ability groupings, engage learners to investigate and write the rule for a given pattern.

Apply these skills to solve real-life problems involving patterns, including tables and charts.

Examples

1. Use the table below to find the number of bananas when $x = 6$ *and* $x = 7$

Pattern Number, x	**Number of bananas**
1	
2	
3	
4	
5	

Identify the Pattern:

Look at how the number of bananas changes as the x values increase.

From $x = 1$ to $x = 2$, the number of bananas changes from 3 to 5 (an increase of 2 bananas).

From $x = 2$ to $x = 3$, the number of bananas changes from 5 to 7 (an increase of 2 bananas).

From $x = 3$ to $x = 4$, the number of bananas changes from 7 to 9 (an increase of 2 bananas).

From $x = 4$ to $x = 5$, the number of bananas changes from 9 to 11 (increase of 2 bananas).

Rule: Each time x increases by 1, the number of bananas also increases by 2.

Therefore, the number of bananas when the x is 6:

If the pattern continues, when x increases from 5 to 6, the number of bananas should increase by 2.

Number of bananas for $x = 5$ is 11.

Add 2 to the last number of bananas

Thus, when $x = 6$, $11 + 2 =$ **13**.

Therefore, the number of bananas when the x is 7:

If the pattern continues, when x increases from 6 to 7, the number of bananas should increase by 2.

The number of bananas for $x = 6$ is 13

Add 2 to the last number of bananas

Thus, when $x = 7$, $13 + 2 =$ **15**.

2. The chart below shows the monthly sales of a product. Analyse the chart and predict the sales for July and August.

Month	Sales (units)
January	35
February	40
March	45
April	50
May	55
June	60

Solution:

Pattern Rule: Sales increase by 5 units each month.

Prediction for July

Add 5 to the sales in June (60)

Next month's sales: 60 + 5 = 65

The prediction for sales for July will be **65 units**.

Prediction for August

Add 5 to the sales in July (65)

Next month's sales: 65 + 5 = 70

The prediction for sales for August will be **70 units**.

Note, sales are notoriously hard to predict and patterns could stop at any time, so this is only a prediction and must be compared with reality to see if the pattern remains true.

3. A student collects books, starting with 7 books and adding 3 more books to the collection each month. Identify the pattern, write the rule and use it to find the first 7 sequences.

Solution

The student starts with 7 books, means the pattern start with 7 books.

Adds 3 more books each month.

The rule: the rule is obtained by adding 3 to the number of books from the previous month.

That is,

Starting month: 7 books

Second month: 7 + 3 = 10 books

Third month: 10 + 3 = 13 books

Fourth month: 13 + 3 = 16 books

Fifth month: 16 + 3 = 19 books

Sixth month: 19 + 3 = 22 books

Seventh month: 22 + 3 = 25 books

So, the first seven sequences are: 7, 10, 13, 16, 19, 22, 25

Learning Task for Practice

1. Learners are tasked to analyse and write the rule for a given real-life problem.
2. Learners are tasked to apply the rule in solving real-life problems.

Pedagogical Exemplars

1. **Experiential learning**: In collaborative and mixed-gender/ability groups, engage learners to analyse and write the rule for a real-life scenario.
2. **Project/Collaborative learning:** Engage learners to apply the rule to predict the subsequent element in the analysed real-life pattern.

Key Assessment

1. Identify the rule and give the next three numbers for the following patterns
   - **i.** 3, 7, 11, 15, ...
   - **ii.** 5, 10, 20, 40, ...
2. The following table shows the population of a town over five years. Identify the pattern and predict the population for Year 6 and Year 7.

Year	Population
1	1,000
2	1,200
3	1,400
4	1,600
5	1,800

3. Bryan is following a weekly exercise routine where he increases the number of minutes he exercises each day. On the first day, he exercises for 10 minutes. Each subsequent day, he adds 5 more minutes to his workout time.
   - **i.** Identify the pattern in Bryan's daily exercise routine.
   - **ii.** Describe the rule that governs the pattern.
   - **iii.** Predict how many minutes Bryan will exercise on the 7th day.
4. In a classroom, the desks are arranged in rows. Each row has two more desks than the previous row. The first row has 3 desks. Find the rule for the pattern and use it to predict the first 4 terms.

Focal Area: Translating Real-Life Situations into Expression

Translating real-life situations into algebraic expressions enables us to model, analyse, and solve more complex problems using variables and constants. While recognising patterns helps identify relationships and predict outcomes, translating real-world scenarios into expressions helps to understand and describe the relationships between the variables and the constants. This process is fundamental for creating mathematical models to address real-life problems effectively.

Identification of quantities, relationships, and operations in real-life contexts.

Examples:

Identify the quantities and the relationships in the following examples.

1. A shopper buys 3 bags of rice, each weighing 2 kg

Solution:

Quantities

3 bags at 2 kg per bag

Relationship: The relationship is that the total weight of the rice is the product of the number of bags and the weight of each bag.

In this case, total weight = 3 bags × 2 *kg* per bag

Operation: total weight = $3 \times 2 = 6kg$

2. A recipe requires 160 grams of sugar for 4 servings.
   How much sugar is required per serving?

Solution:

Amount of Sugar for 4 Servings =160 grams

Relationship:

Amount of Sugar per Serving = Total Amount of Sugar ÷ Number of Servings

= 160÷ 4 = 40grams per serving

So, the relationship is that each serving requires 40 grams of sugar. This tells how much sugar is needed for each individual serving based on the total amount for the recipe.

Translate the relationships between the elements into mathematical expressions

Examples:

Explore and write the mathematical statement or expression for the following real-life scenarios.

1. Sarah saves GH¢ 15 every week. How much will she have saved after *w* weeks?

Solution:

Sarah saves GH¢ 15 every week.

Relationship: Total savings is the weekly savings multiplied by the number of weeks.

Here, S represents the total savings after w weeks.

Therefore the expression for total saving is $S = 15w$

2. A taxi service charges a flat fee of GH¢ 5 plus GH¢ 2 per mile travelled.

   How much will the fare be for a trip of m miles?

Solution:

Total fare for the trip $=F$

The flat fee charged by the taxi service =GH¢ 5

Charge per mile travelled =*GH¢* 2

Number of miles travelled $=m$

Relationship: Total fare is the sum of the flat fee plus the charge per mile.

Here, F represents the total fare for m miles.

$\boldsymbol{F} = 5 + 2\boldsymbol{m}$

So, if the number of miles (m) is known, the total fare (F) can be calculated by adding the flat fee of GH¢ 5 to twice the number of miles travelled.

3. A person planning to buy a book that costs *GH¢ m* but has a coupon for a *GH¢* 4 discount. Write an expression to represent the final cost after applying the discount.

Solution:

The final cost of the book after the discount = C

The original cost of the book = m

Value of the discount coupon = 4

The relationship between the final cost (C) and the original cost (m) can be expressed as

$C = m - 4$

So, if the original cost of the book (m) is known, the final cost (C) can be calculated by subtracting 4 from the original cost.

Note: All the mathematical expression in the above examples are called algebraic expressions.

Learning Task for Practice

1. Learners are tasked to investigate and write the quantities, relationships and operations in a given real-life scenario.
2. Learners are tasked to translate the relationships between the quantities into mathematical expressions and present their results.

Pedagogical Exemplars

1. **Inquiry-Based Learning:** In collaborative and mixed-gender/ability groupings, engage learners to investigate and write the quantities, relationships and operations in a given real-life scenario.
2. **Collaborative learning**: Engage learners to discuss, explain and translate the relationships between the quantities into mathematical expressions and present their results.

Key Assessment

1. A student spends 45 minutes reading and 30 minutes exercising every day.
   - **i.** How many minutes does the student spend reading each day?
   - **ii.** How many minutes does the student spend exercising each day?
   - **iii.** How many total minutes does the student spend on both activities each day?
   - **iv.** How many total minutes does the student spend on both activities in a week?
2. Madam Ellen's monthly budget includes GH¢ 70 for food, GH¢ 100 for utilities and GH¢ 50 for entertainment.
   - **i.** How much money is set aside for food?
   - **ii.** How much money is set aside for utilities?
   - **iii.** How much money is set aside for entertainment?
   - **iv.** What is the total amount of money spent on three in one month?
   - **v.** What is the total amount of money spent on all three in one year?
3. There are p pencils in one box and q pencils in another box.
4. Write an expression to represent how many pencils are there altogether.
5. Sarah buys packs of stickers, each pack containing y stickers.
6. How many stickers does she buy in total?
7. John has 4 more marbles than Tom.
8. Write an expression to represent the number of marbles John has.
9. A store sells n notebooks for GH¢ 2 each and p pencils for GH¢ 1 each.
10. Write an expression to represent how much it will cost to buy n notebooks and p pencils.

Week 8: Operations on Algebraic Expressions

Learning Indicator: *Perform basic operations (add, subtract and multiplication) on algebraic expressions (I)*

Focal Area: Perform Basic Operations on Algebraic Expressions – Addition, Subtraction & Multiplication

Algebraic expressions are essential components of algebra. They consist of variables, constants, and operations that combine to represent mathematical relationships and real-life scenarios. Understanding how to perform basic operations on these expressions is essential for solving equations, modelling situations, and analysing data.

Simplifying Expressions (addition and subtraction).

Example:

1. There are 2 boys in the classroom and 3 more boys join.
2. Find the number of boys in the classroom.
3. In another classroom there are 2 girls and 2 boys.
4. How many boys and girls are in the classroom?

Solution:

1.

From the picture above, we can add 2 boys and 3 boys together to get 5 boys because they are of the same group. Therefore, objects of the same group are called like objects and they can be added together.

Mathematically, we can use variables to represent boys. Thus let x represent boys

Then, 2 boys and 3 more boys joined is the same as $2x + 3x$. This expression $2x + 3x$ is called algebraic expression. The number attached to any variable in algebraic expression is called the **coefficient.** Therefore 2 and 3 are the coefficients of the variable x.

The expression $2x + 3x$ has two terms. Thus $2x$ as the 1st term and $3x$ as the 2nd term. These terms have the same variable and are separated by the operation addition. Therefore, terms of the same variable in an expression are called **like terms**.

Just like we added boys with boys, in algebra, we add the coefficient of the like terms together by adding the like terms and attach one of the variables.

Thus, $2x + 3x = 5x$

2.

From the picture above, we cannot add 2 girls and 2 boys because they are different groups. Therefore, objects of different groups are called **unlike objects**.

Mathematically, we can use variables to represent girls and boys. Thus let x represent boys and y represent girls.

Then, 2 girls and 2 more boys joined is the same as $2x + 2y$.

Algebraic expression: $2x + 2y$

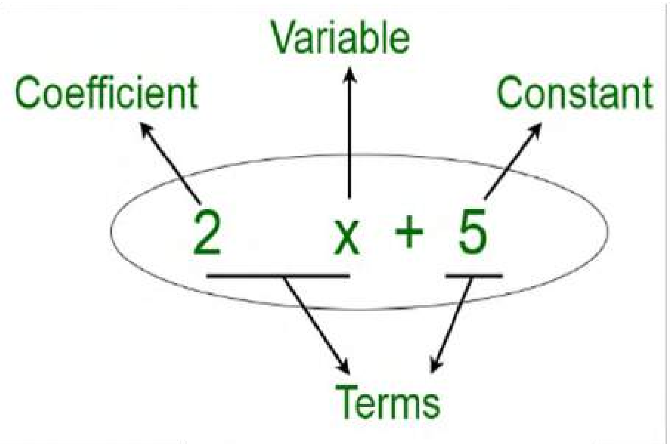
Coefficients for both x and y are 2

The expression $2x + 2y$ has two terms. Thus $2x$ as the 1st term and $2y$ as the 2nd term. These terms have different variable and are separated by the operation addition. Therefore, terms that have different variables in an expression are called **unlike terms**.

Just like we cannot add girls with boys in real life, in algebra, unlike terms cannot be added together.

Thus, $2x + 2y = 5x$

Note: all the terms in an expression without variables are called constants and are simply a number.



Algebraic expression

Examples:

1. Simplify $4a + 7 - 2a + 3$

Solution:

$4a + 7 - 2a + 3 = 4a - 2a + 7 + 3 = 2a + 10$

2. Simplify $6m - 2m + 8$

Solution:

$6m - 2m + 8 = 4m + 8$

3. Simplify $3ab - 2cd + ab + 5cd$

Solution:

$3ab - 2cd + ab + 5cd = 3ab + ab - 2cd + 5cd = 4ab + 3cd$

Multiplication of algebraic expressions

Multiplying a Variable by a Number

Examples:

1. Multiply $2x \times 3$

Solution:

Use the coefficient of x to multiply the constant number.

Thus $2 \times 3 = 6$

The variable x stays the same because you are only multiplying the numbers:

Thus $2x \times 3 = 6x$

2. Multiply 5 by 3p

Solution:

$5 \times 3p = 15p$

Multiplying a Variable by Another Variable

3. What is $2x \times 3y$

Solution:

Multiply the coefficients

Thus is $2 \times 3 = 6$

Multiply the variables

$x \times y = xy$

Combine these results:

Thus $2x \times 3y = 6xy$

4. Multiply $10a \times 4b$

Solution:

$10a \times 4b = 40ab$

Focal Area: Translating and Performing Operations on Real-Life Applications of Algebraic Expressions.

Examples

Create and simplify algebraic expressions from the following real-life scenarios.

1. Ohemaa has x mangoes and buys 7 more.
2. How many mangoes does she have in total?

Solution:

Ohemaa initially has x mangoes.

She buys 7 more mangoes.

The expression gives the total number of mangoes Ohemaa has which is $\boldsymbol{x} + 7$

So, if x represents the number of mangoes Ohemaa initially has, the total number of mangoes she has after buying 7 more is $x + 7$

3. A rectangle has a length of $2t$ and a width of 4.
4. What is the product of the length and width?
5. (And this is the same as the area of the rectangle.)

Solution:

Quantities:

Length = $2t$

Width = 4

To find the product of the length and the width of the rectangle, we need to multiply the given expressions for the length and width

The product of length and width is

$2t \times 4 = 8t$

6. The cost of one unit of material is $GH¢\ x$ and you need n units.
7. What is the total cost?

Solution:

Cost per unit = $GH¢\ x$

Number of units = n

To find the total cost of n units of material, where the cost of one unit is $GH¢\ x$, multiply the cost per unit by the number of units.

$C = x \times n$

Therefore, the total cost of n units of material $C = xn$

Learning task for practice

1. Learners are tasked to perform operations on algebraic expressions.
2. Learners are tasked to translate real-life problems into algebraic expressions and perform operations on them.

Pedagogical Exemplars

1. **Inquiry-Based Learning:** In collaborative and mixed-gender/ability groupings, engage learners to simplify given algebraic expressions.
2. **Problem-based learning**: in groups/pairs learners translate real-life problems into algebraic expressions and perform operations on them.

Key Assessment

1. Simplify the following
   i. $5a + 2b - a + 7b$
   ii. $7m + 2 - 3m + 8$
2. Multiply the following
   i. $3x \times 4$
   ii. $5a \times b$
3. A store offers a GH¢ 1 discount on all items.
4. Write an expression for the discounted price if the original price is *GH¢ p*.
5. A cell phone plan charges a fixed monthly fee of *GH¢ f* plus a per-minute charge of *GH¢ c* dollars. Write an expression for the total cost for m minutes of calls

Section Review

In this section, we explored various aspects of patterns and algebraic expressions, focusing on both visual and mathematical representation. Here's a summary of the key concepts and skills developed:

1. **Representing and Extending Patterns:**
   - **Visual Representation:** We learned to visually represent and extend patterns, recognising how each element builds on the previous one. This involves identifying the rules governing the pattern and applying them to predict future elements.
2. **Describing Patterns Using Mathematical Language:**
   - **Pattern Description:** We practiced describing patterns orally and in writing using precise mathematical language. This includes articulating the rules or relationships within the pattern and predicting subsequent elements based on these rules.
3. **Solving Problems Using Pattern Rules:**
   - **Application of Patterns:** We applied pattern rules to solve problems involving tables and charts. This involved using the identified patterns to make predictions and determine future elements, enhancing our problem-solving skills and understanding of data relationships.
4. **Modelling Real-Life Situations:**
   - **Mathematical Statements:** We learned to model real-life situations using mathematical statements. This skill involves translating real-world scenarios into mathematical expressions or equations, which helps in analysing and solving practical problems.

5. **Performing Basic Operations on Algebraic Expressions:**
    - **Algebraic Operations:** We performed basic operations, including addition, subtraction, and multiplication, on algebraic expressions. This foundational skill is crucial for manipulating and solving algebraic problems effectively.

Additional Reading/Practice

Read and solve more problems on these concepts using the task sheets.

SECTION 4: GEOMETRICAL REASONING AND MEASUREMENT

Strand: **Geometry And Measurement**

Sub-Strand: Shape And Space/Measurement

Content Standards

1. Demonstrate conceptual understanding of geometric shapes and solids.
2. Estimate and measure perimeter of 2-d shapes using centimetres and metres

INTRODUCTION AND SECTION SUMMARY

Understanding and working with geometric shapes is a fundamental aspect of mathematics education. Identifying and sorting 2D shapes according to their attributes helps learners recognise and categorise different geometrical figures. Recognising prisms and pyramids in the environment connects classroom learning to the real world. Constructing nets of prisms and pyramids develops spatial reasoning and visualisation skills. Measuring and recording the perimeter of regular and irregular shapes in centimetres and metres, along with developing and applying formulas for perimeter calculation, enhances s' understanding of geometry. Constructing different rectangles for a given perimeter demonstrates the concept that multiple shapes can share the same perimeter, fostering a deeper comprehension of geometric principles.

The section will cover the following focal areas:

1. *Identify and sort 2D shapes according to their attributes*
2. *Identify and describe prisms and pyramids in the environment*
3. *Construct the nets of prisms and pyramids.*
4. *Measure and record perimeters for regular and irregular shapes in centimetres and metres .*
5. *Develop and apply a formula for determining perimeters of given shapes in centimetres and metres.*
6. *Construct different rectangles for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter*

SUMMARY OF PEDAGOGICAL EXEMPLARS

Effective teaching strategies for these concepts include a mix of hands-on activities, visual aids, and real-world connections.

1. **Identifying and Sorting 2D Shapes:** Use visual aids such as shape cards and manipulatives to help learners identify and sort 2D shapes based on their attributes. Interactive activities like sorting games and group discussions enhance engagement and understanding.
2. **Identifying Prisms and Pyramids:** Take learners on a shape hunt around the school or community to identify prisms and pyramids in the environment. Use photographs and physical models to illustrate these 3D shapes and their characteristics.
3. **Constructing Nets:** Provide learners with templates and materials to construct the nets of prisms and pyramids. Hands-on activities where learners cut and fold paper to create 3D shapes help reinforce their spatial reasoning skills.

4. **Measuring and Recording Perimeter:** Teach learners to measure the perimeter of regular and irregular shapes using rulers and measuring tapes. Practice recording measurements in centimetres and metres, and discuss the importance of accuracy.
5. **Developing Perimeter Formulas:** Guide learners in deriving formulas for the perimeter of various shapes through exploration and pattern recognition. Use examples and non-examples to illustrate the process.
6. **Constructing Rectangles for a Given Perimeter:** Have learners construct different rectangles with the same perimeter using graph paper or geoboards. This activity demonstrates that multiple shapes can have the same perimeter, fostering creativity and deeper understanding.

ASSESSMENT SUMMARY

Assessments should be diverse and address different cognitive levels to ensure comprehensive understanding.

1. **Class Exercises and Tests:** Assess learners' ability to identify and sort 2D shapes based on their attributes. Include tasks that require recognising and describing prisms and pyramids in various contexts.
2. **Hands-On Activities:** Evaluate learners' skills in constructing nets of prisms and pyramids through hands-on projects. Assess their accuracy and ability to follow instructions.
3. **Measurement Tasks:** Test learners' ability to measure and record the perimeter of regular and irregular shapes in centimetres and metres. Include exercises that require the application of perimeter formulas.
4. **Problem-Solving Exercises:** Present problems where learners need to develop and apply formulas for determining the perimeter of given shapes. Assess their ability to derive and use these formulas accurately.
5. **Creative Construction Tasks:** Have learners construct different rectangles for a given perimeter and explain their process. Assess their understanding of the concept that multiple shapes can share the same perimeter.
6. **Presentations:** Require learners to present their constructed shapes, perimeter measurements, and the process of developing formulas. This assesses their understanding, communication skills, and ability to apply concepts to different contexts.

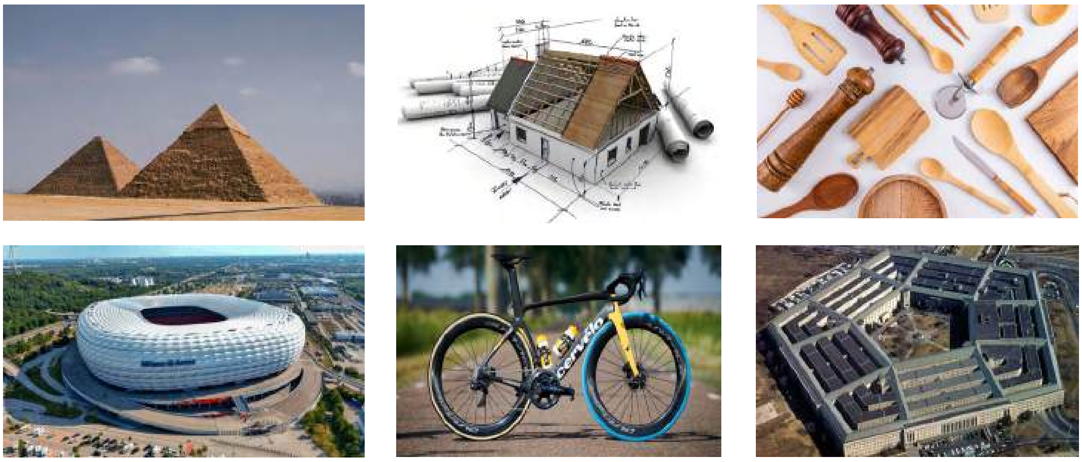
WEEK 9: 2D and 3D Shapes

Learning Indicators

1. *Identify and sort 2D shapes according to their attributes*
2. *Identify and describe prisms and pyramids in the environment*
3. *Construct the nets of prisms and pyramids.*

Focal Area: Identify and Sort 2D Shapes According to their Attributes

Identify common 2D shapes from real-life 3Ds

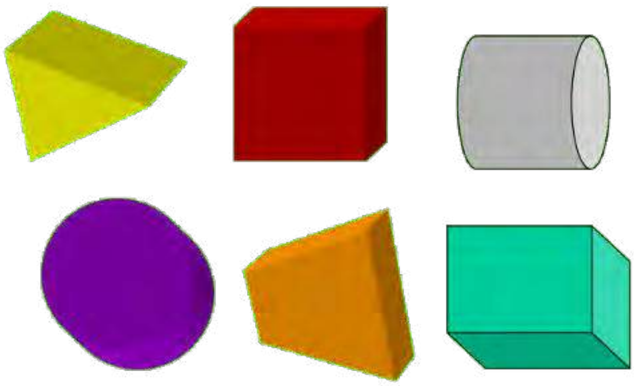


All around us, we find various shapes in the objects that we come into contact with. When we look around us, we find circles, squares, rectangles, triangles, pentagons and many other shapes. In this lesson, we will take a look at some of these everyday life items that have these shapes. and understanding their unique properties and characteristics.

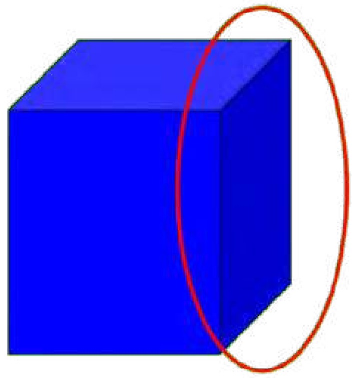
All these real life objects are made up of shapes that we know. Let's us also take a look at these 3D shapes and the 2D shapes that are found within them and their properties.

Three-dimensional Shapes (3D)

These shapes are solid or hollow. They have three dimensions – length, width and height.



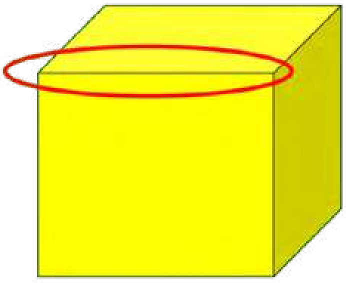
Face



The face is the part of a shape that is flat. (Or curved)

- **E.g.** A cube has 6 faces.

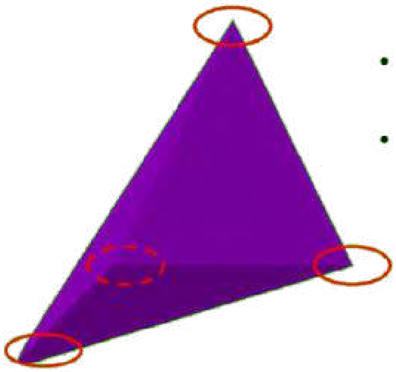
Edge



The edge is the line where two faces meet.

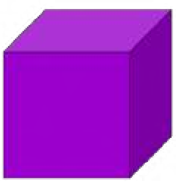
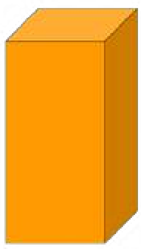
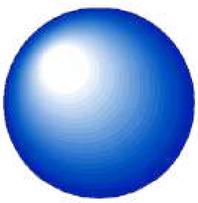
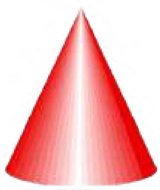
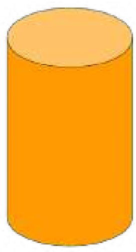
- **E.g.** A cube has 12 edges.

Vertex (Vertices)



A vertex is the place where three or more edges meet.

E.g. This pyramid has 4 vertices.

Cube	Cuboid	Sphere	Cone	Cylinder
				
• 6 square faces all the same size. • 12 Edges all the same length. • 8 Vertices. • It's 2D shape is a **square**	• 6 rectangular faces. • 12 Edges. • 8 Vertices. • It's 2D shape is a **rectangle**.	• A perfectly round 3D shape, like a ball. • It has only one curved face. • It's 2D shape is a **circle**.	• Has a circle at its base and a pointed vertex. • Has 2 faces.	• Circular ends of equal size. • 2 Edges. • 3 Faces. • 0 Vertices

Learning Task for Practice

1. Learners discuss and present on various objects in the school environment and their corresponding 3D shapes.
2. Learners discuss and present on the attributes/features of 3D shapes.
3. Learners investigate other 3D shapes and their attributes and present on them.

Two-dimensional Shapes (2D)

These shapes are flat and can be drawn on paper. They have two dimensions – length and width. They are sometimes called plane shapes.

2D shapes

Rectangle

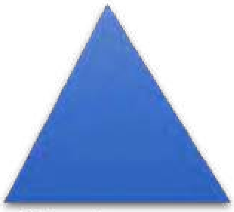
A 2D shape.

- 4 straight sides
- 2 pairs of parallel sides that meet at right angles.

Square *(a special type of rectangle)*

A 2D shape.

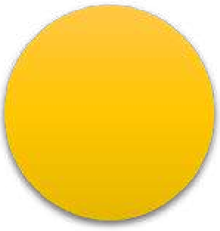
- 4 sides of the same length.
- 2 pairs of parallel sides.



Triangle

A 2D Shape.

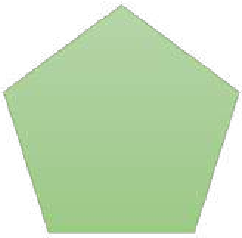
- 3 straight sides.
- 3 Corners/Vertices.
- A regular triangle (equilateral) has 3 lines of symmetry.



Circle

A round flat 2D shape.

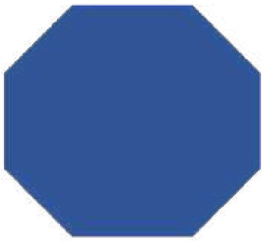
- No Straight sides, only one curved side.
- No corners.



Pentagon

A 2D shape

- 5 straight sides



Octagon

A 2D shape.

- 8 straight sides
- 8 Corners/Vertices.

Hexagon

A 2D shape.

- 6 straight sides
- 6 Corners/Vertices.

Importance of 2Ds and 3Ds in Real-Life Scenarios:

Understanding 2D shapes and their attributes is not just a mathematical exercise; it has real-life applications. For example:

- Architecture and Engineering: Identifying shapes helps in the design and construction of buildings and structures.
- Art and Design: Artists and designers use geometric shapes to create patterns, designs, and artworks.
- Everyday Objects: Recognising shapes helps in identifying and categorising objects in daily life, such as signs, tools, and devices.

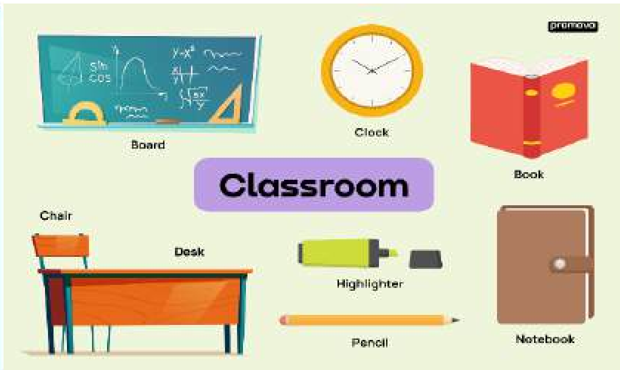
Learning Task for Practice

1. Learners discuss and present on various objects in the school environment and the 2D shapes found in these objects.
2. Learners discuss and present on the attributes/features of 2D shapes.
3. Learners investigate other 2D shapes and their attributes and present on them.

Consider Expanding on these Learning Activities

1. **Shape Hunt**: Engage learners in a shape hunt around the classroom or home to find objects that match specific 2D shapes.
2. **Sorting Games**: Use sorting games where learners group shapes based on their attributes, such as the number of sides or types of angles.

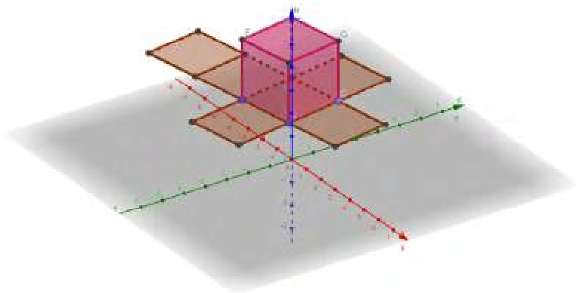
Identify objects in the classroom or the home and indicate the shape found in the objects and sort them. Below are some of the objects you can look for.

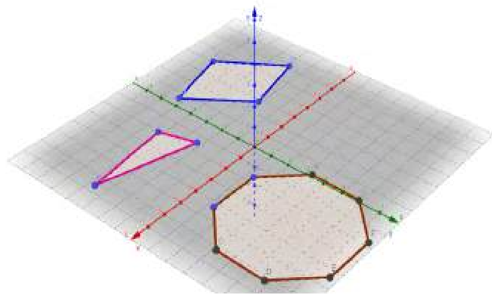


3. **Drawing and Constructing**: Have learners draw different 2D shapes and discuss their properties. They can also use tools like rulers and protractors to measure sides and angles.

   Learners to draw and design simple building plans using cutout papers made of different shapes.

4. **Interactive Technology**: Incorporate educational apps and online games that allow learners to manipulate and explore 2D shapes.





Learning Task for Practice

Learners are to embark on various investigative activities like shape hunting, sorting, drawing and constructing and use of interactive technology to identify 2D shapes from 3D shapes.

Pedagogical Exemplars

1. **Collaborative Learning:** using mixed-ability/gender groupings, learners research and discuss on the various objects in their immediate environment and across the world through the internet to identify items and indicate the various 3D shapes in them.
2. **Problem-based Learning:** In groups/pairs, learners make simple building designs and use cut-out cardboard to design miniature building and any other design items of choice (miniature cars, etc.) that are made up of different shapes.

Focal Area: Identify and Describe Prisms and Pyramids in the Environment

Identify common prisms in the environment (square, rectangular, triangular)

Prisms are three-dimensional solids with two parallel and congruent (identical) bases. Among the most common prisms we encounter are square prisms, rectangular prisms and triangular prisms. Let's explore these shapes, their properties, and where we might find them in our environment.

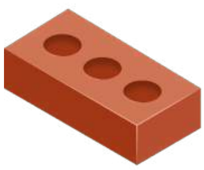
Rectangular Prism

A rectangular prism is a three-dimensional shape with **6 rectangular** faces that meet at right angles. They are also known as cuboids.

Cubes are a special type of rectangular prism where the length, width and height are all equal.

Examples in the Environment:

- Books
- Bricks used in buildings
- Cereal boxes

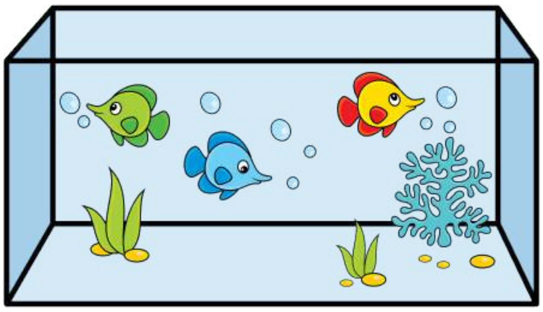


Brick

Box

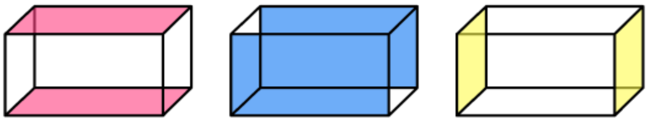
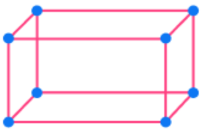
Books

Cereal box



Aquarium

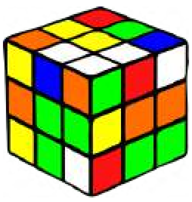
- **Properties/attributes**
  - 6 faces, all rectangles
  - 12 edges, with opposite edges equal in length
  - 8 vertices
  - All interior angles are right angles

	6 faces
	12 edges
	8 vertices

Square Prism

A square prism, also known as a cube, has all sides equal and all are squares. This shape is characterised by all edges being of equal length and right angles. A square prism (= cube) is a special type of a rectangular prism (= cuboid).

- **Examples in the Environment**:
  - Dice used in games
  - Sugar cubes
  - Building blocks in construction



Rubik cube

Suger cube

Choco milo

- **Properties/attributes**:
  - 6 faces, all squares
  - 12 edges of equal length
  - 8 vertices
  - All interior angles are right angles

Triangular Prisms

Triangular prisms have bases that are triangles. These shapes are characterised by their triangular bases and three rectangular faces.

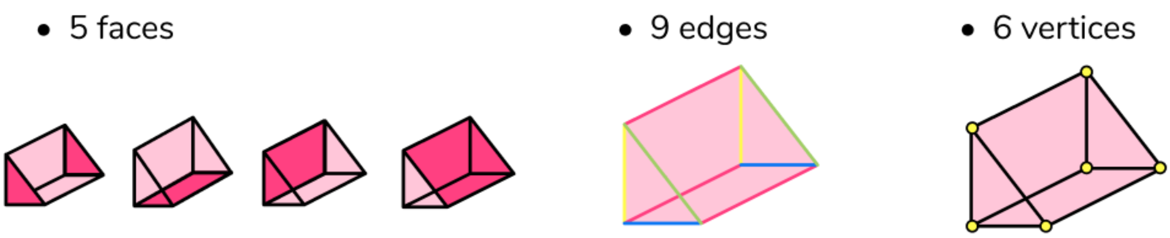
- **Examples in the Environment**:
  - Roofs of houses (often in the shape of triangular prisms)
  - Toblerone chocolate bars
  - Tents

Tent

Roof

Chocolate

- **Properties**:
  - 2 triangular bases
  - 3 rectangular faces
  - 9 edges
  - 6 vertices



Identifying common pyramids in the environment (square, rectangular, triangular)

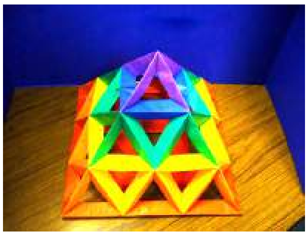
Pyramids are very interesting three-dimensional shapes that we often see in our environments, both in nature and in human-made structures. These shapes consist of a polygonal base and triangular faces that converge at a single point called the apex. Three types of pyramids are: square pyramids, rectangular pyramids, and triangular pyramids.

Square Pyramids

A square pyramid has a square base and four triangular faces that meet at the apex.

- **Examples in the Environment**:
  - The Great Pyramid of Giza
  - Tetrahedra in crystal structures
  - Certain roof designs in architecture



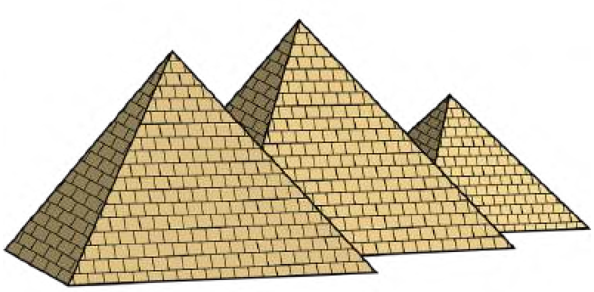


- **Properties/attributes**:
  - 1 square base
  - 4 triangular faces
  - 5 vertices (including the apex)
  - 8 edges

Rectangular Pyramids

A rectangular pyramid has a rectangular base and four triangular faces that converge at the apex.

- **Examples in the Environment**:
  - Pyramidal tents
  - Some modern architectural structures
  - Certain packaging designs

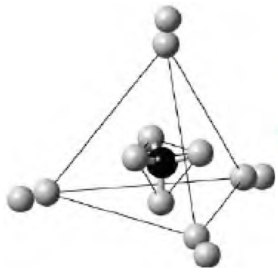



- **Properties/attributes**:
  - 1 rectangular base
  - 4 triangular faces
  - 5 vertices (including the apex)
  - 8 edges

Triangular Pyramids (Tetrahedrons)

A triangular pyramid, or tetrahedron, has a triangular base and three triangular faces that meet at the apex.

- **Examples in the Environment**:
  - Molecules in chemistry, such as methane (CH_4)
  - Certain types of dice
  - Structural elements in geodesic domes



- **Properties/attributes**:
  - 1 triangular base
  - 3 triangular faces
  - 4 vertices (including the apex)
  - 6 edges

Learning Task for Practice

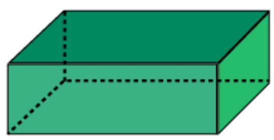
1. Learners are tasked to investigate and make presentations on examples of prisms and pyramids (square, rectangular and triangular) in their environment.
2. Learners are to present on the properties of the prisms and pyramids by demonstrating or showing these properties on a given pyramid or prism.

Construct the nets of prisms and pyramids

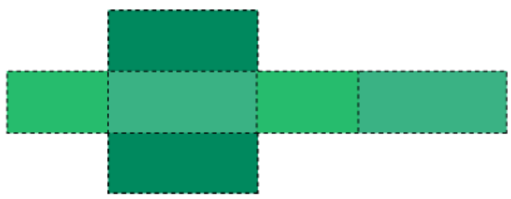
A net is a flat layout of all the faces of a three-dimensional shape. When cut out and folded along the edges, a net forms the corresponding 3D shape. Nets help in visualising the structure of a shape and are useful in understanding the properties of the shape.

Nets of Prisms

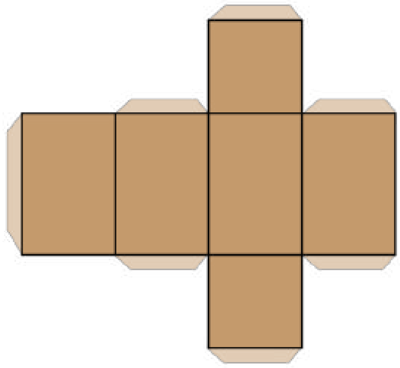
Net of Rectangular Prism



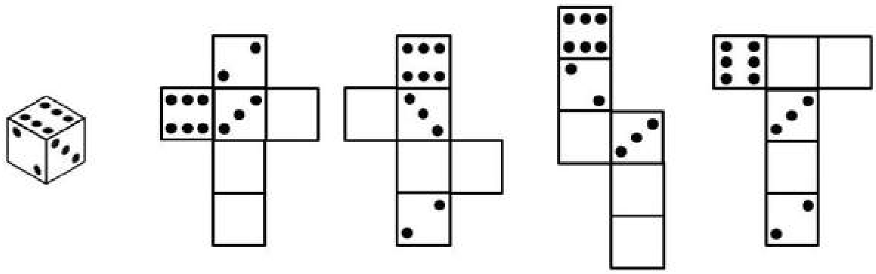
Rectangular Prism



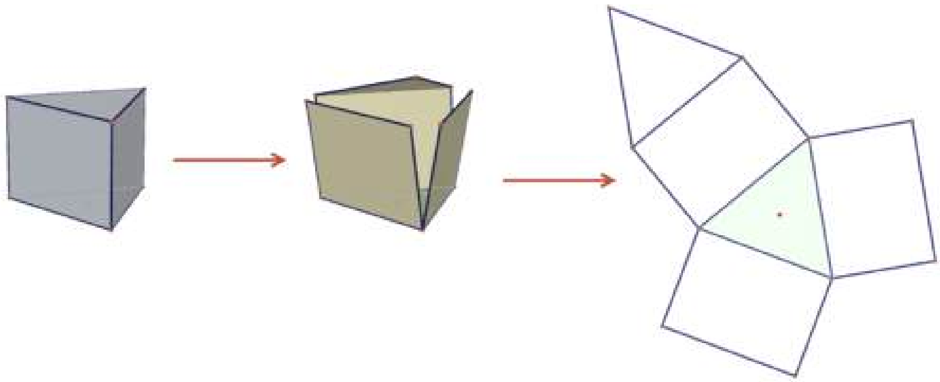
Net of Rectangular Prism



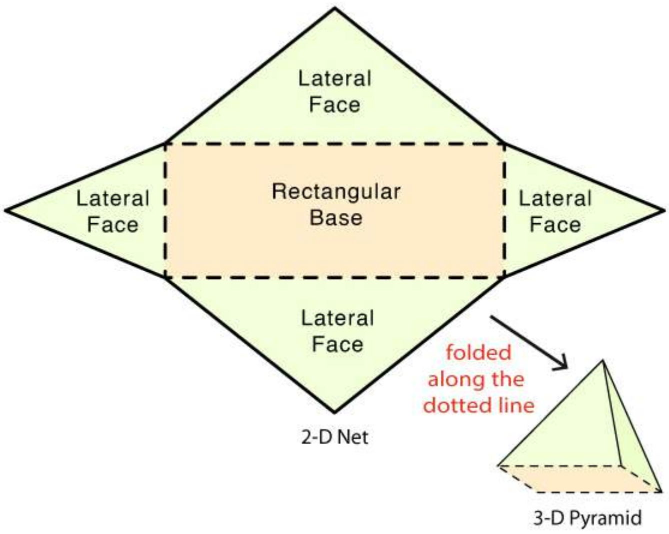
Net of Square Prism (= Cube)



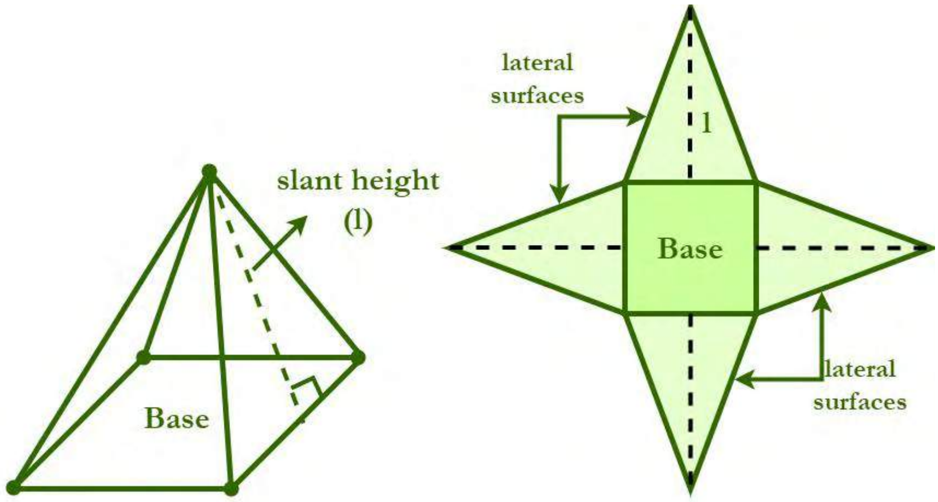
Net of a Triangular Prism



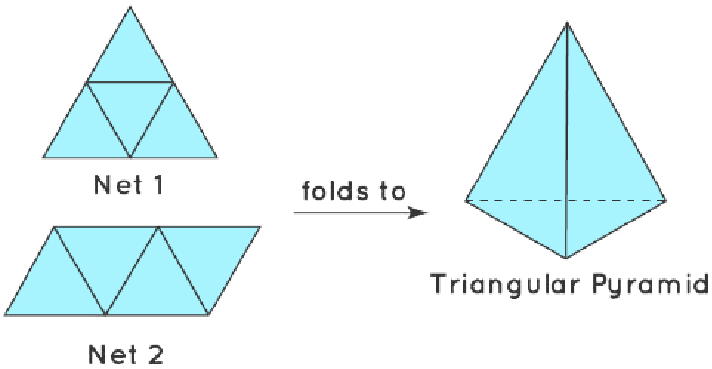
Net of Rectangular Pyramid



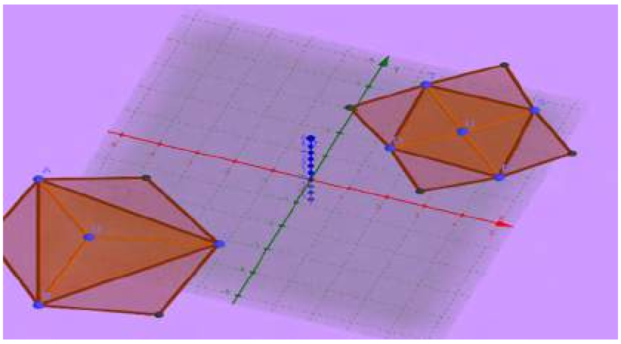
Net of Square Pyramid

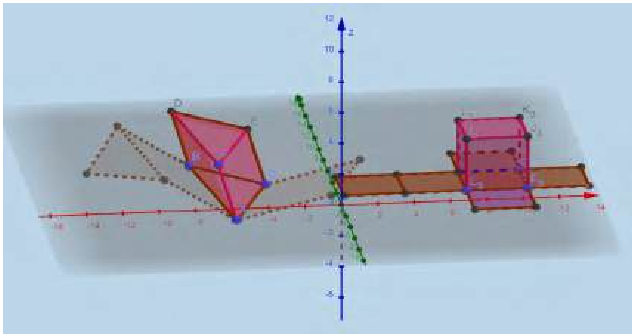


Net of Triangular Pyramid



Learners Explore the nets of prisms and pyramids using applicable computer application software.





Learning Task for Practice

Learners are tasked to engage in practical activities on showing the nets of various prisms and pyramids.

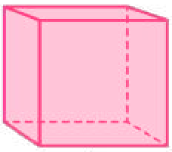
Pedagogical Exemplars

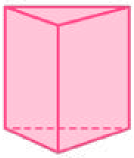
1. **Collaborative Learning:** using mixed-ability/gender groupings, learners embark on an investigative activity on examples of prisms and pyramids (square, rectangular and triangular) in their environment.

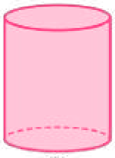
2. **Problem-based Learning:** In groups/pairs, learners are to present on the properties of the prisms and pyramids by demonstrating or showing these properties on a given pyramid or prism.
3. **Experiential Learning:** Learners are tasked in groups to engage in practical activities on showing the nets of various prisms and pyramids. They are also to use computer application programmes such as geogebra to investigate the nets of 3Ds.

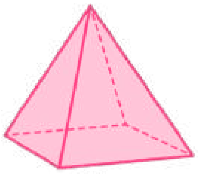
Key Assessment

1. Identify and name the following 3D shapes.


A


B


C

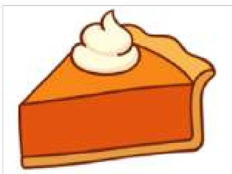

D

2. Identify the 3D shape in the following objects.

A


B

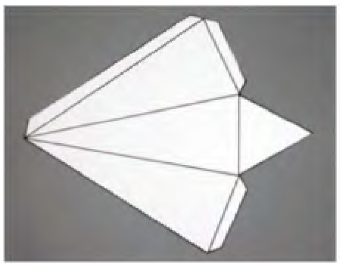

C

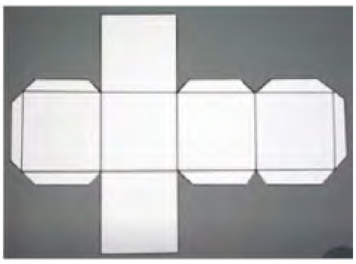

D

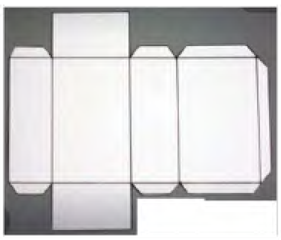

E


F

3.

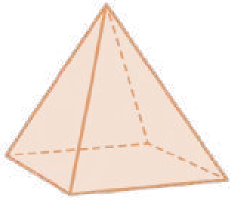

A


B


C


D

4. For each of the following 3D shapes, identify the faces, edges and vertices where applicable.



5. Complete the table below by filling in the number of sides, angles, and lines of symmetry for each shape.

Shape	**Sides**	**Angles**	**Lines of Symmetry**
Square			
Rectangle			
Equilateral Triangle			
Circle			
Regular Pentagon			
Regular Octagon			
Regular Nonagon			

Hint:

- **Sides**: Line segments forming the boundary of the shape.
- **Angles**: The corners where sides meet.
- **Lines of Symmetry**: Imaginary lines that divide the shape into two identical parts.

6. Complete the table below by filling in the number of vertices, edges, and faces for each shape.

Shape	**Vertices**	**Edges**	**Faces**	**Base Shape**
Triangular Prism				
Square Prism				
Rectangular Prism				
Triangular Pyramid				
Square Pyramid				
Rectangular Pyramid				

Hint:

- **Vertices**: Points where edges meet.
- **Edges**: Line segments between vertices.
- **Faces**: Flat surfaces of the shape.
- **Base Shape**: The shape of the base of the prism or pyramid.

Week 10: Measurement of Perimeter I

Learning Indicator: *Measure and record perimeters for regular and irregular shapes in cm and m.*

Focal Area: Measuring Perimeter of Regular and Irregular 2D Shapes

Understanding how to measure the perimeter of both regular and irregular 2D shapes is a fundamental concept in geometry that learners will encounter frequently in both academic and real-life situations. The perimeter is the total distance around the boundary of a shape, and it is important for tasks such as determining the amount of material needed for a fence, the length of trim for a picture frame, or the border of a garden.

For example, to find the perimeter of the fence, you need to measure all the sides of the fence and add the results.

The perimeter of large objects like the fence above is measured in metres (m) and kilometres (km). The perimeter of small objects like your exercise book or the top of your table can be measured in millimetres (mm) and centimetres (cm).

Measure and record perimeter for real life regular shapes in cm and m.

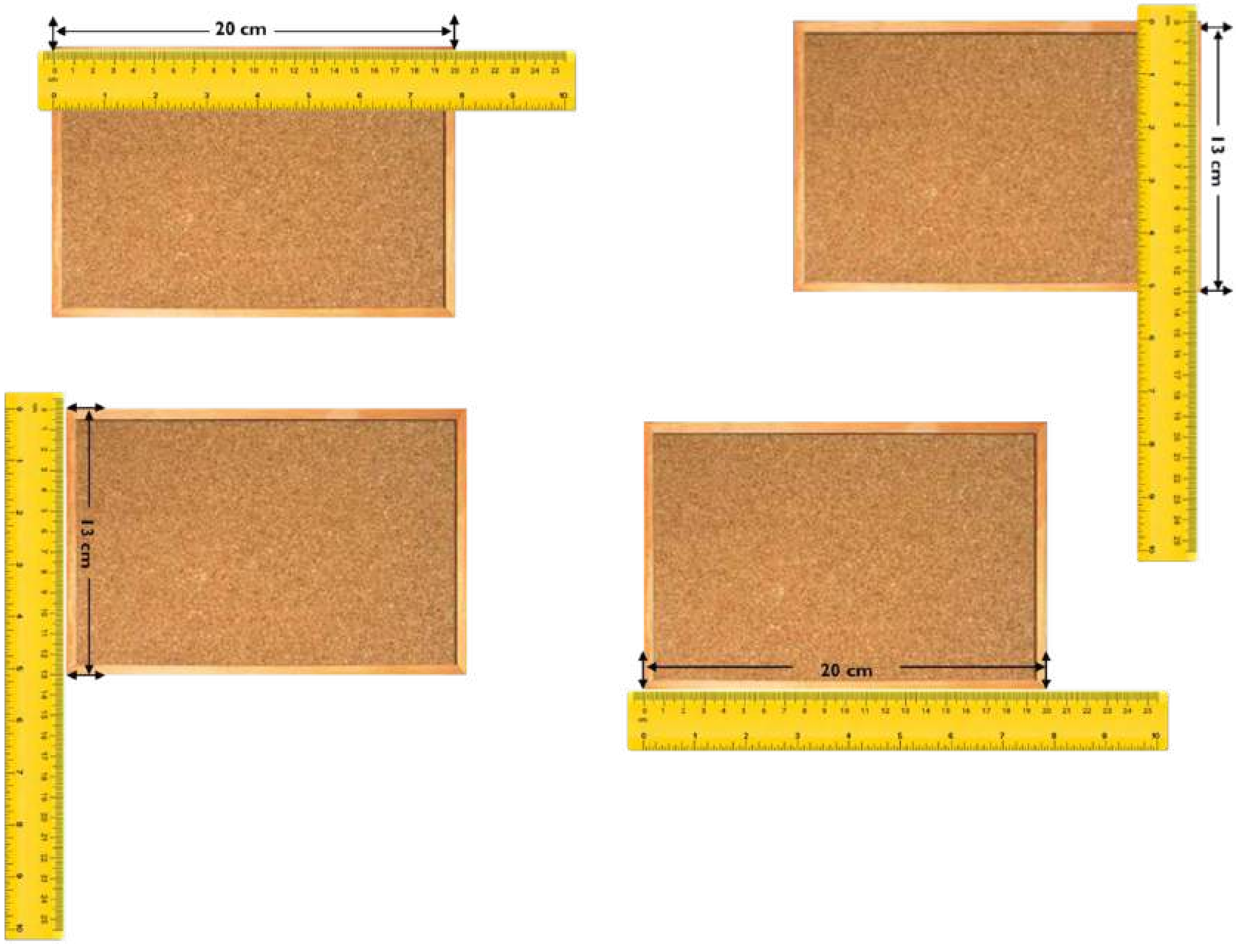
Take a look at the following objects as we find them in our various communities. We will measure and determine the perimeter of these objects.

To find the perimeter of each of these objects, we will use our ruler to measure all the sides of the object and add the results.

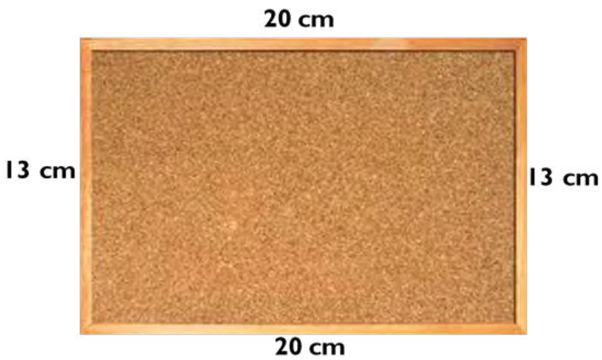
Example:

Determine the perimeter of the object below.

Step 1: Measure the sides of the object



So, these are the measurements of all the sides of our object.

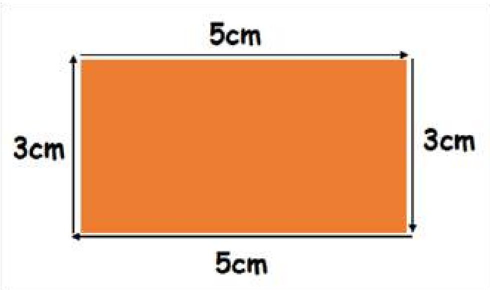


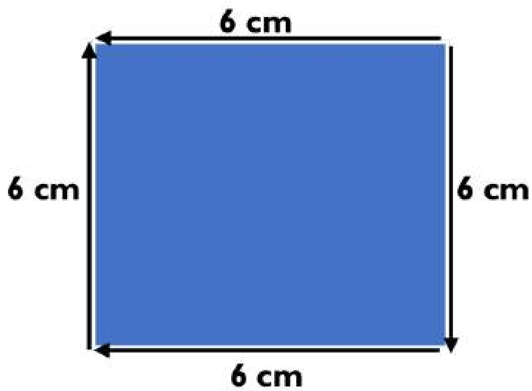
To find the perimeter, we add all the sides. Thus, 20 cm + 20 cm + 13 cm + 13 cm = 66 cm.

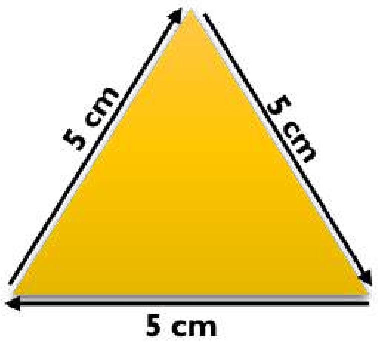
Learning Tasks for Practice

1. Learners are tasked to measure the sides of given objects and determine the perimeter of the objects.
2. Learners embark on out of class activity by measuring the sides of fences or walls and determine the perimeter of these objects/items.

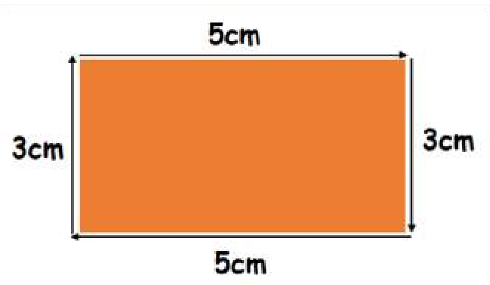
Determine the perimeter of squares, rectangles and triangles





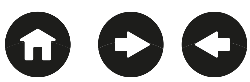


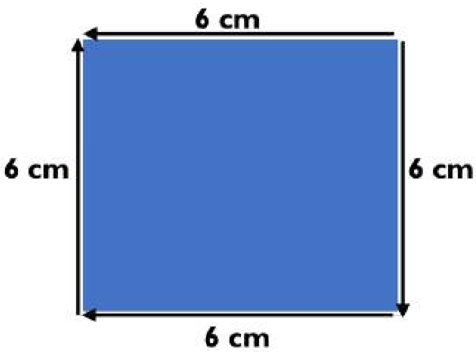
Solutions



Perimeter = $5cm + 3cm + 5cm + 3cm$

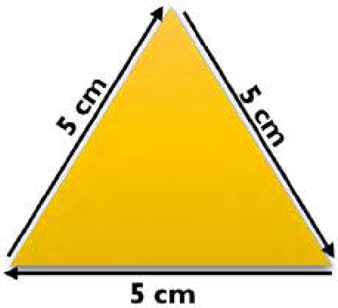
Perimeter = 16cm





Perimeter = $6cm + 6cm + 6cm + 6cm$

Perimeter = 24 cm



Perimeter = $5cm + 5cm + 5cm$

Perimeter = 15 cm

Learning Tasks for Practice

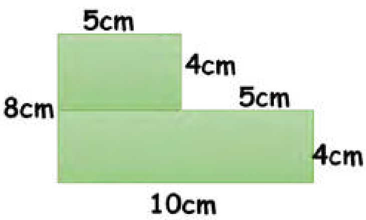
Learners determine the perimeter of the three common shapes squares, rectangles and triangles.

Measure and record perimeter for irregular shapes in cm and m.

Sometimes, we are given shapes that are not common shapes. For such shapes, we will still add the lengths of all the sides to determine the perimeter of the shapes.

Examples:

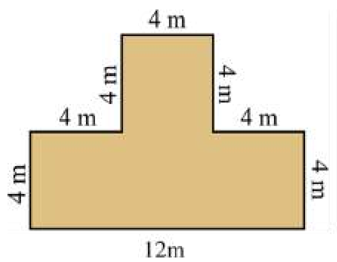
1. This shape is not one of the common shapes that we know, but to find the perimeter we just add all the sides.



5 + 4 + 5 + 4 + 10 + 8 = 36cm

Perimeter of the shape = 36cm

2. Find the perimeter of the shape below

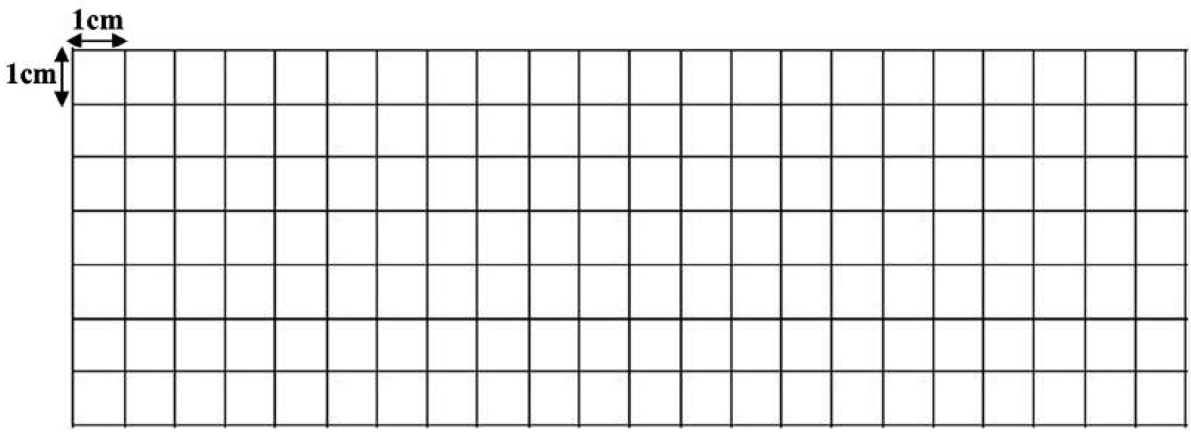


Perimeter = 4m + 4m + 4m+ 4m + 4m + 4m + 4m +12m = 40 m

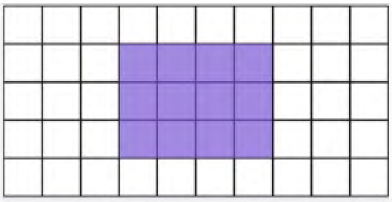
Use grids to determine the perimeter of regular and irregular shapes

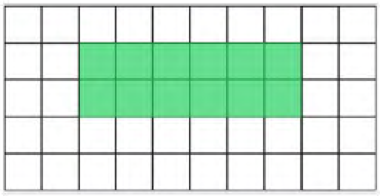
We can also use graph sheets and paper grids to determine the perimeter of 2D shapes.

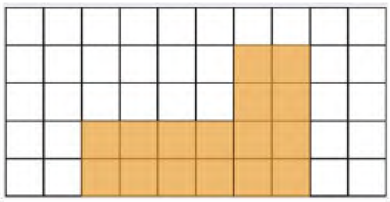
An example of a grid is show below. Usually, a square on a grid is 1 cm by 1 cm.



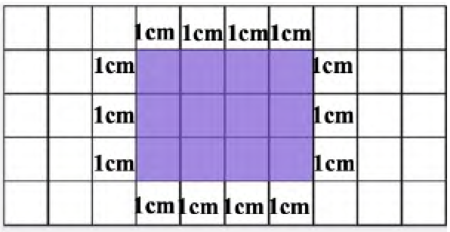
Determine the perimeter of the shapes on the grid.







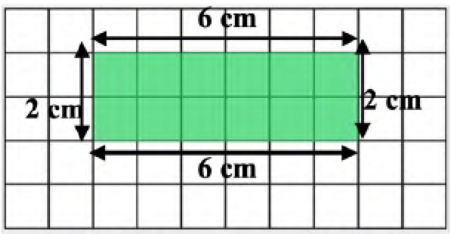
Solution:



Since each side of the grid is 1*cm*, we will count around the shape drawn on the grid.

Perimeter = 4 *cm* + 3 *cm* + 4 *cm* + 3 *cm*

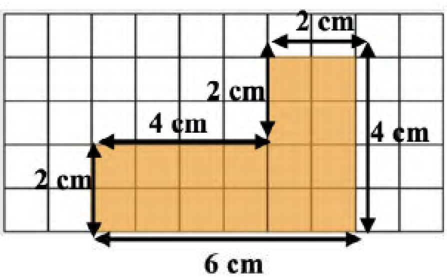
Perimeter = 14 *cm*



Since each side of the grid is *1cm*, we will count around the shape drawn on the grid.

Perimeter = 6 *cm* + 2 *cm* + 6 *cm* + 2 *cm*

Perimeter = 16 *cm*



Since each side of the grid is 1cm, we will count around the shape drawn on the grid.

Perimeter = 6 *cm* + 4 *cm* + 2 *cm* + 2 *cm* + 4 *cm* + 2 *cm*

Perimeter = 20 *cm*

Learning Tasks for Practice

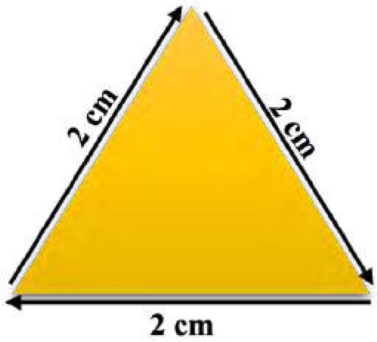
1. Learners determine the perimeter of irregular shapes with all sides given.
2. Learners are tasked to use grid papers to investigate the perimeter of regular and irregular shapes.

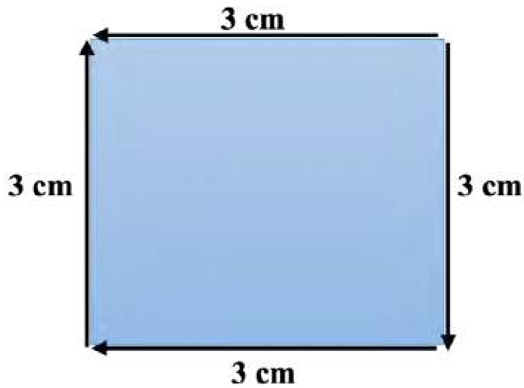
Pedagogical Exemplars

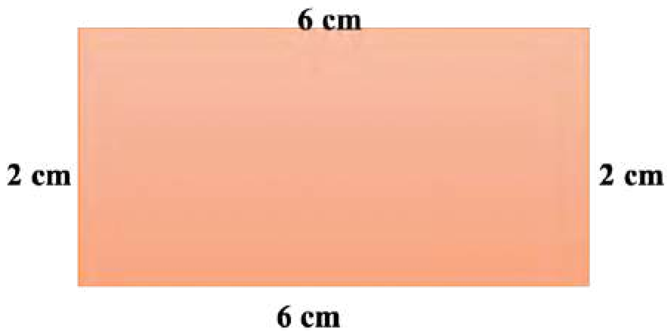
1. **Collaborative/Experiential Learning:** Using mixed-ability/gender groupings, learners measure and record the side lengths of objects and determine the perimeter of the objects.
2. **Problem-Based Learning:** Using mixed-ability/gender groupings, learners calculate the perimeter of squares, rectangles and triangles. Learners also use paper grids to investigate the perimeter of regular and irregular shapes.

Key Assessment

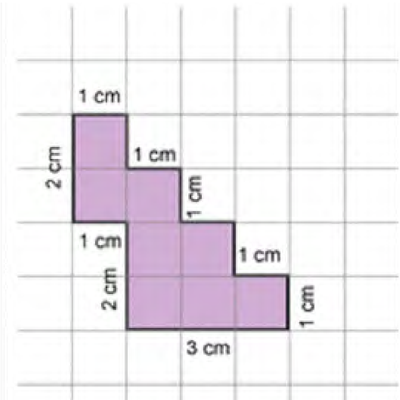
1. Determine the perimeter of the following shapes drawn of the grid.

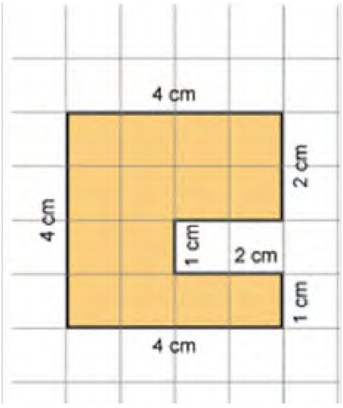


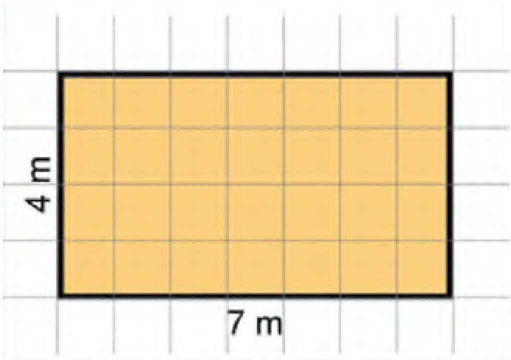


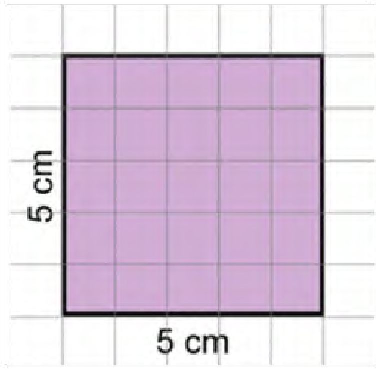


2. Determine the perimeter of the following shapes drawn of the grid.









3. Determine the perimeter of the following objects.

Week 11: Measurement of Perimeter II

Learning Indicators

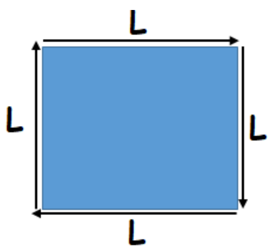
1. *Develop and apply a formula for determining the perimeter of given shapes in centimetres and metres.*
2. *Construct different rectangles for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter.*

Focal Area: Develop and Apply a Formula for Determining Perimeter of Squares and Rectangles.

We can use formula to calculate the perimeter of squares and rectangles. Formulas make our calculations easier and faster.

Square

A square has all four sides equal. Therefore, when adding the sides to find the perimeter, you add the same number four times.



So, for any square, you add the four lengths.

Thus, L + L + L + L

This give 4L

Worked Examples:

1. Find the perimeter of the square below.

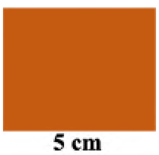
Formula = L + L + L + L = 4L

Since all the sides of a square are the same, we add the 7 cm four times.

Perimeter = 7cm + 7cm + 7cm + 7cm = **28 cm**

Or 4L = 4 (7cm) = **28 cm**.

2. Find the perimeter of the square below.

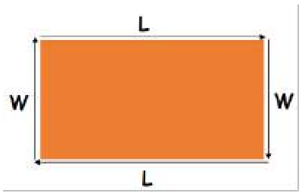


Perimeter = 5cm + 5cm + 5cm + 5cm = **20cm**

Or 4L = 4(5cm) = **20 cm.**

Rectangle

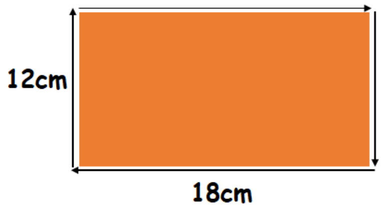
A rectangle has its two opposite sides equal.



Formula = L + W + L + W = 2L + 2W

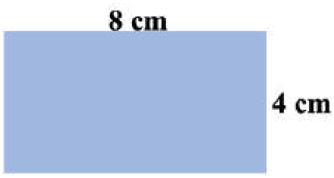
Worked Examples:

1. Find the perimeter of the rectangle below.



Formula = 2L + 2W
2(18cm) + 2(12cm)
36cm + 24cm
60cm

2. Find the perimeter of the rectangle below.



Formula = 2L + 2W
2(8cm) + 2(4cm)
16cm + 8cm
24 cm

Learning Tasks for Practice

Learners develop and use formulas to determine the perimeter of squares and rectangles.

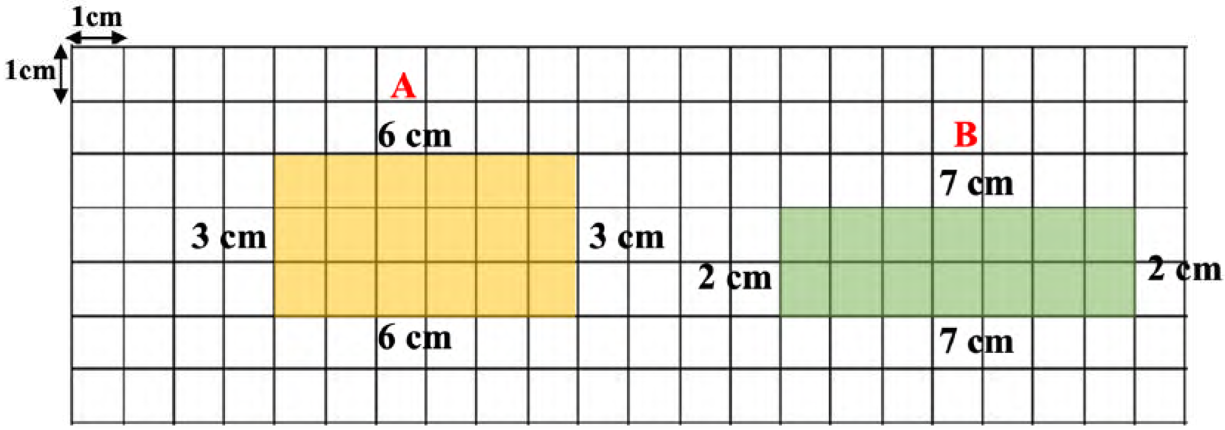
Focal Area: Construct Different Shapes for the Same Perimeter

We can draw different shapes for the same perimeter using the grid paper. Let's take a look at some examples.

Example:

1. Given the perimeter 18 cm, draw two different shapes with different sides for the same perimeter.

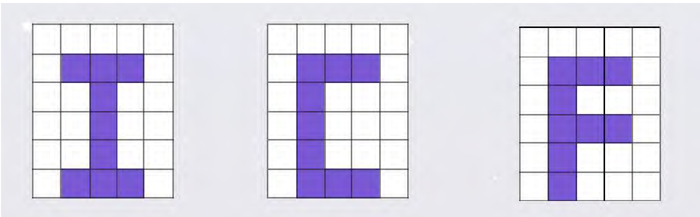
Solution:



Now, comparing the two shapes (Shape A and B), we see that they both have different length sides and are obviously different shapes but they both have the same perimeter as 18 cm.

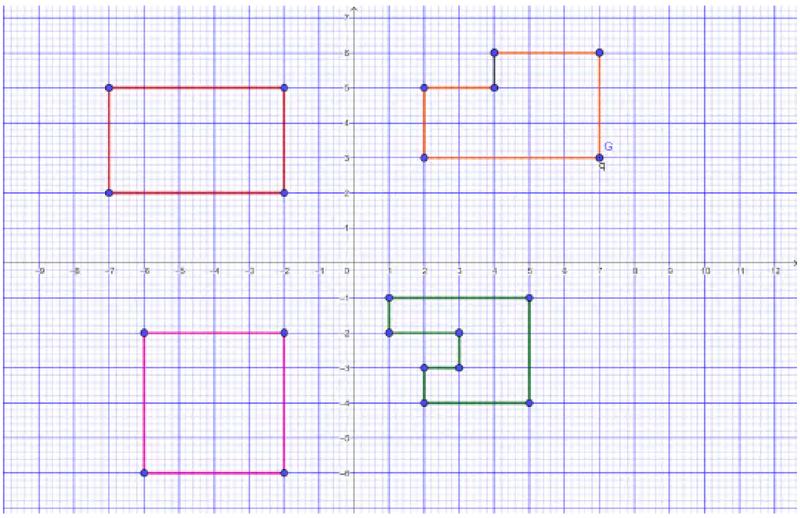
2. Given the perimeter 20 cm, draw three different shapes with different sides for the same perimeter.

Solution



We can also draw different shapes for the same perimeter using the geogebra. Let's take a look at some examples.

Example: Use geogebra to draw four shapes of different shapes with each having the perimeter 16 cm.



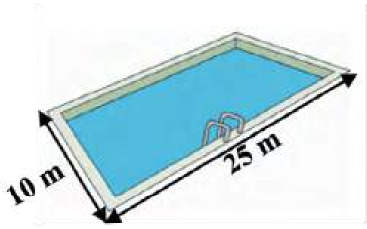
Learning Tasks for Practice

Learners investigate by drawing different shapes for the same given perimeter, including using Geogebra to execute the task.

Focal Area: Solve Word Problems Involving Perimeter

Worked Examples:

1. The rectangular swimming pool in this house has sides of 25m and 10m. What is the perimeter?



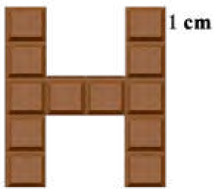
Solution

Formula = L + W + L + W = 2L + 2W

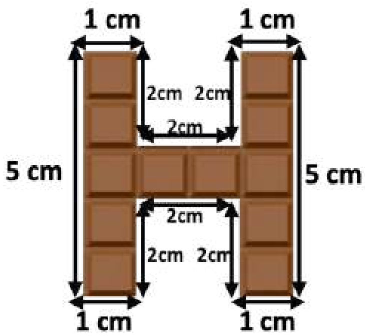
Perimeter = 10m + 25m + 10m + 25m

= 70m

2. Okrakua has a chocolate bar in the shape of a letter H. What is the perimeter of a chocolate bar if the length of one piece of chocolate is 1 cm?



Solution



Perimeter = 5cm + 5cm + 1cm + 1cm + 1cm +1cm + 2cm + 2cm + 2cm + 2cm + 2cm + 2cm

= **26 cm**

3. Akua wants to decorate her birthday card in the shape of a rectangle. She wants to put a tape around the card. The card has sides 13cm long and 9cm wide. How many centimetres of tape does Akua need?

Solution

Formula = L + W + L + W = 2L + 2W

Perimeter = 13cm + 13cm + 9cm + 9cm

= 44cm

Therefore, Akua needs 44cm of tape.

Learning Tasks for Practice

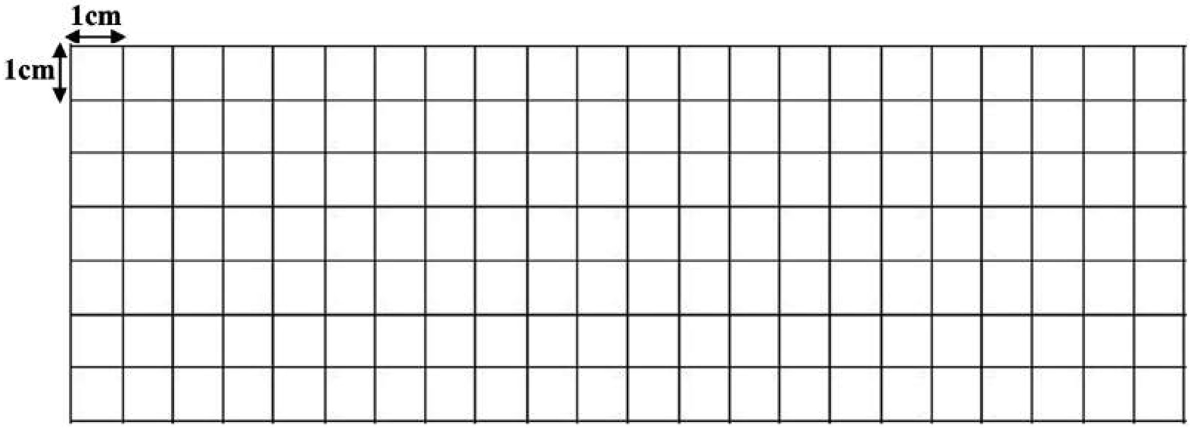
Learners solve simple real life problems involving perimeter.

Pedagogical Exemplars

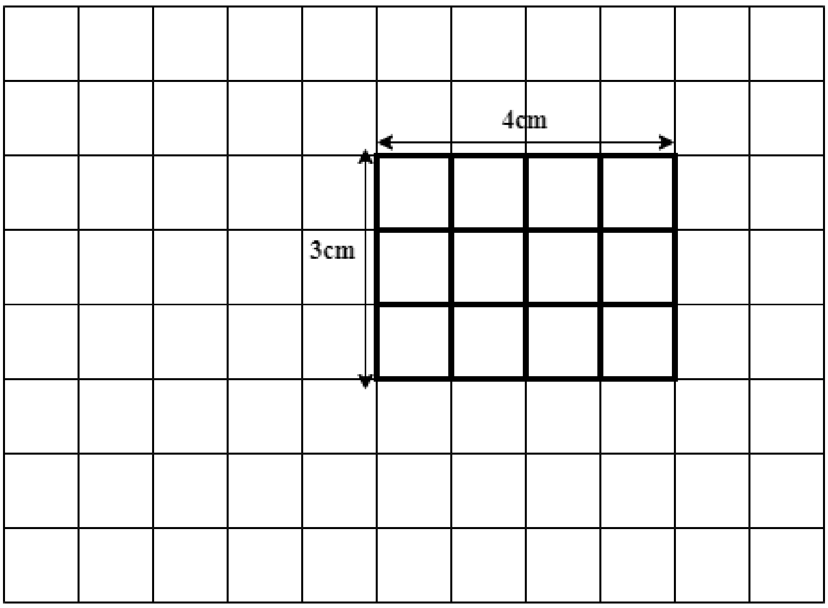
1. **Collaborative/Experiential Learning:** Using mixed-ability/gender groupings, learners develop and use formulae to determine the perimeter of squares and rectangles.
2. **Problem-Based Learning:** Using mixed-ability/gender groupings, learners investigate by drawing different shapes for the same given perimeter, including using geogebra to execute the task.

Assessment Task

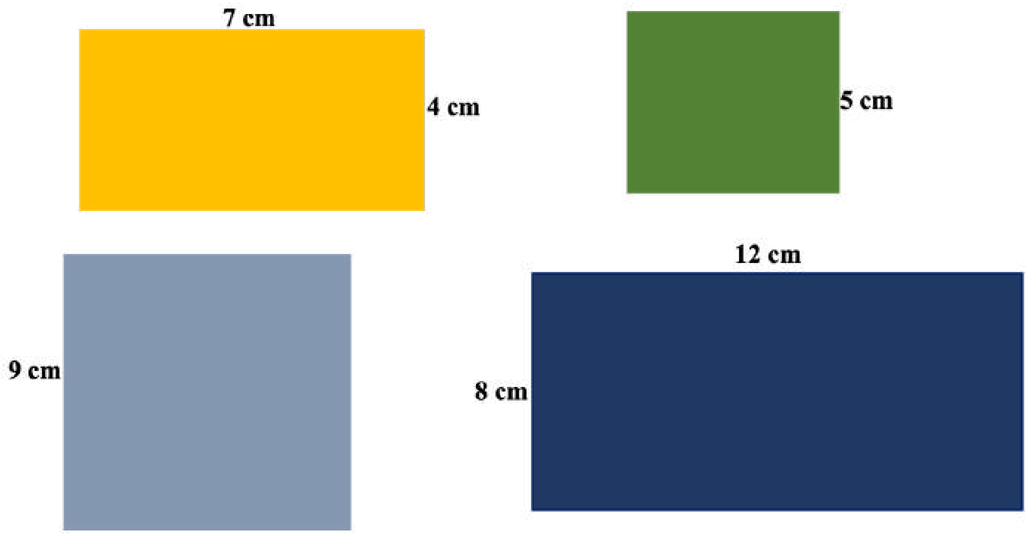
1. Draw three different shapes each with the same perimeter of 22 cm.



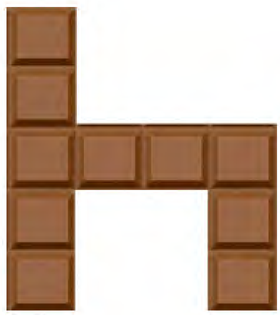
2. Carefully observe the rectangle on the square grid. Draw another rectangle that has the same perimeter but with different side lengths.



3. Find the perimeter of the following shapes.



4. Mark has a chocolate bar in the shape of a letter h. What is the perimeter of a chocolate bar if the length of one piece of chocolate is 1cm?



5. A school built a football field with length of 70 m and a width of 35 m. They will need to draw the boundaries of the field and center line.
What length of line they will need to paint?

6. It is a festive holiday and Larley wants to put lights around the TV. Look at the measurements. What length of lights does she need?

Section Review

In this section, we focused on understanding and working with geometric shapes and their properties. We covered a range of topics including identifying 2D shapes, working with prisms and pyramids, measuring perimeters, and applying geometric formulas. Here are the key ideas we explored:

1. **Identifying and Sorting 2D Shapes:**
   - **Attributes of Shapes:** We learned how to identify and categorise 2D shapes based on their attributes such as the number of sides, angles, and symmetry. Sorting shapes by these characteristics helps in understanding their properties and relationships.
2. **Identifying and Describing Prisms and Pyramids:**
   - **Prisms and Pyramids in the Environment:** We identified and described prisms and pyramids found in everyday life, recognising their geometric features and understanding their spatial properties.
3. **Constructing Nets of Prisms and Pyramids:**
   - **Creating Nets:** We practiced constructing nets for various prisms and pyramids, which are essential for visualising and understanding the 3D structures of these shapes. This process helps in comprehending how 2D shapes can form 3D objects.
4. **Measuring and Recording Perimeter:**
   - **Perimeter Calculation:** We measured and recorded the perimeter of both regular and irregular shapes in centimetres and metres. This practice reinforces our ability to calculate and work with linear dimensions in geometry.
5. **Developing and Applying Perimeter Formulas:**
   - **Formulas for Perimeter:** We developed and applied formulas for determining the perimeter of given shapes, enhancing our understanding of geometric properties and calculation techniques in different units of measurement.
6. **Constructing Rectangles for a Given Perimeter:**
   - **Exploring Different Rectangles:** We constructed various rectangles for a specified perimeter to demonstrate that multiple shapes can have the same perimeter. This activity highlighted the concept of geometric flexibility and the variety of shapes possible with a fixed boundary.

This section deepened our understanding of geometric shapes, their attributes, and measurement techniques, equipping us with the skills to analyse and solve problems involving shapes and perimeters.

Additional Reading/Practice

Solve more problems on these concepts using the task sheets.

SECTION 5: WORKING WITH DATA

Strand: **Collecting And Handling Data**

Sub-Strand: Handling Data

Content Standards

1. Demonstrate understanding of many-to-one correspondence in displaying, and reading or interpreting, graphs
2. Select, justify, and use appropriate methods of collecting data, including questionnaires, interview, observation, experiments, databases, electronic media, etc.

INTRODUCTION AND SECTION SUMMARY

Interpreting and constructing graphs is a vital skill in understanding and representing data. Using an understanding of correspondence to construct and interpret graphs of continuous data enables learners to visualise trends and patterns over time. Learning to interpret double bar graphs, with titles, labeled axes, keys or legends, helps learners compare and analyse multiple sets of data. These skills are crucial for solving problems and making data-driven decisions. Mastering these concepts ensures that learners are well-equipped to handle and present information effectively in various real-world contexts.

The section will cover the following focal areas:

1. *Using understanding of correspondence to construct and interpret graphs of continuous data*
2. *Interpreting double bar graphs, complete with title, labelled axes, key or legend, to represent data collected* (*up to 3 pairs of categories of data and use it to solve problems*).

PEDAGOGICAL SUMMARY

To effectively teach these concepts, a variety of engaging instructional strategies should be employed.

1. **Understanding Correspondence in Graphs:**
   - **Visual Aids and Manipulatives:** Use graph paper, rulers, and markers to help learners plot points and construct graphs of continuous data. Utilise technology such as graphing software to enhance visualisation.
   - **Real-Life Examples:** Present real-life scenarios where continuous data graphs are used, such as tracking temperature changes over time or monitoring daily rainfall.
2. **Interpreting Double Bar Graphs:**
   - **Step-by-Step Instruction:** Teach learners how to read and interpret each element of a double bar graph, including the title, labeled axes, and legend. Use guided practice with examples.
   - **Interactive Activities:** Have learners create their own double bar graphs from collected data. Encourage them to label axes, add titles, and include legends accurately.
   - **Problem-Solving Tasks:** Provide problems that require interpreting double bar graphs to draw conclusions and solve real-world problems.
3. **Collaborative Learning:**
   - **Group Projects:** Assign group activities where learners collect data, construct graphs, and present their findings. This promotes teamwork and enhances understanding through peer learning.

- **Discussion and Reflection:** Facilitate class discussions on the importance and use of graphs in different fields, encouraging learners to reflect on their learning process.

4. **Technology Integration:**
   - **Graphing Tools:** Incorporate graphing software and online tools to allow learners to practice constructing and interpreting graphs digitally. This prepares them for modern data analysis techniques.

ASSESSMENT SUMMARY

Assessments should be varied and comprehensive to evaluate learners' understanding and application of graph-related concepts.

1. **Class Exercises and Tests:**
   - Assess learners' ability to construct and interpret graphs of continuous data. Include tasks that require plotting points, labeling axes, and drawing conclusions from the graphs.
   - Test their skills in reading and interpreting double bar graphs, ensuring they can identify titles, axes labels, legends, and compare data sets.
2. **Hands-On Projects:**
   - Evaluate learners' ability to collect data, construct graphs, and present their findings. Assess their accuracy in labeling and their ability to interpret the graphs they create.
3. **Problem-Solving Tasks:**
   - Present real-life problems that require the use of continuous data graphs and double bar graphs to solve. Assess learners' ability to analyse and draw conclusions from the graphs.
4. **Group Activities:**
   - Engage learners in group tasks where they collaborate to collect data, construct graphs, and interpret the results. Assess their teamwork, problem-solving skills, and ability to communicate their findings.
5. **Presentations:**
   - Have learners present their graphs and explain their interpretation. Assess their understanding, communication skills, and ability to use graphs to support their conclusions.

Week 12: Organising and Presenting Quantitative and Qualitative Data

Learning Indicators

1. *Use an understanding of correspondence to construct and interpret graphs of continuous data*
2. *Interpret double bar graphs, complete with title, labelled axes, key or legend, to represent data collected (up to 3 pairs of categories of data and use it to solve problems).*

Focal Area: Quantitative And Qualitative Data

Data is collected for various reasons and its interpretation is important for the purpose for which it was collected. They are organised by arranging responses systematically into understandable and simple form.

Collecting Data

If we visit any important places such as the palace, museum and Flag Staff House, we do so because we seek information of interest and importance. The answers we get on the reasons we visit such places are called data. So, we can describe data as the set of responses on a variable. Such data can be quantitative or qualitative. Quantitative data is data which can be counted or measured. Qualitative data is data which is not represented by numbers.

Describe these data as quantitative or qualitative:

a. The types of food available during break

b. The ages of your friends

c. Which of your class mates are shorter than you?

d. Which new song is on social media?

e. Which teacher will conduct a test?

In this section, we will focus on **Quantitative Data.** Quantitative data can be counted or measured.

Example:

1. In each picture collect data by counting and by measuring.

Solution

Picture 1:

There are 4 pupils on the left and 3 on the right of the sea-saw (counting)

The pupils on the left are heavier than those on the right (measuring)

Picture 2:

There are 7 blue, 5 yellow, 1 black and 2 red marbles on the playing board (counting)

The length of the playing board is longer than the width (measuring)

2. Match the data to its appropriate mode of collection

A		B
a.	Number of learners in the various class	Counting
b.	Length of my foot	
c.	The weight of a tuber of yam	Measuring
d.	Population of Ghana	
e.	Volume of water in a bottle	

Learning Tasks for Practice

Learners to describe how they will get information from their community and match activities in their community to the process of collecting data as counting or measuring.

Organisation of Data

Data is organised by arranging responses systematically into understandable and simple forms. Effective data organisation ensures that information is structured logically and coherently. We will use frequency distribution table in organising our data. The frequency table has 3 columns as item/variable, tally and frequency.

Steps to organise data

With the help of the example below, let us follow the steps to organise given data.

Examples:

1. To complete a game involving the use of a dice, 30 throws were made and the results recorded.
   4, 5, 1, 3, 4, 2, 3, 2, 6, 4, 2, 6, 4, 3, 4, 5, 1, 6, 3, 5, 2, 4, 2, 3, 6, 5, 4, 4, 5, 6
   Organise the data to make it easy to read at a glance.

Solution:

We can organise the data by:

Step 1 – Identify the smallest and the biggest items

Smallest number = 1

Greatest number = 6

Step 2 – Draw a table of three columns and label

Step 3 – label the first column with the variable under discussion (“number”), second column “Tally” and third column “Frequency”

Number	Tally	Frequency

Step 4 – Write the numbers from the smallest to the biggest in the first column

Step 5 – Use strokes to represent the individual items under the tally column

Step 6 – Represent the number of strokes to each item and record them under the frequency column

Step 7 – Write a title for the table

Fig: Frequency distribution table on numbers showing up in a game

Number	Tally	Frequency
1	||	2
2	𝍸	5
3	𝍸	5
4	𝍸 |||	8
5	𝍸	5
6	𝍸	5
Total		**30**

2. Fifteen learners in Form 1were asked to state the subject they like best and the responses obtained were recorded as below. Organise the data into a frequency distribution table.

   English, ICT, Mathematics, RME, ICT, ICT, Mathematics, Mathematics, ICT, English, English, English, RME, English, RME

Solution:

We identify a particular variable under consideration (favourite subject) and the different elements in the data.

Draw a table of three columns and tittle them; favourite subject, tally and frequency

Complete the table with items and their corresponding frequencies

Fig: Frequency distribution table on subjects that learners prefer

Favourite Subject	**Tally**	**Frequency**
English	~~||||~~	5
Mathematics	|||	3
RME	|||	3
ICT	||||	4
Total		**15**

We can easily see the subject which is highly preferred and the ones which are less preferred. The total can easily be told.

Learning Tasks for Practice

Learners draw a frequency distribution table of a data collected in groups, indicating title and total frequency.

Pedagogical Exemplars

1. **Talk for Learning**: In small groups, learners discuss situations at school and at home for which people seek information. Encourage learners to appreciate others' views and critique without making them feel bad.
2. **Experiential learning**: In small group discussion, learners collect and organise data on a variable (E.g.: day of birth, food preference, favourite subject, etc.)
3. **Collaborative learning**: In small groups, engage learners to discuss a key or scale to be used to draw a graph of their choice. Encourage learners to appreciate others' views and critique without making them feel bad.

Focal Area: Draw and Interpret Bar Graphs

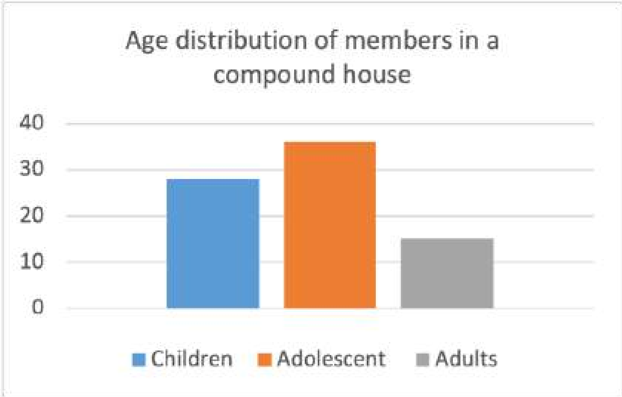
Representing data on graphs

When data is presented pictorially, it is easier to make meaning of the data involved. Some common graphs for representation of data include picture graph, block graph, pie charts, histograms, line graphs, etc. In this work, we will focus on bar graphs.

Key Features of bar graph

- The bar graph has equal width of bars
- It has equal intervals
- The lengths of bars are proportional to their frequencies: the taller the bar, the greater the frequency; the smaller the frequency, the shorter the bar.

We can observe graphs, describe and give our impression on them. The graph below is a bar chart showing distinct bars with horisontal and vertical axes. The title depicts what the graph is all about.



Steps to draw bar graph

To draw a bar graph for data, we need to:

- **a.** identify the items in the data and their corresponding frequencies
- **b.** choose a scale that will helpful in drawing the graph
- **c.** determine the size of the bars to be used and the spacing of the bars
- **d.** use ruler to draw bars for each item with height proportional to the frequency
- **e.** write a title for the graph

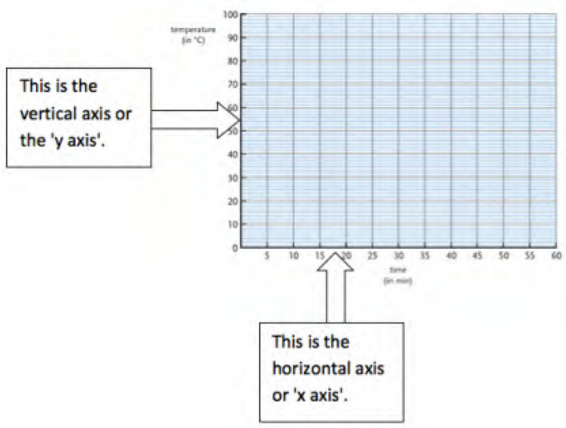
Examples:

1. Represent the number of learners and their favourite subjects on a bar chart.

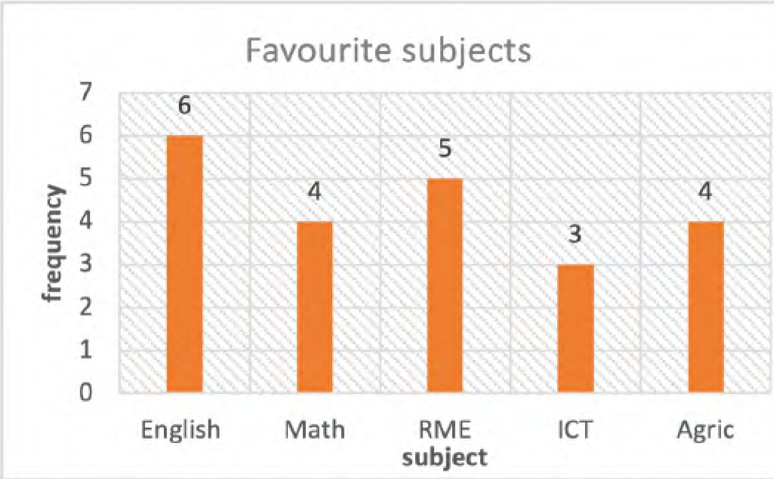
Subject	Number of Learners
English	6
Mathematics	4
RME	5
ICT	3
Agriculture	4

Solution:

We will draw a horizontal axis and a vertical axis on a graph and then label them



We will then draw bars with heights proportional to the frequencies.



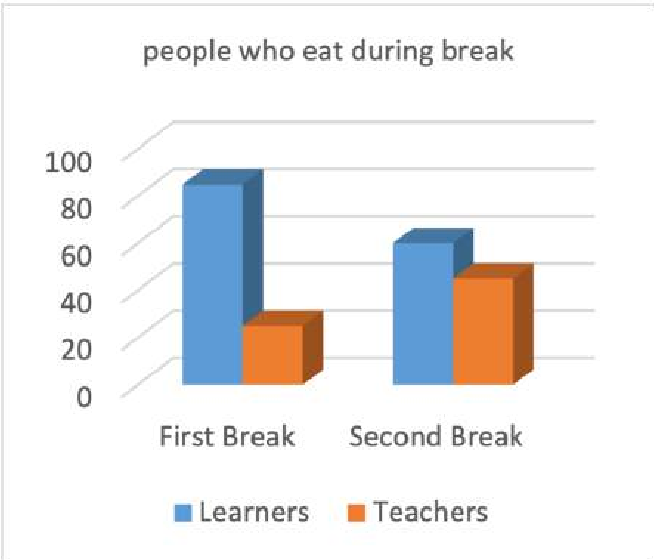
2. The table below shows the number of people who eat during break period. Draw a bar chart to illustrate the information.

	First Break	Second Break
Learners	85	60
Teachers	25	45

Solution:

In this case, we can compare number of learners and teachers who attend first break and then compare the number who go for second break.

We draw a bar for number of learners and a bar for number of teachers for each break.



Learning Tasks for Practice

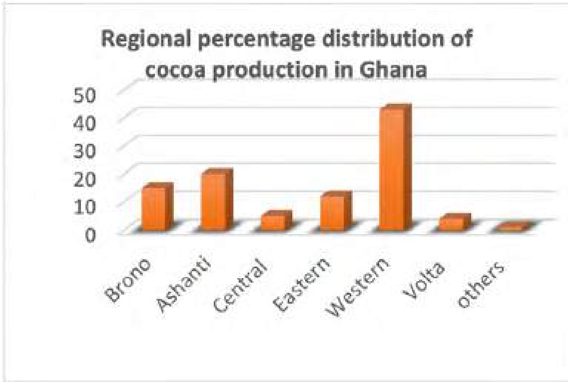
Learners select a page of a news print/a comprehension passage and identify and count the number of verbs, nouns and adjectives in it. Draw a graph to represent the data collected.

Interpreting data

Graphs give appropriate and simplified meaning to data and can be used to predict occurrences for future consideration. They portray information at a glance. Other graphs possess detailed information when probed further.

Example:

1. The bar chart below is on regional percentage distribution of cocoa production in Ghana. Study and respond to the related problems.



   a. Which region produced the greatest percentage of cocoa in Ghana in that year?

   b. Which countries formed "others"?

   c. Which is the second largest cocoa producer in Ghana in that year?

   d. If Western region produced 43% and Bono region produced 15 % of cocoa, what percentage of cocoa was produced by all other regions in Ghana?

   e. What one reason will be the reason for the differences in Cocoa production in different regions in Ghana?

Solution

1.

   a. The region which produced the greatest percentage of cocoa was *Western* as it has the tallest bar.

   b. Regions which formed "others" are all those not represented on the graph. In this case, *Greater Accra*, *Oti*, *Northern*, *Bono East*, *Ahafo*, *Savannah*, *North East*, *Upper East*, Upper West and *Western North*

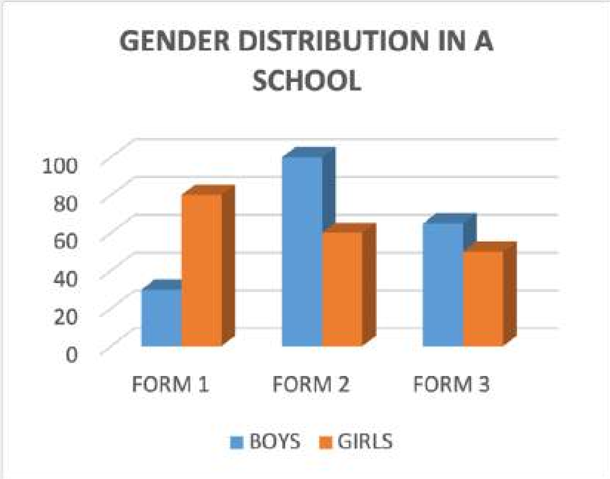
   c. The second largest regional producer of cocoa in Ghana in that year was <u>Ashanti</u>

   d. Production in Western and Bono = 43% +15 % = 58%

   Therefore, all other regions produced = 100 – 58% = **42%**

   e. One reason can be that the different regions in Ghana have different soil types which might be better or worse for the cocoa.

2. Study the bar chart on gender distribution in a school per class and answer the following questions.



a. Which form had the highest number of girls?
b. What is the difference between the number of boys in form 3 and the girls in form 1?
c. How many girls are in the school?
d. How many learners are in the school?
e. If 2 learners sit on a desk, how many desks will be needed for all learners in the school?

Solution

2.

a. Form 1 had the highest number of girls.
b. Boys in form 3 = 65, girls in form 1 = 80. Therefore, the difference is 80 – 65 = 15
c. Number of girls in the school: 80 + 60 + 50 = 190 girls
d. Number of learners in the school: 190 girls + 195 boys = 385 learners
e. Number of desks needed for all learners in the school

   Number of desks needed (Total student ÷ 2) = 385 ÷2 = 192.5

   Therefore, Number of desks needed = 193 desks

Learning Tasks for Practice

Learners to critique graphs, solve problems in relation to the graphs and create simple problems on the graph.

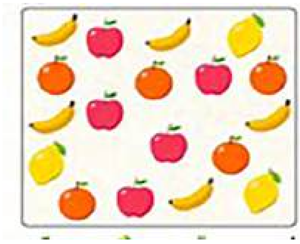
Pedagogical Exemplars

1. **Experiential learning**: In small groups, engage learners to identify and use a key or scale to draw a graph. Use appropriate technology tools such as Microsoft Excel if available and encourage learners to do same.
2. **Collaborative learning**: In groups engage learners to solve a problem in relation to the graph. Encourage learners appreciate other learners' view.
3. **Experiential learning**: Engage learners to individually think through a graph and create a realistic problem for the colleagues to answer orally.

Key Assessment

1. By what means (**counting** or **measuring**) will you collect data on the following?

A

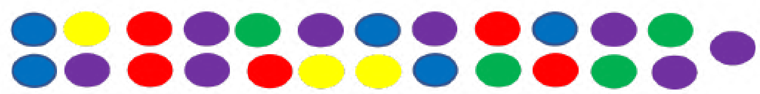


B

C

D

2. Learners of Form one selected coloured toffees (pebbles) from a bag of 20 toffees. Complete the frequency table for the choices of toffees.



Colour of toffee	Number of people

3. When data is organised … (*Tick [√] the correct answer*)

   a. …some information gets lost from it [ ]

   b. …the information is made simple to read [ ]

   c. …the information becomes complicated [ ]

4. The subjects offered in a school and put on the time table are as shown below.

Day	Subject
Monday	Maths, English, Social studies, RME
Tuesday	Maths, ICT, Economics, English, French
Wednesday	Science, Home Economics, Agriculture
Thursday	Maths, ICT, Economics, English, French
Friday	English, Social studies, ICT, Agriculture

   a. On which days are more subjects taught?

   b. Which subjects are taught most in a week?

   c. How many different subjects are taught in the school?

5. Draw a bar chart for the ages of 20 Form 1 learners.
(15, 16, 18, 15, 15, 17, 16, 16, 15, 14, 15, 16, 16, 15, 17, 16, 18, 17, 16, 16)

Section 5 Review

In this section, we explored how to construct and interpret graphs of data and how to work with double bar graphs. We focused on visualising data through graphs and interpreting graphical representations to solve problems. Here are the key ideas we covered:

1. **Constructing and Interpreting Graphs:**
    - **Understanding Correspondence:** We learned how to use an understanding of correspondence to accurately plot and interpret graphs. This involves identifying and representing data points on a graph, recognising trends, and making sense of the data's behavior over time.
    - **Graph Construction:** By constructing graphs, we developed skills in translating data into visual forms, helping us better understand and analyse the data's trends and patterns.
2. **Interpreting Double Bar Graphs:**
    - **Graph Components:** We learned to interpret double bar graphs by focusing on key elements such as the title, labeled axes, and the key or legend. This helps us understand how multiple sets of data are represented and compared within a single graph.
    - **Data Representation and Problem Solving:** We practiced solving problems using double bar graphs by analysing up to three pairs of categories of data. This involved comparing data sets and drawing conclusions based on the visual representation provided by the graph.

Overall, this section enhanced our ability to represent, analyse, and interpret data through graphs, improved our skills in data visualisation and problem-solving.

Additional Reading/Practice

Solve more examples on these concepts using the task sheets.

MODULE 2

SECTION 1: MAKING SENSE WITH NUMBERS

Strand: **Numbers for Everyday Life**

Sub-Strand: Real Number and Numeration System

Content Standard: Demonstrate an understanding of real numbers and its various subsets and use the knowledge and skills to perform operations on the set of real numbers using real-life contexts.

INTRODUCTION AND SECTION SUMMARY

This section delves into categorising real numbers and exploring their subsets. We start by classifying real numbers into different types: natural numbers, whole numbers, integers, rational numbers, and irrational numbers. By understanding these categories, we can better grasp the relationships and distinctions between different types of numbers. We also investigate subsets of natural numbers, such as even and odd numbers, as well as prime and composite numbers, which help us understand the properties and classifications within the number system. Finally, we apply these concepts to real-life contexts, performing operations with real numbers to solve practical problems and demonstrate their relevance in everyday scenarios.

The section will cover the following focal areas:

1. *Categorise real numbers as natural/ counting numbers, whole numbers, integers, and rational and irrational numbers*
2. *Explore the various subsets of counting numbers* (*even and odd, prime and composite*)
3. *Perform operations on the set of real numbers using real-life contexts.*

SUMMARY OF PEDAGOGICAL EXEMPLARS

To effectively teach these concepts:

1. **Categorisation Activities:** Use visual aids, such as number lines or charts, to help students classify numbers into natural numbers, whole numbers, integers, rational numbers, and irrational numbers. Interactive exercises can reinforce these categories.
2. **Exploring Subsets:** Implement activities that involve identifying and working with subsets of natural numbers, such as sorting numbers into even, odd, prime, and composite categories. Use manipulatives like counters or digital tools to visualise these concepts.
3. **Real-Life Applications:** Provide real-life scenarios where students perform operations with real numbers. Examples could include budgeting, measurements, or statistical data analysis, demonstrating the practical use of number classifications.
4. **Problem-Based Learning:** Engage students in solving problems that require the application of number categories and operations. Encourage them to explain their reasoning and how they used number properties to find solutions.

ASSESSMENT SUMMARY

To assess students' understanding:

1. **Categorisation Tests:** Include questions where students classify numbers into natural numbers, whole numbers, integers, rational numbers, and irrational numbers. Use multiple-choice or fill-in-the-blank formats to test their knowledge.

2. **Subset Identification:** Assess students' ability to identify and work with subsets of natural numbers, such as even, odd, prime, and composite numbers. Provide exercises where they sort or categorise numbers and explain their classifications.
3. **Real-Life Problem Solving:** Evaluate students through tasks that involve performing operations on real numbers within real-life contexts. Assess their ability to apply number classifications and operations to solve practical problems.
4. **Application Projects:** Have students create and present projects that demonstrate their understanding of number categories and operations. This could involve solving complex problems or analysing data using the concepts learned.

Week 1: Real Number and Numeration System

Learning Indicators

1. *Categorise real numbers as natural/ counting numbers, whole numbers, integers, and rational and irrational numbers*
2. *Explore the various subsets of counting numbers (even and odd, prime and composite)*

Focal Area: The Real Number System

Introduction

Numbers are the fundamental building blocks of mathematics that form the very foundation of our understanding of the universe. We use numbers in our daily activities. From counting to measuring, comparing amounts and calculations. Numbers are the language of science, engineering, finance, and countless other fields.

Categories of the Real Number system

1. **Natural numbers** are numbers that start from 1 to infinity. They are also known as counting numbers. Counting the number of tables (any available counting objects) in the classroom. In counting you use 1, 2, 3, 4, …

   These set of numbers are called natural numbers. Example: {1, 2, 3, 4, 5, …}

2. **Whole numbers:** When the number 0 is added to the set of natural numbers, we obtain a set of numbers called whole numbers. Example: {0, 1, 2, 3, 4, …}
3. **Positive and negative whole numbers**:

   Examples

   a. The temperature in Aburi yesterday was 20°C In the evening the temperature dropped by 23°C. This means the current temperature will be below 0°C that is −3°C.

   b. If a student is to pay a fee of GH¢ 475.00 but paid GH¢ 470.00 instead, how much is left to be paid? The student now owes GH¢ 5.00. This amount can be written as GH¢ (–5.00).

   These numbers (–3) and (–5) are called negative numbers.

   When we add negative natural numbers to the whole numbers, we have set of numbers called Integers.

   Integers (Z) are positive and negative whole numbers. Example: {…, –4. –3. –2. –1, 0, 1, 2, 3, 4, …}

   Real world examples

Negative Numbers	Positive Numbers
Owing	Owning
Price decrease	Price increase
Temperature below freezing point (0°C)	Temperature above freezing point (0°C)

4. Fractions and Decimal Fractions

Examples

1. Afia is 8 years, six months old. This means, Afia is eight and half years old.
2. The length of an exercise book is 23.7 cm.

When fractions and decimal fractions are added to the set of integers, we have a set of numbers called **rational numbers.**

Example

$Q = \{-4, 3, -2.25, 1, 0, 15.2, \frac{2}{3}, 73\}$

5. Rational numbers have terminating and recurring decimals.

Any number that does not terminate or recur is called an **irrational number.**

Any natural number under square root with the exception of perfect squares (1, 4, 9, 16, 25, etc) are irrational number.

Example

$\sqrt{2} = 1.4142135623730950488016887242097\ldots$

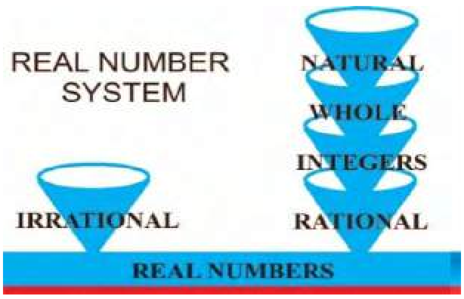
$\sqrt{8} = 2.8284271247461900976033774484194\ldots$

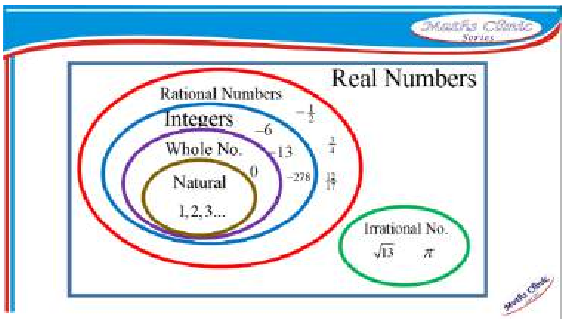
The real value of pi is an irrational number. That is,

$\pi = 3.1415926535897932384626433832795\ldots$

Categories of the Real Number System using models

Establish the relationships between and among the subsets, of real numbers using models (e.g., Venn diagram, number line, etc.)





Learning Tasks for Practice

Learners

1. Use models to categorise the subsets of the real number system.
2. Illustrate the subsets of the real number system using models and give examples where necessary.
3. Analyse the relationships among the various subsets of the real number system.
4. Select one specific area and explore how different types of numbers are applied within that domain. Presentations should include examples, visuals, and explanations to illustrate the relevance of real number concepts in the chosen area.

Application / Importance of numbers

1. Numbers play a crucial role in various aspects of our everyday activities ranging from shopping to cooking, travelling and financial planning etc.
2. Numbers are used when scheduling appointments, estimating travel time and keeping track of deadlines.
3. Numbers help us to follow recipes, accurately measure ingredients and adjust quantities based on the number of people to serve.
4. Medical prescriptions are given based on numbers. That is, taking a specific quantity of drugs a number of times in a day.
5. Numbers helps us to compare unit prices, calculating discounts, determine number of items to buy and which are the best value.

Pedagogical Exemplars

1. **Initiate talk for learning:** Provide opportunities for learners to communicate orally and in writing when discussing the usage and importance of numbers.
2. **Experiential learning**: Engage learners in counting objects available in and around the classroom to build the concepts of natural numbers.
3. **Structuring talk for learning**: Discuss with learners the following scenarios to develop the concept of integers;
   - **a.** Temperature variations in our environment. Temperature variations below freezing point is negative (−) and above freezing point is positive (+)
   - **b.** Age of learners in years and months
   - **c.** Owing and owning something
   - **d.** Elevator floor levels above ground level are positive numbers and floors below ground level are negative numbers.
4. **Collaborative learning**:
   - **a.** Learners in their mixed ability/gender groups measure with ruler and record the length of items like; unsharpened pencils, pens, exercise books etc. to introduce the concept of rational numbers.
   - **b.** Learners in their mixed ability/gender groups list their ages in years and months. Guide learners to convert to fractions.

Key Assessment

1. **Assessment Level 1**:
   - **a.** I am a number that represents the temperature outside. I can be positive or negative, and I don't have any decimal or fractional parts. What category do I belong to?
   - **b.** I am a number that represents half of a whole. I can be expressed as a fraction or a terminating decimal. What category do I belong to?
   - **c.** Categorise the following numbers as natural, integers, rational and irrational.
     $\{\frac{2}{3}, 0.4, \sqrt{2}, 4, -3, \frac{-4}{9}, 37\}$
   - **d.** List any five negative integers.
   - **e.** List the first ten elements of the following subsets of the real number system.
     - **i.** Whole numbers

ii. Natural numbers

2. **Assessment Level 2**
   a. I am a number that can be expressed as a fraction. I am also the square root of 4. What category do I belong to?
   b. I am an infamous number that cannot be expressed as a fraction or a repeating decimal. People often approximate me as 3.14. What category do I belong to?
   c. I am a number that represents the ratio of a circle's circumference to its diameter. I go on forever without repeating. What category do I belong to?
3. **Assessment Level 3**:
   a. Create an artwork using different colours or materials to represent various types of numbers. And use it to explain the relationships between different types of numbers.
   b. In mixed ability/gender groups of 4 or 5, create a presentation (PowerPoint or poster) showcasing how various types of numbers (natural, integers, rational numbers (fractions, decimals) and irrational numbers, are used in:
      i. Cooking
      ii. sports
      iii. construction,
      iv. medicine,
      v. shopping

Focal Area: Exploring Subsets of Counting Numbers (Even and Odd, Prime and Composite)

Introduction

Numbers can be classified into different categories, such as even, odd, composite and prime numbers. Assuming you have a collection of pens. If you can divide them equally into two groups with no pens left over, the total number of pens is **even**.

From the diagram, the number of pens is 8. We can equally divide them into two groups.

If you can't divide them equally, you have an **odd** number of pens. This concept can be applied to any whole number. Even numbers are like having equal players in a football match. While odd numbers are like having one player left out.

Subsets of Counting Numbers

1. **Even numbers** are whole numbers which are divisible by 2 without a remainder.

   Example; {0, 2, 4, 6, 8, 10, …}

2. **Odd Numbers** are whole numbers which give a remainder when divided by 2.

   Example; {1, 3, 5, 7, 9, …}

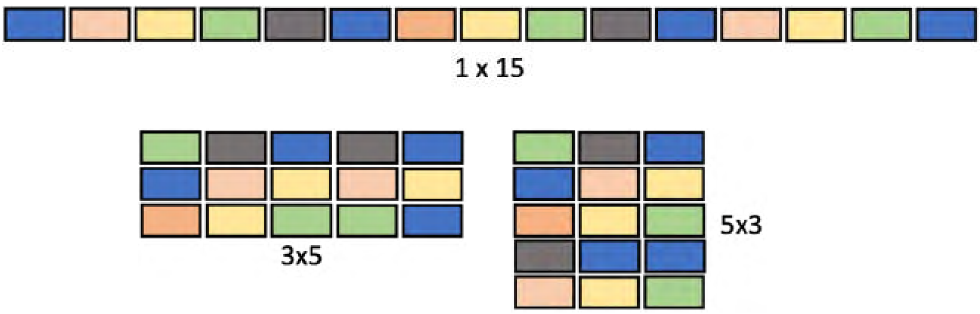
3. **Composite Numbers**

   Examples

   a. Suppose you have a packet containing 12 toffees, and you intend to distribute them among your friends. You can divide these toffees evenly among 2, 3, 4, or 6 friends because 12 can be divided by these numbers without any remainder. Therefore, 12 is termed a composite number since it can be evenly divided into equal groups by numbers other than 1 and, 12, itself.

   b. Again, consider building blocks used to construct different structures. If you have 15 blocks, you can arrange them into rectangles of 1x15, 3x5, or 5x3. These arrangements show that 15 is divisible by 3 and 5, besides 1 and, 15, itself.

   - Therefore, 15 is a composite number because it has more than two factors.



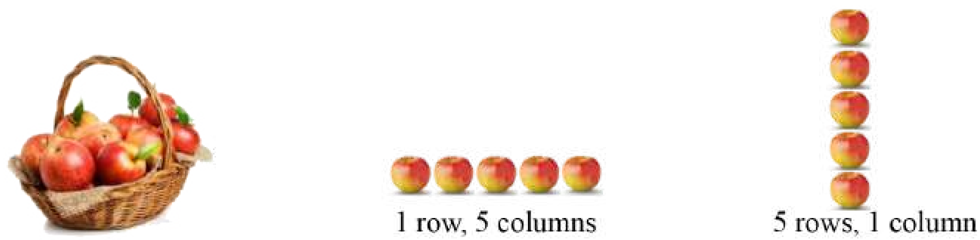
Therefore 12 and 15 are composite numbers.

Composite numbers are numbers with more than two factors.

Example {4, 6, 8, 9, 10, 12, 14, 15, …}

4. **Prime Numbers**

   You have a basket of apples, and you want to arrange them into rows or column. If you have 5 apples, you can only arrange them in a single row or column. You cannot form equal rows or columns other than 1 and 5 apples per row or column. This implies that 5 is a prime number because it has exactly two factors: 1 and itself, 5.



A Prime Number is a number that has only two factors, that is 1 and itself.

Example {2, 3, 5, 7, 11…}

Application of the Concept:

1. Helps to identify and understand patterns and relationships between different types of numbers.
2. Used to generate perfect squares.
3. Used in the Pythagoras's theorem and Number theory.

Learning Tasks for Practice

Limit content expectations to the listing and explaining the various subsets of counting numbers. Extend content expectations to stating the elements in a given set on subsets of counting numbers.

Extend content expectations to creating word problems on odd and even numbers.

1. Provide learners with practice problems to identify even and odd numbers. Offer exercises to determine whether a given number is prime or composite.
2. Encourage students to explore real-life scenarios where even and odd, prime and composite numbers are encountered, fostering a deeper understanding of their relevance.
3. Recognising these properties helps in various mathematical operations and problem-solving. Practice exercises and real-life applications enhance comprehension and applicability.

Pedagogical Exemplars

1. **Experiential / Collaborative learning:**

   a. Engage in a practical scenario of sharing 8 pens between two learners. Following the distribution, learners engage in a discussion to determine the quantity of pens each received. Through this hands-on activity, the concept of equal sharing unfolds. Learners recognise that since both individuals have an equal number of pens, the total count of 8 pens signifies an even number.

   b. Distribute 7 pencils among learners in mixed-ability/ gender groups. The learners are tasked with sharing the pencils within their groups, engaging in discussions about the outcomes. Through this hands-on activity, learners discover that, upon sharing, one pencil remains unallocated. This collective realisation leads to the understanding that 7 is categorised as an odd number.

   c. A dozen toffees were placed in a tray, and the teacher sequentially invited one, two, three, four, six, and twelve learners to take turns sharing the toffees among themselves. The outcomes of each sharing session were diligently recorded by the teacher on the board.

   - One learner receives 12 toffees ($1 \times 12 = 12$). 1 and 12 are factors of 12
   - Two learners each receives 6 toffees ($2 \times 6 = 12$). 2 and 6 are factors of 12.
   - Three learners each receive 4 toffees ($3 \times 4 = 12$). 3 and 4 are factors of 12.
   - Four learners each receive 3 toffees ($4 \times 3 = 12$). 4 and 3 are factors of 12.
   - Six learners each receive 2 toffees ($6 \times 2 = 12$) 6 and 2 are factors of 12.
   - Twelve learners each receive 1 toffee ($12 \times 1 = 12$). 12 and 1 are factors of 12

   This implies that 12 has six factors (i.e. 1, 2, 3, 4, 6, and 12). Therefore, 12 is a composite number because it has more than two factors.

   d. Distribute any 5 items to learners in mixed ability/ gender groupings, direct them to organise the items in an equal number of rows. Upon exploration, the learners recognise that arranging the items is feasible only in two configurations: a single row of five or five rows of one. This means that 5 has only two factors (1 and 5). Therefore, all whole numbers with only two factors are prime.

Key Assessment

1. **Assessment Level 1**

   a. Identify whether the following numbers are even or odd: 7, 14, 19, 22, 78.

   b. Classify the following numbers as prime or composite: 2, 3, 7, 9, 18, 25, 118.

c. Find the first five prime numbers greater than 20.

d. List the first 10 composite numbers

2. **Assessment Level 2**

a. Group the following numbers into even and odd numbers {1, 3, 4. 5, 7, 6, 10, 24, 33}

b. A mother has 14 oranges If she wants to give an equal number of oranges to her 3 children, will there be any leftover oranges?

Explain your answer using even or odd numbers.

c. List all the factors of 9. Is it prime or composite?

3. **Assessment Level 3**

a. Explain why the number one (1) is not composite

b. Find the first six (6) prime numbers greater than 50.

c. The numbers 13 and 31 are prime numbers. Both these numbers have the same digits 1 and 3. Find such pairs of prime numbers up to 100.

d. Create a word problem involving even or odd numbers and solve it.

Week 2: Integers and Operations on Integers [Revision]

Learning Indicator: *Perform operations on the set of real numbers using real-life contexts.*

Focal Area: Addition and Subtraction of Integers

Introduction

In this focal area we will explore the basic arithmetic processes of addition and subtraction involving integers. We will also delve into the rules and techniques in solving real life problems.

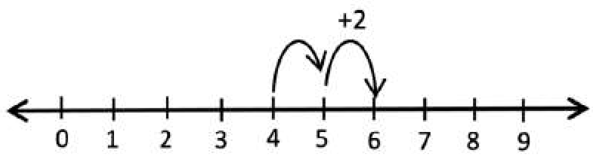
The result of adding two or more numbers is a **sum,** and the result of subtracting a number from another is the **difference.**

Using the number line to performs operations on integers

Addition represents moving towards the right. Subtraction represents moving towards the left.

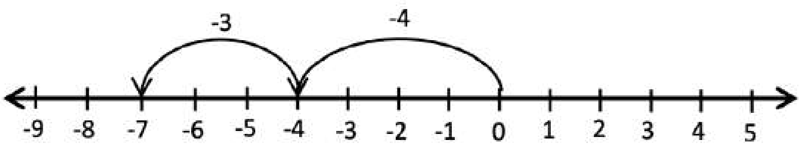
Examples

a. Kwame moved +4 steps from the starting point at 0 and then proceeded to take an additional +2 steps. What is the total number of steps he took?



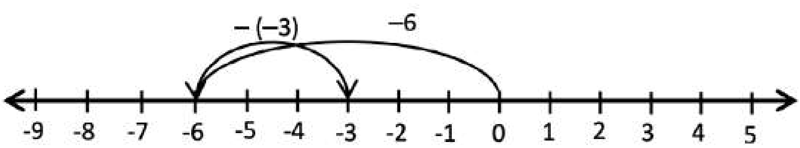
$4 + 2 = 6$

b. Alima borrowed *GH¢* 4.00 from her friend to buy food. She borrowed an extra *GH¢* 3.00 later, how much does she owe?



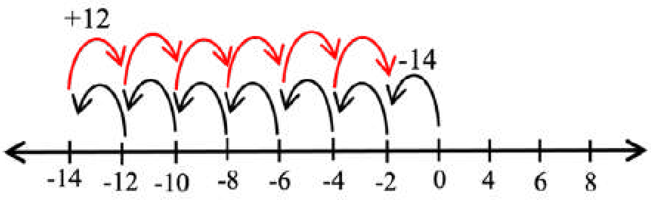
$-4 + -3 = -7$

c. Mawuli borrowed GH¢ 6.00 from Esi, and later returned GH¢ 3.00. The expression (–6) – (–3) which is (–6) + (3) models this situation.



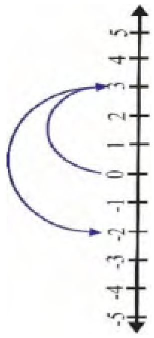
$(-6) - (-3) = -3$

d. Mensima has a jar of toffees, initially, she had 14 fewer toffees than she needed for a party. Kwesi gives her an additional 12. The expression $(-14) + 12$ models this situation.



$$(-14) + 12 = (-2)$$

e. Find $3 + (-5)$



$3 + (-5) = -2$

Learning Tasks for Practice

1. Provide a series of integer addition problems for students to solve, both horizontally and vertically. Include a mix of positive and negative integers to reinforce the rules of adding integers. Limit content expectations to addition and subtraction of integers.
2. Present subtraction problems involving integers, emphasising the concept of "adding the opposite." Include scenarios where students need to subtract a negative number or subtract a positive number from a negative integer. Extend content expectations to solving word problems on addition and subtraction of integers.
3. Present word problems or real-life scenarios where students need to use integer operations to solve problems. For example, scenarios involving finances, temperatures, or distances can help students understand the practical applications of integer operations. Extend content expectations to include writing word problems on addition and subtraction on integers and solving them

Pedagogical Exemplars

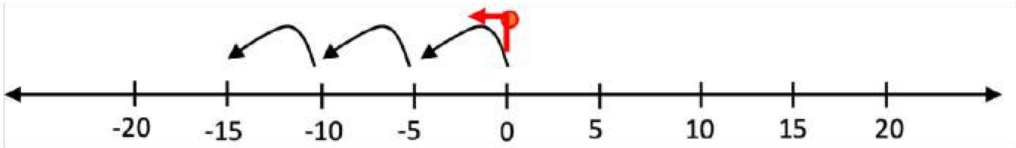
1. **Think-pair-share:** In pairs, learners engage in different problem-solving processes in numbers to perform basic operations on real numbers. Pose a question related to real number operations, such as "How would you explain adding a positive and a negative number?" Allow learners time to think individually, discuss their thoughts with a partner, and then share their ideas with the class. This encourages active engagement and provides a platform for diverse perspectives.
2. **Experiential learning**: Provide concrete manipulatives like number lines, counters, or coloured chips and ask learners to physically manipulate these objects to represent addition and subtraction of real numbers. For example, use counters to model combining positive and negative values.

3. Ask learners to perform addition by moving to the right or forward and subtraction by moving to the left or backward.
4. **Structuring Talk for learning**: Conduct structured mathematical discussions. Present a problem-solving scenario involving real numbers and guide learners in discussing their approaches, strategies, and reasoning. Encourage them to use precise mathematical language to express their thoughts.

Focal Area: Multiplication and Division of Integers

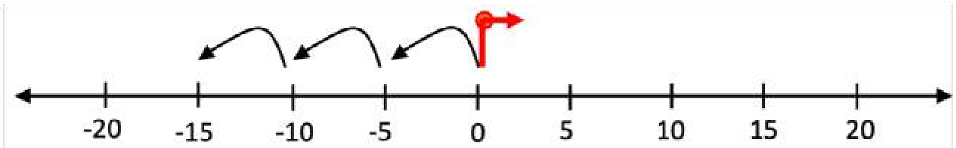
Example:

1. Evaluate –5 × 3



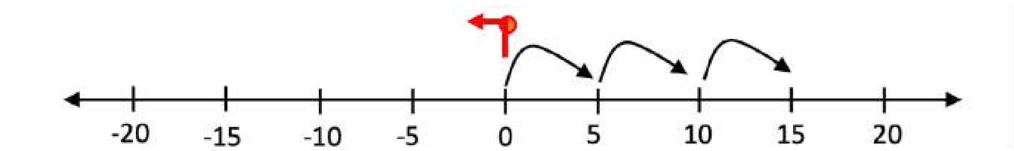
The negative attached to the number 5 indicates the direction you will face at the origin as shown in the diagram above. The 3 indicates the size of the movement from the origin in that direction. This implies that, there is a jump of each one of 5, three times, from the origin. Therefore, –5 × 3 = –15.

2. Evaluate 5 × –3



In the second example, the number 5 is positive and it indicates the direction you will face at the origin as shown in the diagram above. The number 3 is now negative, and it indicates the direction of the movement from the origin. This implies that, there is a jump of each one of 5 backward (negative direction) from the origin. Therefore, 5 × –3 = –15.

3. Evaluate –5 × –3



In the third example, the number 5 is negative and it indicates the direction you will face at the origin as shown in the diagram above. The number 3 is also negative, and it indicates the direction of the movement from the origin. This implies that, there is a jump of each one of 5 backwards (negative direction) from the origin. Therefore, –5 × –3 = 15.

4. If the cost of a notebook is GH¢ 13.00. How much would you pay for, if you buy 16 of them?

Solution

13 × 16 = 208

From the above illustrations, the following rules of multiplication can be deduced.

Rules of multiplication

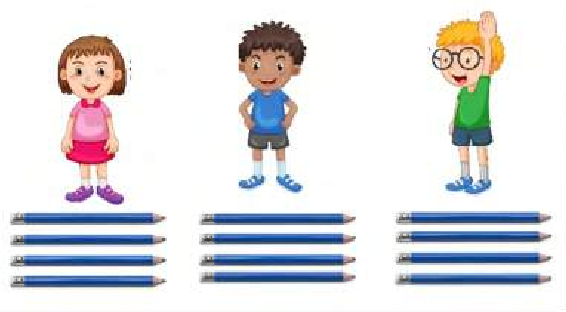
In multiplication, the sign of the product depends on the signs of the numbers being multiplied.

- Positive × Positive = Positive (e.g., 3 × 4 = 12)
- Negative × Negative = Positive (e.g., –5 × –2 = 10)
- Positive × Negative = Negative (e.g., 2 × –7 = –14)
- Negative × Positive = Negative (e.g., −2 × 7 = –14)

Division of Integers

Remember that division is the process of sharing or distributing quantities into smaller groups.

a. Kofi is to share 12 pencils among his three friends. How many pencils will each receive?



From the illustration, each friend receives 4 pencils. Therefore, 12 ÷3 = 4

b. Your class is going on a field trip and the bus can hold 36 passengers There are 108 students in your class. How many buses will be needed to transport all the students?

Solution

This is a division problem: 108 ÷36 = 3

c. A household owes electricity bill of GH¢450.00. This amount was shared among three members equally, how much will each tenant pay?

Solution

Since this is a debt, the amount owe will be written as – 450. Therefore, each member will pay an amount of – 450 ÷ 3 = − 150, which is a debt and can be written as – 150.

d. Again, you owe a friend GH¢30, and you owe another friend GH¢10. "What happens if you divide your total debt (–GH¢40) equally between these two friends?" To divide the total debt (–GH¢40) by the number of friends (–2) gives a positive result, indicating each friend receives a share of the debt.

Mathematically, – 40 ÷ – 2 = 20.

Rules of division

In division, the sign of the division depends on the signs of the numbers being divided.

- Positive ÷ Positive = Positive (e.g., 12 ÷ 4 = 3)
- Negative ÷ Negative = Positive (e.g., –20 ÷–2 = 10)
- Positive ÷ Negative = Negative (e.g., 28 ÷ –2 = –14)
- Negative ÷ Positive = Negative (e.g., −28 ÷ 2 = –14)

Learning Tasks for Practice

1. Use the number line to solve problems on multiplication and division of integers. Solve word problems involving multiplication and division of integers. Model word problems on multiplication and division of integers.
2. Create problem-solving tasks based on real-world contexts, such as budgeting, temperature changes, or distance calculations. Ensure that scenarios are relatable and culturally relevant to students from different backgrounds

Pedagogical Exemplars

1. **Initiating talk for learning:** In a whole class discussion, explore concepts of the operations of integers using multiplication and division.
2. **Collaborative learning:** In mixed ability/gender groups of five, use models to explore the multiplication of integers.

   E.g., pencils, coloured chips, number line.
3. **Managing Talk for learning:** In a whole class discussion, use the number line to investigate the rules of multiplication and division such that:
   - **a.** Positive × Positive = Positive
   - **b.** Negative × Negative = Positive
   - **c.** Positive × Negative = Negative
   - **d.** Negative × Positive = Negative
   - **e.** Positive ÷ Positive = Positive
   - **f.** Negative ÷ Negative = Positive
   - **g.** Positive ÷ Negative = Negative
   - **h.** Negative ÷ Positive = Negative
4. **Experiential learning:** teacher gives 12 pencils to a learner to share among three friends from the larger group. Learners then discuss the number of pencils each receives.
5. **Structuring Talk for learning:** in a whole class discussion create a scenario where a household owes electricity bill of GH¢450.00, and that this amount was shared among three members equally, how much will each member pay? Discuss with learners how the debt of GH¢450.0 will be negative and will be divided by positive 3
6. **Problem based learning:** Put learners in their mixed ability gender groups and create another scenario of a learner owing a friend GH¢10 (negative debt). If he/she pays the friend back GH¢10 (negative payment), The learner's debt is settled (positive outcome), think, ink, and share your ideas.

Application / Importance of operations on real numbers

Operations on real numbers, such as addition, subtraction, multiplication, and division, are fundamental mathematical concepts with widespread applications in various fields.

Here are some key areas where these operations are applied and their importance:

1. Finance and Economics: Operations on real numbers are used to calculate profits, losses, interest rates, and investment returns.

2. Science and Engineering: Operations on real numbers are used for measurements, data analysis, and calculations.
3. Statistics: Operations on real numbers are used in analysing and interpreting data to make informed decisions.
4. Geometry and Mathematics: Operations on real numbers are used to solve geometric problems, calculate angles, and analyse shapes and structures, etc.

Key Assessment

1. **Assessment Level 2**
   - **a.** Using a number line and other models (coloured chips, counters),
     Find the sum of:
     - **i.** 4 and 2
     - **ii.** 5 and – 3
   - **b.** Find the difference between:
     - **i.** 6 and – 3
     - **ii.** – 4 and – 5
   - **c.** You have *GH¢* 25.00 and spend *GH¢* 12.00. How much money do you have left?
   - **d.** The temperature was 35 degrees Celsius. It dropped by 9 degrees.
     What is the new temperature?
   - **e.** The temperature was 20 degrees Celsius. It rose by 8 degrees.
     What is the new temperature?
   - **f.** The temperature was 8 degrees Celsius. It dropped by 10 degrees.
     What is the new temperature?

Section Review

In this section, we explored the categorisation and operations involving real numbers, focusing on the following key concepts:

1. **Categorisation of Real Numbers:** We learned to categorise real numbers into various subsets:
   - **Natural/Counting Numbers:** These are the numbers used for counting {1, 2, 3, ...}.
   - **Whole Numbers:** These include all natural numbers along with zero {0, 1, 2, 3, ...}.
   - **Integers:** These extend whole numbers to include negative numbers
     {-3, -2, -1, 0, 1, 2, 3, ...}.
   - **Rational Numbers:** These are numbers that can be expressed as a fraction where both the numerator and denominator are integers, and the denominator is not zero (e.g. $\frac{1}{2}, -3\frac{}{4}$).
   - **Irrational Numbers:** These cannot be expressed as a simple fraction and have non-repeating, non-terminating decimal expansions (e.g. $\sqrt{2}, \pi$).
2. **Subsets of Counting Numbers:** We examined the different subsets within counting numbers:
   - **Even and Odd Numbers:** Even numbers are divisible by 2, {2, 4, 6, ...}, while odd numbers are not {1, 3, 5, ...}.

- ○ **Prime Numbers:** These are numbers greater than 1 with no divisors other than 1 and themselves {2, 3, 5, 7, ...}.
- ○ **Composite Numbers:** These are numbers greater than 1 that have divisors other than 1 and themselves {4, 6, 8, ...}.

3. **Operations on Real Numbers:** We applied various operations on real numbers in real-life contexts:
   - ○ **Addition and Subtraction:** Combining quantities or finding differences, such as balancing a checkbook **o**r determining the net change in temperature.
   - ○ **Multiplication and Division:** Scaling quantities or distributing items evenly, like calculating total cost or dividing resources among groups.
   - ○ **Application in Real-life Situations:** Practical examples included calculating distances, budgeting expenses, and analysing data sets.

Through engaging activities, we categorised real numbers, explored subsets of counting numbers, and performed operations on real numbers using practical examples. This comprehensive understanding enhances our ability to apply mathematical concepts to real-world scenarios.

SECTION 2: FRACTIONS AND OPERATIONS ON FRACTIONS

Strand: **Numbers for Everyday Life**

Sub-Strand: Number Operations

Content Standard: Demonstrate understanding of basic operation on common and decimal fractions and apply them to solving real-life problems.

INTRODUCTION AND SECTION SUMMARY

This section focuses on understanding and working with fractions, including naming, comparing, and ordering them. We start by identifying and ordering numbers expressed as the quotient of two integers, ensuring that the denominator is not zero. Next, we recognise and name equivalent fractions through visual aids such as pictorial representations and number lines. We also compare and order fractions with the same denominators using pictorial methods and relational symbols (>, <, =). Finally, we apply these concepts to solve problems involving the four basic operations with fractions, providing a comprehensive understanding of how to manipulate and interpret fractions in various mathematical contexts.

The section will cover the following concepts:

1. *Name, compare and order numbers expressed as a quotient of two integers where the denominator is not equal to zero.*
2. *Recognise and name equivalent fractions using pictorial representations and number line.*
3. *Compare and order fractions with like denominators by using pictorial representations and >, < and =.*
4. *Solve problems on fractions involving the four basic operations.*

SUMMARY OF PEDAGOGICAL EXEMPLARS

To effectively teach these fraction concepts:

1. **Fraction Identification and Ordering:** Use visual aids like fraction strips and pie charts to help students name and compare fractions. Interactive activities, such as fraction games or hands-on manipulatives, can make this process engaging and clear.
2. **Equivalent Fractions:** Provide students with pictorial representations and number lines to visually compare and identify equivalent fractions. Encourage them to use these tools to understand the concept of equivalence and reinforce their learning through practice exercises.
3. **Comparison and Ordering with Like Denominators:** Use visual aids and manipulatives to help students compare and order fractions with the same denominators. Activities should involve sorting fractions and using relational symbols to demonstrate their understanding.
4. **Problem Solving with Fractions:** Engage students in solving real-life problems that involve performing the four basic operations on fractions. Provide contextual problems that require addition, subtraction, multiplication, and division of fractions, encouraging students to apply their knowledge in practical situations.

ASSESSMENT SUMMARY

To assess students' understanding of fractions:

1. **Naming and Ordering Fractions:** Include questions where students name fractions expressed as the quotient of two integers, compare and order these fractions. Use various formats such as multiple-choice, short answer, or visual comparison tasks.
2. **Identifying Equivalent Fractions:** Assess students' ability to recognise and name equivalent fractions using pictorial representations and number lines. This can be evaluated through exercises that involve matching or filling in equivalent fractions.
3. **Comparing and Ordering with Like Denominators:** Evaluate students' skills in comparing and ordering fractions with like denominators. Use problems that require them to use pictorial representations and relational symbols ($>$, $<$, $=$) to show their understanding.
4. **Fraction Operations:** Test students' proficiency in solving problems involving the four basic operations with fractions. This could include word problems, calculations, and contextual problems that demonstrate their ability to apply fraction operations accurately.

Week 3: Fractions and Its Applications

Learning Indicators

1. *Name, compare and order numbers expressed as a quotient of two integers where the denominator is not equal to zero.*
2. *Recognise and name equivalent fractions using pictorial representations and number line.*
3. *Compare and order fractions with like denominators by using pictorial representations and >, < and =.*

Focal Area: Concept of Fractions

Introduction

In this week, we will explore fractions, which are critical components of mathematical understanding. Fractions represent parts of a whole, which allows us to express numbers that are not whole quantities.

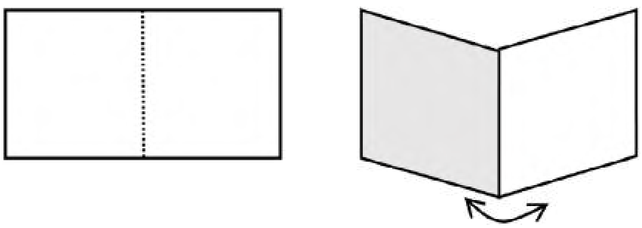
Fractions are important in many real-life scenarios such as dividing bread into slices, measuring ingredients for recipes, and many more.

Fractions are not only about calculations, but they are about understanding relationships and proportion. We will learn to interpret fractions visually, using diagrams and models to represent fractional quantities.

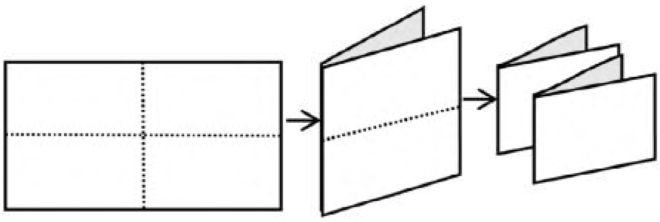
Fractions

Assuming you fold a rectangular piece of paper into two equal parts. How many parts and folds are there? What do we call each part? How many halves are there in a whole?

If you fold the paper into two equal pieces; there will be 2 equal parts and 1 fold. Each part created by folding the paper is called "half" because it represents one of two equal parts. There are two halves in a whole.

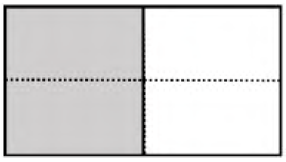


If you later fold the halves of the paper strip how many parts will there be? What do we call each part? Show me two quarters. What is another name for two quarters? Which is larger: one-half or one-quarter? How do you know, explain your answer?



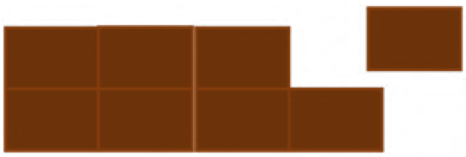
If you later fold the halves of the paper strip into half: you will have four equal parts. Each part created by folding the paper is called a "quarter because it represents one of four equal parts. There are two halves in a whole. To show two quarters, you would fold one half into 2 equal parts and then fold the other half into 2 equal parts. Therefore, two quarters would be represented by two of these resulting parts.

$\frac{2}{4}$



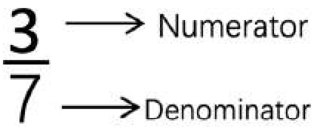
Another name for two quarters is "one half." This is because two quarters equals one half. You can see this by comparing the sizes: visually, two quarters will be the same size as one half when laid side by side. In this case, $\frac{2}{4}$(two quarters) is equivalent to $\frac{1}{2}$ (one half). Therefore, one half is larger than one-quarter.

Assuming you have a chocolate bar with 8 equal pieces and you ate 1 piece, this means you have eaten one-eighth $\left(\frac{1}{8}\right)$.



Therefore, we can say that **fractions** represent part of a whole. Whenever we talk about fractions, we are talking about dividing a whole into smaller, equal parts.

Components of fractions



Numerator represents the number of parts we have out of the whole.

Denominator represents the total number of equal parts into which the whole is divided.

Types of fractions

1. **Proper fractions:** These are fractions where the numerator is smaller than the denominator (e.g. $\frac{1}{2}$, $\frac{3}{10}$, etc.)
2. **Improper fractions:** These are fractions where the numerator is bigger than the denominator (e.g. $\frac{5}{2}$, $\frac{7}{5}$, etc.)
3. **Unit fraction:** These are **fractions where the numerator is one (1). They represent 1 shaded part of all the equal parts of the whole (**e.g. $\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{13}$, etc.)
4. **Mixed fractions:** These are numbers that combine a whole number with a proper fraction.

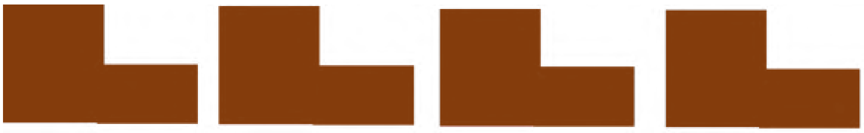
   (e.g. 1 $1\frac{}{4}$, that is, one whole and one-quarter more)

Examples

a. A student shared 3 bars of chocolate among 4 friends. What fraction will each friend receive?

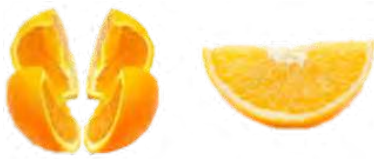
In this case, the student will divide each bar into four equal parts. The total will be 12 parts or pieces.

Each friend will receive a fraction of $\frac{3}{4}$. This is a proper fraction.



b. Hilda's mum is sharing slices of 3 oranges for her and her siblings. She slices each orange into 4 equal parts.

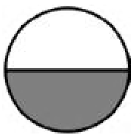
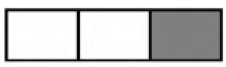
If her siblings first had four slices of one orange and then she had one slice of the same size as the second orange. What is the total fraction of slices of oranges eaten?

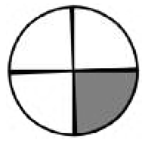
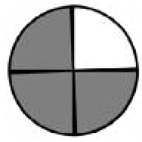
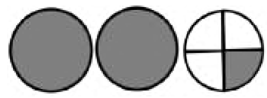


In this case, the total oranges eaten are 5 slices of the orange which is one whole and 1 out 4 slices. ($1\frac{1}{4}$) or ($\frac{5}{4}$). $1\frac{1}{4}$ is a mixed fraction or number and $\frac{5}{4}$ is an improper fraction.

Naming Fractions

Based on the visual representations and the real -life activities above, learners should be able to use the language of fractions in naming and writing them correctly. The table below shows the names and visual representations of some fractions.

Fraction	Name of fraction	Visual representation of a fraction
$\frac{1}{2}$	One half	
$\frac{1}{3}$	One-third	

Fraction	Name of fraction	Visual representation of a fraction
$\frac{1}{4}$	One-quarter or one-fourth	
$\frac{3}{4}$	Three-quarters	
$\frac{3}{5}$	Three-fifths	
$2\frac{1}{4}$	Two and one-fourth	

Learning Tasks for Practice

1. Identify and classify fractions based on their types (proper, improper, mixed), equivalent fractions, and relationships to whole numbers.
2. Represent fractions using visual models, such as fraction circles, bars, and number lines.

Application of fractions

- Recipes often require measurements in fractions, such as $\frac{1}{2}$cup of flour or $\frac{3}{4}$ teaspoon of salt. Understanding fractions helps learners accurately measure ingredients and adjust recipes as needed.
- Fractions are essential for understanding units of measurement. such as metres, centimetres inches, grams and kilograms. Learners use fractions to measure lengths, weights, volumes, and other quantities in real-world situations.
- Understanding fractions helps learners manage money effectively. They can calculate discounts, percentages, and proportions when shopping, budgeting, or comparing prices.
- Learners use fractions to tell time, such as half past 9 (9:30) or a quarter to 10 (9:45).

Pedagogical Exemplars

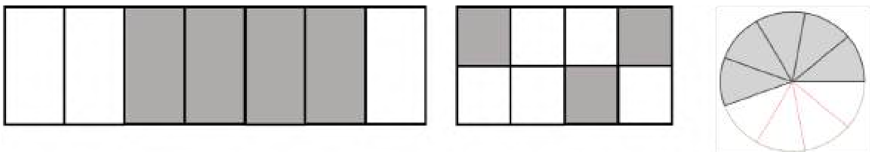
1. **Initiating Talk for learning:**
   a. In a whole class discussion, explore learners' understanding of concepts of fractions by creating a scenario where a piece of paper is folded into two equal pieces and discuss how many parts and folds are there. What do we call each part? How many halves are there in a whole?
   b. Create a scenario where a chocolate bar is divided into 8 equal parts and 1 piece is eaten, Discuss, with learners how you would represent the fraction of the chocolate consumed and what, for example, the 1 and the 8 represent in the fraction.
   c. Ask learners in their groups to fold the halves they have and discuss and ink the number of parts they will get and the name for each part.
2. **Collaborative learning:** In mixed ability/gender small groups, ask learners to fold a piece of paper into two equal pieces. Learners discuss and ink the number of parts, and folds, what each part is called, and the number of halves in a whole.

3. **Experiential learning:**
   a. Use visual models (such as paper folding, fraction bars, circles, pizza, orange etc.) and solve word problems involving fractions.
   b. Based on the folded piece of paper, ask learners to show and ink two-quarters, and give the other name for two-quarters. Learners then discuss which is larger: one-half or one-quarter and how they know.
   c. Using a visual model and representations of some fractions on the board, let learners think-pair and share their thoughts on them.
4. **Problem-based learning:** Learners in their mixed-ability gender small groups, create another scenario on how they can share 3 bars of chocolate among 4 friends and what fraction will each friend receive. Think, ink, and share your ideas.
5. **Structuring Talk for learning:** In a whole class discussion, create a scenario such as, "Hilda's mum is sharing slices of oranges with her and her siblings. If her siblings first had four slices of one orange and then she had one slice of the same size as the second orange. Discuss, with learners the total fraction of slices of oranges eaten".
6. **Problem-based learning:** learners in their mixed-ability gender small groups, create another scenario where Bryan wants to create a study schedule for his upcoming examination. He plans to study for a total of 3 hours each day but wants to break up his study time into two equal sessions, think, ink and share on how long each study session will last.

Key Assessment

1. **Assessment Level 1**
   a. Explain the difference between proper, improper, and mixed fractions.
   b. What fractions are shaded in the following diagrams.

a. b. c.



   c. Draw, shade, and name the fractions below.
      i. $\frac{3}{7}$
      ii. $\frac{3}{5}$
      iii. $\frac{4}{5}$
      iv. $\frac{1}{3}$
2. **Assessment Level 2**
   a. A pizza is divided into 8 equal slices. Tom ate 3 slices.
      What fraction of the pizza did he eat?
   b. Adamu can complete his project work in 6 hours.
      What fraction of the work can he do in 2 hours?
   c. You have $\frac{3}{4}$ cup of flour and a recipe that calls for $\frac{5}{8}$ cup.
      Can you fulfil the recipe with the available flour? Explain your answer using fractions and comparison of quantities.

3. Assessment Level 3

a. Mary has $\frac{2}{3}$ of a cake left. If she wants to divide it equally among 4 friends, how much cake will each friend get?

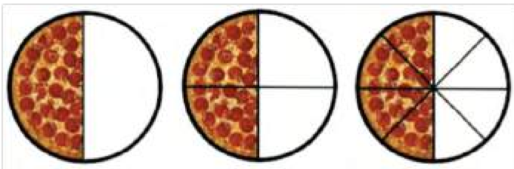
b. Panyin and Kakra are running a race. Panyin completes $\frac{3}{4}$ of the race, while Kakra completes $\frac{5}{8}$ of the race.

Who ran a greater portion of the race?

Focal Area: Equivalent Fractions

Example 1: Using a Pie Chart

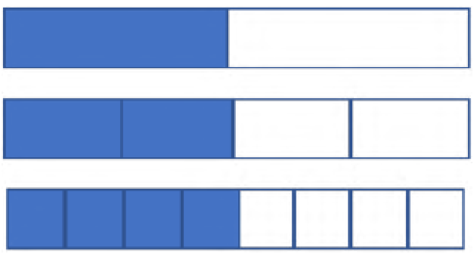
- **Step 1**: Draw three circles (pie charts) of the same size.
- **Step 2**: Divide the first circle into 2 equal parts and shade 1 part. This represents the fraction $\frac{1}{2}$.
- **Step 3**: Divide the second circle into 4 equal parts and shade 2 parts. This represents the fraction $\frac{2}{4}$.
- **Step 4:** Divide the third circle into 8 equal parts and shade 4 parts. This represents the fraction $\frac{4}{8}$.



Observation: All shaded areas are the same, showing that $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$

Example 2: Using Rectangles

- **Step 1**: Draw three identical rectangles.
- **Step 2**: Divide the first rectangle into 2 equal parts and shade 1 part. This represents the fraction $\frac{1}{2}$.
- **Step 3**: Divide the second rectangle into 4 equal parts and shade 2 parts. This represents the fraction $\frac{2}{4}$.
- **Step 4**: Divide the third rectangle into 8 equal parts and shade 4 parts. This represents the fraction $\frac{4}{8}$.



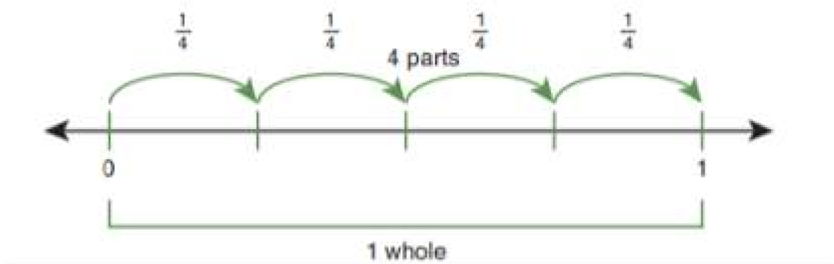
Observation: All shaded areas are the same, showing that $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$

Number Line Representations

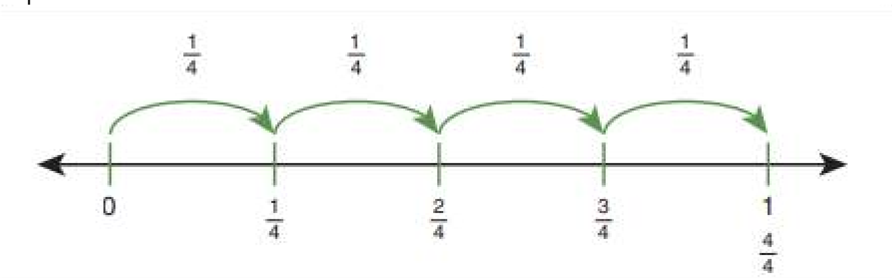
Represent a fraction $\frac{a}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognise that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.

Example

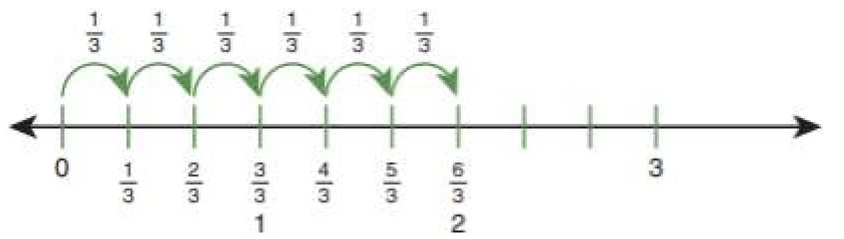
1. Representing $\frac{1}{4}$ on the number line requires learners to understand the distance from 0 to 1 represents one whole. When they partition this distance, the whole, into 4 equal parts, each part has the size of $\frac{1}{4}$. They also reason and justify the location of unit fractions by folding strips or on the number line. Previous work with fraction strips or fraction bars can be extended to developing parts on the number line.



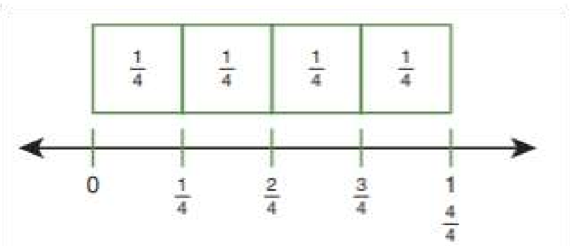
2. Represent the fraction $\frac{3}{4}$ on a number line by marking off lengths of $\frac{1}{4}$ starting at 0. They can explain that 3 pieces of $\frac{1}{4}$ (1 __ 4 + 1 __ 4 + 1 __ 4) or that the distance from 0 to that point represents $\frac{3}{4}$ on the number line.



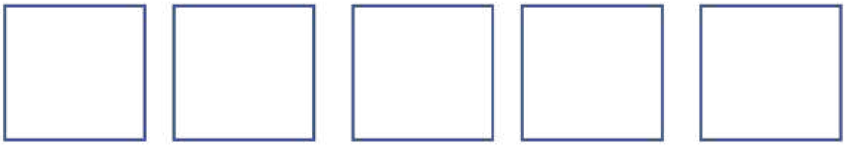
3. This concept can be extended to demonstrate improper fractions.

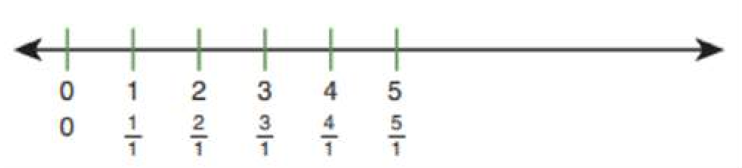


4. Model labelling unit fraction intervals on the number line. And ask students to use the unit fraction intervals to "count" and label the fraction name for each division from zero to one.

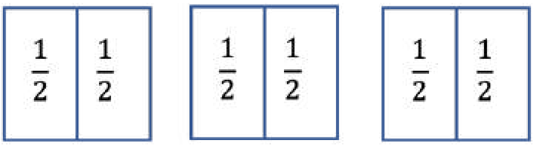


5. Here we can see that we have 5 whole ones, which is equivalent to $\frac{5}{1}$.

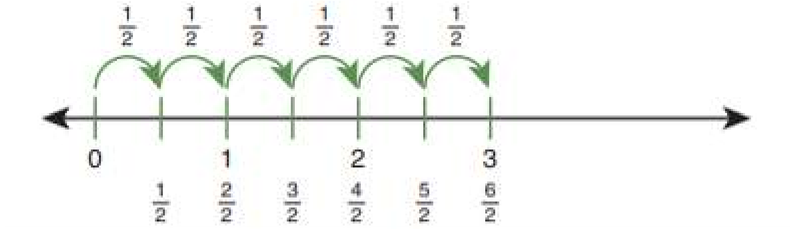




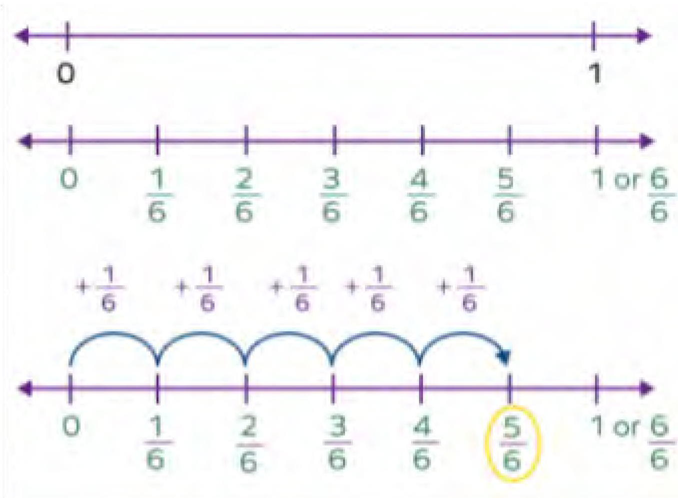
6. Here we have 3 wholes, which is equivalent to $\frac{6}{2}$.



$3 = \frac{6}{2}$ pieces / Parts in each whole



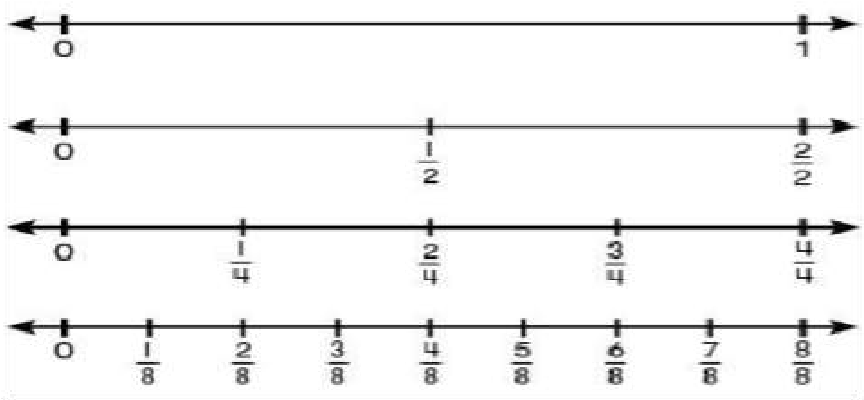
7. Here we are dividing a whole into sixths and how we can represent $\frac{5}{6}$.



Example:

Number Line for $\frac{1}{2}$ and $\frac{2}{4}$

- **Step 1**: Draw four number lines.
- **Step 2**: Label the first one from 0 to 1.
- **Step 2**: Divide the second number line into 2 equal parts. The point at the first division is $\frac{1}{2}$.
- **Step 3** Divide the third number line into 4 equal parts. The point at the second division is $\frac{2}{4}$.
- **Step 4**: Divide the fourth number line into 8 equal parts. The point at the fourth division is $\frac{4}{8}$.



Observation: The points $\frac{1}{2}$, $\frac{2}{4}$ and $\frac{4}{8}$ are at the same location on the number lines, showing that they are equivalent.

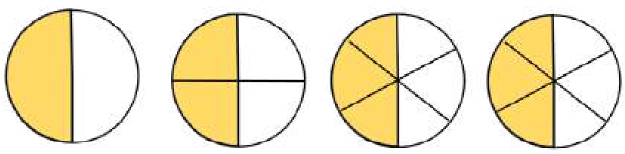
Example

1. $\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$
2. $\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$

Using models to compare fractions

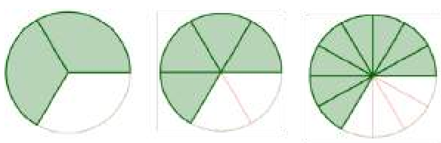
Example

1.



$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$

2.



$\frac{2}{3} = \frac{4}{6} = \frac{8}{12}$

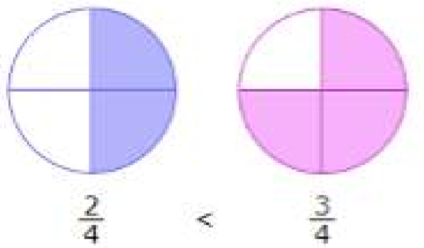
Using the less than "<" and greater than ">" notations in comparing and ordering fractions

Example

1. An exercise book is sold at the canteen at GH¢ 2.50, but it is sold for GH¢ 3.00 at the supermarket. Which is more expensive. Use less than "<" and greater than ">" for the comparison.

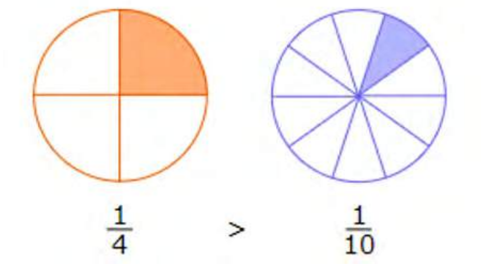
   GH¢ 2.50 is less money than GH¢ 3.00, therefore, GH¢ 2.50 < GH¢ 3.00.

2. Ask learners to observe and identify which shaded fraction represents a larger or smaller portion of the whole using less than "<" or greater than ">"



   When two fractions have the same denominator, the fraction with the bigger numerator is greater.

3. Compare and use the symbol ">" or "<" for your comparison.



   When two fractions have the same numerator, the fraction with the smaller denominator is the greater fraction.

Using equivalent fractions to compare and order fractions.

In comparing unlike fractions, we must use the concept of equivalent fractions to convert all fractions involved to equivalent fractions with common denominators for the purpose of comparing and ordering.

Examples

1. Compare $\frac{3}{4}$ and $\frac{3}{5}$. Which one is greater?

   In order to solve this we must make the two fractions have common denominators. This common denominator will be the lowest common multiple of 4 and 5. In this case that is 20.

   $\frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$

   By multiplying both the numerator and the denominator by 5 we make the common denominator of. 20.

   $\frac{3}{5} = \frac{3 \times 4}{5 \times 4} = \frac{12}{20}$

   By multiplying both the numerator and the denominator by 4 we make the common denominator of. 20.

By comparing the numerators, learners will observe that 12 is less than (<) 15 and 15 is greater than (>) 12.

Therefore, as $\frac{15}{20} > \frac{12}{20}$ then $\frac{3}{4} > \frac{3}{5}$.

2. Which is larger, $\frac{2}{3}$ *or* $\frac{1}{2}$?

   The lowest common multiple of 3 and 2 is 6, so we give the fractions the common denominator of 6.

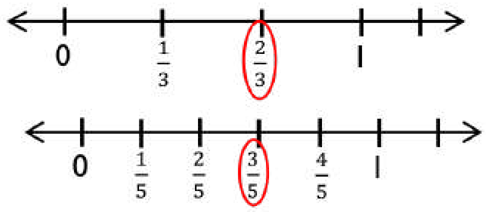
   $\frac{2}{3} = \frac{4}{6}$ and $\frac{1}{2} = \frac{3}{6}$

   As $\frac{4}{6}$ is greater than $\frac{3}{6}$, this means that $\frac{2}{3} > \frac{1}{2}$.

Using number lines to compare and order fractions.

The fraction further to the right is greater.

Compare $\frac{2}{3}$ and $\frac{3}{5}$.



From the number line and comparing the distance from 0 to $\frac{2}{3}$ and that of 0 to $\frac{3}{5}$ it is clear that, $\frac{2}{3} > \frac{3}{5}$.

Learning Tasks for Practice

1. Use visual models (e.g., fraction bars, circles) to compare and order fractions with common denominators.
2. Provide word problems and activities involving comparing and ordering fractions in various contexts (e.g., measurement, money, recipes).
3. Offer additional practice problems and activities involving fractions with different denominators to reinforce understanding for proficient learners.

Pedagogical Exemplars

1. **Think-Pair-Share**: Compare equivalent fractions using cut out sheets of paper representing equivalent fractions. Facilitate a whole-class discussion where learners in pairs share their ideas and strategies.
2. **Experiential Learning:** Learners use shaded fractions on cards to compare various fractions (same numerator as well as those with different denominators).
3. **Problem-Based Learning:** Present students with open-ended problems or scenarios that require them to compare and order fractions. Encourage collaborative problem-solving as students work together to apply their understanding of fractions in meaningful contexts.

Key Assessment

1. Compare the following fractions using $<$, $>$ and $=$

   i. $\frac{1}{3}$ $\quad$ $\frac{1}{4}$

   ii. $\frac{5}{6}$ $\quad$ $\frac{2}{3}$

   iii. $\frac{3}{8}$ $\quad$ $\frac{5}{8}$

2. Order the following fractions from least to greatest: $\frac{2}{5}, \frac{3}{4}, \frac{1}{2}, \frac{5}{6}$

3. Order the following fractions from greatest to least: $\frac{4}{7}, \frac{3}{5}, \frac{5}{8}, \frac{1}{3}$

4. You have 2 loaves of bread. One loaf is divided into 8 slices, and the other loaf is divided into 10 slices. Which loaf has larger slices?

5. Amina has 2 identical chocolate bars. One bar is divided into thirds, and the other is divided into quarters. Which bar has the larger pieces?

Week 4: Operations on Fractions

Learning Indicator: *Solve problems on fractions involving the four basic operations.*

Focal Area: Solve Problems on Fractions Involving the Four Basic Operations

Example:

1. Sarah has $\frac{2}{3}$ of a cup of sugar and needs $\frac{1}{4}$ more for her recipe. How much sugar will she have in total?

Solution

Find a common denominator:

The denominators of the fractions are 3 and 4. The lowest common multiple (LCM) of 3 and 4 is 12. Therefore, we will convert both fractions to have a denominator of 12.

For $\frac{2}{3}$;

$$\frac{2}{3} = \frac{2\times 4}{3 \times 4} = \frac{8}{12}$$

For $\frac{1}{4}$;

$$\frac{1}{4} = \frac{3 \times 3}{4 \times 3} = \frac{3}{2}$$

Now that the fractions have a common denominator, we can add them and simplify, if necessary:

$$\frac{8}{12} + \frac{3}{12} = \frac{8+3}{12} = \frac{11}{12}$$

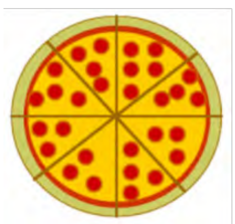
The fraction $\frac{11}{12}$ is already in its simplest form. Therefore, Sarah will have $\frac{11}{12}$ cup of sugar in total.

2. John had $\frac{5}{8}$ of a pizza. He gave $\frac{1}{4}$ of the whole pizza to his friend. How much pizza does he have left?

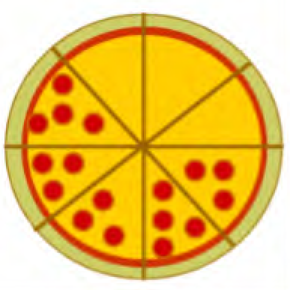
Solution

Draw a circle on the paper plate to represent the pizza.

Divide the pizza into 8 equal slices to represent $\frac{8}{8}$. You can do this by drawing lines through the center of the circle.

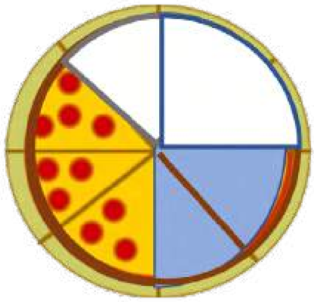


Shade 5 out of the 8 slices to represent $\frac{5}{8}$ of the pizza that John has.



To represent $\frac{1}{4}$ of the pizza, you need to convert it to a fraction with the same denominator (8). So, $\frac{1}{4} = \frac{2}{8}$

Shade 2 out of the 8 slices with a different colour or pattern to show the part that John is giving away.



Count the remaining slices. After removing the 2 slices that John gave away, there should be 3 shaded slices left. These 3 remaining slices represent $\frac{3}{8}$ of the pizza.

To recap on our method:

- Find a common denominator:

  The denominators of the fractions are 8 and 4. The lowest common multiple (LCM) of 8 and 4 is 8 Therefore, we will convert both fractions to have a denominator of 8

  For $\frac{5}{8}$, $\frac{5}{8}$ is already in the desired form with denominator 8.

  For $\frac{1}{4}$, $\frac{1}{4} = \frac{1 \times 2}{4 \times 2} = \frac{2}{8}$

- Now that the fractions have a common denominator, we can subtract them and simplify (if necessary):

  $\frac{5}{8} - \frac{2}{8} = \frac{5-2}{8} = \frac{3}{8}$

  The fraction $\frac{3}{8}$ is already in its simplest form, therefore, John has $\frac{3}{8}$ of pizza left.

3. Add $\frac{1}{3}$ and $\frac{5}{6}$.

Solution

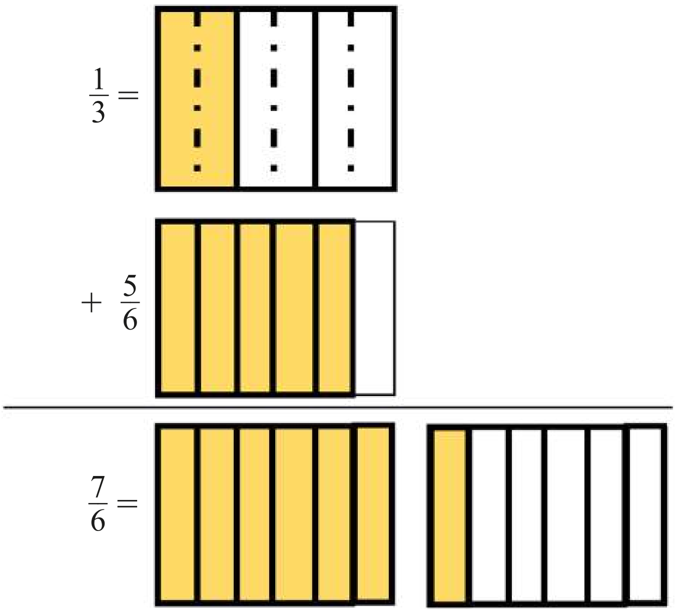
In order to add the fractions we must have a common denominator. The lowest common multiple of 3 and 6 is 6.

$\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$ and $\frac{5}{6}$ already has the denominator of 6.

We can now add our fractions with the common denominators.

$$\frac{2}{6} + \frac{5}{6} = \frac{2+5}{6} = \frac{7}{6} = 1\frac{1}{6}$$

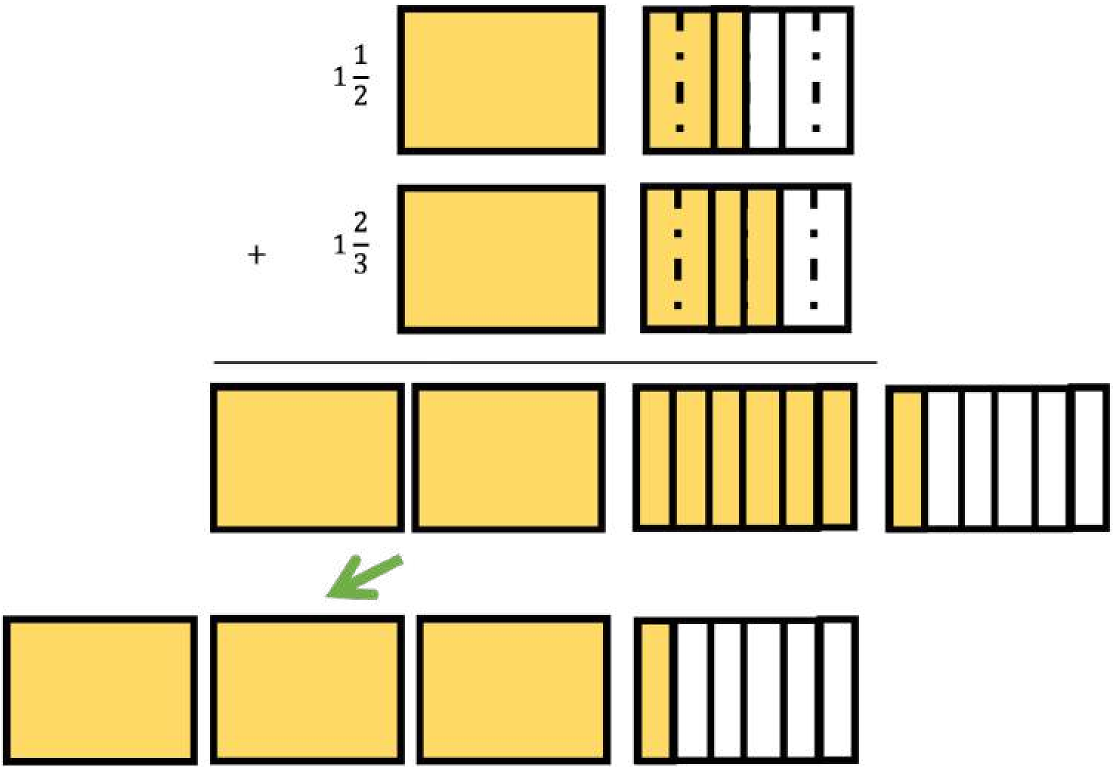
We can also show this with diagrams.



One Whole and One out of six

$= 1\frac{1}{6}$

4. Add $1\frac{1}{2}$ and $1\frac{2}{3}$



Three Wholes and One out of six

$= 3\frac{1}{6}$

This can be interpreted as $1\frac{1}{2} + 1\frac{2}{3} = 1\frac{3}{6} + 1\frac{4}{6}$

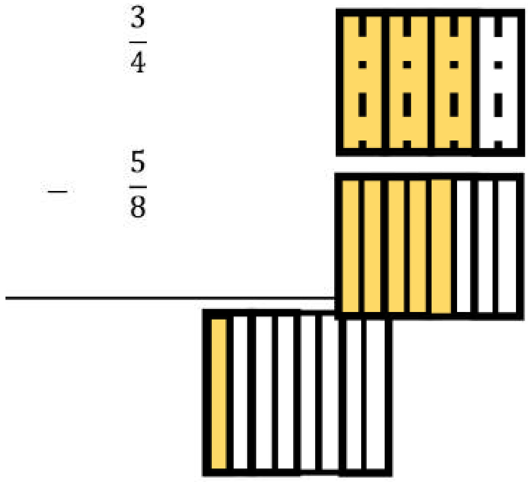
$$= 2\frac{7}{6}$$

$$= 3\frac{1}{6}$$

Subtraction of fractions

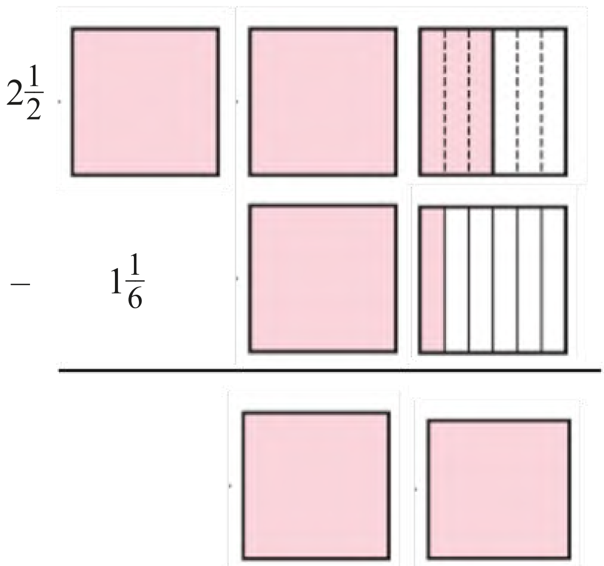
Examples

1. $\frac{3}{4} - 5\frac{}{8}$



This can be interpreted as $\frac{3}{4} - 5\frac{}{8} = 6\frac{}{8} - 5\frac{}{8} = 6\frac{-5}{8} = 1\frac{}{8}$

2. $2\frac{1}{2} - 1\frac{1}{6}$



$2\frac{3}{6} - 1\frac{1}{6} = 1\frac{2}{6} = 1\frac{1}{3}$

Multiplication of Fractions

Multiplying a fraction and a whole

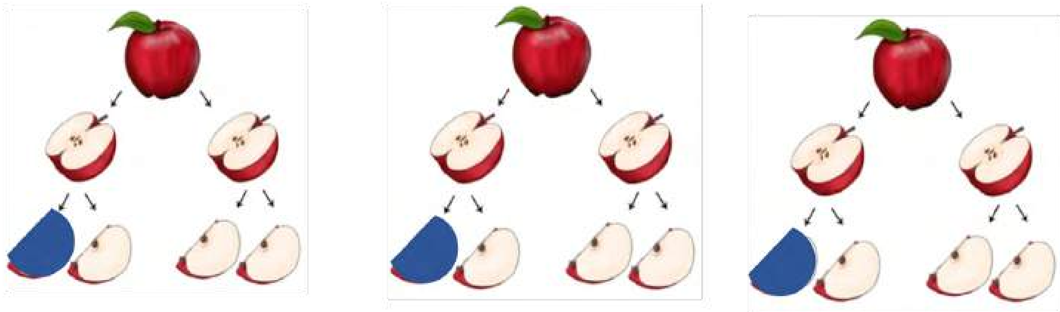
Examples

1. Kwame used $\frac{1}{4}$ of an apple to make one apple pie. He made 3 pies. Let's find out how many apples he used in total.

Solution

Draw a circle to represent an apple and divide it into 4 equal parts and shade $\frac{1}{4}$ of the apple for one pie.

Since he made 3 pies, repeat this process 3 times.



Count all the shaded parts to see that $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$ of an apple is used.

Alternative method:

Multiply $\frac{1}{4}$ by 3.

$$\frac{1}{4} \times 3$$

Represent the whole number 3 as a fraction: $3 = \frac{3}{1}$

Multiply the numerators (top numbers) together and the denominators (bottom numbers) together.

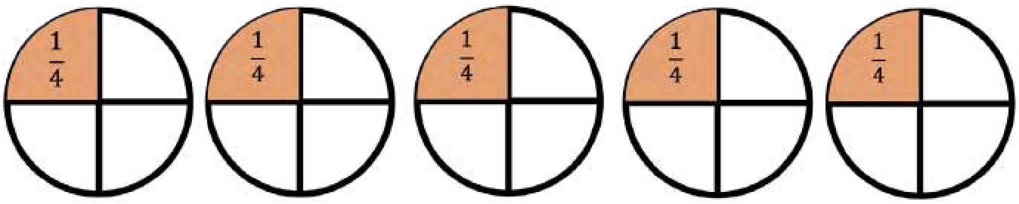
$$\frac{1}{4} \times \frac{3}{1} = \frac{1 \times 3}{4 \times 1} = \frac{3}{4}$$

Simplify the Fraction (if necessary):

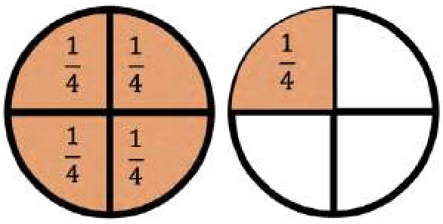
The fraction $\frac{3}{4}$ is already in its simplest form.

Kwame used $\frac{3}{4}$ of an apple in total to make 3 apple pies.

2. If Kwame had to make 5 pies, how many apples would he need?



5 times $\frac{1}{4}$

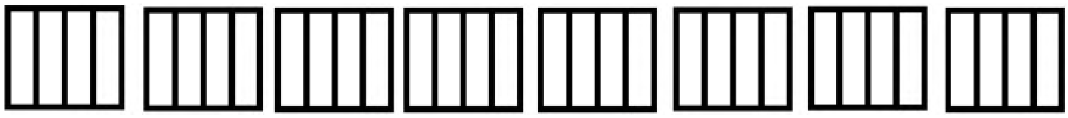


$\frac{5}{1} \times \frac{1}{4} = \frac{5}{4} = 1\frac{1}{4}$

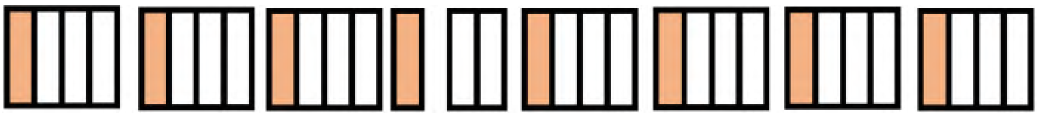
3. Find $\frac{1}{4}$ of 8

Solution

Draw 8 rectangles, each divided into 4 equal parts:



Shade one part in each of the 8 rectangles:



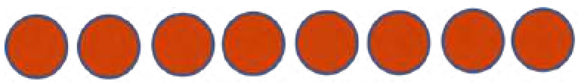
Count the shaded parts:

You have a total of 8 shaded parts out of the total parts.

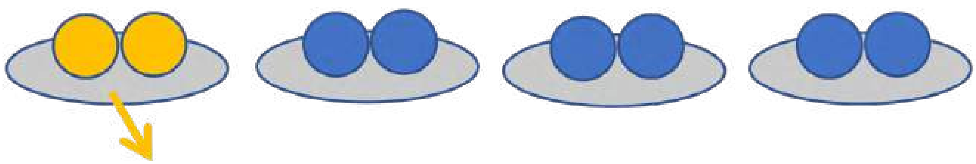
If you combine all the shaded parts, you see that you have a total of $\frac{1}{4} \times 8 = \frac{1 \times 8}{4} = \frac{8}{4} = 2$

So, when you multiply $\frac{1}{4}$ by 8, you end up with 2 whole units.

Alternative solution

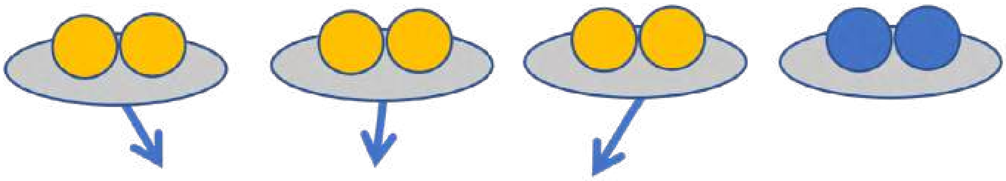


Distribute the marbles equally in 4 groups



One quarter of eight objects $= \frac{1}{4} \times 8 = 2$

4. What is three-quarters of 8?

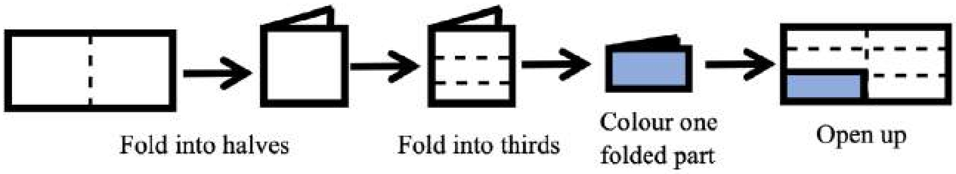


Three quarters of eight objects = $\frac{3}{4} \times 8 = 6$

Multiplying a fraction by a fraction

Example

1. Solve $\frac{1}{3} \times \frac{1}{2}$

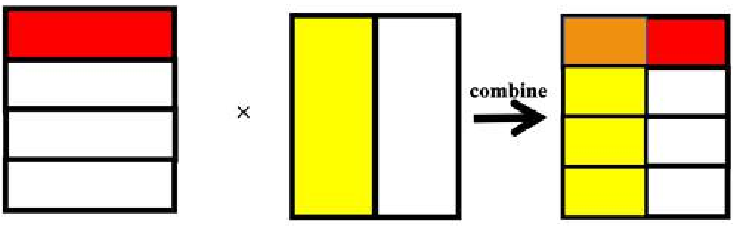


$\frac{1}{3} \times \frac{1}{2} = \frac{1 \times 1}{3 \times 2} = \frac{1}{6}$

The shaded part represents one-third of the half which is one-sixth

2. Find $\frac{1}{4} \times \frac{1}{2}$

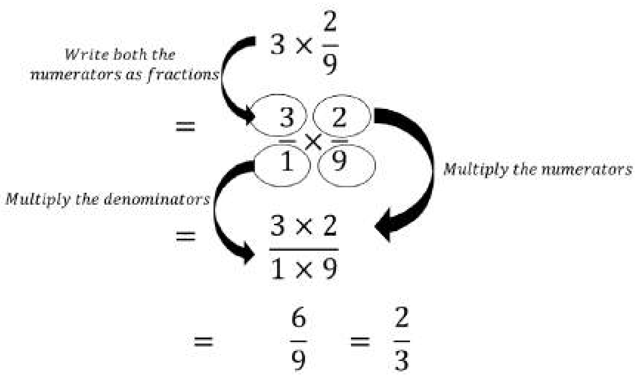
This represents finding a quarter of a half



$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

The fraction which is vertical and shaded in red shows the $\frac{1}{4}$ and the horizontal which is $\frac{1}{2}$. The overlapping area is shaded is orange and shows a quarter of the half, which is one-eighth.

3. Find the product of $3 \times \frac{2}{9}$



$3 \times \frac{2}{9}$

$= \frac{3}{1} \times \frac{2}{9}$

$= \frac{3 \times 2}{1 \times 9}$

$= \frac{6}{9} = \frac{2}{3}$

4. Multiply $\frac{2}{3}$ by $\frac{4}{5}$

Solution

Multiply the Numerators:

$2 \times 4 = 8$

Multiply the Denominators:

$3 \times 5 = 15$

Form the New Fraction:

$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$

Simplify the Fraction, if necessary. In this case, $\frac{8}{15}$ already in its simplest form because 8 and 15 have no common factors other than 1.

$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$

Alternative

Visual Representation

If you want to visualise this, you can use a grid or a pie chart:

Let learners use the grid method by drawing a rectangle and dividing it into 3 equal vertical parts, for $\frac{2}{3}$. Shade 2 out of the 3 parts.

Then divide the same rectangle into 5 equal horizontal parts, for $\frac{4}{5}$. Shade 4 out of the 5 parts.

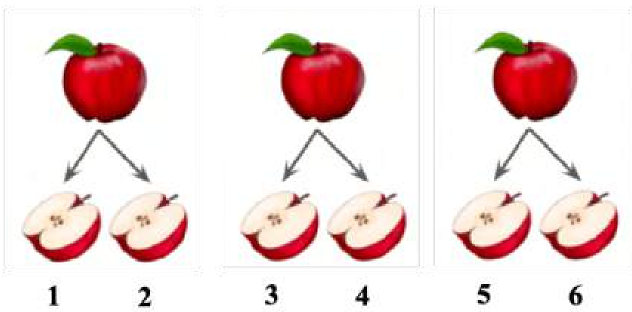
The overlapping shaded area represents the product $\frac{8}{15}$.

Division of fractions

Examples

1. Suppose you have 3 apples, each cut in half. How many people can you distribute these 3 apples if each gets half a piece?

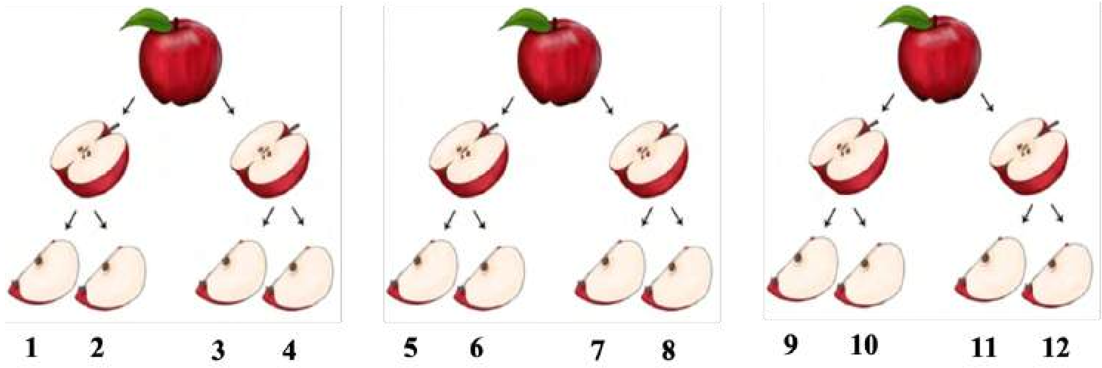
Solution



How many groups of $\frac{1}{2}$ will make 3 wholes?

$3 \div \frac{1}{2} = 6$

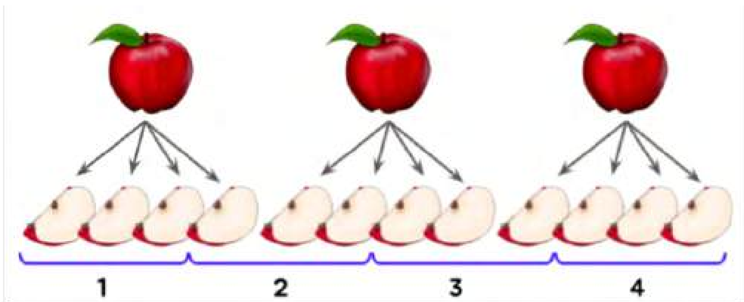
i. Ask learners the number of people who can be served if each person is served a quarter piece?



12 people can be served with 3 apples if each person gets $\frac{1}{4}$ each.

$3 \div \frac{1}{4} = 12$

ii. How many people will be served if each person is given three quarters $\left(\frac{3}{4}\right)$ of the apple



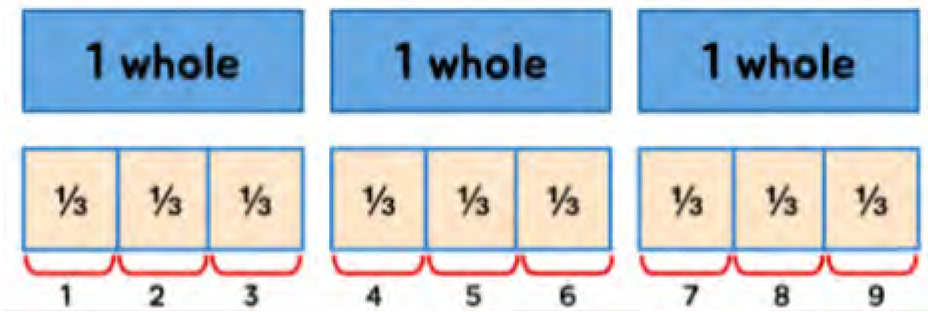
Four (4) people will be served if each person is served three out of four ($\frac{3}{4}$) of an apple.

$3 \div \frac{3}{4} = 4$

2. Find $3 \div 1\frac{}{3}$

Solution

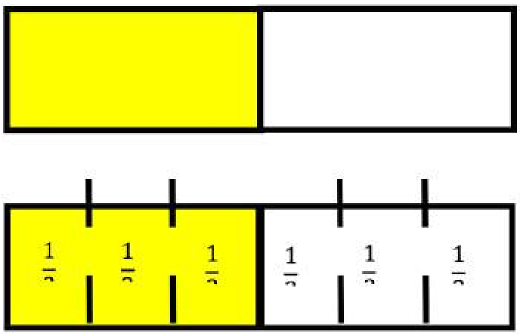
This can be interpreted as how many groups of $\frac{1}{3}$ will make 3 wholes.



The answer is 9. Therefore, $3 \div 1\frac{}{3} = 9$

3. Find $\frac{1}{2} \div 3$.

Solution



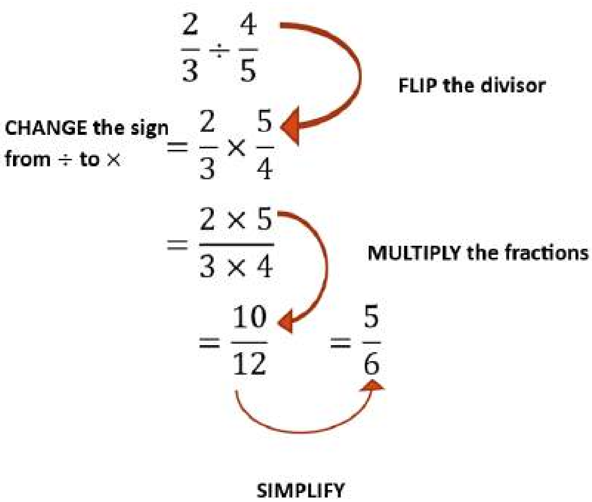
6 groups of $\frac{1}{2}$ are formed

This implies that $\frac{1}{2} \div 3 = 6$

Let learners observe the quotients derived from the models above and see if they can come up with the rule for dividing fractions and then lead them to it formally.

- Flip the divisor fraction.
- Change the sign from ÷ to ×
- Multiply the fractions and simplify.

Example:



Learning Tasks for Practice

1. Learners solve word problems that involve real-life scenarios, such as dividing a pizza among friends, combining ingredients in cooking, or splitting a sum of money.
2. Learners calculate the total cost of items bought in fractional quantities, like $\frac{1}{2}$ kg of apples, $\frac{3}{4}$ kg of oranges, and $\frac{2}{3}$ kg of grapes, practicing all four operations.

Pedagogical Exemplars

1. **Collaborative Learning**: In groups, learners take a recipe and adjust the quantities for different serving sizes, practicing multiplication and division of fractions. They then share their adjusted recipes with the class.

2. **Think-Pair-Share**: Learners individually solve a fraction word problem, pair up to discuss their solutions, and then share their approaches with the class.
3. **Technology Integration**: Learners use calculators or educational apps to practise solving fraction problems involving all four operations.
4. **Project-Based Learning**: Learners create an art project where they must use fractions to divide materials and mix colours, practising addition, subtraction, multiplication, and division of fractions.

Key Assessment

1. Use fraction models to simplify the following
   - **a.** $\frac{1}{2}+\frac{1}{3}$
   - **b.** $\frac{4}{5}\times\frac{3}{7}$
   - **c.** $\frac{3}{4}-\frac{1}{2}$
   - **d.** $\frac{7}{8}\div\frac{2}{3}$
2. Akosua has $\frac{2}{3}$ of a chocolate bar. If she gives $\frac{1}{4}$ of the whole bar to Kwame, how much does she have left?
3. Sarah has $\frac{2}{3}$ of a cup of sugar and needs $\frac{1}{4}$ more for her recipe. How much sugar does she need in total?

Section Review

In this section, we delved into the essential concepts related to fractions, focusing on the following areas:

1. **Naming, Comparing, and Ordering Numbers as Quotients of Two Integers:**
   - **Quotients of Integers:** We practised expressing numbers as quotients of two integers, ensuring that the denominator is not zero (e.g., $\frac{1}{4}, -\frac{5}{8}$).
   - **Comparison and Ordering:** We compared and ordered these fractions by converting them to a common denominator, arranging them in ascending or descending order.
2. **Recognising and Naming Equivalent Fractions:**
   - **Pictorial Representations:** Using visual aids such as pie charts and fraction bars, we identified fractions that represent the same value (e.g., $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}$).
   - **Number Line:** We placed fractions on a number line to Recognise equivalent fractions by their position and spacing.
3. **Comparing and Ordering Fractions with like Denominators:**
   - **Visual Comparisons:** We used pictorial representations to visually compare fractions with like denominators, making it easy to see which is greater or smaller.
   - **Symbols (>, <, =):** We practiced using comparison symbols to express the relationship between fractions (e.g. $\frac{5}{8}>\frac{3}{8}, \frac{4}{9}=\frac{4}{9}$).

4. **Solving Problems on Fractions Involving the Four Basic Operations:**
   - **Addition and Subtraction:** We solved problems that required adding and subtracting fractions with like and unlike denominators, finding common denominators where necessary.
   - **Multiplication and Division:** We tackled problems involving the multiplication and division of fractions, including multiplying by whole numbers and dividing fractions by fractions.

Through a series of engaging exercises and real-life examples, we enhanced our ability to name, compare, and order fractions, recognise equivalent fractions using visual tools, and solve practical problems involving fraction operations. This comprehensive understanding equips us with the skills necessary to handle fractions effectively in various mathematical and real-world contexts.

SECTION 3: REASONING WITH ALGEBRA

Strand: **Algebraic Reasoning**

Sub-Strand: Algebraic Expressions, Equations and Inequalities

Content Standards

1. Demonstrate understanding of algebraic expressions and perform operations on algebraic expressions in real-life contexts.
2. Demonstrate understanding of rearranging a formula from a given context to solve problems.

INTRODUCTION AND SECTION SUMMARY

In this section, we explore how to transform real-life situations into mathematical statements, focusing on applying algebraic operations to solve problems. We start by modeling real-life scenarios, such as financial transactions and everyday decision-making and converting these situations into mathematical expressions and equations. We then practise expanding algebraic expressions by removing brackets and simplifying them using properties of operations. Further, we address how to represent problems symbolically by expressing unknowns as variables in equations, solving these problems through pictorial or symbolic methods. Lastly, we work on creating our own problems based on given equations, which enhances our understanding of how to formulate and solve algebraic expressions.

The section will cover the following focal areas:

1. *Modelling real-life situations into mathematical statements and perform operations on them.*
2. *Expanding by removing brackets and simplifying algebraic expressions using the properties of operations.*
3. *Expressing a given problem as an equation representing the unknown by a letter to the variable, and identify the unknown to solve the problem pictorially or symbolically.*
4. *Creating a problem for a given equation.*

SUMMARY OF PEDAGOGICAL EXEMPLARS

To teach these concepts effectively, use the following strategies:

1. **Real-Life Modeling:** Encourage learners to identify and model real-life situations, such as budgeting or planning, into mathematical statements. This helps learners see the relevance of algebra in everyday life.
2. **Hands-On Algebra:** Use interactive tools and manipulatives to help learners practise expanding and simplifying algebraic expressions. This includes activities where learners physically manipulate algebraic tiles or use algebra software.
3. **Symbolic Representation:** Teach learners to represent problems symbolically by assigning variables to unknowns and solving equations. Provide visual aids like diagrams or flowcharts to illustrate how to translate a problem into an equation.
4. **Problem Creation:** Challenge learners to create their own problems based on given equations. This promotes deeper understanding and application of algebraic concepts and encourages creative problem-solving.

ASSESSMENT SUMMARY

To assess learners' understanding of these concepts, consider the following methods:

1. **Application Tasks:** Provide learners with real-life scenarios and assess their ability to model these situations into mathematical statements. Evaluate their accuracy and reasoning in performing operations on these models.
2. **Expansion and Simplification Exercises:** Include exercises where learners expand algebraic expressions by removing brackets and simplify them. Assess their proficiency in applying properties of operations.
3. **Equation Solving:** Assess learners' skills in expressing problems as equations, solving for unknowns pictorially or symbolically, and interpreting their solutions.
4. **Problem Creation Projects:** Evaluate learners' ability to create meaningful problems based on given equations. This can be done through written assignments or presentations where learners explain their problems and solutions.

Week 5: Algebraic Expressions

Learning Indicator: *Model real-life situations into mathematical statements and perform operations on them II*

Focal Area: Concept of Algebraic Expressions

Introduction

Algebraic expressions are foundational components of algebra, providing a means to represent mathematical relationships and patterns symbolically. These expressions involve variables, constants and operations which allows for the representation of unknown quantities and relationships between them.

Variables in algebraic expressions represent unknown quantities or values that can vary. They are typically represented by letters, such as x, y, or z. Constants, on the other hand, are fixed values that do not change and are represented by numerical values.

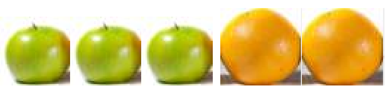
Like and Unlike terms

Like and unlike terms are fundamental concepts in algebra, representing terms that can be combined or simplified based on their similarity.

Examples

1. Consider the image below. How many fruits are there? How many apples and oranges can you identify?

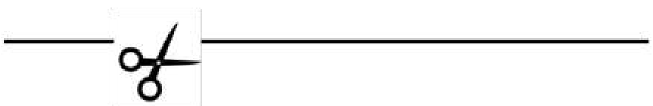
There are five fruits; which includes 3 apples and 2 oranges.



In the absence of known quantities such as apples and oranges, learners can represent these quantities with variables. That is ***a*** for apples and ***o*** for oranges.

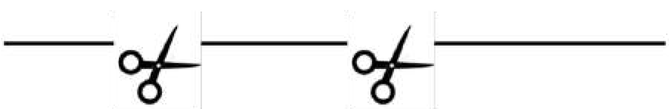
This implies that there are ***3a*** and ***2o***

2. Give each learner /group a piece of string and ask them to cut through it. Ask them how many pieces they have now?



Answer = 2 pieces

Now make another cut on one of the pieces. How many pieces are there now?



Answer = 3 pieces

Continue making more cuts (3, 4, 5) and ask the same question – how many pieces are there now?

Then we can leap to, how many pieces would there be if they made 57 cuts?

Answer = 58 cuts

Learners should have identified a rule that tells them how many pieces they will have after any number of cuts. They should come up with something like Pieces = Cuts plus one

Now introduce symbols for Pieces and cuts, for example, **P = C+I**

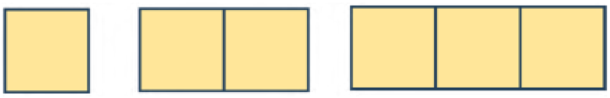
3. Find the perimeter of a unit square with length 1cm.

Perimeter of a square is **L + L + L + L**

P = 1 + 1 + 1 + 1

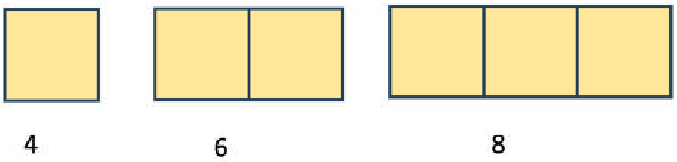
P = 4 cm

4. Find the perimeter of the unit squares below

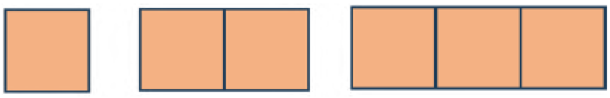


The length of the side of a unit square is 1. The perimeter, as indicated in example 2 above is 4.

Therefore, the perimeter of the squares will be 4, 6 and 8 respectively.



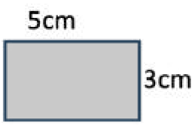
5. Find the perimeter of the squares below with length unknown.



Since the length of the side is not known, let a variable, **L** represent the length.

Therefore, the Perimeter will be **4L, 6L** and **8L** respectively.

6. Find the perimeter of a rectangle with length, 5cm and breadth 3cm.



Let the length be represented by L and breadth by B

Perimeter = L + L + B + B

P = 2L + 2B

Therefore, P = 2(5) + 2(3)

P = 10 + 6

P = 16 cm

7. Find the perimeter of the rectangles below with length 5 cm and breadth 3cm

The perimeter, as indicated in example 6 above is 16 cm.

Therefore, the perimeter of the rectangles will be: 16, 26 and 36 respectively.

Definition of terms

Constant: A fixed numerical value that does not change. For example, in the expression $3x + 2$, the constant term is 2.

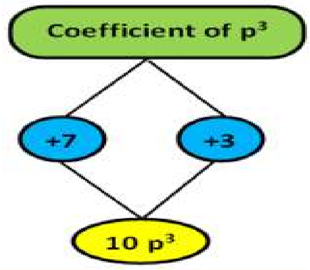
Variable: A symbol that represents an unknown or changing quantity. In the expression $3x + 2$, x is the variable.

Coefficient: The numerical factor that multiplies a variable. In $3x + 2$, 3 is the coefficient of the variable x.

Example

$7p^3 + 3p^3 = 10p^3$

The coefficient of p^3 is 7 and 3. This means that $7p^3 + 3p^3$ which makes the sum, $10p^3$



Exponent: A number that indicates the power to which a variable is raised. For example, in x^2 , 2 is the exponent of x.

Like Terms: Terms that have the same variables raised to the same powers. For instance, $3x$ and $5x$ are like terms.

Unlike Terms: Terms that have different variables or different powers of the same variable. For example, $3x$ and $5y$ are unlike terms.

Examples

Regroup the following according to like and unlike terms:

1. $4x^2$, $3xy, -2x^2$, $5y$, $7x, -3xy$

Solution

The like terms are, $4x^2$ and $-2x^2$, $3xy$ and $-3xy$ but $5y$ and $7x$ are unlike terms.

2. $4a$, $2b$, $3,5,7a$, $6a$

Solution

The like terms are $4a$, $7a$, and $6a$.

The unlike terms are $2b$, $3a^2$ and $5b^2$

3. In the expression, $2x + 5y - 3x + 7y$, identify the coefficients of the terms

Solution

The coefficients of x, y, x and y are 2, 5, −3 and 7 respectively.

Learning Tasks for Practice

1. Identify and define variables, constants, and coefficients in algebraic expressions.
2. Identify like terms in given algebraic expressions.
3. Combine like terms to simplify algebraic expressions.
4. Differentiate between like and unlike terms in expressions containing multiple variables.
5. Apply the concept of like and unlike terms to solve problems involving simplification and evaluation of expressions.

Application / Importance of algebraic expressions

1. Algebraic expressions provide a systematic way to represent and solve mathematical problems. They allow us to translate real-world situations into mathematical models, enabling us to analyse and solve problems efficiently.
2. Algebraic expressions are used to model relationships between variables.
3. Algebraic expressions use symbols and variables to represent quantities that may vary. This symbolic representation allows for generalisation and abstraction, making it easier to analyse complex relationships and patterns.

Pedagogical Exemplars

1. **Structuring Talk for learning:** Present a sequence of squares to learners. Encourage them to observe the patterns and relationships between consecutive numbers and find the perimeter of each figure.
2. **Think-Pair-Share:** Present algebraic expressions containing both like and unlike terms and ask learners to individually identify and categorise the terms. Then have learners discuss their findings with a partner before sharing their conclusions with the class.
3. **Experiential learning:** Engage learners in hands-on activities where they physically manipulate algebraic expressions.
   - **i.** Provide sets of algebraic expressions on cards or tiles and ask learners to group them based on whether the terms are like or unlike. Encourage discussions among learners to justify their groupings.
   - **ii.** Learners can work in groups/ pairs to arrange objects (e.g., counters, blocks) to form squares and triangles, reinforcing the concept visually.

Key Assessment

1. **Assessment Level 1 and Level 2**
   - **a.** Given the following expressions, identify and combine like terms.
     - **i.** $3x + 2y - 5x + 4z$
     - **ii.** $2x + 3y - 5x + 4z$

b. Identify the like terms in the expressions below and group them accordingly.

i $2x^2 + 3x + 5 - x^2 + 4x - 2$

ii $4a - 2b + 3a - 5b$

c. Write an expression representing the total cost of n pens and m pencils if a pen costs *GH¢* **2.00 and a pencil costs** *GH¢* **3.00.**

d. If you have 3 red balls and 5 blue balls, write an expression for the total cost of balls if each red ball is sold at *GH¢* 10.00 and each blue ball is sold at *GH¢* 30.00

Focal Area: Operations On Algebraic Expressions

Introduction

Revise learner's previous knowledge of algebraic expressions and use the idea to build and interpret simple algebraic expressions.

An algebraic expression consists of constants, variables, and mathematical operations, such as addition, subtraction, multiplication, and division. For example:

1. $2n$ — 2 is the coefficient and n is the variable.
2. $2x + 6$ — 2 is the coefficient, 6 is the constant and x is the variable.
3. $2(y + 4)$ — 2 and 4 are the constants and y is the variable.

Examples

Write the algebraic expressions for the following statements:

a. A number 4 more than a given number

This could be expressed in many ways, for example, a number plus 4, 4 added to a number, the sum of 4 and a number, 4 more than a number.

Algebraically it looks like this, $x + 4$, the number is x.

b. A number less than a given number

Again it could be expressed in many ways, 17 minus a number, a number subtracted from 17, the difference of 17 and a number, a number less than 17.

Algebraically it would be this, $17 - b$, the number is b.

c. Several times of a certain number

This could also be written as twice a certain number.

Algebraically it would be written as $2x$

Or we could have 5 times a number, the product of 5 and a number, etc.

Algebraically this would be written as $5x$.

d. The quotient of a given number

Example: 16 divided by a number.

Algebraically this would be written as $\frac{16}{x}$

Examples:

Investigate the following and write mathematical expressions for them:

Addition

i.	The sum of x and y	will be $x + y$
ii.	7 more than m	will be $m + 7$
iii.	The total a, b and c	will be $a + b + c$
iv.	p increased by 6	will be $p + 6$

Subtraction

i.	m subtracted from n	will be $n - m$
ii.	7 less than m	will be $m - 7$
iii.	The difference between a and b	will be $a - b$ or $b - a$
iv.	x decreased by 4	will be $x - 4$

Multiplication

i.	The Product of x and y	will be $\boldsymbol{x} \times \boldsymbol{y} = xy$
ii.	8 of q	will be $8 \times q = 8q$
iii.	Twice x	will be $x + x = 2x$
iv.	4 times y	will be $4 \times y = 4y$

Example

1. Akua went shopping and bought n apples, If each apple costs 8 cedis, write an algebraic expression to represent, the total cost.

   Solution

   n is the number of apples bought.

   Cost for one apple is 8 cedis

   Total cost 8*n cedis.*

Addition and subtraction of algebraic expressions

Examples

1. Kwaku has 3 plantains and 2 yams and his sister has 5 plantains and 4 yams.

   Express these using symbols and add the two expressions.

   Solution

   In this case, you let the p represent the plantain and y represent the yam.

   Then Kwaku's expression is $3p + 2y$

   Kwaku's sister's expression is $5p + 4y$.

   To add these, we can only add the like terms together

   $3p + 5p + 2y + 4y = 8p + 6y$

   Together they have 8 plantains and 6 yams.

2. Simplify $3x + 2y - 5x + 4y$

Solution

$3x - 5x + 2y + 4y = -2x + 6y$

3. There are two groups in a class, 8 learners offer mathematics and 5 offer English in the first group. In the second group, 3 learners offer mathematics and 3 offer English.

   i. Write mathematical expressions for both groups.

   ii. How many more maths / English learners are there in the first group than the second group.

Solution

Let m represent mathematics and e represent English

Expression for the first group is $8m + 5e$

Expression for the second group is $3m + 3e$

$8m + 5e - (3m + 3e) = 8m + 5e - 3m - 3e$

$8m - 3m + 5e - 3e = 5m + 2e$

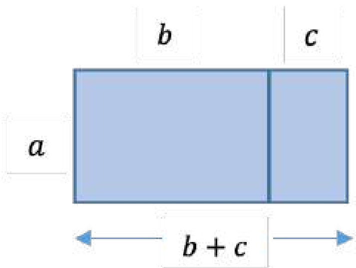
Multiplication of algebraic expressions

Examples

1. Draw a rectangular board of length $b + c$ and width a. Find the overall area of the rectangular board using geometric demonstration.

Solution

Using geometric demonstration



The overall area of the rectangular board is $length \times width = a(b + c)$

However, from the diagram above, the area can be find by adding the areas of the two small rectangles. i.e. $ab + ac$

Therefore, the overall area of a rectangular board of length $b + c$ and width a is

$a(b + c) = ab + ac$

2. Using algebraic tiles, simplify $4x^2 - 3x + 2 - 2x^2 - 2x - 4$

Solution

Here arrange the tiles according to the expression given. See fig 1.

Note that in this activity, the Yellow tile represents positive and red, negative. Feel free to use any colour of your preference for positive and negative.

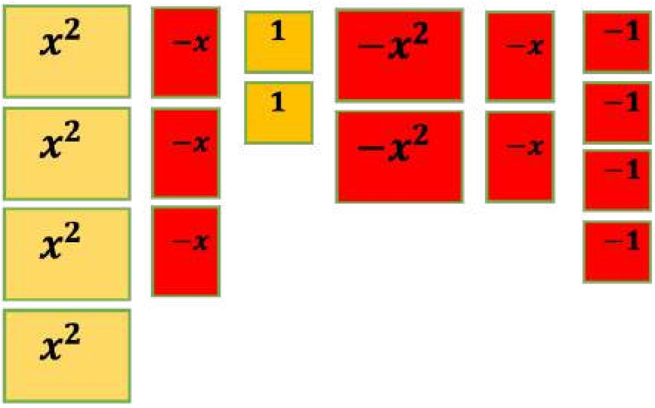


Fig. 1

Group like terms by arranging the tiles as it is done in figure 2 and perform the operation on them.

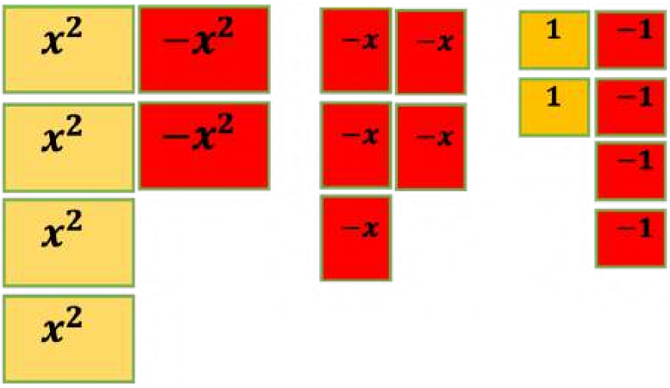


Fig 2

From Fig. 2 we can see that the answer is $2x^2 - 5x - 2$

3. Simplify, 2a + b + 5a − 3b

   Solution

   2a + 5a + b − 3b = 7a − 2b

Learning Tasks for Practice

1. Simplify algebraic expressions.
2. Write mathematical expressions from a given statement.
3. Solve real-life problems involving algebraic expressions.

Application/ Importance of algebraic expressions.

1. Algebra allows us to analyse and solve problems in various areas of mathematics, science, engineering, economics, and many other fields.
2. It provides tools for expressing and solving problems in a general and abstract way

Pedagogical Exemplars

1. **Talk for learning:** In a whole class discussion, review learners' previous knowledge of algebraic expressions through questioning and answering.

2. **Initiating talk for learning:** in a whole class discussion through questioning and answering guide learners to write the mathematical expressions of given statements on the board.
3. **Collaborative learning:** In mixed ability/gender groups of five, provide statements on cards to each group. Ask Learners to discuss and ink the mathematical expressions for each statement on the cardboard.
4. **Talk for learning:** in a whole class discussion, explain how you will perform operations on algebraic expressions.
5. **Problem-based learning:** give learners in their mixed-ability gender groups a scenario, e.g., "Kwaku has 3 plantains and 2 yams and his sister also has 5 plantains and 4 yams" Ask learners to think, ink, and share their ideas on how they will write the mathematical expressions for Kwaku and his sister and add their expressions.
6. **Talk for learning:**
   - **i.** In a whole class discussion, present a question on the multiplication of algebraic expressions; "Draw a rectangular board of length, $b + c$ and width a. Find the overall area of the rectangular board using geometric demonstration.
   - **ii.** In a whole class discussion, through questioning and answering, draw a rectangular board of length, $b + c$ and width, a on the board.
   - **iii.** In a whole class discussion, ask probing questions to assist in building learners' understanding and help them connect the visual representation to the mathematical process of finding the area of the rectangular board.
7. **Collaborative learning:** In small mixed ability/gender groups, engage learners to investigate the question under example 2, and encourage them to discuss how they arrived at their answers.
8. **Talk for learning:** in a whole class discussion, through questioning and answering, introduce and explain how to simplify algebraic expressions using algebraic tiles.
9. **Problem-based learning:** learners in their small mixed-ability gender groups, provide a series of questions on the simplification of algebraic expressions and ask learners to simplify. Encourage them to discuss their strategies, share ideas, and support each other in finding the solution.

Key Assessment

Assessment Level 1

1. Simplify the following
   - **i.** $2x + x + 10x$
   - **ii.** $15x + 3y - x$
   - **iii.** $9m - 4b - 3b + 2m$
   - **iv.** $2x(3 + h)$
   - **v.** $4(10p - x - 2p + 2x)$

Assessment Level 2

1. A motorcycle travels at a constant speed of 60 miles per hour for m hours. Write an expression to represent the total distance travelled by the motorcycle.
2. A rectangular board has a length of l meters and a width of w. Write an expression to represent the perimeter of the board.
3. A school charges a monthly fee of GH 100 plus an additional GH x per hour for extra classes. Write an expression to represent the total cost with p hours of extra classes.

Assessment Level 3

1. The sum of two numbers is 25, and their difference is 7. Form an algebraic expression for the statement.
2. The cost of $'x'$ apples is *GH*₵ 2.00 and the $'y'$ bananas are *GH*₵ 3.00 each. Model the problem and form an algebraic expression.
3. Hannah earned *GH*₵ 5.00 for every book she sold, and she also received *GH*₵ 10.00 as a bonus. If she sold b books, what is her total earnings? Write an algebraic expression for the word problem.

Week 6: Simplification of Algebraic Expressions

Learning Indicator: *Expand by removing brackets and simplify algebraic expressions using the properties of operations.*

Focal Area: Expand and Simplify Algebraic Expressions

Introduction

Expansion is an essential operation in algebra that involves simplifying or multiplying out algebraic expressions. When an expression contains brackets / parentheses, expansion allows us to remove them and write the expression as a sum or difference of terms. The process of expansion is also known as "distributing" or "applying the distributive property."

Expanding brackets means multiplying each term inside the brackets by the terms outside the brackets.

To successfully expand and simplify algebraic expressions one must be systematic and careful with the calculations.

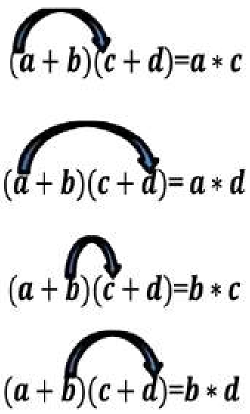
The properties of operations and the order of operations (PEMDAS/BODMAS) when working with multiple operations is of much importance.

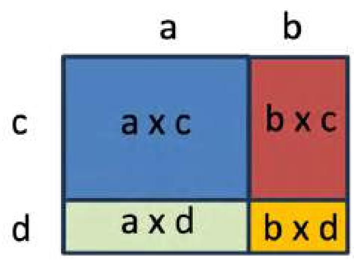
Examples

1. Expand $a(b + c)$.

   Therefore $a(b + c) = (a \times b) + (a \times c) = ab + ac$

2. Expand $(a + b)(c + d)$





   Therefore $(a + b)(c + d) = ac + ad + bc + bd$

3. Expand $(x + 2)(x + 3)$ using the distributive property.

   The expansion of $(x + 2)(x + 3)$ is the same as the product or multiplication of $(x + 2)$ *and* $(x + 3)$.

   Take $(x + 3)$ as a single quantity and apply the distributive property over addition.

   $(x + 2)(x + 3) = x(x + 3) + 2(x + 3)$

   Multiply the x outside the bracket over $(x + 3)$ and 2 over$(x + 3)$.

   $= x(x) + x(3) + 2(x) + 2(3)$

   $= x^2 + 3x + 2x + 6$

   $= x^2 + 5x + 6$

4. $(x-4)(2x+5)$

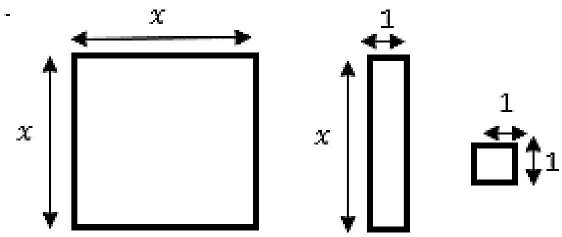
$= x(2x+5) - 4(2x+5)$

$= 2x^2 + 5x - 8x - 20$

$= 2x^2 - 3x - 20$

5. In a rectangular garden with dimensions length = $(x + 2)$ meters and breadth =$(x + 3)$ metres. Expand the expression using algebraic tiles, to find the area of the garden.

Take the algebraic tiles by size $(x \times x)(x \times 1)$ and (1×1) as shown below.



Area of big square = $x \times x = x^2$

Area of rectangle = $x \times 1 = x$

Area of small square = $1 \times 1 = 1$

Therefore, for this question, arrange the tiles to take a rectangular form as shown below Fig. A

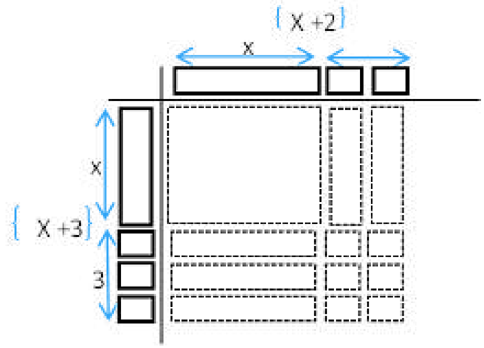
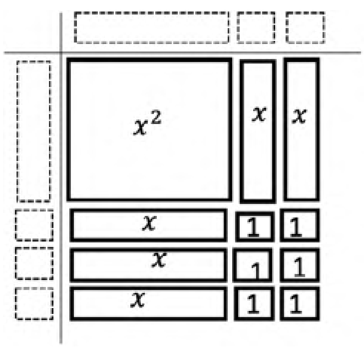


Fig. A

Find the area of the fig. A with the dimensions $(x + 2)$ and $(x + 3)$



Fill the space created under the table with the tiles x^2, x and 1 to form a rectangle as shown below

Find the area of the rectangle created by adding the number of tiles under the table by their areas.

$(x^2 + x + x + x + x + x)+(1 + 1 + 1 + 1 + 1 + 1) = x^2 + 5x + 6$

Thus $(x + 2)(x + 3) = x^2 + 5x + 6$

6. You have gone to the supermarket to buy some items. You wanted to buy a number of items (a), each originally priced at b cedis, but now there was a discount of c cedis.

The total cost of the items bought can be calculated as shown below: $a(b - c) = (a \times b) - (a \times c) = ab - ac$.

7. Ama sold a dress for m cedis and realised that she was making a loss, she decided to double the price to $2m$ cedis. Unfortunately, no one bought the dress so she decided to reduce the new cost by 5 cedis.

Kofi sold a different dress for m cedis and realised he was making a loss. He then decided to sell it for four times the cost of m. He realised the patronage was encouraging so decided to increase it by 7 cedis.

If $m = 10$, what will be the total cost of buying 3 of Ama's dresses and 2 of Kofi's.

Ama's dress will be $(2m - 5)$

Kofi's dress will be $(4m + 7)$

3 times Ama's and 2 times Kofi's will be: $3(2m - 5) + 2(4m + 7)$

$= 6m - 15 + 8m + 14 = 14m - 1$ Given $m = 10$, we have: $14 \times 10 - 1 = 139$, or we can think of it like this:

$3[2(10) - 5] + 2[4(10) + 7]$

$= 3(20 - 5) + 2(40 + 7)$

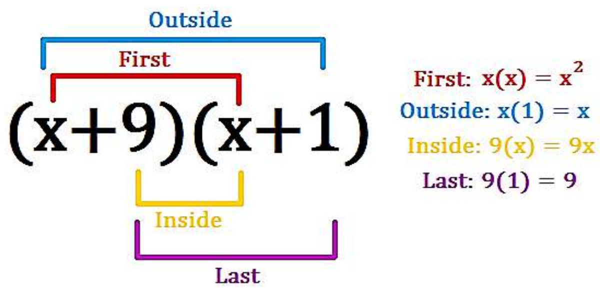
$= 3(15) + 2(47)$

$= 45 + 94$

$= 139$

The total cost of buying 3 of Ama's dresses and 2 of Kofi's will be 139 cedis.

8. Expand $(x + 9)(x + 1)$ using the FOIL method as shown below;



So $(x + 9)(x + 1) = x^2 + x + 9x + 9 = x^2 + 10x + 9$

Special Product Formula (Difference of two squares)

Example

Expand $(a + b)(a - b)$

Solution

Simplifying:

$(\boldsymbol{a} + \boldsymbol{b})(\boldsymbol{a} - \boldsymbol{b})$

$=\boldsymbol{a}(\boldsymbol{a} - \boldsymbol{b}) + (\boldsymbol{ba} - \boldsymbol{b})$

$=\boldsymbol{a}^2 - \boldsymbol{ab} + \boldsymbol{ab} - \boldsymbol{b}^2$

$=\boldsymbol{a}^2 - \boldsymbol{b}^2$

Application / Importance of algebraic expressions

Algebraic Expressions are very useful in many areas including finance and Economics (Budgeting and Financial Planning), Engineering, Computer Science, Architecture and Educational Research.

Learning Task for practice

1. Perform operations on algebraic expressions involving brackets.
2. Perform operations on algebraic expressions involving distributive property.
3. Model and solve problems on real–life scenarios on algebraic expressions.

Pedagogical Exemplars

1. **Initiate Talk for learning:** In a whole class discussion review the previous lesson on algebraic expressions with learners.
2. **Group work/collaborative learning:**
   - **i.** In mixed ability/gender groups distribute a scenario on worksheets for learners in their groups. Let learners think-ink-share their findings. Discuss their findings with them.
   - **ii.** In mixed ability/gender groups, let learners find the area of the rectangle created by adding the number of tiles under the table multiplied by their respective areas. Give assistance to learners or groups who are finding it difficult to find the area. Discuss with learners their findings.
3. **Talk for Learning**
   - **i.** Discuss with learners how to expand and simplify algebraic expressions involving parenthesis.
   - **ii.** Discuss with learners how to use the distributive property to expand algebraic expressions using real life scenarios.
   - **iii.** Discuss with learners the expansion of the difference of two squares. Give assistance to learners who are finding it difficult to get the concept.
4. **Experiential Learning:** Let learners in groups explore the worksheets labelled fig. A and B. Guide them to identify the relationship between fig. A and Fig. B in terms of area. (Learners should be able to say the areas are the same.

Key Assessment

Assessment Level 2

1. In planning a birthday party, each bottle of Coca-Cola costs *GH*₵ 1.50 and $(2x + 5)$ bottles need to be bought. Form an algebraic expression with this statement to find the total cost of Coca-Cola needed for the party.
2. In a rectangular garden with dimensions length = $(x + 3)$ metres and width = $(2x − 1)$ metres, expand the expression using algebraic tiles or the FOIL method, to find the area of the garden.
3. You are setting up tables for a party. Each table can seat $x + 4$ guests. If you have 4 tables, how many guests can be seated in total?

Assessment Level 3

1. The length of a rectangle is 4 metres more than twice its width.

   If the area of the rectangle is 48 square metres, find the dimensions of the rectangle.
2. Sarah has a rectangular garden with dimensions 2x + 6 metres by 3x + 9 metres. She wants to find the area of the garden. How should this be done?
3. Consider the expression $2x(3x + 4) − 4(2x + 4)$. Simplify the expression and explain the steps you took? What strategies did you use to manipulate the terms and arrive at the final result?

Week 7: Algebraic Equations

Learning Indicators

1. *Express a given problem as an equation representing the unknown by a letter to the variable and identify the unknown to solve the problem pictorially or symbolically.*
2. *Create a problem for a given equation.*

Focal Area: Translating Word Problems into Algebraic Equations and Solving for The Unknown Pictorially and Symbolically

Understanding how to translate real-life problems into algebraic equations and solve for the unknown is a key skill in algebra. This process involves recognising patterns, identifying variables, and using pictorial and symbolic methods to find solutions. Pictorial methods, like drawing diagrams, help learners see the problem. Symbolic methods, like writing equations, provide a more abstract approach. By combining these methods, learners will better understand how to turn real-life problems into algebraic equations and solve them.

Identifying the unknown in real-life scenarios and translating it into equations

In algebra, unknowns are variables that represent numbers we do not yet know. These variables are usually denoted by letters such as x, y, *or* z. An unknown is a placeholder for a value that can change or that we need to find. This reinforcement activity will help the learner to identify the unknown in a given real-life problem

Examples

1. Andam has some mangoes. If she buys 4 more, she will have 12 mangoes. Write an equation that represents this situation.

   Solution

   Identify the unknown

   Andam currently has an unknown number of mangoes. Let the unknown number of mangoes be x.

   Now, when Andam buys 4 more mangoes, she adds 4 mangoes to her current number of mangoes so she now has $x + 4$ mangoes.

   After buying 4 more mangoes, the total number of mangoes Andam will have is 12, represented by $x + 4 = 12$.

   Therefore, the equation is given by $\boldsymbol{x} + 4 = 12$

2. Sam has some books. If he gets 5 more, he will have 17 books. Write an equation that represents this situation.

   Solution

   Sam currently has an unknown number of books. Let's call this number p

   After getting 5 more books, the total number of books becomes $\boldsymbol{p} + 5$

   After getting 5 more books, the total is equal to 17, represented by $\boldsymbol{p} + 5 = 17$

   So, the equation is: $\boldsymbol{x} + 5 = 17$

3. Bryan has some money. If, he spends GHC10, he will still has GHC30 left. Write an equation that represents this situation.

Solution

Bryan currently has an unknown amount of money. Let's represent the unknown amount to be x.

Bryan spends GHC10, so we subtract 10 from his original amount to get $\boldsymbol{x} - 10$

After spending GHC10, Bryan has GHC30 left, represented by $x - 10 = 30$

So, the equation is: $\boldsymbol{x} - 10 = 30$

Learning Task for Practice

1. Learners are tasked to discuss and identify the unknown in a given real-life scenario.
2. Learners are tasked to discuss and translate real-life scenarios into an equation. Learners discuss and give examples of growing and repeating patterns.

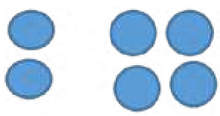
Pedagogical Exemplars

1. **Experiential learning**: In collaborative and mixed-gender/ability groups, engage learners to discuss and identify the unknown in a given real-life scenario.
2. **Collaborative learning**: using mixed-gender/ability grouping, learners discuss, and translate real life problems into an equation.

Solve the equations using pictorial and symbolic methods

Solving equations pictorially

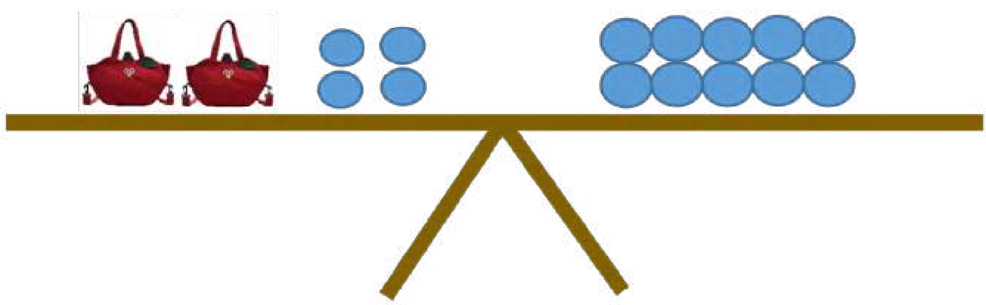
We can use pictures to represent the arithmetic that we do. For example, we can use the below pictures to draw the problem, 2+ 4:



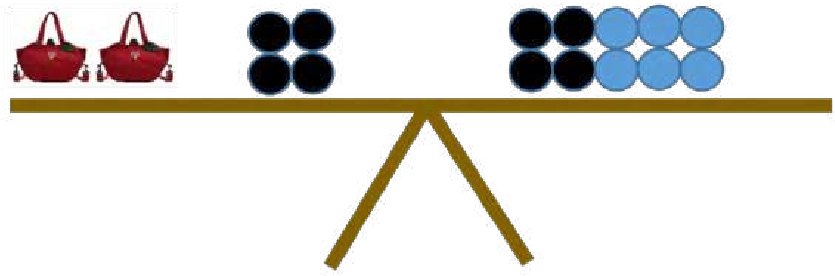
In this picture, each circle represents a chip

We can do the same thing for algebraic expressions when we assume an unknown number of chips, x, are contained in a bag. For example: $2x + 4$:

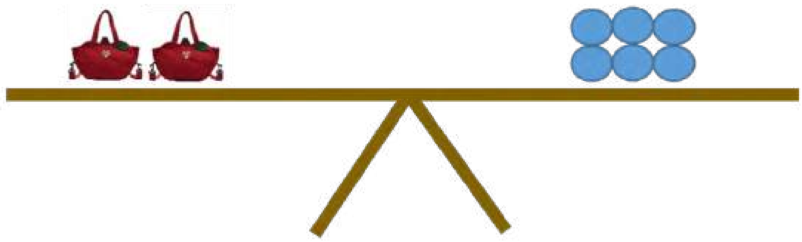
We can do the same for an equation assuming both sides are equal. So we do that by showing them on a balance. For example, $2x + 4 = 10$ could be shown as:



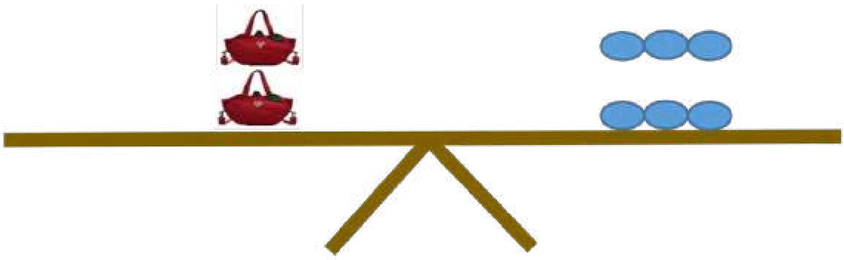
Using the picture below, we can solve this equation by subtracting 4 from both sides. That is $2x + 4 - \mathbf{4} = 10 - \mathbf{4}$.



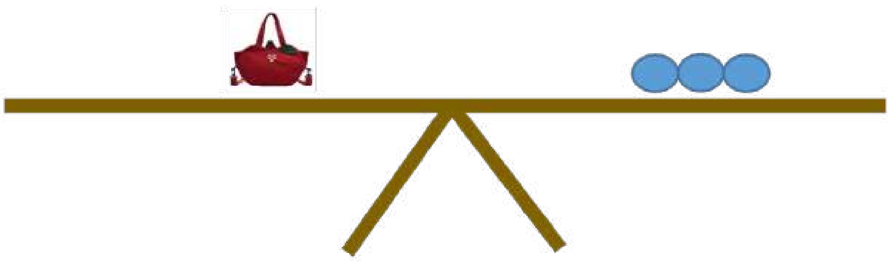
This implies that $2x = 6$



This gives $x + \boldsymbol{x} = 3 + \mathbf{3}$



Therefore $\boldsymbol{x} = 3$



Solving equation symbolically

Example

1. Solve $3x = x + 1$ symbolically.

   Solution

   Subtract x from both sides of the equation, to leave the x's on one side of the equation only.

   $3x - x = x - x + 1$

   $2x = 1$

   Divide both sides of the equation by 2

   $\frac{2x}{2} = \frac{1}{2}$

   $\boldsymbol{x} = \frac{1}{2}$

2. Solve $5x + 1 = 2x + 7$ symbolically.

Solution

Subtract 2x from both sides of the equation, to leave the x's on one side of the equation only.

$5x - 2x + 1 = 2x - 2x + 7$

$3x + 1 = 7$

Subtract 1 from both sides of the new equation to leave the x's on their own.

$3x + 1 - 1 = 7 - 1$

$3x = 6$

Divide both sides of the equation by 3 to solve for a single x.

$\frac{3x}{3} = 6\frac{}{3}$

$\boldsymbol{x} = 2$

Learning Task for Practice

1. Learners are tasked to discuss and solve equations using pictorially.
2. Learners are tasked to discuss and solve equations symbolically.

Pedagogical Exemplars

1. **Experiential learning**: In collaborative and mixed-gender/ability groups, engage learners to discuss how to solve equations using the pictorial method.
2. **Problem-based learning**: in groups/pairs, engage learners to discuss and solve equations symbolically.

Key Assessment

1. A learner has \$50 and wants to buy notebooks that cost \$3 each. How many notebooks can they buy? Identify the unknown, form an equation and solve it.
2. A car travels at a constant speed of 60 km/h. How long will it take to cover a distance of 240 km? Identify the unknown, form an equation and solve it.
3. A person saves \$20 every week. How many weeks will it take to save \$200? Identify the unknown, form an equation and solve it.
4. A class is making posters. Each poster requires 3 sheets of paper. The class wants to make 12 posters. How many sheets of paper do they need?
5. There are 3 boxes, each containing the same number of apples. In total, there are 15 apples. How many apples are in each box?
6. Solve for the unknown in the following equations:

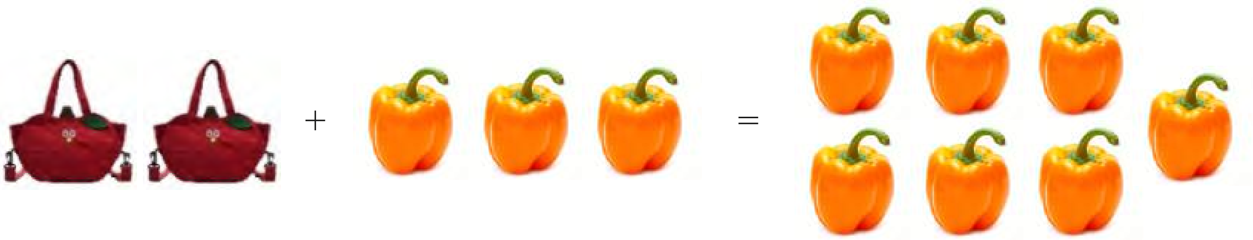
   a. $3n - 1 = 5$

   b. $y + 5 = 12$

   c. $5p - 9 = 2p + 6$

   d. $2y - 7 = 13$

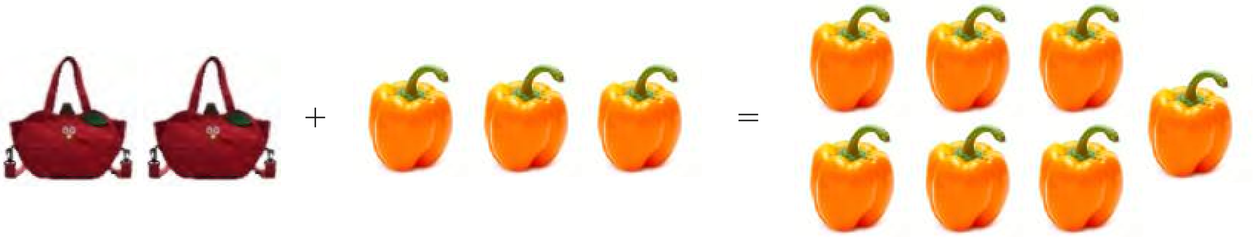
Focal Area: Creating Word Problems from a Given Equation



From the above illustration, we assume that there is an unknown number of bell peppers in the bag that can be added to the 3 peppers to give a total of 7.

Now, let us use x to represent this unknown number of peppers in the bag.

This can be represented as:



$2x$ 3 7

Therefore, the equation $2x + 3 = 7$ can be written from the illustration above and then solved to find x.

We can also use words to explain the obtained equation from the above illustrations:

Sarah had some bell peppers. Her friend gave her 4 more bell peppers. Sarah now has a total of 7 bell peppers. How many bell peppers did Sarah have to begin with?

Examples

1. Use words to explain this the equation $x - 2 = 4$

   Solution

   There are many options here, but here is an example.

   Jake had some marbles. He gave away 2 marbles to his friend, and now he has 4 marbles left. We could then solve the equation to find out how many marbles Jake had at the start.

   Explanation:

   In this problem, the equation $x - 2 = 4$ represents the situation:

   x is the unknown number of marbles Jake had initially, before giving any away.

   Jake gave away 2 marbles, which is represented by the "-2" in the equation.

   After giving away the marbles, Jake has 4 marbles left, which is represented by the " $= 4$" in the equation.

   So, to find out how many marbles Jake had initially, we need to determine the value of x in the equation $x - 2 = 4$

2. Use words to explain this the equation $5x - 2 = 4 + 2x$

Solution

For example, Emily is collecting stickers. She had x stickers at the beginning.

Now she has 2 fewer stickers than 5 times the number of stickers she started with.

This is the same as if she adds 4 stickers to her collection, plus 2 times the amount she started with.

How many stickers did Emily start with?

Explanation:

In this problem, the equation $5x - 2 = 4 + 2x$ represents the situation:

x is the total number of stickers Emily started with.

The term $5x - 2$ represents the number of stickers Emily has now, which is 2 fewer than 5 times the number she started with.

The term $4 + 2x$ is equal to the number of stickers Emily has, with is 2 times her starting number plus 4.

To find out how many stickers Emily started with, we need to solve the equation

$5x - 2 = 4 + 2x$.

Learning Task for Practice

1. Learners are tasked to discuss and analyse the relationships in a given equation.
2. Learners are tasked to discuss and explain a given equation in words.

Pedagogical Exemplars

1. **Inquiry-Based Learning:** In collaborative and mixed-gender/ability groupings, engage learners to discuss and the relationships in a given equation
2. **Collaborative learning**: Engage learners to discuss and explain a given equation in words.

Key Assessment

1. Write a word problem for the equation $2 + 4 = y$
2. Write a word problem that the equation $\boldsymbol{x} + 7 = 12$.
3. Write a word problem for the equation $3y - 5 = 16$.
4. Write real-life situation where the equation $2x + 3 = 15$ could apply.

Section Review

In this section, we focused on key algebraic concepts and skills, applying them to both mathematical and real-life contexts:

1. **Model Real-Life Situations into Mathematical Statements and Perform Operations:**
   - **Real-Life Situations:** We practised translating real-world scenarios into mathematical statements. For instance, converting a shopping list into an algebraic expression to calculate the total cost.
   - **Operations:** We performed operations (addition, subtraction, multiplication, and division) on these mathematical statements to find solutions to practical problems.

2. **Expand by Removing Brackets and Simplify Algebraic Expressions:**
   - **Removing Brackets:** We expanded algebraic expressions by applying the distributive property. For example, expanding $3(x + 4)$ to $3x + 12$.
   - **Simplification:** We simplified the expanded expressions by combining like terms and applying properties of operations to reach a more concise form.
3. **Express a Given Problem as an Equation and Solve:**
   - **Formulating Equations:** We learned to express problems as equations, identifying the unknown variable. For example, if a person has x apples and buys 5 more, we represent the total as $x + 5$.
   - **Solving Equations:** We prasticed solving these equations either pictorially, using visual aids, or symbolically, using algebraic methods, to find the value of the unknown variable.
4. **Create a Problem for a Given Equation:**
   - **Problem Creation:** We developed skills in creating real-life problems that match given equations. For instance, given the equation $2x + 3 = 11$, we might create a scenario where the total cost of x items, each costing 2 units plus an additional fee of 3 units, equals 11 units.

By engaging with these concepts, we improved our ability to model real-world situations algebraically, perform and simplify operations on expressions, and solve and create algebraic equations. This not only enhances our mathematical proficiency but also equips us with practical problem-solving skills applicable to everyday life.

SECTION 4: GEOMETRICAL REASONING AND MEASUREMENT

Strand: **Geometry and Measurement**

Sub-Strand: Shape and Space/ Measurement

Content standards

1. Demonstrate conceptual understanding of lines (parallel, perpendicular, complementary, supplementary angles, vertical and transversal)
2. Estimate and measure the area of 2-D shapes using centimetre and metre squared
3. Demonstrate understanding of time taken by events in minutes and hours

INTRODUCTION AND SECTION SUMMARY

This section focuses on various mathematical concepts related to geometry and measurement, with an emphasis on applying these concepts to real-life contexts. We start by exploring the relationships between angles and lines, including parallel, perpendicular, complementary and supplementary angles and the effects of a transversal cutting through parallel lines. We then move to practical applications, such as measuring and recording the area of regular and irregular shapes using grid sheets and applying formulas to determine areas in square centimetres and square metres. We also cover volume calculations, including determining how many 1 cm^3 cubes fit into a box and finding different box sizes with the same volume. Additionally, we calculate time for events, including determining the duration and the starting or ending time based on given information.

The section will cover the following focal areas:

1. *Identifying and applying parallel, perpendicular, complementary, supplementary angles, vertical and parallel lines cut by transversal in real-life contexts.*
2. *Measuring and recording area for regular and irregular shapes in squared cm and squared m using grid sheets.*
3. *Developing and applying a formula for determining area of given shapes in centimetres and metres squared.*
4. *Determining the volume of boxes by finding how many cubes of sizes 1cm^3 each contains*
5. *Determining different sizes of boxes that have the same volume.*
6. *Determining the time taken to conduct an event.*
7. *Determining the starting or ending time of events given a duration*

SUMMARY OF PEDAGOGICAL EXEMPLARS

To effectively teach these concepts, consider the following strategies:

1. **Real-Life Applications:** Use real-world examples to illustrate the application of angles and lines. For example, examine the angles and lines in everyday objects such as books, windows, or road signs.
2. **Hands-On Activities:** Employ grid sheets and manipulatives to help learners measure and record areas. Have learners practise measuring and calculating areas of various shapes, both regular and irregular.

3. **Volume Exploration:** Use cubes and other solid objects to demonstrate volume calculations. Allow learners to physically count the number of 1 cm^3 cubes in different boxes and explore how different box sizes can contain the same volume.
4. **Capacity and Time Measurement:** Engage learners in practical exercises involving containers of various sizes to understand capacity. Use timing activities, such as timing a class activity, to teach how to calculate the duration and determine start and end times.
5. **Interactive Tools:** Integrate technology, such as graphing software or online measurement tools, to enhance understanding and provide visual representations of these concepts.

ASSESSMENT SUMMARY

To assess learners' grasp of these concepts, employ the following strategies:

1. **Problem-Solving Exercises:** Assess learners' ability to apply knowledge of angles and lines in real-life contexts through problem-solving exercises that require them to identify and analyse different types of angles and lines.
2. **Practical Measurement Tasks:** Evaluate learners' skills in measuring and recording areas by providing various shapes for them to measure and calculate areas using grid sheets and formulas.
3. **Volume and Capacity Problems:** Test learners' ability to determine volume by counting cubes and solving problems related to finding different box sizes with the same volume. Assess their understanding of capacity through similar container exercises.
4. **Time Calculations:** Include tasks that require learners to calculate the duration of events and determine start or end times based on given durations. Assess their accuracy in calculating and recording time-related information.

Week 8: The Concept of Lines and Its Applications

Learning Indicator: *Identify and apply parallel, perpendicular, complementary, supplementary angles, vertical and parallel lines cut by transversal in real-life contexts.*

Focal Area: The Concept of Lines

Introduction

Lines are like the basic building blocks of shapes and space. They are everywhere, from the straight roads we walk along to the edges of the things we see. This week we will explore lines, types of lines and characteristics of lines and examples of lines in real life.

Review the learner's previous knowledge of lines, line segments and rays.

The fundamental elements in geometry that help us understand geometric shapes and structures are lines, line segments and rays. These elements have distinct characteristics and properties.

Exploring the concepts of lines, line segments and rays

We can use the diagram below to explore the characteristics of the line, line segments and rays.

Diagram A is a line which has two arrows indicating that there are no endpoints. Therefore, a line is an infinite set of points that extends infinitely in both directions.

Diagram B is a ray which has only one arrow indicating its directions. Therefore, a ray is a part of the line that has one end point and extends in one direction.

Diagram C has two distinct endpoints. The distance between them is the line segment. Therefore, a line segment is a part of a line with two distinct endpoints.

Relationships between line, line segments and rays.

A line segment and a ray are subsets of a line and the differences between them are their length, endpoints and directional properties.

Exploring types of lines using real-life objects

We can use objects around us to explore the types of lines. Lines can come in many shapes and orientations. Some are straight, like a ruler. Others can curve, like a snake's path. Horizontal lines run flat across, while vertical lines go straight up and down. Diagonal lines slant and intersecting lines cross each other.

For example, a straight line could be represented by a ruler, a curved line by a snake's path, a horizontal line by a table's surface, a vertical line by a doorframe, a diagonal line by a ramp, intersecting lines by a plus sign, parallel lines by railroad tracks, and perpendicular lines by the corners of a square.

Type of Lines

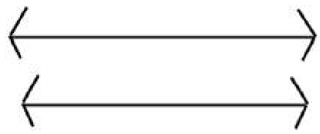
Straight Line: A line that does not change direction.

Ray: A line that starts at a specific point (the endpoint) and extends infinitely in one direction.

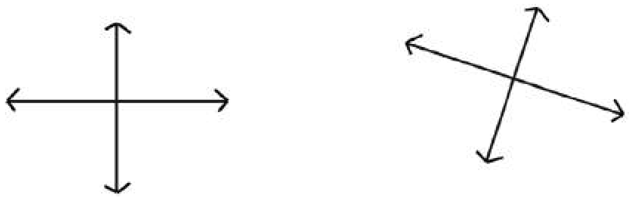
Line Segment: the shortest distance between two points.

Lines can be classified based on their intersection with other lines or the angles they form. The following are some examples of such lines:

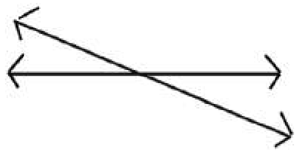
Parallel Lines: Lines that are in the same plane and never intersect, even when extended infinitely. Real-life examples of parallel lines are railroad tracks etc.



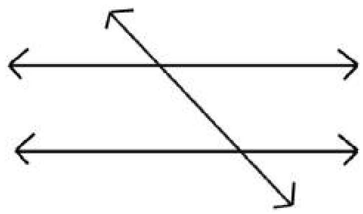
Perpendicular Lines: Lines that intersect at a right angle (90°). A real-life example of a perpendicular line is the corner of a square.



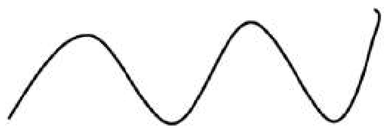
Intersecting Lines: Lines that share a common point and cross each other at that point. For example a plus sign, corners of a desk or the crossing paths on a grid.



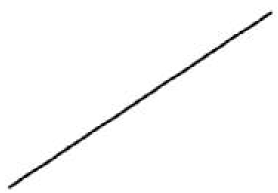
Transversal Line: A line that intersects two or more parallel lines, creating corresponding angles, alternate interior angles, alternate exterior angles, and consecutive interior angles.



Curves: Curved lines bend or arc in different directions. For example, the contour of a circle or the edges of a leaf, snake's path.



Diagonals: lines slant or lean in any direction between horizontal and vertical. For example, the edge of a staircase.



Learning Tasks for Practice

1. Identify examples of lines in pictures. Label the types of lines on a worksheet or diagram and match descriptions of lines with their corresponding visual representations.
2. Create a poster or visual representation that illustrates the properties of different types of lines and classify lines based on their properties in the sorting activity.
3. Analyse and compare the properties of different types of lines to determine similarities and differences.
4. Create a real-world scenario where knowledge of lines is applied to solve a problem or make decisions.

Application /importance of lines

In the classroom, straight lines are drawn on the floor, the entrance, the window, and the zebra crossing on the roadside.

Lines have numerous applications in various fields, including mathematics, engineering, architecture, physics, and everyday life.

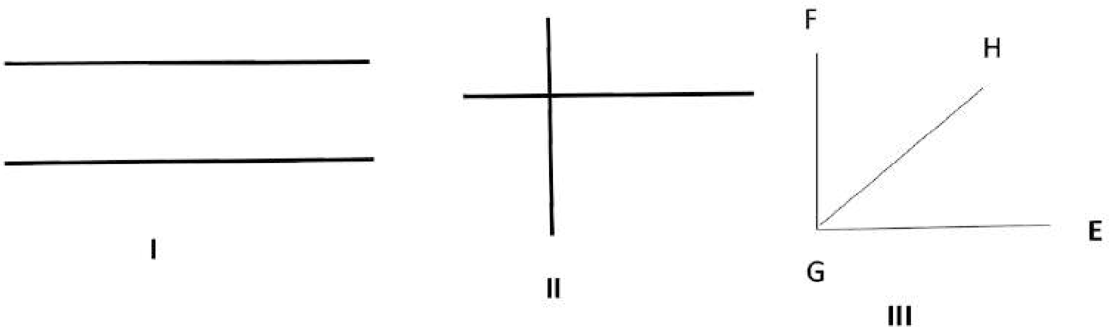
Pedagogical Exemplars

1. **Initiating Talk for Learning:** In a whole class discussion, explore learners' understanding of the concepts of lines, line segments and rays by reviewing their previous knowledge.
2. **Collaborative learning:** In mixed ability/gender groups, encourage learners to explore, describe and ink the properties of the types of lines on the board.
3. **Structuring Talk for learning:** in whole class discussion, through questioning and answering, explain the properties and encourage learners to come up with their definitions of lines, line segments and rays.
4. **Experiential learning:** ask learners to identify and draw any type of line in the classroom.
5. **Problem-based learning:** provide learners in their mixed-ability gender groups, a variety of lines. Ask learners to think, ink and share the properties of each line using the guiding questions.

Key Assessment

Level 1

1. Name two types of lines commonly used in geometry.
2. Look at the diagram below and state the types of lines they represent.



3. State whether the following are TRUE or FALSE.

   i. A line is a set of points in a straight path that extends in opposite directions without ending.

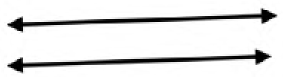
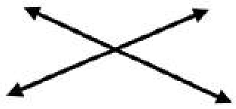
ii. A line has a fixed length.

iii. Horizontal lines are parallel to the $y-$axis

iv. A ray extends in one direction.

Level 2

Match the following types of lines.

a.		Vertical line
b.		Curved line
c.		Intersecting lines
d.		Parallel lines

Level 3

1. Compare and contrast a straight line and a curved line.
   What are the main differences and similarities?
2. What is the difference between parallel lines and perpendicular lines?
   Provide an example of each.
3. Compare a vertical line and a horizontal line.
4. Identify 6 types of lines and give their properties.

Level 4

1. What are the applications of lines and their properties in real life?
2. Create a real-life situation using properties of lines.

Week 9: Measurement of Area

Learning Indicators

1. *Measure and record area for regular and irregular shapes in squared cm and squared m using grid sheets.*
2. *Develop and apply a formula for determining area of given shapes in centimetres and metres squared.*

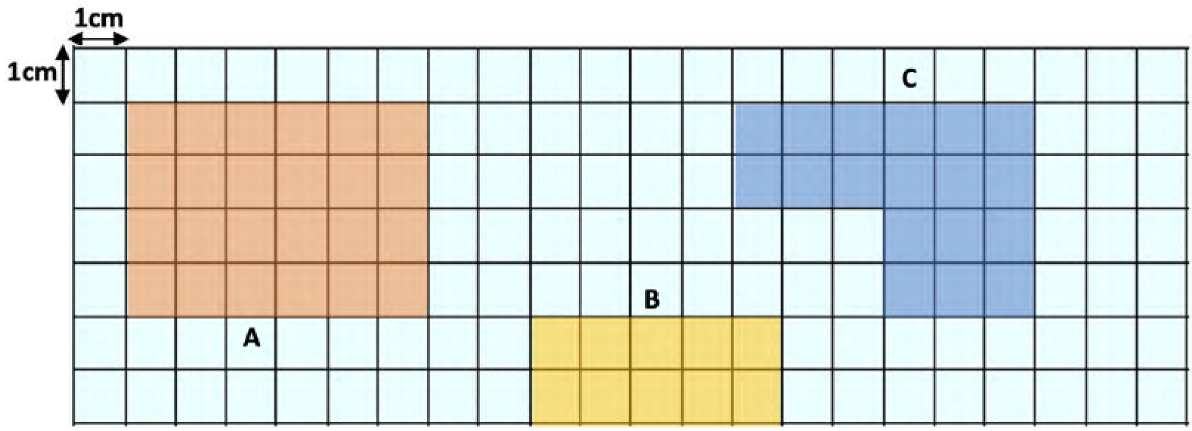
Focal Area: Measurement of Area of Regular and Irregular 2D Shapes

Understanding how to measure the area of both regular and irregular 2D shapes is a fundamental concept in geometry. This knowledge is crucial for solving practical problems in various fields such as architecture, engineering, and everyday life scenarios. Teaching this concept involves helping learners grasp the methods and formulas for calculating the area of different shapes and developing their ability to apply these methods to solve real-world problems.

Key Concepts:

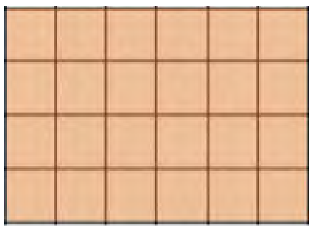
1. **Definition of Area**
   - The area is the amount of space inside a two-dimensional shape.
   - Area is measured in square units, such as square centimetres (cm^2), square metres (m^2), or square inches (in^2).

As in the definition of area, we can measure square units. For example, take a look at the grid below;



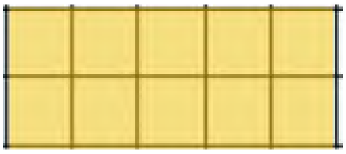
To find the area of any of the three shapes on the grid, we can count the number of squares covered by the shape. So, let's find the area of each shape.

Solution

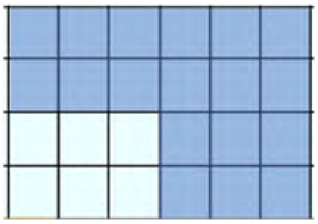


Area of shape A = number of squares in the shape
= 24 squares.

Now, since a square on the grid are 1cm by 1cm, the area of the shape is **24 cm^2**.



Area of shape B = number of squares in the shape
= 10 squares.
Therefore, area of Shape B =**10 cm²**.



Area of shape C = number of squares in the shape
= 18 squares.
Therefore, area of Shape C =**18 cm²**.

Measure and record areas for real-life regular shapes in cm^2 and m^2

In our various schools and homes, we can measure the area of places such as floors, walls, table tops, compound, and many others. We can measure the area of such places and objects using the idea of a square grid.

Assuming you are measure the area of the floor of a room with tiles. One way to do this is to measure the side lengths of one of the tiles, then count how many of the tiles covers the floor. Take a look at this activity.

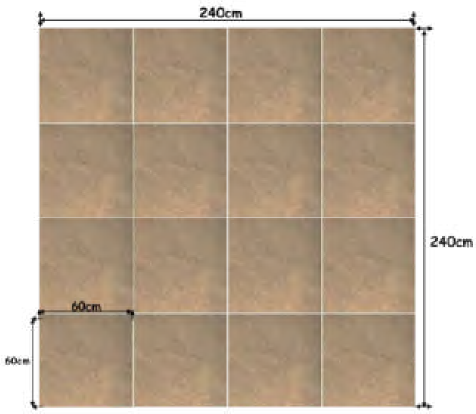
Practical Activity:

Floor tiles

Measuring the sides of 1 tile

Tile measures 60cm × 60cm

Determine the area of this floor



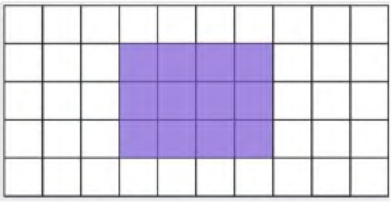
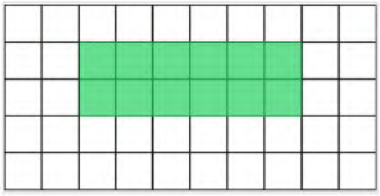
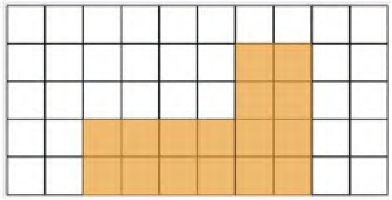
Area of floor = 240cm × 240cm
= 57,600cm2

OR

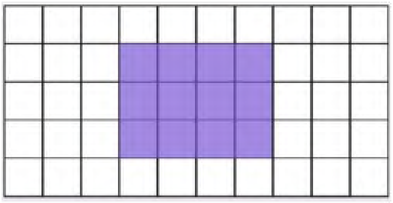
Area of floor = area of 1 tile × number of tiles
= (60cm × 60cm) × 16
= $3600cm^2 \times 16$
= $57,600cm^2$

Worked Examples

Determine the area of the following shapes if a square in each of the shapes is 1cm by 1cm.

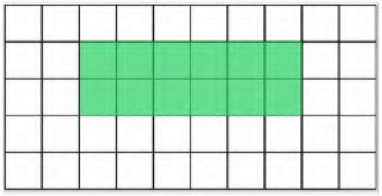




Solution



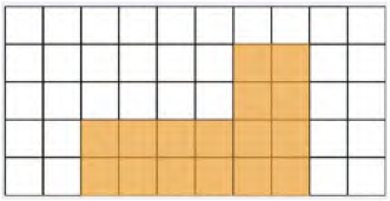
Area of shape = 12 squares

= 12 cm^2



Area of shape = 12 squares

= 12 cm^2



Area of shape = 16 squares

= 16 cm^2

Learning Tasks for Practice

Learners engage in practical activities to measure the area of places and objects.

Focal Area: Develop and Apply a Formula for Determining the Area of Squares and Rectangles in Centimetre and Metre Squared

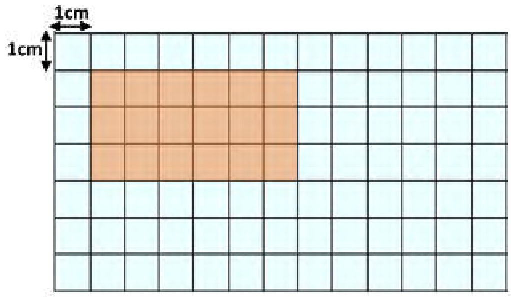
We can use formulas to determine the area of squares and rectangles. The formulas help to make our calculations faster.

Formula for Area of a Rectangle

Assuming I want to calculate the area of the rectangle on the grid below.

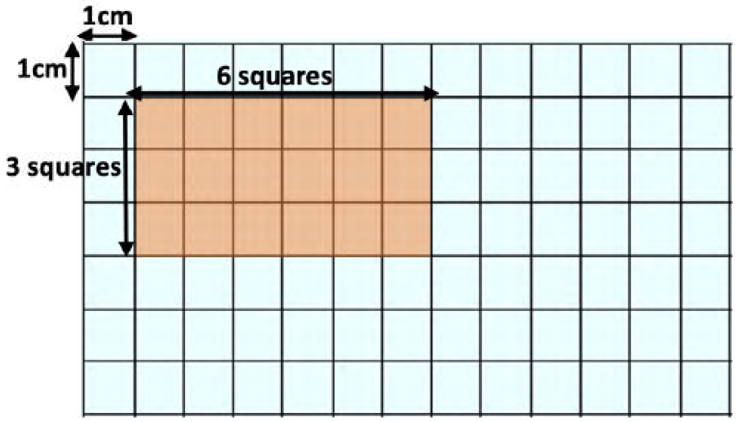
We just have to count the number of squares covered by the shape.

When we count, we get 18 squares.

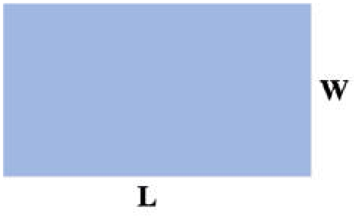


We can also multiply the number of square on the vertical side of the shape by the horizontal side.

Therefore, we have 3 × 6 = 18 squares.



Therefore, for a given rectangle, the area is given by the product of the length by the width.



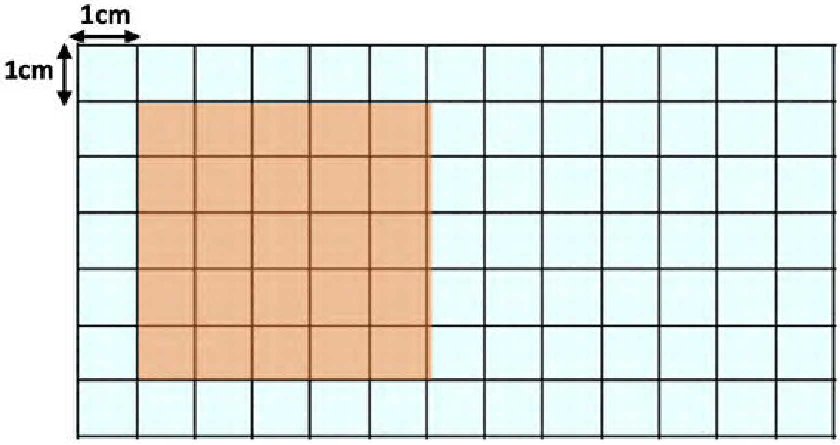
Area of rectangle = L x W

Formula for Area of a Square

To calculate the area of the square on the grid below.

We just have to count the number of squares covered by the shape.

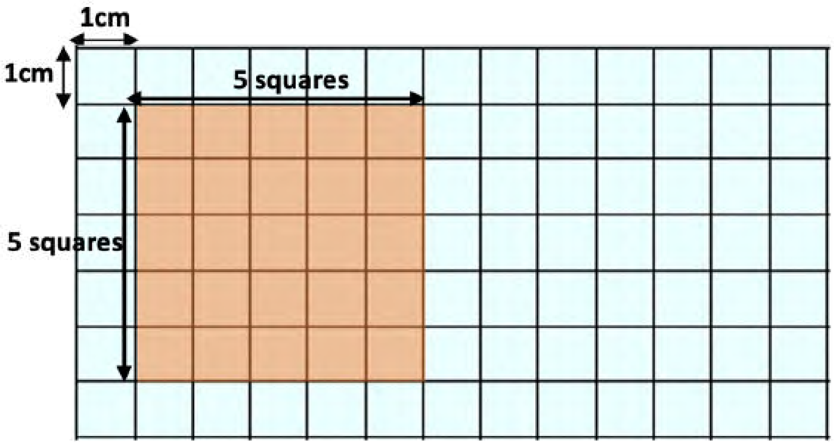
When we count, we get 25 squares.



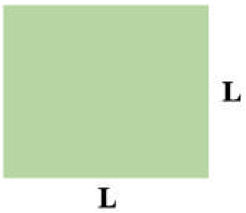
We can also multiply the number of squares on the vertical side of the shape by the horizontal side. But, since it is a square, all the sides are equal. Therefore, we can count the number of squares on any of the sides, then multiply this by itself.

Therefore, we have 5 × 5 = 25 squares.

Calculate the area of given 2D shapes



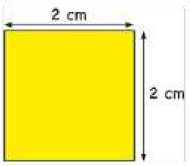
Therefore, for a given square, the area is given by the product of the length by the length.

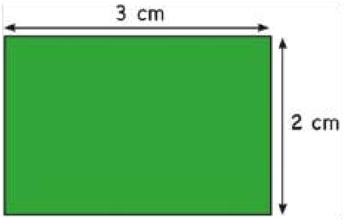


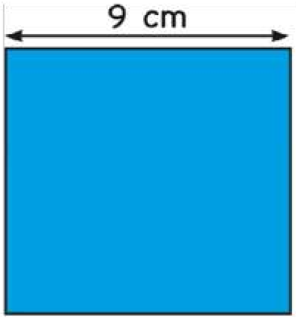
Area of square = L x L

Worked Examples

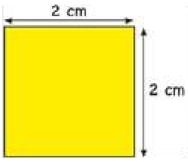
Calculate the area of the following shapes.





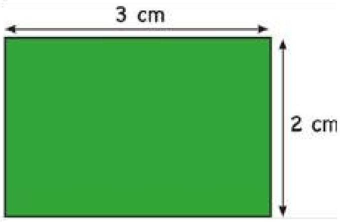


Solution



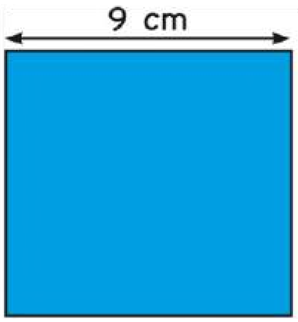
Area of square = L × L

Area of square = 2 cm × 2 cm = 4 cm^2



Area of rectangle = L × W

Area of square = 2cm × 3 cm = 6 cm^2



Area of square = L × L

Area of square = 9 cm × 9 cm = 81 cm^2

Learning Tasks for Practice

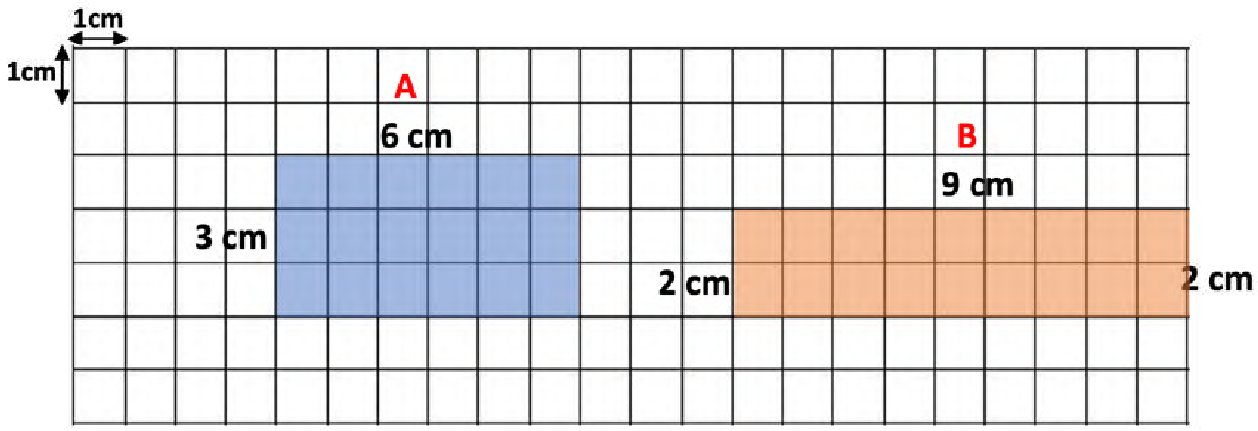
Learners discuss the formulas for calculating the area of squares and rectangles and solve problems on them.

Focal Area: Drawing Different Shapes for the Same Area

We can draw different shapes for the same area using the grid paper. Let's take a look at some examples.

Examples

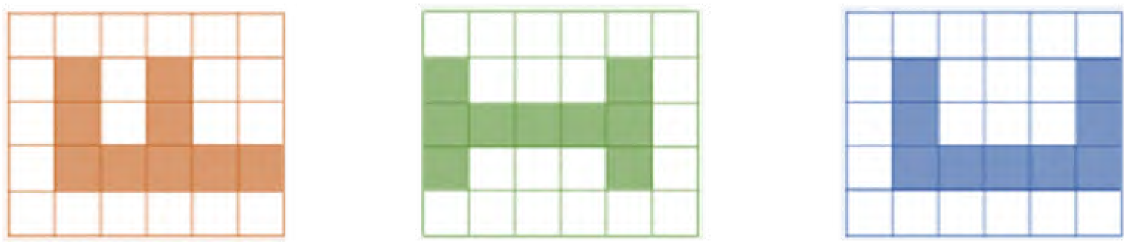
1. Given the area 18 cm^2, draw two different shapes with different length sides for the same area.



Now, comparing the two shapes (Shape A and B), we can see that they both have different side lengths and are obviously different shapes but they both have the same area of 18 cm^2.

2. Given the area 9 cm^2, draw three different shapes with different sides for the same area.

Solution



Focal Area: Solving Word Problems on Area Measurement

Examples

1. A rectangular carpet is 6 m long and 4 m wide. Find its area.

Solution

Area of rectangle = L × W

Area = 6 m × 4 m = 24 m^2

2. Find the area of the floor of a square room in square metres with each side measuring 8 m.

Solution

Area of square = L × L

Area of square = 8 m × 8 m = 64 m^2

3. A laptop's rectangular screen measures 22 cm by 18 cm. What is the area of the laptop screen?

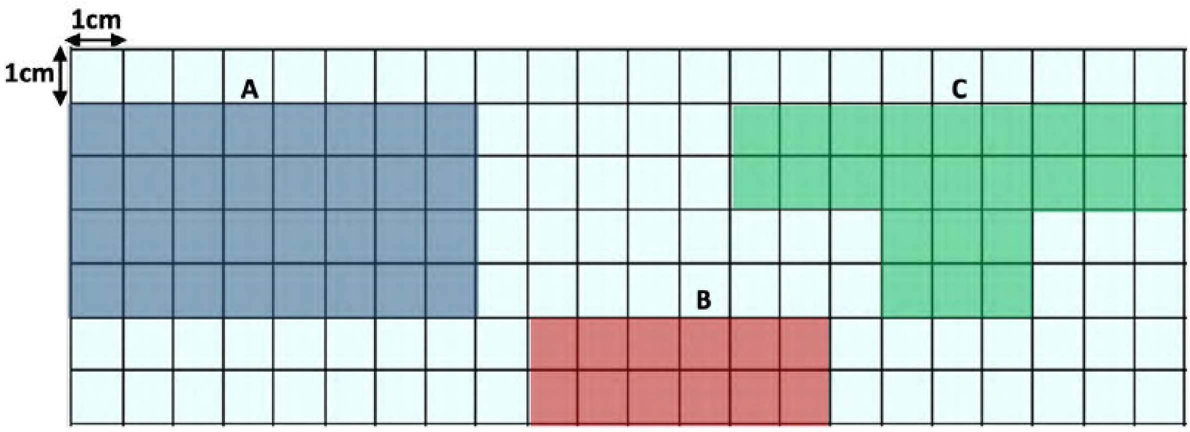
Solution

Area of rectangle = L × W

Area = 22 cm × 18 cm = 396 cm^2

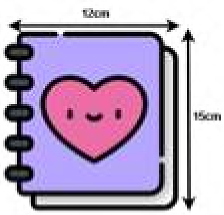
Key Assessment

1. Determine the area of the following shapes



2. Solve the following word problems:

Korkor wants to cover the entire face of her pocket notebook with a decorative sticker. If the face of the notebook has dimensions 15cm by 12cm, calculate the area of the sticker needed to cover the notebook.



3. A large rectangular dining table has sides of 7 m and 4 m.
What is the perimeter of the table? What is its area?

4. A square-shaped playground has length 23 m. What is the area of the playground?

Week 10: Measurement of Volume

Learning Indicators

1. *Determine the volume of boxes by finding how many cubes of sizes 1cm³ each contains*
2. *Determine different sizes of boxes that have the same volume.*
3. *Determine different sizes of containers that have the same capacity.*

Focal Area: Determining the Volume of Solid Shapes

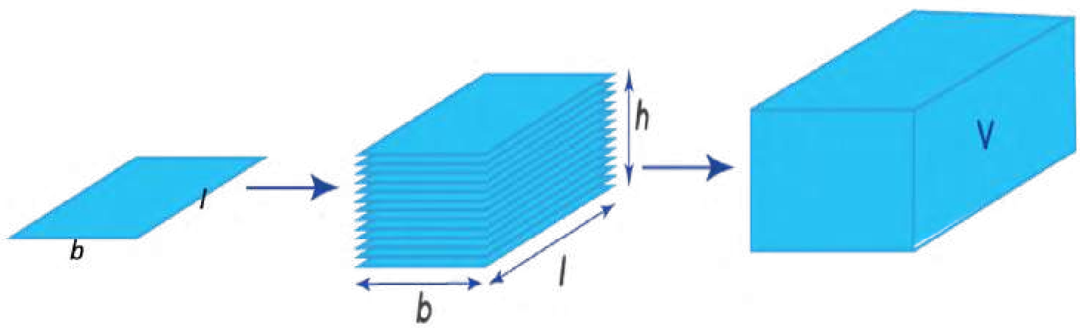
Volume measurement is essential in various real-world applications, including engineering, construction, packaging, and everyday life. Learning this concept involves understanding the basic principles of volume, introducing the formulas for different solid shapes, and providing learners with practical examples to apply their knowledge.

Volume is the amount of space occupied by a three-dimensional object. We make use of the concept of volume in our everyday life. When we fill our cups with water to drink or pack books into boxes, we can talk about the amount of space that has been taken up by the substances. That amount of space is volume. It is measured in cubic units, such as cubic centimetres (cm^3), cubic meters (m^3), or cubic inches (in^3).

Demonstrating Volume

Volume of a cuboid

Suppose we have some rectangular sheets with length **'l'** and width '**b**'. If we stack them one on top of the other up to height **'h'**, we get a cuboid of dimensions **l, b, h**. This can be seen in the following figure which shows the length, width (breadth), and height of the cuboid thus formed.



To calculate the amount of space enclosed by this cuboid, we use the formula:

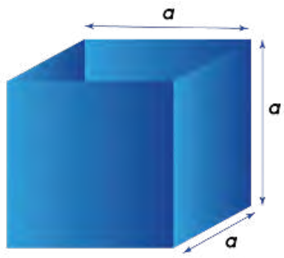
Volume of a Cuboid = $l \times b \times h$

Volume of a cube

A cube is a special case of a cuboid where all three sides are equal in length. If we represent this equal value as '*a*', then the volume of this cube can then be calculated with the formula:

Volume of a Cube = $a \times a \times a = a^3$

Observe the following figure to see the equal sides of a cube and the space it occupies.



Find the volume of boxes including finding how many cubes of sizes 1cm^3 each contains

Examples

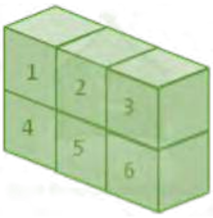
1. What is a cubic unit?

1 cubic unit is the amount of space that a unit cube occupies.

Or simply,

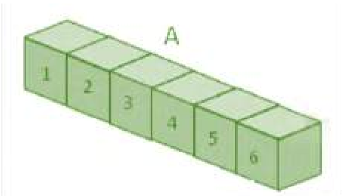
1 cubic unit = the volume of a unit cube = 1 unit^3

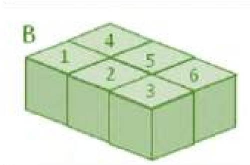
The object below is made of 6 unit cubes.

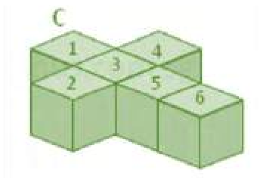


Its volume is 6 cubic units.

Here are some other solids that have the same volume (6 cubic units).



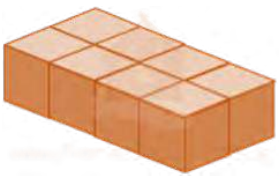




2. What is 1 cm^3?

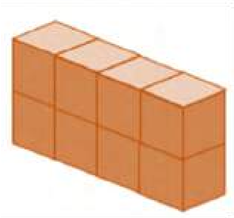
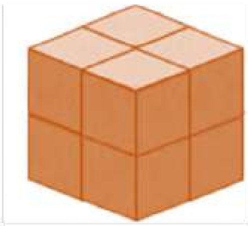
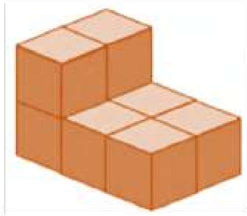
1 cm^3 = the volume of a cube of 1cm sides

The object below is made of eight 1cm^3 units.

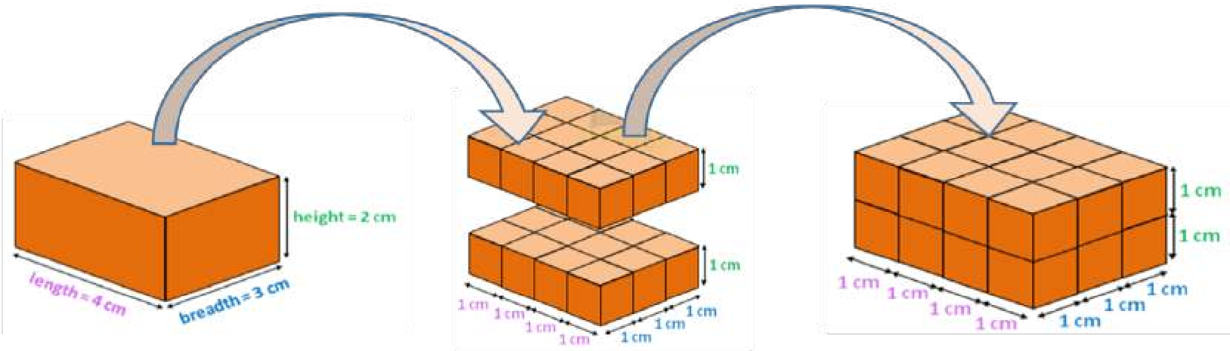


Its volume is 8 cm^3 or 8 cubic centimetres.

Here are some other solids that have the same volume (8 cm^3).

3. What is the volume of the cuboid below?



Each layer in the cuboid above has **4** × **3** = 12 cubes

Since there are **2** layers in the cuboid, there are

2 × 12 = **24 cubes altogether**.

Each of the 24 cubes has a volume of **1 cm³**.

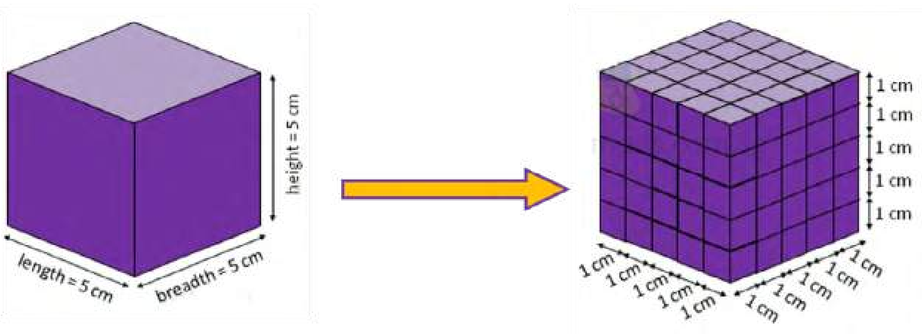
So, the volume of the cuboid is 24 × 1 cm^3 = **24 cm³**.

Volume of a cuboid = length × breadth × height

Applying this formula in the example above, we get:

Volume of the given cuboid = **4 cm** × **3 cm** × **2 cm** = 24 **cm³**

4. What is the volume of the cube below?

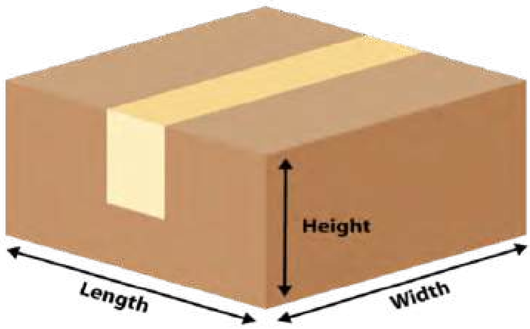


A cube is a cuboid with the same length, breadth and height.

Volume of a cube
= length × breadth × height
= **length × length × length**

Volume of the given cube
= 5 cm × 5 cm × 5 cm
= 125 cm³

5. Calculate the volume of the box with length 40 cm, height 10 cm and width 20 cm.



Solution

Volume of the given box = 40 cm × 10 cm × 20 cm

= 4 × 1 × 2 × 1000 = 8000 (multiply the first digits and attach the 3 zeros) **= 8000 cm³**

Learning Tasks for Practice

Learners determine the volume using cubes and cuboids including real life objects that are considered cubes and cuboids.

Focal Area: Constructing Different Solid Shapes with Different Dimensions but the Same Volume

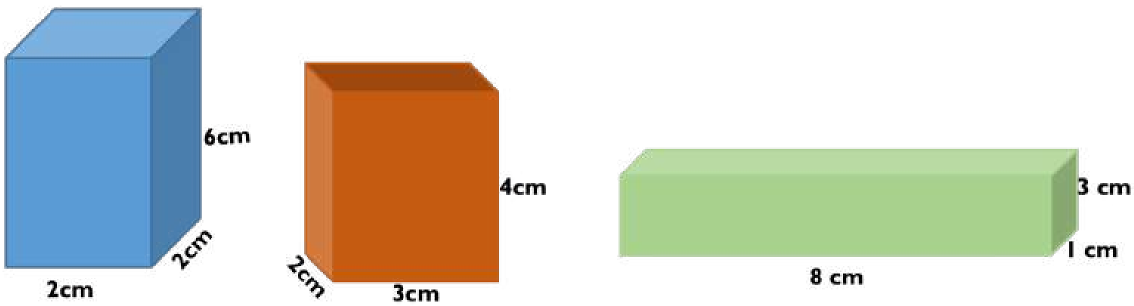
Understanding that different solid shapes can have the same volume despite having different dimensions is an important concept in geometry. This principle highlights the diversity and flexibility in three-dimensional space and deepens learners' comprehension of volume. Teaching this concept involves comparing various solid shapes and showing how they can occupy the same amount of space even when their dimensions differ significantly.

Different shapes, like a cylinder, cube, rectangular prism, cone and sphere, can have different dimensions yet occupy the same volume. This concept demonstrates the variability in shapes that can contain the same amount of space.

Examples

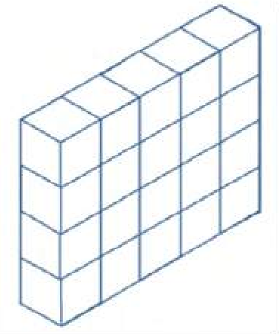
1. Given the volume 24 cm³, construct three different shapes for this volume.

Solution

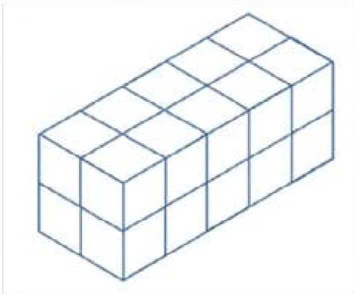


2. Given the volume 20 cm³, construct two different shapes for the given volume.

Solution



Volume = 20 cm³



Volume = 20 cm³

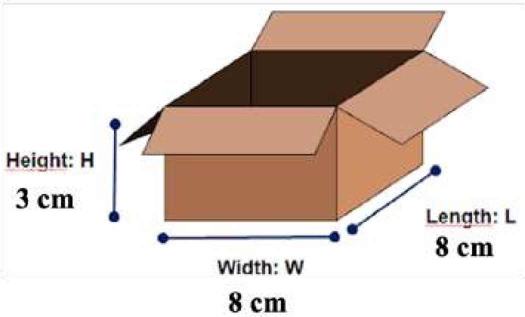
Focal Area: Determine Different Sizes of Boxes that have the Same Volume.

In our everyday lives, we come into contact with containers and boxes that look obviously different in terms of their structure but they have the same volume.

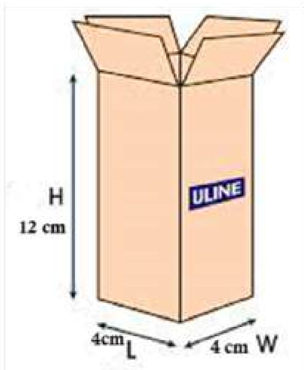
Let's take a look at this example.

The two boxes look different and have different dimensions. However, their volumes are the same. Let's calculate to confirm.

A



B



Solution

Volume of Box A = $3cm \times 8cm \times 8cm = 192\ cm^3$

Volume of Box B = $12cm \times 4cm \times 4cm = 192\ cm^3$

Hence, we can conclude that even though the two boxes are of different shapes and have different dimensions, they have the same volume.

Learning Tasks for Practice

Learners construct different solid shapes with different dimensions but the same volume.

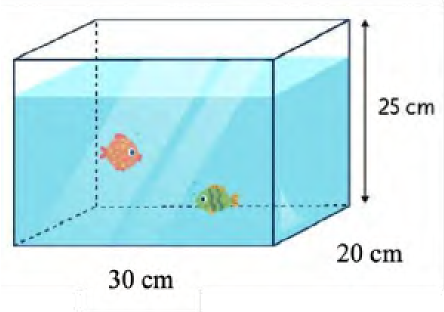
Pedagogical Exemplars

1. **Collaborative learning: In convenient mixed-ability groups,** learners discuss the meaning of volume and establish how it can be measured.

2. **Problem-Based Learning: In mixed-ability groups,** learners discuss and solve problems on volume of cubes and cuboids including real life objects that are considered cubes and cuboids.
3. **Experiential learning: In mixed-ability/gender groups,** learners construct different solid shapes with different dimensions but the same volume.

Focal Area: Solving Word Problems Involving Volume of Cubes and Cuboids

1. Sarah has a small fish tank in the shape of a rectangular prism. The tank measures 30 centimetres in length, 20 centimetres in width, and 25 centimetres in height. She wants to fill the tank with water.



Calculate the volume of the Fish Tank:

- How many cubic centimetres of water will Sarah need to fill the tank completely?

Solution

Calculating the Volume:

- Volume of a cuboid = Length × Width × Height
- Volume = 30 cm × 20 cm × 25 cm
- Volume = 15,000 cubic centimetres

Sarah will need 15,000 cubic centimetres of water to fill the tank completely.

2. Sarah has a big storage box in the shape of a cube. Each side of the cube measures 2 metres. Find the volume of Sarah's storage box.

Solution

Volume of a cube = Side length × Side length × Side length

Volume = 2 m × 2 m × 2 m

Volume = 8 cubic metres

2. John wants to fill his new sandbox with sand. The sandbox is in the shape of a cuboid and measures 3 metres in length, 2 metres in width, and 0.5 metres in height.

Solution

Volume of a rectangular prism = Length × Width × Height

Volume = 3 m × 2 m × 0.5 m

Volume = 3 cubic metres or $3m^3$

Learning Tasks for Practice

Learners solve real-life problems on volume of cubes and cuboids.

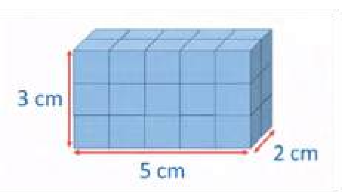
Pedagogical Exemplars

Collaborative/Problem-Based learning: In convenient mixed-ability groups, learners solve real-life problems on volume of cubes and cuboids. Learners also write their own problems for their classmates to solve.

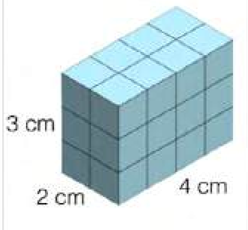
Key Assessment

1. Determine the volume of the following shapes

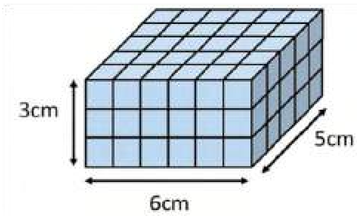
A



B



C



2. Calculate the volume of the following boxes.

3. Solve the following word problems:

i. Mansa has an aquarium in the shape of a cuboid. The aquarium measures 2 metres in length, 1 metre in width, and 1.5 metres in height.

What is the volume of the aquarium in cubic metres?

ii. John uses a rectangular storage container for his toys. The container has a length of 3 metres, a width of 2 metres, and a height of 1.5 metres.

What is the volume of the storage container in cubic metres?

iii. A shipping company uses a rectangular boxes to send packages. The dimensions of the box are 300 cm in length, 200 cm in width, and 150 cm in height.

What is the volume of the shipping box in cubic centimetres?

iv. A community centre has a rectangular swimming pool. It measures 25 metres in length, 10 metres in width, and 2 metres in depth.

What is the volume of the swimming pool in cubic metres?

v. A cereal box is in the shape of a cuboid, with a length of 20 centimetres, a width of 8 centimetres, and a height of 30 centimetres.

What is the volume of the cereal box in cubic centimetres?

Week 11: Measuring Time in Everyday Life

Learning Indicators

1. *Determine the time taken to conduct an event.*
2. *Determine the starting or ending time of events given a duration.*

Focal Area: Measurement of Time

Determine the time taken to conduct an event.

Understanding how to determine the time taken to conduct an event is a fundamental skill in mathematics that has numerous practical applications in daily life and various professional fields. Whether planning a schedule, coordinating activities, or analysing processes, accurately calculating the duration of events is essential. This concept helps learners develop their ability to manage time efficiently and solve real-world problems.

When we talk about determining the time taken for an event, we focus on several key aspects:

1. **Start Time and End Time:** Identifying when the event begins and when it ends.
2. **Time Intervals:** Calculating the difference between the start and end times to find the total duration.
3. **Units of Time:** Understanding and converting between different units of time such as seconds, minutes, hours, and days.
4. **Tools and Methods:** Using various methods, such as clocks, stopwatches, and mathematical calculations, to determine the time taken.

What is Time?

Time is a fundamental concept that measures the ongoing sequence of events from the past through the present to the future. It is a continuous, irreversible progression that allows us to order and compare events, understand durations and coordinate activities. In everyday life, time helps us structure our day, plan our activities, and keep track of when things happen.

Units of Time and their relationships

Time is measured in various units, each serving different purposes and scales of duration. The basic units of time include seconds, minutes, hours, days, weeks, months and years. Here's how these units relate to each other:

- **Seconds (s):** The smallest standard unit of time commonly used in everyday activities. It is the base unit in the International System of Units (SI).

- **Minutes (min):** 1 minute is equal to 60 seconds.
- **Hours (h):** 1 hour is equal to 60 minutes or 3,600 seconds.
- **Days:** 1 day is equal to 24 hours.
- **Weeks:** 1 week is equal to 7 days.
- **Months:** Typically, 1 month is around 30 or 31 days, but it varies (e.g. February has 28 or 29 days).
- **Years:** 1 year is equal to 12 months or approximately 365.25 days, accounting for leap years.

Relationships Among the Units:

- 1 minute = 60 seconds
- 1 hour = 60 minutes = 3 600 seconds
- 1 day = 24 hours = 1 440 minutes = 86 400 seconds
- 1 week = 7 days = 168 hours = 10 080 minutes = 604 800 seconds
- 1 month = 30 days (varies) = 720 hours = 43 200 minutes = 2 592 000 seconds
- 1 year = 12 months = 365.25 days = 8 766 hours = 525 960 minutes = 31 557 600 seconds

Converting Between Units: Understanding the relationships between these units allows us to convert time from one unit to another.

Examples

1. **Converting Hours to Minutes**
   Problem: Convert 3 hours to minutes.

   Step-by-Step Solution:

   Understand the conversion factor:

   1 hour = 60 minutes

   Multiply the number of hours by the conversion factor:

   3 hours × 60 minutes/hour = 180minutes

   Answer: 3 hours is equal to 180 minutes.

2. **Converting Minutes to Seconds**
   Problem: Convert 45 minutes to seconds.

   Step-by-Step Solution:

   Understand the conversion factor:

   1 minute=60 seconds

   Multiply the number of minutes by the conversion factor:

   45 minutes × 60 seconds/minute = 2 700 seconds

   Answer: 45 minutes is equal to 2 700 seconds.

3. Converting Seconds to Hours

Problem: Convert 7 200 seconds to hours.

Step-by-Step Solution:

Understand the conversion factor:

1 hour=3 600 seconds

Divide the number of seconds by the conversion factor:

7 200 seconds ÷ 3 600 seconds/hour = 2 hours

Answer: 7 200 seconds is equal to 2 hours.

4. Converting Hours to Seconds

Problem: Convert 1.5 hours to seconds.

Step-by-Step Solution:

Understand the conversion factors:

1 hour=60 minutes

1 minute = 60 seconds

First, convert hours to minutes:

1.5 hours × 60 minutes/hour = 90 minutes

Then, convert minutes to seconds:

90 minutes × 60 seconds/minute = 5 400 seconds

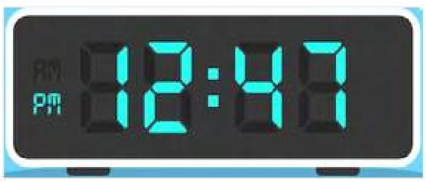
Answer: 1.5 hours is equal to 5 400 seconds.

Focal Area: Calculating the Difference Between the Start and End Times to Find the Total Duration

We can calculate the time spent for an event when we know the starting and ending times of the events.

For example, study the clocks carefully;

Start Time



End Time

The two clocks show a start time and an end time. To calculate the time spent we will follow these steps:

Step-by-Step Solution:

Write down the start time and end time:

- Start time: 7:12
- End time: 12:47

Calculate the difference in minutes:

- From 7:12 to 12:12 is 5 hours (300 minutes)
- From 12:12 to 12:47 is 35 minutes.

Add the minutes together:

300 minutes + 35 minutes = 335 minutes

Convert minutes to hours and minutes (optional):

335 minutes = 5 hours and 35 minutes

Answer: The time duration is 5 hours and 35 minutes.

Let's take a look at another example. Take a look at the two clocks carefully;

Start Time

End Time

For clocks, your first task is to be able to read the clock. The start time is 6:25 a.m. and the end time is 9:00 a.m.

Solution

Starting time is 6:25 a.m. and ending time is 9:00 a.m. How long is the time duration?

Step-by-Step Solution:

Write down the start time and end time:

- Start time: 6:25 a.m.
- End time: 9:00 a.m.

Calculate the difference in minutes:

- From 6:25 a.m. to 8:25 a.m. is 2 hours (120 minutes).
- From 8:25 a.m. to 9:00 a.m. is 35 minutes.

Add the minutes together:

120 minutes+35 minutes=155 minutes

Convert minutes to hours and minutes (optional):

155 minutes = 2 hours and 35

Answer: The time duration is 2 hours and 35 minutes.

The use of clocks can be very tricky. You must always take into consideration the use of a.m. and p.m.

In our above example, the ending time is 9:00 a.m. Assuming the end time was 9:00 p.m., the calculation would have been different. Let's take a look at the calculation.

Starting time is 6:25 a.m. and ending time is 9:00 p.m. How long is the time duration?

Step-by-Step Solution:

Write down the start time and end time:

- Start time: 6:25 a.m.
- End time: 9:00 p.m.

Understand the difference between a.m. and p.m.:

- a.m. stands for "ante meridiem," which means before midday (midnight to noon).
- p.m. stands for "post meridiem," which means after midday (noon to midnight).

Calculate the time duration in two parts:

- From 6:25 a.m. to 12:00 p.m. (noon) is the first part.
- From 12:00 p.m. to 9:00 p.m. is the second part.

Calculate the difference in the first part:

- From 6:25 a.m. to 12:00 p.m. is 5 hours and 35 minutes (6:25 a.m. to 7:25 a.m. is 1 hour, 7:25 a.m. to 8:25 a.m. is 1 hour, 8:25 a.m. to 9:25 a.m. is 1 hour, 9:25 a.m. to 10:25 a.m. is 1 hour, 10:25 a.m. to 11:25 a.m. is 1 hour, 11:25 a.m. to 12:00 p.m. is 35 minutes).

Calculate the difference in the second part:

- From 12:00 p.m. to 9:00 p.m. is 9 hours.

2. **Add the two parts together:** 5 hours and35 minutes + 9 hours = 14 hours and 35 minutes.

   Answer: The time duration is 14 hours and 35 minutes.

   Explanation:

   - When the end time is 9:00 a.m., it means the calculation is within the same a.m. period, spanning just a few hours.
   - When the end time is 9:00 p.m., it spans from the a.m. period into the p.m. period, covering almost the entire day. This adds significantly more hours to the duration.

Focal Area: Word Problems Involving Amount of Time Used to Complete Events

1. A group of boys started walking from school at 10:25 a.m. to the community park to play. They got there at 11:50 a.m. How long did they walk?

   Solution:

   Write down the start time and end time:

   - Start time: 10:25 a.m.
   - End time: 11:50 a.m.

   Convert the times to a 24-hour format (optional):

   - Start time: 10:25
   - End time: 11:50

   Calculate the difference in minutes:

   - From 10:25 to 11:25 is 60 minutes (1 hour).
   - From 11:25 to 11:50 is 25 minutes.

Add the minutes together:

60 minutes + 25 minutes = 85 minutes

Convert minutes to hours and minutes (optional):

85 minutes = 1 hour and 25 minutes

Answer: They walked for 1 hour and 25 minutes.

2. Lisa started her homework at 4:15 p.m. and finished at 6:05 p.m. How long did she spend on her homework?

Step-by-Step Solution:

Write down the start time and end time:

- Start time: 4:15 p.m.
- End time: 6:05 p.m.

Convert the times to a 24-hour format (optional):

- Start time: 16:15
- End time: 18:05

Calculate the difference in minutes:

- From 16:15 to 17:15 is 60 minutes (1 hour).
- From 17:15 to 18:05 is 50 minutes.

Add the minutes together: 60 minutes + 50 minutes = 110 minutes

Convert minutes to hours and minutes (optional):

110 minutes = 1 hour and 50 minutes

Answer: She spent 1 hour and 50 minutes on her homework.

3. A movie started at 7:45 p.m. and ended at 9:20 p.m. How long was the movie?

Step-by-Step Solution:

Write down the start time and end time:

- Start time: 7:45 p.m.
- End time: 9:20 p.m.

Convert the times to a 24-hour format (optional):

- Start time: 19:45
- End time: 21:20

Calculate the difference in minutes:

- From 19:45 to 20:45 is 60 minutes (1 hour).
- From 20:45 to 21:20 is 35 minutes.

Add the minutes together: 60 minutes + 35 minutes = 95 minutes

Convert minutes to hours and minutes (optional):

95 minutes = 1 hour and 35 minutes

Answer: The movie was 1 hour and 35 minutes long.

Calculating Start and End Time of Events

Sometimes, we are required to determine the start or end time of events. In this case you are provided with either the end or start time and the duration (length) of the event.

Let's take a look at some examples.

1. A family plans a picnic which will take 4 hours and 30 minutes.

   If they want to finish their picnic by 3:00 p.m., what time should they start?

   Step-by-Step Solution:

   Write down the end time and the duration of the event:

   - End time: 3:00 p.m.
   - Duration: 4 hours and 30 minutes.

   Subtract the duration from the end time to find the start time:

   Subtract the minutes first:

   - From 3:00 p.m., subtract 30 minutes:
     - 3:00 p.m. − 30 minutes = 2:30 p.m.

   Subtract the hours next:

   - From 2:30 p.m. subtract 4 hours:
     - 2:30 p.m. − 4 hours = 10:30 a.m.

   Answer: The family should start their picnic at 10:30 a.m. to finish by 3:00 p.m.

2. A movie starts at 7:15 p.m. and lasts for 2 hours and 45 minutes. What time will the movie end?

   Step-by-Step Solution:

   Write down the start time and the duration of the event:

   - Start time: 7:15 p.m.
   - Duration: 2 hours and 45 minutes.

   Add the duration to the start time to find the end time:

   Add the minutes first:

   - From 7:15 p.m., add 45 minutes:
     - 7:15 p.m. + 45 minutes = 8:00 p.m.

   Add the hours next:

   - From 8:00 p.m. add 2 hours:
   - 8:00 p.m. + 2 hours = 10:00 p.m.

   Answer: The movie will end at 10:00 p.m.

Learning task for Practice

Learners should read clocks and solve problems on elapsed time.

Pedagogical Exemplars

Collaborative/Problem-Based learning: In convenient mixed-ability groups, learners convert between the various units of time and solve real-life problems on time.

Key Assessment

1. Deledem left home for school at half past six in the morning. She walked for 55minutes to get to school.

   What time did Deledem get to school?
2. Sarah started her homework at 4:20 p.m. and finished it at 6:55 p.m.

   How long did she spend on her homework?
3. A football match started at 3:30 p.m. and lasted for 1 hour and 45 minutes.

   What time did the match end?
4. John began his morning run at 6:45 a.m. He ran for 1 hour and 20 minutes.

   What time did he finish his run?
5. The school bus picks up learners at 7:15 a.m. and drops them off at school at 8:05 a.m.

   How long is the bus ride?
6. A workshop is scheduled to last for 3 hours and 15 minutes.

   If it needs to end by 2:30 p.m., what time should it start?

Section Review

In this section, we covered a variety of geometric and measurement concepts, focusing on their practical applications and problem-solving techniques:

1. **Identify and Apply Parallel, Perpendicular, Complementary, Supplementary Angles, Vertical and Parallel Lines Cut by Transversal in Real-Life Contexts:**
   - **Real-Life Contexts:** We identified and applied these geometric relationships to real-world scenarios, such as the layout of streets, architectural designs, and various engineering projects.
   - **Angles and Lines:** We explored how parallel and perpendicular lines, as well as various types of angles, interact in different contexts, and used transversal lines to analyse these interactions.
2. **Measure and Record Area for Regular and Irregular Shapes in Squared cm and Squared m Using Grid Sheets:**
   - **Area Measurement:** We measured and recorded the area of both regular and irregular shapes using grid sheets, ensuring accuracy in squared centimetres (cm^2) and squared metres (m^2).
   - **Grid Sheets:** Utilising grid sheets helped visualise and calculate the area, especially for irregular shapes.

3. **Develop and Apply a Formula for Determining Area of Given Shapes in Centimetres and Metres Squared:**
    - **Formulas for Area:** We developed and applied formulas for determining the area of various shapes, such as rectangles, triangles, and circles, ensuring we could calculate areas accurately in both cm^2 and m^2.
4. **Determine the Volume of Boxes by Finding How Many Cubes of Sizes $1cm^3$ Each Contains:**
    - **Volume Calculation:** We calculated the volume of boxes by determining the number of $1cm^3$ cubes each box could contain, enhancing our understanding of volume measurement.
5. **Determine Different Sizes of Boxes that Have the Same Volume:**
    - **Volume Equivalence:** We explored how different box dimensions can yield the same volume, reinforcing the concept of volume conservation.
6. **Determine the Time Taken to Conduct an Event:**
    - **Time Calculation:** We calculated the duration of events by measuring the start and end times, ensuring accurate time management.
7. **Determine the Starting or Ending Time of Events Given a Duration:**
    - **Time Management:** Given the duration of an event, we determined either the starting or ending time, enhancing our skills in scheduling and time allocation.

SECTION 5: WORKING WITH DATA

Strand: **Collecting and Handling Data**

Sub-Strand: Handling Data

Content Standard: Select, justify, and use appropriate methods of collecting data, including questionnaires, interview, observation, experiments, databases, electronic media, etc.

INTRODUCTION AND SECTION SUMMARY

In this section, we will explore the essential skills in data collection, focusing on selecting appropriate methods and designing effective tools for gathering information. We will begin by learning how to choose a suitable method for collecting data to address specific questions and justify our choices based on the context. Next, we will design and administer questionnaires and interviews, essential tools for collecting data in various settings. This process involves crafting clear and relevant questions, conducting the data collection, and recording the results accurately. These skills are crucial for conducting thorough and reliable research and for making informed decisions based on collected data.

The section will cover the following focal areas:

1. *Select a method for collecting data to answer a given question and justify the choice*
2. *Design and administer a questionnaire/interview for collecting data to answer a given question(s) and record the results*

PEDAGOGICAL SUMMARY

To effectively teach these data collection concepts, a variety of instructional strategies can be employed:

1. **Hands-on Practice:** Engage learners in selecting methods for real-world data collection scenarios. This could involve discussing different methods such as surveys, interviews, and observations, and practicing their application in various contexts.
2. **Design Workshops:** Guide learners in designing their own questionnaires and interviews. Provide examples of well-constructed questions and facilitate practice in creating tools that are clear, relevant, and aligned with the research objectives.
3. **Role-Playing:** Use role-playing exercises to simulate data collection, allowing learners to experience both administering questionnaires and conducting interviews. This practical approach helps in understanding the nuances of data collection.
4. **Analysis and Reflection:** After data collection, have learners analyse and reflect on their methods and results. This encourages critical thinking about the effectiveness of their chosen methods and the quality of the collected data.

ASSESSMENT SUMMARY

To assess learners' understanding and application of data collection methods, a range of assessment strategies should be utilised:

1. **Project-Based Assessment:** Evaluate learners' ability to select appropriate data collection methods and justify their choices through project-based tasks. Assess their rationale and the relevance of their selected methods to the given questions.

2. **Questionnaire/Interview Design:** Review the effectiveness and clarity of learners' designed questionnaires or interviews. Assess their ability to craft questions that are relevant, unbiased, and capable of eliciting useful information.
3. **Data Collection and Recording:** Assess the accuracy and organisation of the recorded results. This includes checking the completeness of the data and ensuring that it is recorded systematically.
4. **Presentation and Reflection:** Require learners to present their data collection process and results. Evaluate their ability to discuss their methodology, reflect on their findings, and make informed conclusions based on the collected data.

Week 12: Data Collection Methods and Their Uses

Learning Indicators

1. *Select a method for collecting data to answer a given question and justify the choice*
2. *Design and administer a questionnaire/interview for collecting data to answer a given question(s) and record the results*

Focal Area: Selecting and Justifying a Method for Collecting Data

Method of collecting data

To collect data, we can, for example:

- **a.** ask appropriate persons orally, by phone or face to face
- **b.** listen to the news
- **c.** read from the news print
- **d.** request from people through a written sets of questions
- **e.** observe the occurrence of situations
- **f.** search through the internet

These means of getting information become our ***methods of collecting data***.

Some methods of collecting data are interviews, observations, questionnaires, internet searches, existing documents (Databases), experiments, surveys, etc.

We will focus on interviews, observations and questionnaires.

Interviews

The process where information or data is obtained through verbal or oral means is termed an **interview**. **Interviews can take place in-person, by phone or online between the interviewer and the interviewee (respondents).** In any of these cases, a guide to support the line of questioning will be needed to keep the interviewer and the interview session on track.

The advantages and disadvantages of an interview:

- **a.** Helps to get needed information promptly.
- **b.** Further details and clarity of information can be obtained.
- **c.** It is very good for fewer respondents.
- **d.** Responses can be recorded on tape.
- **e.** It is not easy when there are many people to interview and some do not have the time to respond.

Observations

This is what we call the process where information is obtained by carefully watching a situation. We need to use our senses such as sight, smell, hearing and taste. In order not to forget or lose some elements of what we want to know while we observe, we are guided by an **observational check list.**

Questionnaires

The process where information is obtained by writing the questions on paper and the response is collected later is called a **questionnaire.** The tool we use (the set of written/typed questions) is termed the questionnaire.

This method works well if we want information from a large number of people, or we wish to gather information from an individual who is scarcely available, etc. In any of these cases, this forms a guide to support us to get the exact information we need, from the people we want.

Example

1. What type of data collection method could be involved in the pictures below?




2. If I get information from my phone or through my laptop. The method is … (*Tick [√] the correct answer*)
   - **a.** Phone/laptop seller [ ]
   - **b.** Electronic media /internet search [√]
   - **c.** Calling/messaging friends [ ]
3. Which of these can I easily get data on? (*Tick [√] as many as applicable*)
   - **a.** Number of workers at a farm in a day [√]
   - **b.** Type of complexion of class members [√]
   - **c.** Grades that Form 1 learners' got during BECE [√]

Focal Area: Designing and Administering a Questionnaire, an Observation or an Interview

Obtaining useful data requires conscious planning and structuring of the tool to be used. The tool for collecting these data is prepared (***designed***) with the purpose of collecting data and the kind of respondents in mind. A well-designed tool is then distributed or used in the interviews. In distributing or using the tool, we say, we are ***administering*** the tool.

Designing a data collection tool

The method we will use depends on our aim for the information we want, how easily we can obtain the information, who we want responding to the questions, the availability of the person, etc.

For example, to collect data on what happens at the palace, we might get the needed information orally (by ***interview***). To obtain the same information, we might be able to go to the palace and observe proceedings (an ***observation***) or submit a set of questions (by a ***questionnaire***) which could be responded to at the convenience of the palace clerk.

To design a tool;

- **i.** First we must write a brief introduction to what we want to find and what we are planning on doing with the data we collect.

ii. Then write the questions that will help us obtain the required data.

iii. Arrange our questions in a sensible and logical order.

iv. Try the items by answering them ourselves and try it with another person other than the actual person(s) who will be answering it to check it obtains the answers we want without causing upset or offence.

1. A sample interview guide

This interview guide is to collect information on learners' courses and the related interest in learning. It is just for academic purposes and would not be published anywhere.

Date of interview: ……………………….

Gender of student
M ( )   F ( )

What form are you?
Form 1 ( ) Form 2 ( ) Form 3 ( )

What course do you offer?
……………………………………………………………………………..

What is your favourite core subject in your course area?
English   ( )
Mathematics   ( )
Science   ( )
Agriculture   ( )

Why do you like the subject?
………………………………………………………………………………
………………………………………………………………………………

2. A sample observational check list

This observation is to collect information on a learner's behaviour during break time. It is just for academic purposes and would not be published anywhere.
Date of observation: ………………………….. Code for learner: ……………………

S. NO.	Item Description	Very active	Fairly active	Not active
1	Playful			
2	Adventurous			
3	Communicative			
4	Personal reading /Visiting the library			
5	Creating problems / bullying			
6	Solving problems / helping others			

3. A sample questionnaire

This questionnaire is to collect information on teachers' expectations of learners during lessons. It is just for academic purposes and would not be published anywhere.

Your responses will be kept confidential.

Please ask for clarification if you are not clear about anything.

Gender of teacher
M ( ) F ( )

What form do you teach?
Form 1 ( ) Form 2 ( ) Form 3 ( )

What subject do you teach?

………………………………………………………………………………..

What are some of the behaviour learners put up during lesson?
Active in answering questions ( )
Distracted ( )
Non-participatory ( )
Collaborative ( )

How does appropriate learner behaviour support good academic perform?
Very well ( )
Somehow well ( )
Not supportive ( )

What is your best moment about learners' attitude during lessons?

…………………………………………………………………………………………………

…………………………………………………………………………………………………

Administer questionnaire/observation/interview to appropriate respondents
The questionnaire must be distributed and interviews conducted to the required respondents to obtain the required data.

Key notes on administering questionnaires and conducting interviews
It is important to give clear instructions to respondents on what is expected of them and the mode of submission.

Questionnaires can be distributed personally, through other persons, through Google forms or through a social media platform.

In conducting interview, ask questions in an orderly manner as this will help you see patterns in responses. Keep to the line of questioning as much as possible and yet be flexible during interviews.

Learning Tasks for Practice

Learners, in groups, select observations, questionnaires or interviews and design a tool to collect data from the school or home environment and represent their responses on graphs as a project.

Pedagogical Exemplars

1. **Experiential learning:** In small groups, engage learners to discuss the means (methods) through which data on real life activities could be collected and complete questions (eg, questionnaire or interview guides).
2. **Experiential learning:** In small groups, engage learners to distribute questionnaires or conduct interviews or observations.

Focal Area: Recording and Graphing Responses from Questionnaires/Interviews

It is important to follow up and collect all questionnaires at the stipulated time so that no data is lost. When data is collected, it should be organised to make it concise for further consideration. The organised work is then represented on graphs or charts for easier reflection and judgement.

Example

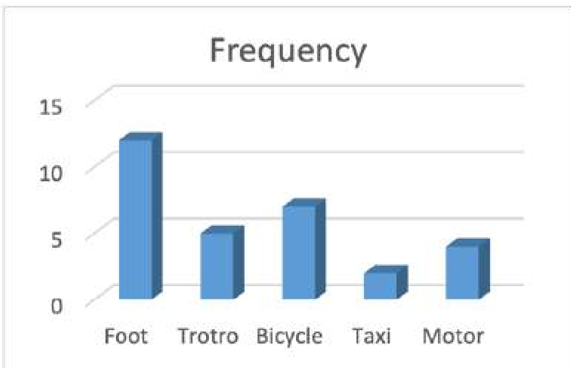
The data below were responses obtained through interview which was conducted on learners' means of transport to school.

Foot, Taxi, Trotro, Foot, Motor, Bicycle, Foot, Bicycle, Foot, Motor, Foot, Trotro, Foot, Taxi, Foot, Motor, Foot, Bicycle, Trotro, Foot, Bicycle, Motor, Bicycle, Foot, Foot, Trotro, Bicycle, Foot, Trotro, Bicycle

Represent the data on a graph.

Solution

Means of Transport	Frequency
Foot	12
Trotro	5
Bicycle	7
Taxi	2
Motor	4
Total	30



Learning Tasks for Practice

Learners, in groups, present responses on a preferred graph and share views on how graphs could be used at home or in the school.

Pedagogical Exemplars

1. **Experiential learning:** In small groups, engage learners to draw a graph of their choice and post for a gallery.
2. **Talk for learning:** Encourage learners to share positive comments about their classmate's graph and discuss how such graphs could be useful in their community.

Key Assessment

1. Match the activities in 'A' to appropriate method of collecting data in 'B'.

A	B
Looking at who becomes first in athletics	Questionnaire
Ask appropriate persons	Observation
Taking information using written questions	Interview

2. **a**. Write one place you would want to visit outside your community?

   …………………………………………..……………………..

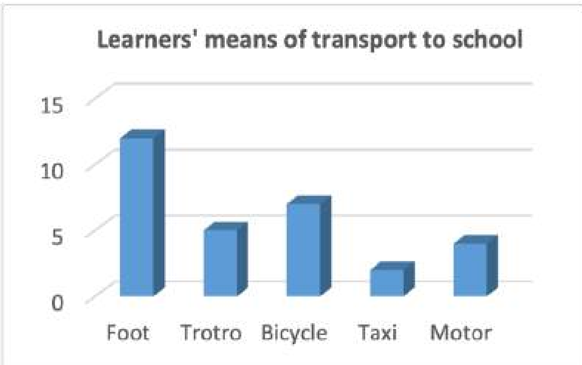
   b. What will you want to know about that place?

   ……………………………………………………..…...………

   c. What method will you use to get that information?

   ………………………………………………………..………

3. Study the graph and answer the questions that follow.



   a. What is the most popular means of transport to school?

   b. What will be the best reason why learners choose the most popular means of transport to school?

   (*Tick* [√] *the correct answer*)

   i. Learners stay far from the school [ ]

   ii. Learners like walking [ ]

   iii. Learners stay close to the school [ ]

   iv. There is no car in the town [ ]

   c. Write one reason why in your view, few people come to school with taxi.

   ………………………………………………………………………….

Section Review

In this section, we explored essential data collection methods and their applications:

1. **Select a Method for Collecting Data to Answer a Given Question and Justify the Choice:**
    - **Method Selection:** We learned how to choose the most appropriate data collection method for answering specific questions, considering factors such as the nature of the question, the type of data needed, and the target population.
    - **Justification:** We practiced justifying our choices by evaluating the advantages and limitations of various methods, such as surveys, observations, experiments, and secondary data analysis. This ensures that our data collection is reliable, valid, and relevant to the research question.
2. **Design and Administer a Questionnaire/Interview for Collecting Data to Answer a Given Question(s) and Record the Results:**
    - **Questionnaire/Interview Design:** We developed skills in designing effective questionnaires and interview guides. This involved formulating clear, unbiased questions that elicit meaningful responses, and structuring them logically to maintain flow and coherence.
    - **Administration:** We learned best practices for administering questionnaires and conducting interviews, ensuring ethical standards are met, and respondents are comfortable and willing to provide honest answers.
    - **Recording Results:** We practiced accurate and systematic recording of responses to maintain data integrity and facilitate subsequent analysis.

MODULE 3

SECTION 1: SETS AND OPERATIONS ON SETS

Strand: **Numbers for Everyday Life**

Sub-Strand: Number Sense

Learning Outcome: *Describe the relationship between subsets of real numbers and perform operations on them.*

Content Standard: Demonstrate understanding of number concepts and basic operations.

INTRODUCTION AND SECTION SUMMARY

Sets are mathematical concepts that are essential for understanding various operations in mathematics. It is a well-defined collection of objects. These objects, called elements, are unique and have no specific order. Sets are used to group objects that share a common characteristic. In this section, we are going to explore the concepts of sets and the operations performed on them. Through a structured approach, learners will understand the concept of sets and how to represent them using set notations. They will explore types of sets (finite, infinite, empty, subsets and universal sets) and perform basic operations on sets, including union, intersection, and complements. Learners will demonstrate understanding and application of set theory concepts in various mathematical contexts and real-world situations.

The section will cover the following concepts:

1. *Identifying properties of operations on sets and apply them in solving real life problems*
2. *Performing operations on fractions with like and unlike denominators and operate and approximate decimals*

SUMMARY OF PEDAGOGICAL EXEMPLARS

This section will make use of various pedagogical approaches, emphasising practical activities that will elicit learners' interest in sets and their operations and applying them in real life. Engaging learners to work in various mixed ability groups fosters collaborative learning.

Initiating talk for learning, through questioning and answering, creates the opportunity for the learners to come out with their views on sets and the ways of describing sets and representing sets.

Experiential learning offers the learners the opportunity to classify elements depending on their kinds, sharing their findings among themselves which helps in building teamwork among learners.

Problem based learning allows learners to come out with their own scenarios of problems in real life situations involving sets and find solutions to them. Through problem solving learning, learners will appreciate the importance of application of sets in real life.

ASSESSMENT SUMMARY

Assessing learners' comprehension of sets and operations on sets involves various strategies to gauge their understanding and application of key concepts. Firstly, learners' grasp of set notation, encompassing elements, subsets, intersections, and unions, is evaluated through tasks requiring identification and description of sets using appropriate notation. This extends to assessing their problem-solving capabilities by presenting real-world scenarios or word problems involving sets, wherein learners analyse the problem, identify relevant sets, and apply set operations to find solutions. Various modes of participation, including oral/written presentations, class exercises, reports, homework and hands-on demonstrations are Utilised and recorded for continuous assessment records.

Week 1: Sets and Its Operations

Learning Indicator: *Identify properties of operations on sets and apply them in solving real life problems.*

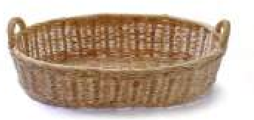
Focal Area: Sets and Operations on Sets

Introduction

A set is a well-defined collection of numbers, variables or objects. Sets help to organise information, compare groups, and solve problems involving elements or finding out the relationships between them. Set operations are performed on two or more sets to obtain a combination of elements according to the operation performed on them. In set theory, there are operations performed on sets, such as: Union of sets (∪), Intersection of sets (∩), complement of sets, etc. These operations allow mathematicians to manipulate sets and study their relationships.

Sets

1. Fruit basket. What goes into it?



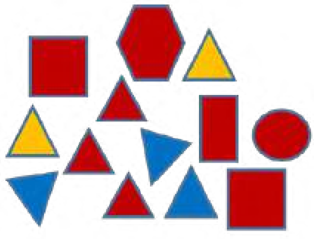
2. Sort the following fruits into their kinds (colour, shape, etc.)

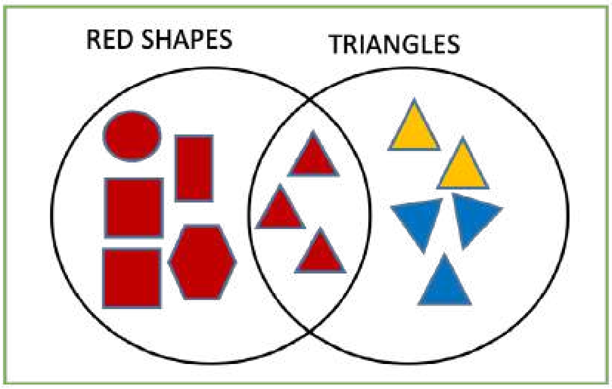
3. Group of learners born on a particular day

Sunday	7
Monday	9
Tuesday	6
Wednesday	4
Thursday	5
Friday	8
Saturday	9

4. Sort the numbers in the playing cards into even and odd.

5. Sort the following into two distinct sets (e.g., shapes and colours) using two circles representing the two sets with the left circle labelled "red shapes" and the right circle labelled "triangles.".





Set Notation

A set is denoted by a capital letter that is, $A = \{a, b, c, d\}$. The objects or components in the set are called **elements** or **members**

Ways of Describing Set

Sets can be described as

- Listing of members in the Sets

  $A = \{2, 3, 5, 7, 11\}$

- Word Description of Sets

  $P = \{\text{Prime numbers less than } 12\}$

- Set-Builder Notation

  $B = \{x : 1 < x < 12\}$ where x is a prime number.

Types of Set

1. **Finite Set:** A set whose last member can be found or counted. For example, a set of natural numbers from 1 to 9.

   $N = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

2. **Infinite Set:** A set whose last member cannot be found. For example, a set of numbers divisible by 3.

   $D = \{3, 6, 9, 12, 15, ...\}$

3. **Null (Empty) Set:** A set which has no element or member. It is denoted by a Greek letter phi φ.

   $D = \{\ \}$ or $D = \varphi$

4. **Universal Set (U):** Is a set of all objects under discussion. It's a set that contains everything under discussion.

For example, the set of learners in a class can be the universal set if, and only if, we can create other set(s) within the class.

5. **Equivalent Sets:** Two sets are said to be equivalent if they have the same number of elements.

   For example, the set $A = \{3, 6, 7\}$ and $B = \{11, 12, 13\}$. Both sets have 3 elements, so Sets A and B are equivalent

6. **Equal Sets:** Two sets are said to be equal if they have the same elements and the same number of elements.

   For example, the set $F = \{3, 6, 7\}$ and $G = \{6, 7, 3\}$are equal

7. **Disjoint Sets:** Two sets are said to be disjoint if they have no member(s) in common, i.e. if their intersection is empty

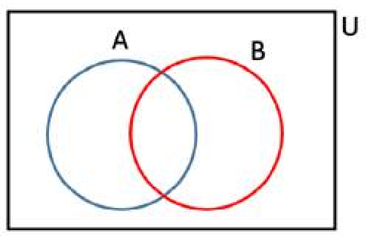
   For example, given the set $M = \{1, 3, 7, 9\}$ and $N = \{2, 4, 6, 8\}$, $N \cap M = \{\}$. This implies that the sets M and N are disjoints.

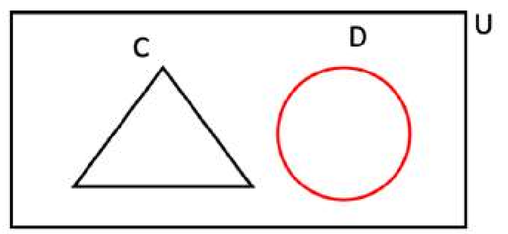
8. **Subset:** If all the members of set **A** belong to a set **B**, then the **A** is said to be a subset of **B**, or **B** contains **A.**

Representation of sets in a Venn diagram

Sets can be represented using a Venn diagram. A Venn diagram is a visual tool that uses plane geometric shapes to show the logical relationship between two or more sets of items.

Example:





Operations on Sets

Union of Sets

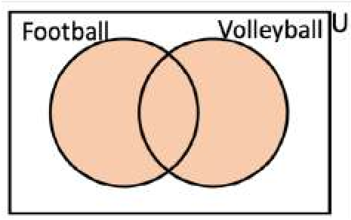
We have a basket containing apples and a basket containing oranges. When we put them together, we have a set of apples and oranges. This is the union of sets.

On a school field, 13 play football, 8 play volleyball and 5 play both football and volleyball.

The **union** will involve students who play football and volleyball and those who engage in both sports.

The **union** (∪) of two sets **A** and **B** is the set of elements that can be found in **either A** or **B** or both.

This is written as $A \cup B$. It can be represented using a Venn diagram as;



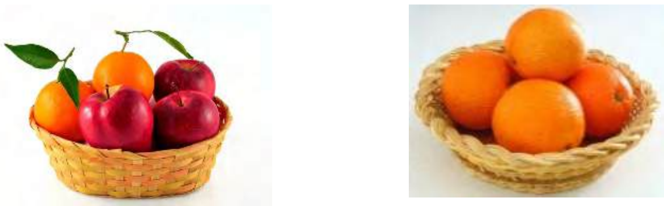
The shaded region represents the union of the set of football and volleyball players.

Example

Given the sets $A = \{1, 3, 7, 9, 11\}$and $B = \{3, 6, 9, 12\}$, $A \cup B = \{1, 3, 6, 7, 9, 11, 12\}$.

Intersection of Sets

Basket A contains apples and oranges and basket B contains only oranges. What is common in the two baskets is the intersection. In this case, orange is the intersection.



A B

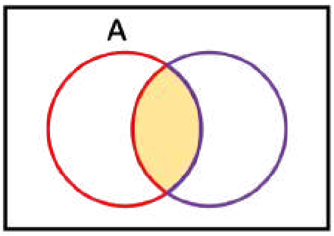
Examples

1. In a class of 30 students, 17 play only football, 12 play only volley ball and 5 play both football and volleyball.

   The five students who played both football and volleyball are the **intersection** of the set of football and volleyball students.

   The intersection (Ç) of two sets **A** and **B** is the set of elements common to **both A** and **B**.

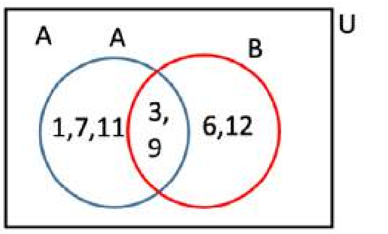
   This is written as $A \cap B$. It can be represented using a Venn diagram as



   The shaded portion is the intersection of A and B.

2. Given the sets $A = \{1, 3, 7, 9, 11\}$and $B = \{3, 6, 9, 12\}$, $A \cap B = \{3, 9\}$.

   This can be represented using a Venn diagram as;



3. Given $A = \{2,4,6,8\}$, $B = \{2,3,7,9\}$ and $C = \{1,3,5,7\}$

$A \cap B = \{2\}$

$A \cup B = \{2,3,4,6,7,8,9\}$

$A \cap C = \{\ \}$

$B \cap C = \{3,7\}$

$A \cup C = \{1,2,3,4,5,6,7,8\}$

$B \cup C = \{1,2,3,5,7,9\}$

Properties of operations on sets

We now know the operation on sets of union and intersection. Just like addition and multiplication of numbers have certain rules, operations on sets also follow specific properties. These properties help us simplify problems and makes working with sets easier.

1. **Commutative Property**

Combining the two set R and set B

This property says the order in which we perform the operation does not affect the result.

That is, $A \cap B = B \cap A$ or $A \cup B = B \cup A$

Given the set $A = \{1,2,3,4,5\}$ and $B = \{2,4,6\}$From the set A and B above $A \cap B = \{2, 4\}$ and $B \cap A = \{2, 4\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$ and $B \cup A = \{1, 2, 3, 4, 5, 6\}$

2. **Associative Property**

The grouping of sets does not affect the result. That is:

(A U B) U C = A U (B U C) or (A ∩ B) ∩ C = A ∩ (B ∩ C)

Example

Given the Set A = {1, 2, 3}, Set B = {3, 4} and Set C = {5, 6, 7}. The operation U is commutative if (A U B) U C = A U (B U C)

- (A U B) U C = ({1, 2, 3} U {3, 4}) U {5, 6, 7}
  = {1, 2, 3, 4} U {5, 6, 7}
  = {1, 2, 3, 4, 5, 6, 7}
- A U (B U C) = {1, 2, 3} U ({3, 4} U {5, 6, 7})
  = {1, 2, 3} U {3, 4, 5, 6, 7}
  = {1, 2, 3, 4, 5, 6, 7}

Since both results are the same, {1, 2, 3, 4, 5, 6, 7}, operation U is commutative

Similarly,

- $(A \cap B) \cap C = (\{1, 2, 3\} \cap \{3, 4\}) \cap \{5, 6, 7\}$
  $= \{3\} \cap \{5, 6, 7\}$
  $= \{\ \}$
- $A \cap (B \cap C) = \{1, 2, 3\} \cap (\{3, 4\} \cap \{5, 6, 7\})$
  $= \{1, 2, 3\} \cap \{\ \}$
  $= \{\ \}$

Since both results are the same, { }, operation ∩ is commutative

3. **Distributive Property**

This property applies to both union and intersection and helps us deal with multiple sets at once.

- For Union: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- For Intersection: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Given the Set A = {1, 2, 3}, Set B = {2, 3, 4}, and Set C = {4, 5, 6, 7}.

- $A \cap (B \cup C) = \{1, 2, 3\} \cap (\{2, 3, 4\}) \cup \{4, 5, 6, 7\})$
  $= \{1, 2, 3\} \cap \{2, 3, 4, 5, 6, 7\}$
  $= \{2, 3\}$
- $(A \cap B) \cup (A \cap C) = (\{1, 2, 3\} \cap \{2, 3, 4\}) \cup (\{1, 2, 3\} \cap \{4, 5, 6, 7\}$
  $= \{2, 3\} \cup \{\ \}$
  $= \{2, 3\}$

Since the LHS = RHS, operation ∩ is distributive over operation U.

Application of the Concept:

Venn diagrams are graphical representations that use set operations to illustrate the relationships between sets. They are commonly used in logic, probability, and statistics to visualise overlapping and non-overlapping regions between sets.

Learning Tasks for Practice

1. Present sets of objects, numbers, or elements and ask learners to identify and classify them based on common attributes or characteristics. For example, categorise sets based on colour, shape, size, or numerical properties.
2. Venn diagrams are visual tools for representing sets and their relationships. Provide sets and guide learners in creating Venn diagrams to illustrate set operations and comparisons. Encourage learners to interpret and analyse the diagrams to draw conclusions about set relationships.
3. Accept fully completed tasks including creating some sets with given elements, identifying the sets and investigating whether some sets could be duplicated.

Pedagogical Exemplars

1. **Initiate talk for learning:**
   a. In whole class, through questioning and answering, ask learners to discuss among themselves what can possibly go into the basket.

   b. In whole class discussion, through questioning and answering, ask learners to mention the days on which they were born and group themselves according to the days they have mentioned. M = {Learners born on Monday} etc.

   c. Using learners' findings, explain the types of sets. Guide learners to come to the conclusion that a set is a well-defined, distinct collection of objects of the same kind.

2. **Experiential Learning:**
   a. Ask learners to sort a given basket full of fruits according to their colours.

   b. In a whole class discussion, provide two baskets, one containing apple and the other containing oranges.


   Ask learners to put them together in one basket discuss how you will name the resulting set.

3. **Collaborative learning:**
   a. Put learners in small mixed ability/gender groups and ask them to sort the numbers in playing cards into even and odd**.**

   b. In mixed ability/gender groups, identify at least two universal sets in real life and identify at least three subsets under each universal set. For example: U = {learners in the foundation mathematics class} is a universal set.

   The following are subsets: A = {Boys in the class} B = {Girls in the class}

4. **Problem based learning:** in small mixed ability/gender groups ask learners to think, ink and share ideas on answering the following questions to introduce types of sets.
   a. List the set of natural numbers from 1 to 14.

   b. List a set of numbers divisible by three

   c. Write a set containing numbers that appear on a six-sided die greater than 7

   d. Write a set that has prime numbers more than 7 but less than 13.

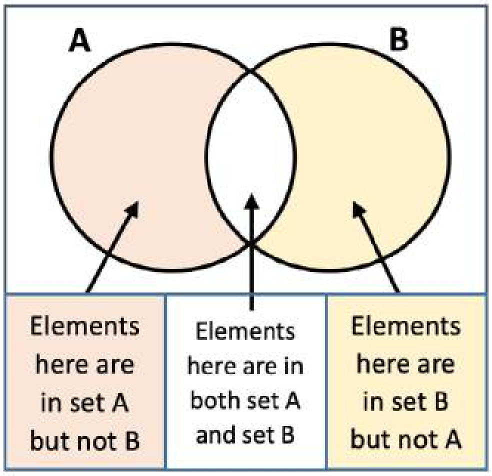
5. **Structuring Talk for learning:**

   a. In a whole discussion, provide two baskets named basket A and B. If Basket A contains apples and oranges and basket B contains only oranges, discuss with learners what is common in the two baskets and the name of the resulting set.

A

B

b. Discuss with learners the concept of representing sets in a Venn diagram.



Key Assessment

1. **Assessment Level 1**:

   a. Describe the following sets

   i. {January, February, March, April, May}

   ii. {1, 3, 5, 7, 9}

   iii. {a, b, c, d, e, f}

   b. Which of the following statements will produce an infinite set?

   i. Letters in the English alphabet

   ii. Odd numbers

   iii. Two digit prime numbers.

   iv. Numbers less than 12

   c. Find the union of the of sets; A= {2, 4, 6}and B= {1, 2, 3}

   d. Given the sets A = {a, b, c, d, e, f} and B = {a, c, e, g, h i}. Find:

   i. $A \cup B$

   ii. $A \cap B$

   e. Given the sets; P= {2, 4, 6}and Q = {1, 2, 3}and R= {1, 2, 3, 4, 5}. Find:

   i. $P \cup R$

   ii. $P \cap Q$

   iii. $Q \cap P \cap R$

   iv. $Q \cup P \cup R$

   f. Identify the following sets:

   i. {Monday, Tuesday, Wednesday}

   ii. {2, 4, 6, 8, 10}

   iii. {apple, orange, banana}

   iv. { }

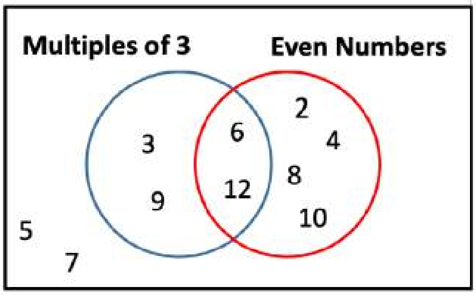
2. **Assessment Level 2**:
   a. Perform the following operations on sets:
      i. A = {1, 2, 3}, B = {3, 4, 5}. Find A ∪ B.
      ii. A = {red, blue, green}, B = {green, yellow}. Find A ∩ B.
      iii. A = {1, 2, 3, 4, 5}, B = {3, 4, 5, 6, 7}. Find A ∩ B and A ∪ B.
   b. Create your own set with at least 3 elements. What type of set is it (finite, infinite, empty etc,)?
   c. Perform the union operation on sets A = {red, blue} and B = {blue, green}.
3. **Assessment Level 3**:
   Can a set have duplicate elements? Explain your answer.

Focal Area: Two Sets Problems

Two set problems often involve comparing, combining, or contrasting different sets of elements to show patterns and relationships hidden within them.

Given each of the following numbers on cardboard, where does each belong to on the Venn diagram?

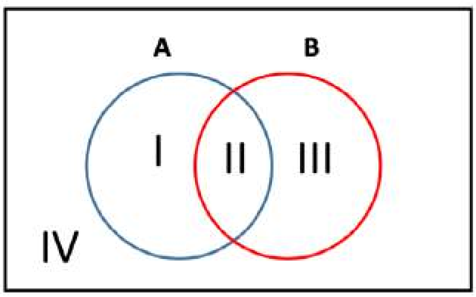
2, 3, 4, 5, 6, 7, 8, 9, 10, 12



From the diagram, learners observe that, the numbers in the set;

- {3, 9} are **multiples of 3** only and are not **even numbers**.
- {6, 12} are both **multiples of 3** and **even numbers.**
- {2, 4, 8, 10} are only **even numbers** and not **multiples of 3**.
- {5, 7} are neither **multiples of 3** nor **even numbers**.

From the above illustration, the following can be concluded:



Region I: A only = $A \cap B'$Region II: Both A and B = $A \cap B$

Region III: B only = $A' \cap B$

Region IV: Not in A and B = $A' \cap B'$

Note, B' is the complement of B, which means all the elements which are NOT in B.

Examples

1. In a class of 30 students, 19 students like basketball, 15 students like football and 8 students like both sports.

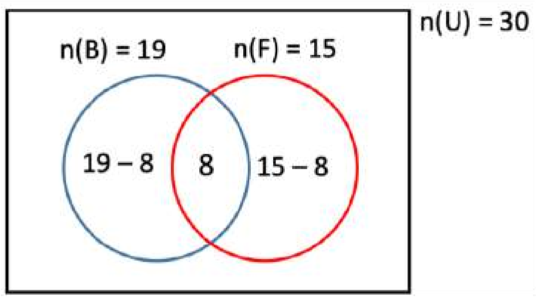
   How many students like

   a. only basketball?

   b. only football?

   c. neither basketball nor football?

   Solution

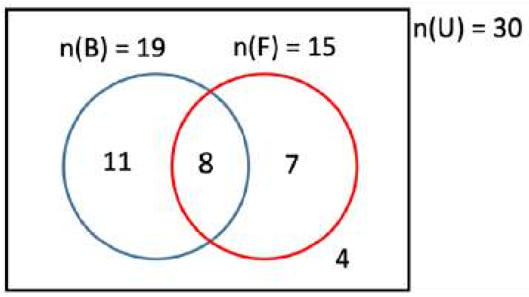


From the Venn diagram:

a. The number of students who liked basket only is 19 – 8 = 11

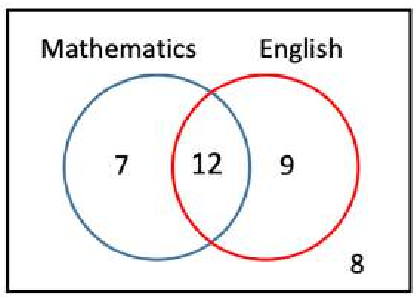
b. The number of students who liked football only is 15 – 8 = 7

c. The number of students who did not like either game is 30 – (11 + 7 + 8) = 4



2. A group of learners in class were asked whether they like Mathematics or English as their favourite subject.

   The results are illustrated in the Venn diagram below.



a. Explain what each region in the Venn diagram represents

b. How many learners like only one subject?

c. How many learners like Mathematics and/or English?

d. How many learners are in the class?

Solution

a. The region with 7, indicates that 7 learners like only mathematics

The region with 12, indicates that 12 learners like mathematics and English

The region with 9, indicates that 9 learners like only English

The region with 8, indicates that 8 learners in the class don't have mathematics or English as their favourite subjects.

b. The number of learners who like only one subject are 7 + 9 = 16

c. The number of learners who like Mathematics and/or English are 7 + 9 +12 = 28

d. The number of learners in the class are 7 + 9 +12 + 8 = 36

Learning Tasks for Practice

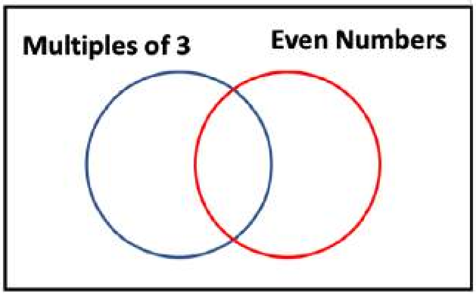
1. Put learners in pairs or in small groups to find the values of a given data involving two sets.
2. Provide examples of problems with incorrect solutions or errors and ask learners to work in pairs to identify the errors, explain what went wrong, and provide the correct solutions.

Applications of sets and operations of sets

- Sets and operations on sets are used to solve problems in probability, such as calculating the probability of an event or finding the intersection of two events.
- The properties of operations on real numbers, such as the commutative, associative and distributive properties.

Pedagogical Exemplars

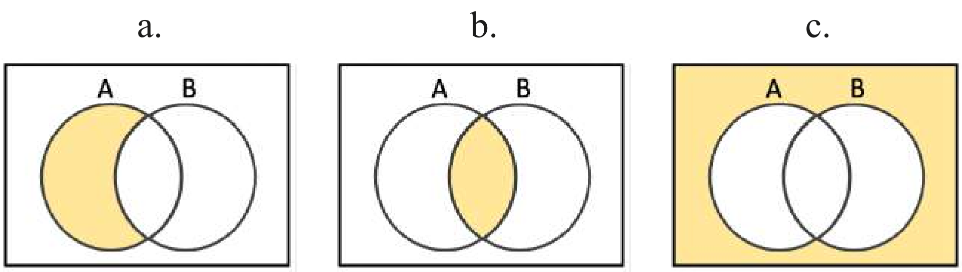
1. **Collaborative learning:** in smal mixed ability/gender groups, give learners sets of numbers on cardboard; 2, 3, 4, 5, 6, 7, 8, 9, 10, 12 and asked them to classify them in Multiples of 3 and Even numbers. Ask them to think, ink and share their findings. This is to introduce the representation of elements on a Venn diagram.



2. **Initiate talk for learning:** in a whole class discussion, guide learners to identify the various regions in a Venn diagram.
3. **Talk for learning:** in a whole class discussion, guide learners to solve problems involving two sets.
4. **Problem solving learning:** in small mixed ability/ gender groups, give learners story problems involving two sets and let learners think, ink and share their findings leading to a whole class discussion.

Key Assessment

1. **Assessment Level 1**:
   a. Identify and explain the following regions in the diagram below.



   b. In a group of 60 learners, 40 learners like dogs as their pet, 35 like cats as their pet, and 25 like both.

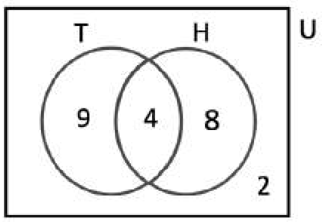
   How many learners like only cats?

2. **Assessment Level 2**:
   a. At a bookstore, there are 90 learners. 60 learners like Literatures, 45 like motivational books, and 30 like both.
      i. Illustrate this information on a Venn diagram
      ii. How many learners like only literature?

   b. In a school club, there are 50 learners. 30 learners like painting, 25 like singing, and 15 like both.
      i. Illustrate this information on a Venn diagram
      ii. How many learners like only singing?
      iii. How many learners like neither singing nor painting?

3. **Assessment Level 3**:
   a. In a group of 70 students, 45 students play basketball, 30 play soccer, and 15 play neither basketball nor soccer.
      i. Illustrate this information on a Venn diagram
      ii. How many students play both games?
      iii. How many played only soccer.

   b. The Venn diagram shows the number of people in a sporting club who play tennis (T) and hockey (H). Find the number of people:
      i. in the club
      ii. who play hockey
      iii. who play at least one of these sports
      iv. who play tennis but not hockey.



Week 2: Fractions and Decimals

Learning Indicator: *Perform operations on fractions with like and unlike denominators and operate and approximate decimals.*

Focal Area: Operations on Fractions

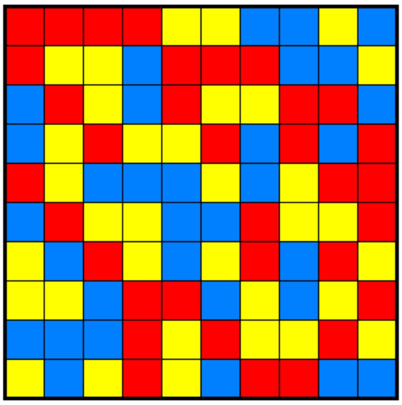
Introduction

This focal area will explore calculations involving fractions and decimals equipping learners with the tools to perform addition, subtraction, multiplication, and division on these number forms. We will provide comprehensive guidance on teaching these operations effectively, including strategies, examples, and activities to engage learners and reinforce their understanding of fractions and decimals to solve problems effectively.

Example

1. Using the grid below, find the fractions of the various colours to the total number of square units in the grid.

   The diagram represents a whole with 100 equal division.



   1. What fraction does the colour red represent?
   2. What fraction does the colour blue represent?
   3. What fraction does the colour yellow represent?

In this case, we count the total squares in the entire grid and the total number of red squares. Divide the total number of red squares by the total number of squares.

Total number of small squares = 100

Total number of red squares = 34

The fraction of red square units = $\frac{34}{100} = \frac{17}{50}$

a) The fraction of the colour blue in the square represents $\frac{32}{100} = \frac{16}{50} = \frac{8}{25}$.

b) The fraction of the colour yellow in the square represents $\frac{34}{100} = \frac{17}{50}$.

Addition and subtraction of fractions with common denominators

Examples

Add and subtract the following fractions on the grid paper.

1.

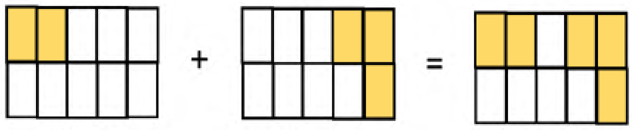


Fig. 1 Fig. 2 Fig. 3

Here the fraction for Fig 1 is $\frac{2}{10}$ and the fraction for Fig 2 is $\frac{3}{10}$. Therefore, to add the fractions in fig 1 and 2, we count the number of shaded parts in both fig 1 and 2 to get fig 3.

This implies that $\frac{2}{10} + \frac{3}{10} = \frac{5}{10}$.

2.

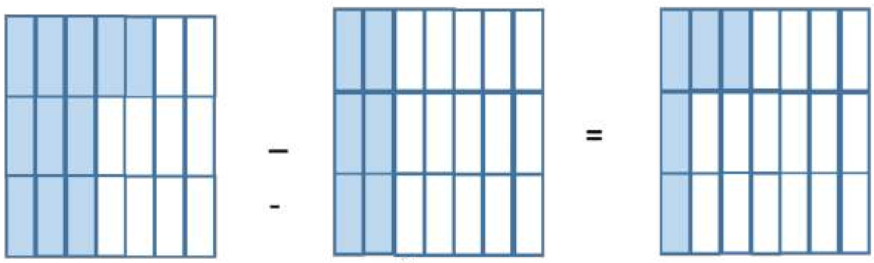


Fig 1 Fig 2 Fig 3

Here, the fraction for Fig 1 is $\frac{11}{21}$ and the fraction for Fig 2 is $\frac{6}{21}$ therefore to subtract the fraction in fig. 2 from fig. 1, we take away the shaded parts in fig 2 from fig. 1 to get fig 3.

This implies that, $\frac{11}{21} - \frac{6}{21} = \frac{5}{21}$.

From the above activities we can conclude that, fractions with the same denominators can be added or subtracted directly by keeping the denominator the same and adding or subtracting the numerators. Therefore, the number 21 in the fraction $\frac{5}{21}$ is the Lowest Common Multiple (LCM) of the fractions given in the question

3. Add the following fractions:

$\frac{3}{8} + \frac{2}{8}$

Solution

The lowest common multiple is 8

$\frac{3}{8} + \frac{2}{8} = \frac{3+2}{8} = \frac{5}{8}$

4. Subtract the following fractions:

$\frac{17}{24} - \frac{8}{24}$

Solution

The lowest common multiple is 24

$\frac{17}{24} - \frac{8}{24} = \frac{17-8}{24} = \frac{9}{24}$

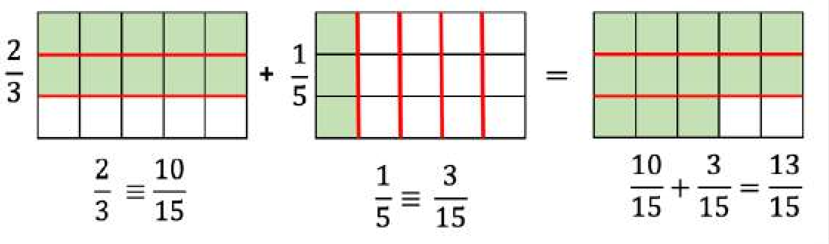
Addition and subtraction of fractions with different denominators

Examples

Add and subtract the following fractions using grid paper.

1. $\frac{2}{3} + \frac{1}{5}$

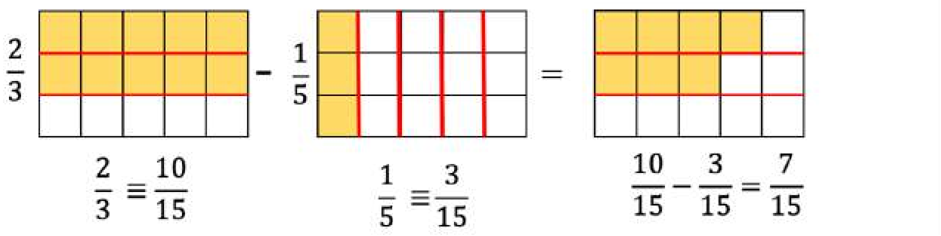
In this case, the denominators in the question given will determine the size of the grid. So, in the above question, the size of the grid is the lowest common multiple of 3 and 5. Or we can use the product of the two numbers. In this case, the lowest common multiple is the product of 3 and 5 (i.e. 3 × 5 = 15). This means that there are small equal parts in a 3 by 5 grid.



From the above diagram, $\frac{2}{3} + \frac{1}{5} = \frac{2 \times 5}{3 \times 5} + \frac{1 \times 3}{5 \times 3} = \frac{10}{15} + \frac{3}{15} = \frac{10 + 3}{15} = \frac{13}{15}$.

2. $\frac{2}{3} - \frac{1}{5}$

Again the 3 and the 5 indicate that the size of the grid must be 3 by 5, or a size 15 grid.



From the above diagram, $\frac{2}{3} - \frac{1}{5} = \frac{2 \times 5}{3 \times 5} - \frac{1 \times 3}{5 \times 3} = \frac{10}{15} - \frac{3}{15} = \frac{10 - 3}{15} = \frac{7}{15}$.

Based on the above activities, we can conclude that, fractions with different denominators can be added or subtracted by finding a common denominator. The common denominator is the lowest common multiple (LCM) of the individual denominators.

Evaluate the following:

1. $\frac{4}{8} + \frac{1}{4}$

2. $\frac{7}{12} - \frac{3}{4}$

Solution

1. $\frac{4}{8} + \frac{1}{4}$

The lowest common multiple of 8 and 4 is 8

$\frac{4}{8} + \frac{1}{4} = \frac{4}{8} + \frac{1 \times 2}{4 \times 2} = \frac{4 + 2}{8} = \frac{6}{8} = \frac{3}{4}$

2. $\frac{7}{12} - \frac{3}{4}$

The lowest common multiple of 12 and 4 is 12

$\frac{7}{12} - \frac{3}{4} = \frac{7}{12} - \frac{3 \times 3}{4 \times 3} = \frac{7 - 9}{12} = \frac{-2}{12} = \frac{-1}{6}$

Multiplication and division of fractions

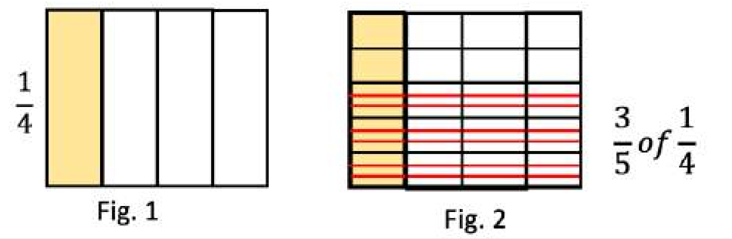
Multiplying fractions allows us to combine different parts of wholes while dividing fractions enables us to distribute quantities proportionally. We will use visual tools, practical scenarios, and engaging activities to enhance our understanding.

Multiplication of fractions

Examples

1. Evaluate $\frac{1}{4} \times \frac{3}{5}$

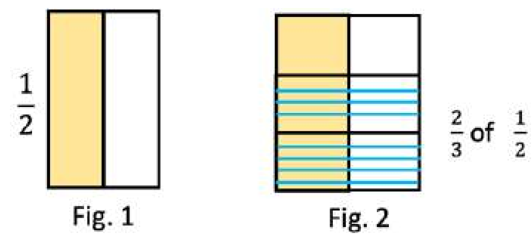
- Shade rectangular shape into $\frac{1}{4}$
- Shade the $\frac{3}{5}$ of the $\frac{1}{4}$ rectangle
- Count the double shaded divided by the total divisions in fig. 2.



Therefore; $\frac{1}{4} \times \frac{3}{5} = \frac{1 \times 3}{4 \times 5} = \frac{3}{20}$

2. Evaluate $\frac{1}{2} \times \frac{2}{3}$

- Shade rectangular shape into $\frac{1}{2}$
- Shade the $\frac{2}{3}$ of the $\frac{1}{2}$ rectangle.
- Count the double shaded divided by the total divisions in fig. 2.



Therefore; $\frac{1}{2} \times \frac{2}{3} = \frac{1 \times 2}{2 \times 3} = \frac{2}{6}$

Multiplication rule

Therefore, from the above, we can conclude that when multiplying fractions, we multiply the numerators together to get the numerator of the product and multiply the denominators together to get the denominator of the product.

Examples

Solve the following

1. $\frac{21}{4} \times \frac{3}{7}$

2. A recipe calls for $\frac{3}{5}$ of a cup of sugar, and Sarah wants to make $\frac{1}{2}$ of the recipe.

How much sugar does she need?

Solution

1. $\frac{21}{4} \times \frac{3}{7} = \frac{63}{28} = 2\frac{9}{28}$
2. To find $\frac{1}{2}$ of the recipe, we multiply $\frac{3}{5}$ cups by $\frac{1}{2}$

   That is, $\frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$

   Therefore, Sarah needs $\frac{3}{10}$ cups of sugar.

Division of Fractions

Examples

1. Akua has $\frac{2}{3}$ of a chocolate bar and wants to share it with her friends, so that each gets $\frac{1}{6}$ of the chocolate bar.

   How many friends can she share with?

   Mathematically this implies, $\frac{2}{3} \div \frac{1}{6}$. If $\frac{2}{3} \div \frac{1}{6}$, then, the size of the chocolate is 3 by 6 whole.

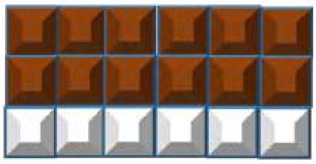
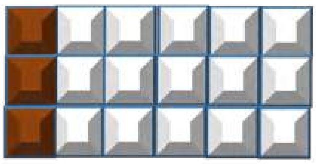
$\frac{2}{3}$  ÷  $\frac{1}{6}$

Fig. 1 Fig. 2

   How many of the one-sixth in fig. 2 can be found in two-thirds in fig 1?

   There are four one-sixth in two-thirds of the chocolate.

   Hence four friends shared two-thirds of the chocolate.

   Therefore, $\frac{2}{3} \div \frac{1}{6} = 4$

   From the above activity, we can conclude that division of fractions involves finding out how many times one fraction is contained within another.

   This question can also be solved by reciprocating the divisor to change the division sign to multiplication sign.

   Therefore, $\frac{2}{3} \div \frac{1}{6} = \frac{2}{3} \times \frac{6}{1} = \frac{12}{3} = 4$

Division rule for fractions.

When dividing one fraction by another, we multiply the first fraction by the reciprocal of the second fraction.

Mathematically $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

Example

Evaluate the following:

a. $\frac{1}{2} \div \frac{1}{4}$

b. $\frac{5}{7} \div \frac{3}{4}$

c. $\frac{3}{14} \div \frac{6}{7}$

Solution

a. $\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1}$

$= \frac{4}{2}$

$= 2$

b. $\frac{5}{7} \div \frac{3}{4} = \frac{5}{7} \times \frac{4}{3}$

$= \frac{20}{21}$

c. $\frac{3}{14} \div \frac{6}{7} = \frac{3}{14} \times \frac{7}{6}$

$= \frac{21}{84}$

$= \frac{1}{4}$

Focal Area: Rounding Off Decimal Fractions

Rounding decimals to the nearest whole number

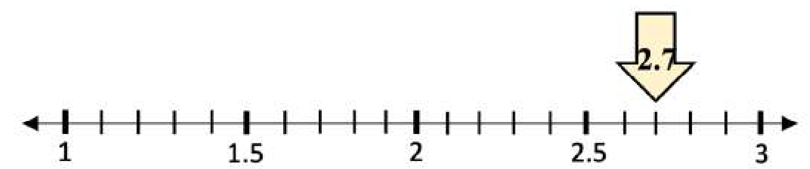
Imagine, you are embarking on a road trip, navigating through a scenic highway. Along the way, you come across road signs showing distances to your destination in decimal fractions, like 2.70 kilometres.

Rounding decimal fractions is like finding your distance to your destination to the nearest whole number as you navigate on your road trip. Rounding allows you to simplify these distances to whole numbers, providing a clearer understanding of your progress towards your destination. Rounding decimal fractions plays an important role in our everyday life such as estimating distances, quantities, or measurements.

Throughout this lesson, we will explore rounding decimal fractions to the nearest whole number, tenth, hundredth, etc. using visual models such as the number line.

Examples

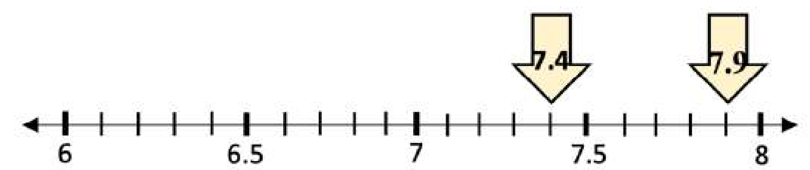
1. Round 2.70 kilometres to the nearest kilometre using the number line.



From the number line above, compare the distance 2.70 km is to the distance 2km and 3km and check which one is closer.

Therefore, 2.70 rounded to the nearest whole number is 3 because it is closer to 3km, than to 2km.

2. Round **7.4** and **7.9** to the nearest integers.



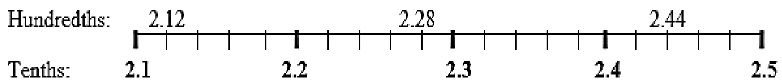
Solution

From the diagram 7.4 is between 7 and 8 and is closer to the number 7. Therefore, 7.4 rounds to 7 to the nearest integer.

Again 7.9 is closer to the number 8. Therefore, 7.9 rounded off the nearest integer is **8.**

Rounding off decimals to the nearest tenth.

The number line below has a graduation that increases by 0.02. The numbers below the line are tenths, the numbers above the line are in hundredths.



From the diagram, we can see that, **2.12** is close to **2.1** and is therefore rounded to 2.1 to the nearest tenth.

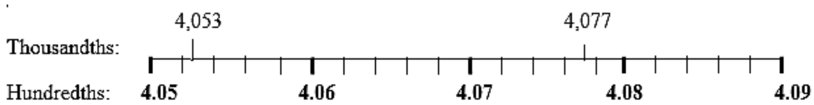
Then **2.28** is closer to **2.3** and is therefore rounded to 2.3 to the nearest tenth.

2.44 is closer to **2.4** and is therefore rounded to 2.4 to the nearest tenth.

Rounding to the nearest hundredth

To round to the nearest hundredth using a number line, look at the position value of the hundredth closest to the number you want to round.

Round **4.053** and **4.077** to the nearest hundredth.



The number 4.053 is between 4.05 and 4.06.

Therefore, **4.053** is closer to **4.05** and is rounded to 4.05.

The number 4.077 is between 4.07 and 4.08. Therefore, **4.077** is closer to **4.08** and is therefore rounded to 4.08.

Example

Round the following numbers to the nearest integer using the diagram below;

12.2 12.8 13.4 14.6 15.4

Solution

1. 12.2 is closer to 12 and is therefore rounded to 12.

   15.4 is closer to 15 and is therefore rounded to 15.

   14.6 is closer to 15 and is therefore rounded to 15.

   13.4 is closer to 13 and is therefore rounded to 13.

   12.8 is closer to 13 and is therefore rounded to 13.

From the above examples, to round a decimal to a specific place value, we first identify the place value we need and check the next place value. If the next place value is 5 or greater (i.e. 5, 6, 7, 8, and 9), we add 1 to our targeted place value. If it is less than 5 (i.e. 1, 2, 3, and 4) we discard it and write our decimal to the targeted place value.

Learning Tasks for Practice

1. Perform operations of fractions involving like denominators and round decimal fractions to the nearest whole, tenth and hundredth.
2. Perform operations of fractions involving unlike denominators.
3. Perform operations of fractions involving mixed fractions
4. Solve real-life problems involving fractions.

Pedagogical Exemplars

1. **Initiating Talk for learning:**
   - i. In a whole class discussion, review learners' previous knowledge of fractions through questioning and answering.
   - ii. Explain the rule of adding and subtracting fractions with the same denominator**.**
   - iii. Draw two rectangular shapes on the board, each divided into thirds to represent $\frac{2}{3}$. Let learners work in pairs to shade in half of each circle to represent $\frac{1}{2}$ of $\frac{2}{3}$.
2. **Collaborative learning:**
   - i. In small mixed ability/gender groups, provide printed coloured grid papers with 100 equal divisions to each group.

     Ask learners to discuss and write the fractions that each of the colours represents.
   - ii. In small mixed ability/gender groups, provide learners with grid papers. Instruct them to represent the size of the chocolate bar on the grid paper and indicate the fractions $\frac{2}{3}$ and $\frac{1}{6}$. Encourage them to discuss their strategies, share ideas, and support each other in finding the number of friends who can share it.
3. **Experiential learning:** in a whole class, paste a fractional bar with shaded fractions on the board and ask learners to come forward to write the fraction for each fractional bar and ask learners what they have observed.
4. **Problem-based learning:** give learners in their mixed-ability gender groups another fractional bar with shaded fractions. Ask learners to think, ink and share their ideas on how they will add and subtract fractions with different denominators.
5. **Problem-based learning:** learners in their mixed-ability gender groups to solve problems. Encourage them to discuss their strategies, share ideas, and support each other in finding the solution.
6. **Initiating talk for learning:** in a whole class discussion, present a scenario, "Akua has $\frac{2}{3}$ chocolate bars and wants to share it with her friends so that each gets $\frac{1}{6}$ of the chocolate." How many friends can she share with? Through questioning and answering ask learners the size of the chocolate and how they can represent the fractions $\frac{2}{3}$ and $\frac{1}{6}$.
7. **Initiating talk for learning:**
   - i. In a whole class discussion, present a scenario; "you are embarking on a road trip, navigating through a scenic highway. Along the way, you came across road signs showing distances to your destination in decimal fractions, like 2.70 kilometers." Through questioning and answering ask learners to share their ideas and strategies for locating their destination to the nearest kilometers. Explain the importance of rounding numbers and give examples in our everyday life as well as how to round 2.70 to the nearest whole using the number line.
   - ii. In a whole class discussion, through questioning and answering, introduce and explain how to round decimal fractions to the nearest tenth, hundredth etc. using the number line.

iii. In a whole class discussion, ask probing questions to assist in building learners' understanding and help them connect the visual representation to the mathematical process of rounding off decimal fractions.

8. **Collaborative learning:** In small mixed ability/gender groups, give a learning task to learners to round to the nearest whole, tenth, hundredth etc. using the number line Encourage them to discuss their strategies, share ideas, and support each other in rounding the decimal fractions. Interact with the groups to offer guidance and support as needed.

Key Assessment

Assessment Level 1

1. Perform the following operations and simplify your answers:

   i. $\frac{3}{4} + \frac{3}{4}$

   ii. $\frac{5}{6} - \frac{4}{6}$

   iii. $\frac{2}{3} \times \frac{7}{3}$

   iv. $\frac{2}{5} \div \frac{4}{5}$

   v. Round off the following decimal fractions to the nearest whole number.

   - 2.1
   - 10.7
   - 6.9

2. Solve the following fractional operations:

   i. $\frac{1}{3} + \frac{5}{4}$

   ii. $\frac{2}{5} - \frac{1}{3}$

   iii. $\frac{2}{3} \times \frac{3}{5}$

   iv. $\frac{4}{6} \div \frac{2}{3}$

   v. Round off the following decimal fractions to the nearest tenth and whole numbers.

   - 3.75
   - 5.23
   - 9.15

Assessment Level 2

3. Evaluate the following fractions.

   i. $2\frac{2}{3} + 3\frac{3}{5}$

   ii. $5\frac{3}{4} - 2\frac{1}{3}$

   iii. $1\frac{1}{3} \times 3\frac{3}{5}$

   iv. $7\frac{2}{3} \div \frac{4}{5}$

v. Round off the following decimal fractions to the nearest, hundredth, tenth and whole numbers.

- 8.953
- 7.819
- 4.366

Assessment Level 3

4. Solve the following questions.

i. A pizza is divided into 8 equal slices. Tom ate 3 slices. What fraction of the pizza did he eat?

ii. Mary has $\frac{2}{3}$ of a cake left. If she wants to divide it equally among 4 friends, how much of the whole cake will each friend get?

iii. Jack and Jill are participating in a race. Jack finishes $\frac{3}{4}$ of the race, whereas Jill finishes $\frac{5}{8}$. Who completes a larger fraction of the race?

Section Review

In this section, we delved into sets and their operations, beginning with understanding sets as collections of distinct elements represented using set notation. Key operations such as union and intersection were explored, alongside various set relationships. Visualising sets through Venn diagrams aided comprehension. The pedagogical exemplars equipped learners to manipulate sets, analyse relationships, and apply operations effectively across diverse contexts. Through collaborative learning and practical applications, learners gained a solid understanding of sets and their significance in various fields, laying the groundwork for further mathematical exploration. Assessments, including quizzes and discussions, ensured mastery.

Again, the concept of fractions was explored. Operations on fractions, addition, subtraction, multiplication, and division, were examined, alongside strategies like finding common denominators and simplification techniques. Real-life applications of fractions were explored, emphasising their significance in practical contexts. Learners also explored rounding fractions to given number of decimal places.

References

https://www.123rf.com/photo_127228667_four-apples-in-a-basket-on-a-blue-wooden-table.html

https://www.freeimages.com/photo/oranges-in-basket-1-1330213

SECTION 2: RATIO AND PERCENTAGES

Strand: **Numbers for Everyday Life**

Sub-Strand: Proportional Reasoning

Learning Outcome: *Perform operations on ratio and percentages and interpret the results*

Content Standard: Demonstrate understanding of ratio and percentage concepts and apply it in solving real life problems.

INTRODUCTION AND SECTION SUMMARY

In this section, we delve into the core concepts of ratio and percentages, exploring their definitions, representations, and applications. We begin by defining ratios and percentages and explaining their relevance in everyday life. Next, we explore different ways to represent ratios and percentages, including fractions, decimals, and visual models. We then delve into various applications of ratios and percentages, such as solving proportion problems, calculating percentages of quantities and interpreting data presented in tables and graphs. Throughout the section, emphasis is placed on practical examples and real-life scenarios to facilitate understanding and application. By the end, learners should have a solid grasp of ratio and percentage concepts and be equipped to apply them effectively in various problem-solving situations.

The section will cover the following concepts:

1. *Compare and estimate quantities in a given ratio*
2. *Express one quantity as a percentage of another and vice versa.*

SUMMARY OF PEDAGOGICAL EXEMPLARS

Utilising hands-on manipulatives, such as ratio bars and percentage models, fosters concrete understanding. Visual representations, like diagrams and charts, aid in conceptualisation, while problem-based learning tasks encourage critical thinking and application. Collaborative learning environments promote peer interaction and collective problem-solving, enhancing comprehension and retention. Moreover, differentiated instruction ensures that diverse learning needs are met, with scaffolded support for struggling learners and extension tasks for advanced learners.

ASSESSMENT SUMMARY

Assessments may include tasks such as identifying and describing ratios, calculating percentages, solving ratio and percentage problems, and interpreting data presented in tables and graphs. Multiple-choice questions, short-answer responses, problem-solving tasks, and performance assessments are Utilised to assess learners' comprehension and proficiency. Additionally, real-life scenarios or word problems involving ratios and percentages are presented to evaluate learners' ability to apply these concepts in practical situations. Various modes of participation, including oral/written presentations, class exercises, reports, homework and hands-on demonstrations, are utilised and recorded for continuous assessment records.

Week 3: Ratios and Its Applications

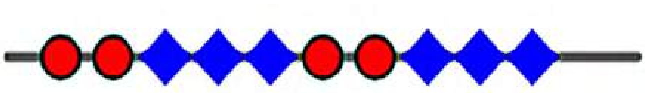
Learning Indicator: *Compare and estimate quantities in a given ratio*

Focal Area: The Concept of Ratio

Introduction

This week, we will explore the language of ratios and learn how to express them in fractions and colon (:) form. As we explore deeper, we will compare and estimate quantities in a given ratio and discuss the importance of ratios in everyday life. By the end, learners will be able to compare quantities and analyse and interpret real-life problems.

Study the diagram below and come up with the pattern.



The pattern follows that, for every two red circles there are three blue squares. This can be written as 2 red circles to 3 blue squares (2:3).

Examples

1. If there are 28 boys and 23 girls in the class. The ratio of girls to boys in the class is 23:28.

   Discussion

   In this case, we are comparing the number of girls to boys in the class. To express the ratio of girls to boys, we place the number of girls first followed by the number of boys separated by colon (:).

   This ratio tells us that for every 23 girls in the class, there are 28 boys. It represents the relative sizes of the two groups, allowing us to understand the distribution of gender in the class. As you can see, order is important here.

2. Assuming you spend GH¢ 1 every day and your friend Kofi spends GH¢2. What is the ratio of your expenditure to that of your friend?

   Solution

   The ratio is 1:2, in that, for every GH¢ 1 you spend, Kofi spends GH¢ 2.

   This ratio indicates that for every GH1 you spend, your friend Kofi spends GH2, representing the relative amounts spent by each person.

3. Ohemaa has 2 pens and Bryan has 4 pens. Express this in a ratio form.

   Solution

   The ratio of Ohemaa's pens to that of Bryan's is 2:4.

   Note, as both of these numbers are multiples of 2, we can divide by 2 to simplify the ratio.

   Therefore, the ratio 2:4 is the same as the ratio 1:2.

From the above activities, we can conclude that a ratio is the relationship between two or more quantities or amounts.

Note the following:

1. To express two quantities as a ratio, they must have the same units of measure.
   For example, express the ratio 30cm to 9m.

   The ratio should be 30*cm* : 900*cm*

   To put the ratio in its simplest form, divide both sides by the lowest common multiple, i.e. 30

   $\frac{30}{30} : \frac{900}{30} = 1:30$

2. Ratio with "**:**" sign has no unit. For example, the ratio 5kg to 2kg is written as 5 : 2.

Equivalent Ratios

Two ratios are said to be equivalent if one can be considered as a multiple of the other or they are ratios that have the same value when simplified. An example is 2:6 when simplified is 1:3. Therefore, the ratio 1:3 is equivalent to 2:6.

Relating ratios to fractions

Ratios in the form **a : b** can be expressed in the fractional form as $\frac{a}{b}$.

Examples:

1. Samuel has 5 apples and Cynthia has 10 apples. Express this as a ratio and write it in terms of fractions.

   Solution

   Samuel: Cynthia = 5 : 10 = $\frac{5}{10} = 1:2$

   Comparing, for every 5 apples that Samuel has, Cynthia has 10. This can be expressed as 5:10 and written in the fractional form as $\frac{5}{10}$, This means that Samuel has half as many apples as Cynthia and in simplified form is 1 : 2.

2. Express the ratio of 20cm to 15m in the form 1:*n*

   Solution

   First, we need to convert both measurements into matching units:

   Converting 15m to cms: 15m ≡ 1500cm

   As a ratio : 20 cm: 1500cm

   $\frac{20}{20} : \frac{1500}{20}$

   1: 75

3. In a class of 30 learners, 12 are girls. What is the ratio of boys to girls?

   Solution

   Total number of learners = 30

   Girls = 12. The boys will be 30 – 12 = 18

   Therefore, the ratio will be 18 : 12 = 3 : 2

Comparing and estimating quantities in a given ratio

Comparing ratios involves understanding the relationship between different quantities, while estimating ratios involves making educated guesses or approximations about those quantities.

Examples

1. A bag contains some quantities of pens and pencils. The ratio of **pens to pencils is 3:5.**

   If there are 60 pencils in the bag, calculate the number of pens in the bag.

   Solution

   The ratio is 3:5, which means, for every 3 pens, there are 5 pencils.

   Given that there are 60 pencils. Let the number of pens be x.

   By comparing the ratios

   Pens: Pencil = $3:5 = x : 60$

   $$\frac{3}{5} = \frac{x}{60}$$

   $$5x = 3 \times 60$$

   $$x = \frac{3 \times 60}{5}$$

   $$x = 36$$

   Therefore, there are 36 pens.

2. The ratio of boys to girls in a class is 2:5. If there are 10 boys in the class, how many girls are there in the class?

   Solution

   $$2:5 = \text{boys: girls}$$

   $$\frac{2}{5} = \frac{\text{number of boys}}{\text{number of girls}}$$

   $$\frac{2}{5} = \frac{10}{\text{number of girls}}$$

   $$\text{number of girls} = \frac{5 \times 10}{2}$$

   number of girls = 25

   It means that for the 10 boys, there are 25 girls.

Learning Tasks for Practice

1. Write given quantities in ratio form
2. Compare given quantities and express them in the ratio form
3. Solve story problems involving estimating quantities given a ratio.

Pedagogical Exemplars

1. **Initiating talk for learning:**
   - **a.** In small mixed ability/gender groups, display a chart showing a pattern of red circles and blue squares and ask learners to observe in their groups and discuss their findings with them**.** Explain to learners that for every 2 red circles, there are three red squares.
   - **b.** In a whole class discussion, give a scenario of the total number of learners in the class, and ask learners to give you the total number of boys and the total number of girls. So, in comparing quantities, for every quantity ‘a’, there is a quantity ‘b’.

2. **Experiential learning:** in a whole class, ask two learners to come forward and give one 2 pens and the other 4 pens. Ask learners to think, ink and share their observations and mention the ratio of the pens each learner has.
3. **Collaborative learning:** In small mixed ability/gender groups, let learners create a real-life scenario to compare quantities and write their findings. Discuss their findings with them.
4. **Talk for learning:** in a whole class discussion, explain to learners what a ratio is and the forms in which ratios can be expressed.
5. **Experiential learning:** in small mixed ability/gender groups, give some worksheets on ratios to learners. Let learners observe the ratios, compare and discuss among themselves their findings. Discuss with learners their findings. Explain equivalent ratios with learners.
6. **Collaborative learning:** in mixed ability/gender groups give some tasks on ratios and equivalent ratios to learners to solve. Encourage learners to assist their friends. Discuss learners' findings with them.
7. **Initiating talk for learning:** in a whole class discussion through questioning and answering guide learners to compare and estimate quantities with a given ratio.

Key Assessment

1. In a class of 30 learners, the ratio of boys to girls is 4:6.

   Find the number of boys to girls.
2. A rectangular garden has a length of 12 meters and a width of 8 meters.

   What is the ratio of the length to the width of the garden in its simplest form?
3. The ratio of apples to oranges in a basket is 5:3.

   If there are 20 apples, how many oranges are in the basket?
4. In a bag containing toffees, the ratio of red to blue toffees is 3:2.

   If there are 24 red toffees, how many blue toffees are there?
5. The ratio of the length to the width of a rectangle is 3:2.

   If the width is 10 meters, what is the length of the rectangle?
6. The lengths of two sides of a rectangle are in a ratio of 4:5.

   If the shorter side is 12 meters, find the length of the longer side.
7. A construction firm mixes cement and sand in a ratio of 1:4 to make concrete.

   If they need 500kg of cement, how many kg of sand should they use?
8. Solve for the value of x in $4:(x + 5) = 1:2$

Week 4: Working With Percentages

Learning Indicator: *Express one quantity as a percentage of another and vice versa.*

Focal Area: Percentages (Expressing One Quantity as a Percentage of Another)

Introduction

- If your class test score is 8 out of 10, what percentage is your score?
- If a bookshop offers a 25% discount on a novel worth GH¢ 50, how much money is saved?

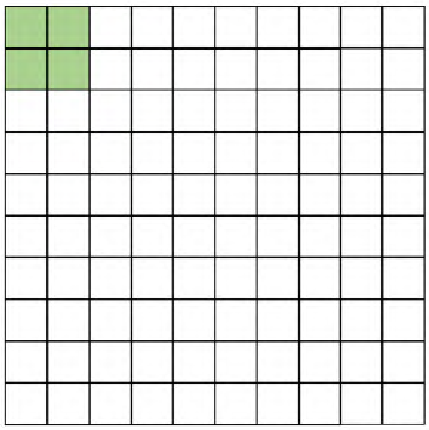
In answering these questions and many others, we need to understand the concept of percentages.

Percentage means "per hundred" or "out of 100" which is used to express a part of a hundred and is denoted by %. "**Percent**" comes from the Latin word ***Per Centum***. ***Centum*** means **100**, for example, a Century is 100 years. Expressing one quantity as a percentage of another involves representing a part of a whole as a percentage.

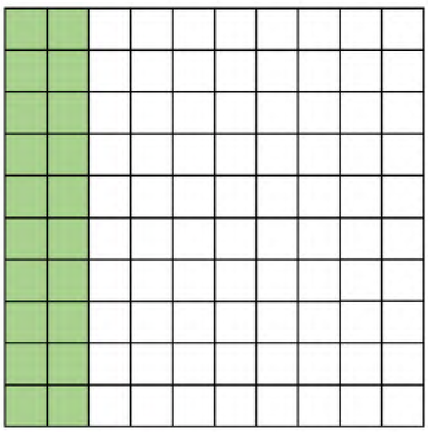
Using models to represent percentages

Examples

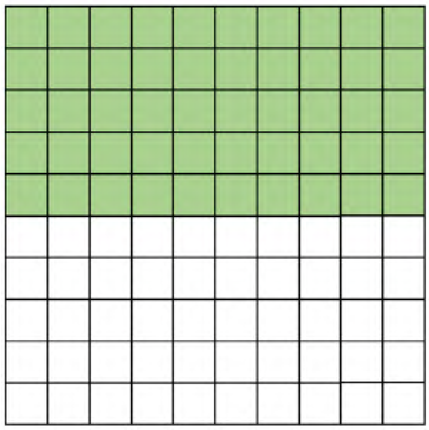
1. This is 10 by 10 grid which is made up of 100 square units. 4 out of 100 is shaded, representing 4% or $\frac{4}{100}$



2. This is 20 out of 100. That is $20\,\% = \frac{20}{100}$.



3. In a class of 100 learners, 50 of them are girls, what percentage of the class are girls?



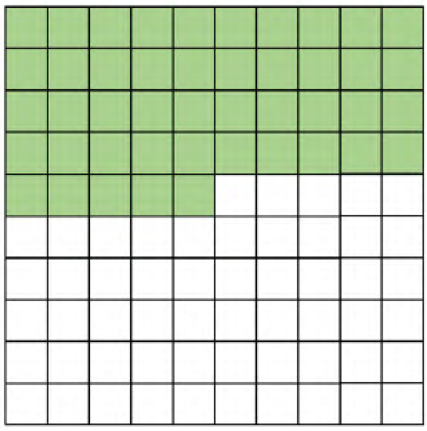
One-half of the learners are girls, so that is 50% of the learners in the class. 50% means 50 per 100, 50 out of 100, $\frac{50}{100}$.

In summary; 70% $=\frac{70}{100}$ 98% $=\frac{98}{100}$ 120% $=\frac{120}{100}$

Expressing Percentages as Fractions and Decimals

Express 45% as a fraction

This can be represented in the diagram as;



From the diagram, 45 square units out of 100 has been shaded. This implies; $\frac{45}{100}=\frac{9}{20}$.

Therefore, $45\% = \frac{9}{20}$

This implies that, converting percentages to fractions simply means divide the said percentage value by 100 and reduce the fraction to its lowest form.

Examples

1. Express 30% as a fraction.

   $30\% = \frac{30}{100} = \frac{3}{10}$

2. Kwame scored 75% in a test. Express his score in fraction form.

   $75\% = \frac{75}{100} = \frac{3}{4}$

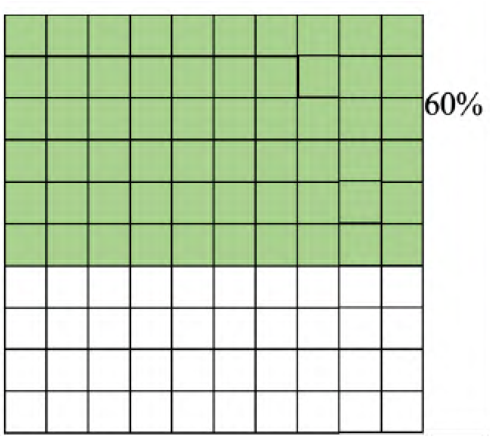
3. Express $20\frac{1}{2}\%$ as a fraction.

$$20\frac{1}{2}\% = \frac{20\frac{1}{2}}{100} = \frac{41}{2} \times \frac{1}{100} = \frac{41}{200}$$

Expressing Percentages as Decimals

Examples

1. Express 60% as a decimal



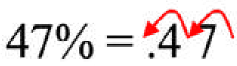
Based on our previous lessons, we learnt that:

$\frac{1}{10} = 0.1$ $\frac{1}{100} = 0.01$ $\frac{1}{1000} = 0.001$ $\frac{4}{100} = 0.04$

Therefore, from the diagram, 60% means $\frac{60}{100} = \frac{6}{10} = 0.6$

In general terms, to convert a percentage to decimal, simply move the decimal point from left to right twice.

2. Express 47% as a decimal.



$$47\% = \frac{47}{100} = 0.47$$

3. Convert the following percentages to fractions and then to decimals:

a. 73%

b. 82.4%

c. 129%

d. 8.6%

Solution

a. $73\% = \frac{73}{100} = 0.73$

b. $82.4\% = \frac{82.4}{100} = 0.824$

c. $129\% = \frac{129}{100} = 1.29$

d. $8.6\% = \frac{8.6}{100} = 0.086$

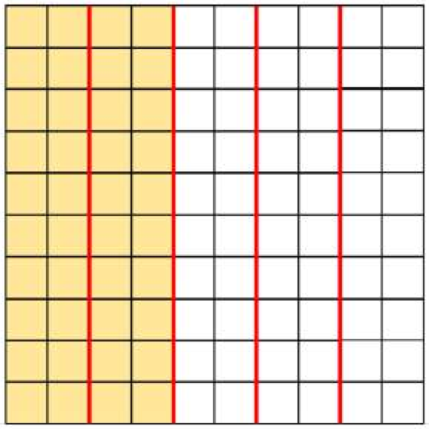
Converting Fractions and Decimals to Percentages

Examples

1. Express $\frac{2}{5}$ as a percentage

- Draw a grid of 10 by 10 to get 100 small units.
- Divide the grid into 5 equal parts and shade 2 as shown below.

$\frac{2}{5}$

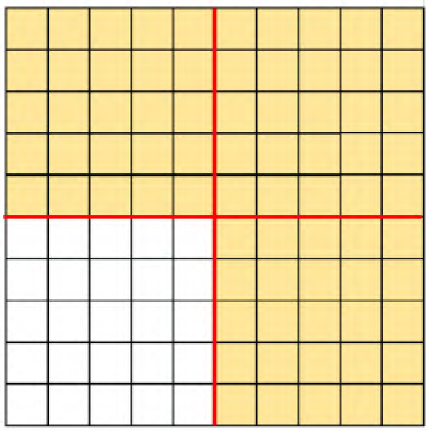


- Count the number of square units shaded. There are 40 square units shaded out of 100 square units.
- This implies that $\frac{2}{5} = \frac{40}{100} = 40\%$

2. Express $\frac{3}{4}$ as a percentage

- From the 10 by 10 grid, divide the grid into 4 equal parts and shade 3 as shown below.

$\frac{3}{4}$



- Count the number of square units shaded. There are 75 square units shaded out of 100 square units.
- This implies that $\frac{3}{4} = \frac{75}{100} = 75\%$

From the two illustrations above, fractions can be converted to percentages by multiply the fraction by 100.

That is:

$\frac{2}{5} \times 100 = \frac{200}{5} = 40\%.$

Again:

$\frac{3}{4} \times 100 = \frac{300}{4} = 75\%$

3. Express the following fractions as percentage.

   a. $\frac{3}{5}$

   b. $\frac{1}{3}$

   c. $1\frac{3}{4}$

Solution

a. $\frac{3}{5} = \frac{3}{5} \times 100$

$= \frac{300}{5} = 60\%$

b. $\frac{1}{3} = \frac{1}{3} \times 100$

$\frac{100}{3} = 33.3\%$

c. $1\frac{3}{4} = \frac{1 \times 4 + 3}{4} = \frac{7}{4} \times 100$

$= \frac{700}{4} = 175\%$

Converting Decimals to Percentages

In general, to convert a decimal to a percentage, you multiply by 100. This is the same as moving the decimal point two places to the right.

Example

1. Express the following decimals as percentages:

   a. 0.8

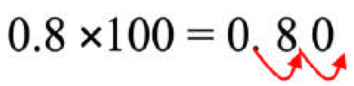
   b. 0.45

   c. 2.05

   d. 0.07

Solution

a. $0.8 \times 100 = 80\%$



b. $0.45 \times 100 = 45\%$

c. $2.05 \times 100 = 205\%$

d. $0.07 \times 100 = 7\%$

Expressing One Quantity as a Percentage of Another

From the onset, we established that percentages are used in everyday life. For example, sales discounts, population growth rates or exam scores etc. In this aspect, we want to find what percentage one quantity represents of another.

To express one quantity as a percentage of another,

- write a fraction with the 'part amount' as the numerator and the 'whole amount' as the denominator.
- convert the fraction to a percentage by multiplying by 100.

Examples

1. Akosua score 16 out of 20 in a test. Express her mark as a percentage.

   Solution

   The 16 represents the '**part**' we are to express as a percentage out of 20, which is the '**whole**'.

   $\frac{16}{20} \times 100 = \frac{1600}{20}$

   $= 80\%$

2. If a class has 40 learners and 22 of them are studying Mathematics, what percentage of the class is studying Mathematics?

   Solution

   The 22 represent the '**part**' and the 40 is the '**whole**'.

   $\frac{22}{40} \times 100 = \frac{2200}{40}$

   $= 55\%$

3. Akoto weighs 18kg. His father weighs 74kg. What percentage of her father's weight does Akoto weigh, correct to one decimal place?

   Solution

   The 18kg represent the '**part value**' and the 74kg is the '**whole**'.

   $\frac{18}{74} \times 100 = \frac{1800}{74}$

   $= 24.3\%$

Application / Importance of percentages

Percentages are useful in many areas including the calculation of discounts, grading scores, tax calculations, inflation and interest rate among others.

Learning Tasks for Practice

Convert percentages to fractions, decimals, and vice versa and solve real life problems involving percentage calculations. Apply percentage calculation skills to real-life situations.

Pedagogical Exemplars

1. **Managing Talk for learning**:

   a. Tailor instruction to meet the diverse needs of learners by providing additional support or extension activities.

   b. Ensure that all learners have opportunities to access the content in a way that best suits their learning preferences and abilities.

   c. Offer scaffolded tasks for struggling learners and challenging problems for advanced learners.

2. **Experiential Learning:**
   a. Use a bar model to represent a quantity and its corresponding percentage visually, showing how the percentage is a portion, or fraction, of the whole. This model allows learners to see the concept concretely and make connections between the numerical representation and the visual representation.
   b. Let learners work in pairs to identify the relationship between percentages, fractions, decimals, and the quantities they represent using concrete manipulatives like fraction bars or circles to provide hands-on learning experiences.
3. **Collaborative Learning:** Foster collaborative learning environments where learners work together to explore concepts of percentages, discuss strategies, and solve problems as a team. Collaborative activities promote communication, critical thinking, and peer learning.

Key Assessment

1. **Assessment Level 1**
   a. Express the following percentages as fractions, in their simplest form:
      i. 55%
      ii. 98%
      iii. 25%
      iv. 150%
      v. $45\frac{1}{2}\%$
   b. Express the following percentages as decimals:
      i. 50%
      ii. 99%
      iii. 72%
      iv. 120%
      v. 132%
2. **Assessment Level 2**
   a. Convert the following decimal to percentages:
      i. 0.05
      ii. 0.65
      iii. 0.75
      iv. 0.09
      v. 0.40
   b. Convert the following fractions to percentages:
      i. $\frac{4}{10}$
      ii. $\frac{2}{5}$
      iii. $\frac{1}{4}$
      iv. $\frac{3}{4}$
      v. $4\frac{1}{2}$

c. Koshie got 72 out of 100 points in her Mathematics exam. Express her score as a percentage.

d. In a survey of 200 learners, 120 prefer online classes. What percentage of learners prefer online classes?

e. In a survey of 200 people, 50 said they prefer tea over coffee. What percentage of the surveyed population preferred tea?

f. A company has 800 employees, and 240 of them work in the marketing department. What percentage of the employees work in marketing?

g. Out of a group of 80 learners, 32 play a musical instrument. What percentage of the learners play a musical instrument?

h. On a 120-question test, a learner got 96 correct answers. What percent of the problems did the learner work correctly?

Week 5: Application of Percentages

Learning Indicator: *Apply the concept of percentages to solve everyday life problems*

Focal Area: Use Percentages to Solve Real-Life Problems

Introduction

As we have previously learnt, a percentage represents a fraction out of 100. When we talk about a percentage of a whole quantity, it signifies the equivalent fraction of that quantity. For instance, if we say 20% of a given quantity, it means 20 out of every 100 parts of that quantity. You can determine a percentage of a quantity by converting the percentage into either a decimal or a fraction, then multiplying it by the quantity. Various methods are available for calculating a percentage of a quantity such as converting to a decimal or converting to a fraction.

Finding Percentage of a quantity

Examples

1. Find 30% of *GH¢* 200.00;

   Solution

   $30\% = \frac{30}{100} = 0.30$

   Multiply the fraction or the decimal by the quantity:

   $\frac{30}{100} \times 200 = GH¢\ 60.00$

   In other words:

   $0.30 \times 200 = GH¢\ 60.00$

   Therefore, 30% of *GH¢* 200.00 is *GH¢* 60.00

2. In a class of 80 learners, 75% passed a test.

   a) How many learners passed?

   b) How many learners failed?

   Solution

   a) $\frac{75}{100} \times 80 = 60$ learners passed.

   b) *Number who failed = Total number of students − Number of students of students who passed.*

   = 80 − 60 = 20 learners failed.

3. Of the pupils in a class 46% are males. If there are 50 pupils altogether, how many of them are females?

   Solution

   $Number\ of\ males = \frac{46}{100} \times 50$

   $= 23$ males

   $Number\ of\ females = Total\ number\ of\ pupils - Number\ of\ males$

   $Number\ of\ females = 50 - 23 = 27\ females$

Use Percentages to Compare and Order Fractions

To compare and order fractions using percentages, first, convert each fraction into its equivalent percentage. To convert a fraction to a percentage, divide the numerator by the denominator, then multiply by 100.

To convert a fraction to a percentage, multiply the fraction by 100.

$\frac{a}{b} \times 100 = \frac{100a}{b}$

To write a decimal as a percentage, multiply the decimal by 100.

Examples

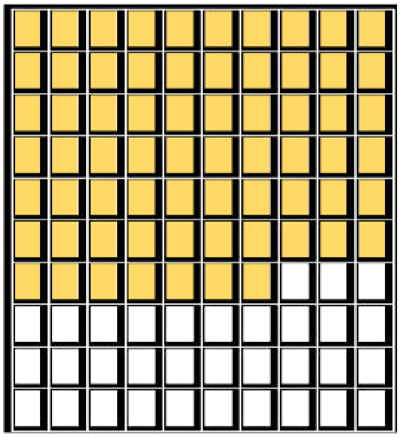
1. Compare $\frac{2}{3}$ and $\frac{3}{4}$

Solution

First, express $\frac{2}{3}$ and $\frac{3}{4}$ as percentages

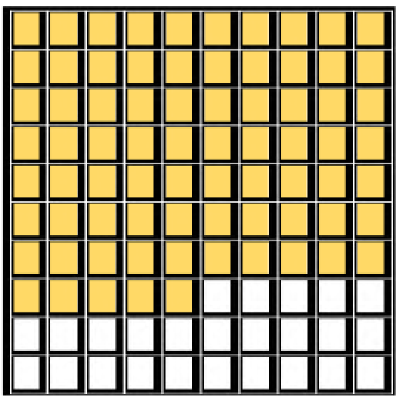
$\frac{2}{3}$ can be written in decimal form as $0.66667 \approx 0.67$ $(2 \div 3)$

$0.67 \times 100 = 67\%$



= 67%

$\frac{3}{4}$ can be written in a decimal form as $0.75 = 0.75 \times 100\% = 75\%$



Since 67% is smaller than 75%, it implies that $\frac{2}{3} < \frac{3}{4}$ or $\frac{3}{4} > \frac{2}{3}$

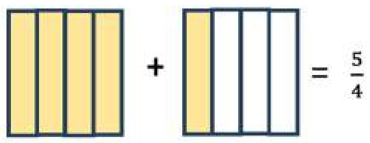
2. Compare $1\frac{1}{4}$ and $\frac{1}{4}$

Solution

Since $1\frac{1}{4}$ is a mixed fraction, first convert it to an improper fraction.

$1\frac{1}{4} = \frac{1 \times 4 + 1}{4} = \frac{5}{4}$

$1\frac{1}{4}$ can be modelled as:



Then express $\frac{5}{4}$ as a percentage, $\frac{5}{4} \times 100\% = 1.25 \times 100\% = 125\%$

$\frac{1}{4} \times 100\% = 25\%$

$1\frac{1}{4} > \frac{1}{4}$ as $125\% > 25\%$

Finding the Percentage Increase or Decrease of a Given Quantity

Percentage increase or decrease is a concept used to measure the change in a quantity relative to its original value over time.

Percentage Increase = $\left(\frac{\textit{Increase in value}}{\textit{Original value}}\right) \times 100\%$

Percentage Decrease = $\left(\frac{\textit{Decrease in value}}{\textit{Original value}}\right) \times 100\%$

Examples

1. If the price of fuel increased from *GH¢* 50.00 to *GH¢* 60.00, the increase in value is *GH¢* 10.00. Find the percentage increase.

Solution

Percentage Increase $= \frac{10}{50} \times 100\% = 20\%$

2. The price of fuel decreased from *GH¢* 60.00 to *GH¢* 50.00, the decrease in value is *GH¢* 10.00. Find the percentage decrease.

Solution

Percentage Decrease $= \frac{10}{60} \times 100\% = 16.67\%$

Increasing or Decreasing a Quantity by a Given Percentage

To increase or decrease a quantity by certain percentage; Assume the original quantity as 100%. If the percentage increase is x%, then the new quantity will be $(100\% + x\%)$ of the original.

Increasing a Quantity: To increase a quantity by a certain percentage, we add the percentage of the original quantity to the original quantity. The formula for increasing a quantity by a percentage is:

Method 1

$$New\ Quantity = \frac{(100 + Percentage\ Increase)}{100} \times Original\ Quantity$$

For example, if we want to increase a quantity by 20%, we add 20% to 100 and divide by 100. This gives us the percentage multiplier, 1.2. If the original quantity is 100, the new quantity would be:

$$New\ Quantity = \left(\frac{100 + 20}{100}\right) \times 100$$

$$1.2 \times 100 = 120$$

or

Method 2

$$New\ Quantity = Original\ Quantity + \left(\frac{Percentage\ Increase}{100} \times Original\ Quantity\right)$$

For example, if we want to increase a quantity by 20%, we add 20% of the original quantity to the original quantity. If the original quantity is 100, the new quantity would be:

$$New\ Quantity = 100 + \left(\frac{20}{100} \times 100\right)$$

$$100 + 20 = 120$$

Example

1. A pack of exercise books which costs *GH¢* 250.00 is increased by 60%, what is the new price of the exercise books?

   Solution

   Method 1

   $$New\ Price = \frac{(100 + Percentage\ Increase)}{100} \times Original\ Price$$

   $$New\ price = \frac{100 + 60}{100} \times GH¢\ 250$$

   $$New\ Price = \frac{160}{100} \times GH¢\ 250$$

   $$New\ Price = GH¢\ 400.00$$

   Method 2

   $$New\ price = Original\ Price + \left(\frac{Percentage\ Increase}{100} \times Original\ price\right)$$

   $$New\ price = GH¢\ 250 + \left(\frac{60}{100} \times GH¢\ 250\right)$$

   $$New\ price = GH¢\ 250 + GH¢\ 150$$

   $$New\ price = GH¢\ 400.00$$

Decreasing a Quantity: To decrease a quantity by a certain percentage, we subtract the percentage of the original quantity from the original quantity. The formula for decreasing a quantity by a percentage is:

Method 1

$$New\ Quantity = \frac{(100 - Percentage\ Decrease)}{100} \times Original\ Quantity$$

For example, if we want to decrease a quantity by 10%, we subtract 10% from 100 and divide by 100. This gives us the percentage multiplier, 0.9. If the original quantity is 200, the new quantity would be:

$$New\ Quantity = \left(\frac{100 - 10}{100}\right) \times 200$$

$$0.9 \times 200 = 180$$

Method 2

$$New\ Quantity = Original\ Quantity - \left(\frac{Percentage\ Decrease}{100} \times Original\ Quantity\right)$$

For instance, if we want to decrease a quantity by 10%, we subtract 10% of the original quantity from the original quantity. If the original quantity is 200, the new quantity would be:

$$New\ Quantity = 200 - \left(\frac{10}{100} \times 200\right)$$

$$200 - 20 = 180$$

Example

1. A pack of exercise books costs *GH¢* 250.00. This is decreased by 60%. What is the new price of the exercise books?

   Solution

   Method 1

   $$New\ price = \frac{(100 - Percentage\ Decrease)}{100} \times Original\ price$$

   $$New\ price = \frac{(100 - 60)}{100} \times GH¢\ 250$$

   $$New\ price = \frac{40}{100} \times 250$$

   $$New\ price = GH¢\ 100.00$$

   Method 2

   $$New\ price = Original\ price - \left(\frac{Percentage\ Decrease}{100} \times Original\ price\right)$$

   $$New\ price = GH¢\ 250.00 - \left(\frac{60}{100} \times GH¢\ 250.00\right)$$

   $$New\ price = GH¢\ 250.00 - GH¢\ 150.00$$

   $$New\ price = GH¢\ 100.00$$

Application / Importance of percentages

1. To calculate the increase/ decrease in cost of items. For example, if an item cost *GH¢* 250.00 and there's a 60% increase, how much will the item cost after the increase?
2. To explain grading systems. Show how a score of 8 out of 10 on a quiz translates to a percentage grade.
3. Discussions on sports statistics. For instance, if a basketball player made 7 out of 10 free throws, what percentage of free throws did they make?
4. Percentages are used in discussing recipes and ingredients. If a recipe calls for 3/4 cup of flour, what percentage of a full cup is this?
5. Time management. For example, if a learner spends 30 minutes out of an hour studying, what percentage of the hour did they spend studying?

Learning Tasks for Practice

1. Learn to analyse survey data and calculate percentages.
2. Practise calculating percentages within their small groups.
3. Provide additional support, simplified tasks, guided examples and step-by-step instructions on how to calculate percentages.

Pedagogical Exemplars

1. **Collaborative learning**
   a. Let learners think, pair and share simple real-life examples involving basic percentages, such as calculating percentage increase/ decrease in prices on everyday items.
   b. Foster group discussions or cooperative learning activities where learners work together to solve percentage-related problems.
2. **Experiential learning**
   a. Use concrete visual aids like diagrams or bar models and interactive visual tools or graphs such as pie charts, histograms etc. to illustrate percentages visually. Bar models or pie charts help learners visualise percentages as parts of a whole.
   b. Use everyday scenarios like shopping, examination scores to demonstrate the relevance of percentages.
   c. Provide access to diverse resources (videos, images etc.) to cater for the varying preferences of learners.
3. **Problem-Based learning**
   a. Allow learners to work in mixed ability/ gender groups to generate problem-solving scenarios with guided prompts to build confidence and develop basic percentage skills.
   b. Present open-ended real-world problems learners to apply percentage concepts independently and develop problem-solving skills.

Key Assessment

Assessment Level 2

1. If you have *GH¢* 50.00 and you spend 20% of it on snacks, how much money do you have left?
2. At the Winneba market, $\frac{1}{3}$ of the fruits are oranges, $\frac{1}{4}$ are bananas, and $\frac{1}{6}$ are apples. Which fruit is the most common? Use percentages to compare.
3. If a shopkeeper reduces the price of a shirt by 15% and it initially costs *GH¢* 80.00, what is the new price of the shirt?

Assessment Level 3

1. If the price of a loaf of bread increased from *GH¢* 2.00 to *GH¢* 2. 50, what was the percentage increase?
2. At a school, $\frac{1}{5}$ of the students play football, $\frac{1}{3}$ play basketball, and $\frac{1}{4}$ play volleyball. Arrange the sports in ascending order of popularity using percentages.
3. A company wants to increase its workforce by 20%. If it currently has 500 employees, how many new employees should they hire?

Section Review

The section on ratio and percentages serves as a comprehensive exploration of fundamental mathematical concepts essential for practical applications. Beginning with an introduction to ratios and percentages, learners delve into understanding their definitions, representations, and relevance in everyday life.

Through various examples and scenarios, learners grasp the concepts of ratios and percentages, learning to express them in different forms such as fractions, decimals, and visual models. Problem-solving tasks and real-world applications deepen comprehension, while collaborative activities foster peer learning and critical thinking.

The section emphasises practicality, with a focus on applying ratio and percentage concepts to solve proportion problems, calculate percentages, and interpret data. Through differentiated instruction and varied assessment methods, educators ensure that all learners engage effectively and demonstrate proficiency in ratio and percentage skills.

SECTION 3: REASONING WITH ALGEBRA

Strand: **Algebraic Reasoning**

Sub-Strand: Expressions and Equations

Learning Outcome: *Apply various methods to factorise algebraic expressions and use the inverse property to rearrange a formula in one or two steps to change the subject.*

Content Standard: *Demonstrate conceptual understanding of factorising and evaluating of algebraic expressions.*

INTRODUCTORY SUMMARY

In algebra, mastering factorisation and the use of inverse properties are crucial skills that enable learners to manipulate and simplify expressions. This section focuses on identifying and applying various methods of factorisation, such as factorising out the greatest common factor, factorising by grouping, and factorising quadratic expressions. Additionally, learners will explore the inverse property to rearrange formulas, a skill that is particularly useful when changing the subject of an equation. These concepts are fundamental for solving more complex algebraic problems and are widely applicable in both academic and real-life contexts.

The section will cover the following focal areas:

1. *Identify and apply the methods of factorising algebraic expressions*
2. *Use the inverse property to rearrange a formula in one or two steps to change the subject.*

PEDAGOGICAL SUMMARY

Teaching these algebraic concepts will involve a step-by-step approach to ensure a deep understanding. For factorisation, learners will begin with identifying common factors and progress to more complex techniques like grouping and factorising quadratics. Hands-on practice with a variety of problems will solidify these methods. The concept of the inverse property will be introduced through practical examples, showing how it can be used to rearrange formulas in one or two steps. Interactive activities, such as peer tutoring and group problem-solving sessions, will encourage collaboration and enhance comprehension. Visual aids like factor trees and algebra tiles can also be employed to help learners visualise the factorisation process.

ASSESSMENT SUMMARY

Assessment will include both formative and summative elements to gauge learners' understanding of factorisation and the use of inverse properties. Formative assessments, such as classwork, group activities, and peer reviews, will focus on the application of different factorisation methods and the correct use of the inverse property in rearranging formulas. Summative assessments will consist of quizzes and tests with questions that require learners to factorise various algebraic expressions and rearrange formulas to change the subject. Real-life application problems will also be included to assess their ability to use these algebraic techniques in practical situations.

Week 6: Factorisation

Learning Indicator: *Identify and apply the methods of factorisation algebraic expressions*

Focal Area: Factorisation of Algebraic Expressions

Introduction

Factorisation is an essential technique in algebra that involves breaking down an algebraic expression into simpler expressions or factors. It is the reverse process of expansion. We find the common terms among the expression and express it as a product of these common factors.

Algebraic expressions can be factorised using the common factor method, regrouping like terms together, and also by using algebraic identities.

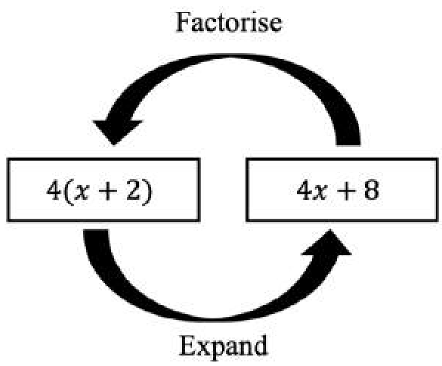
Common Factor:

- **a.** Identify if there is a common factor that can be factorised out from all the terms of the expression.
- **b.** Divide each term by the common factor.
- **c.** Rewrite the expression as the common factor multiplied by the remaining terms.

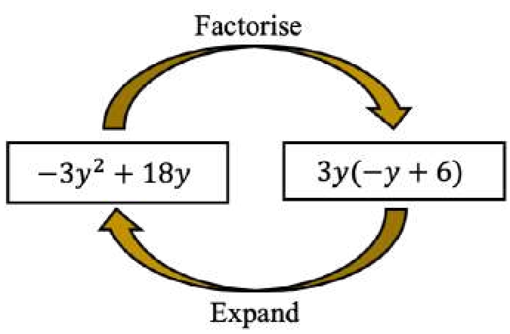
Examples

Factorise the following:

1. Factorise the expression $4x + 8$



2. Factorise $-3y^2 + 18y$



3. Factorise $4a + 12b$

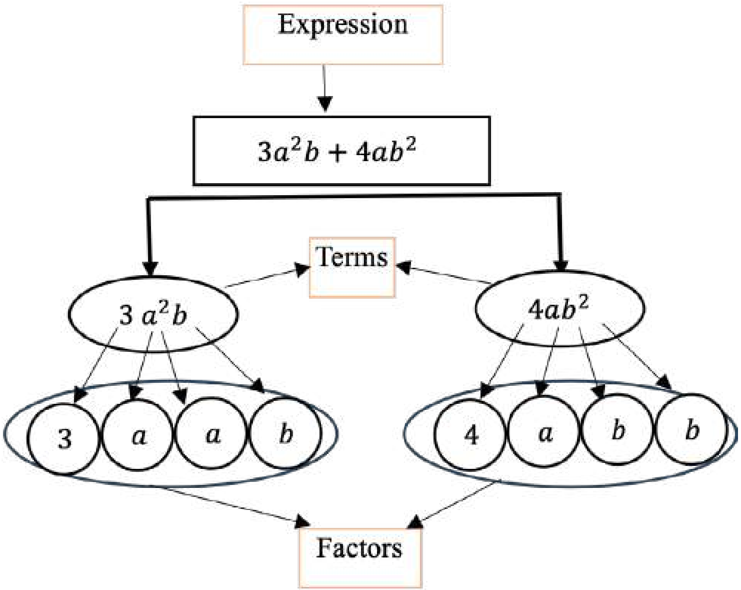
$= 4(a+3b)$

4. $ab + bc$

$= ba + bc$

$= b(a + c)$

5. Find the factors of $3a^2b + 4ab^2$



That is; $(3 \times a \times a \times b) + (4 \times a \times b \times b)$

Pick out the common factors by pairing them off

$(3 \times \boldsymbol{a} \times a \times \boldsymbol{b}) + (4 \times \boldsymbol{a} \times \boldsymbol{b} \times b)$

$= ab(3a + 4b)$

Difference of two squares

a^2 and b^2 are perfect squares and so $(a^2 - b^2)$ is called a difference of two squares.

An expression in the form $(a + b)(a - b)$
can be expanded as $a(a - b) + b(a - b)$
$= a^2 - ab + ab - b^2$
$= a^2 - b^2$

Example

If $6^2 - 5^2$ can be simplified as $36 - 25 = 11$
since $6^2 - 5^2$ is a difference of two squares, this implies that:
$6^2 - 5^2$ can be factored as $(6 + 5)(6 - 5)$ and can be expanded as $6(6 - 5) + 5(6 - 5)$
$= 36 - 30 + 30 - 25 = 36 - 25 = 11$

Perfect Square Trinomial:

Examples

1. Expand the expression $(a + b)^2$

$(a + b)^2 = (a + b)(a + b) = a(a + b) + b(a + b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$

Therefore, $a^2 + 2ab + b^2 = (a + b)^2$

2. Expand the expression $(a - b)^2$

$(a - b)^2 = (a - b)\,(a - b) = a\,(a - b) - b\,(a - b) = a^2 - ab - ab + b^2 = a^2 - 2ab + b^2$

Therefore, $a^2 - 2ab + b^2 = (a - b)^2$

This implies that an expression in the form of $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$, can be factored as $(a + b)^2$ or $(a - b)^2$ respectively.

Be guided by the steps used in the perfect square expansion:

First, square the first term add twice the product of the first and the last terms and add to the square of the last term.

Grouping:

For expressions with four terms, look for pairs of terms that have common factors. Factor out the common factors from each pair. Look for common factors in the resulting binomials and factor them out as well.

Examples

1. $ab + ac + bd + cd$

$= a\,(b + c) + d\,(b + c)$ *factorising each pair separately removing the common factor*

$= (b + c)\,(a + d)$

Many expressions with four terms cannot be factorised using this approach. There is therefore the need to reorder or regroup the terms first.

2. $6ax - 2y + 3ay - 4x$

$= 6ax + 3ay - 4x - 2y$ *Group like terms and remove the common factor*

$= 3a(2x + y) - 2(2x + y)$

$= (3a - 2)(2x + y)$

3. $3ab + d + 3ad + b$

$= 3ab + b + 3ad + d$

$= b(3a + 1) + d(3a + 1)$

$= (3a + 1)(b + d)$

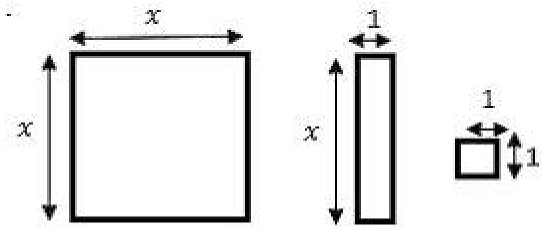
Quadratic Trinomial:

Any mathematical expression of the form $ax^2 + bx + c$, where a, b and c are constants and the coefficient of x^2, $a \neq 0$ is called a quadratic trinomial or expression. One can use various methods like trial and error, grouping, or the quadratic formula to factorise it.

Examples

1. Factorise $x^2 + 5x + 6$

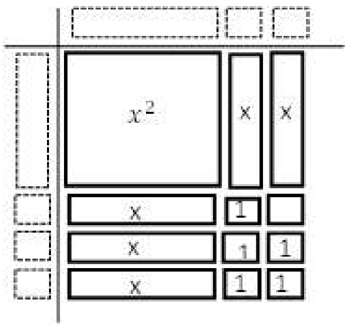
Use concrete algebraic tiles or draw them as shown below.

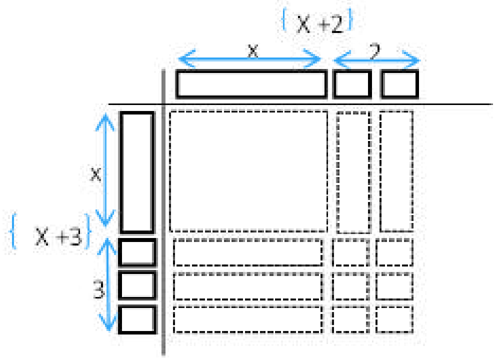


Area of big square = $x \times x = x^2$

Area of rectangle = $x \times 1 = x$

Area of small square = $1 \times 1 = 1$





What is the relationship between the two diagrams in terms of area? (Learners should be able to say the areas are the same.)

Thus $x^2 + 5x + 6 = (x + 2)(x + 3)$

Factorisation by using the factors of a quadratic expression or trinomial

To find the factors of a quadratic expression in the form $x^2 + bx + c$, where the coefficient of , x^2 is 1; first find all the possible factors of the constant term, c, such that the product of a pair of the factors is equal to the constant term, c and the sum of that same pair is equal to the coefficient of x, i.e. b.

For instance, if the factors of ab are a and b, then the quadratic expression will be;

$x^2 + (a + b)x + ab$

That is $x^2 +$ (*sum of factors*)x + *product of factors*

$= (x+a)(x+b)$

Example:

1. Factorise the expression $x^2 + 5x + 6$

   First, find the possible pair of the factors of the constant term + 6. These are (2, 3) and (-2, -3)

   Identify the sum of the factors that gives the coefficient of x which is 5 That is +2 and +3

   $= x^2 + 2x + 3x + 6$

   $=(x^2 + 2x) + (3x + 6)$ *Group the terms*

   $= x(x+2) + 3(x+2)$ *Factorise, using the common factor*

   $= (x+2)(x+3)$

   Note that the factorisation can be done in a straightforward manner if the factors are known.

2. Factorise the expression $x^2 + 6x + 9$

   The pair of the product of the factors that gives the constant term + 9 are; (+ 3, + 3) and (-3, -3)

   The sum of the factors that gives + 6 are + 3 and + 3

   $x^2 + 6x + 9 = x^2 + 3x + 3x + 9 = (x^2 + 3x) + (3x + 9) = x(x + 3) + 3(x + 3) = (x + 3)(x + 3)$

3. Factorise the expression $x^2 - 5x + 6$

   The pair of the product of the factors that gives the constant term +6 are; (+2, +3) and (-2, -3)

   The sum of the factors that gives -5 are -2 and -3.

   $x^2 - 5x + 6 = x^2 - 2x - 3x + 6 = (x^2 - 2x) - (3x + 6) = x(x - 2) - 3(x - 2) = (x - 3)(x - 2)$

Application of algebraic expressions

1. Factorisation is often used to solve polynomial equations. By factorising an expression and setting each factor equal to zero, one can find the roots or solutions of the equation.
2. Factorisation helps in simplifying complex algebraic expressions. By factorising out common terms or grouping like terms, expressions can be made more manageable and easier to work with.

Learning Tasks for Practice

1. Identify factors and terms in algebraic expressions.
2. Recognise expressions with common factors.
3. Identify binomial expressions that can be factorised by the difference of two squares.
4. Factorise quadratic expressions using algebraic tiles, trial and error or grouping.
5. Apply factorisation to solve equations and simplify expressions.

Pedagogical Exemplars

1. **Think-Pair-Share:** Begin by presenting a problem or expression requiring factorisation. Allow learners time to individually think about and attempt the factorisation. Then, pair them up to

discuss their approaches and solutions. Finally, facilitate a whole-class discussion where pairs share their strategies and findings.

2. **Collaborative Learning:** Organise learners into small groups and provide them with algebraic expressions to factorise collaboratively. Encourage group members to work together, share their understanding and support each other in identifying factorisation methods. This collaborative approach fosters peer learning and allows learners to benefit from diverse perspectives.
3. **Structured Talk for Learning:** Use structured talk protocols to guide classroom discussions on factorisation. Provide sentence stems or prompts to scaffold learners' discussions, such as "I noticed that..." or "One strategy we can use is...". Encourage all learners to actively participate in the discussion and articulate their reasoning behind factorisation steps.
4. **Experiential Learning:** Design hands-on activities or manipulative-based tasks where learners physically manipulate algebraic expressions or algebra tiles to explore factorisation concepts.
5. **Problem-Based Learning:** Present learners with real-world problems or contextualised scenarios that require factorisation to find solutions. Encourage learners to analyse the problem, identify relevant factors, and apply appropriate factorisation techniques.
6. **Technology Integration:** Utilise interactive software or online platforms that offer virtual manipulatives or interactive tutorials for factorisation. Allow learners to explore factorisation concepts independently or collaboratively using technology tools, providing them with immediate feedback and opportunities for self-directed learning.

Key Assessment

1. **Assessment Level 1**
   - **a.** Identify the common factors in the expression $3x^2 + 9x$
   - **b.** Simplify the expression $6ab+12a$ by factorising.
2. **Assessment Level 2**

   Match each expression in Column A with its factorisation in Column B.

Column A	**Column B**
a. $26x – 13y$	$2x(15 – 8x)$
b. $30x - 16x^2$	$2(x + 2y)$
c. $x^2y - 2xy - 3y^3$	$13(2x - y)$
d. $2x + 4y$	$y(x^2 - 2x - 3y^2)$

3. **Assessment Level 2**
   - **a.** Identify and correct the error in the factorisation of $2x^2 - 4y^2$ as $(x + 4y)(x - 4y)$.
   - **b.** A farmer has a rectangular field with dimensions $2x + 4$ and $6x + 6$.

     What is the total area of fencing needed to enclose the field?
4. **Assessment Level 3**
   - **a.** Write a step-by-step explanation of how you factorised the expression $3x^2 + 15x$ using the greatest common factor.
   - **b.** A company is manufacturing rectangular tiles with an area represented by the expression $3x^2 + 9x$.

     Factorise the expression to determine the dimensions of the tiles if the length must be $3x$.

Week 7: Substitution and Change of Subject

Learning Indicator: *Use the inverse property to rearrange a formula in one or two steps to change the subject.*

Focal Area: Change of Subject and Substitution (Applying the Inverse Property to Rearrange a Formula)

Introduction

In a formula with an equal sign (=), the subject of the formula is the letter or variable which is expressed in terms of the other variables. The subject of a formula is the variable that stands on its own.

The inverse property can be used to rearrange a formula by applying the inverse operation to both sides of the equation.

Substitution involves replacing a variable (usually denoted by a letter) with a specific value or expression. It allows us to find the value of an algebraic expression by using known values.

Change of subject is very important in dealing with so many topics in mathematics such as volumes and surface area of solid figures, linear equations and inequalities and so on,

In the equation:

$A = \pi r^2$, A is the subject of the equation.

$C = 2\pi r$, C is the subject of the equation.

A and C have been expressed in terms of π and r.

Exploring the steps to changing the subject in an equation

Examples

1. Solve this equation, by making x the subject, $2x + 3 = 9$

   Solution

   By making x the subject, we are finding the value of x that satisfies the equation $2x + 3 = 9$.

   $2x + 3 = 9$ — *Start with the Original Equation*

   $2x + 3 - 3 = 9 - 3$ — *Isolate the Term with* x by subtracting 3 from both sides of the equation to move the constant term to the other side .

   $2x = 6$

   $\frac{2x}{2} = \frac{6}{2}$ — *Remove the coefficient* by dividing both sides by the coefficient of x, which is 2, to isolate a single x

   $x = 3$

2. The volume of a rectangular shape (V), with length (l), width (w) and height (h) is given by the formula $V = lwh$.

   We are interested in finding the height (h) of the rectangular shape. To do this we need to re-arrange the formula $V = lwh$.

Solution

$V = lwh$

$\frac{V}{l \times w} = \frac{l \times w \times h}{l \times w}$

$\frac{V}{l \times w} = h$

3. The equation of a straight line is given by $y = mx + c$, where m is the gradient and c is the intercept on the y-axis.

In finding the gradient, we need to make m the subject of the equation.

Solution

$y = mx + c$

$y - c = mx$

$\frac{y - c}{x} = m$

4. Find the area of a pizza when the radius of the pizza is given as 2.82 inches? Use $\pi = 3.142$

Solution

The area of a circle is given as $A = \pi r^2$, where r is the radius of the circle.

To find the area of the pizza, there is the need to substitute the formula to solve for A.

$A = 3.142 \times 2.82^2$

$A = 3.142 \times 7.9524$

$A = 24.9864408$

$A = 25$ square inches

5.

a. Kwame put *GH₵* 500.00 into his bank account. The interest rate was 5% per year. If he is paid simple interest, how much interest will he earn in two years. (Given $I = P \times R \times T$)

Solution:

Given $I = P \times R \times T$,

Principal, P = 500

Rate, R= 5% = 0.05

Time, T= 2

By substituting , Interest, $I = 500 \times 0.05 \times 2$

$I =$ *GH₵* 50.00

b. Kwame put an amount of money into his bank account. The simple interest rate was 20%. After two years he had earned 150 cedis as interest. What amount did he originally put in the bank?

Solution

If I = PRT, to find the original amount (Principal), make P the subject of the equation and find the value of P where I = 150, R= 20% = 0.20 and T = 2

$\frac{I}{RT} = \frac{PRT}{RT}$, *Divide both sides of the equation by RT*

$$\frac{I}{RT} = P$$

By substituting, $P = \frac{150}{0.20 \times 2}$

$P = GH₵\ 375.00$

This implies that Kwame put *GH₵* 375.00 into his bank account.

Application / Importance of algebraic expressions

1. Physics: When solving equations for motion, such as velocity, acceleration, or distance, substitution and change of subject are commonly used. For instance, in the equation, where v is final velocity, u is initial velocity, a is acceleration, and t is time, we can rearrange the equation to find any of the variables based on the given values.
2. In finance, the simple interest formula is commonly used to calculate the interest earned or paid on a principal amount over a certain period of time. The formula for simple interest is I = PRT.

Learning Task for practice

1. Given algebraic expressions or word problems, identify the variables representing different quantities.
2. Practice substituting given values into algebraic expressions or equations to evaluate them.
3. Given algebraic equations, practice rearranging them to make a different variable the subject.
4. Solve word problems that require substituting values into equations or changing the subject to find unknown quantities.

Pedagogical Exemplars

1. **Think pair share**: Begin with a brief explanation of substitution and changing the subject. Then, allow learners time to think individually about a given problem. Next, pair them up to discuss their approaches and solutions.
2. **Collaborative Problem-Solving**: Divide learners into small groups and assign them problems related to substitution and change of subject. Encourage collaboration as they work together to solve the problems, discussing different approaches and strategies.

Key Assessment

Assessment Level 1

1. What is the value of x in the equation $3x + 2 = 14$
2. Solve for y in the equation $5y - 3 = 7$

Assessment Level 2

In a recipe for making pancakes, the total cost of ingredients is represented by the equation $C = 2f + e$ where f is the cost of flour, and e is the cost of eggs. Rearrange the formula to make f the subject.

Assessment Level 3

1. A company sells T-shirts for *GH₵* 15.00 each. The total revenue R from selling x T-shirts is given by the equation $R = 15x$. If the company's revenue from selling 20 T-shirts is *GH₵* 300.00, how many T-shirts did they sell?

Section Review

In this section, we delved into the fundamental concepts of algebraic expressions, substitution, and changing subjects. We began by exploring algebraic expressions, understanding variables, constants, coefficients, and terms. Through various pedagogical exemplars such as think-pair-share and collaborative learning, learners engaged actively, identifying like and unlike terms and simplifying expressions.

We then moved on to substitution, where learners learned to replace variables with specific values and evaluate expressions. This process was reinforced through hands-on activities and real-life problem-solving tasks, ensuring a deeper understanding of substitution strategies.

Finally, we introduced the concept of changing subjects, where learners learned to manipulate equations to isolate a specific variable. Through scaffolded instruction and guided practice, learners developed proficiency in rearranging equations and solving for a desired variable.

Throughout the section, a variety of assessment strategies, including formative quizzes, performance tasks, and summative tests, were employed to gauge learners' comprehension and mastery. By engaging in these activities, learners were able to develop a solid foundation in algebraic reasoning and problem-solving skills, preparing them for further exploration in mathematics.

SECTION 4: GEOMETRIC REASONING AND MEASUREMENT

Strand: **Geometry and Measurement**

Sub-Strand: Shape, Space and Measurement

Learning Outcomes

1. *Determine the value of various angles including complementary, supplementary angles, vertical and parallel lines cut by transversal in real-life contexts.*
2. *Measure the perimeter, area, surface area and volume of 2D and 3D shapes and solve real-life problems on them.*

Content Standards

1. Demonstrate conceptual understanding of angles and its types and use the knowledge to solve problems.
2. Demonstrate knowledge and understanding of the measurement of perimeter, area, surface area and volume of 2Ds and 3Ds.

INTRODUCTORY SUMMARY

Understanding geometric relationships and measurements is crucial for solving real-world problems. In this section, learners will explore how to identify and apply concepts such as parallel and perpendicular lines, complementary and supplementary angles, as well as vertical angles and parallel lines cut by a transversal. These concepts will be applied in real-life contexts, such as architecture and engineering. Additionally, learners will develop and apply strategies to determine the perimeter and area of plane figures, including circles, as well as strategies for calculating the surface area and volume of prisms. These skills are vital for various practical applications, from designing objects to understanding spatial relationships.

The section will cover the following focal areas:

1. *Identifying and applying parallel, perpendicular, complementary, supplementary angles, vertical and parallel lines cut by transversal in real-life contexts.*
2. *Developing and applying strategies determining the perimeter and area of plane figures* (*circles*)
3. *Developing and applying strategies for determining the surface area and volume of prisms.*

PEDAGOGICAL SUMMARY

Teaching these concepts will involve a combination of direct instruction, hands-on activities, and real-life applications. For angle relationships and lines, learners will use tools like protractors and rulers to measure and identify angles and lines in classroom settings and around their environment. Interactive activities, such as creating geometric models and exploring patterns in architecture, will help learners connect the concepts to real life. When covering perimeter, area, surface area, and volume, learners will engage in problem-solving tasks that require the application of formulas and strategies to solve practical problems. Visualisation tools, such as grid paper and 3D models, will aid in understanding and calculating these measurements.

ASSESSMENT SUMMARY

Assessment for this section will include a variety of methods to ensure a comprehensive understanding of the geometric concepts. Formative assessments, such as in-class exercises, group projects, and hands-on activities, will monitor learners' ability to identify and apply angle relationships, lines, and calculate perimeter, area, surface area, and volume. Summative assessments will include quizzes and tests with questions that require learners to solve problems involving these concepts. Real-life application tasks will also be used to assess learners' ability to apply these geometric principles in practical situations, such as calculating the materials needed for construction or determining the space within a given area.

Week 8: Angles & Its Applications

Learning Indicator: *Identify and apply parallel, perpendicular, complementary, supplementary angles, vertical and parallel lines cut by transversal in real-life contexts.*

Focal Area: Angles

Review the learner's previous knowledge of lines, line segments and rays which are the fundamental elements in geometry that help understand geometric shapes and structures.

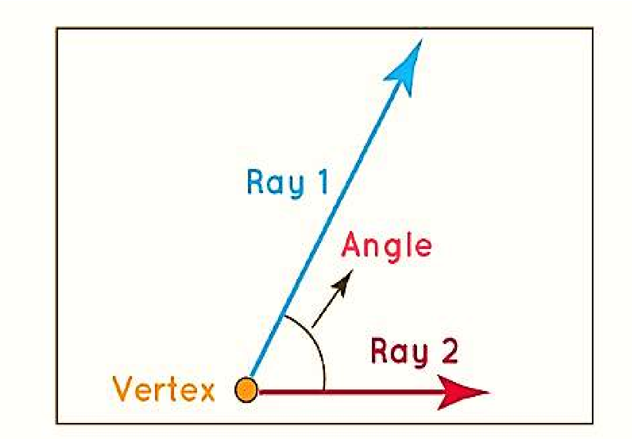
Introduction

An angle is like a twist or a turn. It is the amount of rotation between two lines that meet at a common point, which we call the vertex. Angles are everywhere in our daily lives. These help us to describe shapes, navigate our environment, and understand the relationships between objects. Angles help in building a house or flying a kite. They play a crucial role in how things move and interact.

Now that we have a basic understanding of what angles are and why they matter, let us dive deeper into our exploration to help identify and understand angles better.

Exploring angles and examples of angles in the environment using visual aids, interactive activities and concrete examples.

Imagine holding two straight lines, or rays, with one end connected at a point and then rotating one of the lines around that point. The space between the lines as they rotate is what we call an angle.



From the above diagram, we can conclude that an angle is formed when two rays have a common endpoint called the vertex**.** The rays are known as the sides (or arms) of the angle. It is a measure. Angles are measured in degrees (°).

In the diagram, if ray 1 is A, the vertex is B and ray 2 is C then the angle form at the vertex or B is mathematically written as ∠ABC) or ∠B. The tool used to measure angles is a protractor.

Parts of angles

An angle has three parts

i. **Vertex:** the common endpoint where the two rays meet.

ii. **Sides or arms:** the two rays that form the angle.

iii. **Measure:** the amount of rotation between the two rays**.**

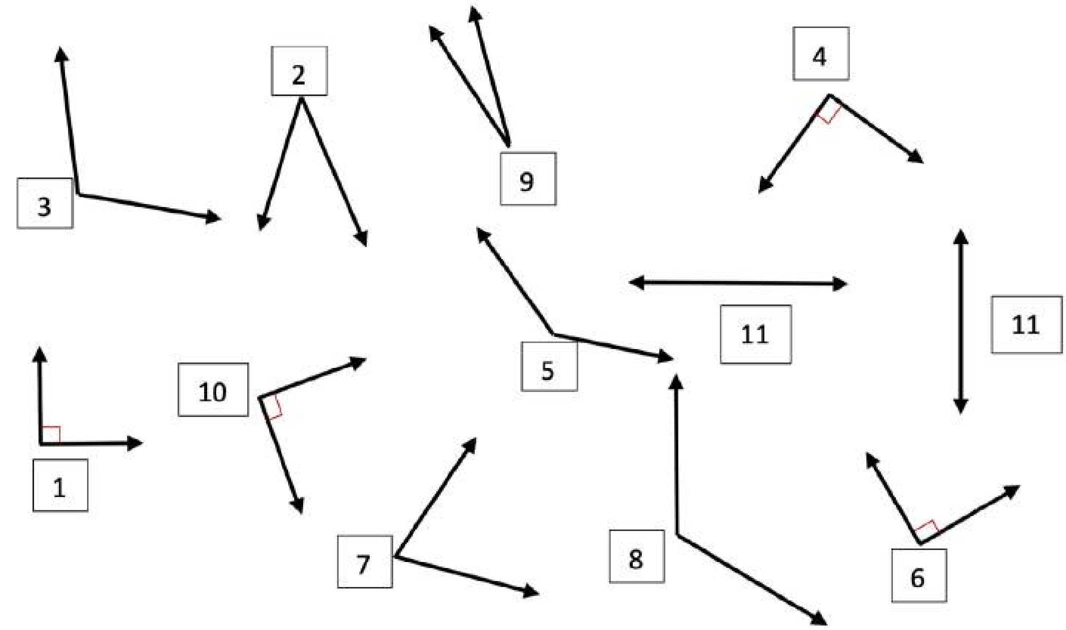
Examples of angles in the environment

Types of angles

We can use diagrams to explore the characteristics of the types of angles.

Types of angles have been drawn to help determine the characteristics of each of them with the help of a protractor.

Group those with the same characteristics as one group.



Angles can be classified based on their measurement and relationship to other angles.

From the diagram above,

i. the angles of the numbers 1, 4, 6 and 10 are 90° by measuring.

ii. The angles of the numbers 2, 7 and 9 are less than 90°

iii. The angles of the numbers 3, 5 and 8 are more than 90° but less than 180°

iv. The angles of the numbers 11 and 12 are exactly 180°

Types of Angles

Right angle: an angle that measures exactly 90°. Therefore, the angles of the numbers 1, 4, 6 and 10 are all right angles.

Acute angle: an angle that measures less than 90°. Therefore, the angles of the numbers 2, 7 and 9 are all acute.

Obtuse angle: an angle that measures more than 90° but less than 180°. Therefore, the angles of the numbers 3,5 and 8 are all obtuse.

Straight angles: an angle that measures exactly 180°. Therefore, the angles of the numbers 11 and 12 are straight.

Reflex Angle: an angle that measures between 180° and 360°.

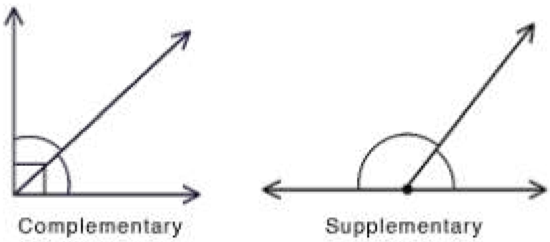
Exploring types of angle pairs.

Review learners' previous knowledge of angles and parallel lines to help explain the types of angle pairs.

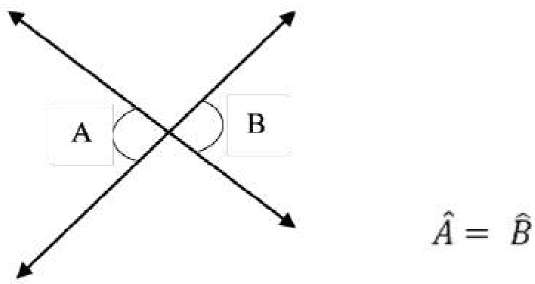
Angle pairs are simply two angles.

Complementary Angles: Two angles are complementary if their sum is exactly 90°.

Supplementary Angles: Two angles are supplementary if their sum is exactly 180°.

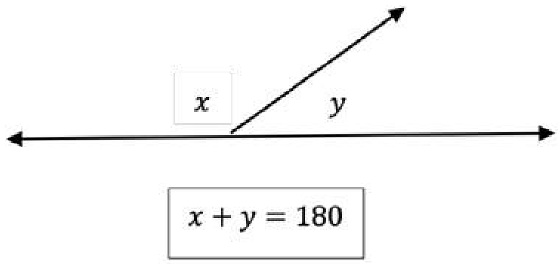


Vertically opposite angles: vertically opposite angles are equal.

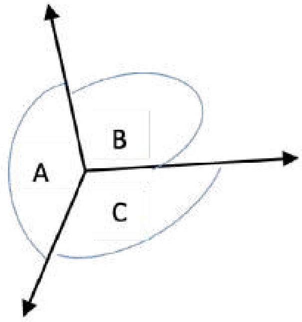


[image 11]

Adjacent angles: Adjacent angles on a straight line add up to 180° .



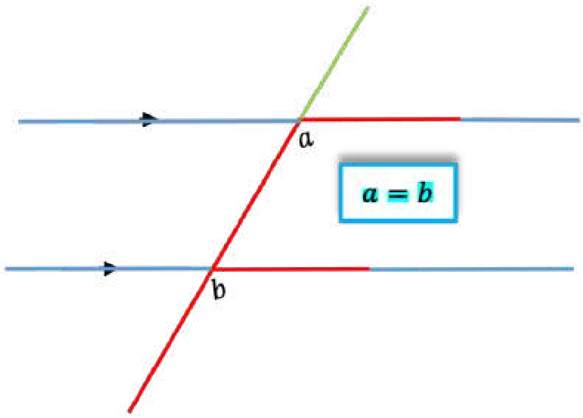
Angles around a point: Angles around a point add up to 360°



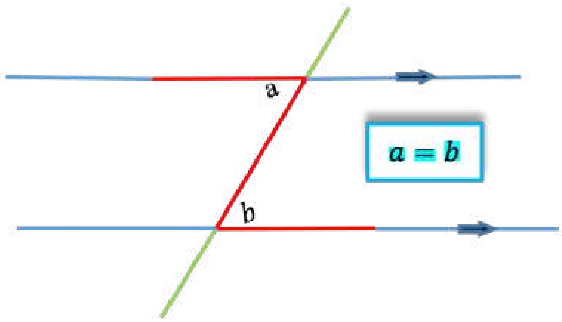
$\hat{A} + \hat{B} + \hat{C} = 360°$

If parallel lines are cut by a transversal, corresponding, alternate and co-interior angle are formed:

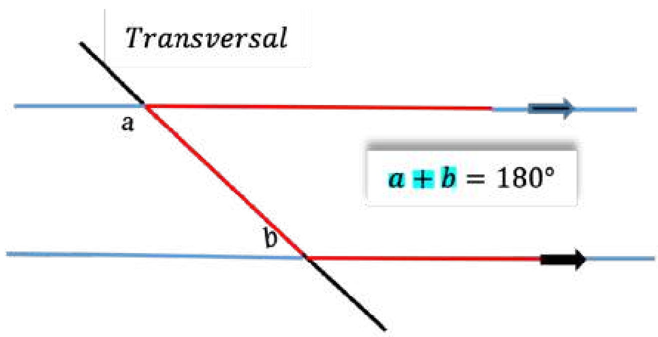
1. Corresponding angles



2. Alternate angles



3. Co-interior angles



If parallel lines are cut by a transversal, the co-interior angles are supplementary.

Learning Tasks for Practice

1. Identify and label angles as acute, obtuse or right angles.
2. Calculate the missing angles in each figure provided.
3. Analyse and justify how to determine the measure of a specific angle given a set of angles within a geometric figure.

Application / importance of angles

Angles are employed in the construction of buildings; they are also interconnected with fields such as physics and chemistry.

Angles have numerous applications in various fields, including mathematics, engineering, architecture, physics, and everyday life.

In our daily routines, we encounter angles when measuring objects, planning layouts, arranging furniture, and navigating through spaces.

They play a significant role in understanding shapes, and spatial relationships and solving real-world problems involving measurement directions.

Pedagogical Exemplars

1. **Initiating Talk for Learning:** In a whole class discussion, review learners' previous knowledge of lines, line segments and rays to introduce angles.
2. **Talk for learning:** in a whole class discussion, draw on the board and engage learners to describe and define angles.
3. **Experiential learning:** In mixed ability/gender groups, engage learners to explore the immediate school environment to investigate examples of angles in the environment.
4. **Collaborative learning:** In small mixed ability/gender groups, encourage learners to explore, describe and ink the properties of the types of angles on the board.
5. **Structuring Talk for learning**: in whole class discussion, through questioning and answering, explain the properties and encourage learners to come out with their definitions of types of lines as an acute, obtuse straight line, etc.
6. **Experiential learning:** ask learners to identify and draw any type of line in the classroom.
7. **Problem-based learning:** provide learners in their mixed-ability gender groups, a variety of angle pairs. Ask learners to think, ink and share the properties of each angle pair using diagrams and guiding questions.
8. **Talk for Learning**: In a whole class, explain and work examples on the angle pair types using their properties.
9. **Experiential learning:** ask learners to create a real-world scenario where knowledge of angles is applied.

Key Assessment

1. Which of the following angles measure exactly 90°
   - **i.** Acute
   - **ii.** Obtuse
   - **iii.** Straight line

iv. Right-angle

2. Given that $(12x - 100)°$ and $(9x + 20)°$ are vertically opposite angles. Calculate:

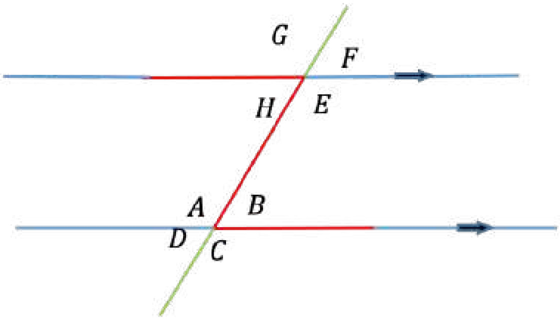
i. the value of x

ii. the value of $(12x - 9x)°$

iii. What is the supplementary angle of $(12x - 9x)°$?

3. Give the complement of 65°.

4. Using the model below



Identify the following angles and justify your answer.

i. Alternate angles

ii. Corresponding angles

iii. Co-interior angles

iv. Complementary angles

Week 9: Measurement of Perimeter

Learning Indicator: *Develop and apply strategies determining the perimeter and area of plane figures*

Focal Area: Perimeter of Plane Figures

Introduction

For these lessons, we will delve into the fundamental concept of perimeter. Perimeter is not just about solving equations but also about exploring the world around us and uncovering hidden patterns and shapes that form part of our everyday life. We will also learn how to apply the knowledge of perimeter to solve real-world problems.

In Module 1, we looked at the perimeter of regular (square, rectangle, triangle, and other polygons) and irregular shapes. In this module, we will explore the perimeter of circles and solve some more word problems on perimeter of 2D shapes as well as problems involving reverse operations.

Perimeter of a circle (Circumference)

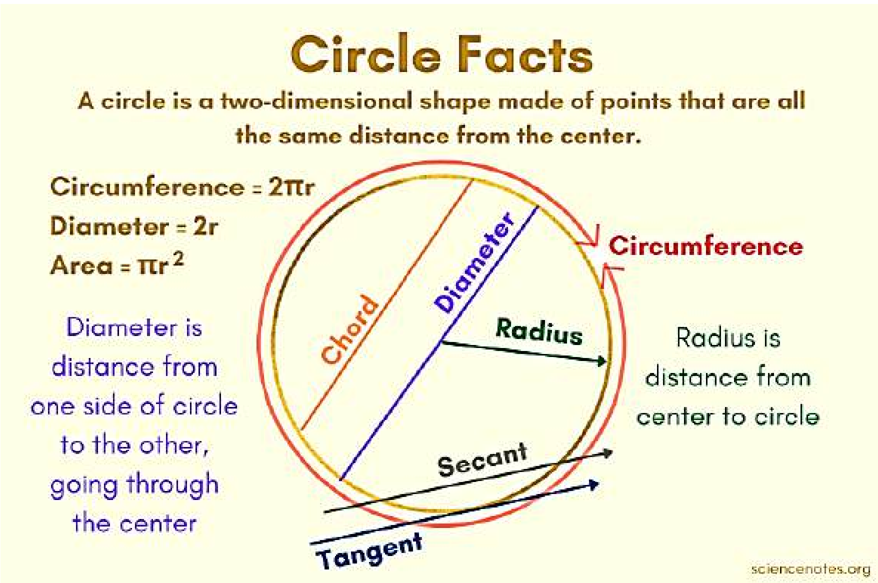
A circle is a two-dimensional shape made of points that are all the same distance from the centre.

Take a look at the diagram of the circle and identify the following:

Circumference: the distance round the circle.

Diameter: any straight line segment that passes through the center of the circle. The diameter divides the circle into two equal halves.

Radius: the distance from the centre to the circle



Establishing the relationship between circumference and the diameter

The relationship between the circumference (C) of a circle and its diameter (D) is one of the fundamental principles in geometry. This relationship is expressed through the mathematical constant known as **pi (π)**, which is approximately equal to 3.14159.

Key Concepts

1. **Circumference (C)**:
   - The circumference is the distance around the edge or boundary of a circle. It can be thought of as the perimeter of the circle.
2. **Diameter (D)**:
   - The diameter is the distance across the circle, passing through the centre. It is twice the length of the radius (r), where the radius is the distance from the centre of the circle to any point on its circumference.
3. **Pi (π)**:
   - Pi (π) is a special mathematical constant representing the ratio of a circle's circumference to its diameter. No matter the size of the circle, this ratio remains constant.

The Relationship

The relationship between the circumference and the diameter of a circle is given by the formula: $C = \pi \times D$

This formula tells us that if you know the diameter of a circle, you can find the circumference by multiplying the diameter by π.

Let's take a look at this practical activity:

1. **Measurement**:
   - Take several circular objects of different sizes, such as lids, hoops, or coins.
   - Use a piece of string to measure the circumference by wrapping it around the object and then measuring the length of the string.
   - Measure the diameter by placing a ruler across the center of the object.
2. **Calculation**:
   - Divide the measured circumference by the diameter for each object.
   - You will notice that for each object, the result is approximately the same value, which is π (about 3.14).

This consistent ratio demonstrates that the circumference of any circle is always π times its diameter, regardless of the circle's size.

Example

A circle has a radius of 3 units.

What is the circle's circumference (perimeter)? *Take* $\pi = \frac{}{7}$

Solution

Circumference of a circle = $2\pi r$

Circumference of this circle = $2 \times \frac{22}{7} \times 3 = \frac{132}{7} = 18.8571$ *units*

Learning Tasks for Practice

1. Find the perimeter of some given plane shapes (including reverse operations) such as circles, squares, rectangles, irregular shapes, etc.
2. Solve word problems involving perimeter of plane shapes.
3. Model real-life situation involving perimeter and solve them.

Application /importance of perimeter of plane figures

The concept of the perimeter of plane shapes, which is essentially the measure of the boundary of a two-dimensional figure, finds extensive applications in various fields of life. That is from the clothes we wear to the houses we live in, and even the sports we play. It can be applied in construction and architecture that is in determining the quantity of materials needed, in the world of sports, textile and fashion industry, mapping and surveying, art and design, agriculture among others.

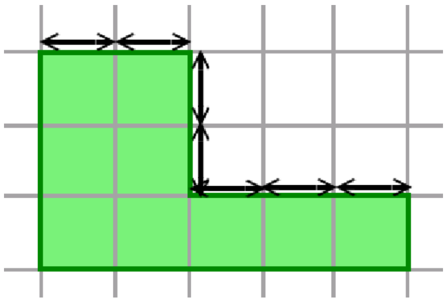
Pedagogical Exemplars

1. **Initiating Talk for Learning:** In a whole class discussion, explore learners' understanding of concepts of shapes and mention the names of some shapes.
2. **Talk for learning:** in a whole class discussion, let learners explain in their own words what perimeter is and its importance.
3. **Collaborative learning:** In small mixed ability/gender groups, ask learners to explore these shapes using hands-on manipulatives. Encourage them to describe the properties they observe with the plane shapes provided.
4. **Experiential learning:** In small mixed ability/gender groups, ask learners to Think- ink-share the values of the perimeter they have after measuring the plane shapes given to them. Discuss their findings with them.
5. **Problem-based learning:** Provide learners with some worksheets on perimeter of plane shapes and guide them to solve the problems.
6. **Talk for learning:** Show learners a chart of a circle and let learners identify the parts of the circle. In whole class discussion, discuss with learners the parts of the circle.
7. **Experiential Learning:** In small mixed ability/gender groups, ask learners to use strings to measure the various circular shapes given to them and write their findings in the table provided. Go round and assist learners have difficulties with the measurement.
8. **Structuring Talk for learning:** In a whole class discussion let learners come out with their observation from measuring the circular shapes. Discuss the idea of pi (π) with learners.
9. **Problem-based learning:** Provide learners with some questions involving perimeter of a circle and let learners in their small mixed/ gender groups, Think-ink-share their findings. In a whole class discuss learners' findings with them.

Key Assessment

1. What is a perimeter?
2. If a square has a side of length 5 units, what is the perimeter?
3. What is the perimeter of a rectangle, if its length and breadth are 36m and 24m, respectively?
4. Find the perimeter of a circle whose diameter is 14 cm. [Note: Take $\pi = 3.14$]
5. Find the breadth of the rectangular field, if its perimeter is 240 m and its length is 90 m.
6. Determine the perimeter of a parallelogram, whose adjacent sides are 4 cm and 7 cm.

7. Find the perimeter of the plane shape drawn in cm on the grid paper below.



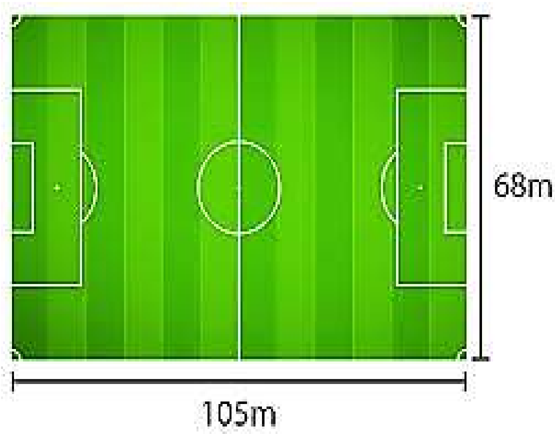
8. Draw a rectangle with a length of 7 cm and a width of 3 cm. What is the perimeter of the rectangle?

9. Calculate the perimeter of a triangle with sides measuring 6 cm, 8 cm, and 10 cm.

10. A rectangular garden has a length that is twice its width. If the perimeter of the garden is 36 meters, what are the dimensions of the garden?

11. A regular hexagon and a square have the same perimeter. If each side of the square measures 8 cm, what is the length of each side of the hexagon?

12. Find the perimeter of the football field provided below



13. A city planner has been tasked to design a new rectangular park. The city has allocated 60m of fencing for the park. What dimensions should the planner use for the park, to maximize the area?

14. A circular garden has a circumference of 100 meters. What is its radius.

15. A farmer wants to enclose a rectangular field with a perimeter of 300 meters using fencing material that costs GH¢6.00 per meter. How much will the fencing cost the farmer?

16. A triangular park has a perimeter of 120 meters. One side measures 40 meters and another measures 50 metres.

What is the length of the other side?

Focal Area: Area of Plane Figures (Triangle and Circle)

Introduction

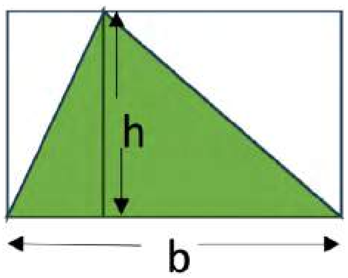
Perimeter, area, and volume are essential concepts in geometry that help us quantify and compare the sizes of two-dimensional and three-dimensional shapes. Perimeter refers to the distance around the boundary of a two-dimensional shape, while area represents the space enclosed by the shape. Volume, on the other hand, is the amount of space occupied by a three-dimensional object. Understanding these concepts is essential for solving real-world problems involving measurement and spatial reasoning. Mastery of perimeter and area allows us to perform real-life activities such as fencing required for a garden or the amount of paint needed to cover a wall.

Area

Area is the measure of the space enclosed by a 2D shape. Area is measured in square units.

Using formula for determining area of triangle and circle.

Area of a triangle



Area of triangle is $A = \frac{1}{2}b \times h$

Where b = base, h is the height

If b = 10 cm and h = 8cm

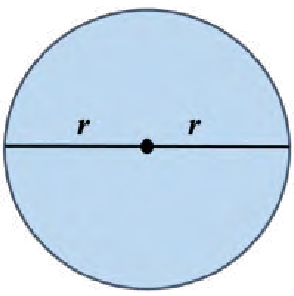
$A = \frac{1}{2}(10) \times 8$

$A = 5 \times 8$

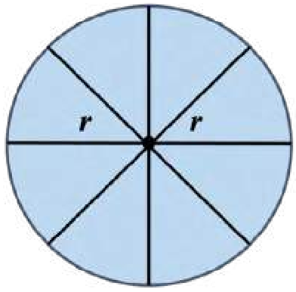
$A = 40\ cm^2$

Area of a circle

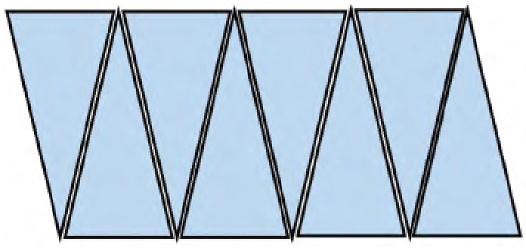
Draw a circle on a card, of radius *r,* and divide it into two equal parts as shown below.



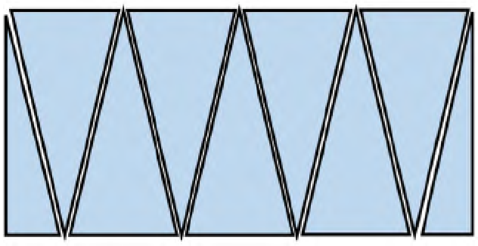
Divide the circle into n equal sectors. The more sectors you use, the closer the approximation to the actual area. This activity makes use of 8 sectors



Each sector will almost form a triangle. Then rearrange the ‘triangles’ so they look like this:

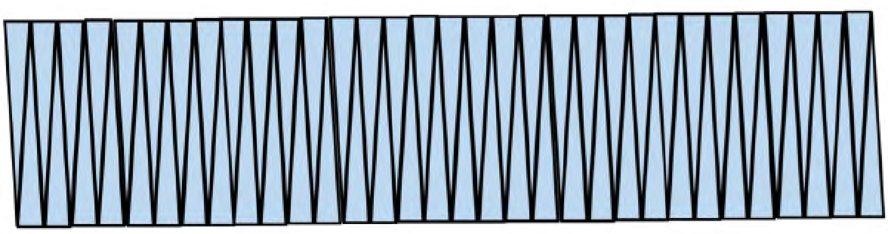


The shape looks like a parallelogram. Shape it into a rectangle by cutting part of the end ‘triangle’ to fill the slant edge so it looks like this:



When dividing the circle into n equal sectors, each sector forms a triangle when rearranged. As n increases, the angle of each sector decreases, making the triangle’s base (length of the sector’s arc) approach very small values and height approaches the radius of the circle. As the number of sectors (n) increases, the approximation of the circle’s area becomes more accurate. In the formula for the area of the circle, π represents the ratio of the circumference of the circle to its diameter.

As n approaches infinity, the sectors become very small, and the resulting shape closely resembles a rectangle. This concept illustrates the fundamental relationship between the circumference and area of a circle, providing a geometric interpretation of π



The role of π becomes apparent in this context. It represents the ratio of the circumference of a circle to its diameter, a fundamental constant in geometry. As such, when calculating the area of a circle using sectors to form a rectangle, π emerges naturally as a factor in the formula.

The height of each triangle is the radius of the circle, and the base of the triangle is equal to the length of the sector’s arc.

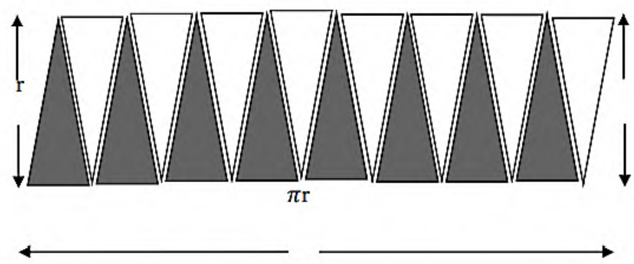
The area of each triangle is given by the formula $\frac{1}{2}$ × **base** × **height**

The total area of all the triangles (sectors) will approximate the area of the circle.

Since the shape formed by the rearranged sectors is a rectangle, the area of the rectangle will be the same as the total area of the sectors.

Using the area of a rectangle, L x B as a reference, arrange the sectors into a rectangle.

With the L as πr and the breadth/width as r, the area of the circle can therefore be deduced as πr^2



Area = $r \times \frac{1}{2}$ circumference

But circumference = $2\pi r$

i.e. Area = $r \times \frac{1}{2} 2\pi r$

$A = \pi r^2$

But $d = 2r$

$r = \frac{d}{2}$

Therefore, the area of a circle given the diameter, d

$$A = \pi\left(\frac{d}{2}\right)^2$$

$$A = \frac{\pi d^2}{4}$$

Application/ Importance of area of plane figures

Understanding the concepts of perimeter and area is essential for solving real-world problems involving measurement and spatial reasoning. Mastery of perimeter and area allows us to perform real-life activities such as fencing required for a garden or the amount of paint needed to cover a wall.

Learning Tasks for Practice

1. Measure the perimeter of various polygons drawn on graph paper.
2. Calculate the area of simple polygons using the formula: $Area = base \times height$.
3. Solve word problems involving the perimeter and area of plane figures.
4. Explore real-life scenarios, such as fencing a garden or painting a wall, and determine the required perimeter or area.
5. Collaborate in groups to design blueprints for houses or rooms, incorporating calculations for perimeter and area.

Pedagogical Exemplars

1. **Experiential learning:** In mixed-gender/ability groups, engage learners to explore the immediate school environment to investigate referents for the measuring various items. Examples could include referent for a centimetre, metre, kilometre, etc. Use their experiences from their investigations on referents for linear and area measurements to estimate the perimeter or area of a given object.

   Note: This activity should focus on learners' ability to estimate and not necessarily getting absolute figures. Encourage shy learners to share their findings from the investigations.

2. **Problem-based learning:** In small groups, engage learners to investigate the area of 2-D shapes (both regular and irregular) and estimate the perimeter using geodot.

3. **Experiential learning:** In convenient mixed-ability groups, task learners to select items within the classroom environment and determine the area and perimeter of the shapes in the objects. Learners are to make presentations using the criteria; kind of shape in the object, regular or irregular, strategy for measuring [formulas, graphical, etc.]. Alternatively, this activity, including that of the investigation of referents, could be given to learners ahead of the lesson schedule. This is to help free some time for teaching the rest of the concepts.

4. **Group & pair activities:** Using think-pair- share, task learners to solve problems, using formulas for determining the perimeter/area of regular and irregular 2-D shapes, including circles. Learners may use digital mathematics tools where applicable and available.

   Using mixed-ability groups, engage learners to write a given perimeter/area measurement expressed in one SI/imperial unit in another SI/ imperial unit.

5. **Problem-based group learning:** Using mixed-ability groups, present learners with task sheets on perimeter and area, including word/real-life problems to solve.

6. **Whole Class discussions and demonstrations:** Lead the class to discuss the main ideas of the lesson and take the opportunity to demonstrate [or learners volunteer to demonstrate] challenging concepts, including resolving all misconceptions.

7. **Individual tasks:** Present learners with individual worksheets to complete. Alternatively, allow learners to take home the tasks for later submission.

Key Assessment

Solve the following problems:

1. A rectangular field has a length of 18 meters and a width of 10 meters.

   Calculate the area of the field.

2. A circular swimming pool has a radius of 6 meters. Find the area of the pool.

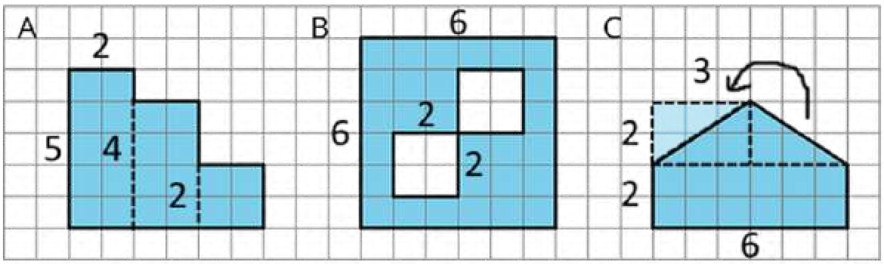
3. Akrofi has a triangular goat pen with a perimeter of 12 ft. He made it bigger by doubling the length of each of the three sides. What is the perimeter now?

4. Adomakoa has a rectangular table with an area of $2m^2$. She is going to buy another table that has dimensions that are double the first table. What is the area of Adomakoa's new table?

5. A small pizza at Eddy's Pizza has an area of 29 in^2. A large pizza has a radius that is triple the radius of a small pizza.

   What is the area of a large pizza?

6. Investigate the perimeter and area of the following shapes. Take a square on the grid as 1cm by 1cm.



Note: for figure B, two small squares have been cut out.

Week 10 & 11: Measurement of Surface Area and Volume of Prisms

Learning Indicator: *Develop and apply strategies for determining the surface area and volume of prisms.*

Focal Area: Surface Area and Volume of Prisms

Introduction

A prism is a polyhedron in which all the faces are flat, and the bases are parallel to each other. It is a solid object with flat faces, identical ends, and the same cross-section along with its length. The area and volume of prisms are fundamental concepts in geometry and are essential for understanding three-dimensional shapes.

The area of a prism refers to the total surface area that covers all its faces, while the volume represents the space enclosed within the prism.

To calculate the surface area of a prism:

i. Find the area of each face (including the bases and lateral faces).

ii. Sum the areas of all the faces to get the total surface area.

The formula for the total surface area of a prism depends on its shape.

To calculate the volume of a prism:

i. Find the area of the base (often a polygon) by using the appropriate formula.

ii. Multiply the area of the base by the height of the prism.

Mathematically, it is defined as the product of the area of the base and the height.

Therefore,

Volume of a Prism* = *Base Area* × *Height

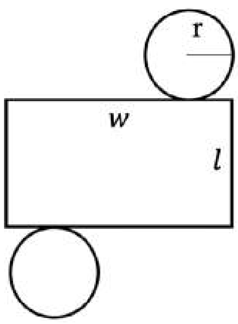
Surface area of a cylinder

To determine the surface area of a right cylinder, the shapes that make up the cylinder must be known.

If you look at the net of a right cylinder, you will find that the shapes of the right cylinder are two circles (if there is a top and a bottom) and a rectangle.

Example:

A possible net of a right cylinder looks like this:



Area of rectangle

$= Length \times Width\ (l \times w)$

Area of circle

$= \pi r^2$

Surface area of cylinder = Area of rectangle + Area of the 2 circles = $(l \times w) + 2(\pi r^2)$

Surface area is measured in square units, written as cm^2, m^2, etc.

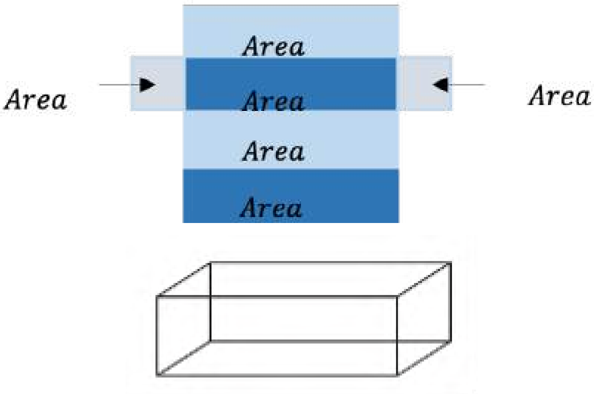
Note: The width of the rectangle is the same as the circumference of the circle.

Surface area of a rectangular prism (cuboid)

To determine the surface area of a rectangular prism, the shapes that make up the rectangular prism must be known. If you look at the net of a rectangular prism, you will find that the shapes of the rectangular prism are six rectangles, with opposite sides of the boxes the same.

Example

A possible net of a rectangular prism looks like this:



Area of each rectangle

$= Length \times Width\ (l \times w)$

To determine the surface area of the rectangular prism (Area A = Area C, Area B = Area D, Area E = Area F), you need to determine the areas of all six rectangles.

Since opposite sides are equal, you only have to calculate the area of three rectangles, double each area, and add them. Surface Area of Rectangular Prism = 2(Area A) + 2(Area B) + 2(Area E) = $2(L_A + W_A) + 2(L_B + W_B) + 2(L_E + W_E)$

Note: L_A is the length of rectangle A, while L_B is the length of rectangle B. The length of rectangle A may or may not be the same as the length of rectangle B. Learners need to be careful to use the correct dimensions to find each area. They are not expected to use the notation L_A.

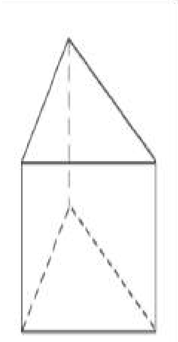
Surface Area of a Triangular Prism

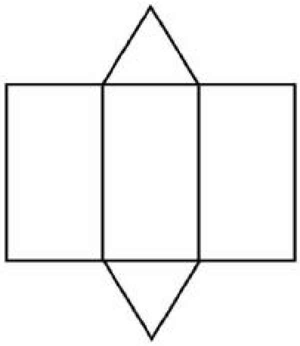
To determine the surface area of a triangular prism, the shapes that make up the triangular prism must be known.

If you look at the net of a right triangular prism, you will find that the shapes of the right triangular prism are three rectangles and two triangles, with the opposite triangles being the same size.

Example

A possible net of a right triangular prism looks like this:





The rectangles may or may not be the same size, depending on the type of triangle the base is made from.

Area of each rectangle = $l \times w$

Area of triangle = $\frac{b \times h}{2}$

To determine the surface area of the triangular prism, you need to determine the area of the two triangles and the area of the three rectangles. You may be able to combine some areas if they contain the same measurements.

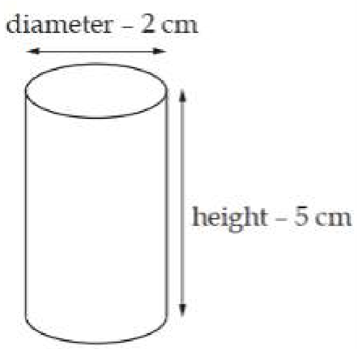
A general formula for determining the surface area of a right triangular prism is as follows:

Surface Area of a Triangular Prism = (area of rectangle 1) + (area of rectangle 2) + (area of rectangle 3) + 2(area of triangle)

$= (l \times w) + (l \times w) + (l \times w) + \frac{b \times h}{2} + \frac{b \times h}{2}$

Volume of a cylinder

The volume of a cylinder is determined by multiplying the area of the base by the height of the cylinder.



Area of base = πr^2

Radius (r) = 1cm

Area = $\pi(1cm)^2$

Area =3.14 cm^2

Volume = Area of base × height

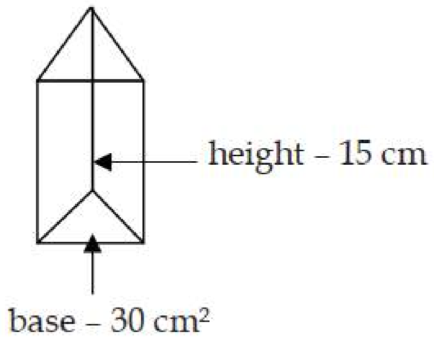
= 3.14 cm^2 × 5 cm

= 15.7cm^3

Volume of a triangular prism

The volume of a triangular prism is determined by multiplying the area of the base of the triangular prism by the height of the triangular prism.

Example

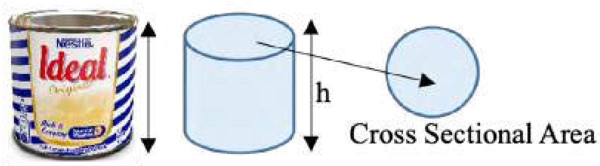


Volume of a triangular prism = area of base × height

$=30cm^2 \times 15cm = 450cm^3$

Example

The figure below is a milk tin with height 15cm and the diameter of the cross section is 10cm. Calculate the volume of the tin.



Solution

To find the volume of the milk tin, we use the formula for the volume of the prism (cylinder)

Volume = Base Area (cross section) × Height

Volume = $\pi r^2 \times$ h

Where r is the radius of the circular cross-section and h is the height of the cylinder.

Given that the diameter is 10 cm, the radius r is half of that,

so $r = \frac{10}{2} = 5$cm

Substituting the given dimensions:

$V = \pi \times (5\,cm)^2 \times 15$ cm

$V = \pi \times 25\,cm^2 \times 15$ cm

$V = 375\pi\,cm^3$

But $\pi = \frac{22}{7}$

$V = 375 \times \frac{22}{7}\,cm^3$

So, the volume of the milk tin is $375\pi\,cm^3$, which is approximately $1178.1\,cm^3$ when rounded to one decimal place.

Learning Tasks for Practice

1. Calculate the surface area of different prisms.
2. Determine the volume of rectangular prisms by multiplying the length, width, and height:
3. Solve word problems involving the area and volume of prisms, such as finding the amount of paint needed to cover a rectangular box.
4. Explore real-life scenarios where prisms are used, like packaging boxes, and calculate their surface area and volume.
5. Create models of prisms using building blocks or other materials and measure their surface area and volume as a hands-on activity.

Application / Importance of area and volume of solid figures.

Transportation and Logistics: Transportation companies use volume calculations to optimise cargo space in vehicles and containers. Understanding the volume of shipping containers, trucks, and warehouses helps maximize efficiency in loading, storage, and transportation of goods.

Construction and Architecture: Architects and engineers use calculations of area and volume to design buildings, bridges, and other structures. Understanding the volume of concrete needed for a foundation or the surface area of walls helps ensure accurate construction estimates and efficient material usage.

Pedagogical Exemplars

1. **Experiential learning**:
   a. Provide physical models or manipulatives of prisms, such as wooden blocks or plastic shapes, that learners can touch, feel, and manipulate. Allow them to build prisms and visually explore their properties, including faces, edges, and vertices.
   b. Use visual aids such as diagrams, illustrations, or pictorial representations to illustrate the concepts of area and volume in prisms. Break down complex concepts into simpler, step-by-step visual explanations that highlight key features and relationships.
   c. Engage learners in hands-on activities where they can directly measure and compare the dimensions of different prisms. Provide worksheets or guided practice exercises that involve counting squares to find the area of faces or using cubic units to calculate volume.
2. **Problem based learning**: Relate the concepts of area and volume to real-world contexts that are familiar to learners, such as measuring the volume of water in a fish tank or calculating the surface area of a gift box. Encourage learners to make connections between mathematical concepts and everyday experiences.
3. **Collaborative learning:** Foster collaborative learning environments where learners can work together in pairs or small groups to solve problems and discuss strategies. Encourage peer teaching and peer support, allowing learners to learn from each other and share their insights and approaches.
4. Differentiate instructions to meet the diverse needs of struggling learners. Provide additional support through one-on-one assistance, small-group instruction, or targeted interventions that address specific areas of difficulty in understanding area and volume concepts.

Key Assessment

1. Given a rectangular prism has dimensions 4 cm × 6 cm × 8 cm, calculate its surface area.

2. A cylindrical water tank has a radius of 5 metres and a height of 10 metres.

   Calculate the volume of water the tank can hold.

3. A triangular prism has a base with sides of length 6 cm, 8 cm, and 10 cm, and a height of 12 cm.

   Calculate the surface area of the prism and then determine the volume if it is filled with water up to half its height.

Section Review

In this section, learners delved into important geometric concepts and their practical applications, focusing on:

- **Angle Relationships and Lines:** Identifying and applying parallel, perpendicular, complementary, supplementary angles, vertical angles, and parallel lines cut by a transversal in various contexts.
- **Perimeter and Area:** Developing and applying strategies for calculating the perimeter and area of plane figures, including circles.
- **Surface Area and Volume:** Learning and applying methods to determine the surface area and volume of prisms.

These concepts are integral to understanding and solving real-world problems, particularly in fields like construction, design, and engineering, where precise measurements and spatial reasoning are crucial.

SECTION 5: PROBABILITY OF EVENTS

Strand: **Collecting and Handling Data**

Sub-Strand: Probability

Content Standard: *Describe the likelihood of a single outcome occurring using words such as impossible, possible, and certain*

INTRODUCTORY SUMMARY

Understanding probability is essential in analysing and predicting outcomes in everyday life. In this section, learners will explore how to classify various situations as impossible, possible (likely or unlikely), or certain. By engaging with real-life scenarios, they will develop an intuition for assessing the likelihood of different events. Additionally, learners will design and conduct experiments to test the probability of specific outcomes. Through repeated trials, they will record results and interpret the data to draw meaningful conclusions. This exploration of probability not only builds critical thinking skills but also provides a foundation for understanding more complex statistical concepts in the future.

The session will cover the following focal areas:

1. *Classify the occurrence of everyday life situations as impossible, possible, or certain*
2. *Design and conduct an experiment in which the likelihood of a single outcome occurring is impossible, possible* (*likely or unlikely*), *certain.*
3. *Conduct a given probability experiment a number of times, recording the outcomes, and explaining the results.*

PEDAGOGICAL SUMMARY

To effectively teach these probability concepts, a hands-on, experiential learning approach will be emphasised. Learners will begin by discussing and identifying events in their daily lives that are impossible, possible, or certain. This will be followed by guided activities where they design simple experiments to observe these outcomes. Using manipulatives like dice, coins, or spinners, learners will conduct experiments multiple times, record outcomes, and analyse the data to understand probability patterns. Collaborative group work will encourage peer discussion and deeper understanding, while visual aids such as probability trees or charts will help in illustrating concepts. Real-life examples will be used to connect these ideas to situations outside the classroom, enhancing their relevance and applicability.

ASSESSMENT SUMMARY

Assessment of these concepts will involve a combination of formative and summative approaches. Learners will be evaluated on their ability to classify situations correctly based on their likelihood. This will include classroom discussions, written exercises, and real-time observation during experiments. Practical assessments will focus on the design and execution of probability experiments, with emphasis on accuracy in recording outcomes and interpreting results. Group projects may also be utilised to assess collaborative skills and the ability to explain concepts to peers. Quizzes and tests will include questions on the classification of events, as well as problem-solving tasks requiring the application of probability to predict outcomes.

Week 12: Probability

Learning Indicators

1. *Classify the occurrence of everyday life situations as impossible, possible, or certain*
2. *Design and conduct an experiment in which the likelihood of a single outcome occurring is impossible, possible (likely or unlikely), certain.*
3. *Conduct a given probability experiment a number of times, recording the outcomes and explaining the results.*

Focal Area: Describing the Likelihood of a Single Outcome

There are everyday life situations which we cannot tell their occurrence with certainty. Clouds form and we may hope to have rain but sometimes it does not. At childbirth, the child which comes out may be a boy or a girl. In all these cases, we can only guess or predict which likely situations may occur. The value which we assign to the possibility of an event is the **probability.**

Conducting experiments and classifying events as likely or unlikely or certain

An activity whose outcome cannot be predicted with certainty can be described as an experiment. To start a game a coin may be tossed to determine how the game is to start. The umpire/referee will be conducting an experiment. Similarly, during play, we throw a ludo dice wishing for a certain number, but this is frequently not the number we roll. In all these activities, we can say that we are conducting an experiment.





During a ludo game, a single throw is termed a **trial**. Each trial can reveal; 1, 2, 3, 4, 5 or 6. These numbers form the **possible outcomes**. The successful outcome (the number that actually shows up) is our event. The set of all possible outcomes is termed **sample space.**

Example

1. A 20 pesewas coin is tossed once and the outcome observed as the "cocoa" turned up.

   a. What are the possible outcomes?

   b. Write the sample space

   c. What is the event?

   d. Can both cocoa and coat of arms show up at the same time? Why?

   e. Describe such an event

Solution

1.
   a. Possible outcomes = cocoa, coat of arms
   b. Write the sample space = {cocoa, coat of arms}
   c. The event = {cocoa}
   d. Both cocoa and coat of arms cannot show up at the same time because the face turns flat to show only one face at a time
   e. For both cocoa and coat of arms to show up is an impossible situation

2. A game involves selecting pebbles numbered 0 to 10 and summing them to reach 50.

   One pebble is selected at time in turns and observed. The first player to get a sum of 50, wins the game.

   a. What is the sample space for a single selection?
   b. Describe these events as certain, possible, impossible
      i. A pebble numbered '7' will be selected
      ii. A pebble numbered '12' will be selected
      iii. A pebble numbered 1 to 10 will be selected
      iv. A pebble numbered with prime number will be selected
      v. A pebble numbered with a negative number will be selected

Solution

2.
   a. The sample space = **{0,1,2,3,4,5,6,7,8,9,10}**
   b. Describing the events as sure, possible, impossible
      i. A pebble numbered '7' will be selected = **possible event**
      ii. A pebble numbered '12' will be selected = **impossible event**
      iii. A pebble numbered 1 to 10 will be selected = **certain event**
      iv. A pebble numbered with prime number will be selected = **possible event**
      v. A pebble numbered with a negative number will be selected = **impossible event**

3. A bag contains 12 red toffees, 8 green toffees and 16 yellow toffees. One toffee is selected at random.

   Give a reason why you think the event is impossible, certain or possible

   a. The toffee is red or green or yellow.
   b. The toffee is yellow
   c. The toffee will be both red and green
   d. The toffee is blue

Solution

3.

a. It is **certain** that the toffee being red or green or yellow because they are the only colours of toffees in the bag.

b. It is **possible** because there are yellow toffees in the bag.

c. **Impossible** because only one toffee was picked and there is no toffee which has two colours.

d. **Impossible** because there are no blue toffees in the bag

Learning Task for Practice

Learners in small groups, conduct experiments, describe the events as possible (likely), impossible (unlikely) or certain (sure)

Pedagogical Exemplars

1. **Experiential learning:** In small groups, engage learners to throw a ludo die, toss a coin and select a card from a pack. Encourage learners to allow all members to take turns in the experiment.
2. **Experiential learning:** In small groups, engage learners to record all possible occurrences as the outcomes
3. **Talk for learning:** Let learners describe outcomes as possible, impossible or certain.

Focal Area: Computing Probabilities

The chance that a baby being born will be a boy exist. It is a possible outcome. **How much** possibility exists for this to occur?

The "**How much"** is a measure or an amount. The chance of an event occurring can be given a numerical value.

By this we say probability is a measure assigned to the possibility that an event will occur.

Calculating probability of single experiment conducted once

To calculate probability of an event:

a. We identify the sample space count to identify the sample size as $n(P)$

b. We identify the event and count to determine the number of successful outcomes as $n(E)$

c. Describe the chance /probability as the number of successful outcomes out of the total number of elements in the sample space

d. Calculate the ***ratio of number of successful outcomes to the total number of elements in the sample space.***

That is $P(E) = \frac{n(E)}{n(S)}$

Example

1. A ludo dice is thrown once and the outcome observed. Find the probability of

   i. A '5' showing up

   ii. An even number showing up

iii. A number greater than 4 showing up

Solution

The sample space (S) = {1,2,3,4,5,6}

Sample size [n(S)] = 6

i. Event '5' showing up

n(E) = 1, ie. There is only one element in the sample space which is 5

$P(E) = \frac{n(E)}{n(S)}$

P(5 shows up) = $\frac{1}{6}$

ii. Event even number shows up = {2,4,6} n(E) = 3

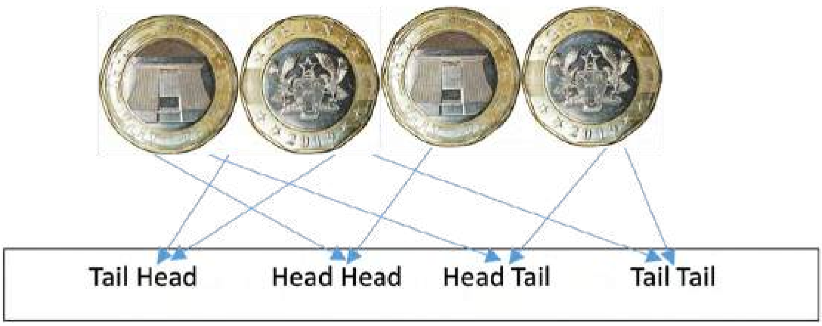
$P(E) = \frac{3}{6} = \frac{1}{2}$

iii. Event (number greater than 4 shows up) = {5,6} n(E) = 2

$P(E) = \frac{2}{6} = \frac{1}{3}$

Calculating probability of experiments conducted multiple times

A 'Zim Zim' game is played with 2 similar coins or flat objects. The 2 coins are thrown at the same time and the faces that show are recorded. After 20 throws, the player with more similar faces showing up in each throw wins the game.



If we represent the faces of the coin as 'Head' (H) and 'Tail' (T), then the possible outcomes will be (HH), (HT), (TH) and (TT).

Example

A coin is thrown twice and the outcome observed.

a. Write the sample space.

b. Calculate the probability that:

i. two Heads are thrown

ii. different faces are thrown.

Solution

a. Sample Space = {(HH), (HT), (TH), (TT)}

b.

i. Sample size = 4

Event, 2 Heads show up = {HH}

Hence, n(E) = 1

$P(E) = \frac{1}{4}$

ii. Event, different faces show up = {(HT), (TH)}

Hence, n(E) = 2

$P(E) = \frac{2}{4} = \frac{1}{2}$

Learning Task for Practice

Learners in small groups and individually, conduct experiments, describe the events and determine the probability of events.

Pedagogical Exemplars

1. **Talk for learning:** Encourage learners to describe probability of an event of an experiment conducted once
2. **Experiential learning:** In small groups, engage learners to throw a coin, throw a die two times or select 2 balls from a bag of coloured balls one at a time and describe the outcomes.
3. **Talk for learning:** Encourage learners to describe orally, the successful outcomes of experiments conducted multiple times.

Key Assessment

1. Which of these words describe '**Probability**'?

   Tick [√] as many as found correct.

   a. Chance [ ]

   b. Possibility [ ]

   c. Problems [ ]

   d. Likelihood [ ]

2. Write an event in school that can be considered *certain*, another which is *impossible*, and finally an event which is *possible*.

   a. Certain ……………………………………………………………….

   b. Impossible …………………………………………………………...

   c. Possible ……………………………………………………………….

3. A ludo dice with faces (1,2,3,4,5,6) is tossed once and the outcome observed.

   a. Write the sample space.

   b. What is the sample size?

   c. Write the event that a prime number shows up.

   d. Describe (in words) the probability that a prime number shows up.

4. Five **(5)** red bottle cups, **4** blue bottle cups and **1** green bottle cup are put in a box. Two bottle cups are selected without looking into the box. What is the probability that they are both blue?

5. A 'Kwasasa' game is played with 2 dice thrown at the same time. Complete the table of sample space with the appropriate outcomes (coloured spaces).

Dice 2 Dice 1

1,1	2,1	3,1	4,2	5,1	6,1
1,2	2,2	3,2	4,3	5,3	6,2
1,3	2,4	3,3	4,4	5,4	6,3
1,4	2,5	3,5	4,5	5,5	6,4
1,6	2,6	3,6	4,6	5,6	6,5

Section Review

In this section, we explored the basic concepts in probability, focusing on classifying events based on their likelihood as impossible, possible, or certain. Learners engaged in practical activities to design and conduct experiments, observing the occurrence of different outcomes. Key takeaways include:

- **Classification of Events:** Understanding and identifying everyday situations as impossible, possible, or certain.
- **Experiment Design:** Creating and executing experiments to test the likelihood of outcomes, such as flipping coins or rolling dice.
- **Data Analysis:** Recording and analysing experimental outcomes to determine probability patterns and explaining the results.