SECTION 2

FRACTIONS AND PERCENTAGES

NUMBER FOR EVERYDAY LIFE

Proportional Reasoning

INTRODUCTION

Hello, learner! In this section, we're going to explore the fascinating world of fractions. Fractions are a fundamental concept in mathematics that represents part-to-whole relationships. Understanding fractions is crucial for problem-solving in various aspects of life; from cooking and shopping to science and engineering.

In this section, you will learn to;

- Establish the concept of fractions and investigate the connections between fractions and decimal numbers.
- Establish basic rules for operations on fractions: addition, subtraction, multiplication and division.
- Investigate the connections between fractions and decimal numbers.
- Establish additive and multiplicative inverses of fractions using multi-purpose model charts.
- Review the concept of fractions and investigate the connections between fractions and decimal numbers.
- Develop models to examine connections between and among fractions, percentages and decimal numbers and generalise.
- Analyse daily activities/issues/businesses involving fractions, percentages and decimals.
- Apply fractions, percentages and decimals to problems involving personal or household finance (such as utility bills, exchange rates, project budgeting, school sees, shopping, etc.)
- Apply fractions, percentages, decimals to real life problems involving Percentage increase and percentage decrease, Profit, and loss,
- Establish appropriate procedures solving problems involving simple and compound interests.

Key Ideas:

- Fractions represent parts of a whole or a ratio between two numbers.
- Fractions consist of a numerator (the top number) representing the part being considered and a denominator (the bottom number) representing the total number of equal parts into which the whole is divided.
- Fractions can also be expressed as decimal numbers, where the numerator is divided by the denominator. For example, $\frac{1}{2}$ is equivalent to 0.5 as a decimal.
- Fractions and decimal numbers represent parts of a whole. Fractions consist of the number of selected parts (numerator) over a total number of equal parts (denominator), while decimals are a division of the numerator by the denominator. They are different notations for the same concept and are interchangeable in many cases.
- Both fractions and decimals can be operated on using similar arithmetic operations.
- Additive and Multiplicative inverses of fractions, and Model charts for Visualization are some of the key concepts that can be used to deepen your understanding of fractions, decimals, and their interconnectedness as well as improve your understanding of operations on fractions.
- Application of Percentages is about using percentages to solve real-world problems, such as discounts, commissions, profit and loss, percentage increase and decrease and percentage profit.
- Systematic approaches to calculate simple interest include identifying the principal amount, interest rate, and period involved.

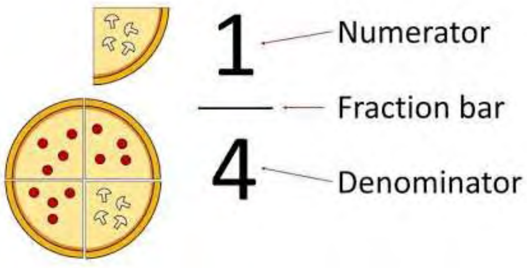
THE CONCEPT AND OPERATIONS OF FRACTIONS 1

In this lesson, we'll delve into the concept of fractions, learn how to perform operations like addition, subtraction, multiplication, and division, and apply these skills to real-world scenarios. You'll discover how fractions connect to other math topics, such as decimals, percentages, and ratios.

The Concept of Fractions

A fraction is a concept which represents a numerical value and defines parts of a whole/unit, a group or a ratio.

Parts of Fractions



Fractions include two parts, the numerator and the denominator.

1. Numerator: It is the upper part of the fraction, that represents the number of sections of the fraction.
2. Denominator: It is the lower, or bottom part, of the fraction that represents the total parts into which the fraction is equally divided.

Example

If $\frac{3}{7}$ is a fraction, then 3 is the **numerator** and 7 is the **denominator**.

Types of Fractions

1. **Proper fractions:** Proper fractions are those where the numerator is less than the denominator. For example, $\frac{2}{5}$ is a proper fraction since "the numerator is less than the denominator".
2. **Improper fractions:** Improper fractions are fractions where the numerator is greater than the denominator. For example, $\frac{7}{5}$ is an improper fraction since "the numerator is greater than the denominator".
3. **Mixed fractions**: Mixed fractions are a combination of the integer part and a proper fraction. These are also called mixed numbers or mixed numerals. For example: $5\frac{3}{7}$, $1\frac{2}{3}$, $9\frac{2}{3}$, $8\frac{11}{12}$
4. **Like fractions:** These are fractions that are alike or the same. For example, take $\frac{1}{3}$ and $\frac{2}{3}$; they are alike since they have same denominator.
5. **Unlike fractions:** Unlike fractions, are those that are dissimilar. They have different denominators. For example, $\frac{1}{2}$ and $\frac{2}{3}$ are unlike fractions.

6. **Equivalent fractions** are fractions that represent the same value or proportion, even though they have different numerators (top numbers) and denominators (bottom numbers). In other words, two fractions are equivalent if, when simplified or converted, they equal the same fraction.

   For example:

   $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$

   All of these fractions represent the same value, which is one-half.

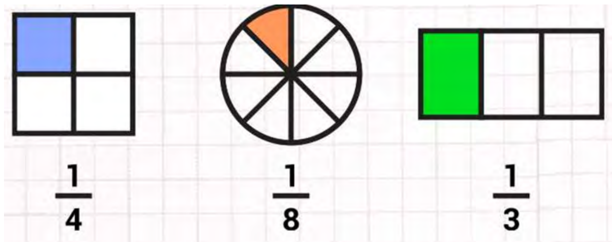
7. **Unit fractions:** A fraction is known as a unit fraction when the numerator is equal to 1.

   i. One half of a whole $= \frac{1}{2}$

   ii. One-third of a whole $= \frac{1}{3}$

   iii. One-fourth of a whole $= \frac{1}{4}$

   One-fifth of a whole $= \frac{1}{5}$



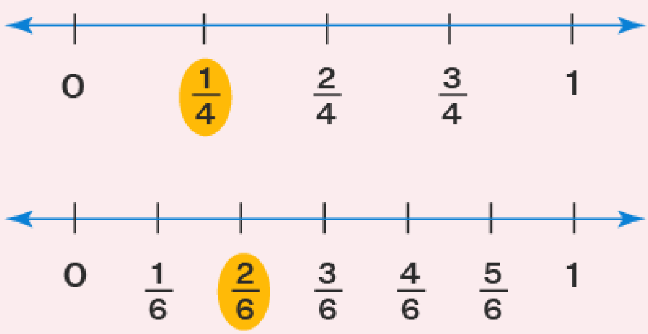
Application of the Concept and Examples

Activity 2.1

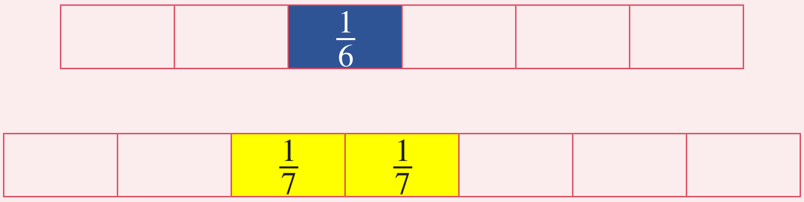
(To be carried out in school with support from the teacher).

1. Model and represent fractions using a number line, paper strips, or Cuisenaire rods.

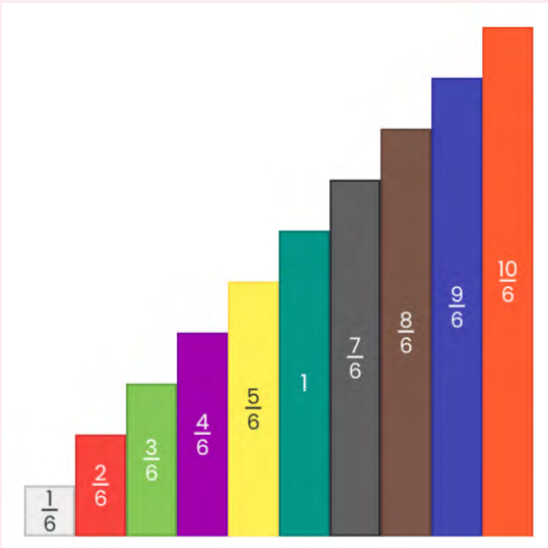
   a) Representing fractions on a number line. If you have to represent $\frac{1}{4}$ and $\frac{2}{6}$ parts of a whole on a line, then we have the figure below.



b) You can also use a paper strip to model and represent the fractions $\frac{1}{6}$ *and* $\frac{1}{7}$ *as*;



c) For Cuisenaire rods, once we know which of the rods represents 1, we can use it to compare and represent other fractions. Take a look at the illustration below.



Now, model fractions of a similar nature, such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{7}$, $\frac{5}{2}$, etc. using paper strips, Cuisenaire rods, number lines, etc.

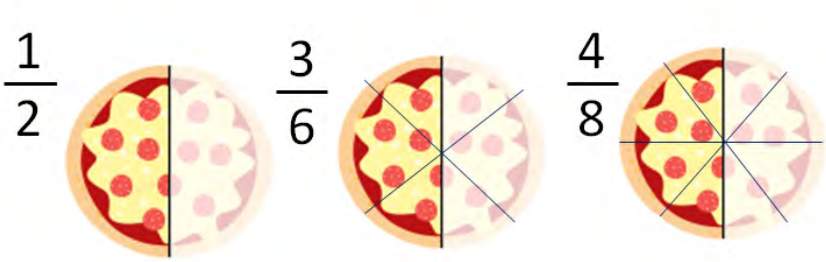
Dear learner! I hope you can now perform the activity using these resources and other appropriate resources.

Benchmark fractions are simple, commonly used fractions that are easy to visualize and serve as reference points for estimating or comparing other fractions. The most frequently used benchmark fractions include:

1. 0 (zero)
2. $\frac{1}{4}$ (One-quarter / one-fourth)
3. $\frac{1}{2}$ (one-half)
4. $\frac{3}{4}$ (Three-fourths)
5. 1 (one whole)

Equivalent Fractions

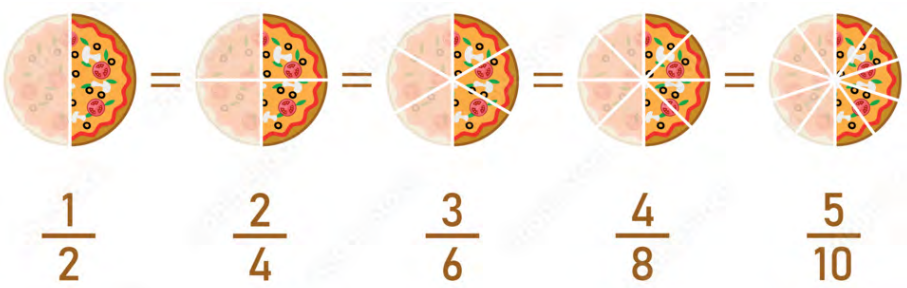
Equivalent fractions are different fractions that represent the same value or proportion. They have different numerators and denominators but simplify to the same fraction. For example, $\frac{1}{2}, \frac{3}{6},$ *and* $\frac{4}{8}$ are all equivalent fractions because they all represent the same portion of a whole.



Looking at the three fractions, you realise that although they all have different numerators and denominators, they are the same since they represent the same portion. Therefore, the three fractions are equivalent.

Establish equivalent fractions from benchmark fractions using interactive approaches.

Take a look at this example, we can create equivalent fractions by multiplying the numerator and the denominator by the same factor.



i. $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10} \cdots$

Take a look at these other examples;

ii. $\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \cdots$

iii. $\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \cdots$

iv. $\frac{1}{5}, \frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \cdots$

v. $\frac{1}{6}, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \cdots$

Alright Learner! I hope you can create more equivalent fractions.

Tip!!

You can also generate equivalent fractions by dividing the numerator and the denominator by a common factor. For example;

$\ldots \frac{12}{36}, \frac{6}{18}, \frac{3}{9}, \frac{1}{3}$

Activity 2.2: Equivalent Fractions Matching Game

Materials:

- Fraction cards (with fractions in various forms)
- Blank cards to write fractions
- Grid sheets for recording results
- Scissors, markers, and rulers (for making cards)

1/2	2/4	3/6
4/8	1/3	2/6
3/9	4/5	8/10

Instructions:

1. **Preparation**:
    - Have a partner or work individually.
    - Obtain fraction cards or design your fraction cards. The cards should contain various fractions (e.g., $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, etc.) and blank cards to allow you to generate your own equivalent fractions during the activity.
    - Note that the fraction cards you design should include some equivalent fractions but mixed with non-equivalent ones.
2. **Matching Game**:
    - **Step 1**: Pick the fraction cards.
    - **Step 2**: Find all the equivalent fractions within their set of cards. They must match fractions that are equivalent by comparing the values (e.g., $\frac{1}{2} = \frac{2}{4}$).
    - **Step 3**: After finding a match, you must explain why the fractions are equivalent by simplifying one fraction or multiplying both the numerator and denominator by the same number/factor.
3. **Challenge Round** (Creating Equivalent Fractions):
    - **Step 1**: Once you have matched all equivalent fractions, take the blank cards and create your own equivalent fractions.

- **Step 2**: Swap their newly created equivalent fractions with someone to verify whether the fractions are truly equivalent.

4. Reflection

- Explain how you understand equivalent fractions and tell how you feel about the activity.

Rules for operations on fractions

Let us now establish some basic rules necessary for operations on fractions.

Rule 1: Addition and *subtraction* of fractions are possible with a common denominator.

Rule 2: When we *multiply* two fractions, then the numerators are multiplied together as well as the denominators are multiplied together.

Rule 3: When we *divide* a fraction by another fraction, we have to find the reciprocal of the second fraction and then use Rule 2 above.

Adding Fractions

Like fractions: The addition of fractions is easy when they have a common denominator, meaning the denominators are the same.

For example,

1. $\frac{3}{7} + \frac{1}{7}$

2. $\frac{2}{3} + \frac{1}{3}$

Solution

We need to add the numerators since the denominators are common (ie, the same). Thus,

1. $\frac{3}{7} + \frac{1}{7} = \frac{3+1}{7} = \frac{4}{7}$

2). $\frac{2}{3} + \frac{1}{3} = \frac{2+1}{3} = \frac{3}{3} = 1,$

Unlike fractions: Adding fractions with different denominators.

Example

1) $\frac{1}{4}+\frac{1}{2}$

Solution

One of the denominators is a factor or multiple of the other and we need to adjust the denominators to be the same before adding. Thus,

$$\frac{1}{4}+\frac{1}{2}=\frac{1}{4}+\frac{1}{2}\times\frac{2}{2}=\frac{1}{4}+\frac{2}{4}=\frac{1+2}{4}=\frac{3}{4}$$

2) $\frac{1}{3}+\frac{3}{4}=?$

Solution

The denominators are neither factors nor multiples of the other and we need to adjust the denominators to be the same before adding. Thus, multiply $\frac{1}{3}$ *by* $\frac{4}{4}$ and $\frac{3}{4}\times\frac{3}{3}$

$$\frac{1}{3}+\frac{3}{4}=\frac{1}{3}\times\frac{4}{4}+\frac{3}{4}\times\frac{3}{3}=\frac{4}{12}+\frac{9}{12}=\frac{4+9}{12}=\frac{13}{12}$$

Using equivalent fractions to make unlike fractions common before adding them.

Example

$\frac{1}{3}+\frac{3}{4}$

Solution

Using equivalent fractions of $\frac{1}{3}=\frac{2}{6}=\frac{3}{9}=\frac{4}{12}=\frac{5}{15}=\frac{6}{18}=\ldots$

Similarly, $\frac{3}{4}=\frac{6}{8}=\frac{9}{12}=\frac{12}{16}=\frac{15}{20}=\frac{18}{24}=\ldots$

Fractions with common denominators are $\frac{4}{12}$ and $\frac{9}{12}$.

Implies $\frac{1}{3}+\frac{3}{4}=\frac{4}{12}+\frac{9}{12}=\frac{13}{12}$

$\therefore \frac{1}{3}+\frac{3}{4}=\mathbf{\frac{13}{12}}$

Using Lowest Common Multiples (LCM) to solve addition of fraction problems:

Add the fractions $\frac{1}{5} + \frac{5}{8}$.

We need to simplify them by finding the LCM of denominators $\frac{1}{5} + \frac{5}{8}$ and then making it common for both fractions.

$$\text{LCM} = \frac{1}{5} + \frac{5}{8} = \frac{8 + 25}{40} = \frac{33}{40}$$

Subtraction of fractions with same denominators

If the denominators are same, so we can simply subtract the numerators.

Example

$$\frac{5}{7} - \frac{2}{7} = \frac{5-2}{7} = \frac{3}{7}$$

Subtraction of fractions with different denominators

If the denominators of two fractions are different, we need to simplify them by finding the least common multiple (LCM) of the denominators and making them the same for both fractions.

Example

1. $\frac{2}{3} - \frac{1}{4}$

The two denominators are 3 and 4. Hence, LCM of 3 and 4 *is* 12

Therefore, multiplying $\frac{2}{3}$ by $\frac{4}{4}$ and $\frac{1}{4}$ by $\frac{3}{3}$, we get; $\frac{8}{12} - \frac{3}{12} = \frac{5}{12}$

Multiplication of Fractions

Example

1. $\frac{2}{5} \times \frac{3}{4}$

Here we simply multiply the numerators together and then the denominators together. There is no need to have a common denominator.

Solution

$$\frac{2}{5} \times \frac{3}{4} = \frac{2 \times 3}{5 \times 4} = \frac{6}{20} = \frac{3}{10}$$

Division of Fractions

Example

1. $\frac{2}{3} \div \frac{3}{4}$

Here we think ***Keep***, ***Change***, ***Flip***.

This means we ***Keep*** the first fraction the same, we ***Change*** the sign from division to multiply and we ***Flip*** the second fraction, so its denominator becomes the numerator and the numerator becomes the denominator.

Solution

$\frac{3}{7} \div \frac{3}{4} = \frac{3}{7} \times \frac{4}{3} = \frac{12}{21} = \frac{4}{7}$

Activity 2.3

1. If I have a fraction bar and I shade in two out of the four equal parts, what fraction of the whole bar is shaded?
2. Can you show on the fraction circle what $\frac{3}{5}$ would look like?
3. If I have a fraction bar that is divided into eight equal parts, and I shade in six of those parts, what fraction of the bar is shaded? Is this fraction in its simplest form?
4. If I have a square and I divide it into four equal parts, what fraction does each part represent?
5. We have a rectangle, and we divide it into six equal parts horizontally. How would you represent $\frac{2}{6}$ of the rectangle?
6. If we partition a circle into ten equal slices and shade in three of them, what fraction of the circle is shaded?

Activity 2.4

1. You have a large pizza with 8 slices, and you want to share it equally among 4 friends. How would you represent the fraction of the pizza that each friend receives? Is this fraction in its simplest form?

2. If you have a chocolate bar divided into 12 equal pieces and you want to share it equally among yourself and two friends, what fraction of the chocolate bar would each person get? Is this fraction in its simplest form?
3. You have a rectangular pizza that is cut into 10 equal slices. If you eat 3 slices, what fraction of the pizza have you eaten?
4. You order a round pizza that is divided into 6 equal slices. If you eat 2 slices and your friend eats 3 slices, what fraction of the pizza has been eaten altogether?
5. If a recipe calls for $\frac{1}{2}$ cup of flour and you want to make a double batch, how much flour would you need in total?
6. If a recipe requires $\frac{3}{4}$ cup of sugar, but you only want to make half of the recipe, how much sugar would you use?

Note: You may replace 'pizza' with appropriate food in your community. An example is Fufu.

Activity 2.5: Fraction Challenge: Mastering Operations with Fractions

You will practice adding, subtracting, multiplying, and dividing fractions through a fun, hands-on activity with your classmates.

Station 1: Addition of Fractions

At this station, you will be given fraction cards. Follow these steps:

1. Pick two cards randomly from the deck.
2. Add the two fractions together. If the denominators are different, find the least common denominator (LCD) and make the denominators the same before adding.
3. Simplify the fraction to its lowest terms.
4. Each group member will complete two rounds of addition.

Example

You pick $\frac{1}{4}$ *and* $\frac{2}{3}$.

- Find the LCD of 4 and 3, which is 12.
- Convert the fractions to $\frac{3}{12}$ *and* $\frac{8}{12}$.
- Add $\frac{3}{12} + \frac{8}{12} = \frac{11}{12}$.

- Your answer is $\frac{11}{12}$.

Station 2: Subtraction of Fractions

Here, you will practice subtracting fractions. Follow these steps:

1. Pick two fraction cards from the deck.
2. Subtract the second fraction from the first. If the denominators are different, find the LCD and adjust the fractions before subtracting.
3. Simplify the fraction to its lowest terms.
4. Each group member will complete two rounds of subtraction.

Example

You pick $\frac{5}{6}\, and\ \frac{1}{3}$.

- Find the LCD of 6 and 3, which is 6.
- Convert the fractions to $\frac{5}{6}$ *and* $\frac{2}{6}$.
- Subtract 5 /6 $- \frac{2}{6} = \frac{3}{6}$.
- Simplify to $\frac{1}{2}$.

Station 3: Multiplication of Fractions

At this station, you will practice multiplying fractions. Follow these steps:

1. Pick two fraction cards from the deck.
2. Multiply the numerators together and the denominators together.
3. Simplify the fraction to its lowest terms.
4. Each group member will complete two rounds of multiplication.

Example

You pick $\frac{2}{5}\, and\ 3\frac{}{4}$.

- Multiply the numerators: $2 \times 3 = 6$.
- Multiply the denominators: $5 \times 4 = 20$.
- The product is $\frac{6}{20}$, which simplifies to $\frac{3}{10}$.

Station 4: Division of Fractions

Here, you will practice dividing fractions. Follow these steps:

1. Pick two fraction cards from the deck.
2. Flip the second fraction (take its reciprocal) and multiply it by the first fraction.
3. Simplify the fraction to its lowest terms.
4. Each group member will complete two rounds of division.

Example

You pick $\frac{3}{8}$ *and* $\frac{4}{5}$.

- Take the reciprocal of $\frac{4}{5}$, which is $\frac{5}{4}$.
- Multiply $38 \times 54 = \frac{15}{32}$.
- Your answer is $\frac{15}{32}$ (already in lowest terms).

Final Challenge: Fraction Word Problems

After all groups have rotated through each station, you will complete a final challenge together. Your teacher will give your group 3 real-life word problems involving addition, subtraction, multiplication, or division of fractions. Work as a team to solve these problems, showing all your steps clearly.

Reflection and Sharing

Once all groups have completed the stations and final challenge, we will come together as a class to discuss what you found challenging and how you solved the different types of problems. You'll also have the opportunity to explain your thinking to the class!

EXTENDED READING

1. Watch this video on the concept of fractions using the link provided https://youtu.be/kZzoVCmUyKg?si=Z28AI72cTWGCB4OC.
2. Watch this video on the concept and operations on fractions using the link provided. https://youtu.be/wrQTYCkmI3c?si=h_Syh7dTcKPyg1n8.
3. Watch this video on operations on fractions using the link provided. https://youtu.be/iabIxg7ET5Q?si=fDdPeOxFj2PDQrSe.

THE CONCEPT AND OPERATIONS OF FRACTIONS 2

In this lesson, we are going to look at the connections between Fractions and Decimals in mathematics.

Fractions and decimals are essential tools for representing quantities that are not whole numbers. In our daily lives, we encounter fractions and decimals in various contexts, such as cooking recipes (where ingredients are often measured in fractions), financial transactions (where prices and quantities are expressed as decimals), and measurements (where lengths, weights, and volumes are commonly represented using fractions or decimals)

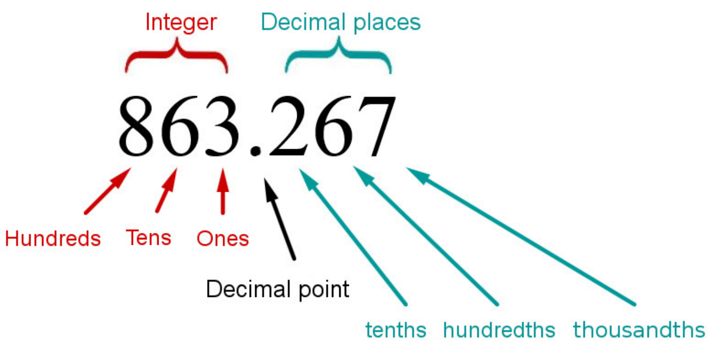
Multi-purpose model charts, such as number lines, grids, and diagrams provide visual representations that aid in understanding the relationships between fractions, decimals, and their inverses. By using these visual aids, you can develop a deeper conceptual understanding of how fractions and decimals relate to each other and how their additive and multiplicative inverses can be determined.

Understanding these concepts is crucial not only for basic arithmetic but also for more advanced mathematical reasoning and problem-solving skills, paving the way for success in future academic and professional endeavours.

Problem-Solving on Common Fractions

Here we will explore how to convert decimals to fractions and vice versa.

Decimals: In mathematics, decimals are numbers that consist of both a whole number part and a fractional part, separated by a decimal point. The dot that divides the whole part from the fractional part is known as the decimal point. Decimals can represent whole numbers (without a decimal point), fractions (with a decimal point), and may either repeat a pattern (repeating decimals) or not (non-repeating decimals).



From the above illustration, 863 represents the whole part and the 267 after the decimal point represents the fractional part.

Examples of decimals include

a) 35.7 (equivalent to $\frac{357}{10}$)

b) 0.5 (equivalent to $\frac{1}{2}$)

c) 3.142 (approximation of pi (π))

d) 0.75 (equivalent to $\frac{3}{4}$)

e) 2.012.

Establishing decimals as fractions

Hello learner! To express decimals as common fractions, below are the general steps to guide you.

Step 1: count the number of decimal places and express it as a power of 10. That is $10^{\mathbf{n}}$, where **n** is the number of decimal places counted.

Step 2: remove the decimal point from the number to make it a whole number

Step 3: divide the whole number by a power of 10 obtained in step 1.

Step 4: express the fraction obtained in its lowest form.

I hope you have carefully studied the steps. Let us now look at some examples;

1. **Convert 0.5 into a fraction**.

   Step 1: count the number of decimal places in 0.5. Since 0.5 has only one decimal place, the power of 10 is written as 10^1 which is the same as 10

   Step 2: remove the decimal point from the number to make it a whole number

   Removing the decimal point from 0.5 gives us 05 which is the same as 5.

   Step 3: divide the whole number by 10^n, where n is the number of decimal places counted

   $0.5 = \frac{5}{10}$

   Step 4: express the fraction obtained in its lowest form.

   $\frac{5}{10} = 1 =$

2. **Express 0.25 as a fraction**

   Removing the decimal point gives us 025 which is the same as 25 and since

0.25 has two decimal places, we divide the whole number by 10^2 which is also the same as 100

$0.25 = \frac{025}{10^2} = \frac{25}{100} = \frac{1}{4}$

3. **Express 2.689 as a fraction**

Solution

Let's count the number of decimal places in 2.689. This gives us $10^3 = 1000$

Removing the decimal point out of the number also gives us 2689

- $\frac{2689}{1000}$ and this is in its simplest form

Establishing fractions as decimals

Dear learner, can you now establish fractions as decimals? Good.

To express a fraction as a decimal number, multiply the numerator and denominator by a common natural number such that the denominator becomes a power of 10. The power of 10 must be a multiple of the denominator and this indicates the number of decimal places the numerator should be expressed.

Click on the link below to watch how to convert common fractions to decimals

https://youtu.be/lh2mp0aqSh8

Here are some worked examples;

- $\frac{1}{2} = \frac{1}{2} \times \frac{5}{5} = \frac{5}{10} = 0.5$
- $\frac{1}{4} = \frac{1}{4} \times \frac{25}{25} = \frac{5}{100} = 0.25$
- $\frac{2}{25} = \frac{2}{25} \times \frac{4}{4} = \frac{8}{100} = 0.08$

Look for other examples, try them and compare your answers to those of your friends.

Activity 2.6

Establish $\frac{1}{2}$ as decimal.

Dear learner here is an activity to guide you.

a) Take a square paper and divide it into 10 equal parts.

b) Consider each square as 0.1

c) fold the paper into two equal parts and shade one part.

d) You will observe that each half contains exactly 5 boxes. This represents 0.5.

Thus, $\frac{1}{2} = 0.5$

I hope you have found this activity interesting. Try more examples.

Percentages

Percentage: it is a number that can be expressed as a fraction out of a hundred.

Percentage also represents the number of parts in every 100. We use fractions and percentages to describe parts of shapes, quantities and measures.

Example

40%, 2.5%, 75%, $2\,3\frac{}{4}$% etc.

Converting Percentages to fractions

Have you tried converting your exam scores from percentage to its fractional value or vice versa?

Study the examples below and carry out the activities that follow.

- $50\,\% = 50 \times \frac{1}{100} = \frac{50}{100} = \frac{1}{2}$
- $25\,\% = 25 \times \frac{1}{100} = \frac{25}{100} = \frac{1}{4}$
- $80\,\% = 80 \times \frac{1}{100} = \frac{80}{100} = \frac{4}{5}$
- $0.4\,\% = \frac{4}{10} \times \frac{1}{100} = \frac{4}{1000} = \frac{1}{250}$

To convert a percentage to a fraction, rewrite the figure of the percentage as "out of 100"

That is $\frac{3}{4}\%$ means "three-fourth of a hundred" or $\frac{3}{4} \div \mathbf{100} = \frac{3}{4} \times \frac{1}{100}$

Activity 2.7

Let us explore some examples in the activities below.

Here are some steps to guide you

1. Convert $12\,1\frac{}{2}$% into a fraction.

Solution:

Consider the steps below:

Step 1: Convert $12\frac{1}{2}\%$ into an improper fraction.

Thus, $12\frac{1}{2}\% = \frac{25}{2}\%$

Step 2: Replace the percent symbol (%) with $\frac{1}{100}$.

So, $12\frac{1}{2}\% = \frac{25}{2} \div 100 = \frac{25}{2} \times \frac{1}{100} = \frac{5}{200}$

Step 3: Reduce it to the lowest form, to get $12\frac{1}{2}\% = \frac{1}{8}$.

Conclude that $12\frac{1}{2}\% = \frac{1}{8}$

2. Convert $\frac{75}{2}\%$ into a fraction

Solution

$\frac{75}{2}\% = \frac{75}{2} \times \frac{1}{100} = \frac{5}{200} = \frac{3}{8}$

3. Your friend spends $\frac{4}{3}\%$ of her pocket money with you during break. What fraction does that represent?

Solution

$\frac{4}{3}\% = \frac{4}{3} \times \frac{1}{100} = \frac{4}{300} = \frac{1}{75}$

Now, can you try converting these fractions back to percentages?

Converting a fraction to a percentage only requires the multiplication of the fraction by 100.

Examples

- $\frac{1}{2} = 0.5 = \frac{5}{10} \times 100 = 50\%$ or $\frac{1}{2} \times 100 = 50\%$
- $\frac{1}{4} = 0.25 = \frac{25}{100} \times 100 = 25\%$
- $\frac{3}{10} = \frac{3}{10} \times 100 = 30\%$
- $\frac{1}{75} = \frac{1}{75} \times 100 = \frac{4}{3}\%$
- $\frac{1}{8} \times 100 = \frac{100}{8} = \frac{25}{2} = 12\frac{1}{2}\%$

Example

Amewu scored $\frac{11}{20}$ in a class test. What would be his score as a percentage?

Solution

$\frac{11}{20} = \frac{11}{20} \times 100 = 55\%$

Activity 2.8

Establish 25% as a fraction

a) Take a 10 by 10 square paper.

b) Consider each square as 1unit.

c) Count and shade 25 boxes. This represents 25 out of a hundred. That is $\frac{25}{100}$

d) You will observe that the 25 boxes represent one-quarter of the square paper. This represents $\frac{1}{4}$

Thus, $25\,\% = \frac{25}{100} = \frac{1}{4}$

Addition and Subtraction of Fractions

Hello learner! In the previous lesson you learned how to perform operations on proper and improper fractions. In this lesson, we shall look at addition and subtraction of fractions where the fractions are mixed numbers.

Let's now consider the addition and subtraction of mixed numbers by following these activities.

Example 1

Add $3\frac{2}{5}$ to $5\frac{2}{3}$,

Solution

Step I: Add the whole numbers and then add the fractions separately as shown below;

$(3 + 5) + \left(\frac{2}{5} + \frac{2}{3}\right) = 8 + \left(\frac{6}{15} + \frac{10}{15}\right) = 8 + \frac{6 + 10}{15}$

Step II: Simplify to get $= 8 + \frac{16}{15}$

Step III: Convert the improper fraction into a mixed number $= 8 + 1 + \frac{1}{15} = 9 + \frac{1}{15}$

Step IV: Simplify to get $9\frac{1}{15}$.

Example 2

In an athletic competition, Jamal covered a distance of $3\frac{1}{2}$km while Ama covered a distance of $3\frac{5}{6}$ km. Determine how far Jamal is behind Ama.

Solution

We need to find the difference in their distances $\Rightarrow 3\frac{1}{2} - 3\frac{5}{6}$

Step 1: Subtract the whole numbers and then subtract the fractions separately as shown below;

$$(3 - 3) + \left(\frac{1}{2} - \frac{5}{6}\right) = 0 + \left(\frac{3}{6} - \frac{5}{6}\right) = 0 + \frac{3-5}{6}$$

Step II: Simplify to get $= 0 + \left(-\frac{2}{6}\right) = -\frac{2}{6} = -\frac{1}{3}$

Therefore, Jamal was behind Ama by $\frac{1}{3}$km

Example 3.

Elinam was asked by her teacher to stand exactly halfway between the numbers $2\frac{6}{7}$ *and* $2\frac{11}{28}$ on the tape measure provided. On what number on the tape measure is the teacher expecting her to stand?

Solution

Halfway between $2\frac{6}{7}$ and $2\frac{11}{28}$

$$\Rightarrow \frac{\left(2\frac{6}{7} + 2\frac{11}{28}\right)}{2}$$

$$\Rightarrow \frac{(2+2) + \left(\frac{6}{7} + \frac{11}{28}\right)}{2}$$

$$\Rightarrow \frac{4 + \left(\frac{24}{28} + \frac{11}{28}\right)}{2}$$

$$\Rightarrow \frac{4 + \frac{35}{28}}{2}$$

$$\Rightarrow \frac{4 + \frac{5}{4}}{2} = \frac{4}{2} + \left(\frac{5}{4} \div 2\right)$$

$\Rightarrow 2 + \frac{5}{8}$

$\Rightarrow 2\frac{5}{8}$

Example 4.

Abubakari has investments in two different rural banks in his community. If one investment yields 3 $1\frac{}{4}$% interest and the other yields $2\frac{3}{4}$% interest annually, what will be his total interest each year?

Solution

Interest on investment one $=3\frac{1}{4}\%$

Interest on investment two $= 2\frac{3}{4}\%$

Total interest $=3\frac{1}{4}\% + 2\frac{3}{4}\%$

$= (3 + 2) + \left(\frac{1}{4} + \frac{3}{4}\right)$

$= 5 + \left(\frac{1 + 3}{4}\right)$

$= 5 + (1) = 6\%$

Therefore, Abubakari's total interest each year is 6%

Additive and Multiplicative Inverses of Fractions Using Multi-Purpose Model Charts

In the previous section, you learned additive and multiplicative inverses of numbers and their examples. In this section, we are going to apply them as we look at the additive inverse and multiplicative inverse of fractions.

Additive inverse(s):

Hello Learner! Have you ever added two fractions and the result is zero (0)?

A number that, when added to the original number, results in a sum of zero is called the additive inverse of the original number.

i.e. If $\frac{a}{b}$ + c-= 0, $\frac{c}{d}$ is additive inverse of $\frac{a}{b}$. Eg. If $\frac{3}{2} + \left(-\frac{3}{2}\right) = 0$, then $\left(-\frac{3}{2}\right)$ is the additive inverse of $\frac{3}{2}$.

Click on the link below to watch a video on additive inverse of fractions:

https://youtu.be/vrTm_fMO47c

Examples

1. The additive inverse of $\frac{1}{2}$ is $-\frac{1}{2}$, because $\frac{1}{2} + \left(-\frac{1}{2}\right) = 0$
2. The additive inverse of $-\frac{3}{5}$ is $\frac{3}{5}$, because $-\frac{3}{5} + \left(\frac{3}{5}\right) = 0$
3. $2\frac{3}{4}$ is the additive inverse of $-2\frac{3}{4}$, because $2\frac{3}{4} + \left(-2\frac{3}{4}\right) = 0$
4. If a balance scale has a weight of $\frac{5}{8}$ pounds on one of its sides, what weight must be added to the other side to balance it?

Solution

Since one side has a weight of $\frac{5}{8}$ pounds, balancing it requires that you take the weight off the scale

$\Rightarrow \frac{5}{8} + \left(-\frac{5}{8}\right) = \frac{5}{8} - \left(\frac{5}{8}\right) = 0$

Multiplicative inverse(s):

Can you think of a pair of fractions whose product is 1?

A number that, when multiplied by an original number, results in a product of 1 is called the multiplicative inverse of the original number.

i.e. if $\frac{a}{b}$ × c-= 1, then $\frac{c}{d}$ is a multiplicative inverse of a-or vice versa

E.g. $\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$, hence $\frac{2}{3}$ is a multiplicative inverse of $\frac{3}{2}$ or vice versa

Study the Multipurpose fractional chart below on multiplicative inverses and establish the pairs of numbers whose product is always 1, some of them are already done for you.

$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$
$\frac{2}{1}$	$\frac{2}{2}$	$\frac{2}{3}$	$\frac{2}{4}$	$\frac{2}{5}$	$\frac{2}{6}$	$\frac{2}{7}$	$\frac{2}{8}$	$\frac{2}{9}$	$\frac{2}{10}$
$\frac{3}{1}$	$\frac{3}{2}$	$\frac{3}{3}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{3}{6}$	$\frac{3}{7}$	$\frac{3}{8}$	$\frac{3}{9}$	$\frac{3}{10}$
$\frac{4}{1}$	$\frac{4}{2}$	$\frac{4}{3}$	$\frac{4}{4}$	$\frac{4}{5}$	$\frac{4}{6}$	$\frac{4}{7}$	$\frac{4}{8}$	$\frac{4}{9}$	$\frac{4}{10}$
$\frac{5}{1}$	$\frac{5}{2}$	$\frac{5}{3}$	$\frac{5}{4}$	$\frac{5}{5}$	$\frac{5}{6}$	$\frac{5}{7}$	$\frac{5}{8}$	$\frac{5}{9}$	$\frac{5}{10}$
$\frac{6}{1}$	$\frac{6}{2}$	$\frac{6}{3}$	$\frac{6}{4}$	$\frac{6}{5}$	$\frac{6}{6}$	$\frac{6}{7}$	$\frac{6}{8}$	$\frac{6}{9}$	$\frac{6}{10}$
$\frac{7}{1}$	$\frac{7}{2}$	$\frac{7}{3}$	$\frac{7}{4}$	$\frac{7}{5}$	$\frac{7}{6}$	$\frac{7}{7}$	$\frac{7}{8}$	$\frac{7}{9}$	$\frac{7}{10}$
$\frac{8}{1}$	$\frac{8}{2}$	$\frac{8}{3}$	$\frac{8}{4}$	$\frac{8}{5}$	$\frac{8}{6}$	$\frac{8}{7}$	$\frac{8}{8}$	$\frac{8}{9}$	$\frac{8}{10}$
$\frac{9}{1}$	$\frac{9}{2}$	$\frac{9}{3}$	$\frac{9}{4}$	$\frac{9}{5}$	$\frac{9}{6}$	$\frac{9}{7}$	$\frac{9}{8}$	$\frac{9}{9}$	$\frac{9}{10}$
$\frac{10}{1}$	$\frac{10}{2}$	$\frac{10}{3}$	$\frac{10}{4}$	$\frac{10}{5}$	$\frac{10}{6}$	$\frac{10}{7}$	$\frac{10}{8}$	$\frac{10}{9}$	$\frac{10}{10}$

Dear learner, I hope you have explored the Multipurpose fractional chart and identified numerous pairs of numbers that are the multiplicative inverses of each other. Now compare your answers to that of your friends.

Examples

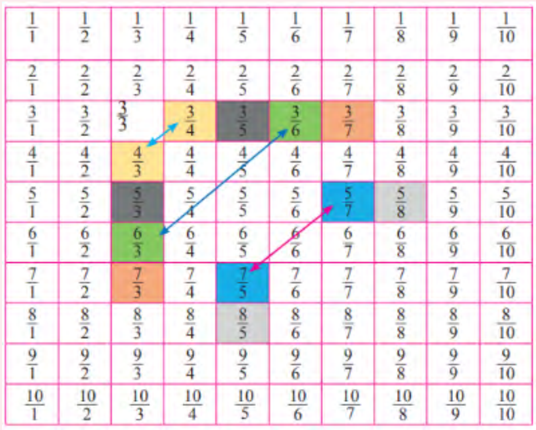
1. $\frac{5}{8} \times \frac{8}{5} = 1$, here multiplicative inverse of $\frac{5}{8}$ is $\frac{8}{5}$ since their product is 1.
2. $\frac{2}{7} \times \frac{7}{2} = 1$, similarly, multiplicative inverse of $\frac{2}{7}$ is $\frac{7}{2}$ since their product is 1.
3. $\frac{1}{3} \times \frac{3}{1} = 1$, also, multiplicative inverse of $\frac{1}{3}$ is $\frac{3}{1}$ since their product is 1.

After going through these examples, I hope you have made some observations and generalizations. Share them with your friend/class teacher.

Activity 2.9: Exploring Additive and Multiplicative Inverses of Fractions Using Multi-Purpose Model Charts

Objective:

You will use multi-purpose model charts to practice finding the additive and multiplicative inverses of fractions. This hands-on activity will help you understand the concept of inverses in fractions and how they work.



Materials:

- Multi-purpose model charts (fraction strips, fraction circles, or grids)
- A set of fraction cards
- Pencils and paper
- Calculator (optional)

Part 1: Additive Inverses of Fractions

The **additive inverse** of a fraction is a number that, when added to the fraction, results in zero.

Steps:

1. **Group Activity**: Form groups of 3-4 students. Each group will receive a set of fraction cards and a multi-purpose model chart (fraction strips or grids).
2. **Pick a Fraction**: Draw a fraction card from the deck. For example, you pick $\frac{2}{5}$.
3. **Visual Representation**: Use your model chart to identify the fraction $\frac{2}{5}$. Thus, colour the $\frac{2}{5}$ on the chart.
4. **Find the Additive Inverse**: Now, find the fraction that, when added to $\frac{2}{5}$, results in zero.
   - o **Hint**: The additive inverse of $\frac{2}{5}$ *is* $-\frac{2}{5}$.
5. **Demonstrate with the Model**: Solve to show that $\frac{2}{5} + \left(-\frac{2}{5}\right) = 0$. Discuss with your group how the two fractions cancel each other out to give zero.

6. **Repeat**: Each group member should pick a different fraction card, represent it on the chart, and find its additive inverse.

Part 2: Multiplicative Inverses of Fractions

The **multiplicative inverse** (or reciprocal) of a fraction is a number that, when multiplied by the fraction, results in 1.

Steps:

1. **Pick a Fraction**: Draw a new fraction card from the deck. For example, you pick $\frac{3}{4}$
2. **Visual Representation**: Use the model chart to represent the fraction $\frac{3}{4}$.
3. **Find the Multiplicative Inverse**: Now, find the fraction that, when multiplied by $\frac{3}{4}$, gives 1.
   - **Hint**: The multiplicative inverse of $\frac{3}{4}$ *is* $\frac{4}{3}$.
4. **Demonstrate with the Model**: Discuss with your group how multiplying $\frac{3}{4} \times \frac{4}{3}$ results in 1. You can use multiplication of the numerators and denominators to show that the product is $\frac{12}{12} = 1$.
5. **Repeat**: Each group member should pick a different fraction, represent it on the chart, and find its multiplicative inverse.

Extra Challenge:

After completing both parts of the activity, as a group, answer the following challenge questions:

1. What is the additive inverse of $\frac{7}{8}$? How do you know?
2. Find the multiplicative inverse of $\frac{5}{6}$. Use multiplication to verify your answer.
3. What happens if you multiply a fraction by its additive inverse? Discuss the result with your group.

EXTENDED READING

1. Baffour Ba Series: Core Mathematics for Schools and Colleges (Pages 40 – 41)
2. Click on the link below to watch a video on the multiplicative inverse of fractions: https://youtu.be/nE7LwfhhPpE

OPERATIONS ON COMMON FRACTIONS

Hello, learner! Today, we're going to explore the exciting world of connections between fractions, percentages, and decimal numbers. This topic is a crucial milestone in our mathematical journey, as it reveals the relationships between these fundamental concepts.

We will delve into the equivalencies and conversions between these representations, discovering how they are used in different contexts. We will build on our previous knowledge of fractions, decimals, and percentages, and see how they fit together like pieces of a puzzle.

We will work together to build a strong foundation, and I encourage you to ask questions, share your ideas, and learn from each other.

Connections Between and Among Fractions, Percentages and Decimal Numbers

Converting percentages into fractions

To convert a percentage into a fraction, write the figure of the percentage as "out of 100"

Example

Convert the following percentages into fractions:

a) 25%

b) 50%

c) 9%

Solution

a) 25%

$$\frac{25}{100} = \frac{25 \div 25}{100 \div 25} = \frac{1}{4}$$

Thus, 25% as a fraction is $\frac{1}{4}$

b) 50%

$$\frac{50}{100} = \frac{50 \div 50}{100 \div 50} = \frac{1}{2}$$

Thus, 50% as a fraction is $\frac{1}{2}$

c) 9%

$\frac{9}{100}$

Thus, 9% as a fraction is $\frac{9}{100}$

Converting a fraction into a percentage

To turn a fraction into a percentage, we multiply the fraction by 100 and express in % form.

Example

Convert the following fractions into percentages:

a) $\frac{2}{5}$

b) $\frac{17}{20}$

c) $\frac{1}{6}$

Solution

a) $\frac{2}{5}$

$\frac{2}{5} \times 100 = \frac{2}{5} \times \frac{100}{1} = \frac{200}{5} = 40\%$

Thus, $\frac{2}{5}$ as a percentage is 40%.

b) $\frac{17}{20}$

$\frac{17}{20} \times 100 = \frac{17}{20} \times \frac{100}{1} = \frac{1700}{20} = 85\%$

Thus, $\frac{17}{20}$ as a percentage is 85%.

c) $\frac{1}{6}$

$\frac{1}{6} \times 100 = \frac{1}{6} \times \frac{100}{1} = \frac{100}{6} = \frac{100 \div 2}{6 \div 2} = \frac{50}{3} = 16.67\%$

Thus, $\frac{1}{6}$ as a percentage is 16.67%.

Converting from decimals to fractions

1. Write the decimal as a fraction:
   Place the decimal number over 1 (e.g., 0.75 becomes 0.75/1).
2. Multiply numerator and denominator: Multiply both by 10 for each digit after the decimal point (e.g., 0.75/1 has two numbers after the decimal point, so we multiply by 100 and it becomes 75/100).
3. Simplify the fraction: Reduce the fraction by dividing both the numerator and the denominator by their greatest common divisor (e.g., 75/100 simplifies to 3/4)

Example

Convert the following decimals to fractions

a) 0.3

b) 0.05

Solution

a) 0.3

$0.3 = \frac{0.3}{1} = \frac{0.3 \times 10}{1 \times 10} = \frac{3}{10}$

Therefore, the decimal 0.3as a fraction is $\frac{3}{10}$

b) 0.05

$0.05 = \frac{0.05}{1} = \frac{0.05 \times 100}{100} = \frac{5}{100}$

Therefore, the decimal 0.05 as a fraction is $\frac{5}{100}$, and this simplifies to

$\frac{5 \div 5}{100 \div 5} = \frac{1}{20}$.

Converting from fractions to decimals

Example

Convert the following fractions to decimals:

a) $\frac{1}{2}$

b) $\frac{1}{5}$

Solution

a) $\frac{1}{2}$

$\frac{1}{2} = 0.5$

Therefore, $\frac{1}{2}$ as a decimal is 0.5 (You may use a calculator and do the calculation $1 \div 2 = 0.5$)

b) $\frac{1}{5}$

$\frac{1}{5} = 0.2$

Therefore, $\frac{1}{5}$ as a decimal is 0.2

Converting from decimals to percentages

To convert a decimal to a percentage, you multiply the decimal by 100 and then add the percentage symbol (%).

Example

Convert the following decimals to percentages:

a) 0.2

b) 0.04

c) 0.52

Solution

a) 0.2

$0.2 \times 100 = 20\%$

Therefore, 0.2 as a percentage is 20%

b) 0.04

$0.04 \times 100 = 4\%$

Therefore, 0.04 as a percentage is 4%

c) 0.52

$0.52 \times 100 = 52\%$

Therefore, 0.52 as a percentage is 52%

Converting from percentages to decimals

To convert a percentage to a decimal, you divide the percentage by 100.

Example

Convert the following percentages to decimals:

a) 75%

b) 25%

c) 9%

Solution

a) 75%

$\frac{75}{100} = 0.75$ (You may use a calculator)

Therefore, 75% as a decimal is 0.75

b) 25%

$\frac{25}{100} = 0.25$

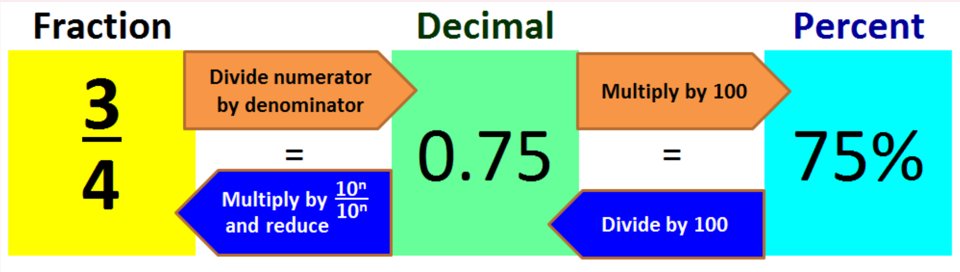
Therefore, 25% as a decimal is 0.25

c) 9%

$\frac{9}{100} = 0.09$

Therefore, 9% as a decimal is 0.09

Activity 2.10: Converting Between Fractions, Decimals, and Percentages



Objective: In this activity, you will practice converting fractions to decimals, decimals to percentages, percentages to decimals, and percentages to fractions. This will help you understand the relationship between these forms and apply them to real-life situations.

Instructions:

1. **Work in pairs or groups** to complete the following tasks. For each task, show your steps clearly.

Task 1: Converting Fractions to Decimals

Convert the following fractions to decimals:

- $\frac{1}{4}$
- $\frac{2}{5}$
- $\frac{7}{8}$
- $\frac{3}{10}$

Steps:

Divide the numerator (top number) by the denominator (bottom number) to get a decimal.

Write down the decimal equivalent of each fraction.

Task 2: Converting Decimals to Percentages

Convert the following decimals to percentages:

- 0.25
- 0.75
- 0.5
- 0.2

Steps:

Multiply the decimal by 100 to convert it to a percentage.

Add a percentage sign (%) to your answer.

Task 3: Converting Percentages to Decimals

Convert the following percentages to decimals:

- 25%
- 40%
- 75%
- 12.5%

Steps:

Divide the percentage by 100 to convert it to a decimal.

Write your answer as a decimal without the percentage sign.

Task 4: Converting Percentages to Fractions

Convert the following percentages to fractions:

- 50%
- 80%
- 25%
- 60%

Steps:

Write the percentage as a fraction over 100.

Simplify the fraction by dividing both the numerator and denominator by their greatest common factor (GCF).

Task 5: Converting Fractions to Percentages

Convert the following fractions to percentages:

- $\frac{1}{2}$
- $\frac{3}{4}$
- $\frac{5}{8}$
- $\frac{7}{10}$

Steps:

Divide the numerator by the denominator to get a decimal.

Multiply the decimal by 100 to convert it to a percentage.

Task 6: Real-Life Application

Scenario: You are helping your parents calculate discounts during a sale at the market. The original price of a bag of rice is 80 cedis, and the store offers a 15% discount.

1. Calculate how much money you will save with the discount.
2. Find the new price of the bag of rice after applying the discount.

Steps:

- Convert the percentage discount to a decimal.
- Multiply the decimal by the original price to find the amount saved.
- Subtract the discount from the original price to find the new price.

Task 7: Convert Between All Forms

Pick any two of the numbers below and convert it into all forms (fraction, decimal, and percentage):

- 0.6
- 35%
- $\frac{4}{5}$

Steps:

- Start with any form (fraction, decimal, or percentage).
- Convert it to the other two forms following the methods you practised in the previous tasks.

Solving fractions involving the four basic operations

When solving problems involving the four basic operations, you must follow the BODMAS or PEDMAS principle carefully. Let us take a look at these examples;

Example 1

Solve the following fraction problem using BODMAS principles:

Problem:

$\frac{3}{4} + \left(\frac{2}{5} \times \frac{7}{8}\right) - \frac{1}{3}$

Step-by-step solution:

1. **Apply the BODMAS principle**:

   BODMAS stands for **Brackets**, **Orders** (powers and roots), **Division**, **Multiplication**, **Addition**, and **Subtraction**. First, we solve the expression inside the brackets and perform any multiplication before addition and subtraction.

2. **Solve the expression inside the brackets**:

   Inside the brackets, we have $\frac{2}{5} \times \frac{7}{8}$:

   $\frac{2}{5} \times \frac{7}{8} = \frac{2 \times 7}{5 \times 8} = \frac{4}{40} = \frac{7}{20}$

3. **Rewrite the expression**: Now that we have solved the multiplication inside the brackets, the expression becomes:

   $\frac{3}{4} + \frac{7}{20} - \frac{1}{3}$

4. **Find a common denominator**:

   The denominators are 4, 20, and 3. The least common denominator (LCD) of 4, 20, and 3 is 60.

   - Convert 3/4 to a fraction with a denominator of 60:

     $\frac{3}{4} = \frac{3 \times 15}{4 \times 15} = \frac{45}{60}$

   - Convert $\frac{7}{20}$ to a fraction with a denominator of 60:

     $\frac{7}{20} = \frac{7 \times 3}{20 \times 3} = \frac{21}{60}$

   - Convert $\frac{1}{3}$ to a fraction with a denominator of 60:

     $\frac{1}{3} = \frac{1 \times 20}{3 \times 20} = \frac{20}{60}$

5. **Rewrite the expression with the common denominator**:

   $\frac{45}{60} + \frac{21}{60} - \frac{20}{60}$

6. **Perform the addition and subtraction**:

   - First, add $\frac{45}{60} + \frac{21}{60}$:

     $\frac{45 + 21}{60} = \frac{66}{60} = \frac{11}{10}$

   - Now, subtract $\frac{11}{10} - \frac{20}{60}$: Convert $\frac{11}{10}$ to a fraction with a denominator of 60:

     $\frac{11}{10} = \frac{11 \times 6}{10 \times 6} = \frac{66}{60}$

     Then: $\frac{66}{60} - \frac{20}{60} = \frac{66 - 20}{60} = \frac{46}{60} = \frac{23}{30}$

7. **Final answer**: The result is $\frac{23}{30}$.

Example 2

Solve the following fraction problem using BODMAS principles:

Problem:

$\frac{5}{6} \div \frac{2}{3} + \left(\frac{1}{4} \times \frac{3}{5}\right) - \frac{1}{2}$

Step-by-step solution:

1. **Apply the BODMAS principle**:

   First, solve the expression inside the brackets and then handle division and multiplication before addition and subtraction.

2. **Solve the expression inside the brackets**:

   Inside the brackets, we have $\frac{1}{4} \times \frac{3}{5}$:

   $\frac{1}{4} \times \frac{3}{5} = \frac{1 \times 3}{4 \times 5} = \frac{3}{20}$

3. **Rewrite the expression**:

   After solving the multiplication in the brackets, the expression becomes:

   $\frac{5}{6} \div \frac{2}{3} + \frac{3}{20} - \frac{1}{2}$

4. **Perform the division**:

   Now, we perform the division $\frac{5}{6} \div \frac{2}{3}$

   When dividing fractions, multiply by the reciprocal of the divisor:

   $\frac{5}{6} \div \frac{2}{3} = \frac{5}{6} \times \frac{3}{2} = \frac{5 \times 3}{6 \times 2} = \frac{5}{12} = \frac{5}{4}$

5. **Rewrite the expression**: Now the expression becomes:

   $\frac{5}{4} + \frac{3}{20} - \frac{1}{2}$

6. **Find a common denominator**:

   The denominators are 4, 20, and 2. The least common denominator (LCD) is 20.

   - Convert $\frac{5}{4}$ to a fraction with a denominator of 20:

     $\frac{5}{4} = \frac{5 \times 5}{4 \times 5} = \frac{25}{20}$

   - Convert $\frac{1}{2}$ to a fraction with a denominator of 20:

     $\frac{1}{2} = \frac{1 \times 10}{2 \times 10} = \frac{10}{20}$

7. **Rewrite the expression with the common denominator**:

   $\frac{25}{20} + \frac{3}{20} - \frac{10}{20}$

8. **Perform the addition and subtraction**:

   First, add $\frac{25}{20} + \frac{3}{20}$:

   $\frac{25+3}{20} = \frac{28}{20} = \frac{7}{5}$

   Now, subtract $\frac{7}{5} - \frac{10}{21}$: Convert $\frac{7}{5}$ to a fraction with a denominator of 20:

   $\frac{7}{5} = \frac{7 \times 4}{5 \times 4} = \frac{28}{20}$

   Then:

   $\frac{28}{20} - \frac{10}{20} = \frac{28-10}{20} = \frac{18}{20} = \frac{9}{10}$

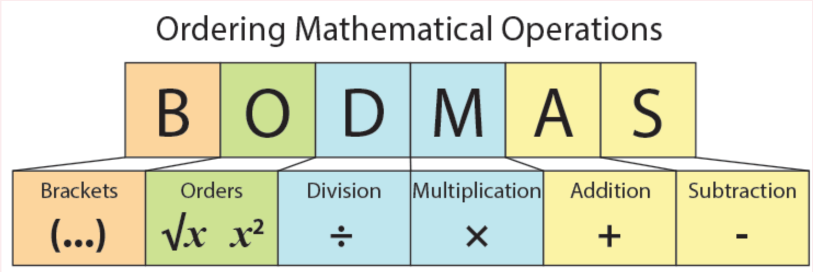
9. **Final answer**: The result is $\frac{9}{10}$.

Activity 2.11: Solving Fractions Using BODMAS Rule

Objective: In this activity, you will practice solving fraction problems that involve all four basic operations (addition, subtraction, multiplication, and division) using the BODMAS/PEDMAS rule. This will help you understand how to apply the correct order of operations when working with fractions.

Instructions:

1. **Work individually, in pairs or in groups** to solve the following problems. Be sure to follow the correct order of operations using the BODMAS rule:



2. **Show all steps clearly** for each problem, and make sure you simplify your answers as much as possible.

Task 1: Problem 1 (Brackets first)

Solve the following expression:

$$\left(\frac{3}{4}+\frac{1}{2}\right)\times\frac{5}{6}$$

Steps:

- First, solve inside the brackets by adding $\frac{3}{4}$ *and* $\frac{1}{2}$
- Then, multiply the result by $\frac{5}{6}$.

Task 2: Problem 2 (Division and Multiplication first)

Solve the following expression:

$$\frac{3}{5}\div\frac{1}{4}\times\frac{2}{3}$$

Steps:

- First, perform the division of $\frac{3}{5}$ by $\frac{1}{4}$ by multiplying $\frac{3}{5}$ by the reciprocal of $\frac{1}{4}$.
- Then, multiply the result by $\frac{2}{3}$.

Task 3: Problem 3 (Mixed operations)

Solve the following expression:

$$\frac{5}{8}-\left(\frac{1}{4}\times\frac{2}{5}\right)+\frac{3}{7}$$

Steps:

- First, solve the multiplication inside the brackets.
- Then, perform the subtraction $\frac{5}{8}$ – (result from brackets).
- Finally, add $\frac{3}{7}$ to the result.

Task 4: Problem 4 (Combining all operations)

Solve the following expression:

$$\left(\frac{2}{3}+\frac{1}{6}\right) \div \frac{7}{8} - \frac{1}{4} \times \frac{3}{5}$$

Steps:

- First, solve inside the brackets by adding $\frac{2}{3}$ *and* $\frac{1}{6}$.
- Then, divide the result by $\frac{7}{8}$.
- After that, perform the multiplication $\frac{1}{4} \times \frac{3}{5}$.
- Finally, subtract the result of the multiplication from the division.

Task 5: Problem 5 (Challenging mixed operation)

Solve the following expression:

$$\frac{9}{10} + \left(\frac{1}{3} \times \frac{4}{9}\right) \div \left(\frac{7}{12} - \frac{1}{6}\right)$$

Steps:

- First, solve the multiplication inside the first set of brackets.
- Then, solve the subtraction inside the second set of brackets.
- Perform the division of the two results from the brackets.
- Finally, add $\frac{9}{10}$ to the result.

Task 6: Real-Life Problem

Scenario: You and your friend are sharing a pizza. You ate $\frac{3}{8}$ of the pizza, and your friend ate $\frac{5}{12}$. Later, another friend came and ate $\frac{1}{6}$ of what was left. How much of the pizza is left?

Steps:

- First, add the amounts you and your friend ate: $\frac{3}{8} + \frac{5}{12}$.
- Subtract this sum from 1 (the whole pizza).
- Then, subtract the $\frac{1}{6}$ eaten by your other friend from the remaining amount.

Task 7: Final Reflection

After completing the tasks, **discuss with your group**:

- Which operation was the most difficult to apply?

- How did using the BODMAS rule help you solve these problems?
- Share your strategies for simplifying fractions and working through multi-step problems.

Convert fractions from one form into other forms

When you are asked to convert fractions from one form to another, it typically means transforming the fraction from one representation to another while maintaining its equivalent value.

Improper Fractions to Mixed Fractions

To change an improper fraction to a mixed fraction, you need to follow these steps:

a) Divide the numerator by the denominator.

b) The quotient (result of the division) will be the whole number part of the mixed fraction.

c) The remainder will be the numerator of the fractional part.

d) The denominator of the fractional part will be the same as the original denominator.

Example

Convert the following improper fraction into a mixed fraction

a) $\frac{5}{3}$

b) $\frac{7}{2}$

Solution

a) Given: $\frac{5}{3}$

$\frac{5}{3} = 5 \div 3 = 1, \textit{remainder } 2, = 1\,\frac{2}{3}$

Therefore, the improper fraction $\frac{5}{3}$ can be expressed as the mixed number 1 $\frac{2}{3}$

b) Given: $\frac{7}{2}$

$\frac{7}{2} = 7 \div 2 = 3, \textit{remainder } 1 = 3\frac{1}{2}$

Therefore, the improper fraction $\frac{7}{2}$ can be expressed as the mixed number 3

$1\frac{}{2}$

Mixed fractions to improper fractions

To change a mixed fraction to an improper fraction, you need to follow these steps:

a) Multiply the whole number part (the integer) by the denominator (the bottom number).

b) Add the numerator (the top number) to the product from step 1.

c) Keep the denominator (the bottom number) the same.

Example

Convert the following mixed fractions into improper fractions

a) $3\frac{1}{4}$

b) $5\frac{3}{4}$

Solution

a) Given: $3\frac{1}{4}$

$3\frac{1}{4} = \frac{3 \times 4 + 1}{4} = \frac{13}{4}$

Therefore, the mixed fraction $3\frac{1}{4}$ can be expressed as the improper fraction $\frac{13}{4}$

b) Given: $5\frac{3}{4}$

$5\frac{3}{4} = \frac{5 \times 4 + 3}{4} = \frac{23}{4}$

Therefore, the mixed fraction $5\frac{3}{4}$ can be expressed as the improper fraction $\frac{23}{4}$

Activity 2.12

Otonko has $\frac{1}{4}$ of a piece of *Kenkey* left over from last night's dinner. His friend, Raymond, brings over $\frac{1}{6}$ of a piece of *Kenkey* that he didn't finish from his lunch.

a) How much Kenkey will Otonko and Tinyor have in total if they combine their leftovers?

b) Convert the total fraction of their leftovers to a percentage.

c) Convert the total fraction of their leftovers to a decimal.

Steps:

i. *Identify the fractions*

ii. *Find the lowest common multiple (LCM) of 4 and 6*

iii. *Convert each fraction to have a common denominator*

iv. *Add the fractions*

v. *Multiply the results by 100 to convert to a percentage*

vi. *Divide the numerator by the denominator to convert to a decimal*

Activity 2.13

Amina wants to prepare *shito* for her son, Abu, for school. The usual recipe for shito requires $\frac{3}{4}$ tablespoon of pepper. However, Amina wants to make only $\frac{2}{3}$ of the usual quantity. How much pepper will she need?

Steps

i. *Identify the quantities involved.*

ii. *Determine the fractions to multiply.*

iii. *Multiply the numerators and denominators.*

iv. *Reduce the fraction to its simplest form if necessary*

Activity 2.14

The library at Nchumuruman Community SHS has a bookshelf that is $\frac{3}{4}$ occupied by books. If the bookshelf can hold a total of 24 books, how many more books can be added to the bookshelf?

Steps:

Step 1: Understand the Problem:

- The bookshelf can hold a total of 24 books.
- Currently, $\frac{3}{4}$ of the bookshelf is occupied by books.
- You need to find out how many more books can be added to the bookshelf.

Step 2: Calculate the Number of Books Currently on the Bookshelf:

- To find out how many books are currently on the bookshelf, multiply the total capacity by the fraction that is occupied.

Step 3: Perform the Multiplication:

- Multiply the total number of books (24) by the fraction occupied $\frac{3}{4}$

Step 4: Calculate the Number of Books that Can Still Be Added:

- Subtract the number of books currently on the bookshelf from the total capacity

EXTENDED READING

1. Baffour – Ba Series: Core Maths for Schools and Colleges, (Pages 28 - 30)
2. Akrong Series: Core Mathematics for Senior High Schools (6th ed.), (Pages 30 – 31)
3. Click on the link attached to watch a video on how to convert percentages to fractions and follow a step-by-step demonstration of this calculation. https://youtu.be/gUlbT4-NEdg
4. Click on the link attached to watch a video on how to convert fractions to decimals using long division method and follow a step-by-step demonstration of this calculation: https://youtu.be/do_IbHId2Os
5. Click on the link attached to watch a video on how to divide fractions and follow a step-by-step demonstration of this calculation: https://youtu.be/4lkq3DgvmJo
6. Click on the link attached to watch a video on how to multiply fractions and follow a step-by-step demonstration of this calculation: https://youtu.be/vn7AC43cmZ0

APPLICATION OF PERCENTAGES 1

Hello, learner! We are going to explore the powerful world of percentages. Percentages are a fundamental concept in mathematics that helps us understand proportions, changes, and relationships in various aspects of life. We will delve into the application of percentages, learning how to calculate and interpret them in real-world contexts. You will discover how percentages are used in finance, business, science, and everyday life to make informed decisions, compare data, and measure growth.

1. **Percentage increase** refers to how much a value has grown, expressed as a percentage of its original value. It measures the relative increase from an initial amount to a new amount.

   Percentage increase $= \frac{increase}{original\ price} \times 100\%$

2. **Percentage decrease** refers to the reduction in a value, expressed as a percentage of its original value. It measures the relative decrease from an initial amount to a new, smaller amount.

   Percentage decrease $= \frac{decrease}{original\ price} \times 100\%$

3. **Commission** is a fee or percentage of a sale amount paid to a salesperson or agent for facilitating the sale of goods or services. It is usually calculated as a percentage of the total sales made.
4. **Discount**: A discount is a reduction in the original price of a product or service. It is often offered to encourage purchases and is usually expressed as a percentage of the original price.
5. **Profit**: Profit is the financial gain made when the selling price of a product or service is greater than its cost price. It is the difference between the revenue from sales and the total expenses incurred.
6. **Percentage profit** is the profit expressed as a percentage of the cost price. It is calculated using the formula:

   Percentage profit $= \left(\frac{profit}{cost\ price}\right) \times 100\%$

7. **Loss**: Loss occurs when the selling price of a product or service is less than the cost price. It is the amount of money lost in a transaction when the cost exceeds the revenue generated from the sale.

Application of the Concept in Daily Activities and Examples

Examples

1. In July 2021 there was an increment in the price of diesel per litre from GH¢9.50 to GH¢13.20, which later declined to GH¢12.50. Find

a. percentage increase

b. percentage decrease.

Solution

$$\text{Percentage increase} = \frac{increment}{original\ price} \times 100\,\%$$
$$= \frac{13.2 - 9.5}{9.5}$$
$$= \frac{3.7}{9.50} \times 100\,\%$$
$$= 38.95\,\%.$$

$$\text{Percentage decrease} = \frac{decrement}{original\ price} \times 100\,\%$$
$$= \frac{13.2 - 12.5}{13.2}$$
$$= \frac{0.7}{13.20} \times 100\,\%$$
$$= 5.30\%$$

2. Safia deposited $GH¢4{,}000.00$ into a bank account and the annual simple interest rate is $8\,\%$. How much interest is added to the account after 4 years?

Solution

Principal $= GH¢4{,}000.00$, rate $= 8\,\%$, time $= 4$ years

$$\text{Interest} = \frac{PTR}{100} = \frac{Principal \times Time \times Rate}{100} = \frac{GH¢\ 4{,}000.00 \times 8 \times 4}{100} = GH¢1{,}280$$

Profit and loss

1. A shopkeeper bought $1kg$ of apples for $GH¢100.00$ and sold it for $GH¢120.00$ per kg. Calculate the profit gained by the shopkeeper?

Solution

Cost Price for apples is GH¢100.00,

Selling Price for apples is GH¢120.00,

Then profit gained by shopkeeper is; $P = SP - CP$

$P = 120 - 100 = \text{GH¢}20.00$

2. Referring to the above example, calculate the percentage of the profit gained by the shopkeeper.

Solution

We know, Profit percentage $= \left(\frac{profit}{cost\ price}\right) \times 100\%$

Therefore, Profit percentage $= \left(\frac{\text{GH¢}20.00}{\text{GH¢}100.00}\right) \times 100\% = 20\,\%$.

3. At Maame Adisa's retail shop, the marked price of an item is GH¢ 1700.00. If a customer pays GH¢1540.00 for the item after discount, then calculate the discount.

Solution

$Given$: $Marked\ Price = GH¢\ 1700$

$Selling\ Price = GH¢\ 1540$

By using the formula,

$Selling\ price = Marked\ price - Discount$

So, $discount = Marked\ Price - Selling\ Price$

$Discount = 1700 - 1540 = GH¢\ 160$

$\therefore$ The discount is GH¢ 160.00

4. Fafa bought a fan for GH¢1000 and sold it at a loss of 15%. Calculate the selling price of the fan?

Solution

Cost Price of the fan is GH¢1000.00

Loss percentage is 15%

As we know, Loss percentage = $\left(\frac{profit}{cost\ price}\right) \times 100\%$

$15 = \left(\frac{LOSS}{1000}\right) \times 100\%$

Therefore, Loss = GH¢150.00

As we know,

$Loss = Cost\ Price - Selling\ Price$

So, $Selling\ Price = Cost\ Price - Loss$

$= 1000 - 150$

$Selling\ Price =$ GH¢850.00

Alternatively, $Loss = 15\%\ of\ 1000 = 150$ therefore, $SP = 1000 - 150 = 850$

Calculating Commission and Discount

1. An item is marked at GH¢ 990.00. If a percentage discount allowed on the item is 10%, find the selling price.

Solution

Given: Marked price = GH¢ 990.00

$discount = 10\%$

Let the percentage of discount be x

$\therefore x = 10\%$

By using the formula,

Discount = $\frac{(\text{Marked price} \times \text{discount }\%)}{100\%} = \frac{990 \times 10}{100\%} = GH¢\ 99.00$

Now, $Selling\ price = Marked\ price - Discount = 990 - 99 = GH¢\ 891.00$

$\therefore$ The selling price is $GH¢$ 891.00

Note This!!

Alternatively, we can find 10% and subtract or find 90% of the amount as we pay 100 – 10, = 90% of the full amount.

2. At Melcom Shopping Mall, a discount of 20% is allowed on an item selling at GH¢ 900.00. Find the marked price.

Solution

Given: Selling $Price = GH¢$ 900.00

Discount = 20%

Now, let us consider the marked price as GH¢ ×.

Given discount is 20% on the marked price.

Discount $= \frac{20x}{100} = 0.2x$

By using the formula,

Selling Price = Marked Price – Discount

$900 = \times - 0.2x$

$900 = 0.8x$

$x = \frac{900}{0.8}$

$= GH¢1125$

∴ The marked price is $GH¢$ 1125.00

Alternatively, 80% = 900 [100 - 20% = 80%] So 100% = 900 * 100/80 = $GH¢$ 1125.00

Alternatively, we use proportions to find the marked price as follows

80% = 900 [100 – 20% = 80%]

So 100% = 900 * 100/80 = $GH¢$ 1125.00

3. Christopher sold food grains for $GH¢9200$ as an agent. He received a commission at a rate of 2%. How much commission did he get?

Solution

Given: $Selling\ Price = GH¢\ 9200.00$

Commission rate = 2%

By using the formula, $Commission = Commission\ Rate \times Selling\ Price$

$= (2/100) \times 9200$

$=$ GH¢ 184.00

∴ The agent got a commission of GH¢ 184.00

4. Kande sold flowers worth GH¢ 15,000.00 by giving 4% commission to the agent. Find

a. the commission she paid,

b. the amount received by Kande.

Solution

Given: Selling Price = GH¢ 15000.00

Commission rate = 4%

By using the formula,

Commission = Commission Rate × Selling Price

$$\text{Commission} = \frac{4}{100} \times 15000$$
$$= \text{GH¢ } 600$$

Kande paid a commission of $GH¢$ 600.00

Amount received by Kande = Selling Price – Commission

$Amount\ received = 15000 - 600 = GH¢\ 14400.00$

∴ The amount received by Kande is GH¢14400.00

Activity 2.15

The price of a laptop was initially GH¢ 4,800. After a 20% discount, the laptop's price was reduced. What is the new price of the laptop after the discount?

Follow the steps to solve the problem:

GH¢4,800

Step 1: Find the amount of the discount

= 20% of GH¢4,800

= (20/100) × GH¢4,800

= GH¢960

Step 2: Subtract the discount from the original price

= GH¢4,800 - GH¢960

= GH¢3,840

Therefore, the new price of the laptop after the discount is GH¢3,840.

Here's the summary:

Original price = GH¢4,800

Discount = 20% of GH¢4,800 = GH¢960

New price = GH¢4,800 - GH¢960 = GH¢3,840

Alternatively, for percentage decrease, we can subtract to get $100\% - 20\%$ $= 80\% = 0.8$

Now, by multiplying $0.8\times$ $GH¢4{,}800.00 = GH¢3{,}840.00$

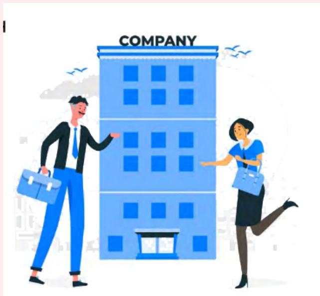
Activity 2.16

A company experienced a 15% increase in sales this year compared to the previous year. If the sales for the previous year were GH¢8 million, what were the sales for the current year?

Follow the steps to solve the problem:

Step 1: Identify the original value (previous year's sales) = GH¢8,000,000

Step 2: Identify the percentage increase = 15%



15% INCREASE

Step 3: Calculate the increase in value = Original value × Percentage increase

= GH¢8,000,000 × (15/100)

= GH¢1,200,000

Step 4: Calculate the new value (current year's sales) = Original value + Increase in value

= GH¢8,000,000 + GH¢1,200,000

= GH¢9,200,000

Therefore, the sales for the current year are GH¢9,200,000.

Alternatively, for a percentage increase, we can add to get $100\% + 15\% = 115\% = 1.15$

Now, by multiplying 1.15× *GH¢*8,000,000= *GH¢*9,200,000

Activity 2.17

A market stall in Korkorse offers a 15% discount on all student bags. If a bag originally costs GH¢50, how much will you pay after the discount?

Follow the steps below to solve this problem:

Step 1: Identify the original price = GH¢50

Step 2: Identify the discount percentage = 15%

Step 3: Calculate the discount amount = Original price × Discount Percentage

= GH¢50 × (15/100)

= GH¢7.50

Step 4: Calculate the new price = Original price - Discount amount

= GH¢50 - GH¢7.50

= GH¢42.50

Answer: You will pay GH¢42.50 for the bag after the discount.

Activity 2.18

Mr. Gregory Gbandi who is a yam farmer in Kpassa made a profit of GH¢10,000 from his yam farm after sales. If he has to pay a 15% tax on his profit, how much tax will he pay?

Follow the steps below to solve this problem:

Step 1: Identify the profit = GH¢10,000

Step 2: Identify the tax percentage = 15%

Step 3: Calculate the tax amount = Profit × Tax percentage

= GH¢10,000 × (15/100)

= GH¢10,000 × 0.15

= GH¢1,500.00

Answer: Mr Gregory Gbandi will pay GH¢1,500 as tax.

Activity 2.19

Mr. Tinyor Raymond who is a shoemaker in Prestea sold a pair of shoes for GH¢600 and incurred a cost of GH¢480 to produce them. Calculate the profit or loss.

Follow the steps below to solve this problem:

Step 1: Identify the selling price = GH¢600

Step 2: Identify the cost price = GH¢480

Step 3: Calculate the profit = Selling price - Cost price

= GH¢600 - GH¢480

= GH¢120

Answer: Mr. Tinyor Raymond made a profit of GH¢120.00

Activity 2.20: Application of Percentages in Real-Life Scenarios

Objective: In this activity, you will apply the concepts of percentages to solve real-life problems related to discounts, commissions, percentage increases and decrease, profit, loss, and their percentages. By completing the tasks, you will understand how percentages are used in everyday situations like shopping, sales, and business.

Materials Needed:

- Calculator
- Notebook
- Pen or pencil

Task 1: Calculating **Discount**

Problem: You want to buy a new phone that originally costs GHS 1,200, but the store is offering a 15% discount on all electronics.

1. Calculate the amount of the discount.
2. Determine how much you will pay after the discount is applied.

Task 2: Determining **Commission**

Problem: A real estate agent sells a house for GHS 250,000. The agent earns a 3% commission on the sale.

1. Calculate the commission the agent will earn.
2. If the agent's monthly target is to earn GHS 10,000 in commissions, how many houses of this price must they sell to meet their target?

Task 3: Percentage Increase in Rent

Problem: Your rent last year was GH¢ 800 per month, but this year your landlord increases it by 12%.

1. Calculate the new rent after the increase.
2. By how much has your rent increased in cedis?

Task 4: Calculating **Profit**

Problem: You buy 100 shirts at GH¢ 20 each and sell them for GH¢ 30 each.

1. Calculate the total cost of buying the shirts.
2. Calculate the total revenue from selling the shirts.
3. Determine the total profit made from selling all the shirts.

Task 5: Calculating Loss

Problem: You bought 50 pairs of shoes for GH¢ 80 each but could only sell them for GH¢ 60 each.

1. Calculate the total cost of buying the shoes.
2. Calculate the total revenue from selling the shoes.
3. Determine the total loss.

EXTENDED READING

1. Aki – Ola Series: Core Mathematics for SHS, Revised Edition, (Pages 295 – 325)
2. Akrong Series: Core Mathematics for Senior High Schools, 6th ed. (Pages 352 – 387)
3. Baffour Ba Series: Core Mathematics for Schools and Colleges (Pages, 363, 458)
4. Click on the link below to watch how to calculate Percentage increase and decrease. https://youtu.be/MTxNPBupKMo
5. https://youtu.be/FdSVZ5cvhi4

SIMPLE AND COMPOUND INTERESTS

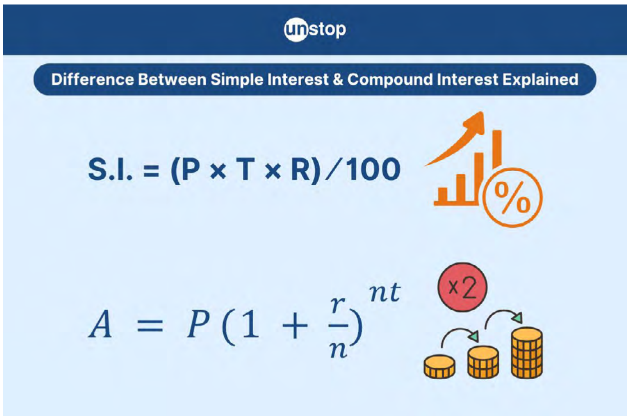
Hello, learner! We are going to explore the world of interest - Simple and Compound Interest. This topic is a vital milestone in our mathematical journey, as it helps us understand how money grows and how financial decisions impact our lives.

Simple and compound interest are fundamental concepts in personal finance, economics, and business. Understanding how interest works is crucial for making informed decisions about saving, investing, borrowing, and managing debt.

We will be learning how to calculate simple and compound interest, understanding the differences between them, and exploring real-world applications, such as:

Saving for goals, like college or retirement, Investing in stocks or real estate, understanding credit card debt and loans and analyzing the impact of inflation and interest rates on personal finance.

In this lesson, we shall look at Simple and Compound interest and applications involving percentages.



Application of Percentages 2: Problem-Solving Involving Percentages

Hello learner, in week 6 of this section, we learnt the meaning of percentage increase and percentage decrease, profit and loss. In this week, we shall explain the meaning of the following terms: Principal, Interest, Time, Rate, Simple Interest and compound Interest. Problems involving real-life applications of these terms will also be discussed.

Principal: This is the sum of money lent or borrowed or invested initially. The principal is denoted by the letter P.

Interest: This is the extra money paid for taking the money as a loan. This is often expressed as a percentage.

Rate: Rate is the rate of interest at which the principal amount is given to someone for a certain time, the rate of interest can be 5% , 10% , *or* 13% , *etc*. The rate of interest is denoted by the letter R or r.

Time: Time is the duration for which the principal amount is given to someone. Time is denoted by the letter T or t.

Simple Interest

Simple interest is a type of interest that is calculated only on the principal amount of a loan or deposit, without taking into account any interest that has accumulated previously. It is called "simple" because the interest is applied only to the original amount borrowed or deposited, and it doesn't compound over time.

To put it concisely, Simple Interest is a method of interest that always applies to the original, 'principal', amount, with the same rate of interest for every time cycle. It can be calculated with the formula:

$\textit{Simple Interest}, S.I. = \frac{(P \times R \times T)}{100}$ or P × R × T, where

$P = \textit{Principal}$, R *Rate of Interest in % per annum*, and $T =$ number of years. The rate of interest is in percentage is written as $\frac{R}{100}$.

Through change of subject, the formula $S.I. = \frac{(P \times R \times T)}{100}$ can be written as

$P = \frac{(100 \times S.I.)}{R \times T}$, $R = \frac{(100 \times S.I.)}{P \times T}$ Or $T = \frac{(100 \times S.I.)}{P \times R}$

$\textit{Amount in account} = \textit{Principal} + \textit{Simple Interest}$

$\boldsymbol{A} = P + S.I.$

$\boldsymbol{A} = P + PTR.$

$\boldsymbol{A} = P(1 + TR).$

Example 1

Jane invests $GH¢25{,}000.00$ in a building society account. At the end of the year her account is credited with 2% interest. How much interest had her GH¢25,000.00 earned in the year?

Solution

$S.I. = \frac{(P \times R \times T)}{100}$

$$S.I. = \frac{(GH¢25{,}000.00 \times 2 \times 1)}{100} = GH¢500.00$$

Example 2

Adepa invested $GH¢280.00$ in a venture that pays $R\%$ interest. After the first year she received $GH¢5.60$ interest. What is the value of R, the rate of interest?

Solution

After one year, the interest rate is given by

$$R = \frac{(100 \times S.I.)}{P \times T}$$

$$R = \frac{(100 \times GH¢5.60.)}{GH¢280.00 \times 1}$$

$$R = \frac{(GH¢560.00)}{GH¢280.00}$$

$$R = 2$$

Therefore, the interest rate, R is 2%.

Example 3

A student plans to save money for their college education. They have a target to earn GHS 1,500 in interest over 4 years. The bank offers a simple interest rate of 5% per year. How much money should the student invest (as the principal) to reach their target?

Solution

We use the formula for simple interest:

$$I = \frac{P \times R \times T}{100}$$

Where:

- I is the interest earned (**GH¢** 1,500),
- P is the principal (the amount to be calculated),
- R is the rate of interest (5% per year),
- T is the time in years (4 years).

Step 1: Rearrange the formula to solve for P

$$P = \frac{I \times 100}{R \times T}$$

Step 2: Substitute the known values

$$P = \frac{1{,}500 \times 100}{5 \times 4}$$

$P = 150{,}00020$

Step 3: Calculate the value of P

P = 7,500

Final Answer: The student needs to invest **GH¢ 7,500** as the principal to earn GH¢ 1,500 in interest over 4 years at a 5% simple interest rate.

Example 4

A farmer borrows **GH¢** 8,000 from a bank to buy farming equipment. The bank charges a simple interest rate of 6% per year. The farmer pays **GH¢** 2,400 as interest. How long (in years) will it take the farmer to repay the loan with this amount of interest?

Solution

We use the formula for simple interest:

$$I = \frac{P \times R \times T}{100}$$

Where:

- III is the interest paid (**GH¢** 2,400),
- PPP is the principal (**GH¢** 8,000),
- RRR is the rate of interest (6% per year),
- TTT is the time in years (to be calculated).

Step 1: Rearrange the formula to solve for T

$$T = \frac{I \times 100}{P \times R}$$

Step 2: Substitute the known values

$$T = \frac{2{,}400 \times 100}{8{,}000 \times 6}$$

$$T = \frac{240{,}000}{48{,}000}$$

Step 3: Calculate the value of T

$T = 5$ years

Final Answer: It will take the farmer **5 years** to repay the loan with **GH¢** 2,400 in interest at a 6% simple interest rate.

Compound Interest

Compound interest is a type of interest where the interest earned on an investment or loan is added to the principal amount, and subsequent interest calculations are based on this new total. In other words, interest is earned on both the initial principal and the accumulated interest from previous periods. Compound interest allows for exponential growth of an investment or debt over time.

To put it concisely, Compound Interest**:** is an interest calculated on the principal and the existing interest together over a given time. In compound interest, the principal amount with interest after the first unit of time becomes the principal for the next unit.

For example, when compounded annually for 2 years, the principal amount with interest accrued at the end of first year becomes the principal for the second year.

Compound Interest Formula:

Total compounded amount $= p\left(1 + \frac{R}{100}\right)^{nT}$

Where:

- P is the principal amount
- R is the rate of interest
- n is the number of times the interest is compounded annually
- T is the overall tenure.

Compound Interest $= p\left(1 + \frac{R}{100}\right)^{nT} - p$

Amount per annum $= p(1 + r)^{T}$

Amount per semi-annual $= p\left(1 + \frac{r}{2}\right)^{2T}$

Amount per quarter-annual $= p\left(1 + \frac{r}{4}\right)^{4T}$

Amount per month $= p\left(1 + \frac{r}{12}\right)^{12T}$

Amount per day $= p\left(1 + \frac{r}{365}\right)^{365\mathrm{T}}$

Example 1

Calculate the compound interest on GH¢3000 at 5% for 2 years, compounded annually.

Solution

Amount with CI = 3000 (1+ 5/100)^2 = **GH¢3307.50**

CI = 3307.5 – 3000 = GH¢307.50

Therefore, the compound interest is GH¢307.50

Example 2

You invest GH€5000.00 at an annual compound interest rate of 6%. What will be the future value of your investment after 5 years?

Solution

Principal amount (P) = GH€5000.00

Annual interest rate (r) = 6% or 0.06

Time period (t) = 5 years

Formula for future value (FV) with annual compounding:

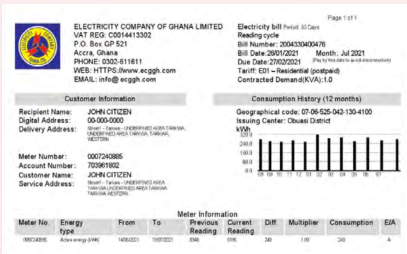
$FV = P(1 + r)^t$

Substituting the values: $FV = 5000 \times (1 + 0.06)^5 \approx$ GH€6691.13

Activity 2.21

Suppose a household has a total monthly income of GH¢3000.00. Their utility bills for the month are as follows:

Water bill: GH¢100.00



Electricity bill: GH¢150.00

Telephone bill: GH¢50.00

To analyze the proportion of each utility bill:

i) Water bill proportion $= \left(\dfrac{\textit{Water bill}}{\textit{Total income}}\right) \times 100$

$$= \left(\frac{100}{3000}\right) \times 100$$

$$= 3.33\%$$

ii) Electricity bill proportion $= \left(\frac{\textit{Electricity bill}}{\textit{Total income}}\right) \times 100$

$$= \left(\frac{150}{3000}\right) \times 100$$

$$= 5\%$$

iii) Telephone bill proportion $= \left(\frac{\textit{Telephone bill}}{\textit{Total income}}\right) \times 100$

$$= \left(\frac{50}{3000}\right) \times 100$$

$$= 1.67\%$$

Conclusion

Proportional reasoning allows for a clear understanding of how utility expenses contribute to the overall budget. By analysing proportions, individuals can identify areas where adjustments may be necessary to achieve better financial balance. This mini-project demonstrates the practical application of proportional reasoning in managing utility bills effectively.

Dear learner! I hope with the above sample activity, you can perform a similar activity using data from your community.

Activity 2.22: Banking Terms Matching

Match the banking terms with their definitions.

Banking Terms	Definitions
Fixed Deposit	The duration for which money is invested or borrowed, typically in years
Savings Account	The initial amount of money deposited or invested
Principal Amount	A type of account where individuals can deposit money and earn interest
Interest Rate	The percentage at which interest is calculated on the principal amount
Time	Financial product where a sum of money is deposited for a specific period at a fixed interest rate

Activity 2.23: Comparing different interest rates for various banks.

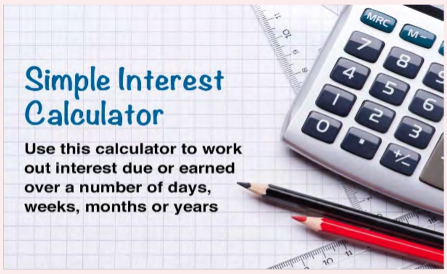
Study the table below and answer the questions that follow.

Bank	Annual Percentage Yield
Absa:	2.5%
Fidelity	3.0%
Bank of Africa	2.2%
Ghana Commercial Bank	2.8%
Bonzali Rural Bank	2.4%
Zenith Bank	3.2%

If Ama plans to start saving towards her future education at the tertiary level, what advice will you give her? Consider the following factors as well: account fees, minimum balance requirements, and customer service.

Activity 2.24: Amina's Savings Growth Challenge.

Calculate the interest Amina will earn for depositing money in a Bank with the information below.



Materials Needed:

- Calculator.
- Pen and paper.

Instructions:

1. Amina's initial deposit amount is GH¢1000.00.
2. Amina's annual interest rate is 4%.
3. Calculate the interest Amina will earn in one year using the formula:

   Interest = Principal × Rate × Time

   In this case, Interest = GH¢1000.00 × 0.04 × 1.
4. Perform the calculation to find the interest earned in one year.
5. To find the simple interest earned over 2 years, multiply the interest earned in one year by 2.

   Note: You may use a calculator to compare with your answer.
6. Write down or calculate the total interest earned over 2 years.

Congratulations! You've successfully completed Amina's Savings Growth Challenge.

EXTENDED READING

1. Watch this video (on simple interest) using the link provided: https://youtu.be/NCYNXkbTTUo
2. Watch this video (on simple interest) using the link provided:

   https://youtu.be/XjD4nl7Llww

REVIEW QUESTIONS

Review Questions 2.1

1. Amina designed a pie with four slices of equal dimensions. She decided to give out two slices of the pie to her son. Represent this with a fraction and put it in its simplest form.
2. If you owe someone $\frac{3}{4}$ of an amount of money and your friend owes $\frac{5}{7}$ of the amount, find the sum of the debt of you and your friend.
3. In a school choir of 45 members, $\frac{3}{5}$ of the members are female. How many female members are in the choir?
4. A bakery has a stock of 76 loaves of bread, and $\frac{3}{4}$ of them are whole wheat bread. How many loaves of whole wheat bread are there?
5. In a company of 120 employees, $\frac{3}{5}$ of them are full-time employees. How many temporary employees are there?
6. A library has a collection of 90 books, and $\frac{2}{3}$ of them are fiction books. How many non-fiction books are in the collection?

Review Questions 2.2

1. Naa–Ayorkor uses 0.75 cups of flour to prepare akple for her household. How many fractions of a cup is this?
2. A wholesale shop is giving out a discount of 0.4% of the original prices of its electrical gadgets. Determine what fraction it is?
3. A seamstress used $\frac{4}{15}$ of her 60 pins to hold a fabric for sewing. How many pins must she remove from the dress after sewing?
4. A carpenter cuts off $\frac{5}{8}$ inch of a piece of wood. Examine the length of wood he must add to return the wood to its original length?
5. Adobea's body temperature has dropped by $\frac{7}{10}$ degree Celsius, how much temperature rise is required to get her back to the original temperature?

6. Semenyo did $\frac{3}{5}$ of his house chores this morning before taking his breakfast. What percentage of the house chores has he done?

7. Using the expanded form, express the following decimal numbers in the form $\frac{a}{b}$, where $b \neq 0$

   Hint $0.75 = \left(75 \times \frac{1}{100} = \frac{75}{100} = \frac{3}{4}\right)$

   a) 0.45

   b) 1.25

   c) 45.5

Review Questions 2.3

1. Express 60% as a fraction in its simplest form.
2. Convert 0.8 to a proper fraction in its simplest form.
3. What is 3 out of 8 expressed as a percentage?
4. If 18 out of 30 students prefer gablee, what percentage of the students prefers gablee?
5. Express $\frac{5}{8}$ as a decimal (correct to two decimal places).
6. Suglo has 20 items on her shopping list. At the market, she realised from her list that she completed 40% of her shopping. Determine how many more items she has to buy?

Review Questions 2.4

1. A company's sales increased by 20% this year compared to the previous year. If the sales for the previous year were GH¢250,000, what were the sales for the current year?
2. A retail store purchases a television set for GH¢800 and plans to sell it with a mark-up of 30%. If the VAT rate is 12.5%, what will be the final selling price of the television set, including VAT?
3. A real estate agent earns a commission of 3% on the sale of a property. If the property was sold for GH¢450,000, how much commission did the agent receive?
4. A clothing store experienced a 25% decrease in sales during the previous year compared to the year before that. If the sales for the year before last were GH¢120,000, what were the sales for the previous year?

5. A household's monthly electricity bill is calculated based on the number of units consumed and the rate per unit. If the household consumed 600 units of electricity in a month and the rate per unit is GH¢0.75, calculate the total electricity bill for that month.
6. A family's monthly water bill consists of a fixed service charge of GH¢15 and a usage charge based on the number of cubic meters consumed. If the usage rate is GH¢2.50 per cubic meter and the family consumed 25 cubic meters of water in a month, calculate the total water bill for that month.
7. A small business has to pay a monthly Internet service fee of GH¢120 and a usage charge based on the amount of data consumed. If the usage rate is GH¢0.10 per gigabyte (GB) and the business consumed 250 GB of data in a month, calculate the total Internet bill for that month.

Review Questions 2.5

1. Aisha purchased a smartphone for GH¢500.00. After using it for a year, she decided to upgrade to a newer model. She sold her old smartphone for GH¢400.00. What was the percentage decrease in the value of Aisha's smartphone over the year?
2. A bakery sells cupcakes for GH¢2.00 each. Due to an increase in the cost of ingredients, the bakery decides to raise the price of cupcakes by 25%. What is the new price of a cupcake?
3. Edith invested GH¢5000.00 in a fixed deposit account that earns an annual interest rate of 4.5%. How much simple interest will she earn after 2 years?
4. John borrowed GH¢10,000.00 from a bank to purchase a car. The bank charges an annual simple interest rate of 6%. If John repays the loan after 3 years, how much interest will he have to pay?
5. Hajara invested GH¢20,000.00 in a rental property. After deducting expenses, she earns a net income of GH¢15,000.00 in a year. What is the effective annual simple interest rate on her investment?
6. Find the compound interest on GH¢10000.00 at 12% rate of interest for 1 year, compounded half-yearly.
7. The difference between SI and CI compounded annually on a certain sum of money for 2 years at 8% per annum is GH¢ 12.80. Find the principal.
8. You deposit GH¢2000.00 into a savings account with a 4% annual interest rate compounded quarterly. How much will you have after 3 years?

9. You borrow GH¢10,000.00 at an annual interest rate of 8% compounded semi-annually. What will be the total amount owed after 2 years?
10. You borrow GH¢10,000.00 at an annual interest rate of 8%. Compare the total amount owed after 5 years if the interest is compounded annually versus calculated as simple interest.
11. You deposit GH¢2000.00 into a savings account with a 6% annual interest rate. Determine the difference in the final amount after 10 years if the interest is compounded annually versus compounded quarterly.
12. A shopkeeper bought a batch of shirts for GH600.00 He sold $\frac{3}{4}$ of them at a price that gave him a profit of 25%, and the rest at a price that resulted in a loss of 10%. What was his overall profit or loss?
13. A company sells a product at a profit of 20%. If the selling price of the product is GH¢240.00, what is the cost price?
14. Kwame's water bill increased from GH¢150.00 to GH¢180.00. What is the percentage increase in his water bill?
15. If Abena's electricity bill decreased from GH¢250.00 to GH¢200.00, what is the percentage decrease in her electricity bill?
16. Yaw's gas bill increased from GH¢120.00 to GH¢150.00. What is the percentage increase in his gas bill?
17. Kofi's internet bill decreased from GH¢80.00 to GH¢60.00. What is the percentage decrease in his internet bill?
18. Ama's total utility bill for water, electricity, and gas increased from GH¢400.00 to GH¢450.00 What is the percentage increase in her total utility bill?

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