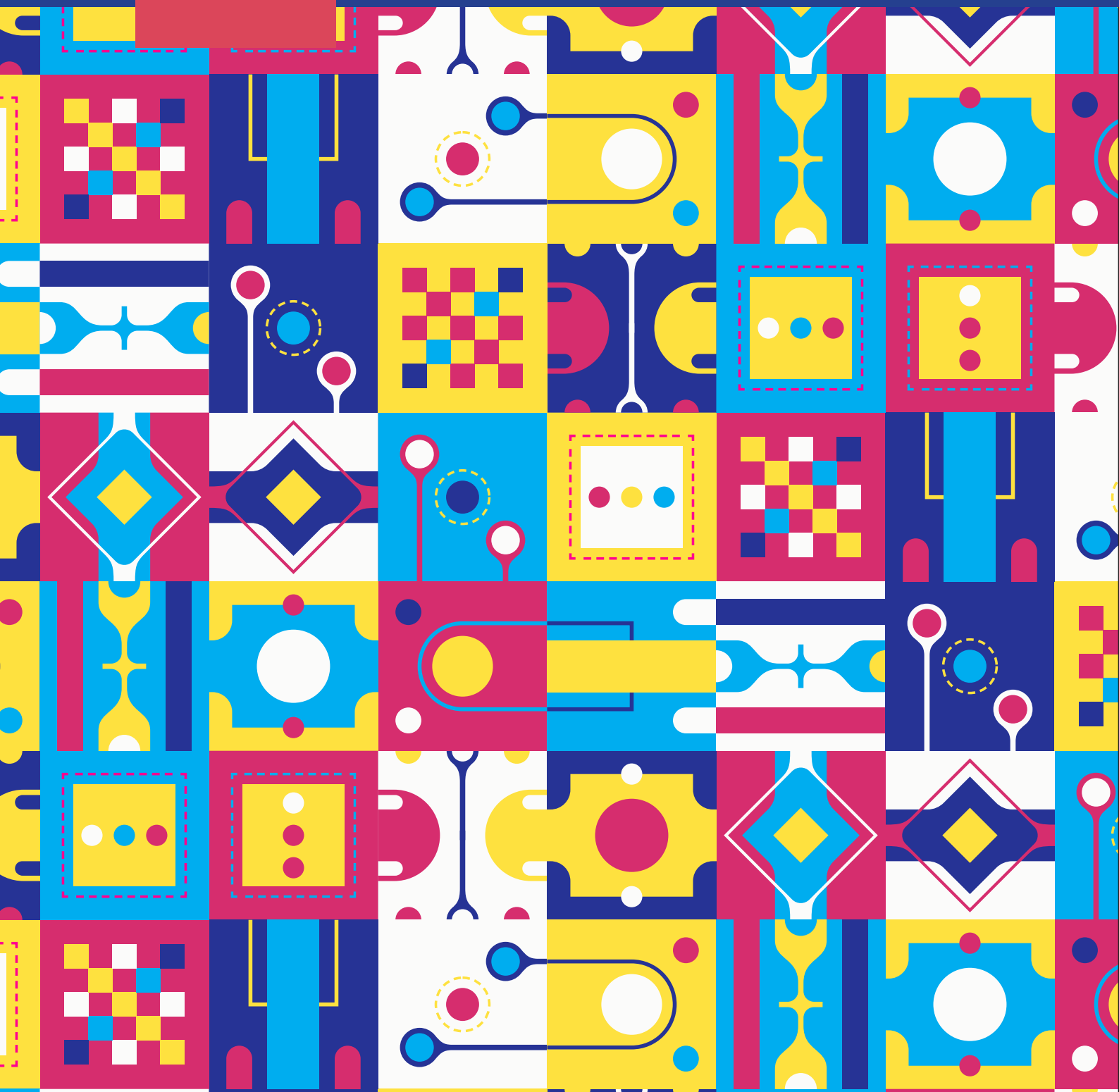
SECTION

3

# REASONING WITH ALGEBRA



# ALGEBRA

## Patterns and Relations; Algebraic Expressions; Variables And Equations

## INTRODUCTION

In this section, you will learn about patterns and algebraic expressions, developing the ability to represent and extend patterns visually and describe the mathematical rules that govern them. By understanding how each element in a pattern differs from the preceding one, you will gain the skill to predict future elements, an important ability for solving problems that involve sequences and trends. You will apply these skills to real-life situations, modelling them as mathematical statements that can be analysed and manipulated to find solutions. Furthermore, you will perform basic operations—addition, subtraction and multiplication—on algebraic expressions, which form the foundation for more advanced mathematical concepts. These skills are essential for making sense of regularities and structures in both everyday life, helping you to think logically and systematically when approaching complex problems.

**In this section, you will learn to;**

1. Represent and extend a given pattern visually and explain how each element differs from the preceding one.
2. Describe, orally or in writing, a given pattern (rule), using mathematical language and predict subsequent elements in the pattern.
3. Solve a given problem (including tables/charts) using a pattern rule to determine subsequent elements (predictions).
4. Model real-life situations as mathematical statements
5. Perform basic operations (addition, subtraction and multiplication) on algebraic expressions.

# EXTENDING PATTERNS VISUALLY

## Focal Area: Represent and Extend A Given Pattern Visually

Patterns are all around us in our daily lives and recognising them helps us understand and predict how things work. Imagine you are decorating a room with tiles. The tiles have a repeating design: one red tile followed by two blue tiles, and this sequence continues. If you want to tile the entire floor, you need to figure out how many red and blue tiles you will need and how the pattern will look as it extends across the room.

By understanding how to represent and extend patterns visually, you cannot only complete your tile design accurately but also create beautiful, consistent designs without making mistakes. Learning to spot and continue patterns like this is useful in many areas, from art and design to solving everyday problems. This concept is essential because it helps us plan, organise, and predict outcomes based on the repeating structures we see around us. Before we explore this concept further, let's go through this reinforcement activity!

### Reinforcement Activities

**Purpose:** To help learners develop an understanding of patterns by recognizing and creating simple patterns using everyday objects.

**Materials Needed:**

- Coloured beads or buttons
- Paper and coloured pencils/crayons
- Small objects like coins, pebbles, or leaves

**Instructions:**

**Step 1: Introduction to Patterns**

1. **Group Activity:** Be in small groups of 3-4 learners.

2. **Discussion:** Start by looking around the classroom and identify any patterns you see (e.g., tiles on the floor, stripes on clothing, sequence of books on a shelf).
3. **Sharing:** Each group should share the patterns they observed with the class.

**Step 2: Creating Patterns with Objects**

1. **Pattern Creation:** Each group will have a set of coloured beads or buttons. Now, create a simple pattern using these objects (e.g., red bead, blue bead, red bead, blue bead).
2. **Extension:** Once you've created the pattern, extend it by adding more beads while following the same sequence.
3. **Sharing:** Each group will then share their pattern with the class and explain how they extended it.

**Step 3: Drawing Patterns**

1. **Visual Representation:** Each learner should have a paper and coloured pencils/crayons. Draw the pattern you created with the beads/buttons.
2. **Challenge:** Create a new pattern using colours, shapes, or numbers and extend it visually.

**Step 4: Exploring Patterns in Nature**

1. **Outdoor Activity:** Go outside the classroom and collect small objects like leaves, pebbles, or twigs. Use these objects to create a pattern on the ground.
2. **Extension and Discussion:** After creating the pattern, extend it and discuss how patterns are present in nature (e.g., the arrangement of leaves on a stem or the markings on an animal).

***Wrap-Up Discussion:***

Come back to the classroom and discuss the importance of recognising and extending patterns. Explain how these skills will help you in the next lesson on "Representing and Extending a Given Pattern Visually."

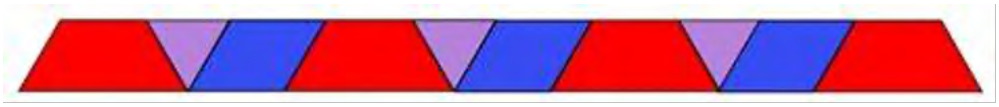
***Expected Outcome:***

By the end of this activity, you should be more comfortable with identifying, creating and extending patterns.

# What is a pattern?

A pattern is a repeated or regular arrangement of numbers, shapes, or objects. They follow a specific rule or set of rules that determine the order and structure of the elements.

## Examples of patterns

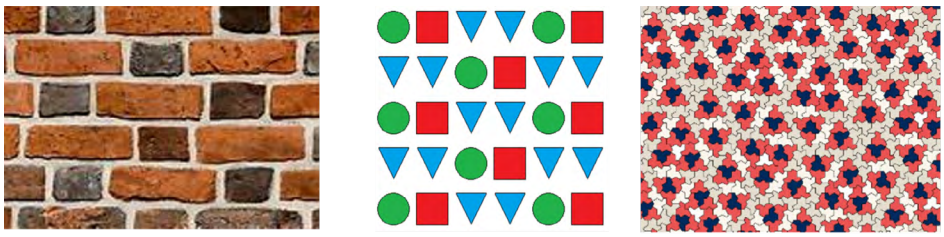


1. Take a look at the pattern made of colours. This is a pattern because it follows a repeating cycle of colours. Thus the colours "red, violet and blue" form a cycle that starts over after every three elements.
2. 3, 6, 9, 12, 15… This is a pattern because each number is obtained by adding 3 to the previous number.
3. Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday. This is a pattern because it represents the continuous and repeating cycle of the days of the week.

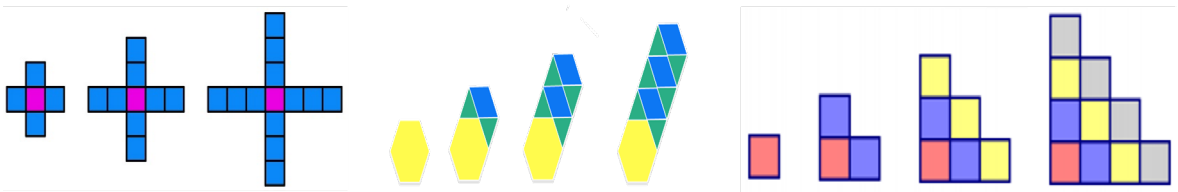
## Types of pattern

1. Repeating patterns: arrangements that follow a specific rule and repeat themselves at regular intervals.

   **Visual representation of repeating patterns in the environment**



2. Growing patterns: arrangements where each subsequent element in the sequence builds upon or increases from the previous elements according to a specific rule.



## Activity 3.1: Individual/Pair/Group Work

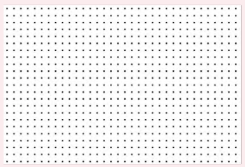
### Pattern Challenge

**Purpose:** Apply your knowledge of repeating and growing patterns to create and extend patterns using different materials.

**Materials Needed:**

- Coloured beads or buttons
- Paper and coloured pencils/crayons
- String or yarn
- Building blocks or cubes
- Grid paper





**Instructions:**

**Step 1: Group Formation**

1. **Group Work:** Divide into groups of 3-4 learners. Each group will receive a set of materials including coloured beads/buttons, paper and pencils, and building blocks.

**Step 2: Creating Repeating Patterns**

1. **Bead/String Pattern:** Using the beads or buttons and a piece of string, create a repeating pattern. For example, you could use a sequence like red, blue, green, red, blue, green.

2. **Extension:** After creating the pattern, extend it by adding more beads in the same sequence.
3. **Presentation:** When you're done, show your pattern to the class and explain how it repeats.

**Step 3: Creating Growing Patterns**

1. **Building Block Pattern:** Using the building blocks, create a growing pattern. For example, start with 1 block, then 2 blocks, then 3 blocks, and so on. The pattern should grow by adding more blocks each time.
2. **Extension:** Continue your growing pattern as far as you can.
3. **Sharing:** Present your growing pattern to the class and explain how it grows.

**Step 4: Drawing and Extending Patterns**

1. **Visual Pattern on Grid Paper:** Use the grid paper and coloured pencils to draw a repeating or growing pattern. For example, you might draw a shape that repeats or a sequence of numbers that increases.
2. **Extension:** After drawing the initial pattern, extend it further on the grid paper.
3. **Group Display:** Attach your paper to a display board or wall, and explain your pattern to the other groups.

**Step 5: Pattern Puzzle**

1. **Challenge Each Other:** Each group will create a pattern puzzle for another group to solve. Create part of a pattern and ask the other group to extend it correctly.
2. **Puzzle Swap:** Swap puzzles with another group and try to solve theirs by correctly extending the pattern.
3. **Class Discussion:** Come together as a class and discuss the different patterns you created, extended, and solved.

**Step 6: Wrap-Up**

1. **Reflect:** Think about how you used the patterns and how you were able to extend them.
2. **Application:** Discuss with your group how patterns are important in everyday life, such as in nature, art and mathematics.

# Focal Area 2: Describe Patterns Orally Using Mathematical Language And Extend The Patterns

Describing patterns is important for you in understanding and communicating mathematical ideas. This activity focuses on using mathematical language to describe patterns, helping you to articulate your observations and reasoning orally or in writing.

## Describing patterns orally

**Examples**

Describe the following patterns orally

1.

The pattern alternates between 2 peppers and 1 tomato, repeating every four items.

2.

The pattern starts with one slice of watermelon and adds one more slice with each step. The pattern begins with a single slice of watermelon, two slices of watermelon, three slices of watermelon, and four slices of watermelon.

3. 2, 4, 6, 8, 10

   The pattern starts with 2, and the next number is obtained by adding 2 to the previous one

4. 40, 20,10,5

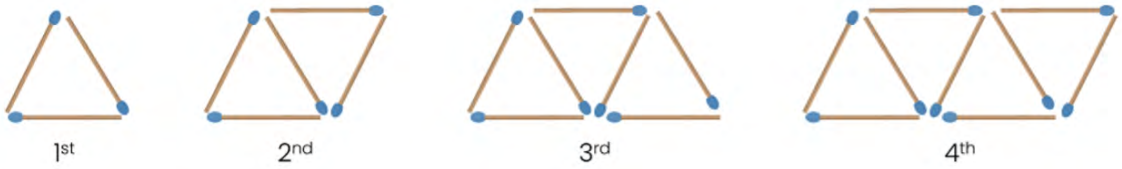
   Each number in this pattern is obtained by dividing the previous number by 2.

## Extend a given pattern.

Visual, numerical, or symbolic patterns can be extended based on how each number relates to the previous one.

## Examples

**1.** Study the pattern and find the number of sticks that could be used in the 5th term.




## Solution:

Look at the number of sticks in each term.

- 1st term: 3 sticks
- 2nd term: 5 sticks
- 3rd term: 7 sticks
- 4th term: 9 sticks

Observe how the number of sticks changes from one term to the next.

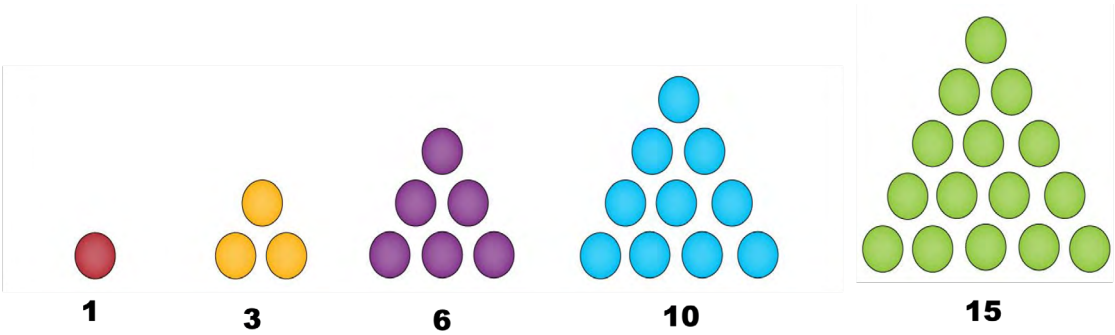
- From 3 to 5, the number of sticks increases by 2.
- From 5 to 7, the number of sticks increases by 2.
- From 7 to 9, the number of sticks increases by 2.

This shows that each term increases by 2 sticks.

To find the $5^{th}$ term, start with the $4^{th}$ term, which has 9 sticks and add 2 sticks to the $4^{th}$ term to get the $5^{th}$

Therefore, the number of sticks used in the $5^{th}$ term is **11**.

**2.** Write the next term in the pattern below



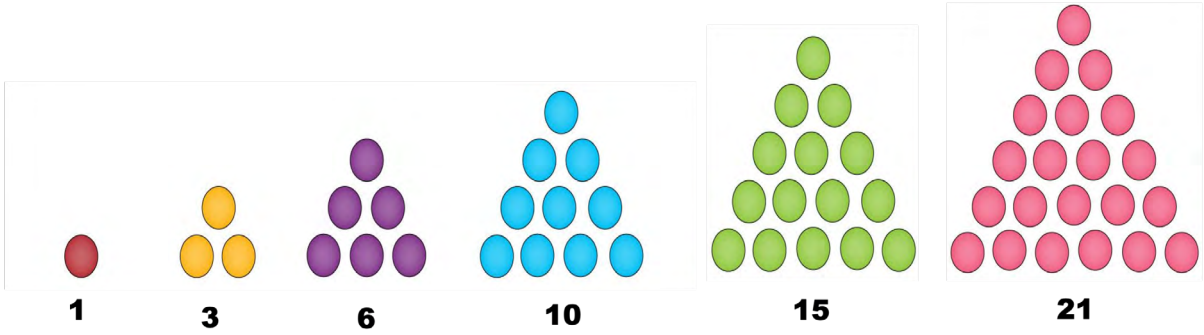
**Solution**

Each number in the pattern is the sum of the first natural numbers. Therefore, we add the next natural number to the last number in the pattern to obtain the next number.

Thus,

- The first number is 1.
- The second number is $1 + 2 = 3$.
- The third number is $1 + 2 + 3 = 6$.
- The fourth number is $1 + 2 + 3 + 4 = 10$.
- The fifth number is $1 + 2 + 3 + 4 + 5 = 15$.
- The sixth number is $1+ 2+ 3+ 4+ 5+ 6 = 21$

Therefore, the pattern extends to 21 as shown in the diagram below.

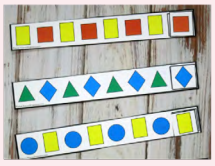


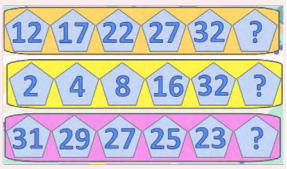

## Activity 3.2: Individual/Pair/Group Work

### Pattern Detective

**Purpose:** Use mathematical language to describe and extend patterns you observe or create.

**Materials Needed:**






- Pattern cards (with sequences of shapes, numbers, or colours)
- Coloured pencils or crayons
- Paper
- Small objects (like beads, buttons, or blocks)

**Instructions:**

**Step 1: Group Formation**

1. **Group Work:** Get into groups of 3-4 learners. Each group will receive a set of pattern cards, coloured pencils, paper, and small objects like beads or buttons.

**Step 2: Observe and Describe**

1. **Pattern Cards:** Look at the pattern cards provided to your group. Observe the pattern carefully.
2. **Oral Description:** Take turns within your group to describe the pattern orally using mathematical language. For example, you might say, "This pattern increases by adding 2 each time" or "The shapes alternate between a circle and a square."
3. **Class Sharing:** After everyone has had a turn describing, choose one pattern to share with the class. Use clear mathematical language to explain the pattern.

**Step 3: Extend the Pattern**

1. **Pattern Extension:** Using the materials provided (coloured pencils, beads, buttons, etc.), extend the pattern you just described. If the pattern involves numbers, continue the sequence on paper; if it involves shapes or colours, use the objects to extend it.
2. **Check with Group:** Show your extended pattern to your group. Have a group discussion to make sure everyone agrees that the pattern has been extended correctly.

**Step 4: Create Your Own Pattern**

1. **Create a New Pattern:** Each group should now create a new pattern using the small objects or drawing on paper. Make sure the pattern has a clear rule, like increasing by 3 each time or alternating colours.
2. **Describe Orally:** Once your group has created the pattern, take turns describing it orally using mathematical language. Explain the rule of the pattern clearly.
3. **Extend the Pattern:** Have another group member extend the pattern you just created and described.

**Step 5: Pattern Swap**

1. **Swap Patterns:** Exchange your pattern with another group. Listen as they describe your pattern using mathematical language.
2. **Extend Each Other's Patterns:** After listening to the description, try to extend the other group's pattern based on their description.
3. **Feedback:** Give feedback to the group on how well they described their pattern and whether it was easy to extend based on their explanation.

**Step 6: Class Discussion**

1. **Reflection:** Come together as a class and reflect on the activity. Discuss how using mathematical language helped you understand and extend patterns more clearly.
2. **Real-Life Application:** Talk about where you might use this skill in real life. For example, in following a recipe, predicting a sequence in a game, or recognizing patterns in nature.

# USING PATTERN RULES TO SOLVE PROBLEMS

## Focal Area 1: Applying Pattern Rules To Solve Problems And Make Predictions

Imagine you're planning a garden, and you want to arrange the flowers in rows. You notice that in each new row, you add two more flowers than in the previous row. If the first row has 3 flowers, the second has 5, and the third has 7, you might wonder: How many flowers will be in the 10th row? Or, if you keep planting in this pattern, how many flowers will you need in total for 10 rows?

This situation is an example of a pattern—a sequence that follows a specific rule. By recognising and applying this rule, you can solve problems like predicting the number of flowers in any row or even determining the total number needed. Patterns like these appear everywhere in real life, from the way we arrange objects to predicting trends in data. Learning to apply pattern rules helps us make informed decisions, solve problems efficiently, and anticipate outcomes in various situations.

Under this focal area, we will describe pattern rules in words. Sometime later in our studies, we will then use algebraic structures to describe these rules. Before we begin, let's have fun with this activity!

### Reinforcement Activities

#### Exploring Simple Patterns

**Purpose:** This activity is designed to help you recognise and describe patterns, which will prepare you for applying pattern rules to solve problems and make predictions.

**Materials Needed:**

- A collection of small objects (like beads, buttons, or blocks) in different colours or shapes
- Paper and pencils

**Activity Steps:**

1. **Grouping Objects:**
   - Get into small groups and pick out 20 small objects from the collection.

- Start by creating a simple pattern with your objects. For example, you might arrange them in a sequence like red-blue-red-blue or square-circle-square-circle.

2. **Identifying the Pattern:**
   - Once your pattern is arranged, write down what you notice about the pattern. What comes next? How can you describe the pattern in words? For example, "The pattern repeats every two objects."
3. **Extending the Pattern:**
   - Continue the pattern for at least 10 more objects. Discuss with your group how you knew what came next in the sequence. Did you use a specific rule?
4. **Exploring Different Patterns:**
   - Create a new pattern using a different sequence or arrangement. For example, you might try a pattern where you add an extra object each time, like one red bead, then two blue beads, then three green beads, and so on.
   - Extend this pattern and write down the rule you used to keep it going.
5. **Reflection:**
   - Share your patterns and the rules you identified with the class. Discuss how recognizing and describing these rules helped you predict what comes next in the sequence.

**Wrap-Up:**

- This activity will help you understand how patterns work and how rules guide these patterns. The skills you practice here will be essential when you start applying pattern rules to solve more complex problems and make predictions in your upcoming lessons.

## Identify and write down the rule that describes the pattern and predicts the subsequent elements.

Identifying and writing the rule that defines a given pattern involves looking at the sequence of numbers, shapes, or symbols to notice any regularity or repetition. Find the relationship by determining how each element changes from one to the next. This could involve addition, subtraction, multiplication, or another operation. Write down the rule that describes this relationship.

**Example 1**

Write the rule for the following sequences and predict the next two numbers.

**Solution**

The number of toffees in each set of our pattern are 4,8,12.

**First, we must find the relationship**:

The difference between consecutive terms:

$8 - 4 = 4$

$12 - 8 = 4$

The difference between each consecutive pair of number of toffees is 4.

**The rule:** We can see that this means that the rule is to add 4 to the previous number of toffees to find the number of toffees in the next term.

Thus,

First number of toffees $= 4$

Second number of toffees $=$ *first number of toffees* $+ 4 = 8$

Third number of toffees $=$ *second number of toffees* $+ 4 = 12$

Therefore, the next two terms:

Fourth number of toffees $=$ *third number of toffees* $+ 4 = 16$

Fifth number of toffees $=$ *fourth number of toffees* $+ 4 = 20$

**Example 2**

Investigate the number pattern and describe the rule.


**Solution**

**Find the relationship by identifying the difference:**

$3 - 1 = 2$

$5 - 3 = 2$

$7 - 5 = 2$

The difference between each consecutive pair of numbers is 2.

**The rule:** From the above, the rule is to add 2 to the previous number to get the next number.

Therefore,

First number = 1

Second number = $1 + 2 = 3$

Third number = $3 + 2 = 5$

Fourth number = $5 + 2 = 7$

Therefore, the next two terms are:

**Therefore, the next two elements are:**

$7 + 2 = 9$

$9 + 2 = 11$

## Apply these skills to solve real-life problems involving patterns, including tables and charts.

**Examples**

**1.** Use the table below to find the number of bananas when $x = 6$ *and* $x = 7$

| Pattern Number ($x$) | Number of bananas |
|---|---|
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |

**Solution**

Identify the Pattern:

Look at how the number of bananas changes as the $x$ values increase.

From $x = 1$ to $x = 2$, the number of bananas changes from 3 to 5 (an increase of 2 bananas).

From $x = 2$ to $x = 3$, the number of bananas changes from 5 to 7 (an increase of 2 bananas).

From $x = 3$ to $x = 4$, the number of bananas changes from 7 to 9 (an increase of 2 bananas).

From $x = 4$ to $x = 5$, the number of bananas changes from 9 to 11 ( an increase of 2 bananas).

**Rule:** Each time $x$ increases by 1, the number of bananas increases by 2.

Therefore, the number of bananas when the $x$ is 6:

If the pattern continues, when $x$ increases from 5 to 6, the number of bananas should increase by 2.

Number of bananas for $x = 5$ is 11.

Add 2 to the last number of bananas

Thus, when $x = 6$, the number of bananas is $11 + 2 = 13$.

Therefore, the number of bananas when the $x$ is 7:

If the pattern continues, when $x$ increases from 6 to 7, the number of bananas should increase by 2.

The number of bananas for $x = 6$ is 13

Add 2 to the last number of bananas

Thus, when $x = 7$, $13 + 2 = 15$.

**Example 2**

The chart below shows the monthly sales of a product. Analyse the chart and predict the sales for July and August

| Month | Sales (units) |
|---|---|
| January | 35 |
| February | 40 |
| March | 45 |
| April | 50 |
| May | 55 |
| June | 60 |

**Solution**

**Pattern Rule:** Sales increase by 5 units each month.

**Prediction for July**

Add 5 to the sales in June (60)

Next month's sales: $60 + 5 = 65$

The prediction for sales for July will be **65 units**

**Prediction for August**

Add 5 to the sales in July (65)

Next month's sales: $65 + 5 = 70$

The prediction for sales for August will be **70 units**.

Note, sales are notoriously hard to predict and patterns could stop at any time, so this is only a prediction and must be compared with reality to see if the pattern remains true.

**Example 3**

A learner collects books, starting with 7 books and adding 3 more books to the collection each month. Identify the pattern, write the rule and use it to find the first 7 sequences.

**Solution**

The learner starts with 7 books, means the pattern start with 7 books.

Adds 3 more books each month.

**The rule**: the rule is obtained by adding 3 to the number of books from the previous month:

Starting month: 7 books

Second month: $7 + 3 = 10$ books

Third month: $10 + 3 = 13$ books

Fourth month: $13 + 3 = 16$ books

Fifth month: $16 + 3 = 19$ books

Sixth month: $19 + 3 = 22$ books

Seventh month: $22 + 3 = 25$ books

So, the first seven numbers in the sequence are: **7, 10, 13, 16, 19, 22, 25**

## ACTIVITY 3.3: Individual/Pair/Group Work

### Pattern Detectives in Action

**Purpose:** In this activity, you will identify and write down the rules that describe patterns and use these rules to predict the next elements. You will then apply these skills to solve real-life problems involving patterns, including those found in tables and charts.

**Materials Needed:**

- Printed tables and charts showing various patterns (e.g., a table of daily temperatures, a sequence of shapes, a chart showing a growing savings account)
- Paper and pencils

- Markers or coloured pencils

### Activity Steps:

1. **Become a Pattern Detective:**
   - Form small groups and choose one of the provided tables or charts ***(refer to tables below)***. Your task is to carefully examine the data or sequence and identify the pattern.
   - Write down the rule that describes how the pattern is formed. For example, if the pattern is in a sequence of numbers, what is the rule that determines the next number? Is it increasing by 2 each time? If the pattern is in a chart, how do the values change?
2. **Predict the Next Elements:**
   - Using the rule you identified, predict the next two or three elements in the pattern. Write down your predictions and explain how you arrived at them using the rule.
3. **Apply to Real-Life Problem:**
   - Now that you've practiced identifying patterns and predicting elements, use these skills to solve a real-life problem.
   - Choose a scenario provided by your teacher, such as predicting how much money will be in a savings account after a certain number of weeks, given a chart showing weekly deposits.
   - Apply the pattern rule to solve the problem and write down your solution.
4. **Create Your Own Pattern:**
   - After solving the problem, create your own pattern using numbers, shapes, or any other sequence. Write down the rule for your pattern.
   - Exchange your pattern with another group and see if they can identify the rule and predict the next elements.
5. **Present Your Findings:**
   - Each group will present their original pattern, the rule they used, and the predictions they made. Discuss how well your predictions matched the actual pattern and what strategies helped you identify the rule.

### 6. Reflection:

- Reflect on how understanding and predicting patterns can help in real-life situations, such as budgeting money, planning schedules, or analysing data in charts.

### Wrap-Up:

- By the end of this activity, you will have a deeper understanding of how to identify rules in patterns and use these rules to solve real-life problems. This skill is essential for recognizing trends, making predictions, and solving practical problems effectively.

**Chart Showing a Growing Savings Account:**

| Week | Savings Balance (GHS) |
|---|---|
| Week 1 | 100 |
| Week 2 | 150 |
| Week 3 | 200 |
| Week 4 | 250 |
| Week 5 | 300 |
| Week 6 | 350 |

**Prediction**: By Week 7, the savings balance will be GHS 400 if the pattern continues.

**Table of Daily Temperatures:**

| Day | Temp. (°C) |
|---|---|
| Monday | 22 |
| Tuesday | 24 |
| Wednesday | 26 |
| Thursday | 28 |
| Friday | 30 |
| Saturday | 32 |
| Sunday | 34 |

**Prediction**: If the pattern continues, Monday of the following week will have a temperature of 36°C.

**Sequence of Shapes:**

| Position | Shape |
|---|---|
| 1 | Square |
| 2 | Circle |
| 3 | Triangle |
| 4 | Square |
| 5 | Circle |
| 6 | Triangle |

**Prediction**: Position 7 will be a Square, Position 8 will be a Circle, and Position 9 will be a Triangle.

**Weekly Sales Growth Chart:**

| Week | Number of Items Sold |
|---|---|
| Week 1 | 20 |
| Week 2 | 25 |
| Week 3 | 30 |
| Week 4 | 35 |
| Week 5 | 40 |

**Prediction:** In Week 6, 45 items will be sold if the pattern continues.

**Number Sequence:**

| Position | Number |
|---|---|
| 1 | 3 |
| 2 | 6 |
| 3 | 9 |
| 4 | 12 |
| 5 | 15 |
| 6 | 18 |

**Prediction**: Position 7 will have the number 21, and Position 8 will have 24.

**Work Hours Chart:**

| Day | Hours Worked |
|---|---|
| Monday | 8 |
| Tuesday | 7 |
| Wednesday | 6 |
| Thursday | 8 |
| Friday | 7 |
| Saturday | 6 |

**Prediction**: If the pattern continues, Sunday would have 8 hours of work.

# Focal Area 2: Translating Real-Life Situations Into Expression

Imagine you're at a supermarket with a budget for your shopping. You need to buy several items: apples, bread, and a carton of milk. As you pick up each item, you mentally keep track of the total cost to ensure you don't exceed your budget. In this situation, you're adding the costs of different items together to get a total, and this can be expressed as an equation.

For example, if apples cost GH¢3, bread costs GH¢2, and milk costs GH¢4, you might write an expression like:

**Total Cost = Cost of Apples + Cost of Bread + Cost of Milk**

In this case, your expression is:

**Total Cost = 3 + 2 + 4**

This expression helps you calculate the total cost quickly and make sure you're staying within your budget. Translating real-life situations into mathematical expressions is a powerful tool. It allows us to simplify and solve problems that we encounter every day. Whether we're budgeting, measuring, or planning, the ability to express real-world scenarios as mathematical expressions helps us make informed decisions, solve problems efficiently, and communicate our thoughts clearly.

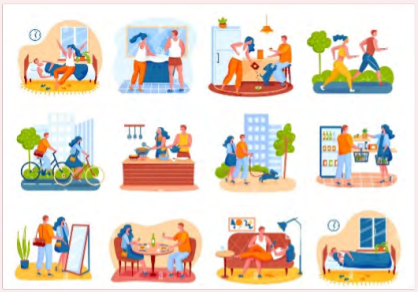
## Reinforcement Activities

### Real-Life Scenarios and Simple Mathematical Operations

**Purpose:** To prepare learners to understand how real-life situations can be represented using basic mathematical operations, laying the foundation for translating these scenarios into mathematical expressions.

**Materials Needed:**

- Picture cards depicting everyday activities (e.g., shopping, cooking, traveling, sports)
- Whiteboard and markers
- Sticky notes or small index cards




**Activity Steps:**

1. **Introduction to the Activity:**
   - Start by asking learners to think about their daily activities, such as buying snacks, counting their school supplies, or sharing something with friends.
   - Explain that many of these activities can be described using simple mathematical operations like addition, subtraction, multiplication and division.

2. **Picture Card Matching:**
   - Divide the learners into small groups.
   - Provide each group with a set of picture cards that depict different real-life scenarios (e.g., a child buying 3 apples, a baker making 2 batches of 5 cookies, a runner covering 4 kilometres each day for 3 days).
   - Ask each group to discuss what is happening in each picture and identify the numbers involved. Then, they should match the scenario with the correct operation (addition, subtraction, multiplication, or division) using sticky notes or index cards.

3. **Group Discussion:**
   - Once the groups have completed the matching task, bring the class together to discuss their findings.
   - Ask questions like:
   - "What operation did you choose for the picture of the child buying apples? Why?"
   - "How would you describe what's happening in the picture using numbers?"

4. **Simple Mathematical Statements:**
   - On the whiteboard, take a few examples from the picture cards and write simple mathematical statements based on the learners' descriptions. For instance:
   - "If a child buys 3 apples for 2 dollars each, how much do they spend in total?"
   - Write: $3 \times 2 = 6$ dollars.

- Encourage learners to come up with their own examples based on their experiences, guiding them to form simple mathematical statements.

5. **Reflection:**
   - End the activity by explaining how these simple operations are the building blocks for translating real-life situations into mathematical expressions.
   - Inform them that in the next lesson, they will learn how to take this a step further by translating more complex real-life scenarios into mathematical expressions.

## Identification of quantities, relationships and operations in real-life contexts

### Examples

Identify the quantities and the relationships in the following examples

**Example 1**

A shopper buys 3 bags of rice, each weighing 2 kg.


**Solution**

### Quantities

3 bags at 2 kg per bag

**Relationships:** The relationship is that the total weight of the rice is the product of the number of bags and the weight of each bag.

In this case, total weight = 3 bags × 2 *kg* per bag

Operation: total weight = $3 \times 2 = 6kg$

**Example 2:**

A recipe requires 160 grams of sugar for 4 servings. How much sugar is required per serving?

**Solution**

**Quantities:**

Amount of Sugar for 4 Servings =160 grams

**Relationship:** Amount of Sugar per Serving = Total Amount of Sugar ÷ Number of Servings

= 160 ÷ 4 = 40grams per serving

So, the relationship is that each serving requires 40 grams of sugar.

This tells how much sugar is needed for each individual serving based on the total amount for the recipe.

## Translate the relationships between the elements into mathematical equations

Explore and write the mathematical statement or equation for the following real-life scenarios.

**Example 1:**

Sarah saves GHC15 every week. How much will she have saved after $w$ weeks?

**Solution**

Sarah saves GHC15 every week.

**Relationship:** Total savings is the weekly savings multiplied by the number of weeks.

Here, S represents the total savings after $w$ weeks.

Therefore, the equation for total saving is $S = 15w$

**Example 2:**

A taxi service charges a flat fee of GH¢5 plus GH¢2 per mile travelled. How much will the fare be for a trip of $m$ miles?

**Solution**

Total fare for the trip $=F$

The flat fee charged by the taxi service =GHC5

Charge per mile travelled $=GHC2$

Number of miles travelled $=m$

**Relationship:** Total fare is the sum of the flat fee plus the charge per mile.

Here, $F$ represents the total fare for $m$ miles.

$$\boldsymbol{F} = 5 + 2\boldsymbol{m}$$

So, if the number of miles ($m$) is known, the total fare ($F$) can be calculated by adding the flat fee of GHC 5 to twice the number of miles travelled.

**Example 3:**

A person is planning to buy a book that costs *GHC m* but has a coupon for a *GHC*4 discount. Write an equation to represent the final cost after applying the discount.

**Solution:**

The final cost of the book after the discount $= C$

The original cost of the book $= m$

Value of the discount coupon $= 4$

The relationship between the final cost ($C$) and the original cost ($m$) can be expressed as

$$C = m - 4$$

So, if the original cost of the book ($m$) is known, the final cost ($C$) can be calculated by subtracting 4 from the original cost.

**Note**: All the mathematical expressions in the above examples are called algebraic equations. Note, they are expressions unless they contain an equals sign when they become equations.

## Activity 3.4: Individual/Pair/Group Work

**Purpose:** To help you practise identifying quantities, relationships, and operations in real-life contexts and translating those relationships into mathematical expressions.image

**Materials Needed:**

- Scenario cards (each card with a real-life situation)
- Whiteboard and markers
- Paper and pencils
- Small group activity sheets

**Activity Steps:**

1. **Introduction:**
   - In this activity, you will work in small groups to explore different real-life scenarios. Your task is to identify the key quantities, relationships, and operations in each scenario and then translate those relationships into mathematical expressions.
2. **Scenario Exploration:**
   - Each group will receive a set of scenario cards.
   - Here's an example scenario:

     *"A bakery sells 5 cupcakes each day. If the price of one cake is 10 cedis, how much money does the bakery make in 3 days?"*
   - Begin by discussing the scenario within your group. Identify the key quantities (like the number of cupcakes, price per cup cake, and number of days) and the relationships between them (like multiplication of the number of cupcakes by the price to find total earnings per day).
3. **Identify Relationships:**
   - Write down the quantities and relationships you find. For the example scenario, you might write:
   - Number of cakes per day: 5
   - Price per cake: 10 cedis
   - Number of days: 3
   - Discuss what mathematical operation (addition, subtraction, multiplication, or division) will help you solve the problem.

4. **Translate to Mathematical Expressions:**

- After identifying the relationships, translate them into a mathematical expression. For the example scenario, the expression could be:
- Total earnings = Number of cakes per day × Price per cake × Number of days
- Expression: $5 \times 10 \times 3 = 150$ cedis
- Write the expression on your group's activity sheet.

5. **Group Presentations:**

- After your group has worked through all the scenario cards, each group will choose one scenario to present to the class.
- Explain how you identified the quantities, relationships, and operations, and how you translated them into a mathematical expression.

6. **Class Discussion:**

- After all groups have presented, we'll discuss as a class how translating real-life situations into mathematical expressions can help solve problems more easily and accurately.
- Think about how these skills could be useful in your daily life, such as when shopping, budgeting, or planning events.

7. **Reflection:**

- Reflect on what you've learned from this activity. How did identifying relationships help you understand the scenarios better? How confident do you feel about translating real-life situations into mathematical expressions?

# OPERATIONS ON ALGEBRAIC EXPRESSIONS

## Focal Area 1: Perform Basic Operations On Algebraic Expressions – Addition, Subtraction & Multiplication

Algebraic expressions are essential components of algebra. They consist of variables, constants, and operations that combine to represent mathematical relationships and real-life scenarios. Understanding how to perform basic operations on these expressions is essential for solving equations, modelling situations, and analysing data.

### Reinforcement Activities

#### Exploring Operations with Simple Numbers

**Purpose:** To help you refresh and strengthen your understanding of basic arithmetic operations (addition, subtraction, and multiplication) with simple numbers, laying the foundation for performing these operations on algebraic expressions.

**Materials Needed:**

- Number cards (with various whole numbers)
- Whiteboard and markers
- Paper and pencils
- Small group activity sheets


**Activity Steps:**

1. **Introduction:**
   - Today, we are going to revisit basic arithmetic operations—addition, subtraction, and multiplication—that you have already learned. This will help you feel more confident when we start working with algebraic expressions.
2. **Number Card Operations:**
   - You will each receive a set of number cards. These cards will have different whole numbers written on them.

- In pairs or small groups, take turns selecting two or three number cards. Use these numbers to perform basic operations: add, subtract, and multiply them.
- For example, if you pick the cards 3 and 7, you might perform the following operations:
  - Addition: $3 + 7 = 10$
  - Subtraction: $7 - 3 = 4$
  - Multiplication: $3 \times 7 = 21$

**3. Discuss the Patterns:**

- After performing the operations, discuss with your group the patterns or observations you notice when adding, subtracting, and multiplying numbers.
- Think about how these operations change when you use different numbers or change the order of the numbers.

**4. Connecting to Algebra:**

- On the whiteboard, we'll collectively solve a few more examples. As we do this, I'll introduce some simple algebraic expressions to show how similar they are to the numbers you just worked with.
- For instance, if you added $3 + 7$ earlier, now imagine adding $3 + x$, where $x$ is a variable that could stand for any number.

**5. Reflection:**

- Write down any observations you have about how these basic operations work.
- Think about how comfortable you feel with addition, subtraction, and multiplication, and consider how these skills might be applied when working with algebraic expressions.

## Simplifying Expressions (addition and subtraction)

**Example 1:**

If there are 2 boys in a class and 3 more boys join, find the number of boys in the classroom.

**Solution**


From the picture above, we can add 2 boys and 3 boys together to get 5 boys because they are of the same group. Therefore, objects of the same group are called **like objects** and they can be added together.

Mathematically, we can use variables to represent boys. Thus let $x$ represent boys.

Then, 2 boys and 3 more boys joined is the same as $2x + 3x$. This expression $2x + 3x$ is called an **algebraic expression**. The number attached to any variable in algebraic expression is called the **coefficient.** Therefore, 2 and 3 are the coefficients of the variable $x$.

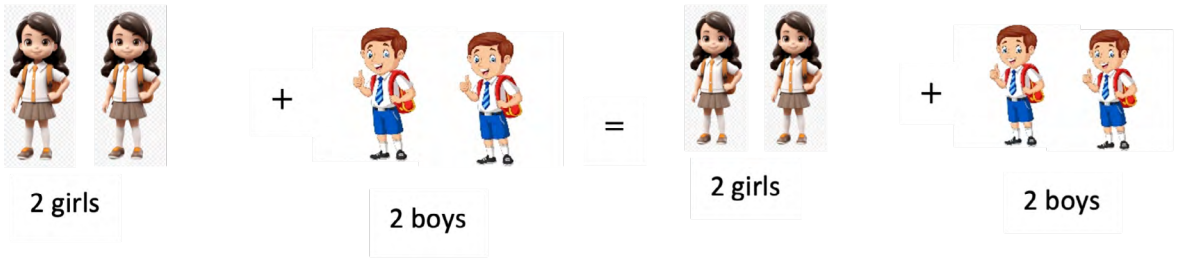
The expression $2x + 3x$ has two terms. Thus, $2x$ is the $1^{st}$ term and $3x$ is the $2^{nd}$ term. These terms have the same variable and are separated by the operation addition. Therefore, terms of the same variable in an expression are called **like terms**.

Just like we added boys with boys, in algebra, we add the coefficient of the like terms together by adding the like terms and attach one of the variables.

Thus , $2x + 3x = 5x$

**Example 2:**

If there are 2 girls and 2 boys in the classroom find the number of boys and girls in the classroom.




2 girls | 2 boys | 2 girls | 2 boys

From the picture above, we cannot add 2 girls and 2 boys because they are different groups. Therefore, objects of different groups are called **unlike objects**.

Mathematically, we can use variables to represent girls and boys. Thus, let $x$ represent girls and $y$ represent boys.

Then, 2 girls and 2 more boys joined is the same as $2x + 2y$.

Algebraic expression: $2x + 2y$

**Coefficient**s for both $x$ and $y$ are 2.

The expression $2x + 2y$ has two terms. Thus, $2x$ is the $1^{st}$ term and $2y$ is the $2^{nd}$ term. These terms have different variable and are separated by the operation addition. Therefore, terms that have different variables in an expression are called **unlike terms**.

Just like we cannot add girls with boys, in algebra, unlike terms cannot be added together, they remain the same.

Thus, $2x + 2y = 2x + 2y$

**Note**: all the terms in an expression without variables are called the constant number

**Example 3:**

Simplify $4a + 7 - 2a + 3$

**Solution**

$4a + 7 - 2a + 3$

$= 4a - 2a + 7 + 3$

$= 2a + 10$

**Example 4:**

Simplify $6m - 2m + 8$

**Solution**

$6m - 2m + 8$

$= 4m + 8$

**Example 5:**

Simplify $3ab - 2cd + ab + 5cd$

**Solution**

$3ab - 2cd + ab + 5cd$

$= 3ab + ab - 2cd + 5cd$

$= 4ab + 3cd$

# Multiplication Of Algebraic Expressions

## Multiplying a Variable by a Number

**Example 1:**

Multiply $2x \times 3$

**Solution**

Use the coefficient of x to multiply the constant number.

Thus $2 \times 3 = 6$

The variable $x$ stays the same because you are only multiplying the numbers:

Thus $2x \times 3 = 6x$

**Example 2:**

Multiply 5 by 3p

**Solution**

$5 \times 3p = 15p$

## Multiplying a Variable by Another Variable

**Example 1:**

What is $2x \times 3y$

**Solution**

Multiply the coefficients

Thus is $2 \times 3 = 6$

Multiply the variables

$x \times y = xy$

Combine these results:

Thus $2x \times 3y = 6xy$

**Example 2:**

Multiply $10a \times 4b$

**Solution:**

$10a \times 4b = 40ab$

**Activity 3.5: Individual/Pair/Group Work**

### Algebraic Operations in Action

**Purpose:** To reinforce your understanding of performing basic operations (addition, subtraction, and multiplication) on algebraic expressions through collaborative problem-solving.

**Materials Needed:**

- Whiteboard and markers
- Worksheets with algebraic expressions
- Calculators (optional)

**Sample worksheet**

3x + 2x
5y - 2y
4a + 3b - 2a
3x - 1 + x
2y × 3x
5x - 3x + 4x
2a + 3b + 4a - b
5xy - 2xy + 3xy

**Activity Steps:**

1. **Introduction:**
   - Now that you have learned how to add, subtract, and multiply algebraic expressions, we are going to practise these skills in groups.

- This activity will help you apply what you've learned and see how these operations work together to simplify and solve algebraic expressions.

2. **Group Problem-Solving:**
   - You will be divided into small groups. Each group will receive a worksheet with a variety of algebraic expressions that need to be simplified using addition, subtraction, or multiplication.
   - Work together to simplify the expressions. Use coloured pencils or markers to highlight each operation step you perform, making it easier to track your process.
3. **Using Algebra Tiles (Optional):**
   - If algebra tiles are available, you can use them to visually represent the algebraic expressions and operations.
   - For example, use tiles to represent terms like $x$ and $x^2$, then physically add, subtract, or multiply them as required.
4. **Solve and Share:**
   - Once your group has simplified the expressions, write your final answers on the whiteboard. Be prepared to explain the steps you took to simplify each expression.
   - Discuss any challenges or strategies that worked well in your group. This will help everyone learn different approaches to solving algebraic expressions.
5. **Challenge Problems:**
   - After completing the initial worksheet, each group will receive a challenge problem that involves more complex operations or a combination of all three operations (addition, subtraction, and multiplication).
   - Work together to simplify the challenge problem, and then share your solution with the class.
6. **Reflection:**
   - Write a brief reflection on what you learned from this activity. Consider which operations were easiest or hardest for you and how working in a group helped you understand the concepts better.

- Think about how you can apply these algebraic operations to solve real-world problems.

## Focal Area 2: Translating And Performing Operations On Real-Life Applications Of Algebraic Expressions

Create and simplify algebraic expressions from the following real-life scenarios.

**Example 1:**

Ohemaa has $x$ mangoes and buys 7 more.

How many mangoes does she have in total?

**Solution**

Ohemaa initially has $x$ mangoes.

She buys 7 more mangoes.

The expression gives the total number of mangoes Ohemaa has which is $\boldsymbol{x} + 7$So, if $x$ represents the number of mangoes Ohemaa initially has, the total number of mangoes she has after buying 7 more is $x + 7$

**Example 2:**

A rectangle has a length of $2t$ and a width of 4.

What is the product of the length and width?

(And this is the same as the area of the rectangle.)

**Solution:**

Quantities:

Length $= 2t$

Width $= 4$

To find the product of the length and the width of the rectangle, we need to multiply the given expressions for the length and width

The product of length and width is

$2t \times 4 = 8t$

**Example 3:**

The cost of one unit of material is $GHCx$ and you need $n$ units.

What is the total cost?

**Solution:**

Cost per unit $= GHC\ x$

Number of units $= n$

To find the total cost of $n$ units of material, where the cost of one unit is $GHC\ x$, multiply the cost per unit by the number of units. $C = x \times n$

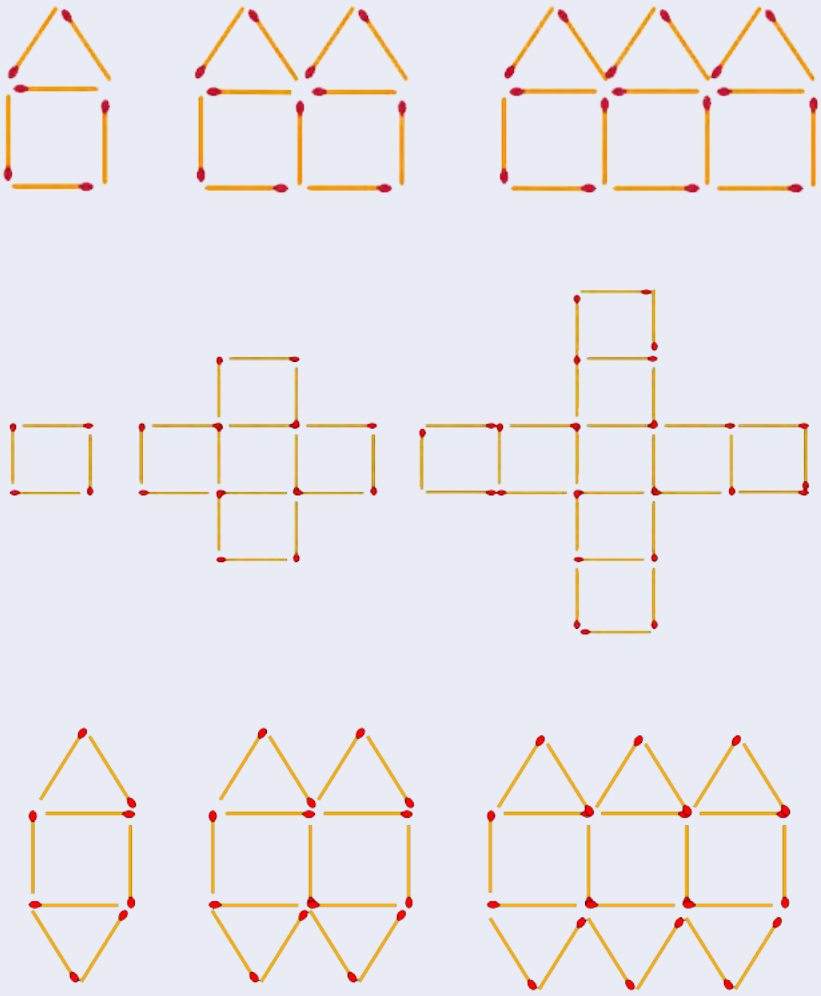
Therefore, the total cost of $n$ units of material $C = xn$

# REVIEW QUESTIONS

## Review Questions 3.1

1. Study each of the patterns carefully.

   For each of them, if the pattern continuous, what will be its fifth pattern and how many sticks will be in each?



**2. Instructions for Learners:**

Look at each pattern carefully.

Determine the rule for the pattern (e.g., adding a specific number, multiplying, etc.).

Extend the pattern by filling in the missing numbers in the sequence.

**a) Simple Addition Patterns**

1. 2, 4, 6, 8, ___, ___, ___
2. 5, 10, 15, 20, ___, ___, ___
3. 12, 15, 18, 21, ___, ___, ___

**b) Simple Subtraction Patterns**

1. 20, 18, 16, 14, ___, ___, ___
2. 30, 25, 20, 15, ___, ___, ___
3. 50, 45, 40, 35, ___, ___, ___

**c) Multiplication Patterns**

1. 3, 6, 9, 12, ___, ___, ___
2. 4, 8, 12, 16, ___, ___, ___
3. 7, 14, 21, 28, ___, ___, ___

**d) Division Patterns**

1. 100, 50, 25, 12.5, ___, ___, ___
2. 80, 40, 20, 10, ___, ___, ___
3. 64, 32, 16, 8, ___, ___, ___

**e) Mixed Patterns**

1. 1, 3, 6, 10, 15, ___, ___, ___
2. 2, 5, 9, 14, 20, ___, ___, ___
3. 50, 45, 40, 35, ___, ___, ___

**f) Growing Patterns**

1. 2, 4, 8, 16, ___, ___, ___
2. 1, 2, 4, 7, 11, ___, ___, ___
3. 10, 20, 40, 80, ___, ___, ___

**g) Pattern Challenge**

1. 3, 9, 27, 81, ___, ___, ___
2. 1, 4, 9, 16, 25, ___, ___, ___
3. 15, 10, 5, 0, ___, ___, ___

3. Solve the following word problems

i. A tile floor has a pattern of red, blue, red, blue, and so on. If the pattern is extended for the next four tiles, the sequence will be red tile, blue tile, red tile, blue tile.

The relationship is to alternate between red and blue tiles.
True / False

The tile floor has a growing pattern. True / False

ii. The schedule of examination timings for some candidates is given below, what is the time of the sixth examination if the pattern continues

| | | | | | | |
|---|---|---|---|---|---|---|
| a. | Examination: | 1st | 2nd | 3rd | 4th | 5th |
| b. | Time: | 1:00 pm | 1:35 pm | 2: 10 pm | 2:45 pm | 3:20 pm |

## Review Questions 3.2

1. Look at the following number sequence: 5, 10, 15, 20, 25.
   - **a.** What is the rule for this pattern?
   - **b.** What are the next two numbers in the sequence?
2. Consider the sequence: 2, 4, 8, 16, 32.
   - **a.** What is the rule for this pattern?
   - **b.** What are the next two numbers in the sequence?
3. The sequence is: 100, 95, 90, 85, 80.
   - **a.** What is the rule for this pattern?
   - **b.** What are the next two numbers in the sequence?
4. The pattern of shapes is as follows: Circle, Square, Triangle, Circle, Square.
   - **a.** What is the rule for this pattern?
   - **b.** What is the next shape in the sequence?

5. The pattern of shapes is as follows: Hexagon, Pentagon, Square, Hexagon, Pentagon.
   a. What is the rule for this pattern?
   b. What is the next shape in the sequence?
6. A gardener notices a pattern in the number of flowers blooming in her garden. On Monday, she has 3 flowers, on Tuesday, she has 6 flowers, and on Wednesday, she has 9 flowers.
   a. What is the rule for this pattern?
   b. How many flowers will she have on Friday?
7. A farmer sells eggs in a sequence of boxes. Each box holds 12 eggs. If he sells 1 box on Monday, 2 boxes on Tuesday, and 3 boxes on Wednesday:
   a. What is the rule for this pattern?
   b. How many eggs will he sell by Thursday?
8. A savings account grows by GH¢10 every month. If the balance in January is GH¢100, in February it is GH¢110, and in March it is GH¢120:
   a. What is the rule for this pattern?
   b. What will the balance be in May?
9. A car rental company rents out cars and notices a pattern in the number of cars rented out each day. On Monday, 5 cars are rented, on Tuesday, 10 cars are rented, and on Wednesday, 15 cars are rented.
   a. What is the rule for this pattern?
   b. How many cars will be rented out on Friday?
10. A runner increases his distance by 2 kilometres each week. In the first week, he runs 5 kilometres, in the second week, he runs 7 kilometres, and in the third week, he runs 9 kilometres.
    a. What is the rule for this pattern?
    b. How far will he run in the fifth week?

## Review Questions 3.3

1. A learner spends 45 minutes reading and 30 minutes exercising every day.
   - i. How many minutes does the learner spend reading each day?
   - ii. How many minutes does the learner spend exercising each day?
   - iii. How many total minutes does the learner spend on both activities each day?
   - iv. How many total minutes does the learner spend on both activities in a week?
2. Madam Ellen's monthly budget includes GHC70 for food, GHC100 for utilities and GHC50 for entertainment.
   - i. How much money is set aside for food?
   - ii. How much money is set aside for utilities?
   - iii. How much money is set aside for entertainment?
   - iv. What is the total amount of money spent on three in one month?
   - v. What is the total amount of money spent on all three in one year?
3. There are $p$ pencils in one box and q pencils in another box.

   Write an expression to represent how many pencils are there altogether.
4. Sarah buys $p$ packs of stickers, each pack containing $y$ stickers.

   How many stickers does she buy in total?
5. John has 4 more marbles than Tom.

   If Tom has $x$ marbles, write an expression to represent the number of marbles John has.

## Review Questions 3.4

1. **Simplify the expression:** $3x + 5x$
2. **Combine the like terms:** $2a + 4b + 3a - 2b$
3. **Simplify the expression:** $7m + 2n + 3m + 5n$
4. Sarah bought 3 pencils for $x$ cedis each and 2 erasers for $y$ cedis each.

   Write an expression for the total cost.
5. **Simplify the expression:** $8x - 3x$
6. **Combine the like terms:** $6p - 2q - 3p + 5q$

7. **Simplify the expression** $9k - 4l - 2k + 3l$
8. John has $x$ apples and he gave away 3 apples.
   Write an expression for the number of apples John has left.
9. **Multiply the expressions:** $4x \times 3y$

**Challenge**

10. **Find the product:** $2a \times (b + 4)$
11. **Multiply the following:** $5m \times (2n - 3)$
12. A gardener plants $x$ rows of flowers, and each row has $y$ flowers.
    Write an expression for the total number of flowers planted.

# MINI-PROJECTS

## Project 1: Mastering Number Concepts and Applications

**Objective:**

In this project, you will engage in activities to enhance your understanding of large numbers, rounding techniques, identifying even and odd numbers, and working with factors and multiples.

### Part 1: Reading, Writing, and Comparing Large Numbers

**Task 1: Understanding Large Numbers**

- Use graph sheets and multi-base blocks to model numbers up to 1 000 000.
- Write the numbers you model in both numerical and word forms. For example, model 235 000 using blocks, then write it as "two hundred thirty-five thousand."
- Compare two large numbers by placing them on a number line or using comparison symbols ($>$ , $<$ , $=$).

**Task 2: Real-Life Application**

- Think of situations where large numbers are used, such as population counts or distances between cities.
- Write down examples and explain how understanding these large numbers can help in making informed decisions in real life.

### Part 2: Rounding Whole Numbers

**Task 1: Rounding Practice**

- Practice rounding numbers up to 100 000 to the nearest ten, hundred, thousand, and ten thousand.
- Use real-life scenarios where rounding is necessary, such as estimating the total cost of items or approximating distances on a map.

**Task 2: Rounding Challenges**

- Create a list of numbers that you will round in different ways. For instance, round 47 823 to the nearest ten (47 820), the nearest hundred (47 800), and so on.

- Discuss how rounding can simplify calculations in everyday tasks, like budgeting or planning trips.

## Part 3: Identifying Even and Odd Numbers

### Task 1: Arrays of Numbers

- Identify even and odd numbers between 1 and 100 by creating arrays of objects. Even numbers can be paired completely, while odd numbers will have one left out.
- Write down the patterns you observe in even and odd numbers.

### Task 2: Practical Application

- Use your understanding of even and odd numbers to solve problems. For example, decide if a group of items can be evenly shared among friends.

## Part 4: Working with Factors and Multiples

### Task 1: Identifying Factors and Multiples

- Choose several numbers and list their factors. For example, for the number 12, the factors are 1, 2, 3, 4, 6, and 12.
- Identify multiples of these numbers up to a certain limit. For example, the first five multiples of 4 are 4, 8, 12, 16, and 20.

### Task 2: Problem-Solving with Factors and Multiples

- Solve word problems involving factors and multiples. For instance, “If you have 24 candies, how can you share them equally among 4 friends?”
- Explore real-life scenarios like organising items into groups or scheduling events using your knowledge of factors and multiples.

### Materials Needed:

- Graph sheets
- Multi-base blocks
- Number line
- Colored pencils or markers
- Objects for arrays (e.g., counters or small blocks)
- Calculator

**Rubric for Assessment:**

| Criteria | Excellent (4) | Good (3) | Fair (2) | Needs Improvement (1) |
|---|---|---|---|---|
| Modelling and Comparing Large Numbers | Accurately models and compares numbers | Mostly accurate with minor errors | Basic understanding with some errors | Incorrect or incomplete modelling |
| Rounding Whole Numbers | Rounds numbers correctly in all cases | Mostly accurate with minor errors | Somewhat accurate with several errors | Incorrect or incomplete rounding |
| Identifying Even and Odd Numbers | Accurately identifies and explains patterns | Mostly accurate with minor errors | Basic understanding with some errors | Incorrect or incomplete identification |
| Identifying Factors and Multiples | Accurately identifies factors and solves problems | Mostly accurate with minor errors | Basic understanding with some errors | Incorrect or incomplete identification |

This project will help you strengthen your understanding of large numbers, rounding, and number patterns, and improve your ability to solve problems using factors and multiples.

# Project 2: Exploring Integers and Fractions in Real-Life Contexts

**Objective:**

This project will help you understand how to use positive and negative numbers to describe real-life situations, and how to work with fractions by modeling, representing, and naming them.

## Part 1: Describing Real-Life Situations with Integers

### Task 1: Understanding Positive and Negative Numbers

- Think of real-life situations where you use positive and negative numbers. For example, temperatures above and below zero, gains and losses in money, or elevations above and below sea level.
- Create a list of at least five scenarios where you would use positive and negative numbers to describe the situation. For instance, "+10°C" could represent a warm day, while "-5°C" could represent a cold day.

### Task 2: Performing Operations on Integers

- Solve word problems that involve adding, subtracting, multiplying and dividing positive and negative numbers. For example, "If you owe GH¢20 (represented as - GH¢20) and earn GH¢50, how much money do you have now?"
- Represent the problem using a number line or by writing out the mathematical operation.

## Part 2: Working with Fractions

### Task 1: Modeling and Representing Fractions

- Use materials like fraction circles, strips, or blocks to model fractions. For example, if you have a pizza divided into 8 slices and you eat 3, you can represent the amount eaten as 3 _ 8.
- Draw pictorial representations of the fractions you model and label them. Show how different fractions represent parts of a whole.

### Task 2: Naming and Representing Quantities as Fractions

- Practise naming fractions by dividing objects into equal parts and writing the fraction that represents a portion of the whole. For instance, divide a piece of paper into 4 equal parts and shade 1 part, then write it as 1 _ 4.

- Find real-life examples where fractions are used, such as in recipes or measuring ingredients. Represent these situations using fractions.

**Materials Needed:**

- Fraction circles, strips, or blocks
- Number line
- Graph paper
- Colored pencils or markers
- Objects for modeling (e.g., paper, blocks, or cut-outs)

**Rubric for Assessment:**

| Criteria | Excellent (4) | Good (3) | Fair (2) | Needs Improvement (1) |
|---|---|---|---|---|
| Describing Situations with Integers | Accurately describes and uses integers in all scenarios | Mostly accurate with minor errors | Basic understanding with some errors | Incorrect or incomplete descriptions |
| Performing Operations on Integers | Correctly solves all problems using integers | Mostly accurate with minor errors | Somewhat accurate with several errors | Incorrect or incomplete operations |
| Modeling and Representing Fractions | Accurately models and represents fractions | Mostly accurate with minor errors | Basic understanding with some errors | Incorrect or incomplete modeling |
| Naming and Representing Fractions | Correctly names and represents all fractions | Mostly accurate with minor errors | Somewhat accurate with several errors | Incorrect or incomplete naming/ representation |

This project will help you understand how to apply integers to real-life situations and work confidently with fractions through modeling and representation.

# Project 3: Exploring Patterns and Algebraic Expressions in Real-Life Contexts

**Objective:**

This project will help you understand how to recognize, describe, and extend patterns, solve problems using patterns, model real-life situations mathematically, and perform basic operations on algebraic expressions.

## Part 1: Working with Patterns

### Task 1: Representing and Extending Patterns

- Look at a pattern provided to you, such as a sequence of shapes, numbers, or colors.
- Draw the pattern and extend it by adding the next three elements.
- For each new element you add, explain how it is different from the element before it. Describe what rule you used to extend the pattern.

### Task 2: Describing Patterns Using Mathematical Language

- Choose a pattern and describe it using mathematical language. For example, "This pattern increases by 2 each time" or "The shapes alternate between squares and circles."
- Predict the next three elements in the pattern based on the rule you have described.

### Task 3: Solving Problems with Patterns

- Solve a problem that involves a pattern. For example, use a table or chart to identify how a pattern progresses, and predict a future outcome.
- You might be given a pattern of numbers and asked to determine the 10th element in the sequence.

## Part 2: Algebraic Expressions in Real-Life Contexts

### Task 4: Modelling Real-Life Situations Mathematically

- Think of a real-life situation that can be expressed as a mathematical statement. For example, "If you have 5 apples and you get 3 more, how many apples do you have?"
- Write this situation as an algebraic expression, such as $x+3$, where $x$ represents the number of apples you start with.

### Task 5: Performing Operations on Algebraic Expressions

- Practise adding, subtracting and multiplying algebraic expressions. For example, add two expressions like $2x + 3$ and $4x + 5$ or multiply an expression like $3(x + 2)$.
- Show your work and explain the steps you took to perform each operation.

**Materials Needed:**

- Graph paper
- Ruler
- Colored pencils or markers
- Tables or charts for recording patterns
- Notebook for writing and solving algebraic expressions

**Rubric for Assessment:**

| Criteria | Excellent (4) | Good (3) | Fair (2) | Needs Improvement (1) |
|---|---|---|---|---|
| Representing and Extending Patterns | Accurately represents and extends patterns | Mostly accurate with minor errors | Basic understanding with some errors | Incorrect or incomplete representations |
| Describing Patterns Using Math Language | Correctly describes patterns and predicts elements | Mostly accurate with minor errors | Basic descriptions with some errors | Incorrect or incomplete descriptions |
| Solving Problems with Patterns | Solves problems accurately using patterns | Mostly accurate with minor errors | Somewhat accurate with several errors | Incorrect or incomplete solutions |
| Modeling Real-Life Situations Mathematically | Accurately models situations using algebra | Mostly accurate with minor errors | Basic understanding with some errors | Incorrect or incomplete modeling |

| Criteria | Excellent (4) | Good (3) | Fair (2) | Needs Improvement (1) |
|---|---|---|---|---|
| Performing Operations on Algebraic Expressions | Correctly performs all operations on expressions | Mostly accurate with minor errors | Somewhat accurate with several errors | Incorrect or incomplete operations |

This project will help you develop skills in recognizing and extending patterns, solving problems using patterns, and working with algebraic expressions in real-life c

# ACKNOWLEDGEMENTS

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