MATHEMATICS

CURRICULUM FOR SECONDARY EDUCATION (SHS 1 - 3)

NaCCA

NATIONAL COUNCIL FOR CURRICULUM & ASSESSMENT OF MINISTRY OF EDUCATION

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FOREWORD

Through the National Council for Curriculum and Assessment (NaCCA), Ghana's Ministry of Education has introduced a series of curriculum reforms to improve the quality and relevance of learning experiences in pre-tertiary schools in the country. These reforms will improve learning through the introduction of innovative pedagogies that encourage critical thinking and problem-solving. For a long time, our learners memorise facts and figures, which does not develop their analytical and practical skills. The Ministry recognises that learners need to be equipped with the right tools, knowledge, skills and competencies to deal with the fast-changing environment and the challenges facing their communities, the nation and the world.

These curriculum reforms were derived from the Education Strategic Plan (ESP 2018-2030), the National Pre-tertiary Education Curriculum Framework (NPTECF) and the National Pre-Tertiary Learning Assessment Framework (NPLAF), which were all approved by Cabinet in 2018. The new standards-based curriculum implemented in 2019 in basic schools, aims to equip learners to apply their knowledge innovatively to solve everyday problems. It also prioritises assessing learners' knowledge, skills, attitudes, and values, emphasising their achievements. The content of the basic school standards-based curriculum was therefore designed to promote a curriculum tailored to the diverse educational needs of the country's youth. It addresses the current curriculum's deficiencies in learning and assessment, especially in literacy and numeracy. These reforms have been carried out in phases. The curriculum for the basic school level – KG, Primary and Junior High School (JHS) – was developed and implemented from 2019 to 2021.

The curriculum for Senior High School (SHS), Senior High Technical School (SHTS) and Science, Technical, Engineering and Mathematics (STEM), which constitutes the next phase, is designed to ensure the continuation of learning experiences from JHS. It introduces flexible pathways for progression to facilitate the choice of subjects necessary for further study, the world of work and adult life. The new SHS, SHTS and STEM curriculum emphasises the acquisition of 21st Century skills and competencies, character development and instilling of national values. Social and Emotional Learning (SEL), Information Communications Technology, Gender Equality and Social Inclusion, have all been integrated into the curriculum. Assessment – formative and summative has been incorporated into the curriculum and aligned with the learning outcomes throughout the three-year programme.

The Ministry of Education's reform aims to ensure that graduates of our secondary schools can successfully compete in international high school competitions and, at the same time, be equipped with the necessary employable skills and work ethos to succeed in life. The Ministry of Education, therefore, sees the Senior High School (SHS) curriculum as occupying a critical place in the education system – providing improved educational opportunities and outcomes for further studies, the world of work and adult life – and is consequently prioritising its implementation.

ACKNOWLEDGEMENTS

This standards-based SHS curriculum was created using the National Pre-Tertiary Learning Assessment Framework (NPLAF), the Secondary Education Assessment Guide (SEAG), and the Teacher and Learner Resource Packs which include Professional Learning Community (PLC) Materials and Subject Manuals for teachers and learners. All the above-mentioned documents were developed by the National Council for Curriculum and Assessment (NaCCA). The Ministry of Education (MoE) provided oversight and strategic direction for the development of the curriculum with NaCCA receiving support from multiple agencies of the MoE and other relevant stakeholders. NaCCA would like to extend its sincere gratitude, on behalf of the MoE, to all its partners who participated in the professional conversations and discussions during the development of this SHS curriculum.

In particular, NaCCA would also like to extend its appreciation to the leadership of the Ghana Education Service (GES), the National School Inspectorate Authority (NaSIA), the National Teaching Council (NTC), the Commission for Technical and Vocational Education and Training (Commission for TVET), West African Examinations Council (WAEC) and other agencies of the MoE that supported the entire process. In addition, NaCCA acknowledges and values the contributions made by personnel from various universities, colleges of education Industry players, Vice Chancellors Ghana, Vice Chancellors Technical Universities as well as educators and learners working within the Ghana education landscape.

Special appreciation is extended to consultants who contributed to development of the curriculum. The development process involved multiple engagements between national stakeholders and various groups with interests in the curriculum. These groups include the teacher unions, the Association of Ghana Industries, and heads of secondary schools.

CONTENTS

THE SHS CURRICULUM OVERVIEW

The vision for this curriculum is to ensure the nation has a secondary education system that enables all Ghanaian children to acquire the 21st Century skills, competencies, knowledge, values and attitudes required to be responsible citizens, ready for the world of work, further studies and adult life. The nation's core values drive the SHS curriculum, and it is intended to achieve Sustainable Development Goal 4: 'Inclusive, equitable quality education and life-long learning for all'. Above all, it is a curriculum enabling its graduates to contribute to the ongoing growth and development of the nation's economy and well-being.

The curriculum is inclusive, flexible, and robust. It was written under the auspices of the National Council for Curriculum and Assessment by a team of expert curriculum writers across Ghana. It reflects the needs of critical stakeholders, including industry, tertiary education, the West African Examination Council, SHS learners, teachers, and school leaders. It has been written based on the National Pre-Tertiary Learning and Assessment Framework and the Secondary Education Policy.

The key features of the curriculum include:

- flexible learning pathways at all levels, including for gifted and talented learners and those with deficiencies in numeracy and literacy, to ensure it can meet the needs of learners from diverse backgrounds and with different interests and abilities.
- the five core learning areas for secondary education: science and technology, language arts, humanities, technical and vocational and business; with emphasis placed on STEM and agriculture as integral to each subject.
- a structured, standards-based approach that supports the acquisition of knowledge, skills and competencies, and transition and seamless progress throughout secondary education, from JHS to SHS and through the three years of SHS.
- a focus on interactive approaches to teaching and assessment to ensure learning goes beyond recall enabling learners to acquire the ability to understand, apply, analyse and create.
- guidance on pedagogy, coupled with exemplars, demonstrating how to integrate cross-cutting themes such as 21st Century skills, core competencies, the use of ICT, literacy and mathematics, Social Emotional Learning, Gender Equality and Social Inclusion as tools for learning and skills for life. Shared Ghanaian values are also embedded in the curriculum.

The curriculum writing process was rigorous and involved developing and using a Curriculum Writing Guide which provided systematic instructions for writers. The process was quality assured at three levels: through (a) evaluation by national experts, (b) trialling curriculum materials in schools and (c) through an external evaluation by a team of national and international experts. Evidence and insights from these activities helped hone the draft's final version. The outcome is a curriculum coherently aligned with national priorities, policies and the needs of stakeholders. A curriculum tailored to the Ghanaian context ensures that all learners benefit from their schooling and develop their full potential.

The following section highlights the details of the front matter of the draft curriculum. The vision, philosophy and goal of the curriculum are presented. This is followed by the details of the 21st Century skills and competencies, teaching and learning approaches, instructional design and assessment strategies. The template for the curriculum frame, which outlines the scope and sequence, the design that links the learning outcomes to particular 21st Century skills and competencies, as well as Gender Equality and Social Inclusion, Social and Emotional Learning and Ghanaian values are presented together with the structure of the lesson frame showing the links between the content standards, learning indicators with their corresponding pedagogical exemplars and assessment strategies.

INTRODUCTION

Effective implementation of this Senior High School (SHS) curriculum is the key to creating a well-educated and well-balanced workforce that is ready to contribute to Ghana's progress by harnessing the potential of the growing youth population, considering the demographic transition the country is currently experiencing (Educational Strategic Plan [ESP] 2018-2030). SHS curriculum aims to expand equitable, inclusive access to relevant education for all young people, including those in disadvantaged and underserved communities, those with special educational needs and those who are gifted and talented. Senior High School allows young people to develop further skills and competencies and progress in learning achievement, building from the foundation laid in Junior High School. This curriculum intends to meet the learning needs of all high school learners by acquiring 21st Century skills and competencies to prepare them for further studies, the world of work and adult life. Changing global economic, social and technological context requires life-long learning, unlearning, and continuous processes of reflection, anticipation and action.

Philosophy of Senior High School Curriculum

The philosophy underpinning the SHS curriculum is that every learner can develop their potential to the fullest if the right environment is created and skilled teachers effectively support them to benefit from the subjects offered at SHS. Every learner needs to be equipped with skills and competencies of interest to further their education, live a responsible adult life or proceed to the world of work.

Vision of Senior High School Curriculum

The vision of the curriculum is to prepare SHS graduates equipped with relevant skills and competencies to progress and succeed in further studies, the world of work and adult life. It aims to equip all learners with the 21st Century skills and competencies required to be responsible citizens and lifelong learners. When young people are prepared to become effective, engaging, and responsible citizens, they will contribute to the ongoing growth and development of the nation's economy and well-being.

Goal of Senior High School Curriculum

The goal of the curriculum is to achieve relevant and quality SHS through the integration of 21st Century skills and competencies as set out in the Secondary Education Policy. The key features to integrate into the curriculum are:

- Foundational Knowledge: literacy, numeracy, scientific literacy, information, communication and digital literacies, financial literacy and entrepreneurship, cultural identity, civic literacy and global citizenship
- Competencies: critical thinking and problem-solving, innovation and creativity, collaboration, and communication
- Character Qualities: discipline, integrity, self-directed learning, self-confidence, adaptability and resourcefulness, leadership, and responsible citizenship.

The JHS curriculum has been designed to ensure that learners are adequately equipped to transition seamlessly into SHS, where they will be equipped with the relevant knowledge, skills and competencies. The SHS curriculum emphasises character building, acquisition of 21st Century skills and competencies and nurturing core values within an environment of quality education to ensure the transition to further study, the world of work and adult life. This requires the delivery of robust secondary education that meets the varied learning needs of the youth in Ghana. The SHS curriculum, therefore, seeks to develop learners to become technology-inclined, scientifically literate, good problem-solvers who can think critically and creatively and are equipped to communicate with fluency, and possess the confidence and competence to participate fully in Ghanaian society as responsible local and global citizens – (referred to as 'Glocal citizens').

The SHS curriculum is driven by the nation's core values of truth, integrity, diversity, equity, discipline, self-directed learning, self-confidence, adaptability and resourcefulness, leadership, and responsible citizenship, and with the intent of achieving the Sustainable Development Goal 4: 'Inclusive, equitable quality education and life-long learning for all'. The following sections elaborate on the critical competencies required of every SHS learner:

Gender Equality and Social Inclusion (GESI)

- Appreciate their uniqueness about others.
- Pay attention to the uniqueness and unique needs of others.
- Value the perspective, experience, and opinion of others.
- Respect individuals of different beliefs, political views/ leanings, cultures, and religions.
- Embrace diversity and practise inclusion.
- Value and work in favour of a democratic and inclusive society.
- Be conscious of the existence of minority and disadvantaged groups in society and work to support them.
- Gain clarity about misconceptions/myths about gender, disability, ethnicity, age, religion, and all other excluded groups in society
- Interrogate and dispel their stereotypes and biases about gender and other disadvantaged and excluded groups in society.
- Appreciate the influence of socialisation in shaping social norms, roles, responsibilities, and mindsets.
- Identify injustice and advocate for change.
- Feel empowered to speak up for themselves and be a voice for other disadvantaged groups.

21st Century Skills and Competencies

In today's fast-changing world, high school graduates must be prepared for the 21st Century world of work. The study of Mathematics, Science, and Language Arts alone is no longer enough. High school graduates need a variety of skills and competencies to adapt to the global economy. Critical thinking, creativity, collaboration, communication, information literacy, media literacy, technology literacy, flexibility, leadership, initiative, productivity, and social skills are needed. These skills help learners to keep up with today's fast-paced job market. Employers want workers with more than academic knowledge. The 21st Century skills and competencies help graduates navigate the complex and changing workplace. Also, these help them become active citizens who improve their communities. Acquisition of 21st Century skills in high school requires a change in pedagogy from the approach that has been prevalent in Ghana in recent years. Teachers should discourage and abandon rote memorisation and passive learning. Instead, they should encourage active learning, collaboration, and problem-solving, project-based, inquiry-based, and other learner-centred pedagogy should be used. As well as aligning with global best practices, these approaches also seek to reconnect formal education in Ghana with values-based indigenous education and discovery-based learning which existed in Ghana in pre-colonial times. This is aligned with the 'glocal' nature of this curriculum, connecting with Ghana's past to create confident citizens who can engage effectively in a global world. Digitalisation, automation, technological advances and the changing nature of work globally mean that young people need a new set of skills, knowledge and competencies to succeed in this dynamic and globalised labour market.

Critical Thinking and Problem-Solving Competency

- Ability to question norms, practices, and opinions, to reflect on one's values, perceptions, and actions.
- Ability to use reasoning skills to come to a logical conclusion.
- Being able to consider different perspectives and points of view
- Respecting evidence and reasoning
- Not being stuck in one position
- Ability to take a position in a discourse
- The overarching ability to apply different problem-solving frameworks to complex problems and develop viable, inclusive, and equitable solution options that integrate the above-mentioned competencies, promote sustainable development,

Creativity

- Ability to identify and solve complex problems through creative thinking.
- Ability to generate new ideas and innovative solutions to old problems.
- Ability to demonstrate originality and flexibility in approaching tasks and challenges.
- Collaborating with others to develop and refine creative ideas
- Ability to incorporate feedback and criticism into the creative process
- Utilising technology and other resources to enhance creativity
- Demonstrating a willingness to take risks and experiment with new approaches
- Adapting to changing circumstances and further information to maintain creativity

- Integrating multiple perspectives and disciplines to foster creativity
- Ability to communicate creative ideas effectively to a variety of audiences

Collaboration

- Abilities to learn from others; to understand and respect the needs, perspectives, and actions of others (empathy)
- Ability to understand, relate to and be sensitive to others (empathic leadership)
- Ability to deal with conflicts in a group
- Ability to facilitate collaborative and participatory problem-solving
- Ability to work with others to achieve a common goal.
- Ability to engage in effective communication, active listening, and the ability to compromise.
- Ability to work in groups on projects and assignments.

Communication

- Know the specific literacy and language of the subjects studied
- Use language for academic purposes
- Communicate effectively and meaningfully in a Ghanaian Language and English Language
- Communicate confidently, ethically, and effectively in different social contexts.
- Communicate confidently and effectively to different participants in different contexts
- Ability to communicate effectively verbally, non-verbally and through writing.
- Demonstrate requisite personal and social skills that are consistent with changes in society
- Ability to express ideas clearly and persuasively, listen actively, and respond appropriately
- Ability to develop digital communication skills such as email etiquette and online collaboration.
- Ability to engage in public speaking, debate, and written communication.

Learning for Life

- Understand subject content and apply it in different contexts
- Apply mathematical and scientific concepts in daily life
- Demonstrate mastery of skills in literacy, numeracy, and digital literacy.
- Develop an inquiry-based approach to continual learning.
- Be able to understand higher-order concepts and corresponding underlying principles.
- Participate in the creative use of the expressive arts and engage in aesthetic appreciation.
- Use and apply a variety of digital technologies
- Be digitally literate with a strong understanding of ICT and be confident in its application.
- Be equipped with the necessary qualifications to gain access to further and higher education and the world of work and adult life
- Ability to apply knowledge practically in the workplace so that they are able to utilise theory by translating it into practice.
- Develop their abilities, gifts and talents to be able to play a meaningful role in the development of the country
- Be able to think critically and creatively, anticipate consequences, recognise opportunities and be risk-takers
- Ability to pursue self-directed learning with the desire to chart a path to become effective lifelong learners.
- Independent thinkers and doers who show initiative and take action.
- Ability to innovate and think creatively, building on their knowledge base so that they take risks to achieve new goals
- Ability to think critically and solve problems so that they become positive change agents at work, in further study and in their personal lives.
- Be motivated to adapt to the changing needs of society through self-evaluation and ongoing training
- Be able to establish and maintain innovative enterprises both individually and in collaboration with others.
- Be able to ethically prioritise economic values to ensure stability and autonomy
- Show flexibility and preparedness to deal with job mobility
- Be committed towards the improvement of their quality of life and that of others
- Feel empowered in decision-making processes at various levels e.g., personal, group, class, school, etc.

- Be able to seek and respond to assistance, guidance and/or support when needed.
- Ability to make and adhere to commitments.
- Adopt a healthy and active lifestyle and appreciate how to use leisure time well.
- Be enthusiastic, with the knowledge, understanding and skill that enable them to progress to tertiary level, the world of work and adult life.
- Ability to transition from school to the world of work or further study by applying knowledge, skills and attitudes in new situations.
- Be independent, have academic and communication skills such as clarity of expression (written and spoken), and the ability to support their arguments.
- Be innovative and understand the 21st Century skills and competencies and apply them to everyday life.

Global and Local (Glocal) Citizenship

- Appreciate and respect the Ghanaian identity, culture, and heritage
- Be conscious of current global issues and relate well with people from different cultures
- Act in favour of the common good, social cohesion and social justice
- Have the requisite personal and social skills to handle changes in society
- Appreciate the impact of globalisation on the society.
- Ability to be an honest global citizen displaying leadership skills and moral fortitude with an understanding of the wider world and how to enhance Ghana's standing.

Systems Thinking Competency

- Ability to recognise and understand relationships
- Ability to analyse complex systems
- Ability to think of how systems are embedded within different domains and different scales
- Ability to deal with uncertainty

Normative Competency

- Ability to understand and reflect on the norms and values that underlie one's actions
- Ability to negotiate values, principles, goals, and targets, in a context of conflicts of interests and trade-offs, uncertain knowledge and contradictions

Anticipatory Competency

- Ability to understand and evaluate multiple futures – possible, probable, and desirable
- Ability to create one's vision for the future.
- Ability to apply the precautionary principle
- Ability to assess the consequences of actions
- Ability to deal with risks and changes

Strategic Competency

- Ability to collectively develop and implement innovative actions that further a cause at the local level and beyond.
- Ability to understand the bigger picture and the implications of smaller actions on them

Self-Awareness Competency

- The ability to reflect on one's role in the local community and (global) society
- Ability to continually evaluate and further motivate one's actions
- Ability to deal with one's feelings and desires

Social Emotional Learning (SEL): Five Core Competencies with Examples

1. Self-Awareness

Understanding one's emotions, thoughts, and values and how they influence one's behaviour in various situations. This includes the ability to recognise one's strengths and weaknesses with a sense of confidence and purpose. For instance:

- *Integrating personal and social identities;*
- *Identifying personal, cultural, and linguistic assets;*
- *Identifying one's emotions;*
- *Demonstrating honesty and integrity;*
- *Connecting feelings, values, and thoughts;*

- *Examining prejudices and biases;*
- *Experiencing self-efficacy;*
- *Having a growth mindset;*
- *Developing interests and a sense of purpose;*

2. *Self-Management*

The capacity to control one's emotions, thoughts, and actions in a variety of situations and to realise one's ambitions. This includes delaying obtaining one's desires, dealing with stress, and feeling motivated and accountable for achieving personal and group goals. For instance:

- *Managing one's emotions;*
- *Identifying and utilising stress-management strategies;*
- *Demonstrating self-discipline and self-motivation;*
- *Setting personal and group goals;*
- *Using planning and organisation skills;*
- *Having the courage to take the initiative;*
- *Demonstrating personal and collective agency;*

3. *Social Awareness*

The capacity to comprehend and care for others regardless of their backgrounds, cultures, and circumstances. This includes caring for others, understanding larger historical and social norms for behaviour in different contexts, and recognising family, school, and community resources and supports. For instance:

- *Recognising others' strengths*
- *Demonstrating empathy and compassion*
- *Caring about others' feelings*
- *Understanding and expressing gratitude*
- *Recognising situational demands and opportunities*
- *Understanding how organisations and systems influence behaviour*

4. *Relationship Skills*

The capacity to establish and maintain healthy, beneficial relationships and adapt to various social situations and groups. This includes speaking clearly, listening attentively, collaborating, solving problems and resolving conflicts as a group, adapting to diverse social and cultural demands and opportunities, taking the initiative, and asking for or offering assistance when necessary. For instance:

- *Communicating effectively;*
- *Building positive relationships;*
- *Demonstrating cultural competence;*
- *Working as a team to solve problems;*
- *Constructively resolving conflicts;*
- *Withstanding negative social pressure;*
- *Taking the initiative in groups;*
- *Seeking or assisting when needed;*
- *Advocating for the rights of others.*

5. *Responsible Decision-Making*

The capacity to make thoughtful and constructive decisions regarding acting and interacting with others in various situations. This includes weighing the pros and cons of various personal, social, and group well-being actions. For example:

- *Demonstrating curiosity and an open mind;*
- *Solving personal and social problems;*
- *Learning to make reasonable decisions after analysing information, data, and facts;*
- *Anticipating and evaluating the effects of one's actions;*
- *Recognising that critical thinking skills are applicable both inside and outside of the classroom;*
- *Reflecting on one's role in promoting personal, family, and community well-being;*
- *Evaluating personal, interpersonal, community, and institutional impacts*

Learning and Teaching Approaches

Learning and teaching should develop learners as self-directed and lifelong learners. Learners must be helped to build up deep learning skills and competencies to develop the ability to acquire, integrate and apply knowledge and skills to solve authentic and real-life problems. Learners need to be exposed to a variety of learning experiences to enable them to collaborate with others, construct meaning, plan, manage, and make choices and decisions about their learning. This will allow them to internalise newly acquired knowledge and skills and help them to take ownership of their education. The 21st Century skills and competencies describe the relevant global and contextualised skills that the SHS curriculum is designed to help learners acquire in addition to the 4Rs (Reading, wRiting, aRithmetic and cReativity). These skills and competencies, as tools for learning and teaching and skills for life, will allow learners to become critical thinkers, problem-solvers, creators, innovators, good communicators, collaborators, digitally literate, and culturally and globally sensitive citizens who are life-long learners with a keen interest in their personal development and contributing to national development.

Given the diverse needs of learners, teachers need to have a thorough grasp of the different pedagogies as they design and enact meaningful learning experiences to meet the needs of different learners in the classroom. The teaching-learning techniques and strategies should include practical activities, discussion, investigation, role play, problem-based, context-based, and project-based learning. Active learning strategies have become increasingly popular in education as they provide learners with meaningful opportunities to engage with the material. These strategies emphasise the use of creative and inclusive pedagogies and learner-centred approaches anchored on authentic and enquiry-based learning, collaborative and cooperative learning, differentiated teaching and learning, holistic learning, and cross-disciplinary learning. They include experiential learning, problem-based learning, project-based learning, and talk-for-learning approaches. Some of the pedagogical exemplars to guide learning and teaching of the SHS curriculum include:

- **Experiential Learning:** Experiential learning is a hands-on approach to learning that involves learners in real-world experiences. This approach focuses on the process of learning rather than the result. Learners are encouraged to reflect on their experiences and use them to develop new skills and knowledge. Experiential learning can take many forms, including internships, service learning, and field trips. One of the main benefits of experiential learning is that it allows learners to apply what they have learned in the classroom to real-world situations. This can help them develop a deeper understanding of the material and make connections between different concepts. Additionally, experiential learning can help learners develop important skills such as critical thinking, problem-solving and communication.
- **Problem-Based Learning:** Problem-based learning is an approach that involves learners in solving real-world problems. Learners are presented with a problem or scenario and are asked to work together to find a solution. This approach encourages learners to take an active role in their learning and helps them develop important skills such as critical thinking and problem-solving. One of the main benefits of problem-based learning is that it encourages learners to take ownership of their learning. By working together to solve problems, learners can develop important skills such as collaboration and communication. Additionally, problem-based learning can help learners develop a deeper understanding of the material as they apply it to real-world situations.
- **Project-Based Learning:** Project-based learning is a hands-on approach to learning that involves learners in creating a project or product. This approach allows learners to take an active role in their learning and encourages them to develop important skills such as critical thinking, problem-solving, collaboration, and communication. One of the main benefits of project-based learning is that it allows learners to apply what they have learned in the classroom to real-world situations. Additionally, project-based learning can help learners develop important skills from each other and develop a deeper understanding of the material.
- **Talk for Learning Approaches:** Talk for learning approaches (TfL) are a range of techniques and strategies that are used to encourage learners to talk by involving them in discussions and debates about the material they are learning. This approach encourages learners to take an active role in their learning and helps them develop important skills such as critical thinking, collaboration and communication and also makes them develop confidence. One of the main benefits of TfL is that it encourages learners to think deeply about the material they are learning. By engaging in discussions and debates, learners can develop a deeper understanding of the material and make connections between different concepts.
- **Initiating Talk for Learning:** Initiating talk for learning requires the use of strategies that would encourage learners to talk in class. It helps learners to talk and participate meaningfully and actively in the teaching and learning process. Apart from developing skills such as communication and critical thinking, it also helps learners to develop confidence. Some strategies for initiating talk among learners are Activity Ball; Think-Pair-Share; Always, Sometimes, Never True; Matching and Ordering of Cards.
- **Building on What Others Say:** Building on what others say is an approach that involves learners in listening to and responding to their classmates'

ideas. This approach encourages learners to take an active role in their learning and helps them develop important skills such as critical thinking and communication. One of the main benefits of building on what others say is that it encourages learners to think deeply about the material they are learning. By listening to their classmates' ideas, learners can develop a deeper understanding of the material and make connections between different concepts. Additionally, building on what others say can help learners develop important skills such as collaboration and reflection. Some of the strategies to encourage learners to build on what others say are brainstorming, concept cartoons, pyramid discussion, and 5 Whys, amongst others.

- **Managing Talk for Learning:** Managing talk for learning requires the use of various strategies to effectively coordinate what learners say in class. Effective communication is a crucial aspect of learning in the classroom. Teachers must manage talk to ensure that learners are engaged, learning, and on-task in meaningful and purposeful ways. Some strategies for managing learners' contributions are debates, think-pair-share, sage in the circle etc.
- **Structuring Talk for Learning:** One effective way to shape learners' contributions is to structure classroom discussions. Structured discussions provide a framework for learners to engage in meaningful dialogue and develop critical thinking skills. Teachers can structure discussions by providing clear guidelines, such as speaking one at a time, listening actively, and building on each other's ideas. One popular structured discussion technique is the "think-pair-share" method. In this method, learners think about a question or prompt individually, and then pair up with a partner to discuss their ideas. Finally, the pairs share their ideas with the whole class. This method encourages all learners to participate and ensures that everyone has a chance to share their thoughts. Another effective way to structure talk for learning is to use open-ended questions. Open-ended questions encourage learners to think deeply and critically about a topic. They also promote discussion and collaboration among learners. Teachers can use open-ended questions to guide classroom discussions and encourage learners to share their ideas and perspectives. Other strategies that can be used are Concept/Mind Mapping, "Know," "Want to Know," "Learned" (KWL); Participatory Feedback; and the 5 Whys.
- **Diamond Nine:** The Diamond Nine activity is a useful tool for managing talk for learning in the classroom. This activity involves ranking items or ideas in order of importance or relevance. Learners work in groups to arrange cards or sticky notes with different ideas or concepts into a diamond shape, with the most important idea at the top and the least important at the bottom. The Diamond Nine activity encourages learners to think critically about a topic and prioritise their ideas. It also promotes collaboration and discussion among group members. Teachers can use this activity to introduce a new topic, review material, or assess student understanding.
- **Group Work/Collaborative Learning:** Group work or collaborative learning are effective strategies for managing talk for learning in the classroom. These strategies encourage learners to work together to solve problems, share ideas, and learn from each other. Group work and collaborative learning also promote communication and collaborative skills that are essential for success in the workplace and in life. To implement group work effectively, teachers must provide clear guidelines and expectations for group members. They should also monitor group work to ensure that all learners are participating and on-task. Teachers can also use group work as an opportunity to assess individual student understanding and participation.
- **Inquiry-Based Learning:** Learners explore and discover new information by asking questions and investigating.
- **Problem-Based Learning:** Learners are given real-world problems to solve and must use critical thinking and problem-solving skills.
- **Project-Based Learning:** Learners work on long-term projects that relate to real-world scenarios.
- **Flipped Classroom:** Learners watch lectures or instructional videos at home and complete assignments and activities in class.
- **Mastery-Based Learning:** Learners learn at their own pace and only move on to new material once they have mastered the current material.
- **Gamification:** Learning is turned into a game-like experience with points, rewards, and competition.

These strategies provide learners with opportunities to engage with the material in meaningful ways and develop important skills such as critical thinking, problem-solving, collaboration, and communication. By incorporating these strategies into their teaching, teachers can help learners develop a deeper understanding of the material and prepare them for success in the real world. Effective communication is essential for learning in the classroom. Teachers must manage talk to ensure that learners are engaged in learning and on-task. Strategies such as structuring talk for learning, using Diamond Nine activities, and implementing group work/ collaborative learning can help teachers manage talk effectively and promote student learning and engagement. By implementing these strategies, teachers can create a positive and productive learning environment where all learners can succeed.

Universal Design for Learning (UDL) in the SHS Curriculum

The design of the curriculum uses UDL to ensure the creation of flexible learning environments that can accommodate a wide range of learner abilities, needs, and preferences. The curriculum is designed to provide multiple means of engagement, representation, and action and expression, so teachers can create a more inclusive and effective learning experience for all learners. UDL is beneficial for all learners, but it is particularly beneficial for learners needing special support and learners who may struggle with traditional teaching approaches. The integration of UDL in the pedagogy is aimed at making learning accessible to everyone and helping all learners reach their full potential. For instance, teachers need to:

- incorporate multiple means of representation into their pedagogy, such as using different types of media and materials to present information.
- provide learners with multiple means of action and expression, such as giving them options for how they can demonstrate their learning.
- consider incorporating multiple means of engagement into their choice of pedagogy, such as incorporating games or interactive activities to make learning more fun and engaging.

By doing these, teachers can help ensure that the curriculum is accessible and effective for all learners, regardless of their individual needs and abilities.

Curriculum and Assessment Design: Revised Bloom's Taxonomy and Webb's Depth of Knowledge

The design of this curriculum uses the revised Bloom's Taxonomy and Webb's Depth of Knowledge (DoK) as frameworks to design what to teach and assess.

The Revised Bloom's Taxonomy provides a framework for designing effective learning experiences. Understanding the different levels of learning, informed the creation of activities and assessments that challenge learners at the appropriate level and help them progress to higher levels of thinking. Additionally, the framework emphasises the importance of higher-order thinking skills, such as analysis, evaluation, and creation, which are essential for success in today's complex and rapidly changing world. This framework is a valuable tool for educators who want to design effective learning experiences that challenge students at the appropriate level and help them develop higher-order thinking skills. By understanding the six levels of learning and incorporating them into their teaching, educators can help prepare students for success in the 21st century. The six hierarchical levels of the revised Bloom's Taxonomy are:

1. **Remember** – At the foundation is learners' ability to remember. That is retrieving knowledge from long-term memory. This level requires learners to recall concepts—identify, recall, and retrieve information. Remembering is comprised of identifying, listing, and describing. Retrieving relevant knowledge from long-term memory includes, recognising, and recalling is critical for this level.
2. **Understand** – At understanding, learners are required to construct meaning that can be shown through clarification, paraphrasing, representing, comparing, contrasting and the ability to predict. This level requires interpretation, demonstration, and classification. Learners explain and interpret concepts at this level.
3. **Apply** – This level requires learners' ability to carry out procedures at the right time in a given situation. This level requires the application of knowledge to novel situations as well as executing, implementing, and solving problems. To apply, learners must solve multi-step problems.
4. **Analyse** – The ability to break things down into their parts and determine relationships between those parts and being able to tell the difference between what is relevant and irrelevant. At this level, information is deconstructed, and its relationships are understood. Comparing and contrasting information and organising it is key. Breaking material into its constituent parts and detecting how the parts relate to one another and an overall structure or purpose is required. The analysis also includes differentiating, organising and attributing.
5. **Evaluate** – The ability to make judgments based on criteria. To check whether there are fallacies and inconsistencies. This level involves information evaluation, critique, examination, and formulation of hypotheses.
6. **Create** – The ability to design a project or an experiment. To create, entails learners bringing something new. This level requires generating information—planning, designing, and constructing.

Webb's Depth of Knowledge (DoK) is a framework that helps educators and learners understand the level of cognitive engagement required for different types of learning tasks. The framework includes four levels. By understanding the four DoK levels, educators can design learning activities that challenge students to engage in deeper thinking and problem-solving. DoK is an essential tool for designing effective instruction and assessments. By understanding the different levels of DoK, teachers can design instruction and assessments that align with what they intend to achieve. DoK is a useful tool for differentiating instruction and providing appropriate challenges for all learners. Teachers can use DOK to identify students who need additional support or those who are ready for more advanced tasks. The four levels of Webb's' DoK assessment framework are:

- **Level 1: Recall and Reproduction –** Assessment at this level is on recall of facts, concepts, information, and procedures—this involves basic knowledge acquisition. Learners are asked specific questions to launch activities, exercises, and assessments. The assessment is focused on recollection and reproduction.
- **Level 2: Skills of Conceptual Understanding –** Assessment at this level goes beyond simple recall to include making connections between pieces of information. The learner's application of skills and concepts is assessed. The assessment task is focused more on the use of information to solve multi-step problems. A learner is required to make decisions about how to apply facts and details provided to them.
- **Level 3: Strategic Reasoning** – At this level, the learner's strategic thinking and reasoning which is abstract and complex is assessed. The assessment task requires learners to analyse and evaluate composite real-world problems with predictable outcomes. A learner must apply logic, employ problem-solving strategies, and use skills from multiple subject areas to generate solutions. Multitasking is expected of learners at this level.
- **Level 4: Extended Critical Thinking and Reasoning –** At this level of assessment, the learner's extended thinking to solve complex and authentic problems with unpredictable outcomes is the goal. The learner must be able to strategically analyse, investigate, and reflect while working to solve a problem, or changing their approach to accommodate new information. The assessment requires sophisticated and creative thinking. As part of this assessment, the learner must know how to evaluate their progress and determine whether they are on track to a feasible solution for themselves.

The main distinction between these two conceptual frameworks is what is measured. The revised Bloom's Taxonomy assesses the cognitive level that learners must demonstrate as evidence that a learning experience occurred. The DoK, on the other hand, is focused on the context—the scenario, setting, or situation—in which learners should express their learning. In this curriculum, the revised Bloom's taxonomy guided the design, and the DoK is used to guide the assessment of learning. The taxonomy provides the instructional framework, and the DoK analyses the assignment specifics. It is important to note that Bloom's Taxonomy requires learners to master the lower levels before progressing to the next. So, suppose the goal is to apply a mathematical formula. In that case, they must first be able to identify that formula and its primary purpose (remember and understand). The cognitive rigour is therefore presented in incremental steps to demonstrate the learning progression. When measuring assessments in DoK, learners move fluidly through all levels. In the same example, while solving a problem with a formula, learners recall the formula (DoK 1) to solve the problem (DoK 2 and DoK 3). Depending on the difficulty of the problem to be solved, the learner may progress to DoK 4.

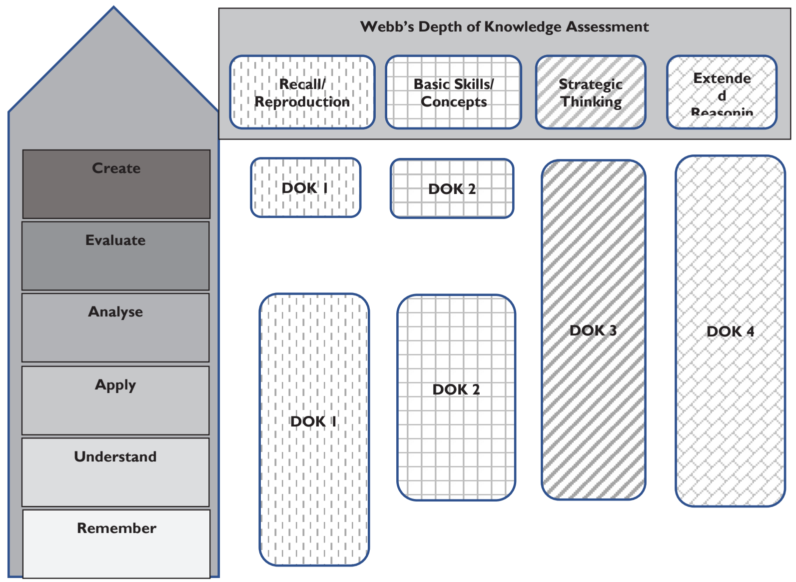

Figure 1: Revised Bloom Taxonomy combined with Webb's Depth of Knowledge for Teaching and Assessment

The structure of teaching and the assessment should align with the six levels of Bloom's knowledge hierarchy and DoK shown in Figure 1. Each level of DoK should be used to assess specific domains of Bloom's Taxonomy as illustrated in the table below:

Depth of Knowledge (DoK) Assessment	Bloom's Taxonomy applied to DoK
• Level 1: Recall and Reproduction	• Remembering, Understanding, Application, Analysis and Creation
• Level 2: Basic Skills and Concepts	• Understanding, Application, Analysis and Creation
• Level 3: Strategic Thinking	• Understanding, Application, Analysis, Evaluation and Creation
• Level 4: Extended Reasoning	• Understanding, Application, Analysis, Evaluation and Creation

In line with the National Pre-Tertiary Learning and Assessment Framework, the Secondary Education Assessment Guide (SEAG) requires that classroom assessments should cover **Assessment as learning (AaL), Assessment of learning (AoL) and Assessment for learning (AfL).** Therefore, teachers should align the Revised Bloom's Taxonomy with the DoK framework of assessment. Formative assessments should include classroom discussions, project-based assignments, and self-reflection exercises, while summative assessments should include standardised tests and rubric-based evaluations of learners' work. It is important to seek feedback from learners themselves, as they may have unique insights into how well they are developing these skills in the classroom.

To assess 21st Century skills and competencies in the classroom, teachers will have to use a combination of both formative and summative assessments to evaluate learners' acquisition of these skills and competencies. For instance:

- Identify the specific 21st Century skills and competencies to be assessed. For instance, you might want to assess *critical thinking, problem-solving, or creativity*.
- Align the skills and competencies with the DoK levels. For example, lower DoK levels might be more appropriate for assessing basic knowledge and comprehension, whereas higher DoK levels might be more appropriate for assessing more complex skills such as *analysis, synthesis, and evaluation*.
- Develop assessment items that align with the DoK levels and the skills and competencies you want to assess. These items should be designed to elicit evidence of learning across the different levels of the DoK framework.
- Administer the assessment and collect data. Analyse the data to gain insights into student learning and identify areas where learners may need additional support or instruction.

The DoK framework is a powerful tool for assessing the acquisition of 21st Century skills and competencies in the classroom, helping teachers to better understand how learners are learning and identify areas for improvement.

Educational success is no longer about producing content knowledge, but rather about extrapolating from what we know and applying the knowledge creatively in new situations.

The overall assessment of learning at SHS should be aligned with the National Pre-Tertiary Learning and Assessment Framework and the Secondary Education Assessment Guide. Formative and summative assessment strategies must be used.

Definition of Key Terms and Concepts in the Curriculum

- **Learning Outcomes:** It is a statement that defines the knowledge, skills, and abilities that learners should possess and be able to demonstrate after completing a learning experience. They are specific, measurable, attainable, and aligned with the content standards of the curriculum. It helps the teachers to determine what to teach, how to teach, and how to assess learning. Also, it communicates expectations to learners and helps them to better master the subject.
- **Learning Indicators:** They are measures that allow teachers to observe progress in the development of capacities and skills. They provide a simple and reliable means to evaluate the quality and efficacy of teaching practices, content delivery, and attainment of learning outcomes.
- **Content Standards:** It is a statement that defines the knowledge, skills, and understanding that learners are expected to learn in a particular subject area or grade level. They provide a clear target for learners and teachers and help focus resources on learner achievement.
- **Pedagogical Exemplars:** They are teaching examples used to convey values and standards to learners. Pedagogical Exemplars are usually demonstrated through teacher behaviour.
- **Assessment:** It is the systematic collection and analysis of data about learners' learning to improve the learning process or make a judgement on learner achievement levels. Assessment is aimed at developing a deep understanding of what learners know, understand, and can do with their knowledge because of their educational experiences. Assessment involves the use of empirical data on learners' learning to improve learning. Assessment is an essential aspect of the teaching and learning process in education, which enables teachers to assess the effectiveness of their teaching by linking learner performance to specific learning outcomes.
- **Teaching and Learning Resources:** Teaching and learning resources are essential tools for teachers to provide high-quality education to their learners. These resources can take various forms, including textbooks, audiovisual materials, online resources, and educational software. It is also important to avoid stereotypes and use inclusive language in teaching and learning resources. This means avoiding language that reinforces negative stereotypes and using language that is respectful and inclusive of all individuals regardless of their background. Using a consistent tone, style, and design is very important.

PHILOSOPHY, VISION AND GOAL OF MATHEMATICS

PHILOSOPHY

Every learner can develop their potential in mathematics through creative and innovative ways to become lifelong learners, apply mathematical skills and competencies to solve everyday problems, further their education and/or proceed to the world of work.

VISION

Mathematically enthusiastic learners who are highly interested in the subject and are capable of reasoning, modelling, representing and making use of mathematical tools and technology to solve problems in real life, further their studies and/or proceed to the world of work.

GOAL

Building on the overall Secondary Education Goals for Young Ghanaians, the fundamental goal of the mathematics curriculum is to educate students to be dynamic, thinking citizens, interpreting the world mathematically and using mathematics to help form their predictions and decisions about personal and financial priorities. Further, it is important that citizens can critically examine social and scientific issues raised or influenced by public opinion by using and interpreting mathematical perspectives in a democratic society. In addition, successful mathematics learning will provide a workforce that is appropriately educated in mathematics to contribute productively in an ever-changing global economy, with both rapid revolutions in technology and global and local social challenges. An economy competing globally requires substantial numbers of proficient workers able to learn, adapt, create, interpret and analyse mathematical information.

Specifically, the goals of teaching and learning mathematics are to encourage and enable students to:

- recognise that mathematics pervades the world around us and appreciate the worth, supremacy and magnificence of mathematics
- develop tolerance and perseverance when solving problems
- use the language, symbols and notation of mathematics appropriately
- develop mathematical curiosity and use inductive and deductive reasoning when solving problems
- develop Ghanaian values and appreciate the existence of diversity in Ghanaian society and the world at large
- develop the knowledge, skills and attitudes necessary to pursue further studies in mathematics
- become assertive in using mathematics to analyse and solve problems both in school and in real-life situations
- develop abstract, logical and critical thinking and the ability to reflect critically upon their work and the work of others
- develop a critical appreciation of information and communication technology in mathematics.

RATIONALE

Learning mathematics enriches lives and creates opportunities for all Ghanaians. Mathematics forms an integral part of our everyday lives, and it is a universal truth that development is hinged on mathematics. Mathematics provides students with essential mathematical skills and knowledge in numbers and algebra, measurement and geometry, and statistics and probability. Mathematics is the backbone of a country's social, economic, political, and physical development. It is a never-ending creative ideology which seeks to promote discovery and understanding. It consists of a body of knowledge which attempts to explain and interpret phenomena and experiences. Mathematics has changed our lives and is thus vital to Ghana's future development, as echoed in the nation's vision to make Ghana a Mathematics-Friendly Nation.

Facilitators must facilitate learning in the mathematics classroom to provide quality mathematics education. This will instil in students an appreciation of the elegance and power of mathematical reasoning, provide the foundations for discovering and understanding the world around us and lay the grounds for mathematics and mathematics-related studies at higher levels of education. Learners should be encouraged to understand how mathematics can be used to explain what is occurring, predict how things will behave and analyse the causes and origin of phenomena in our environment. Digital technologies should be the drive to lead to the expansion of ideas and provide access to new tools for continuing mathematical exploration and invention.

Mathematics is also concerned with developing our national values, attitudes and 21st-century skills, and it is therefore important for all citizens to be mathematically and technologically literate for sustainable development. Mathematics, therefore, ought to be taught using hands-on and minds-on approaches and recognising gender equality and social inclusions (GESI), which will produce enthusiastic learners who will adopt mathematics as a culture.

CONTEXTUAL ISSUES

Assessment

The role of assessment in education has grown greatly over the past few decades, a trend that has a major manifestation. That is the ever-rising importance of assessment to hold systems and their key actors (notably facilitators) accountable for education outcomes.

The Sustainable Development Goals raise the bar for education in terms of equity and how to perceive "quality," which now requires a much more focus on the relevance of assessment. The SDGs stipulate that measuring progress towards these goals will begin with the learning assessment to determine whether students are acquiring the required knowledge and competencies and whether a system is providing students with the appropriate education to acquire these outcomes. However, framers of the goals recognise that while assessment will be vital to this process, there is a severe double risk that systems and their partners will continue to rely excessively on tests to drive their reforms.

This phenomenon is not so different in Ghana. Although the existing SHS mathematics syllabus suggests evaluation exercises in the form of oral questions, quizzes, class assignments, essays, project work, etc., most SHS facilitators' assessment practices have largely been used for summative purposes due to the neglect of formative assessment practices. Thus, assessment strategies are largely paper and pen tests. Besides, assessments are usually focused on recall and, to some extent, understanding, to the neglect of assessments that encourage critical thinking, application, reasoning and construction of knowledge. An increase in criticism of the practice has accompanied the increase in the use of paper and pencil tests. Tests vary in quality, with some being particularly poor. Currently, most facilitator-made assessments lack real-life application of the mathematics concepts delivered in the classroom. They are largely an extension of the same content taught in the lessons and high-stakes examinations organised at the end of the entire school year by external examination bodies.

To this end, there is the need to shift focus to formative assessment practices hinged on the various depths of knowledge, emphasising levels two through to four. The assessment process should operate without bias with respect to gender, social class, ethnicity, language use and religion. Both formative and summative assessments should be authentic, promote reasoning and allow all learners to demonstrate their ability. There is a need to use multiple assessment strategies such as project works, portfolios, performance-based assessments, etc. Assessment contents and forms should be differentiated to allow gifted and talented learners to show their unique gifts/talents.

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SCOPE AND SEQUENCE

Mathematics Summary

S/N	STRAND	SUB-STRAND	SHS1			SHS2			SHS3		
			CS	LO	LI	CS	LO	LI	CS	LO	LI
1	Numbers for everyday life	Real number and Numeration system	2	2	6	3	3	7	-	-	-
		Proportional reasoning	2	2	4	2	2	4	2	2	4
2	Algebraic Thinking	Applications of expressions, equations and inequalities	2	3	6	1	2	4	-	-	-
		Patterns and relationships	2	2	4	1	1	5	1	1	3
3	Geometry around us	Spatial sense	1	1	5	1	1	4	2	2	7
		Measurement	3	3	9	3	3	8	1	1	2
4	Making sense of and using data	Statistical reasoning and its application in real life	3	3	8	3	3	7	2	2	4
		Chance	1	1	3	1	1	2	1	1	3
Total			**16**	**17**	**45**	**15**	**16**	**41**	**9**	**9**	**22**

Overall Totals (SHS 1 – 3)

Content Standards	40
Learning Outcomes	42
Learning Indicators	108

YEAR ONE

Subject MATHEMATICS

Strand 1. NUMBERS FOR EVERYDAY LIFE

Sub-Strand 1. REAL NUMBER SYSTEM

Learning Outcomes	21st-Century Skills and Competencies	GESI[1], SEL[2] and Shared National Values
1.1.1.LO.1		
Apply the relationships and differences between the set of rational and irrational numbers and use them to solve problems.	**Communication and Collaboration:** Learners communicate confidently and effectively to develop appropriate mathematics vocabulary for real numbers through teamwork. **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to application to lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: • Respect individuals of different backgrounds in their groups as they discuss and solve problems in mathematics involving rational and irrational numbers. • Interrogate their stereotypes and biases about the roles and abilities of different individuals in learning and applying mathematics. • Examine and dispel misconceptions/ myths about GESI as they engage in a sustainable discourse while learning sets of rational and irrational numbers. • Value and promote justice as they develop and implement innovative actions in the mathematics classroom and beyond. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies – *Self-*

[1] Gender Equality and Social Inclusion
[2] Socio-Emotional Learning

		Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the Mathematics curriculum, the facilitator should apply Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and confident as they establish relationships and differences in the set of real numbers system. · Exhibiting motivation and SMART goal setting in a mathematics classroom and beyond. · Managing emotions and conflicts as they engage in a mathematical discourse. · Showing empathy and cooperation in a mathematical problem-solving situation. These may be done by the facilitator through modelling emotional self-regulation and decision-making, the promotion of positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork and respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning the real number system. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning

		environment as they debate the relationship between rational and irrational numbers. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination as they engage in a mathematical discourse based on sets of real numbers. **Truth and Integrity:** Reward truth and honesty as strong moral principles as they engage in peer assessment to help them become responsible citizens. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment.
1.1.1.LO.2		
Analyse and solve real world problems involving union, intersection and complements of sets and apply these to three sets of problems using simple surveys.	**Communication and Collaboration:** Learners communicate confidently and effectively to develop appropriate everyday vocabulary related to real numbers through collaborative work. **Technology Literacy Skills:** Initiate mathematical thinking processes to solve challenging problems in an IT-driven environment to develop their potential. **Strategic Competency:** Make conscious efforts to help learners collectively develop and implement innovative actions that promote sustainability at their level, leading to application to lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: · Respect individuals of different backgrounds in their groups as they discuss and solve problems based on union, intersection and complements of sets. · Interrogate their stereotypes and biases about the roles and abilities of different individuals in learning and applying mathematics. · Examine and dispel misconceptions/ myths about GESI as they engage in a sustainable discourse while learning sets of rational and irrational numbers.

	reflect on one's own values, perceptions and actions for decision-making.	· Value and promote justice as they develop and implement innovative actions in the mathematics classroom and beyond. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the Mathematics curriculum, the facilitator should apply Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and confident as they learn about unions, intersection and complements of sets. · Exhibiting motivation and SMART goal setting in a mathematics classroom and beyond. · Managing emotions and conflicts as they engage in a mathematical discourse. · Showing empathy and cooperation in a mathematical problem-solving situation. These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group

		work in the course of learning of union, intersection and complements of sets. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment as they debate the relationship among unions, intersection and complements of sets. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination as they engage in a mathematical discourse based on union, intersection and complements of sets. **Truth and Integrity:** Reward truth and honesty as strong moral principles as they engage in peer assessment to help them become responsible citizens. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment.

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
1.1.1.CS.1	1.1.1.LI.1	1.1.1.AS.1
Demonstrate knowledge and understanding of real number systems and the operations of the various subsets.	**Develop the real number system using the closure property.** **Problem-based Learning, Talk for Learning, Experiential Learning, and Group Work/Collaborative Learning.** Review learners' knowledge of basic concepts of numbers, deal with their misconceptions or preconceptions about such concepts, audit their difficulties and transition to develop the set of real numbers, using closure properties and applying such to solve real life problems. While doing so, encourage learners to be truthful and honest in their responses within their collaborative groups. **Examples:** i. Establish, through a variety of differentiated strategies, **the set of real numbers** (**rational and irrational**) using models such as number lines, number tracks, algebraic tiles, multibase arithmetic blocks, etc., in a socio-emotional learning environment that ensures the development of values such as diversity, equity and respect for others. ii. Extend the closure property to **determine if the subsets of real numbers are closed** with respect to **addition (+), multiplication (*), division (÷) and subtraction (-)**. Include steps for establishing other properties of real numbers (i.e., additive and multiplicative inverses, distributive, etc.). Be mindful of values such as self-confidence, diversity and leadership in achieving strategic critical thinking.	Level 1 Recall **Level 2 Skills of conceptual understanding** Level 3 Strategic reasoning Level 4 Extended critical thinking and reasoning
	1.1.1.LI.2	1.1.1.AS.2
	Distinguish between rational and irrational numbers using the conversion of common fractions to decimals and solve related problems. **Diamond Nine; Group Work/Collaborative Learning** **Using the Diamond Nine strategy,** review subsets of real numbers, establish fractions that lead to recurring and non-recurring decimals, and investigate their applications to real life problems. Promote among learners tolerance, truth, honesty, respect for others' views, etc.	Level 1 Recall Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

Example: Enumerate the subsets of rational and irrational numbers and establish their connections to the real number system.

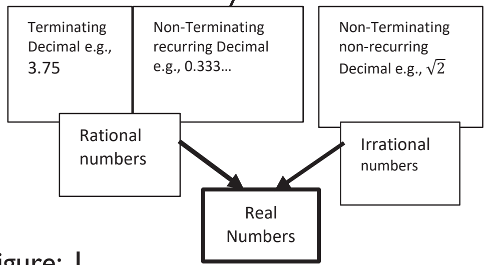


Figure: 1

Through collaborative group activity, use number lines, technology tools, the wheel of Theodorus, etc., to establish rational and irrational numbers. (i.e., $\sqrt{2}$, $\sqrt{3}$, π, etc.).

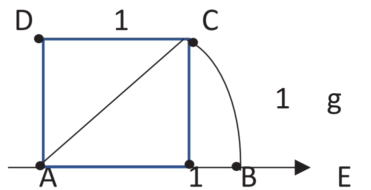


Fig. $|AC| = \sqrt{2}$ is an irrational number.

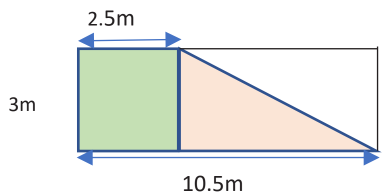
Figure: 2

i. Investigate which of the following real numbers $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{4}$, 2.8, $\sqrt{6}$, can be plotted at exact points on a number line and classify such numbers.

ii. Convert given decimals (recurring, non-recurring, fractions and percentages) to fractions and establish whether they are rational or irrational.

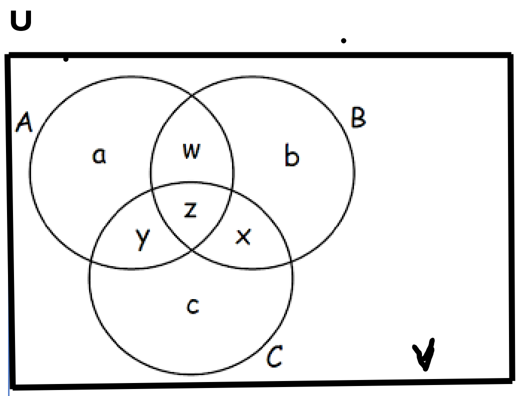
iii. If the ratio $\frac{C}{d} = \pi$, explain why the approximated value of π, " $\frac{22}{7}$ " is not an irrational number (As a group work to be presented in class).

	1.1.1.LI.3	1.1.1.AS.3
	Establish the properties of real numbers with respect to commutative, associative, identity, inverse, distributive, etc. **Talk for Learning, Think-pair-share, and Group Work/Collaborative Learning.** Enumerate the properties of operations with respect to commutative, associative, distributive, identity, inverse, etc., through Think-pair-share and Group work/collaborative activities. Employ differentiated content and ensure learners' tolerance, truth, honesty, respect for others' views, etc. **Example:** Establish (individually, in pairs and in groups) that for any given set of numbers $a, b\ and\ c$ i. $a * b = b * a; \quad and\ a + b = b + a.$ $ie. 2 * 7 = 7 * 2 = 14;\ 2 + 7 = 7 + 2 = 9$ ii. $a * b * c = c * b * a; \quad and\ a + b + c = c + b + a.$ $ie. 2 * 3 * 4 = 4 * 3 * 2 = 24;\ and$ $2 + 3 + 4 = 4 + 3 + 2 = 9$ **Example:** Investigate using multi-base blocks, Geodot and YouTube videos to establish that for a given set of numbers $\boldsymbol{a, b}\ and\ \boldsymbol{c}$; 1 is the *multiplicative identity,* and 0 is the *additive identity*, and extend the knowledge to identity elements, additive and multiplicative inverses as: i. Multiplicative identity element $a * b = b * a = a,$ $ie. 4 * 1 = 1 * 4 = 4$ (1 is multiplicative ID element) ii. Additive identity element $a + b = b + a = a,$ $ie. 4 + (0) = (0) + 4 = 0$ (0 is additive ID element) iii. Multiplicative inverse $a * b = b * a = 1,$ $ie. 4 * \frac{1}{4} = \frac{1}{4} * 4 = 1$ ($\frac{1}{4}$ is the multiplicative inverse of 4) iv. Additive inverse $a + b = b + a = 0,$ $ie. 4 + (-4) = (-4) + 4 = 0$ ((-4) is additive inverse of 4. $ie\ a * 1 = 1 * a = a; \quad and\ a + 0 = 0 + a = a.$ **In small learning groups**, using worksheets, establish the distributive property operation and investigate its applications in other areas of knowledge.	Level 1 Recall Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

	Example 1 i.e., Establish that for any three numbers $\boldsymbol{a}, \boldsymbol{b}$ *and* $\boldsymbol{c}$, the distributive property connects $(*)$ *and* $(+)$ or $(*)$ *and* $(-)$ as follows i. $a * (b + c) = a * b + a * c$, *[* is distributed over +)* $ie. 10 * (6 + 4) = (10 * 6) + (10 * 4) = 100$ ii. $a * (b - c) = (a * b) - (a * c)$, *['*' is distributed over '–').* i.e., solve $108 \div 12$ Rewrite $108 \div 12$ as $(120 - 12) \div 12$ This gives $10 - 1 = 9$ $\therefore 108 \div 12 = 9$ **Example 2:** Find the area of the shaded portion in the figure below using the distributive property of operation.  Applying the distributive property, the area of the shaded portion is realised from A= $h * (a + b) = h * a + h * b$ $\rightarrow 3m\ (2.5m + 4m) = 3m * 2.5m + 3m * 4m$ $\rightarrow 3m(6.5m) = 7.5m^2 + 12m^2$ $19.5m^2 = 19.5m^2$ LHS=RHS **Example 3:** Using distributive property, solve $108 \div 12$ Rewrite $108 \div 12$ as $(108 + 12 - 12) \div 12 = (120 - 12) \div 12$ $= 10 - 1 = 9$ $108 \div 12 = 9$	

Teaching and Learning Resources	• Fractional boards • square or grid paper • multi-base blocks • algebraic tiles	• video clips • cardboards • models • SHS Mathematics curriculum	• Technology tools: computer, mobile phone, calculator, YouTube videos, etc. • Number line • Number track • Wheel of Theodorus.	• Twine • cylinders • Tape measure or rule.

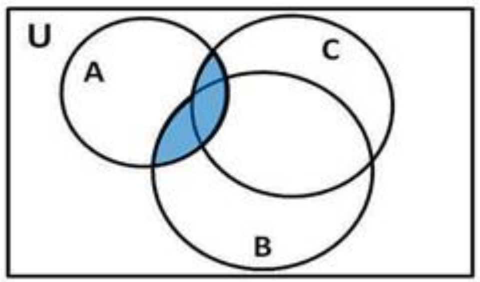
Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
1.1.1.CS.2	1.1.1.LI.1	1.1.1.AS.1
Demonstrate knowledge and understanding of real number systems with respect to the concepts and vocabulary of sets, establish their relationships and carry out simple surveys using the properties of sets.	**Determine the properties of subsets (for two-set problems), their vocabulary and operations and use them to solve real-life problems.** **Building on What Others Say, Managing Talk for Learning, Structuring Talk for Learning, Diamond Nine; Group Work/Collaborative Learning.** **In an interactive activity,** review subsets, unions, intersections, and regions of two sets of Venn diagram and their properties of operations to investigate real life problems involving two sets. Employ differentiated assessment and ensure values such as tolerance, truth, honesty, respect for others' views, etc., among learners. **Example**: Using build on what others say, investigate real-life problems where two sets can be employed to resolve them. Ensure tolerance, discipline and honesty.	Level 1 Recall Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning
	1.1.1.LI.2	1.1.1.AS.2
	Organise information visually to establish the relationship between and among sets of items (three sets) and apply these to conduct mini surveys in the school community and beyond. **Diamond Nine; Group Work/Collaborative Learning**. Apply the concept of the Venn diagram to solve three set problems using Group work/Collaborative Learning, ensuring balance in the differentiation of the various proficiency levels coupled with GESI. **Example**: Create Venn diagrams for three sets (intersecting or non-intersecting) and identify the various sets or regions.	Level 1 Recall Level 2 Skills of conceptual understanding Level 3 Strategic reasoning **Level 4 Extended critical thinking and reasoning**

	 Figure: 4 From the Venn diagram, i. Region (z) represents elements that are common to sets A, B, and C:$A \cap B \cap C$. ii. Region (w) represents elements that belong to sets A and B but not common to C: $(A \cap B) \cap C'$. iii. Region (y) represents elements that belong to sets A and C but not common to B: $(A \cap C) \cap B'$. iv. Region (x) represents elements that belong to sets B and C but not common to A: $(B \cap C) \cap A'$. v. Region (a) represents elements in set A that are neither in set B nor C:$A \cap (B' \cap C')$. vi. Region (b) represents elements in set B that are neither in set A nor C: $B \cap (A' \cap C')$. vii. Region (c) represents elements in set C that are neither in set A nor B: $C \cap (B' \cap A')$. viii. Region (v) represents elements in the universal set U that are not in sets A, B and C: $(A \cup B \cup C)'$.	
	1.1.1.LI.3	1.1.1.AS.3
	Establish the relationship between and among three sets, including set equations and de Morgan's law. **Using Think-pair-share in an interactive and learner-centred classroom,** model and resolve real-life problems using equations involving two and three sets and de Morgan's law. Employ	Level 1 Recall Level 2 Skills of conceptual understanding Level 3 Strategic reasoning

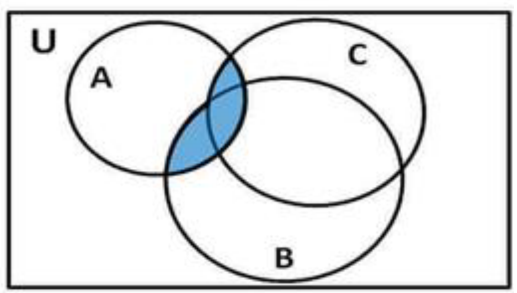
	differentiated assessment and ensure values such as tolerance, truth, honesty, respect for others' views, etc., among learners. **Example** Note: 1. $(A \cup B)' = A' \cap B'$ 2. $(A \cap B)' = A' \cup B'$ 3. $(A \cup B \cup C)' = A' \cap B' \cap C'$ 4. $(A \cap B \cap C)' = A' \cup B' \cup C'$ **In mixed groupings (gender/ability),** engage learners to **e**stablish set identities, including commutative, associative, and distributive properties on set operations, sets algebra and apply them. **Experiential Learning:** Learners will collaboratively be engaged in hands-on activity to create set problems and apply set identities, algebra and operations to solve them. **Example 1**: Use algebra to verify that for any three Sets A, B and C, the following properties are true. i. Commutative Properties $A \cup B = B \cup A$ $A \cap B = B \cap A$ ii. Distributive Properties: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$; $A \cap (B \cup C) = (A \cap B) \cup (A \cap C$ iii. Associative properties: $A \cap (B \cap C) = (A \cap B) \cap C$; $A \cup (B \cup C) = (A \cup B) \cup C)$ iv. Other properties: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ $n(A \cup B \cup C) = n(A) + n(B) + n(C) - [n(A \cap B) + n(A \cap C) + n(B \cap C)] + n(A \cap B \cap C)$ **Example 2**: Learners in small convenient groups (mixed gender, mixed-ability, etc.) investigate and establish set identities and use Venn diagrams to verify that for any three Sets A, B and C:	**Level 4 Extended critical thinking and reasoning**

i. $A \cup B = B \cup A$
ii. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
iii. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Example of illustration of $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ in the Venn diagram



$A \cap (B \cup C)$



$(A \cap B) \cup (A \cap C)$

Inter-group competition: Learners from one group create real-life problems involving up to 3 sets whilst the other group applies set algebra, set operations and set identities to solve the problems. Group switch roles.

Example: A pharmaceutical company is considering manufacturing new toothpaste. They are considering two charcoal flavours, strawberry and mint. In a sample of 74 people, it was found that

45 liked strawberry
37 liked mint
21 liked both types

Create a Venn diagram to model the information.

How many liked only strawberries?

How many liked only mint?

How many liked exactly one of the two (that is, they liked one but not the other)?

<table>
<tr>
<td></td>
<td colspan="2">Using collaborative strategy, engage learners to apply the knowledge of the Venn diagram to solve everyday set-related problems. Encourage differentiation.

Example: Carry out a survey (school or community-based) to establish the blood groups of humans using the antigens A, B and RhD+.
i.e., Blood groups: $U = \{ group\ \boldsymbol{A}, group\ \boldsymbol{B}, group\ \boldsymbol{RhD}\}$
$\boldsymbol{A} = \{ A^{+},\ A^{-},\ AB^{+}\ \ AB^{-}\}$
$\boldsymbol{B} = \{ B^{+},\ B^{-},\ AB^{+}\ \ AB^{-}\}$
RhD $= \{ RhD^{+}\}$
and to present their findings in class to be peer-reviewed by colleagues.

Note: The Biology Department/ health facility needs to be involved in order to further interpret the concepts for lifelong usage.</td>
<td></td>
</tr>
<tr>
<td>Teaching and Learning Resources</td>
<td>• GeoGebra (free software)
• video clips
• cardboards</td>
<td>• models
• SHS Mathematics curriculum
• Fractional boards</td>
<td>• square or grid paper
• multi-base blocks
• algebraic tiles</td>
</tr>
</table>

Subject MATHEMATICS

Strand 1. NUMBERS FOR EVERYDAY LIFE

Sub-Strand 2. PROPORTIONAL REASONING

Learning Outcomes	**21st-century Skills and Competencies**	**GESI, SEL and Shared National Values**
1.1.2.LO.1		
Make connections between fractions and decimals and use them to solve daily problems.	**Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on fractions and decimals using appropriate IT tools to boost their interest and desire to solve more problems on their own. **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to the application of the concept of fractions and decimals and their applications to lifelong learning and further studies. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on fractions and decimals and their applications. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concept of fractions and decimals and their applications to lifelong learning.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: · Respect individuals of different backgrounds in their groups as they discuss and solve problems based on real numbers. · Interrogate their stereotypes and biases about the roles and abilities of different individuals in learning and applying mathematics. · Examine and dispel misconceptions/ myths about GESI as they engage in a sustainable discourse while learning fractions and decimals. · Value and promote justice as they develop and implement innovative actions in the mathematics classroom and beyond. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-*

		Management, Social Awareness, Relationship Skills and Responsible Decisions are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the Mathematics curriculum, the facilitator should apply Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and confident as they establish the relationship and properties of operations of real numbers. · Exhibiting motivation and SMART goal-setting in a mathematics classroom and beyond. · Managing emotions and conflicts as they engage in a mathematical discourse. · Showing empathy and cooperation in a mathematical problem-solving situation. These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the learning of operations of real numbers.

		Diversity: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment as they debate the relationship between rational and irrational numbers. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination as they engage in a mathematical discourse based on properties of real numbers. **Truth and Integrity:** Reward truth and honesty as strong moral principles as they engage in peer assessment to help them become responsible citizens. **Tolerance:** Model tolerance among learners by creating opportunities for collaborative learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment.
1.1.2.LO.2		
Create strategies for solving problems involving percentages of personal or household finances.	**Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on percentages and their applications in life. **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to them developing various strategies for dealing with percentages and their applications to lifelong learning and further studies.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: · Respect individuals of different backgrounds in their groups as they discuss and solve problems based on percentages.

	Integrated Problem-solving Competency: Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on percentages and their applications. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concept of percentages and its applications to lifelong learning.	· Interrogate their stereotypes and biases about the roles and abilities of different individuals in learning and applying mathematics. · Examine and dispel misconceptions/ myths about GESI as they engage in a sustainable discourse while learning percentages. · Value and promote justice as they develop and implement innovative actions in the mathematics classroom and beyond. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the Mathematics curriculum, the facilitator should apply Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and confident as they solve problems in mathematics. · Exhibiting motivation and SMART goal-setting in a mathematics classroom and beyond. · Managing emotions and conflicts as they engage in a mathematical discourse. · Showing empathy and cooperation in a mathematical problem-solving situation. These may be done by the facilitator through modelling emotional self-regulation and

		decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork and respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning about percentages and their applications. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment as they debate on percentages and their applications. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination as they engage in a mathematical discourse based on percentages and their applications. **Truth and Integrity:** Reward truth and honesty as strong moral principles as they engage in peer assessment to help them become responsible citizens. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative

		Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment.

<table>
<tr><th>Content Standards</th><th>Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI</th><th>Assessment</th></tr>
<tr><td>1.1.2.CS.1</td><td>1.1.2.LI.1</td><td>1.1.2.AS.1</td></tr>
<tr><td>Demonstrate an understanding of proportional reasoning involving fractions and its operations and use it to solve real-life problems, including rounding off (decimal places and significant figures).</td><td>

Establish the concept of fractions and investigate the connections between fractions and decimal numbers.

Problem-based Learning; Collaborative Learning:
In an interactive and collaborative grouping, establish the connections among fractions, decimals and percentages, then model and solve problems on daily transactions using these connections. Employ differentiated assessment and ensure values such as tolerance, truth, honesty, respect for others' views, etc., among learners.

Example: Identify decimal names of given fractions; convert fractious to decimals (and vice versa); and convert decimals to percentages (and vice versa)

1. Convert fractions to decimals and percentages (and vice versa) and use these representations in estimations, computations, and applications. i.e.

i. If $0.5 = \frac{0.5}{1} = \frac{5}{10} = \frac{1}{2}$; then $0.25 = ?$

ii. If $\frac{1}{2} = \frac{1}{2} * \frac{5}{5} = \frac{5}{10} = \frac{0.5}{1} = 0.5$; then $\frac{1}{4} = ?$

iii. Convert 0,5454… into a fraction.

2. Establish percentage as the number of parts in every 100; use fractions and percentages to describe parts of shapes, quantities and measures. i.e.

a) $\frac{1}{2} = 0.5, =;\ 0.5 * \frac{100}{100} = 50\%$

b) $\frac{1}{4} = 0.25, ;\ 0.25 * \frac{100}{100} = 25\%$

i. Convert $\frac{5}{9}$, $\frac{15}{75}$, 1.45, $3\frac{1}{4}$ into percentages.

ii. Suglo has 20 items on her shopping list. She checks her list and finds that she has completed 40% of her shopping. Determine how many more items she has to buy.

Through collaborative group activity, enumerate the steps for converting mixed number percent into fractions and solve real life problems involving mixed numbers.

</td><td>Level 1 Recall
Level 2 Skills of conceptual understanding
Level 3 Strategic reasoning
Level 4 Extended critical thinking and reasoning</td></tr>
</table>

	Example 1: Convert $12\frac{1}{2}\%$ into a fraction. We consider: • Step 1: Convert it into an improper fraction. • Thus, $12\frac{1}{2}\% = \frac{25}{2}\%$ • Step 2: Drop the percent symbol%, multiplying by $\frac{1}{100}$. So, $12\frac{1}{2}\% = \frac{25}{2} * \frac{1}{100} = \frac{25}{200}$ • Step 3: Reduce it to the lowest form to get $12\frac{1}{2}\% = \frac{1}{8}$. **Example 2** i. Add, subtract, multiply, and divide rational numbers (integers, fractions). Express positive rational numbers to whole-number powers. i.e., To add mixed fractions, such as. $2\frac{2}{5}$ and $1\frac{2}{3}$, we first add the whole numbers and then add the fractions; i.e. $2 + 1 + \frac{2}{5} + \frac{2}{3} = 3 + \frac{6}{15} + \frac{10}{15} = 3\frac{6+10}{15} = 3\frac{16}{15} = 4\frac{1}{15}$ ii. Solve a variety of word problems involving addition or subtraction of fractions.	

	1.1.2.LI.2	1.1.2.AS.2
	Establish additive and multiplicative inverses of fractions using multi-purpose model charts. Through collaborative group activity, find the multiplicative inverse of a chosen number. **Example:**	Level 1 Recall **Level 2 Skills of conceptual understanding** Level 3 Strategic reasoning Level 4 Extended critical thinking and reasoning

$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$
$\frac{2}{1}$	$\frac{2}{2}$	$\frac{2}{3}$	$\frac{2}{4}$	$\frac{2}{5}$	$\frac{2}{6}$	$\frac{2}{7}$	$\frac{2}{8}$	$\frac{2}{9}$	$\frac{2}{10}$
$\frac{3}{1}$	$\frac{3}{2}$	$\frac{3}{3}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{3}{6}$	$\frac{3}{7}$	$\frac{3}{8}$	$\frac{3}{9}$	$\frac{3}{10}$
$\frac{4}{1}$	$\frac{4}{2}$	$\frac{4}{3}$	$\frac{4}{4}$	$\frac{4}{5}$	$\frac{4}{6}$	$\frac{4}{7}$	$\frac{4}{8}$	$\frac{4}{9}$	$\frac{4}{10}$
$\frac{5}{1}$	$\frac{5}{2}$	$\frac{5}{3}$	$\frac{5}{4}$	$\frac{5}{5}$	$\frac{5}{6}$	$\frac{5}{7}$	$\frac{5}{8}$	$\frac{5}{9}$	$\frac{5}{10}$
$\frac{6}{1}$	$\frac{6}{2}$	$\frac{6}{3}$	$\frac{6}{4}$	$\frac{6}{5}$	$\frac{6}{6}$	$\frac{6}{7}$	$\frac{6}{8}$	$\frac{6}{9}$	$\frac{6}{10}$
$\frac{7}{1}$	$\frac{7}{2}$	$\frac{7}{3}$	$\frac{7}{4}$	$\frac{7}{5}$	$\frac{7}{6}$	$\frac{7}{7}$	$\frac{7}{8}$	$\frac{7}{9}$	$\frac{7}{10}$
$\frac{8}{1}$	$\frac{8}{2}$	$\frac{8}{3}$	$\frac{8}{4}$	$\frac{8}{5}$	$\frac{8}{6}$	$\frac{8}{7}$	$\frac{8}{8}$	$\frac{8}{9}$	$\frac{8}{10}$
$\frac{9}{1}$	$\frac{9}{2}$	$\frac{9}{3}$	$\frac{9}{4}$	$\frac{9}{5}$	$\frac{9}{6}$	$\frac{9}{7}$	$\frac{9}{8}$	$\frac{9}{9}$	$\frac{9}{10}$
$\frac{10}{1}$	$\frac{10}{2}$	$\frac{10}{3}$	$\frac{10}{4}$	$\frac{10}{5}$	$\frac{10}{6}$	$\frac{10}{7}$	$\frac{10}{8}$	$\frac{10}{9}$	$\frac{10}{10}$

Figure:6

Teaching and Learning Resources	• fractional boards • square or grid paper • multi-base blocks	• algebraic tiles • video clips • cardboards	• models • SHS Mathematics curriculum

<table>
<tr><th>Content Standards</th><th>Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI</th><th>Assessment</th></tr>
<tr><td>1.1.2.CS.2</td><td>1.1.2.LI.1</td><td>1.1.2.AS.1</td></tr>
<tr><td rowspan="3">Demonstrate a conceptual understanding of proportional reasoning on percentages and use it to solve everyday life problems, including simple interest, discount, profit, loss, commission, etc.</td><td>Develop models to examine connections between and among fractions, percentages and decimal numbers and make generalisations.

Through Think-pair-share, explore properties of rational numbers from diagrams (such as Figure 6 above) to establish strategies for reviewing the connection between fractions and decimal numbers and solve real-life problems involving them.

Employ differentiated assessment and ensure values such as tolerance, truth, honesty, respect for others' views, etc., among learners.

Examples:
i. Convert fractions to decimals and percentages and use these representations in estimations, computations, and applications.
ii. Play mental mathematics games: Learners use simple mental strategies to perform the following.
a) Addition using words and phrases like plus, add, calculate the sum, increase a number by, and find the total.
b) Subtraction using words and phrases like minus, from a number take, find the difference, and what must be added to make.
c) Multiplication using words and phrases like 'times', 'multiply', 'find the product', 'square', and 'what must be divided by … to give ….'
d) Division using words and phrases like 'divide', 'share', 'how many times does it go into?', and 'what must be multiplied by … to give …'</td><td>Level 1 Recall
Level 2 Skills of conceptual understanding
Level 3 Strategic reasoning
Level 4 Extended critical thinking and reasoning</td></tr>
<tr><td>1.1.2.LI.2</td><td>1.1.2.AS.2</td></tr>
<tr><td>Analyse daily activities/issues/businesses involving percentage change, including simple problems involving personal or household finance (such as utility bills, exchange rates, project budgeting, school sees, shopping, etc.)</td><td>Level 1 Recall
Level 2 Skills of conceptual understanding</td></tr>
</table>

	Using think-pair-share activities, engage learners to undertake a project or group activities on proportional reasoning involving percent, simple/compound interests, discount, profit, loss, and tax, through problem-solving techniques. Employ differentiated assessment and ensure values such as tolerance, truth, honesty, respect for others' views, etc., among learners. **Examples**: 1. Calculate the percentage increase and decrease of a quantity, i.e., if the Public Utility and Regulatory Authority announced an increment in the price of diesel per litre from GHC9.50.00 to GHC13.20, which later declined to GHC12.50. Find the percentage increase and as percentage decrease. **Solution**: Percentage increase = $\frac{increment}{original\ price} * 100\% = \frac{3.7}{9.50} * 100\%$ $= 38.95\%.$ Percentage decrease = $\frac{decrement}{original\ price} * 100\% = \frac{0.7}{13.20} * 100\%$ $= 5.30\%$ 2. Gladys deposited $\mathrm{GHC}4{,}000.00$ into a bank account, and the annual interest rate is 8%. How much is the interest after 4 years?	**Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

Teaching and Learning Resources	• Boards • Square or grid paper • Multi-base blocks	• Algebraic tiles • Video clips	• Cardboards • Fractional boards	• Models • SHS Mathematics curriculum, etc.

Subject MATHEMATICS

Strand 2. ALGEBRAIC REASONING

Sub-Strand 1. APPLICATIONS OF EXPRESSIONS, EQUATIONS AND INEQUALITIES

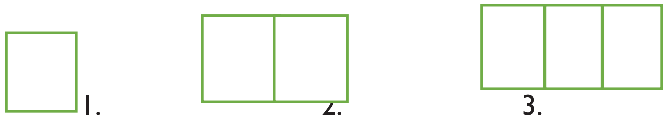
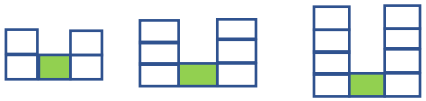
Learning Outcomes	21st-century Skills and Competencies	GESI, SEL and Shared National Values
1.2.1.LO.1		
Formulate algebraic expressions using patterns to create models and solve real life problems (e.g., linear and quadratic models).	**Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on fractions and decimals using appropriate IT tools to boost their interest and desire to solve algebraic expressions using patterns to create model problems on their own. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on algebraic expressions using patterns to create models and their applications in life. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on algebraic expressions and their applications. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concept of algebraic expressions and their applications to lifelong learning.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: • Respect individuals of different backgrounds in their groups as they discuss and solve problems based on algebraic expressions using patterns. • Interrogate their stereotypes and biases about the roles and abilities of different individuals in learning and applying mathematics. • Examine and dispel misconceptions/ myths about GESI as they engage in a sustainable discourse while learning algebraic expressions. • Value and promote justice as they develop and implement innovative actions in the mathematics classroom and beyond. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies – *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the Mathematics curriculum, the facilitator should

		apply Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and confident as they formulate algebraic expressions using patterns. · Exhibiting motivation and SMART goal-setting in a mathematics classroom and beyond. · Managing emotions and conflicts as they engage in a mathematical discourse. · Showing empathy and cooperation in a mathematical problem-solving situation. These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork and respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning how to formulate algebraic expressions using patterns. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment as they debate how to formulate algebraic expressions using patterns. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted

		segregation or discrimination as they engage in a mathematical discourse based on formulating algebraic expressions using patterns. **Truth and Integrity:** Reward truth and honesty as strong moral principles as they engage in peer assessment to help them become responsible citizens. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment.
1.2.1.LO.2		
Model and solve linear equations and inequalities in one variable, including problems in real life.	**Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on linear equations and inequalities using appropriate IT tools to boost their interest and desire to solve linear equations and inequalities and solve problems on their own. **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to developing various strategies for linear equations and inequalities to lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on own one's values, perceptions and actions for decision-making as they engage in group and individual activities on linear equations and inequalities and their applications in life.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: · Respect individuals of different backgrounds in their groups as they discuss and solve problems based on linear equations and inequalities. · Interrogate their stereotypes and biases about the roles and abilities of different individuals in learning and applying mathematics. · Examine and dispel misconceptions/ myths about gender as they engage in a sustainable discourse while learning mathematics. · Value and promote justice as they develop and implement innovative actions in the mathematics classroom and beyond. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-*

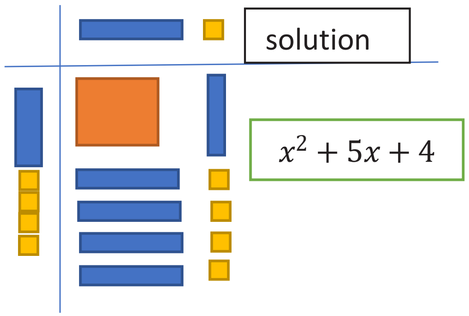
	Integrated Problem-solving Competency: Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on linear equations and inequalities and their applications. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concept of linear equations and inequalities and their applications to lifelong learning.	*Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the Mathematics curriculum, the facilitator should apply Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and confident as they solve problems on linear equations and inequalities. · Exhibiting motivation and SMART goal-setting in a mathematics classroom and beyond. · Managing emotions and conflicts as they engage in a mathematical discourse. · Showing empathy and cooperation in a mathematical problem-solving situation. These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork and respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning to solve linear equations and inequalities. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning

		environment as they debate linear equations and inequalities. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination as they engage in a mathematical discourse in solving linear equations and inequalities. **Truth and Integrity:** Reward truth and honesty as strong moral principles as they engage in peer assessment to help them become responsible citizens. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment.

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
1.2.1.CS.1	1.2.1.LI.1	1.2.1.AS.1
Demonstrate knowledge and understanding of algebraic expressions and solve real-life problems on them.	**Use numbers, patterns, and variables to formulate mathematical expressions and apply the algebraic order of the four operations to solve problems.** **Group Work/Collaborative learning, initiating Talk for Learning and Problem-based Learning.** **Collaborative learning**: Using mixed-ability groups, learners formulate mathematical expressions using patterns and variables. Ensure values such as patience, truthfulness, respect for others' views, etc. **Example**: i. Squares are arranged as shown below. Investigate and write an algebraic expression if the pattern is continuous.  1. 2. 3. Fig.1 Fig.2 Fig.3 ii. Investigate and write an algebraic expression for the patterns and explain your answer.  Fig.1 Fig.2 Fig.3 **Using think-pair-share,** brainstorm how to translate real-life statements into a mathematical expression. Employ differentiated assessment and ensure values such as tolerance, truth, honesty, respect for others' views, etc., among learners.	Level 1 Recall Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

	Example: Discuss and translate statements, including real-life statements, into mathematical expressions. i.e. a) Nine more than a certain number $x+9$ b) Quantity less than five. c) There are 25 oranges in a bag. Write the algebraic expression for the number of oranges in x number of bags. **Using Talk for Learning** approach in a whole class discussion, learners work collaboratively to solve problems on operations on algebraic expressions (**addition**, **subtract, multiply** and **divide).** **Examples**: i. Simplify the following expressions. a) $x+2y+5x-y$ b) $5p-c-9c$ iii. Simplify the following expressions. a) $4x \times 2y$ b) $x^2(x^3-3y)$ c) $10b \div 2bb$	
	1.2.1.LI.2	1.2.1.AS.2
	Factorise algebraic expressions involving quadratic trinomials. **Group Work/Collaborative learning, initiating Talk for Learning and Problem-based Learning** **Using Collaborative Learning:** Learners identify, expand and simplify two binomial expressions using algebraic tiles. Employ differentiated assessment and ensure values such as tolerance, truth, honesty, respect for others' views, etc., among learners. **Example**: Identify binomial expressions and work in groups to expand and simplify the two binomial expressions using algebraic tiles. i.e., $(x+4)(x+2)$. **Solution**. **Steps**:	Level 1 Recall Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

- The blue tile is $+x$, the orange is x^2 and the yellow tile is 1. Along one side of the chart, fill in the expression $x + 4$ using the algebraic tiles, and across the top, fill in the expression $x + 1$
- The product is the result inside the chart when we multiply the terms outside. Now count the number of x^2 ,x, and the 1 and sum them up to give the solution.



Problem-based Learning: In mixed gender/ability groups, learners expand and simplify algebraic expressions.

Example: Expand and simplify the following expressions

i. $(x + 4)(x - 4)$
ii. $(4x + 1)^2$
iii. $(p - q)^2$
iv. $(x + 9)^2$

Investigate, in mixed-ability/gender groups, the concept of factorisation and apply the idea to factorise expressions, including algebraic trinomials.

Example: Factorise $ac + bc + ad + bd$

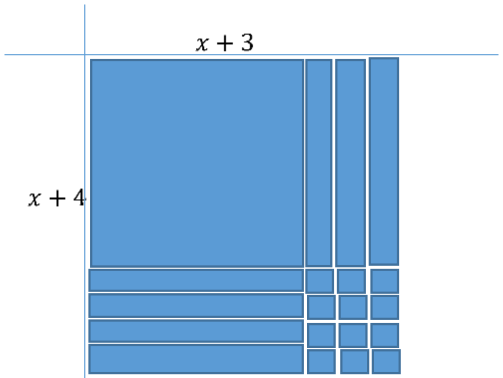
Solution: First, put the four expressions into two groups of two and find the common factors from each. i.e., $(ac + bc)(ad + bd)$

$c(a+b)+d(a+b)$, add the outside terms and take one of the common terms to be the final answer.
$(c$ +d)(a+b)

Using Talk for Learning in a whole class discussion, review with learners the concepts of finding the area of a rectangle and use the idea to factorise the trinomial expressions using algebraic tiles.

Example: Factorise $x^2 + 7x + 12$ using algebraic tiles.

Solution
Use algebraic tiles



$x^2 + 7x + 12 = (x+3)(x+4)$

From the above diagram, the product of -2 and 1 is -2, the sum of -2 and 1 is -1, and the product of the x is the x^2. Therefore, the sum of the root is **b,** and the product of the root is **c,** which gives a quadratic expression.

	1.2.1.LI.3	1.2.1.AS.3
	Recognise perfect squares and apply the idea to solve problems, including the difference of two squares of binomials.	Level 1 Recall

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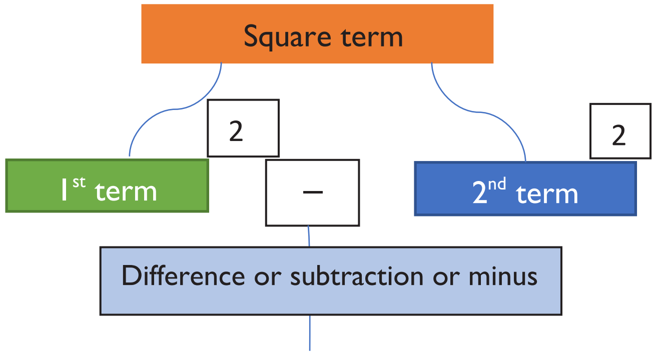
Group Work/Collaborative Learning, Problem-based Learning and self-paced learning.

In convenient groups, learners explore the concept of perfect squares using whole numbers.

Example: 0+1=1, 1+3=4, 4+5=9, etc.
Working collaboratively, learners brainstorm the meaning of perfect squares, identify differences of two squares, and use problem-solving strategies to apply the concepts of perfect squares.
Employ differentiated assessment and ensure values such as tolerance, truth, honesty, respect for others' views, etc., among learners.



For example, $x^2 - 4$, $y^2 - 1$

Note that whenever you have a binomial with each term being squared (i.e. having an exponent of 2) and subtraction as the middle sign, then you are guaranteed to have the case of difference of two squares.

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Level 2 Skills of conceptual understanding
Level 3 Strategic reasoning
Level 4 Extended critical thinking and reasoning

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Analyse and apply operations on simple algebraic fractions involving monomial and binomial denominators and determine the conditions under which an algebraic fraction is zero or undefined.

Group discussions, Problem-based Learning and Collaborative Learning.

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Level 1 Recall
Level 2 Skills of conceptual understanding
Level 3 Strategic reasoning

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	Using Talk for Learning strategy, the class discuss and explain what an algebraic fraction is and use the idea to solve problems. **Example 1:** Explain how operations involving algebraic fractions are applied; that is, when fractions are multiplied, their numerators are multiplied, and denominators are also multiplied, etc. **E.g.** $\frac{m}{n} \times \frac{a}{b} = \frac{ma}{nb}$ When a fraction is divided by another, multiply the first fraction by the reciprocal of the second fraction. Thus $\frac{m}{n} \div \frac{a}{b} = \frac{m}{n} \times \frac{b}{a} = \frac{mb}{na}$ etc. **Example 2**: Extend the idea to perform operations on algebraic fractions, including monomial and binomial denominators. **E.g.:** Multiply and divide algebraic fractions. Simplify the following: i. $\frac{2x}{1} \times \frac{2y}{1}$ ii. $\frac{6x+8}{4} \div \frac{x^2+3}{5x^2}$ **Example 2:** Add and subtract algebraic fractions with binomial denominators. Simplify the following. i. $\frac{10}{x-4} + \frac{2}{x+1}$ ii. $\frac{-1}{3x+4} - \frac{6}{2x-1}$ iii. $\frac{x}{x+1} - \frac{2x}{x+2}$ iv. $\frac{x}{x^2-5x+6} + \frac{1}{x-2} + \frac{3}{x-3}$ **Collaborative Learning:** Using think-pair-share in mixed-ability groups, learners identify and explain the condition under which an algebraic fraction is zero or undefined.	Level 4 Extended critical thinking and reasoning

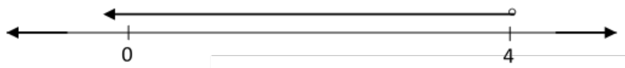
	Example 1: Learners, after discussions, come out with the idea that an algebraic fraction is said to be undefined or have no meaning if the denominator is equal to zero and also zero if the numerator is zero. **E.g**., Determine the condition under which $\frac{3a}{a-4}$ is zero or undefined and determine the value of x that makes the expression undefined. **Solution** For $\frac{3a}{a-4}$ to be zero, then $3x = 0$ For $\frac{3a}{a-4}$ to be undefined, $a - 4 = 0$ To find the value of a that makes the expression undefined, we take the denominator and solve for a Therefore, the expression is undefined if $a = 4$. **Example 2:** Determine the condition under which an algebraic expression is undefined. Find the value of x that makes the expressions undefined. i. $\frac{3}{x-1}$ ii. $\frac{(2x-1)(x-4)}{4x^2-1}$ $$\frac{2x+5}{x^2+5x-20}$$	

Teaching and Learning Resources	• Algebraic tiles • Patterns • calculator • technology tools such as	• computer • mobile phone • YouTube videos, etc. • Paper grids	• Maths posters • YouTube videos • Whiteboard • Pan balance	• Videos • mini whiteboards or laminated white paper • Dry-erase markers and erasers

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
1.2.1.CS.2	1.2.1.LI.1	1.2.1.AS.2
Demonstrate knowledge and understanding of equations and inequalities in one variable and apply it in solving real-life problems.	**Construct and interpret formulae for a given task and apply them to problems involving change of subjects.** **Group/Collaborative Learning, Problem-based Learning:** **Using think-pair-share activity,** perform calculations in a change of subject and substitute values into formulae and use it to solve problems**.** Employ differentiated assessment and ensure values such as tolerance, truth, honesty, respect for others' views, etc., among learners. **Example** 1: Change subjects in a given formula. i. Make c the subject of the relation $y = mx + c$ **Solution:** To make c the subject, subtract mx from both sides of the equation. $\boldsymbol{mx} - \boldsymbol{mx} + c = y - mx$ $\therefore \boldsymbol{c} = \boldsymbol{y} - \boldsymbol{mx}$ ii**.** From the equation $3c + 2r = md + k$, make r the subject. **Solution** Given $3c + 2r = md + k$, Make 2r the subject: ☐ $2r = md + k - 3c$ Divide both sides by 2. ☐ $r = \frac{md + k - 3c}{2}$ **Example 2**: The relation between energy *E,* mass *m*, and velocity of light *v* is given by $E = mv^2$. Find the value v when $E = 20$ and $m = 5$	Level 1 Recall **Level 2 Skills of conceptual understanding** Level 3 Strategic reasoning Level 4 Extended critical thinking and reasoning

	Solution Given that E = mv^2 ☐ $v^2 = \frac{E}{m}$ $\therefore v = \sqrt{\frac{E}{m}}$ $v = \sqrt{\frac{20}{5}}$ $\therefore v = 2$	
	1.2.1.LI.2	1.2.1.AS.2
	Solve linear equations and inequalities in one variable for a given problem and relate it to real life situations. **Group/Collaborative Learning, initiating Talk for Learning, Problem-based Learning.** **Using the think-pair-share activities,** learners discuss and explain a linear equation in one unknown (variable) as an equation of the form ax+b=c, where a, b and c are real numbers and a≠0 has exactly one solution. **Example 1**: Solve for the variable indicated in the following equations: i. $3x - 12 = 21$ ii. $7(x-6) = 3(x+9)$ **Solution** i. To solve $3x - 12 = 21$, add 12 to both sides of the equation, $3x = 33$ then divide both sides of the equation by 3 to make *x* the subject $x = \frac{33}{3} = 11$	Level 1 Recall Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

	ii. To solve the equation $7(x-6)=3(x+9)$, First, multiply the brackets on both sides of the equation, $7x-42=3x+27$ Then subtract 3*x* from both sides $4x-42=27$ Then, add 42 to both sides of the equation. $4x=69$ Divide both sides of the equation by 4. $x=\frac{69}{4}=17\frac{1}{4}$ **In mixed-ability groups,** learners extend the idea of linear equation to explain linear inequality, including real life activities to enable conceptual understanding. **Example:** Learners establish that a linear equation is of the form $ax+b<c, ax+b\leq c, ax+b>c, ax+b\geq c.$ They also establish that folding the human right arm resembles the idea of a greater than (>) symbol, and that of the left arm also resembles the idea of a less than (<) symbol and gives expressions that involve these signs. i.e. $4<5$ (4 is less than 5) $6>4$ (6 is greater than 4) $x\leq 4$ (Depending on the values of *x*, which is an unknown variable) Suppose we are given an inequality of the form $x > -5$; the solution set for an inequality (as it is for an equation) is the set of all values for the variable that make the inequality a true statement. An appropriate way to picture the solution set is by a graph on a number line or the use of a geodot to generate conjectures. **Discuss graphs of inequalities and graph the** set; $\{x: x < 4\}$	



Explanations: We want to include all real numbers less than 4, that is, to the left of 4 on the number line. The open circle is used to indicate that the point corresponding to 4 is not included in the graph. It is called **an open half line**; it extends to the left and does not include 4.
Two other symbols, as shown in the introduction, $\leq$ and $\geq$, are also used in writing inequalities. In each case, they combine the inequality symbols for less than or greater than with the symbol for equality.

The following explains the use of these symbols. The expression ***a*** $\leq$ ***b*** is read as "***a*** is less than or equal to ***b.***"

Note that this combines the symbols '< 'and '=' and means that either ***a*** < ***b*** or ***a*** = b.
Similarly, a ≥ b reads "a is greater than or equal to b". Implying either ***a*** > ***b*** or ***a*** = ***b***. etc.

Example 1: Find the solution set of the following:
Solve for the truth set of $\frac{1}{2}x-\frac{1}{3}(x+4)$ > 4***x*** + $\frac{2}{3}$
Dividing both sides by 23 yields $x<\frac{-12}{23}$
Hence, the truth set is $\left\{x:x<\frac{-12}{23}\right\}$.

Using think-pair-share in mixed-ability groups, learners discuss word problems involving linear equations and inequalities and translate them into mathematics statements and solve.

Example 1: If Kofi's age now is 30 years, what will be his age in 5 years' time?
Explore: What facts are you given?
* The fact already given is 30 years
* The fact yet to be found is 5 years' time
What fact do you need to find?
* Thus, how many years together would Kofi be in the coming 5 years (future)?
Plan: Write an equation

	Let **m** be Kofi's age now His age in 5 years time $\Rightarrow$ m = 30 + 5 $\Rightarrow$ m = 35 years. **Example** 2**:** If Ama is 40 years old now, what was her age 4 years ago? Let n be her age now = 40 years. (given fact) Her age 4 years ago (past years) $\Rightarrow (n-4)$ $\Rightarrow 40-4$ $= 36$ years			
Teaching and Learning Resources	• Algebraic tiles • Patterns • calculator • technology tools such as	• computer • mobile phone • YouTube videos, etc. • Paper grids	• Maths posters • YouTube videos • Whiteboard • Pan balance	• Videos • mini whiteboards or laminated white paper • Dry-erase markers and erasers

Subject **MATHEMATICS**

Strand **2. ALGEBRAIC REASONING**

Sub-Strand **2. PATTERNS AND RELATIONS**

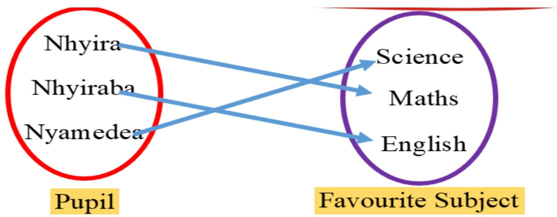
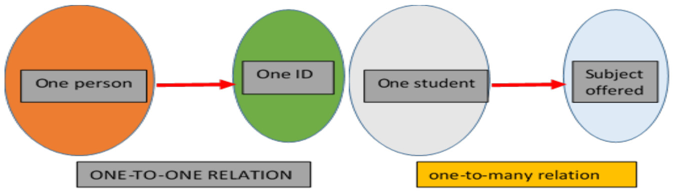
Learning Outcomes	21st-century Skills and Competencies	GESI, SEL and Shared National Values
1.2.2.LO.1		
Distinguish between relations and functions, determine the rules, then draw graphs of functions and interpret them.	**Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for distinguishing between relations and functions, including determining the rules for functions, then drawing graphs of functions and interpreting them. **Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on fractions and decimals using appropriate IT tools to boost their interest and desire to distinguish between relations and functions, including determining the rules for functions, then drawing graphs of functions and interpreting them. **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to distinguishing between relations and functions, including determining the rules for functions, then drawing graphs of functions and interpreting them. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on relations and functions, including determining	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: · Respect individuals of different backgrounds in their groups as they discuss and solve problems based on functions. · Interrogate their stereotypes and biases about the roles and abilities of different individuals in learning and applying mathematics. · Examine and dispel misconceptions/ myths about GESI as they engage in a sustainable discourse while learning relations and functions, including determining the rules for functions. · Value and promote justice as they develop and implement innovative actions in the mathematics classroom and beyond. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies – *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome

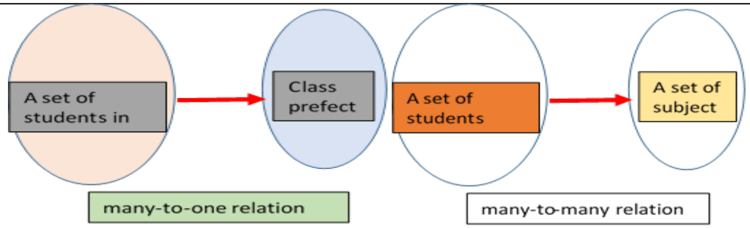
	the rules for functions, then draw graphs of functions and interpret them.	in the Mathematics curriculum, the facilitator should apply Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and confident as they establish relations and functions, including determining the rules for functions. · Exhibiting motivation and SMART goal-setting in a mathematics classroom and beyond. · Managing emotions and conflicts as they engage in a mathematical discourse. · Showing empathy and cooperation in a mathematical problem-solving situation. These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of determining the rules for functions, drawing graphs of functions and interpreting them. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment.

		Equity: Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination as they engage in a mathematical discourse based on the functions. **Truth and Integrity:** Reward truth and honesty as strong moral principles as they engage in peer assessment to help them become responsible citizens. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability groupings within a differentiated mathematics classroom instruction and assessment.
1.2.2.LO.2		
Determine the gradient and equation of a straight line and find the distance between two points on a straight line.	**Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on determining the gradient and equation of a straight line and finding the distance between two points. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on determining the gradient and equation of a straight line and finding the distance between two points. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concept of determining the gradient and equation of a	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: · Respect individuals of different backgrounds in their groups as they discuss and solve problems based on gradients and equations of straight lines and find the distance between two points on a straight line. · Interrogate their stereotypes and biases about the roles and abilities of different individuals in learning and applying mathematics. · Examine and dispel misconceptions/ myths about GESI as they engage in a sustainable discourse while learning mathematics. · Value and promote justice as they develop and implement innovative actions in the mathematics classroom and beyond.

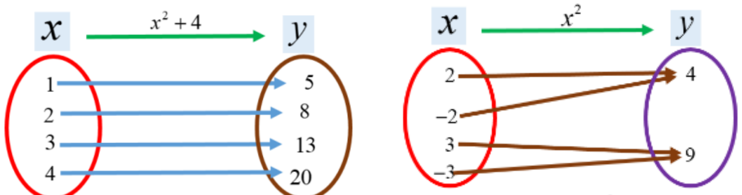
	straight line and finding the distance between two points and their applications to lifelong learning.	**SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the Mathematics curriculum, the facilitator should apply Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and confident as they establish gradients and equations of straight lines and find the distance between two points on a straight line. · Exhibiting motivation and SMART goal-setting in a mathematics classroom and beyond. · Managing emotions and conflicts as they engage in a mathematical discourse. · Showing empathy and cooperation in a mathematical problem-solving situation. These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; and respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work.

		Diversity: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment, as they discuss the gradients and equations of straight lines and find the distance between two points on a straight line. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination as they engage in related mathematical discourse. **Truth and Integrity:** Reward truth and honesty as strong moral principles as they engage in peer assessment to help them become responsible citizens. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment.

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
1.2.2.CS.1	1.2.2.LI.1	1.2.2.AS.1
Demonstrate an understanding of mapping, relations, and functions and the ability to interpret graphs of a function and its applications in real life.	**Distinguish between relations and functions using models such as graphs, investigate relationships between two number sets and determine rules for mappings or functions.** **Using Talk for Learning strategy in mixed-ability groups,** learners discuss the meaning of relations and functions with examples and types of relations. **Example**: A Relation is a relationship between one group or set (input) and another group or set (output).  **Types of Relations** 	Level 1 Recall Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning



Using Talk for Learning in the whole class, learners identify functions out of the relation and establish the meaning of a function as a relation which derives one OUTPUT for each given INPUT and give the types of function as one-to-one and one-to-many function.



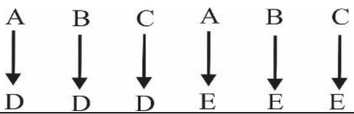
One-to-one function

Many-to-many function

In collaborative groups, learners brainstorm and establish that mapping is the same as function, which maps elements in one set to a unique element in another set. Represent these relations on mapping diagrams.

Example: Ama has three jumpers and two skirts. Combine these in six possible ways using an arrow diagram.

Solution: Let the jumpers be A, B, and C
Let the skirts be D and E.



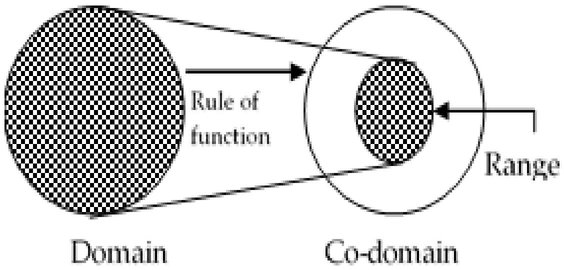
Make an arrow diagram showing the given relation between A and B.

$A = \begin{Bmatrix} \text{The Gambia, Ghana, Liberia,} \\ \text{Nigeria, Sirerra Leone} \end{Bmatrix}$ $B = \begin{Bmatrix} \text{Accra, Banjul, Freetown,} \\ \text{Lagos, Monrovia} \end{Bmatrix}$

Relation: Country $\rightarrow$ Capital of that Country.

A subset of the ***co-domain*** that is actually used by the function is called the ***range*** of the function. This is illustrated in the figure below.

A subset of the ***co-domain***, the ***range*** of the function is shown below:



Talk for Learning: In mixed-gender groups, learners should extend their idea of mapping to discuss linear mapping and establish the rule for linear mapping. Employ differentiated assessment and ensure values such as tolerance, truth, honesty, respect for others' views, etc., among learners.

Example: **Linear Mapping**
A mapping is said to be linear if the difference between the consecutive elements in both the domain and the co-domain is constant.

i.e.,

x	1	2	3	4	5 ...
y	1	3	5	7	9 ...

x	0	2	4	6	8 ...
y	4	8	12	16	20 ...

Example: Rule for Linear Mapping

The rule is of the form $y = mx + c$

Where $y = ax + b$

$$m/a = \frac{\text{constant difference of the co} - \text{domain}}{\text{constant difference in the domain}}$$

And b is the constant.

E.g. What is the rule of the mapping shown below?

x	0	2	4	6	8 ...
y	4	8	12	16	20 ...

Solution

The rule of the mapping is of the form $y = ax + b$

$$a = \frac{8-4}{2-0}$$
$$a = 2$$

Put the value of a into the equation $y = 2x + b$ ... (1)

Now take any coordinates, say (0,4), and put them into Equation (1): $x = 4$

And $y = 4$,

$4 = 2(0) + b$

$b = 4$

Put b back into Equation (1) to give the rule for the mapping above.

$\therefore y = 2x + 4$

	Talk for Learning: In convenient groups, learners should discuss **exponential mapping** and establish the rule of exponential mapping. **Example: Exponential Mapping** A mapping is said to be Exponential if the ratio between the consecutive elements in the co-domain is constant. i.e. $\begin{array}{ccccc} x & 1 & 2 & 3 & 4 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ y & 2 & 4 & 8 & 16 \end{array}$ $\qquad$ $\begin{array}{ccccc} x & 1 & 2 & 3 & 4 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ y & \frac{1}{3} & \frac{1}{9} & \frac{1}{27} & \frac{1}{81} \end{array}$ Put the values of a, b and c into the form $y = ax^2 + bx + c$ $\therefore y = 2x^2 + 3x + 1$	
	1.2.2.LI.2	1.2.2.AS.2
	Draw graphs of linear functions and interpret them. **Initiating Talk for Learning in a whole class discussion,** review the form of a linear function as $\mathrm{y = mx + c}$ where m and c are constant (include the form $\mathrm{ax + by + c = 0}$). **Experiential Learning:** In small groups, learners use any of the available IT tools to research and come out with an explanation as to why the graph of a linear function is a straight line. **Example**: Mrs. Avotris asks Ama to identify whether the given equation $3x - 7y = 16$ forms a linear graph without plotting its values. **Solution:** First, Ama needs to identify the type of equation. Next, she needs to remember that any linear equation in two variables always represents a straight line. Therefore, the above equation represents a straight line. **Collaborative learning*:*** In pairs, task learners to draw a straight line given a gradient and justify their answer.	Level 1 Recall Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** **Level 4 Extended critical thinking and reasoning**

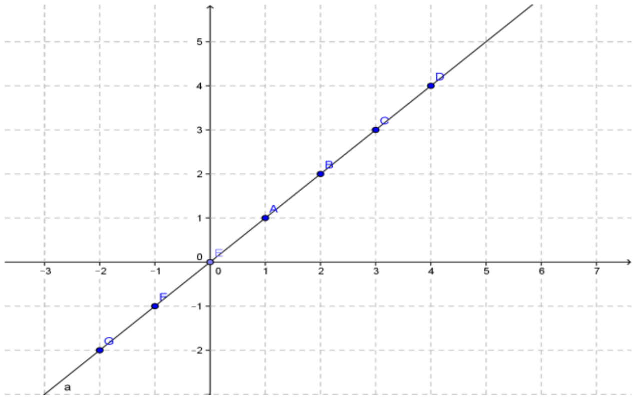
Example: Draw the graph of a straight line with the following gradient and explain your answer.

i. 1

ii. -1

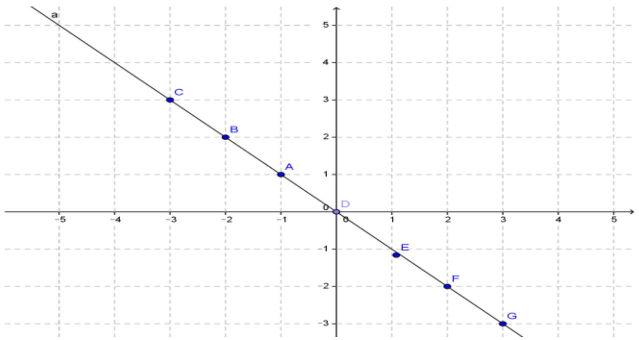
Solution

i.



The graph with the gradient 1 passes through the origin.

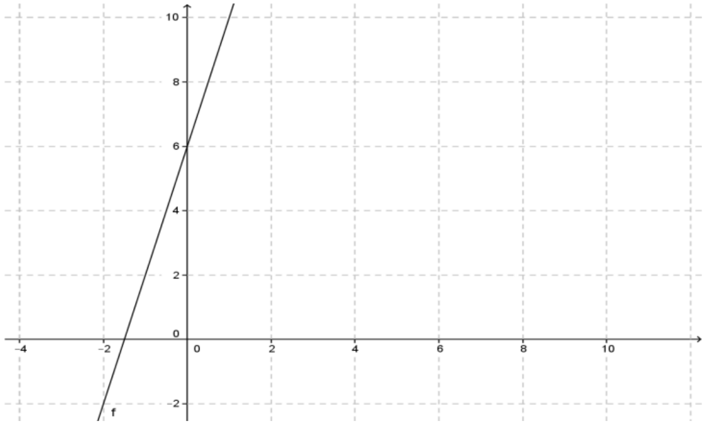
ii.



Collaborative Learning: In pairs, task learners to brainstorm on how to draw a straight line using the slope-intercept form (I.e., $y = mx + c$) and investigate what happens if the constant c (i.e., y-intercept) keeps on changing in a particular equation.

Example: Draw the graph of $y = 4x + 6$ and explain what happens if the constant 6 is changed to 1.

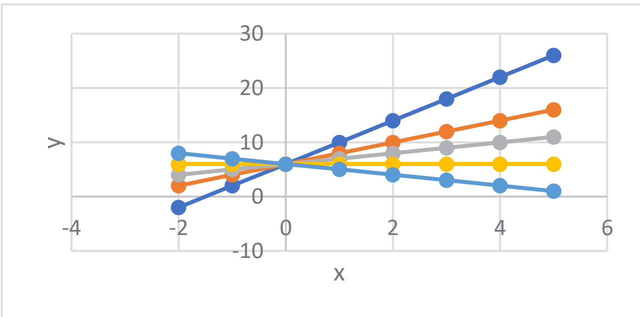
Solution



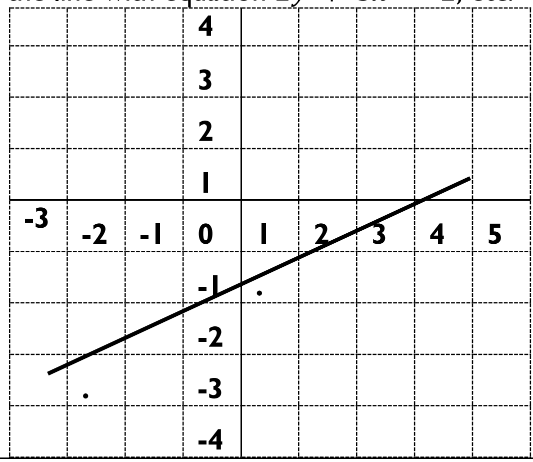
Collaborative Learning: In pairs, task learners to brainstorm on how to draw a straight line, using equations in the form $y = mx + c$ and investigate what happens if the coefficient of x keeps on changing in a particular equation.

Example: Draw the graph of $y = 4x + 6$ and explain what happens if the coefficient 4 is changed to 2,1, 0 and -1, respectively.

Solution: As the co-efficient decreases, the line is rotated clockwise about (0, 6)



	Talk for Learning: In a whole class discussion, review how to draw a linear function with a given interval.			
Teaching and Learning Resources	• GeoGebra • Algebraic tiles • Graph boards	• Patterns • Calculator	• Technology tools such as • Computer	• Mobile phone • YouTube videos, etc.

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
1.2.2.CS.2	1.2.2.AS.1	1.2.2.LI.1
Demonstrate understanding of the gradient and equation of a straight line, the magnitude of a line segment, and its applications in real-life situations.	**Extend the knowledge of the coordinates of two points to find the gradient and equation of a straight line**. **Using think-pair-share strategies,** discuss the midpoint of a line segment using everyday real objects e.g., book lengths, classroom length, etc. **Examples**: i. Compute the midpoint of the line segment whose endpoints are P (8, -16) and Q(4,-4). ii. Examine the value of *b* if the midpoints of P(3,7) and Q(5,b) are (4,-1), etc. Extend to evaluate and discuss the gradient of the specified form of the equation of a straight line and apply them to solve problems. iii. Compute the equation of a line which passes through the points (-2, -3) and (1, -1). iv. Write the equation of the line that passes through the point (3, -1) with a slope of 3. v. Determine the equation of a straight line that passes through the point $(-2, 4)$ with gradient $-\frac{1}{2}$. vi. Evaluate the gradient of the line with equation $2y + 3x = 2$, etc.  	Level 1 Recall Level 2 Skills of conceptual understanding Level 3 Strategic reasoning **Level 4 Extended critical thinking and reasoning**

<table>
<tr><td></td><td>1.2.2.AS.2</td><td>1.2.2.LI.2</td></tr>
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Recognise and interpret two points on a straight line and use it to find the distance between them.

In groups, task learners to investigate between parallel and perpendicular lines and establish their relationships through problem-solving.

Example 1:

i. Are the lines L_1 through (2, 3) and (4, 6) and L_2 through (-4, 2) and (0, 8) parallel, or do they intersect? Explain.

ii. Show that the following pairs of lines are parallel.
(a) $3y = 6x + 9$ and $2y + 12 = 4x$
(b) $5y + 3x = 2$ and $15y = -9x – 12$

iii. Examine the equation of the line which passes through the point (2, 3) and parallel to the line $2y – x = 3$

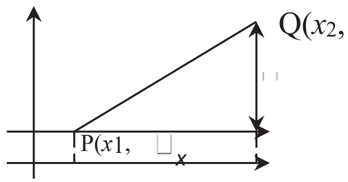
iv. Show that the graphs of $3x+4y=4$ and $-4x+3y=12$ are perpendicular lines.

v. Are lines L_1, through points (-2, 3) and (1, 7) and L_2 through points (2, 4) and (6, 1) perpendicular? Explain.

vi. Determine the equation of the line perpendicular to $2y+3x= 6$ through the point (5, 2) and explain your result.

Example 2: Deduce, through discussions, the magnitude of a line segment and determine the distance between two points using everyday activity.

E.g.



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Level 1 Recall
Level 2 Skills of conceptual understanding
Level 3 Strategic reasoning
Level 4 Extended critical thinking and reasoning

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The magnitude of a line, also known as "length", "distance", or "modulus" of a line, describes how long a line links to two points.
Relating the length of objects discussed from activities, if P and Q have coordinates (x_1, y_1) and (x_2, y_2) from the above figure,

$\Delta x = x_2 - x_1$ $\Delta x = x_2 - x_1$

$\Delta y = y_2 - y_1$

Where Δ means a change
By Pythagoras theorem,

$|PQ|^2 = \Delta x^2 + \Delta y^2$

$|PQ| = \sqrt{\Delta x^2 + \Delta y^2}$

$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

So, if P(x_1, y_1) and Q(x_2, y_2) are two points in the oxy plane, then the distance between P and Q is

$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Note: The symbol **(☐) called delta,** as used here, implies a change in $\mathbf{x_2}$, $\mathbf{x_1}$, y_2, and y_1 or the differences in their values.

Example 3: Determine the distance between the points.
(a) P(2, 1) and Q(5, 5)   (b) A(7, -3) and B(-1, 5) (c) D(4, 1) and E(-3, -5)

Solution

The distance between two points is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(a) P(2, 1) and Q(5, 5)

$|PQ| = \sqrt{(5-2)^2 + (5-1)^2} = \sqrt{3^2 + 4^2}$

$\Rightarrow |PQ| = \sqrt{25} = 5$units.

(b) A(7, -3) and B(-1, 5)

	$\Rightarrow \lvert AB \rvert = \sqrt{(-1-7)^2 + (5+3)^2}$ $\Rightarrow \lvert AB \rvert = \sqrt{(-8)^2 + (8)^2} = \sqrt{128}$ $\Rightarrow \lvert AB \rvert = 8\sqrt{2}$ units. (c) D(4, 1) and E(-3, -5) $\Rightarrow \lvert DE \rvert = \sqrt{(-3-4)^2 + (-5-1)^2}$ $\Rightarrow \lvert DE \rvert = \sqrt{(-7)^2 + (-6)^2} = \sqrt{49+36}$ $\Rightarrow \lvert DE \rvert = \sqrt{85}$ units, etc. **Example:** Determine the length of the line joining P(-5, 1) and Q(7, -4) and explain why the result is a perfect square, etc. Encourage applications to day-day problem-solving.			
Teaching and Learning Resources	• GeoGebra • Algebraic tiles • Graph boards	• Patterns • Calculator	• Technology tools such as • Computer	• Mobile phone • YouTube videos, etc.

Subject **MATHEMATICS**

Strand **3. GEOMETRY AROUND US**

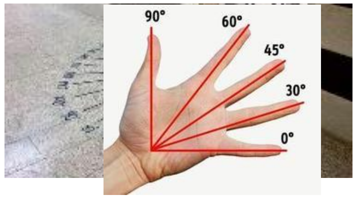
Sub-Strand **1. SPATIAL SENSE**

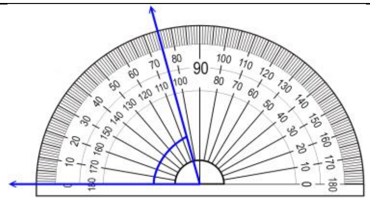
Learning Outcomes	**21st-century Skills and Competencies**	**GESI, SEL and Shared National Values**
1.3.1.LO.1		
Draw and describe angles of various measures; solve problems on the Pythagorean theorem, parallel lines, perpendicular lines and transversal; use the exterior angle theorem of a triangle and calculate the sums of interior and exterior angles of polygons.	**Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on fractions and decimals using appropriate IT tools to boost their interest and desire to develop various strategies for solving problems on the Pythagorean theorem, parallel lines, perpendicular lines and their applications to lifelong learning and further studies **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to developing various strategies for solving problems on the Pythagorean theorem, parallel lines, perpendicular lines and their applications to lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on algebraic expressions using patterns to create models and their applications in life. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on the Pythagorean theorem, parallel lines, and perpendicular lines and their applications.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: • Respect individuals of different backgrounds in their groups as they discuss and develop various strategies for solving problems on the Pythagorean theorem, parallel lines, perpendicular lines and their applications. • Interrogate their stereotypes and biases about the roles and abilities of different individuals in learning and applying mathematics. • Examine and dispel misconceptions/ myths about GESI as they engage in a sustainable discourse while learning mathematics. • Value and promote justice as they develop and implement innovative

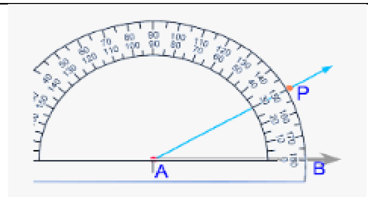
	Innovation and Creativity: Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concept of the Pythagorean theorem, parallel and perpendicular lines and their applications to lifelong learning.	actions in the mathematics classroom and beyond. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the Mathematics curriculum, the facilitator should apply Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and confident as they develop innovative strategies for solving problems on the Pythagorean theorem, parallel lines, perpendicular lines and their applications to lifelong learning and further studies. · Exhibiting motivation and SMART goal-setting in a mathematics classroom and beyond. · Managing emotions and conflicts as they engage in a mathematical discourse. · Showing empathy and cooperation in a mathematical problem-solving situation. These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates,

		encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork and respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning the Pythagorean theorem. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment as they debate on developing various strategies for solving problems on the Pythagorean theorem, parallel lines, perpendicular lines and their applications to lifelong learning and further studies. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination as they engage in a mathematical discourse based on the set of real numbers. **Truth and Integrity:** Reward truth and honesty as strong moral principles as they engage in peer assessment to help them become responsible citizens.

		Tolerance: Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment.

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
1.3.1.CS.1	1.3.1.LI.1	1.3.1.AS.1
Demonstrate a conceptual understanding of spatial sense with respect to angles, parallel lines, transversal and polygons, and apply their properties to solve everyday life problems.	**Draw and describe angles with various measures, including acute, right, straight, obtuse and reflex angles and solve problems on them.** **Experiential Learning:** In convenient groups, engage learners to identify referents for angles. **Example**: A referent is an object/item that can be used to help understand/represent a concept. Some referents of angles are corners of rooms and doors, the human palm, tree branches, and adjustable chairs.  **Experiential Learning:** In convenient groups, engage learners to sketch a given angle. **Example**: Make a free-hand sketch of angles such as acute, right, straight, obtuse and reflex angles and verify the closeness of their sketch with the actual angle. **Experiential Learning:** Using 30°, 45°, 60°, 75°, 90° and 180° as referent angles, engage learners in convenient groups or pairs/squares to make estimations of the measure/size of given angles. **Using think-pair-share activities:** In pairs, task learners to measure angles in various orientations using a protractor. **Examples**	Level 1 Recall Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning



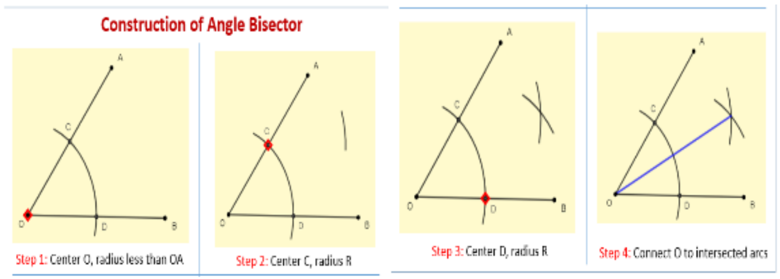


Experiential Learning:
In mixed-ability groups, learners explain and illustrate how angles can be replicated in a variety of ways, e.g., Mira, protractor, compass and straightedge, carpenter's square, and geometry software (e.g., GeoGebra).

In mixed groupings (gender/ability), learners replicate angles, with and without technology, in various ways.

In mixed-ability groups, engage learners to use various methods (including the use of feasible IT tools) to bisect given angles.

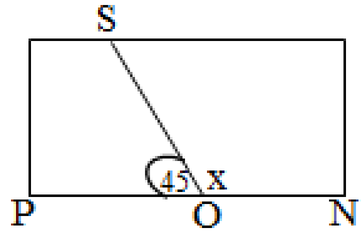
Example



In pairs, learners should solve a contextual problem that involves angles.

Example: Kwabena cut a piece of wood to make a 45^0 angle. At what angle was the other piece cut?

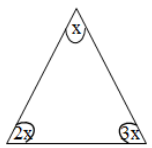
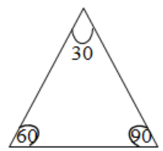
Solution

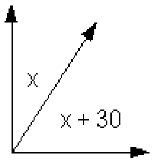
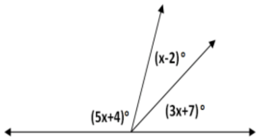
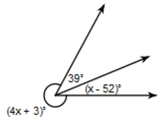
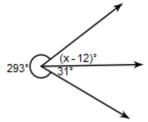
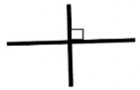


Think:
We need to find $\angle SON$
Since PON is a straight line, its angle is 180^0.
Therefore: $\angle PON = \angle POS + \angle SON$
$\angle PON - \angle POS = \angle SON$
$180º - 45° = 135°$
Therefore, the angle was the other piece cut is 45^0.

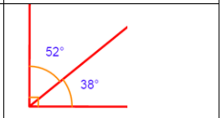
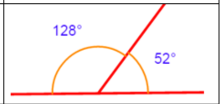
Example 2: In a triangle, one angle is twice the measure of the second angle. The third angle is three times the measure of the second angle. What is the measure/size of each angle?

Solution

	**<u>Think:</u>** The sum of the angles in a triangle is 180°. $x + 2x + 3x = 180º$ $6x = 180º$ $x = 30º$	

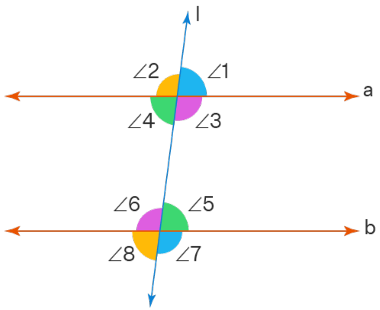
	Example: Determine the measure of the angles represented by x 	
	1.3.1.LI.2	1.3.1.AS.2
	Solve problems that involve parallel lines, perpendicular lines transversal, and pairs of angles formed between them. **Experiential Learning, Group discussions, think-pair-share activities, Talk for Learning (use of blended experiences that include both group and individual learning experiences).** **Experiential Learning:** In convenient groups, engage learners to sort a set of lines as perpendicular, parallel or neither, and justify it. **Activity 1:** <table><tr><td></td><td></td><td></td></tr><tr><td>Parallel lines</td><td>Neither</td><td>Perpendicular lines</td></tr></table> **In a collaborative group activity,** engage learners to illustrate and describe complementary and supplementary angles. Then, learners should go on to identify, in a set of angles, adjacent angles that are not complementary or supplementary.	Level 1 Recall Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

Activity 2 Identifying and naming pairs of angles.

Type of Angles	Description	Example
Complementary Angles	Angles that add up to 90°	
Supplementary Angles	Angles that add up to 180°	

In mixed-ability group activities, engage learners to identify and name pairs of angles formed by parallel lines and a transversal, including corresponding angles, vertically opposite angles, alternate interior angles, and alternate exterior angles, interior angles on the same side of transversal and exterior angles on the same side of the transversal.

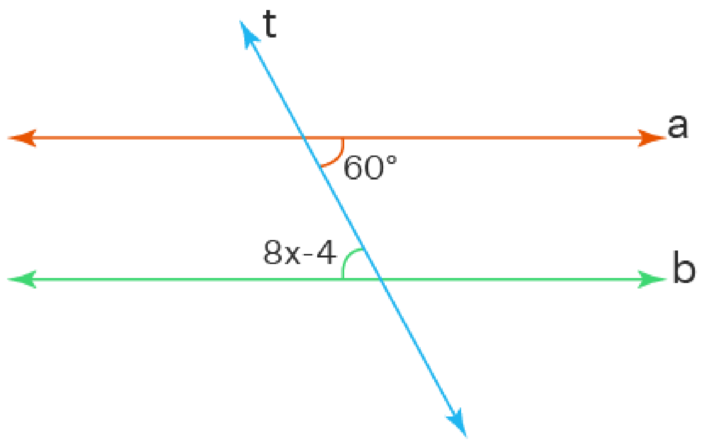
Example

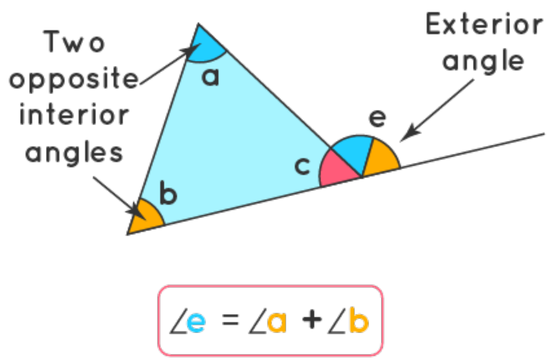


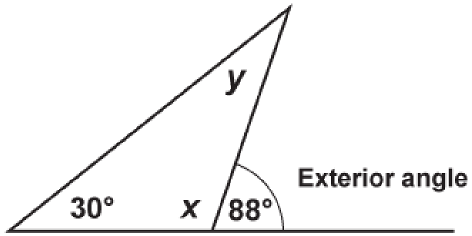
In mixed-ability group activities, Engage learners to explain and illustrate the relationships between angles formed by parallel lines and a transversal.

Using think-pair-share activities, learners should explain, using examples, why the angle relationships do not apply when the lines are not parallel.

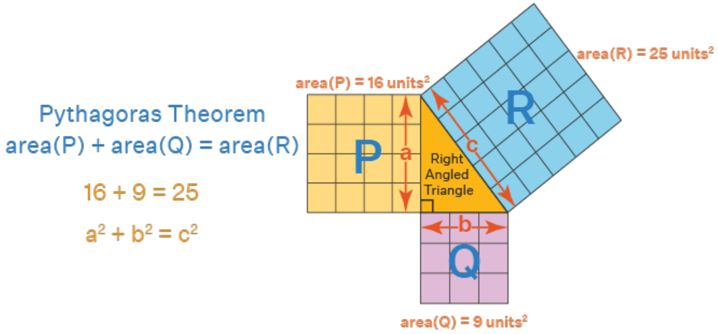
Using think-pair-square activities, learners should determine if lines or planes are perpendicular or parallel. **E.g.,** A wall perpendicular to the floor, and describe the strategy used.

	Using think-pair-share activities, learners should determine the measures of angles involving parallel lines and a transversal, using angle relationships. **Example**: Find the value of x in the given parallel lines 'a' and 'b', cut by a transversal 't'.  **Solution**: The given parallel lines are cut by a transversal; therefore, the marked angles in the figure are the alternate interior angles, which are equal in measure. This means, $8x - 4° = 60°$, and $8x = 64°$, $x = 8°$. Therefore, the value of $x = 8°$. **In convenient groups**, engage learners to solve a contextual problem that involves angles formed by parallel lines and a transversal (including perpendicular transversal).	
	1.3.1.LI.3	1.3.1.AS.3
	State and apply the exterior angle theorem of a triangle to solve problems and identify various properties of special triangles. **Using talk for Learning Strategy,** task learners in convenient groups to discuss the **Exterior Angle Theorem** of a triangle and establish the formula to solve problems on triangles.	Level 1 Recall **Level 2 Skills of conceptual understanding**

	Activity 1: Exterior Angle Theorem According to the exterior angle theorem, the exterior angle that results from stretching a triangle's side is equal to the sum of the dimensions of the triangle's two opposed interior angles. The theorem can be used to find the measure of an unknown angle in a triangle. **For instance**: Three internal angles in a triangle always add up to 180 degrees. This theorem is applied to each of the outer angles, which total si. As they constitute a linear pair of angles, take note that an exterior angle is supplementary to the neighbouring interior angle. Exterior angles are those that are created between a polygon's side and its extended neighbouring side.  **Activity 2: Proof of the exterior angle theorem.** We can verify the exterior angle theorem with the known properties of a triangle. Consider a Δ ABC. The three angles a + b + c = 180 (angle sum property of a triangle) ----- Equation 1 c= 180 - (a+b) ----- Equation 2 (rewriting equation 1) e = 180 - c----- Equation 3 (linear pair of angles) Substituting the value of c in equation 3, we get e = 180 - [180 - (a + b)] e = 180 - 180 + (a + b) e = a + b Hence verified.	**Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

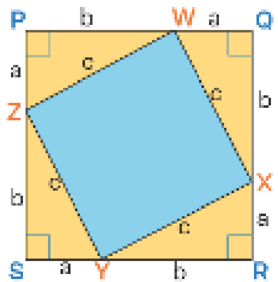
	Example 3: Use the properties of exterior angles of a triangle to determine the measure of the angles represented by x and y.  **Solution** $x + 88º = 180º$ $x + 88º - \mathbf{88º} = 180º - \mathbf{88º}$ $x = 92$ $x + y + 30º = 180º$ $92 + y + 30º = 180º$ $y + 122º = 180°$ $y + 122º - \mathbf{122º} = 180º - \mathbf{122º}$ $y = 58º$	
	1.3.1.LI.4	1.3.1.AS.4
	Solve problems on Pythagorean theorem by identifying situations that involve right triangles, verify the formula and apply it. **Collaborative learning:** In small groups, engage learners to research and discuss to come out with an explanation, using illustrations, why the Pythagorean theorem only applies to right triangles. **Collaborative Learning:** In groups, using examples and counterexamples, engage learners to discuss and verify the Pythagorean theorem, including drawings, concrete materials and the use of technology.	**Level 1 Recall** **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

Activity 1: Pythagoras' theorem using the following figure shows that the area of the square formed by the longest side of the right triangle (the hypotenuse) is equal to the sum of the area of the squares formed by the other two sides of the right triangle.



Thus, if AB and AC are the sides and BC is the hypotenuse of the triangle, then $BC^2 = AB^2 + AC^2$. In this case, AB is the base, AC is the altitude or the height, and BC is the hypotenuse.

Activity 2: Using convenient groups, task learners to investigate the Pythagorean theorem formula using the algebraic method.
Use the values a, b, and c as shown in the following figure and follow the steps given below:



Step 1: Arrange four congruent right triangles in the given square PQRS, whose side is a + b. The four right triangles have 'b' as the base, 'a' as the height and 'c' as the hypotenuse.
Step 2: The 4 triangles form the inner square WXYZ, as shown, with 'c' as the four sides.
Step 3: The area of the square WXYZ by arranging the four triangles is c^2.

Step 4: The area of the square PQRS with side $(a + b)$ = Area of 4 triangles + Area of the square WXYZ with side 'c'. This means $(a + b)^2 = = [4 \times 1/2 \times (a \times b)] + c^2$. This leads to $a^2 + b^2 + 2ab = 2ab + c^2$. Therefore, $a^2 + b^2 = c^2$. Hence proved.
Experiential Learning: In convenient groups, task learners to research and make presentations on the real-life (both historical and contemporary) applications of the Pythagorean theorem.

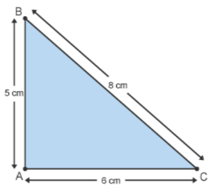
Activity 3: Real-life uses of the Pythagorean theorem

- The Pythagorean Theorem is useful for two-dimensional navigation.
- Painting on a Wall: To paint tall structures, painters make use of ladders, and they frequently employ Pythagoras' theorem to carefully position the base away from the wall so it does not topple over.
- What Size of TV Should You Buy: The size of a television is always specified in terms of its diagonal. If a television is specified as 43 inches in size, its true size is the diagonal's or hypotenuse's measurement.

Think-pair-share: In pairs, task learners to determine if a given triangle is a right-angled triangle using the Pythagorean theorem.

Activity 4: Pythagoras' theorem can be used to determine whether a triangle has a right angle. The triangle contains a right angle if the squares of the two shorter sides equal the square of the hypotenuse.

E.g. Does the triangle ABC contain a right angle?

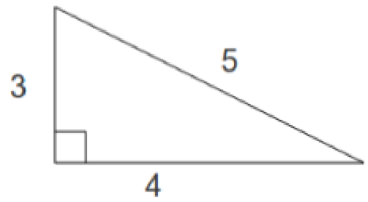


$a^2 + b^2 = c^2$
$5^2 + 6^2 = c^2$
$61 = c^2$
The hypotenuse of the triangle is 8.
$8^2 = 64$
61 does not equal 64. Therefore, the triangle does **not** contain a right angle.

Think-pair-share: In pairs, task learners to brainstorm and come out with an explanation on why a triangle with a side length ratio of 3:4:5 is a right triangle.

Example: We can prove this by using the Pythagorean Theorem as follows:

$\Rightarrow a^2 + b^2 = c^2$
$\Rightarrow 3^2 + 4^2 = 5^2$
$\Rightarrow 9 + 16 = 25$

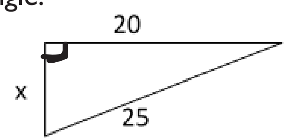


Think-pair-share: In pairs, engage learners with task sheets to solve problems using the Pythagorean theorem.

Example: Solve the following problems using the Pythagorean theorem.

i. A rectangular playing field is 20 metres long. A straight path is cut across the field along one of its diagonals. If the length of the path in metres is 25m, how wide is the playing field?

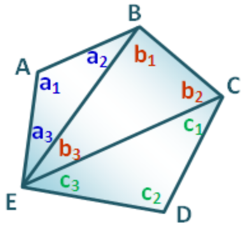
Solution: This is a right-angled triangle.
Let x = with
202 + x2 = 25
x2 = 625 – 400 ⇒ x2 = 125
x = 15m



ii. An animal shed with a pent roof needs to have some new roof beams fitted. The width of the shed is 10m, and the height of the pent roof is 1.3m. Work out the length of the roof beams needed.

iii. An army captain is on a hunt for a criminal. Her GPS tells her that she is 50m away from the criminal. She walks 34m due west. The GPS compass now tells her that the criminal is due south from where she is standing. How far south does she need to go to find the criminal?

iv. An anchor line for a tower needs to be replaced. The tower is 96ft tall. The anchor line is 105ft long. How far from the tower can it be placed to the nearest foot?

	1.3.1.LI.5	1.3.1.AS.5
	State and use the properties of quadrilaterals and calculate the sums of interior angles and exterior angles of a polygon. **Experiential Learning:** In small groups, learners discuss with models to come out with a generalisation/formula for determining the sum of the interior angles of polygons. **Example**: Determine the sum of the interior angle of a pentagon. **Solution**: Calculating the angle sum of pentagon ABCDE we have; Pentagon Angle Sum $= \overset{A}{\downarrow}{a_1} + \overbrace{(a_2 + b_1)}^{B} + \overbrace{(b_2 + c_1)}^{C} + \overset{D}{\downarrow}{c_2} + \overbrace{(c_3 + a_3 + b_3)}^{E}.$  $= (a_1 + a_2 + a_3) + (b_1 + b_2 + b_3) + (c_1 + c_2 + c_3)$ $= 180° + 180° + 180°$ $= 540°$ **Explanation:** Realise that the angle measures in the first line of our equation are just a rearrangement of the measures of the interior angles of the three triangles. Hence, the sum of the interior angles of the pentagon is equal to the angle sum of the three triangles. Therefore, we can conclude that the sum of the interior angles of a polygon is equal to the angle sum of the number of triangles that can be formed by dividing it using the method described above. Using this conclusion, we will now relate the number of sides of a polygon, the number of triangles that can be formed by drawing diagonals and the polygon's angle sum.	Level 1 Recall **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

Polygon	Number of Vertices (n)	Number of triangles	Sum of Angles (m°)
Triangle	3	1	1(180)=180
Quadrilateral	4	2	2(180)=360
Pentagon	5	3	3(180)=540
Hexagon	6	4	4(180)=720
Heptagon	7	5	5(180)=900
…	…	…	…
Decagon	10	8	8(180)=1440
100-gon	100	?	?
n-gon	n	n-2	(n-2)180

From the table, we observe that the number of triangles formed is 2 less than the number of sides of the polygon. This is true because n – 2 triangles can be formed by drawing diagonals from one of the vertices to n – 3 non-adjacent vertices. Therefore, the angle sum m of a polygon with n sides is given by the formula $m = 180(n - 2)$.

Teaching and Learning Resources	• Mathematical sets. • Technology tools such as computers, mobile phones, etc.	• Computer software applications like GeoGebra. • Tape measure, carpenters square, compass, clock face, etc.

Subject **MATHEMATICS**

Strand **3. GEOMETRY AROUND US**

Sub-Strand **2. MEASUREMENT**

Learning Outcomes	21st-century Skills and Competencies	GESI, SEL and Shared National Values
1.3.2.LO.1		
Interpret information about real-world applications of vectors and recognise vectors with the same magnitude and direction but different positions as equal vectors.	**Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on fractions and decimals using appropriate IT tools to boost their interest and desire to applications of vectors and recognise vectors with the same magnitude and direction. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on applications of vectors and recognise vectors with the same magnitude and direction and apply such knowledge in real life. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on applications of vectors and recognise vectors with the same magnitude and direction.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: · Respect individuals of different backgrounds in their groups as they discuss and solve problems based on applications of vectors and recognise vectors with the same magnitude and direction. · Interrogate their stereotypes and biases about the roles and abilities of different individuals in learning and applying mathematics. · Examine and dispel misconceptions/ myths about GESI as they engage in a sustainable discourse while learning mathematics. · Value and promote justice as they develop and implement innovative actions in the mathematics classroom and beyond.

		SEL: Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the Mathematics curriculum, the facilitator should apply Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and confident as they learn about applications of vectors and recognise vectors with the same magnitude and direction. · Exhibiting motivation and SMART goal-setting in a mathematics classroom and beyond. · Managing emotions and conflicts as they engage in a mathematical discourse. · Showing empathy and cooperation in a mathematical problem-solving situation. These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities.

		National Core Values: **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning applications of vectors and recognising vectors with the same magnitude and direction. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment as they debate the relationship between rational and irrational numbers. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination as they engage in a mathematical discourse based on vectors. **Truth and Integrity:** Reward truth and honesty as strong moral principles as they engage in peer assessment to help them become responsible citizens. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated

		mathematics classroom instruction and assessment.
1.3.2.LO.2		
Investigate and determine the trigonometric functions of special angles and solve problems using the three primary trigonometric ratios.	**Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on fractions and decimals using appropriate IT tools to boost their interest and desire to learn trigonometric functions of special angles. **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to the developing various strategies for trigonometric functions of special angles using patterns to create models and their applications to lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on their own values, perceptions and actions for decision-making as they engage in group and individual activities on trigonometric functions of special angles using patterns to create models and their applications in life.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: · Respect individuals of different backgrounds in their groups as they discuss and solve problems based on trigonometric functions of special angles. · Interrogate their stereotypes and biases about the roles and abilities of different individuals in learning and applying mathematics. · Examine and dispel misconceptions/ myths about GESI as they engage in a sustainable discourse while learning mathematics. · Value and promote justice as they develop and implement innovative actions in the mathematics classroom and beyond. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness,*

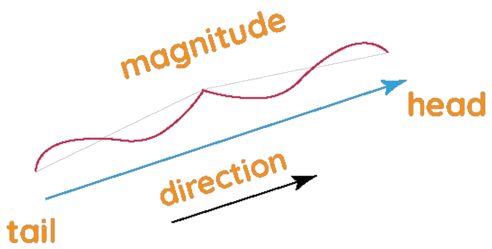
		Self-Management, Social Awareness, Relationship Skills and Responsible Decisions are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the Mathematics curriculum, the facilitator should apply Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and confident as they apply vectors and recognise vectors with the same magnitude and direction. · Exhibiting motivation and SMART goal-setting in a mathematics classroom and beyond. · Managing emotions and conflicts as they engage in a mathematical discourse. · Showing empathy and cooperation in a mathematical problem-solving situation. These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:**

		Leadership and Respect for others' views: Inculcate the habit of leadership through teamwork; and respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning applications of trigonometric functions of special angles. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment as they debate issues on the trigonometric functions of special angles. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination as they engage in a mathematical discourse based on trigonometric functions of special angles. **Truth and Integrity:** Reward truth and honesty as strong moral principles as they engage in peer assessment to help them become responsible citizens. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment.

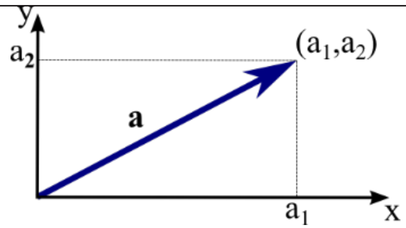
1.3.2.LO.3		
Identify and compare referents for SI and imperial area measurements, estimate the perimeter and area of 2D shapes [kites, parallelogram, rhombus and trapezoids], and volume of prisms, and solve problems that involve a given regular, composite or irregular 2D shapes.	**Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on fractions and decimals using appropriate IT tools to boost their interest and desire to solve problems involving measurements. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on measurement. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on a measurement. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concept of measurements and their applications to lifelong learning.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: · Respect individuals of different backgrounds in their groups as they discuss and solve problems based on measurements. · Interrogate their stereotypes and biases about the roles and abilities of different individuals in learning and applying mathematics. · Examine and dispel misconceptions/ myths about GESI as they engage in a sustainable discourse while learning mathematics. · Value and promote justice as they develop and implement innovative actions in the mathematics classroom and beyond. **SEL**: Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the Mathematics curriculum, the

		facilitator should apply Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and confident as they learn about measurements. · Exhibiting motivation and SMART goal-setting in a mathematics classroom and beyond. · Managing emotions and conflicts as they engage in a mathematical discourse. · Showing empathy and cooperation in a mathematical problem-solving situation. These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; and respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning about measurements.

		Diversity: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment as they debate on measurements. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination as they engage in a mathematical discourse based on measurements. **Truth and Integrity:** Reward truth and honesty as strong moral principles as they engage in peer assessment to help them become responsible citizens. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment.

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
1.3.2.CS.1	1.3.2.LI.1	1.3.2.AS.1
Demonstrate knowledge and understanding of the concept of measurement with respect to bearings and vectors.	**Recognise a vector as a quantity with both magnitude and direction, and identify, gather, and interpret information about real-world applications of vectors.** **Using Talk for Learning strategy,** lead the class to discuss (dwelling on their experiences from JHS) the concept of vectors. **Example**: Vectors define the movement of objects from one point to another. Vectors carry a point A to point B. The length of the line between the two points A and B is called the magnitude of the vector, and the direction of the displacement of point A to point B is called the direction of the vector AB.  **Using Talk for Learning strategy,** engage learners in a class discussion on some real-life examples of vectors. **Examples**: Vectors play an important role in physics. For instance, velocity, displacement, acceleration, and force are all vector quantities that have a magnitude as well as a direction. **Real-life uses of vectors** · Vectors can be used to find the direction in which the force is applied to move an object. · The concept of vectors aids in understanding how gravity uses a force of attraction on an object to work. · Vectors can be used to obtain the motion of a body which is confined to a plane. · Vectors help in defining the force applied on a body simultaneously in the three dimensions.	**Level 1 Recall** **Level 2 Skills of conceptual understanding** Level 3 Strategic reasoning Level 4 Extended critical thinking and reasoning

	· In the field of Engineering, for a structure not to collapse, vectors are used where the force is much stronger than the structure will sustain. · Vectors are used in various oscillators. **Using Talk for Learning approaches,** discuss the representation of and types of vectors with examples. **Example** Zero Vectors — Negative Vector Unit Vectors — Parallel Vectors Position Vectors — Orthogonal Vectors Equal Vectors — Co-initial Vectors	
	1.3.2.LI.2	1.3.2.AS.2
	Represent a vector in two-space geometrically as a directed line segment, with directions expressed in different ways (e.g., 320°, N40°W) and algebraically; then recognise vectors with the same magnitude and direction but different positions as equal vectors. **Using Talk for Learning strategy,** learners in pairs discuss the representation of vectors. **Example: Vectors in the plane** By this stage, learners are familiar with the standard (x, y) Cartesian coordinate system in the plane. That is, each point P in the plane is identified with its x and y components: P (p_1, p_2). To determine the coordinates of a vector a in the plane, the first step is to translate the vector so that its tail is at the origin of the coordinate system. Then, the head of the vector will be at some point (a_1, a_2) in the plane. We call (a_1, a_2) the coordinates or the components of the vector a. We often write a $\in R^2$ to denote that it can be described by two real coordinates.	Level 1 Recall Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

	 Using the Pythagorean Theorem, we can obtain an expression for the magnitude of a vector in terms of its components. Given a vector a = (a_1, a_2), the vector is the hypotenuse of a right triangle whose legs are length a_1 and a_2. Hence, the length of the vector a is $\|a\| = \sqrt{a_1^{\ 2} + a_2^{\ 2}}$ **Example:** Consider the vector represented by the line segment that goes from the point (1, 2) to the point (4, 6). Calculate the coordinates and the length of this vector. **Solution:** To find the coordinates, translate the line segment one unit left and two units down. The line segment begins at the origin and ends at (4−1, 6−2) = (3, 4). Therefore, a = (3, 4). The length of a is $\|a\| = \sqrt{3^2 + 4^2} = 5$ **Using think-pair-share activities,** engage learners to discuss vectors with the same magnitude and direction but different positions as equal vectors. **Examples**	

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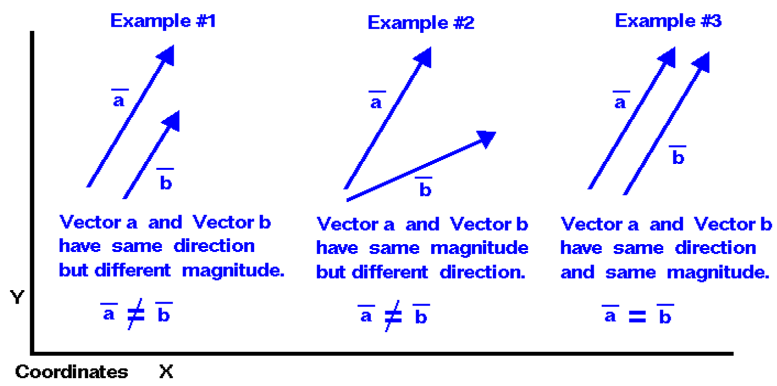
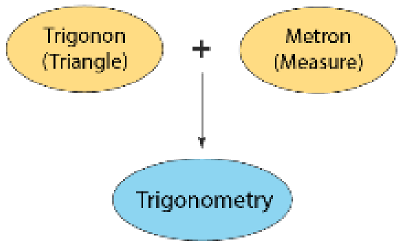
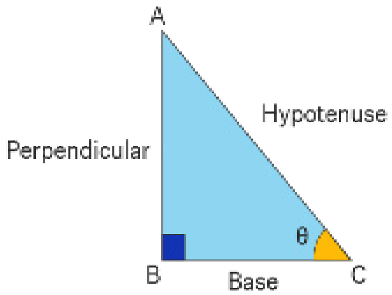
<tr><td></td><td>
</td><td></td></tr>
<tr><td>Teaching and Learning Resources</td><td>• Mathematical sets.
• Technology tools such as computers, mobile phones, etc.</td><td>• Computer software applications like GeoGebra.</td></tr>
</table>

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
1.3.2.CS.2	1.3.2.LI.1	1.3.2.AS.1
Demonstrate a conceptual understanding of the primary trigonometric ratios and apply it to solve problems that involve right triangles.	**Investigate the three basic trigonometric ratios (tangent, sine and cosine) of an acute angle in degrees.** **Using think-pair-share activities:** Engage learners in a discussion to recall the pre-requisite concepts on trigonometry**.** **Example**: Engage learners on the following pre-requisite concept: · Recall learners' familiarity with Pythagoras' theorem. · Review learners' basic knowledge of congruence and similarity of triangles. · Knowledge of the basic properties of triangles, squares and rectangles. · Familiarity with the use of a calculator. **Using think-pair-share activities,** task learners to discuss the meaning of the concept of trigonometry. **Example**: i. Trigonometry is the study of the relation between the sides and angles of a right-angled triangle. It thus helps in finding the measure of unknown dimensions of a right-angled triangle using formulas and identities based on this relationship. 	Level 1 Recall **Level 2 Skills of conceptual Understanding** Level 3 Strategic reasoning Level 4 Extended critical thinking and reasoning

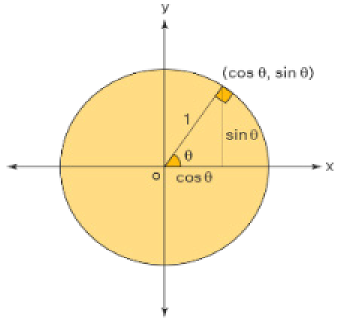
	ii. Using ideas from the concept of Pythagoras' theorem, deduce the basic six ratios in trigonometry that help in establishing a relationship between the ratios of sides of a right triangle with the angle. Use geodot to explore the basic ratios. **Example**: If θ is the angle in a right-angled triangle formed between the base and hypotenuse, then sin θ = Perpendicular/Hypotenuse cos θ = Base/Hypotenuse tan θ = Perpendicular/Base The value of the other three functions, cot, sec, and cosec, depend on tan, cos and sin, respectively, as given below. cot θ = 1/tan θ = Base/Perpendicular sec θ = 1/cos θ = Hypotenuse/Base cosec θ = 1/sin θ = Hypotenuse/Perpendicular 	

1.3.2.LI.2

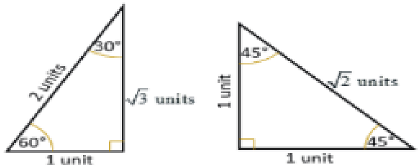
Find the trigonometric functions of special angles 30^0, 45^0 and 60^0, including using the calculator to determine the values of sine, cosine and tangent of angles up to 360^0.

Using convenient group activities: Engage learners to investigate the trigonometric functions of special angles 30^0, 45^0 and 60^0, including using the calculator to determine the values of sine, cosine and tangent of angles up to 360^0.

Example 1: Engage learners in convenient groups to explore the use of unit circles to calculate the values of basic trigonometric functions: sine, cosine, and tangent. The following diagram shows how trigonometric ratios sine and cosine can be represented in a unit circle.



Example 2: Engage students to do activities on the following special angles shown below on how to derive the trig ratios of 30°, 45° and 60° from the 30-60-90 and 45-45-90 special triangles.



	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

1.3.2.AS.2

Level 1 Recall
Level 2 Skills of conceptual understanding
Level 3 Strategic reasoning
Level 4 Extended critical thinking and reasoning

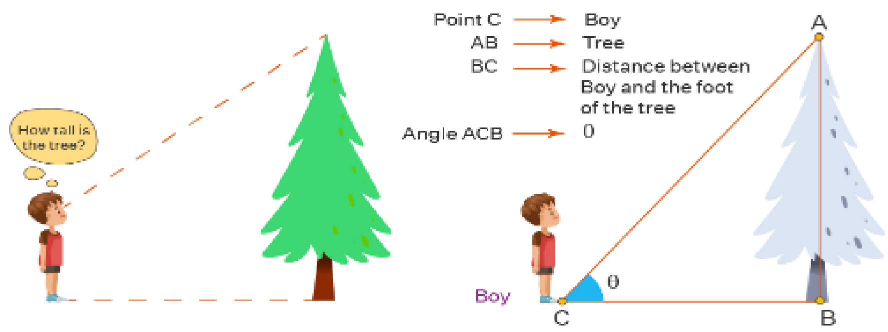
<table>
<tr><td></td><td>1.3.2.LI.3</td><td>1.3.2.AS.3</td></tr>
<tr><td></td><td>

Solve problems using the three primary trigonometric ratios for angles from 0° to 360° in standard position.

Using Think-Pair-Share activities: Task learners to investigate and create real-life problems and solve them.

Example: A boy is standing near a tree. He looks up at the tree and wonders, “How tall is the tree?”

Solution: The height of the tree can be found without actually measuring it. What we have here is a right-angled triangle, i.e., a triangle with one of the angles equal to 90 degrees.



It is determined using the tangent function, such as the tan of angle is equal to the ratio of the height of the tree and the distance. Let us say the angle is θ, then tan θ = Height/Distance between object and tree
Distance = Height/tan θ

Let us assume that the distance is 30m and the angle formed is 45 degrees, then.
Height = 30/tan 45°
Since, tan 45° = 1
So, Height = 30 m

</td><td>

Level 1 Recall
Level 2 Skills of conceptual understanding
Level 3 Strategic reasoning
Level 4 Extended critical thinking and reasoning

</td></tr>
</table>

	The height of the tree can be found out by using basic trigonometry formulas.	
Teaching and Learning Resources	· Mathematical sets. · Technology tools such as computers, mobile phones, etc.	· Computer software applications like GeoGebra

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
1.3.2.CS.3	1.3.2.LI.1	1.3.2.AS.1
Demonstrate conceptual understanding of the measurement of perimeter and area of circles and quadrilaterals.	**Solve problems that involve identifying and comparing referents for SI and imperial area measurements of regular, composite and irregular 2D shapes, including decimal and fractional measurements and verify the solutions.** **Experiential Learning:** In small groups, learners identify and compare referents for area measurements in SI and imperial units. **Example**: Learners in groups investigate to validate t you tube he following referents in the tables. **Referents for Linear Measurement**	Level 1 Recall **Level 2 Skills of conceptual understanding** Level 3 Strategic reasoning Level 4 Extended critical thinking and reasoning

Imperial Measurement	Referent
Inch	Thumb length; Thickness of a hockey puck;
Foot	A standard floor tile in a classroom
Yard	An arm span from the tip of the nose, a yardstick length of a guitar
Mile	Distance walked in 20 minutes, lights to the railway crossing.

SI Measurement	Referent
millimetres	Thickness of a dime or fingernail
centimetres	width of a fingernail, black keys on a piano, crayon, paper clip, or AA battery
meter	distance from a doorknob to the floor, width of a volleyball net, meter stick, waist height
kilometre	Distance walked in 15 minutes, lights to the bridge.

Referents for Area

Referent	Measurement
Area of a floor tile	≈1 ft^2
Area of a postage stamp	≈1 in^2
Area of a fingernail	≈1 cm^2
Area of an exterior house door	≈ 2 m^2
Area of exercise notebook	≈ 93.5 in^2 or 600 cm^2
Area of an ice rink surface	≈ 1500 m^2 or 17 000 ft^2
Area of a sheet of plywood	≈ 32 ft^2 or 3 m^2

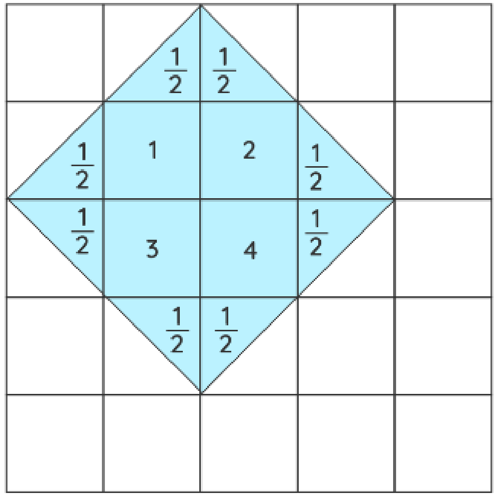
Experiential Learning: In mixed-gender/ability groups, engage learners to use their experiences from their investigations on referents for linear and area measurements to estimate an area measurement using a referent.

1.3.2.LI.2

Estimate the perimeter and area of a given regular, composite or irregular 2D shape [kites, parallelogram, rhombus and trapezoids].

Experiential Learning: In small groups, engage learners to investigate the area of 2-D shapes (both regular and irregular) and estimate the perimeter using geodot.

Example 1: Find the area of the shape on the grid:

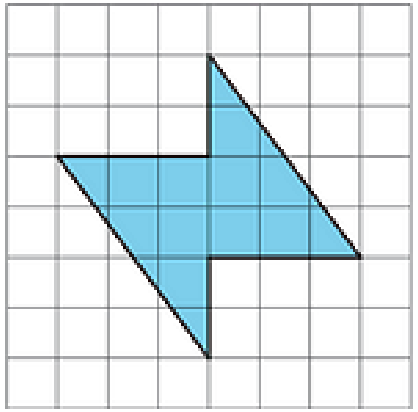


Solution: For this problem, some part of the shape does not occupy complete squares. Because of that, we need to approximate its perimeter. If it occupies about 1/2 of the unit square, we can combine two such halves to form an area of 1 square unit.

Example 2: Find the area of the shape:

1.3.2.AS.2

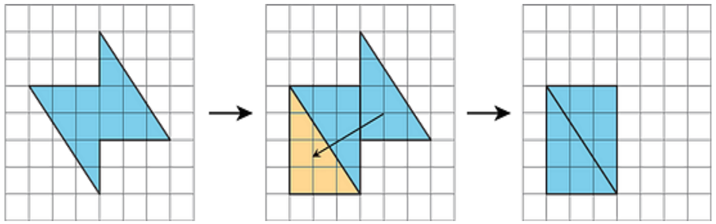
Level 1 Recall
Level 2 Skills of conceptual understanding
Level 3 Strategic reasoning
Level 4 Extended critical thinking and reasoning



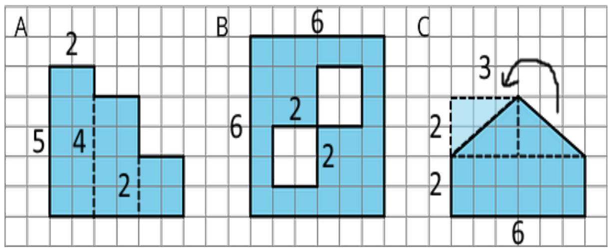
Solution:

<u>Think</u>!

We will first have to decompose it and rearrange the pieces into a shape that will make it easy to determine the area.



Example 3: Investigate the perimeter and area of the following shapes.



<table>
<tr><td></td><td>1.3.2.LI.3</td><td>1.3.2.AS.3</td></tr>
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Solve a contextual problem that involves the perimeter and area of a regular, a composite or an irregular 2-D shape [kites, parallelogram, rhombus and trapezoids].

Using think-pair-share activities, engage learners in pairs to write a given perimeter/area measurement expressed in one SI/imperial unit in another SI/ imperial unit.

Example: Discuss the formula for determining the area of the following 2D shapes:

<table>
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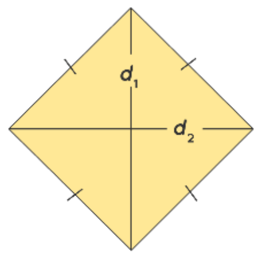
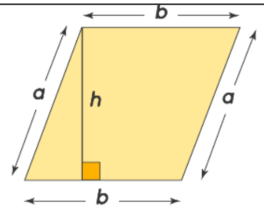


Area of a rhombus = 12×(d1)×(d2) square units

</td><td>



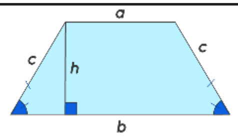
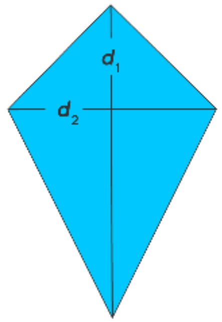
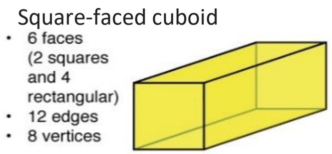
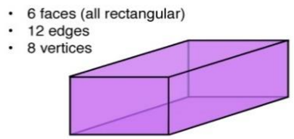
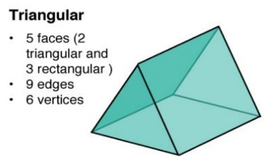
Area of a parallelogram = base × height = b × h square units

</td></tr>
</table>

</td><td>

Level 1 Recall
Level 2 Skills of conceptual understanding
Level 3 Strategic reasoning
Level 4 Extended critical thinking and reasoning

</td></tr>
</table>

	 Area of an isosceles trapezoid = 12(a+b)h square units  Area of a kite = 12×(d1)×(d2)square units **Using think-square-share activities,** present learners, in pairs, with task sheets on perimeter and area, including word problems to solve. **Using Talk for Learning strategy**, learners in groups explain, using examples, the effect of changing the measurement of one or more dimensions on the area and perimeter of rectangles.	
	1.3.2.LI.4	1.3.2.AS.4
	Determine the volume of prisms and solve everyday life problems on them. **Using think-square-share activities,** task learners to solve problems using formulas for determining the volume of prisms. **Examples of prisms include;**    	Level 1 Recall Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

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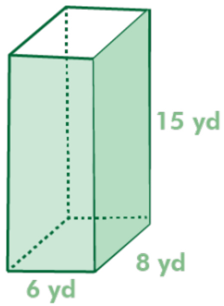
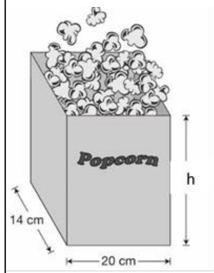


Rectangular-faced cuboid

Example 1

Find the volume of the prism shown in the figure.



Example 2

A box of popcorn holds 7000 cubic centimetres of popcorn. The length and width of the base of the box are 14cm and 20cm, respectively. Find the height of this box of popcorn.
</td>
<td></td>
</tr>
<tr>
<td>Teaching and Learning Resources</td>
<td>• Mathematical sets.
• Technology tools such as computers, mobile phones, etc.</td>
<td colspan="2">• Computer software applications like GeoGebra.</td>
</tr>
</table>

Subject **MATHEMATICS**

Strand **4. MAKING SENSE OF AND USING DATA**

Sub-Strand **1. STATISTICAL REASONING AND ITS APPLICATION IN REAL LIFE**

Learning Outcomes	21st-Century Skills and Competencies	GESI, SEL and Shared National Values
1.4.1.LO.1		
Decide whether or not a selected data collection method is appropriate given a particular data, justify responses, and collect both qualitative and quantitative data with the appropriate methods.	**Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on data using appropriate IT tools to boost their interest and desire to solve algebraic expressions using patterns to create and model problems on their own. **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, enabling them to develop various strategies for collecting qualitative and quantitative data and applying them in lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on data using patterns to create models and their applications in life. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities involving data and their applications.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: · Respect individuals of different backgrounds in their groups as they discuss and solve problems based on data. · Interrogate their stereotypes and biases about the roles and abilities of different individuals in learning and applying mathematics. · Examine and dispel misconceptions/ myths about gender as they engage in a sustainable discourse while learning mathematics. · Value and promote justice as they develop and implement innovative actions in the mathematics classroom and beyond. **SEL**: Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the Mathematics curriculum, the facilitator should

	Innovation and Creativity: Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concept of data and their applications to lifelong learning.	apply Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and confident as they establish the concept of data · Exhibiting motivation and SMART goal-setting in a mathematics classroom and beyond. · Managing emotions and conflicts as they engage in a mathematical discourse. · Showing empathy and cooperation in a mathematical problem-solving situation. These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; and respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning about data. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment as they debate on the various types of data. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination as they engage in a

		mathematical discourse based on data types and collection methods. **Truth and Integrity:** Reward truth and honesty as strong moral principles as they engage in peer assessment to help them become responsible citizens. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment.
1.4.1.LO.2		
Organise and present data (grouped/ungrouped) using frequency tables, line graphs, pie charts, multiple bar graphs, infographics, etc.; generate 3D graphs/charts with appropriate digital technology (where available) and solve problems on them.	**Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems using frequency tables, line graphs, pie charts, multiple bar graphs, and infographics using appropriate IT tools to boost their interest and desire to solve algebraic expressions using patterns to create and model problems on their own. **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to developing various strategies for using frequency tables, line graphs, pie charts, multiple bar graphs, and infographics using patterns to create models and their applications to lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities using frequency tables, line graphs, pie charts,	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: · Respect individuals of different backgrounds in their groups as they discuss and solve problems based on using frequency tables, line graphs, pie charts, multiple bar graphs, and infographics. · Interrogate their stereotypes and biases about the roles and abilities of different individuals in learning and applying mathematics. · Examine and dispel misconceptions/ myths about gender as they engage in a sustainable discourse while learning mathematics. · Value and promote justice as they develop and implement innovative actions in the mathematics classroom and beyond. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-*

	multiple bar graphs, infographics using patterns to create models and their applications in life. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on algebraic expressions and their applications. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of using frequency tables, line graphs, pie charts, multiple bar graphs, and infographics.	*Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the Mathematics curriculum, the facilitator should apply Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and confident as they establish the process of using frequency tables, line graphs, pie charts, multiple bar graphs, and infographics. · Exhibiting motivation and SMART goal-setting in a mathematics classroom and beyond. · Managing emotions and conflicts as they engage in a mathematical discourse. · Showing empathy and cooperation in a mathematical problem-solving situation. These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning about frequency tables, line graphs, pie charts, multiple bar graphs, infographics.

		Diversity: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment as they debate on the frequency tables, line graphs, pie charts, multiple bar graphs, and infographics. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination as they engage in a mathematical discourse based on frequency tables, line graphs, pie charts, multiple bar graphs, and infographics. **Truth and Integrity:** Reward truth and honesty as strong moral principles as they engage in peer assessment to help them become responsible citizens. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment.
1.4.1.LO.3		
Design and execute a project by posing and refining questions to collect, analyse and interpret quantitative and/or qualitative data directly from the school community and beyond, draw useful conclusions and make recommendations.	**Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on collecting, analysing and interpreting quantitative or qualitative data using appropriate IT tools to boost their interest and desire to solve algebraic expressions using patterns to create models problems on their own. **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to developing various strategies for collecting, analysing and interpreting quantitative or	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: · Respect individuals of different backgrounds in their groups as they discuss and solve problems based on collecting, analysing and interpreting quantitative or qualitative data.

	qualitative data and their applications to lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on collect, analyse and interpret quantitative or qualitative data using patterns to create models and their applications in life. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on collect, analyse and interpret quantitative or qualitative data. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concept of algebraic expressions and their applications to lifelong learning.	· Interrogate their stereotypes and biases about the roles and abilities of different individuals in learning and applying mathematics. · Examine and dispel misconceptions/ myths about GESI as they engage in a sustainable discourse while learning mathematics. · Value and promote justice as they develop and implement innovative actions in the mathematics classroom and beyond. **SEL**: Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the Mathematics curriculum, the facilitator should apply the Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and confident as they collect, analyse and interpret quantitative or qualitative data. · Exhibiting motivation and SMART goal-setting in a mathematics classroom and beyond. · Managing emotions and conflicts as they engage in a mathematical discourse. · Showing empathy and cooperation in a mathematical problem-solving situation. These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities.

		National Core Values: **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning to collect, analyse and interpret quantitative or qualitative data. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment as they debate on methods of collection, analysis and interpretation of quantitative or qualitative data. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination as they engage in a mathematical discourse based on collecting, analysing and interpreting quantitative or qualitative data. **Truth and Integrity:** Reward truth and honesty as strong moral principles as they engage in peer assessment to help them become responsible citizens. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment.

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
1.4.1.CS.1	1.4.1.LI.1	1.4.1.AS.1
Demonstrate a conceptual understanding of the appropriateness of data collection methods to collect everyday life data.	**Classify data (primary and secondary) as quantitative (discrete and continuous), qualitative (nominal and ordinal), numerical, categorical, grouped, ungrouped, etc.** **Using project-based learning activities,** learners research and make presentations on primary data (gathered by the researcher himself, e.g., surveys, interviews, experiments, etc.) and secondary data (collected by someone else earlier). **Example 1**: Using data hunting game, write a variety of data types on different cards and task learners to sort the cards under given headings and justify their reason for the sorting. **Using project-based learning activities**, engage learners to research a number of existing documents on/from the internet/textbooks/magazines/newspapers and describe, with reasons, the type of data used. **Example 1**: Discuss and draw out the differences, with examples, between discrete and continuous data. **Hint**: **1.** Discrete data includes discrete variables that are finite, numeric, countable, and non-negative integers. **E.g.:** i. The number of students who have attended the class. ii. The number of customers who have bought different products. iii. The number of groceries people are purchasing every day. **Hint 2.** Continuous data is the unspecified number of possible measurements between two presumed points. · The weather temperature. · The wind speed. · The weight of the kids.	**Level 1 Recall** **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

	Using project-based learning activities, task learners to research and make presentations on the importance of grouped and ungrouped data, discrete and continuous data, etc. in the areas of marketing and advertising, research, population analysis, etc.	
	1.4.1.LI.2	1.4.1.AS.2
	Identify and validate quantitative data collection methods (Survey/Questionnaire, Interviews, Observation, Existing Data, and Probability) and use them to collect everyday-life data. **Project-based learning, Group discussions, think-pair share activities.** **Using project-based learning activities**, students sample out numerous data they can collect and justify a particular data collection method(s) that is appropriate for collecting that quantitative data. **Example**: i. **Experiential Learning:** Using convenient groups, students research and select an existing questionnaire, interview guide, observation guide, etc., then discuss its features, validate it usefulness and collect quantitative data with it in the classroom or within the school community. ii. **Experiential Learning:** Using convenient groups/pairs, design a mini project where they choose a data collection method of choice and collect real quantitative data with it.	**Level 1 Recall** **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** **Level 4 Extended critical thinking and reasoning**
	1.4.1.LI.3	1.4.1.AS.3
	Identify and validate qualitative data collection methods (interviews, observations, focus groups, oral histories, online tracking, social media monitoring, etc.) and use them to collect everyday-life data. **Using think-pair-share activities,** students sample out numerous data they can collect and justify a particular data collection method(s) that is appropriate for collecting that qualitative data. **Experiential Learning:** · Using convenient groups, students research and select an existing questionnaire, interview guide, observation guide, etc., then discuss its features, validate its usefulness and collect qualitative data with it.	**Level 1 Recall** **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** **Level 4 Extended critical thinking and reasoning**

	· Experiential Learning: Using convenient groups/pairs, design a mini project where they choose a data collection method of choice and collect real qualitative data with it.		
Teaching and Learning Resources	· Sample questionnaire · interview guides	· observation guide · Computer-assisted telephone interview guide	· mail survey

<table>
<tr><th>Content Standards</th><th>Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI</th><th>Assessment</th></tr>
<tr><td>1.4.1.CS.2</td><td>1.4.1.LI.2</td><td>1.4.1.AS.2</td></tr>
<tr><td>Demonstrate conceptual understanding of data organisation and presentation for grouped and ungrouped data, including 3D graphs/charts with appropriate digital technology.</td><td>Organise and present data (grouped/ungrouped) using frequency tables, line graphs, pie charts, multiple bar graphs, infographics, etc., including generating 3D graphs/charts with appropriate digital technology (where available) and solving problems on them.

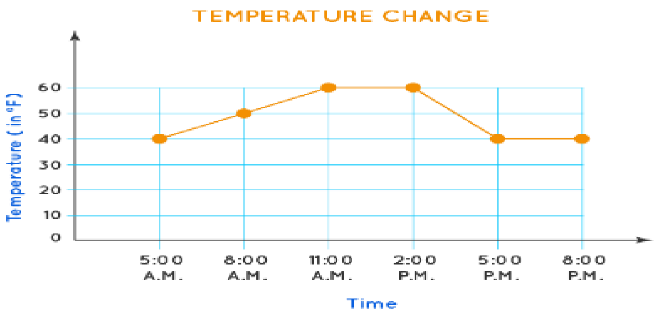
Using think-pair-share activities, learners are tasked to Organise a given/chosen raw data into a frequency distribution table.

Example 1: Ungrouped data
A jar containing beads of different colours- red, green, blue, black, red, green, blue, yellow, red, red, green, green, green, yellow, red, green, yellow. Organise the data in a frequency table.
<table>
<tr><th>Categories</th><th>Tally Marks</th><th>Frequency</th></tr>
<tr><td>Red</td><td>卌</td><td>5</td></tr>
<tr><td>Green</td><td>卌 |</td><td>6</td></tr>
<tr><td>Blue</td><td>||</td><td>2</td></tr>
<tr><td>Black</td><td>|</td><td>1</td></tr>
<tr><td>Yellow</td><td>|||</td><td>3</td></tr>
</table>

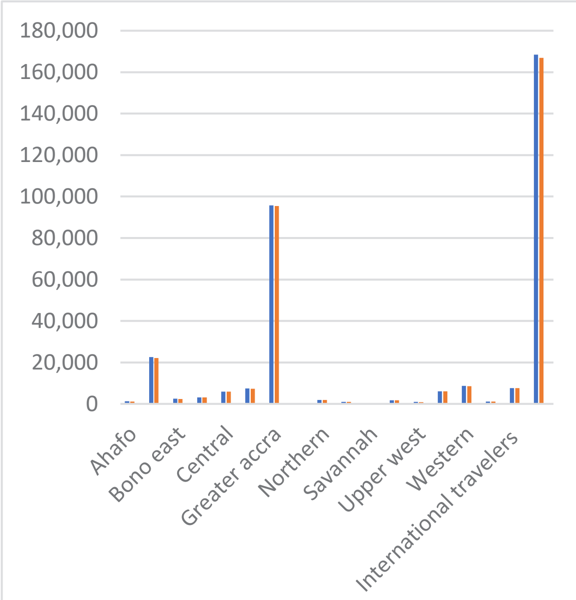
Example 2: Grouped data
One hundred schools decided to plant 100 tree saplings in their gardens on World Environment Day. Represent the given data in the form of frequency distribution and find the number of schools that are able to plant 50% of the plants or more.
95, 67, 28, 32, 65, 65, 69, 33, 98, 96, 76, 42, 32, 38, 42, 40, 40, 69, 95, 92, 75, 83, 76, 83, 85, 62, 37, 65, 63, 42, 89, 65, 73, 81, 49, 52, 64, 76, 83, 92, 93, 68, 52, 79, 81, 83, 59, 82, 75, 82, 86, 90, 44, 62, 31, 36, 38, 42, 39, 83, 87, 56, 58, 23, 35, 76, 83, 85, 30, 68, 69, 83, 86, 43, 45, 39, 83, 75, 66, 83, 92, 75, 89, 66, 91, 27, 88, 89, 93, 42, 53, 69, 90, 55, 66, 49, 52, 83, 34, 36.</td><td>Level 1 Recall
Level 2 Skills of conceptual understanding
Level 3 Strategic reasoning
Level 4 Extended critical thinking and reasoning</td></tr>
</table>

Number of plants survived	Tally Marks	Number of schools (frequency)
20 - 29	III	3
30 - 39	~~IIII~~ ~~IIII~~ IIII	14
40 - 49	~~IIII~~ ~~IIII~~ II	12
50 - 59	~~IIII~~ III	8
60 - 69	~~IIII~~ ~~IIII~~ ~~IIII~~ III	18
70 - 79	~~IIII~~ ~~IIII~~	10
80 - 89	~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ III	23
90 - 99	~~IIII~~ ~~IIII~~ II	12
Total		100

Example 3: Line Graphs
The temperature of a city from 5 a.m. to 8 p.m. on a day was recorded in the form of a line graph, as shown below. Study the graph and answer the following questions.
a) At which time of the day was the temperature 40° F?
b) What was the maximum recorded temperature?



Example 4: Multiple Bar Graphs
The chart below is the summary of cases and recoveries of the coronavirus pandemic recorded in various regions in Ghana from March 2020 to 11th August 2022.

	 From the graph: 1. Which region recorded the least cases over the period? 2. What is the percentage difference between cases recorded in Greater Accra and the total cases?	
	1.4.3.LI.2	1.4.1.AS.2
	Analyse (including using appropriate computer applications) and interpret data using descriptive statistics (i.e., measures of central tendency/location and minimum and maximum values) and justify which of the averages best represents the data. **Using collaborative activities,** task learners to discuss the concepts mean, median and mode and how to obtain them by dwelling on ideas from JHS. Then, solve some problems to consolidate their ideas on these concepts - mean, median and mode for grouped data. **Example 1:** Mode for grouped data is found using the following mode formula.	**Level 1 Recall** **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** **Level 4 Extended critical thinking and reasoning**

$$Mode = L + h\frac{(f_m - f_1)}{(f_m - f_1) + (f_m - f_2)}$$

Where:
•L is the lower limit of the modal class.
•'h' is the size of the class interval.
•'fm' is the frequency of the modal class.
•'f1' is the frequency of the class preceding the modal class.
•'f2' is the frequency of the class succeeding the modal class.

The median formula for grouped data is given as,

$$\boldsymbol{median = L_m + \left[\frac{\frac{n}{2} - F}{F_m}\right] i}$$

Where:
n = total frequency
F = cumulative frequency
F_m= frequency of other class median
I= class width
L_m = lower boundary of the class median

Similarly, we have a mean formula for grouped data. Which is expressed as:

$$mean = \frac{\sum fx}{\sum f}$$

Where:
∑ = is the summation sign
x = the mean value of the set of given data.
f = frequency of the individual data

Example 2: Establish the relation between mean, median and mode.

The three measures of central values, i.e., mean, median, and mode, are closely connected by the following relations (called an empirical relationship).

2Mean + Mode = 3Median

For example, we have data whose mode = 65 and median = 61.6.

Then, we can find the mean using the above mean, median, and mode relation.

2Mean + Mode = 3 Median

$\therefore$2Mean = 3 × 61.6 - 65

$\therefore$2Mean = 119.8

$\Rightarrow$ Mean = 119.8/2

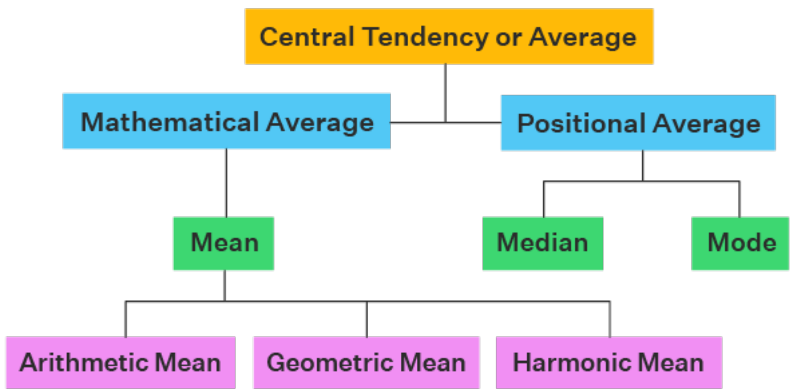
$\Rightarrow$ Mean = 59.9

Example 3: Establish that the mean is a form of average

The term average is frequently used in everyday life to denote a value that is typical for a group of quantities. Average rainfall in a month or the average age of employees of an Organisation is a typical example.

- Average is the value that indicates what is most likely to be expected.
- They help to summarise large data into a single value.

An average tends to lie centrally with the values of the observations arranged in ascending order of magnitude. So, we call it an average measure of the central tendency of the data. Averages are of different types. What we refer to as mean, i.e., the arithmetic mean, is one of the averages. Mean is called the mathematical average, whereas median and mode are positional averages.



Using collaborative activities, task learners on the difference between mean and median.

Example: A department of an Organisation has 5 employees, which include a supervisor and four executives. The executives draw a salary of GH 10,000 per month, while the supervisor gets GH 40,000.

Solution: Mean = (10000 + 10000 + 10000 + 10000 + 40000)/5 = 80000/5 = 16000
Thus, the mean salary is GH 16,000.

To find the median, we consider the ascending order: 10000, 10000, 10000, 10000, 40000.
n = 5, so, (n + 1)/2 = 3
Thus, the median is the 3rd observation.
Median = GH10,000

Thus, the median is GH10,000 per month.

Now, let us compare the two measures of central tendencies. We can observe that the mean salary of GH 16,000 does not give even an estimated salary of any of the employees, whereas the median salary represents the data more effectively. One of the weaknesses of the mean is that it gets affected by extreme values.

Using collaborative activities, task learners to discuss the effect of an extreme value on the mean.

Example: The following graph shows how extreme values affect mean and median:

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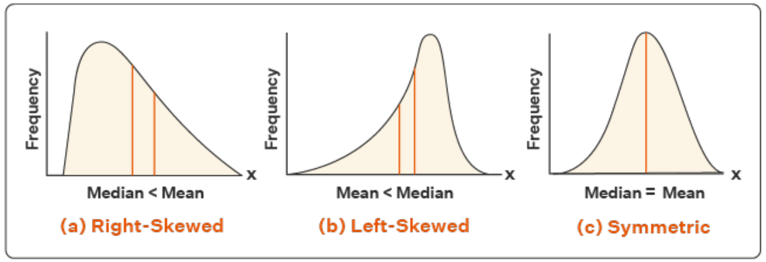
- **Symmetric Data**
  - Data sets whose values are evenly spread around the centre
- **Skewed Data**
  - Data sets that are not symmetric



So, mean is to be used when we don't have extremes in the data. If we have extreme points, then the median gives a better estimation.,
</td>
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<td></td>
<td>1.4.1.LI.3</td>
<td>1.4.1.AS.3</td>
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Use mathematical arguments to support personal choices as well as incorporate the views and perspectives of others to assess and make inferences from data presented in everyday life (including class discussions, school debates, textbooks, school clubs, etc.)

Group discussions and debates, oral and written presentations, and enquiry project activities. Experiential Learning.

In collaborative groups for Experiential Learning, learners embark on a project by collecting, analysing and presenting real-life data and use mathematical arguments and inferences from their data to discuss the findings.

Example: The table below is the summary of cases and recoveries of coronavirus recorded in various regions in Ghana from March 2020 to 11th August 2022. Use the information from the table to make recommendations to the government on resource allocation to the various regions.
</td>
<td>

Level 1 Recall
Level 2 Skills of conceptual understanding
Level 3 Strategic reasoning
Level 4 Extended critical thinking and reasoning
</td>
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</table>

Regions	Cases	Recovered and discharged	Active cases
Ahafo	1,210	1174	1
Ashanti	22,527	22130	8
Bono east	2523	2422	4
Brong Ahafo	3171	3078	17
Central	5911	5852	10
Eastern	7425	7267	5
Greater Accra	95723	95362	43
North East	407	396	0
Northern	1882	1850	0
Oti	1015	1006	0
Savannah	316	305	7
Upper east	1801	1738	0
Upper west	945	891	10
Volta	6140	6048	2
Western	8655	8573	7
Western north	1162	1138	10
International travellers	7644	7642	2
Total	**168,457**	**166,872**	**126**

	Examples of recommendations and their justifications **Recommendation 1:** More resources should be allocated to the Greater Accra region and Ashanti region since they recorded the highest cases, i.e., Greater Accra 95723 (56.8%) and Ashanti 22,527 (13.4%). **Recommendation 2:** More resources should be allocated to the Greater Accra region and Brong Ahafo region since they have the highest number of active cases, i.e., Greater Accra 43 (34%) and Brong Ahafo 17 (13.5%). All the two recommendations are based on the data. Each of the recommendations has a good basis per the data in the table. Students must learn to make such recommendations based on data and accept alternative views of others since it can also be useful.			
Teaching and Learning Resources	· Graph sheets · mathematical sets · computer with data organising software like MS Excel, MS PowerPoint, etc. · A4, A3 papers · flip charts	· markers · colour pens, etc. · Reports from analysed data · manila cards	· worksheets · posters · teaching presentations · enquiry project-template	

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
1.4.1.CS.3	1.4.1.LI.1	1.4.1.AS.1
Demonstrate the ability to embark on a project involving the collection, analysis and interpretation of quantitative and qualitative data within the school environment.	**Develop and execute a project with a team by collecting and analysing data within the school environment and giving useful conclusions and recommendations (including the use of appropriate computer applications, e.g., Excel).** **Using an enquiry project activity,** assign projects to learners in mixed-gender/ability groups to develop and execute a project by obtaining data within the school environment. **Example** · In convenient groups, obtain the WASSCE result of your school for the past five years and analyse it by looking at the overall differences in the performance by years, by programme/courses/subject. · As part of the analysis, obtain the frequencies and percentages, then draw charts (line, pie, bar, etc.) for the data. · Also, from the analysis, make conclusions and give recommendations to the school. · Design **a** Student Course Evaluation Questionnaire and use it to collect data from students in your school. · As part of the project, decide on the sample and justify it appropriateness for generalisation to the entire population. · Analyse the data and make conclusions and recommendations based on the results.	Level 1 Recall **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning
	1.4.1.LI.2	1.4.1.AS.2
	Present a project report to your class or at a school forum, including the use of presentation software such as PowerPoint, infographics, etc., and publish the report in a school magazine, school notice board, school social media platforms, etc. **Using an enquiry project activity,** task learners to embark on a publishing activity by using a creative means (including an IT tool) to present the findings of their research project and publish it on any of the available school platforms. **Example**:	Level 1 Recall **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

	i. Make summaries of your results, conclusions and recommendations of your project and present them using a PowerPoint, infographics design, Microsoft Word, or handwritten to the class or at a mini forum (including school clubs) in the school. ii. Make oral presentations on the project by explaining the choice of project topic and its relevance, choice of data collection method(s) and the analysis and talk about the challenges faced. iii. Publish the summary of the findings and recommendations of the project on the school notice board, or magazines, school social media platforms, etc.	
Teaching and Learning Resources	• Real-life data samples from both the school community and outside the school community. • Data match-up cards • Memory cards, etc. • Graph sheets • mathematical sets • computer with data organising software like MS Excel, MS PowerPoint, etc., • A4 and A3 papers	• Flip charts • Markers • Colour pens, etc. • Technology tools such as computers, mobile phones, etc. • Sample questionnaire, interview guides, observation guide, computer-assisted telephone interview guide, mail survey, computer-assisted personal interview guide. • Manila cards

Subject **MATHEMATICS**

Strand **4. MAKING SENSE OF AND USING DATA**

Sub-Strand **2. PROBABILITY/CHANCE**

Learning Outcomes	**21st-century Skills and Competencies**	**GESI, SEL and Shared National Values**
1.4.2.LO.1		
Determine the sample space for simple and compound probability experiments involving independent events; express the probabilities of given events as fractions, decimals, percentages and/or ratios and solve problems everyday life problems.	**Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on data using appropriate IT tools to boost their interest and desire to develop sample space for simple and compound probability experiments involving independent events. **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to developing various strategies for probability of events and their applications to lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions to reflect on their own values, perceptions and actions for decision-making as they engage in group and individual activities on fractions, decimals, percentages and/or ratios. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on probabilities and their applications. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: · Respect individuals of different backgrounds in their groups as they discuss and solve problems based on developing sample space for simple and compound probability experiments involving independent events. · Interrogate their stereotypes and biases about the roles and abilities of different individuals in learning and applying mathematics. · Examine and dispel misconceptions/ myths about GESI as they engage in a sustainable discourse while learning mathematics. · Value and promote justice as they develop and implement innovative actions in the mathematics classroom and beyond. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the Mathematics curriculum, the facilitator should

	creative actions that reflect their level for application of the concept of probability applications to lifelong learning.	apply Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and confident as they collect, analyse and interpret quantitative or qualitative data. · Exhibiting motivation and SMART goal-setting in a mathematics classroom and beyond. · Managing emotions and conflicts as they engage in a mathematical discourse. · Showing empathy and cooperation in a mathematical problem-solving situation. These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning to develop sample space for simple and compound probability experiments involving independent events. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment as they debate on developing sample space for simple and compound probability experiments involving independent events.

		Equity: Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination as they engage in a mathematical discourse based on collecting, analysing and interpreting quantitative or qualitative data. **Truth and Integrity:** Reward truth and honesty as strong moral principles as they engage in peer assessment to help them become responsible citizens. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment.

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
1.4.2.CS.1	1.4.2.LI.1	1.4.2.AS.1
Demonstrate conceptual understanding of simple and compound probability experiments involving two independent events.	**List the elements of the sample space from a simple or compound experiment involving two independent events.** **Using think-pair square activities,** learners engage in probability games to review compound probability and help reinforce understanding of the concepts (Hands-on probability games, QR Code Game, scavenger hunt, deal or no deal? Knockout Game) for the whole). **Using think-pair-share activities**, discuss and write short notes (with examples) on the terminologies relating to the concept of probability. **Example 1:** i. Experiment ii. Random Experiment iii. Trial iv. Sample space v. Event vi. Equally Likely Events vii. Exhaustive Events viii. Favourable Events ix. Additive Law of Probability **Example 2**: Discuss the concept of conditional probability as a prerequisite to understanding probability with independent events. **Example 3:** Conditional probability $P(A\|B)$ is the probability of event A given the information that B has already taken place. For any two events A and B, $P(A\|B)$ is given as $P(A\|B) = (P(A \cap B))/(P(B))$. **Example 4**: Perform independent experiments and list the sample space **For instance,** i. Two events, A and B, are independent if $P(A \cap B) = P(A)\ P(B)$.	**Level 1 Recall** **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

	Consider tossing a coin three times in a row. Since each of the throws is independent of the other two, we consider all 8 (= 23) possible outcomes as equiprobable and assign each the probability of 1/8. Here is the sample space of a sequence of three tosses: {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}. ii. A fair die is rolled twice. List the sample space for the experiment. **Solution:** Sample space for two dice (outcomes): | | 1 | 2 | 3 | 4 | 5 | 6 | | 1 | (1, 1) | (1, 2) | (1, 3) | (1, 4) | (1, 5) | (1, 6) | | 2 | (2, 1) | (2, 2) | (2, 3) | (2, 4) | (2, 5) | (2, 6) | | 3 | (3, 1) | (3, 2) | (3, 3) | (3, 4) | (3, 5) | (3, 6) | | 4 | (4, 1) | (4, 2) | (4, 3) | (4, 4) | (4, 5) | (4, 6) | | 5 | (5, 1) | (5, 2) | (5, 3) | (5, 4) | (5, 5) | (5, 6) | | 6 | (6, 1) | (6, 2) | (6, 3) | (6, 4) | (6, 5) | (6, 6) |	
	1.4.2.LI.2	1.4.2.AS.2
	Determine the probabilities of independent events and express the results as fractions, decimals, percentages and/or ratios. **Using think-pair-share activities,** engage learners to discuss and solve problems on probabilities of independent events. **Example 1**: Let us suppose there are ten balls in a bo. Four balls are Green (G), and six balls are Red(R). If we draw two balls, one at a time, with replacement, find the probability of the following events: 1. Both Balls are Green. 2. The first ball is Red, and the second is Green. 3. At least one ball is Red. **Solution:** Let G1 and R1 be the events that the first ball is Green/Red, respectively. Similarly, let G2 and R2 be the events that the second ball is Green/Red. Since we are dealing with sampling with replacement, so P(G1) = P(G2) = $\frac{4}{10} = \frac{2}{5}$ and P(R1) = P(R2) = $\frac{6}{10} = \frac{3}{5}$	**Level 1 Recall** **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

	1. P(both balls are green) = P(G1 and G2) = P(G1∩ G2). Since the trials are independent, so P(G1∩ G2) = P(G1) × P(G2) = $\frac{2}{5} \times \frac{2}{5} = \frac{4}{2}$. 2. P(first Red and Second Green) = P(R1 and G2) = P(R1 ∩ $G2$). since the trials are independent, so P(R1 ∩ $G2$)= P(R1) × P(G2) = $\frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$. 3. We use the fact that P(at least one ball is Red) = 1 – P(both balls are Green). Hence, P(at least one ball is Red) = $1 - \frac{4}{25} = \frac{21}{25}$. **Example 2**: A poll finds that 72% of the youth in Gushegu consider themselves football fans. If you randomly pick two people from the population, what is the probability that • the first person is a football fan and the second is as well. • the first one is, and the second one isn't? **Solution**: One person being a football fan does not have an effect on whether the second randomly selected person is. Therefore, the events are independent, and the probability can be found by multiplying the probabilities together: - First and second are football fans: P(A∩B) = P(A) · P(B) = .72 * .72 = .5184. - First one is a football fan, the second one isn't: P(A∩B) = P(A) · P(B) = .72 * 1 – 0.72) = 0.202. In the second part, I multiplied by the complement. As the probability of being a fan is .72, then the probability of not being a fan is 1 – .72, or .28. Events A and B are independent if the equation P(A∩B) = P(A) and P(B) holds true. You can use the equation to check if events are independent by multiplying the probabilities of the two events together to see if they equal the probability of them both happening together.	
	1.4.2.LI.3	1.4.2.AS.3
	Solve everyday life problems involving the probability of two independent events. Put learners in convenient groups and offer them appropriate and adequate resources to create and solve some real-life problems.	**Level 1 Recall** **Level 2 Skills of conceptual understanding**

	Example 1: A message is transmitted from Node-A to Node-B through three intermediate nodes. The message will be successfully transmitted only if all the intermediate nodes are working. The probability that an intermediate node will fail is 1%. All nodes are independent of each other. What is the probability that you will not successfully transmit the message? **Solution:** We first find the probability that you will successfully transmit the message. For successful transmission, we need all nodes to be working. The probability that a node will not fail is P(Node does not fail) = 1– P(Node fails) = 1 – 0.01 = 0.99. Since all the nodes are independent, the probability that node 1 AND node 2 AND node 3 do not fail = 0.99 × 0.99 × 0.99 = 0.97 Accordingly, P(message is not successful) = 1 – P(message is successful) = 1-0.97 = 0.03 = 3%. **Example 2:** You are travelling from location A to location B using a bus and a train. The probability that the bus will get delayed is 10%, and the probability that the train will get delayed is 5%. Both events are independent. Find the probability that: · You will experience a delay during your travelling. · You will get on time to location B. **Solution**: Let E1 represent the event that the bus gets delayed, and E2 is the event that the train gets delayed. We will experience a delay if either the bus gets delayed, or the train gets delayed, or both get delayed. So, we are interested in finding the probability P(E1 OR E2) = P(E1 ∪ E2). Using the formulae for the independent events, we can write. P(E1 ∪ E2) = P(E1) + P(E2) – P(E1)P(E2) P(E1 ∪ E2) = 0.1 + 0.05 – (0.1)(0.5) = 0.145 = 14.5% To get on time, we need neither the bus gets delayed, nor the train gets delayed. Hence, we need to find the probability. P(NOT E1 AND NOT E2) = P(NOT E1 ∩ NOT E2) P(NOT E1) = 1 – P (E1) = 1 – 0.1 = 0.9. P(NOT E2) = 1 – P(E2) = 1 – 0.05 = 0.95. Since both events are independent P(NOT E1 ∩ NOT E2) = P(NOT E1) x P(NOT E2).	**Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

	P(NOT E1 ∩ NOT E2) = 0.9 × 0.95 = 0.855 = 85.5%	
Teaching and Learning Resources	• Manipulative (dice, coins, spinners, playing cards, counters, digit cards), • Simple Probability Mazes (Printable & Digital), • Worksheets • Task Cards	

YEAR TWO

Subject **MATHEMATICS**

Strand **1. NUMBERS FOR EVERYDAY LIFE**

Sub-Strand **1. REAL NUMBER AND NUMERATION SYSTEM**

Learning Outcomes	**21st-century Skills and Competencies**	**GESI[3], SEL[4] and Shared National Values**
2.1.1.LO.1		
Evaluate the relationships between the laws and properties of surds, indices and logarithms and apply them to solve problems with radicands.	**Communication and Collaboration:** Learners communicate confidently and effectively to develop appropriate mathematics vocabulary for the concept of surds, indices and logarithms. **Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on surds, indices and logarithms using appropriate IT tools to boost their interest and desire to solve more problems on their own. **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to application of the concept of surds, indices and logarithms to lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on surds, indices and logarithms.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: • Respect individuals of different backgrounds in their mathematics groups and beyond. • Embrace diversity and practise inclusion in the mathematics classroom and beyond. • Examine and dispel misconceptions/ myths about gender as they relate to the learning of mathematics, home management and human development. • Interrogate their stereotypes and biases about gender and the role members in a group play in the mathematics classroom and in home management. • Identify injustice, especially in recognition of the contributions of different groups and individuals to the effective management and maintenance of the mathematics classroom and home.

[3] Gender Equality and Social Inclusion

[4] Socio-Emotional Learning

	Integrated Problem-solving Competency: Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on surds, indices and logarithms. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concept of surds, indices and logarithms to lifelong learning.	· Sensitive to the inter-relatedness of the various aspects of life even as they engage with others in the mathematics classroom and beyond. · Value and promote justice in the mathematics classroom, at home and in society. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the mathematics curriculum, the facilitator should apply Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and finding confidence · Exhibiting motivation and SMART goal-setting · Managing emotions and conflicts · Showing empathy and cooperation These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group

		work in the course of learning of surds, indices and logarithms. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination among learners. **National Core Values**: Develop tolerance, friendliness, open-mindedness, patience, hard work, and humility in learners as they interact with their peers in the mathematics classroom. **Truth and Integrity:** Reward truth and honesty as strong moral principles in the learning of mathematics, leading to responsible citizenship. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment. **Discipline and Honesty:** Encourage learners to behave and work in a controlled way, which involves obeying mathematical rules, principles and standards, leading to self-directed learning.

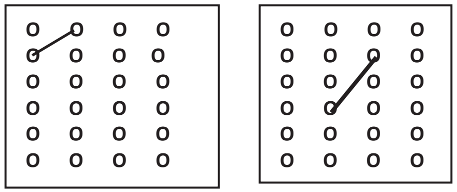
2.1.1.LO.2		
Apply the laws and properties of indices and logarithms to solve real life problems.	**Communication and Collaboration:** Learners communicate confidently and effectively to develop appropriate mathematics vocabulary for the concept of surds, indices and logarithms. **Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on surds, indices and logarithms using appropriate IT tools to boost their interest and desire to solve more problems on their own. **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to the application of the concept of surds, indices and logarithms to lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on own one's values, perceptions and actions for decision-making as they engage in group and individual activities on surds, indices and logarithms. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on surds, indices and logarithms. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: · Respect individuals of different backgrounds in their mathematics groups and beyond. · Embrace diversity and practise inclusion in the mathematics classroom and beyond. · Examine and dispel misconceptions/ myths about gender as they relate to the learning of mathematics, home management and human development. · Interrogate their stereotypes and biases about gender and the role members in a group play in the mathematics classroom and in home management. · Identify injustice, especially in recognition of the contributions of different groups and individuals to the effective management and maintenance of the mathematics classroom and home. · Sensitive to the inter-relatedness of the various aspects of life even as they engage with others in the mathematics classroom and beyond. · Value and promote justice in the mathematics classroom, at home and in society. **SEL**: Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage

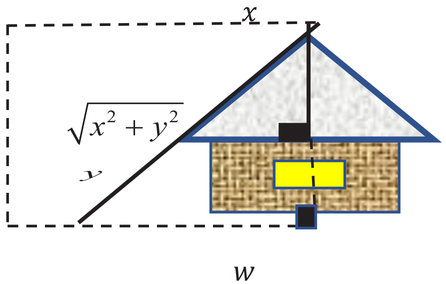
	the concept of surds, indices and logarithms to lifelong learning.	inclusion. As part of achieving each learning outcome in the mathematics curriculum, the facilitator should apply the Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and finding confidence · Exhibiting motivation and SMART goal-setting · Managing emotions and conflicts · Showing empathy and cooperation These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning of surds, indices and logarithms. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination among learners. **National Core Values**: Develop tolerance, friendliness, open-mindedness, patience, hard work,

		and humility in learners as they interact with their peers in the mathematics classroom. **Truth and Integrity:** Reward truth and honesty as strong moral principles in the learning of mathematics, leading to responsible citizenship. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment. **Discipline and honesty:** Encourage learners to behave and work in a controlled way, which involves obeying mathematical rules, principles and standards, leading to self-directed learning.
2.1.1.LO.3		
Establish the connections between number bases and modular arithmetic and apply these relationships to the concept of place value.	**Communication and Collaboration:** Learners communicate confidently and effectively to develop appropriate mathematics vocabulary for number bases and modular arithmetic through teamwork. **Leadership and Respect for others' views**: Inculcate the habit of leadership through teamwork; respect individuals' views, different beliefs, religions, and cultures. **Strategic Competency:** Make conscious efforts to help learners collectively develop and implement innovative actions that promote sustainability at their level, leading to application to lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: · Respect individuals of different backgrounds in their groups as they discuss and solve problems based on the concept of number bases, modular arithmetic and place value. · Interrogate their stereotypes and biases about the roles and abilities of different individuals in learning and applying mathematics. · Examine and dispel misconceptions/ myths about GESI as they engage in a sustainable discourse while learning number bases and modular arithmetic.

	reflect on one's own values, perceptions and actions for decision-making.	· Value and promote justice as they develop and implement innovative actions in the mathematics classroom and beyond. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies – *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the Mathematics curriculum, the facilitator should apply Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and confident as they establish the relationship and find differences among the concepts of number bases, modular arithmetic and place value. · Exhibiting motivation and SMART goal setting in a mathematics classroom and beyond. · Managing emotions and conflicts as they engage in a mathematical discourse. · Showing empathy and cooperation in a mathematical problem-solving situation. These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and

		cultures through interactive and collaborative/group work in the course of learning number bases and modular arithmetic. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment as they debate the relationship between number bases and modular arithmetic. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination as they engage in a mathematical discourse based on number bases and modular arithmetics. **Truth and Integrity:** Reward truth and honesty as strong moral principles as they engage in peer assessment to help them become responsible citizens. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment.

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
2.1.1.CS.1	2.1.1.LI.1	2.1.1.AS.1
Demonstrate knowledge and understanding of surds, indices and logarithms and establish their laws and properties.	**Carry out operations on surds and rationalize monomial denominators.** **Through think-pair-share activities,** find lengths of segments on Geodot (or grid paper) to introduce the concept of surds (including conditions for surds, examples, non-examples and counterexamples). Ensure participation by all learners and tolerate one another, leading to the development of showing empathy and willingness to work with people of different backgrounds.  Figure 2.1 Geodot illustrations **Problem-based Learning:** In interactive and collaborative groups, solve problems involving arithmetic operations with surds (I.e., Adding, subtracting, multiplying, dividing, squaring and rationalizing). Allow for innovation and creativity in learners' responses and solutions, leading to the development of self-awareness and independence in learners. **Examples** 1. i.e., $n\sqrt{a} \pm n\sqrt{b} = n(\sqrt{a} \pm \sqrt{b})$ where a and b are non-negative 2. $m\sqrt{a} \times n\sqrt{b} = mn\sqrt{ab}$ 3. $m\sqrt{a}.n\sqrt{b} = mn\sqrt{\ }$ 4. $m\sqrt{a}.n\sqrt{b} = mn\sqrt{\ }$	Level 1 Recall Level 2 Skills of conceptual understanding Level 3 Strategic reasoning **Level 4 Extended critical thinking and reasoning**

	Using talk-for-learning, investigate real-life problems involving surds. Encourage and ensure fair and impartial opportunities for learners to help learners become aware of the diversity in real life and the need to treat everyone fairly. **Examples:** How architects apply radicals to find slope lengths of roofs, determine rates of flows in pipes and the number of offspring in the breeding cycles of animals (e.g., rabbits with Fibonacci sequence).  Figure 2.2 Slope length of roofs	
	2.1.1.LI.2	2.1.1.AS.2
	Explain the concepts of indices and logarithms with examples. **Talk for Learning:** In interactive, collaborative and gender-responsive groupings, describe the concept of indices through repeated multiplication of a given base. Learners carry out these activities to promote the use of appropriate technology (calculators, computers, and other tools), leading to the development of learners' technology Literacy Skills for everyday life, further studies and/or the world of work. **Examples**	Level 1 Recall Level 2 Skills of conceptual understanding Level 3 Strategic reasoning **Level 4 Extended critical thinking and reasoning**

	$a^1 = a$ $a^2 = a \times a$ $a^3 = a \times a \times a$ $a^4 = a \times a \times a \times a$ $\vdots$ $a^n = a \times a \times a \times \cdots \times a$ $3^3 \times 3^4 = (3\times3\times3)\times(3\times3\times3\times3) = 2^7$ Compose the various indicial laws such as 1. Addition Product Law 2. Subtraction-Quotient Law 3. Index (Power) Law 4. product-index law 5. Quotient – index law 6. Fractional Power (or Root) Law 7. Zero Exponent or Index 8. Negative Integer Exponents and use them to solve real life problems.	
	2.1.1.LI.3	2.1.1.AS.3
	Compose and decompose logarithm laws and properties with exponents and apply the concepts to solve real-life problems. **Using group work and think-pair-share activities**, extend the idea of exponents to logarithms in base ten. Then, review and investigate the concept of logarithm as an exponent to which a base is raised to produce a given number or power. Encourage Talk for Learning and draw learners' attention to the importance of listening to one another and respecting each other's views. **In collaborative and gender-responsive groups**, investigate to discover the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and	Level 1 Recall Level 2 Skills of conceptual understanding Level 3 Strategic reasoning **Level 4 Extended critical thinking and reasoning**

exponents. In doing so, ensure diversity and offer equal opportunities for learners to interact with the lesson and learning materials.

Examples: i.e. $\log_b x =^y$,
E.g., for $x = b^y$, y is the index or exponent, and b is the base. *The logarithm y is the exponent to which b is to be raised to get* . In other words, the log of a number "*x*" to a given base "*b*" is equal to the exponent "*y*".

$\log_{10} 2 = 0.30102999566398119521373889472449$

$\Rightarrow 10^{0.30102999566398119521373889472449} = 2$

No.	0	1	2	3	4
Powers	2^0	2^1	2^2	2^3	2^4
Base 2	1	2	4	8	16
⋮	⋮	⋮	⋮	⋮	⋮
Base 10	1	10	100	1000	10000

Figure 2.3

Using Project-based Learning, task learners to establish the laws and properties of indices for any real number a and natural numbers m and n. The laws and properties are applicable:

Examples

a. Addition Product Law, The Subtraction-Quotient Law, The Index (Power) Law, The product-index law, the Product – index law Quotient – index law, Fractional Power (or Root) Law, The Zero Exponent or Index, Negative Integer Exponents, etc.

b. The Addition-Product Law $\log_k^a + \log_k^b = \log_k^{ab}$.

The Addition-Product Law $\log_k a + \log_k b = \log_k ab$

The Subtraction-Quotient Law $\log_k a - \log_k b = \log_k \frac{a}{b}$

The exponent law: $\log_a b^m = m \log_a b$

Same base law: $\log_b a = 1$

	Change of base law: $\log_a b = \frac{\log_k b}{\log_k a}$ The characteristic is 0 and the mantissa is 0.5441. Discuss the logarithms of numbers less than 1.			
Teaching and Learning Resources	· Geodot · Rubber bands · Computer	· GeoGebra · Google search · Mobile phone	· Calculator · YouTube videos, etc. · Geodot,	· Rubber bands · Cardboards · Measuring instruments

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
2.1.1.CS.2	2.1.1.LI.1	2.1.1.AS.1
Demonstrate knowledge and understanding of the laws and properties of indices and logarithms and their applications to solving real-life problems.	**Investigate real life problems using laws and properties of indices and logarithms.** • Develop a variety of strategies for solving problems involving indices and logarithms and their properties through interactive activities in mixed-ability groupings. **Hint**: Possible strategies include establishing certain clues and properties before solving problems on indices and logarithms and, looking for patterns in the results, finding whether the equations have equal exponents and bases and determining the unknowns. • Encourage learners to collaborate with team/group members and appreciate the worth of persevering to complete tasks successfully. **Examples:** If $x^a = y^a$, then $x = y$ and if $x^a = x^b$, then $a = b$ Thus, exponents are the same since the bases are the same. **In mixed-ability groups,** learners investigate and generalise radicals, discuss observations and use these to determine the range of permissible change of the selected numbers. **Example**: $2\sqrt{\frac{2}{3}} = \sqrt{2\frac{2}{3}}$, $5\sqrt{\frac{5}{24}} = \sqrt{5\frac{5}{24}}$, etc.	Level 1 Recall Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning
	2.1.1.LI.2	2.1.1.AS.2
	Use mathematical connections to explore the relevance of surds, indices and logarithms and their applications to scientific concepts. **In collaborative groups:** • Assign tasks to help learners apply the understanding of common logarithms to solve real-life problems (i.e., relating to chemistry) if necessary, in a collaborative teaching with the experts (i.e., chemistry facilitator). For instance, calculating for pH values.	Level 1 Recall Level 2 Skills of conceptual understanding Level 3 Strategic reasoning

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- Relate connecting Math to other subjects as a way of also connecting with other learners that although they are all different, there are some common values we all share.

Through a mini project, investigate the connections of logarithms to concepts such as algebraic equations. i.e., Logarithms can be used to solve algebraic equations where the unknown variable appears as a power

Example: Find the value of x if $200(1.1)^x = 20000$
Step 1. Simplify by dividing across by 200.
Step 2. Take logs of both sides $(1.1)^x = 100$
Step 3. Simplify as $x = \frac{\log(100)}{\log(1.1)}$
Step 4. Conclusion $x = \frac{\log(100)}{\log(1.1)} = \frac{2}{0.0414} = 48.32$

</td>
<td>Level 4 Extended critical thinking and reasoning</td>
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<tr>
<td>Teaching and Learning Resources</td>
<td>• Geodot
• Rubber bands
• Computer</td>
<td>• GeoGebra
• Google search
• Mobile phone</td>
<td>• Calculator
• YouTube videos, etc.
• Geodot,</td>
<td>• Rubber bands
• Cardboards
• Measuring instruments</td>
</tr>
</table>

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
2.1.1.CS.3	2.1.1.LI.1	2.1.1.AS.1
Demonstrate understanding of the concepts of modulo arithmetic and solve real life problems on them.	**Investigate the concept and existence of modulo arithmetic in learners' environment and introduce it as the arithmetic of remainders**. **In an interactive and differentiated learning environment,** undertake a brief review of sorting and divisibility rules of integers and determine their connections to modular arithmetic using models such as number arrays. **Examples**: i. Sort integers into odd and even blocks (mod2 and mod3): i.e., For modulo 2, the integers are sorted into 2 distinct subsets as, · {. . . , −4, −2, 0, 2, 4, 6, 8, . . .} and · {. . . , −3, −1, 1, 3, 5, 7, . . .}. (Remember that the set of integers is used in its entirety). Similarly, in modulo 3, the set of integers is sorted into 3 distinct subsets, such as · {. . . , −6, −3, 0, 3, 6, 9, 12, . . .}, · {. . . , −5, −2, 1, 4, 7, 10, 13, . . .}, and · {. . . , −4, −1, 2, 5, 8, 11, 14, . . .}. ii. Use repeated subtraction/addition to determine the modulo of negative numbers. I.e., $-8\ (mod\ 3)$ $-8\ (mod\ 3) = -8 + 3 = -5$ $= -5 + 3 = -2$ $= -2 + 3 = 1$	Level 1 Recall Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning
	2.1.1.LI.2	2.1.1.AS.2
	Model and solve real-life problems involving modular arithmetic **Experiential Learning, Problem-based Learning, Group Work/Collaborative Learning.** **In an interactive and differentiated learning environment, m**odel simple situations involving modular arithmetic concepts, connect the ideas to real world problems and solve them using	Level 1 Recall Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning**

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appropriate models and technology. Employ differentiated assessment and ensure values such as tolerance, truth, honesty, respect for others' views, etc., among learners.

Example 1: Determine the appropriate connections between number bases and modular arithmetic.

In GESI-responsive group activities, initiate reflective thinking (or discourse) on the application of modulo-arithmetic concepts in everyday business:

Modulo in Everyday Activities

1. Travelling and Vehicle capacity,
2. Modelling time (using a 12-hour clock),
3. The market day arithmetic,
4. Restaurants and feeding capacity.
5. Modulo arithmetic in music (i.e., base 8 for octave).

The idea is that a and b are "equivalent" when they leave the same remainder upon division by mod number.
Create number arrays using spreadsheets.

-9	-8	-7	-6	-
-4	-3	-2	-1	0
1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

Knowing that $9 \equiv 4(mod 5)$, and $7 \equiv 2(mod 5)$
Then, $9 + 7 \equiv 4 + 2 \equiv 1\ (mod 5)$

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<td>Level 4 Extended critical thinking and reasoning
rational and irrational numbers</td>
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Example 2: Explore the relationships that will lead to the following generalisation.

$$a \equiv b\ (mod\ m), \qquad c \equiv d\ (mod\ m)$$
$$a + c \equiv b + d\ (mod\ m)$$
$$a - c \equiv b - d\ (mod\ m)$$
$$a \times c \equiv b \times d\ (mod\ m)$$

Example 3: Investigate the following properties of operation involving modulo arithmetic.
Commutativity:
Associativity:
Distributive
Existence of Identity Elements
Existence of Additive Inverses:
E.g. 02 = 0 ≡ 0 (mod 4)
12 = 1 ≡ 1 (mod 4)
22 = 4 ≡ 0 (mod 4)
32 = 9 ≡ 1 (mod 4) 2
42 = 16 ≡ 0 (mod 4)

$$n^2 = 0(mod4) \equiv or\ n^2 = 1(mod4)$$

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<tr>
<td>Teaching and Learning Resources</td>
<td>• Fractional boards
• Square or grid paper
• Multi-base blocks
• Algebraic tiles
• Video clips
• Cardboards</td>
<td>• Models
• SHS Mathematics curriculum, etc.
• Technology tools
• computer
• mobile phone
• calculator</td>
<td>• Youtube videos, etc.
• Number line
• Number track
• Wheel of Theodorus
• twine, cylinders
• Tape measure or rule</td>
<td colspan="2">• GeoGebra (free software)
• video clips
• cardboards
• models
• SHS Mathematics curriculum, etc.</td>
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</table>

Subject **MATHEMATICS**

Strand **1. NUMBERS FOR EVERYDAY LIFE**

Sub-Strand **2. PROPORTIONAL REASONING**

Learning Outcomes	21st-century Skills and Competencies	GESI, SEL and Shared National Values
2.1.2.LO.1		
Establish the similarities among ratios, rates and proportions and use these to solve problems.	**Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on ratios, rates and proportions using appropriate IT tools to boost their interest and desire to solve more problems on their own. **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to the application of the concept of ratios, rates and proportions to lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on ratios, rates and proportions. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concept of ratios, rates and proportions to lifelong learning.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: • Respect individuals of different backgrounds in their mathematics groups and beyond. • Embrace diversity and practise inclusion in the mathematics classroom and beyond. • Examine and dispel misconceptions/ myths about gender as they relate to the learning of mathematics, home management and human development. • Interrogate their stereotypes and biases about gender and the role members in a group play in the mathematics classroom and in home management. • Identify injustice, especially in recognition of the contributions of different groups and individuals to the effective management and maintenance of the mathematics classroom and home. • Sensitive to the inter-relatedness of the various aspects of life even as they engage with others in the mathematics classroom and beyond.

		· Value and promote justice in the mathematics classroom, at home and in society. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the mathematics curriculum, the facilitator should apply the Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and finding confidence · Exhibiting motivation and SMART goal-setting · Managing emotions and conflicts · Showing empathy and cooperation These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning ratios, rates and proportions.

		Diversity: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination among learners. **National Core Values**: Develop tolerance, friendliness, open-mindedness, patience, hard work, and humility in learners as they interact with their peers in the mathematics classroom. **Truth and Integrity:** Reward truth and honesty as strong moral principles in the learning of mathematics, leading to responsible citizenship. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment. **Discipline and honesty:** Encourage learners to behave and work in a controlled way, which involves obeying mathematical rules, principles and standards, leading to self-directed learning.
2.1.2.LO.2		
Analyse the relevance of ratios, rates and proportions in solving date-to-day problems.	**Communication and Collaboration:** Learners communicate confidently and effectively to develop appropriate mathematics vocabulary for the concept of ratios, rates and proportions. **Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on ratios,	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to:

	rates and proportions using appropriate IT tools to boost their interest and desire to solve more problems on their own. **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to the application of the concept of ratios, rates and proportions to lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on ratios, rates and proportions. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on ratios, rates and proportions. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concept of ratios, rates and proportions to lifelong learning.	· Respect individuals of different backgrounds in their mathematics groups and beyond. · Embrace diversity and practise inclusion in the mathematics classroom and beyond. · Examine and dispel misconceptions/ myths about gender as they relate to the learning of mathematics, home management and human development. · Interrogate their stereotypes and biases about gender and the role members in a group play in the mathematics classroom and in home management. · Identify injustice, especially in recognition of the contributions of different groups and individuals to the effective management and maintenance of the mathematics classroom and home. · Sensitive to the inter-relatedness of the various aspects of life even as they engage with others in the mathematics classroom and beyond. · Value and promote justice in the mathematics classroom, at home and in society. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the mathematics curriculum, the facilitator should apply the Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and finding confidence · Exhibiting motivation and SMART goal-setting · Managing emotions and conflicts

		· Showing empathy and cooperation These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning ratios, rates and proportions. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination among learners. **National Core Values**: Develop tolerance, friendliness, open-mindedness, patience, hard work, and humility in learners as they interact with their peers in the mathematics classroom. **Truth and Integrity:** Reward truth and honesty as strong moral principles in the learning of mathematics, leading to responsible citizenship.

		Tolerance: Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment. **Discipline and honesty:** Encourage learners to behave and work in a controlled way, which involves obeying mathematical rules, principles and standards, leading to self-directed learning.

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
2.1.2.CS.1	2.1.2.LI.1	2.1.2.AS.1
Demonstrate knowledge and understanding ratios, rates and proportions and use it to solve real-world problems.	**Explain the concepts ratios and rate as a comparison of quantities.** **Using think-pair-share activities,** explain the concepts ratio and rate as a comparison of measures of quantities with similar and/or different dimensions using related mathematical statements and real-life situations. Encourage the use of appropriate mathematics vocabulary for ratio and rate whiles respecting and appreciating how much contributions each individual can make to the lesson, leading to developing the value of tolerance and appreciation for diversity. **Using a blend of whole-class and group activities, r**eview and model the concepts of ratio, rates and proportional reasoning to establish among their parallels by engaging all learners to work with partners to discover the following: ***Ratio versus Rate*** i. Establish a ratio by comparing two quantities of the same unit of measure. This ratio can be expressed as **a: b or** $\frac{a}{b}$ and read as $'a$ to b'. E.g., Write a ratio for the number of males to females in a country. ii. Establish the rate by comparing two quantities of the different units of measure. E.g., If Ananga earns GHȻ5000.00 for working 24 hours, then his average rate of pay per hour is worked as iii. $\frac{\text{GHs}5000.00}{24hours} = \frac{GHs308.33}{1hour} = GHs308.33$ iv. Proportion: A proportion is a statement that compares two ratios or rates to be equal. Application of ratio, rate and proportions a) Reflections: If a function reflects in the line $x = y$, then the function is 1:1 b) Rate of currency exchange between US dollars ($) and Ghana cedis (GHȻ) is GHȻ 10.04 per US dollar. How many GHȻ are equivalent to 5,275$? $\frac{GHs}{\$} = \frac{10.04}{1} = \frac{x}{5,275}$	Level 1 Recall Level 2 Skills of conceptual understanding Level 3 Strategic reasoning **Level 4 Extended critical thinking and reasoning**

	$\Rightarrow x = 10.04 \times 5,375 = GHs53,965.00$ In the course of leading these activities, model the behaviour of gender responsiveness and have the ability to tackle injustice among learners, which learners emulate, leading to self/social awareness and responsible decisions.	
	2.1.2.LI.2	2.1.2.AS.2
	Establish the relationships among ratio, rates and proportions. · **Using Collaborative Learning**, initiate small group discourse on additional fractional quantities. Ensure participation by all and the correct language usage. Champion the course for respecting each other's views and contributions even if they differ from yours. **Example**: Cross Products Investigate the cross-products of ratios and make generalisations**.** If $\frac{a}{b} : \frac{c}{d}$ then the cross products $\boldsymbol{ad}$ and $\boldsymbol{bc}$ are equal. Also $\frac{a}{c} : \frac{b}{d}$ · In mixed groupings, learners investigate four numbers in a proportion, to demonstrate that if any three of these numbers are known, the fourth can be found. **Example**: i. Solve for x in the proportion $\frac{36}{x} : \frac{7}{3}$ ii. Solve the equation $\frac{a-2}{5} : \frac{a+1}{3}$ iii. Similar triangles are said to be similar if they have the same shape (but not necessarily the same size). Similar triangles have sides that are proportional.	Level 1 Recall Level 2 Skills of conceptual understanding Level 3 Strategic reasoning **Level 4 Extended critical thinking and reasoning**

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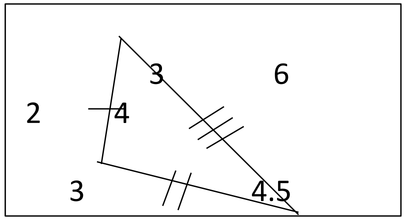

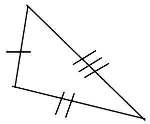

Figure: Similar triangles.

Notice that the ratios of the corresponding sides are all equal $\frac{3}{2}=\frac{3}{2}, \frac{6}{4}=\frac{3}{2}, \frac{4.5}{3}=\frac{3}{2}$

Extend the application of ratios to angles, Pythagoras's theorem and some algebra application of rates to speed: distance-time graphs.</td>
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<td>Teaching and Learning Resources</td>
<td>· Geodot
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<td>· Calculator
· YouTube videos, etc.
· Geodot,</td>
<td colspan="2">· Rubber bands
· Cardboards
· Measuring instruments</td>
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Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
2.1.2.CS.2	2.1.2.LI.1	2.1.2.AS.1
Demonstrate an understanding of proportional reasoning using mathematical connections among ratios, rates and proportions to solve daily problems, including compound interest, tax (VAT, E-Levy), utilities, depreciation, etc.	**Apply the concept of ratios, rates and proportions to solve problems in financial mathematics, health, sports, etc.** **Collaborative group activities:** Using think-pair-share strategies, apply the understanding of the concepts ratio in fraction, percent, or decimal form to solve daily problems in financial mathematics, health, sports, etc. As learners collaborate, bring up scenarios that require learners to be sensitive to the inter-relatedness of the various aspects of life. **Example**: Using proportions to solve percent problems. **Exponential Percentages** Using mini survey groupings, in collaboration with the Science Department, find the body mass indices of humans around us. i.e., Establish the formula for finding the BMI of bodies as $\text{BMI} = \frac{\text{Weight}}{(\text{Height})^2}$ and use the results to advise on appropriate weight control among community members.	Level 1 Recall Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning
	2.1.2.LI.2	2.1.2.AS.2
	Establish the relevance of ratios, rates and proportions in their day-to-day activities, make generalisations and apply them to solve real-world problems. **In a collaborative and gender-responsive group**, model strategies for extending the ideas of proportional reasoning to inter-country exchange rates. Create scenarios to bring out moral lessons where learners come to the realisation that there is a reward in truth and honesty leading to responsible citizenship. **Using Project-based and Experiential Learning, task learners to;** i. Visit and collect exchange rates-related data from banks and forex bureaus to solve current financial problems.	Level 1 Recall Level 2 Skills of conceptual understanding Level 3 Strategic reasoning **Level 4 Extended critical thinking and reasoning**

	ii. One country's currency can be changed to another using exchange rates at banks or forex bureau. Explain that exchange rates are not fixed, and they change almost every day, etc.				
Teaching and Learning Resources	· Geodot · Rubber bands · Computer	· GeoGebra · Google search · Mobile phone	· Calculator · YouTube videos, etc. · Geodot,	· Rubber bands · Cardboards · Measuring instruments	

Subject **MATHEMATICS**

Strand **2. ALGEBRAIC REASONING**

Sub-Strand **1. APPLICATIONS OF EXPRESSIONS, EQUATIONS AND INEQUALITIES**

Learning Outcomes	21st-century Skills and Competencies	GESI, SEL and Shared National Values
2.2.1.LO.1		
Solve linear equations in two variables using the elimination, substitution and graphical methods.	**Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to the application of the concept of linear equations in two variables for lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on linear equations in two variables. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on linear equations in two variables. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concept of linear equations in two variables to lifelong learning.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: • Respect individuals of different backgrounds in their mathematics groups and beyond. • Embrace diversity and practise inclusion in the mathematics classroom and beyond. • Examine and dispel misconceptions/ myths about gender as they relate to the learning of mathematics, home management and human development. • Interrogate their stereotypes and biases about gender and the role members in a group play in the mathematics classroom and in home management. • Identify injustice, especially in recognition of the contributions of different groups and individuals to the effective management and maintenance of the mathematics classroom and home. • Sensitive to the inter-relatedness of the various aspects of life even as they engage with others in the mathematics classroom and beyond.

		· Value and promote justice in the mathematics classroom, at home and in society. **SEL**: Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the mathematics curriculum, the facilitator should apply the Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and finding confidence · Exhibiting motivation and SMART goal-setting · Managing emotions and conflicts · Showing empathy and cooperation These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning linear equations in two variables.

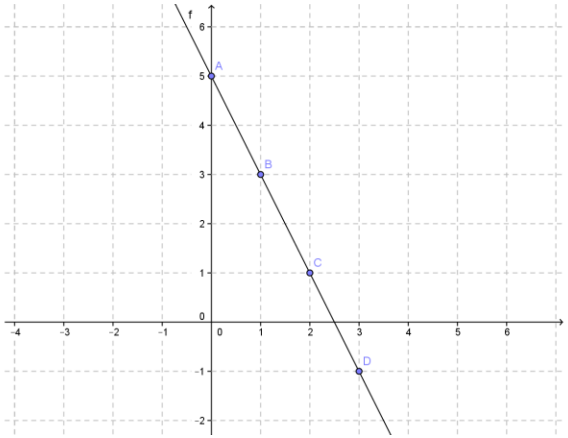
		Diversity: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination among learners. **National Core Values**: Develop tolerance, friendliness, open-mindedness, patience, hard work, and humility in learners as they interact with their peers in the mathematics classroom. **Truth and Integrity:** Reward truth and honesty as strong moral principles in the learning of mathematics, leading to responsible citizenship. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment. **Discipline and honesty:** Encourage learners to behave and work in a controlled way, which involves obeying mathematical rules, principles and standards, leading to self-directed learning.
2.2.1.LO.2		
Analyse, model, and solve simultaneous linear equations involving real-life problems.	**Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on simultaneous linear equations using appropriate IT tools to boost their interest in and desire to solve more problems on their own. **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: · Respect individuals of different backgrounds in their mathematics groups and beyond.

	the application of the concept of simultaneous linear equations in lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on simultaneous linear equations. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concept of simultaneous linear equations to lifelong learning.	· Embrace diversity and practise inclusion in the mathematics classroom and beyond. · Examine and dispel misconceptions/ myths about gender as they relate to the learning of mathematics, home management and human development. · Interrogate their stereotypes and biases about gender and the role members in a group play in the mathematics classroom and in home management. · Identify injustice, especially in recognition of the contributions of different groups and individuals to the effective management and maintenance of the mathematics classroom and home. · Sensitive to the inter-relatedness of the various aspects of life even as they engage with others in the mathematics classroom and beyond. · Value and promote justice in the mathematics classroom, at home and in society. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the mathematics curriculum, the facilitator should apply the Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and finding confidence · Exhibiting motivation and SMART goal-setting · Managing emotions and conflicts · Showing empathy and cooperation

		These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning of simultaneous linear equations. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination among learners. **National Core Values**: Develop tolerance, friendliness, open-mindedness, patience, hard work, and humility in learners as they interact with their peers in the mathematics classroom. **Truth and Integrity:** Reward truth and honesty as strong moral principles in the learning of mathematics, leading to responsible citizenship. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning

		through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment. **Discipline and honesty:** Encourage learners *to behave and work in a controlled way which involves obeying mathematical rules, principles and standards leading* to self-directed learning.

Content Standards	**Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI**	**Assessment**
2.2.1.CS.1	2.2.1.LI.1	2.2.1.AS.1
Demonstrate knowledge and understanding of the concept of simultaneous equations involving two variables and apply it to solve every day-life problem.	**Solve simultaneous linear equations involving two variables using the graphical method and interpret them.** **Using group work/Collaborative Learning,** review learners' knowledge on linear equations, deal with their misconceptions about such concepts, and extend the ideas to two linear equations using GeoGebra (where available). Bring up scenarios for learners to respond to situations that require empathy and cooperation. Learners should discuss the need to be discipline with the use of technological tools in their learning. **Using Talk for Learning strategy,** engage learner in a whole class, to explain simultaneous equation and the solution of simultaneous equations in two variables as one point or value that satisfies both equations. (i.e. the intersection of the two lines) and the strategies for solving them. (I.e., graphical, elimination, and substitution method.) Offer fair and equitable opportunity for all learners including showing tolerance for each one's contributions. **Using Talk for Learning strategy, engage learner** in a whole class to discuss and explain the graphical method of solving simultaneous equations using the slop-intercept form. I.e., $y = mx + c$ where m is the slope, and c is the y- intercept. Learners should show respect for each other. Steps: i. Write the two equations in the slope intercept form. (e.g., the equation, $y + 2x = 5$ in the slope-intercept form is $y = -2x + 5$ ii. Use the y-intercept and the gradient to draw the line $y = -2x + 5$. I.e.,	**Level 1 Recall** Level 2 Skills of conceptual understanding Level 3 Strategic reasoning Level 4 Extended critical thinking and reasoning



In small ability mixed groups find the point or value that satisfies the simultaneous equation using the graphical method

. $y + x = 3$

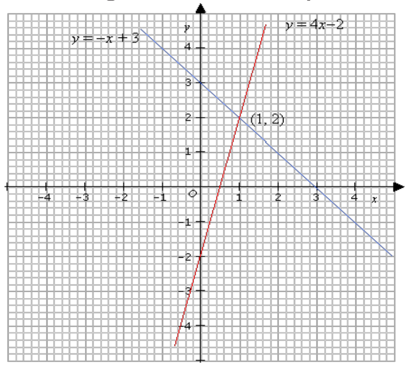
$y = 4x - 2$

Solution: Draw the two lines graphically by writing the two equations in the slope intercept form. I.e.,

$y = -x - 3$

$y = 4x - 2$

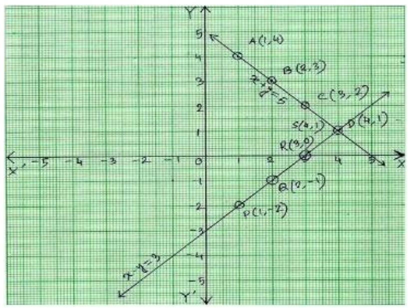
Use the gradient and the y-intercept to draw the two lines.



From the graph, the point of intersection is (1, 2) and this is the point or value that satisfies the two equations.

Problem-based Learning: In a small mixed-ability/gender group, draw the lines of the equations below, investigate the patterns in the graph and find the solution that satisfies both equations, and justify your answer. (I.e. x – y = 3 and $x+y=5$.)

Solution: Write the equations in the slope-intercept form.
(i.e., $y=x-3$ and $y=-x+5$).
On the same graph sheet, draw the two lines.



The patterns for the line $y=x-3$ are (-1, -4), (-2,0), (1, -2), (2, -1), (3,0), (4,1), (5,2) and the patterns for the line $y=-x+5$ are (0,5), (1,4), (2,3), (4,1), (5,0).

	From the patterns of both lines, the point (4,1) is common, and this is the point of intersection that becomes the solution for both equations. **Problem-based Learning:** In small mixed-ability/gender groups, task learners to solve direct and indirect questions consisting of simultaneous equations using a graphical method. Encourage learners to respect the diversity among themselves in the classroom, leading to the promotion of respect for divergent views and inclusivity in the mathematics learning environment and beyond.	
	2.2.1.LI.2	2.2.1.AS.2
	Analyse two linear equations in two variables and solve them using the elimination and substitution methods. **Using talk-for-learning**, review with **the** whole class the basic concepts of substitution and change of subject. **Example 1**: If $v = u + at$ make u e subject and find the value of u when $a = 2$, $v = 10$, and $t = 4$ **Solution:** Subtract at from both sides $v - at = u + at - at$ $u = v - at$ Substitute the values of a, v, t into $u = v - at$ $u = 10 - (2 \times 4)$ $u = 2$ **Example 2**: The volume of a cuboid is the product of the length, breadth, and height of the cuboid. Make breadth the subject of the equation. The expected answer is $v/(l \times h)$ **In small groups**, engage learners to discuss to come out with the process of solving simultaneous equations using the substitution method. Learners must talk about the need to collaborate and acknowledge the diversity among themselves.	Level 1 Recall Level 2 Skills of conceptual understanding Level 3 Strategic reasoning Level 4 Extended critical thinking and reasoning

Example: When using the substitution method, choose any of the equations and make any of the variables the subject and substitute the expression of the variable you made the subject into the second equation and solve the value of the variable in the obtained equation. Now, substitute the value of the solved variable into any of the two equations to find the other variable. The value of the two variables is the point of intersection for the two equations.

Collaborative learning: In a small mixed gender/ability group, solve two linear equations using the substitution method.

Example: Solve $x + y = 3$………(1) and $5x + y = 15$………(2) simultaneously using the substitution method.

Solution
Steps.
- Chose any of the equations and make any of the variables the subject (I.e., $x + y = 3$, $x = 3 - y$)
- Substitute the expression of x into the second equation and solve the value of the variable in the obtained equation. (I.e., $5(3 - y) = 15$ therefore $y = 0$)
- Substitute the value obtained for y into any of the equations to find for . (I.e., taken equation 1, $x = 3$).

The solution of the two equations is 3 and 0.

Collaborative Learning: In small groups, engage learners to discuss and come out with the process of solving simultaneous equations using the elimination method. Learners must discuss the need to persevere as they strive to think critically to come up with solutions to their assigned tasks.

Example: When using the elimination method, write the two equations in the standard form and use the additive property of equality to eliminate one of the variables in both equations and solve for the remaining variable. Now, substitute the value of the solved variable into any of the two equations and solve the other variable. The value of the two variables is the point of intersection for the two equations.

Collaborative learning: In a small mixed-gender/ability group, solve two linear equations using the elimination method.

	Example: Find the value of x and y that satisfies the following equations $y + 3\text{x} = 12 \ldots\ldots (1)$ and $2x - y = 13 \ldots\ldots (2)$ using the elimination method. **Solution** **Steps:** · Write the two equations in the standard form and use the additive property of equality to eliminate one of the variables in both equations. **Note** that the coefficient of the y-term is additive inverse, so when we add the two equations together, the y-terms add to 0, and we have one equation with one variable. i.e., $3\text{x}+\text{y} = 12 \ldots . (1)$ $2x - y = 13 \ldots (2)$ $3x + y = 12$ $2x - y = 13$ $5x = 25$ • Make x the subject (I.e., $= 5$) and substitute the value into any of the two equations. (i.e., taken equation (1), $3(5) + y = 12$) the value of $y = -3$) • Therefore, the values of x and y that satisfy both equations are 5 and -3, respectively. **Problem-based Learning:** In mixed-gender groups, solve direct and indirect problems consisting of simultaneous equations using both substitution and elimination methods. Engage learners to discuss the need to treat each gender equally and with respect, leading to the promotion of respect for divergent views and inclusivity in the mathematics learning environment and beyond.	
	2.2.1.LI.3	2.2.1.AS.3
	Analyse, model, and solve word problems of simultaneous linear equations involving numbers, age, etc. **Using Talk for Learning strategy**, review with the whole class the concept of linear equations involving word problems. **Collaborative Learning**: Using think-pair-share activity, task learners to analyse, model and solve real-life problems involving numbers and age using any of the methods. Appreciate the need to treat each other fairly.	Level 1 Recall Level 2 Skills of conceptual understanding Level 3 Strategic reasoning Level 4 Extended critical thinking and reasoning

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Example: The difference of twice one number and three times a second number is equal to 2, and the sum of these numbers is 11. Find the two numbers.

Solution: Learners in the various groups, through discussion, will come out with the equations.

i.e. let x be the first number, and y be the second number.
Twice one number (the first number) is $2x$
Three times the second number is $3y$
The difference equals 2 gives $2x - 3x = 2$ ... (1)
The sum of these numbers is $x + y = 11$ ... (2)

Solving these equations simultaneously, the numbers are $x = 7$ and $y = 4$.

Example: The sum of the ages of Kwaku and Ama is 32. In two years' time, Kwaku will be three times as old as Ama. How old are they now?

Solution: Let Kwaku's age be x and Ama's age be y
The sum of their ages $x + y = 32$
This means that $x = 32 - y$ and $y = 32 - x$
In two years' time, Kwaku's age will be $+2$. (I.e., $32 - y + 2 = 34 - y$)
In two years time, Ama's age will be $y + 2$. (I.e., $32 - x + 2 = 34 - x$)
In two years time, Kwaku's age will be equal to three times Ama's age
i.e., $x + 2 = 3(34 - x)$

By simplifying $x = 25$ this is Kwaku's age.
To find Ama's age, substitute Kwaku's age into any of the equations. Therefore, Ama's age is 7.

Project-based Learning: In small mixed-ability/gender groups, solve direct and indirect real word problems consisting of simultaneous equations in two variables using all three methods.

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Subject **MATHEMATICS**

Strand **2. ALGEBRAIC REASONING**

Sub-Strand **2. PATTERNS AND RELATIONS**

Learning Outcomes	21st-century Skills and Competencies	GESI, SEL and Shared National Values
2.2.2.LO.1		
Explore patterns of a sequence using plane figures and find the nth and the sum of the nth term of an arithmetic and geometric progression. Analyse, model, and solve real-life problems involving financial mathematics and exponential growth.	**Communication and Collaboration:** Learners communicate confidently and effectively, to develop appropriate mathematics vocabulary for the concept of patterns of a sequence. **Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on patterns of a sequence using appropriate IT tools to boost their interest and desire to solve more problems on their own. **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to the application of the concept of patterns of a sequence to lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on patterns of a sequence. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concept of patterns of a sequence to lifelong learning.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: • Respect individuals of different backgrounds in their mathematics groups and beyond. • Embrace diversity and practise inclusion in the mathematics classroom and beyond. • Examine and dispel misconceptions/ myths about gender as they relate to the learning of mathematics, home management and human development. • Interrogate their stereotypes and biases about gender and the role members in a group play in the mathematics classroom and in home management. • Identify injustice, especially in recognition of the contributions of different groups and individuals to the effective management and maintenance of the mathematics classroom and home. • Sensitive to the inter-relatedness of the various aspects of life even as they engage with others in the mathematics classroom and beyond.

		· Value and promote justice in the mathematics classroom, at home and in society. **SEL**: Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the mathematics curriculum, the facilitator should apply the Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and finding confidence · Exhibiting motivation and SMART goal-setting · Managing emotions and conflicts · Showing empathy and cooperation These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning patterns of a sequence.

		Diversity: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination among learners. **National Core Values**: Develop tolerance, friendliness, open-mindedness, patience, hard work, and humility in learners as they interact with their peers in the mathematics classroom. **Truth and Integrity:** Reward truth and honesty as strong moral principles in the learning of mathematics, leading to responsible citizenship. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment. **Discipline and Honesty:** Encourage learners to behave and work in a controlled way, which involves obeying mathematical rules, principles and standards, leading to self-directed learning.

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
2.2.2.CS.1	2.2.2.LI.1	2.2.2.AS.1
Demonstrate understanding of patterns and relations involving sequence and series, generate strategies for algebraic formulas, and use them in solving real-life problems.	**Explore patterns of a sequence using plane figures and continue with more terms.** **Using Talk for Learning approaches**, learners discuss the meaning of sequence as a set of objects, like numbers, which follow a particular pattern, whilst series is the sum of all the items in the sequence. **Collaborative learning:** In convenient groups, task learners to investigate, discuss and identify real-life patterns in the environment, including numerical and non-numerical sequences (shapes, fabric, etc.) and write the algebraic expression from the pattern. Encourage learners to justify their ideas and respect ideas from others, leading to the development of the character of tolerating others and respecting individual variance. **Example**: Learners investigate and talk about: i. **Seating around tables.** (In a restaurant, a rectangular table fits 6 people. When two rectangular tables are put together, 10 people can be seated. Put 3 rectangular tables together, and now 14 people are seated …).  ii. **Stacking chairs, bowls, etc**., to compare the number of objects to the height of the object. 	**Level 1 Recall** **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** **Level 4 Extended critical thinking and reasoning**

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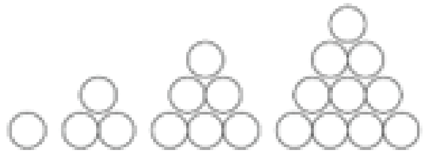
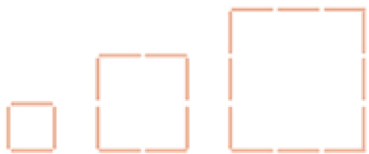
<td>Experiential Learning: In small ability groups, task learners to use sticks, matchsticks, symbols, and numerals to create a sequence and write the algebraic expression if the sequence continues.

Example 1


Example 2: In the match-stick pattern, a sequence is modelled with 1 side matchstick, 2 sides matchsticks and 3 sides matchsticks. Study the pattern and draw,
i. The next two squares of the pattern and create a table showing the perimeter of all the squares.
ii. How many matchsticks will make the 6^{th} square?
iii. Write the rule (algebraic expression) to describe your matchsticks if the pattern continues.


Match-stick pattern

In convenient groups, task learners to find the rule for a given sequence in both direct and indirect questions. Show tolerance to each other as you share diverse ideas in your groups.</td>
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<td>2.2.2.LI.2</td>
<td>2.2.2.AS.2</td>
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<td>Recognise and find the nth term and the sum of the nth term of an arithmetic progression (AP) or linear sequence.

Using Talk for Learning approaches, engage learners in a whole class to discuss the types of sequences as arithmetic (linear) and geometric (exponential) sequences. Encourage learners to volunteer to lead group discussions, leading to the development of leadership qualities for national development.</td>
<td>Level 1 Recall
Level 2 Skills of conceptual understanding
Level 3 Strategic reasoning</td>
</tr>
</table>

	Using Collaborative Learning, create convenient groups and task learners to investigate, discuss and brainstorm the meaning of arithmetic progression as a sequence where the differences between every two terms are the same or every term is obtained by adding a fixed number (positive or negative or zero) to its previous term. (e.g., 3, 6, 9, 12, 15, ...) and establish the general rule for the nth term as $Un = a + d(n-1)$ where *a* is the first term, *d* is the common difference, and *n* is the number of terms and the sum of the first n terms as $S_n = \frac{n}{2}(2a + (n-1)d$ by creating different numerical sequences and investigating the patterns. **Example 1**: Find the formula for the nth term of the arithmetic sequence 2, 5, 8, ... and find the 6th term. **Solution** $a = 2$ and $d = 3$ So $U_n = 2 + 3(n - 1)$ $\therefore U_n = 3n - 1$ $n = 6$ $\therefore U_n = 3(6) - 1 = 17$ Therefore, the 6th term of the above sequence is 17. **Example 2:** Find the sum of the first 10 terms of the arithmetic sequence 3, 7, 11, ... **Solution** $S_n = \frac{n}{2}(2a + (n-1)d$ $n = 10, a = 3\ and\ d = 4$ $S_{10} = \frac{10}{2}(2 \times 3 + (10-1) \times 4$ $S_{10} = 210$	**Level 4 Extended critical thinking and reasoning**
	2.2.2.LI.3	2.2.2.AS.3
	Identify geometric progression or exponential sequence and find the algebraic expression for the general term.	Level 1 Recall Level 2 Skills of conceptual understanding

	Collaborative Learning: In small ability groups, task learners to investigate, discuss and brainstorm the meaning of geometric sequence (exponential sequence) as a sequence in which each term is obtained by multiplying the previous term by a constant factor. (e.g., 6, 12, 24, 48…), and establish the general rule for the *n*th term as $U_n = ar^{n-1}$ where a is the first term, r is the common ratio n is the number of terms, and the sum of the first n terms as $S_n = \frac{a(r^n-1)}{r-1}$ where $r > 1$ and $S_n = \frac{a(1-r^n)}{1-r}$ where $r < 1$, by creating different numerical sequences and investigating the patterns. **Example 1**: Find the 8th term of the exponential sequence 2, 6, 18, 54… **Solution** $U_n = ar^{n-1} \quad a = 2, r = 3, \ n = 8$ $U_8 = 2 \times 3^{8-1}$ $U_8 = 4372$ **Example 2**: Find the sum of the first 7 terms of the G P $\frac{1}{2}, 1, 2, 4, \ldots$ **Solution** $S_n = \frac{a(r^n-1)}{r-1}$ since $r > 1 \quad r = 2, a = \frac{1}{2}, n = 7$ $S_7 = \frac{\frac{1}{2}(2^7-1)}{2-1}$ $S_7 = \frac{1}{2}(256 - 1)$ $S_7 = 127.5$ **Example 3**: The second and the fourth terms of an exponential sequence (GP) are 9 and 4, respectively. Find the sequence. **Solution** $U_2 = ar \quad = 9 \ \ldots\ldots.(1)$ $U_4 = ar^3 \ = 4 \ldots\ldots.(2)$ Solving the equation simultaneously $r = \frac{2}{3}$ and $a = 13.5$ Using a, ar, ar^2, ar^3 … the sequence is $13.5, 9, 6, \ldots$	**Level 3 Strategic reasoning** **Level 4 Extended critical thinking and reasoning**

	2.2.2.LI.4	2.2.2.AS.4
	Analyse, model, and solve real-life problems involving financial mathematics. **Put learners in convenient groups and engage them** to analyse and model real-life problems of a sequence involving finances. Initiate conversation on the need to be honest in handling money at home and in school. **Example 1**: A man saves GHC16500 in ten years. In each year after year, he saved GHC100 more than he did in the previous year. How much did he save in the first year? **Solution:** Sequence: a, a+100, a+200, a+300, … $d = (a + 100) - a = 100.$ $n = 10, \quad S_{10} = 16500$ $S_n = \frac{n}{2}(2a + (n-1)d$ $16500 = \frac{n}{2}(2a + (10-1)100$ $a = 1200$ Hence, he saved GHC1200 in the first year. **Example 2**: A job pays a salary of GHC25,000 for the first year. For the next 19 years, the salary increases by 5% each year. What is the total amount of money earned over the twenty years? **Solution** . $r = \frac{100+5}{100} = 1.05,\ a = 25000\ n = 20$ $S_{20} = 25000 + 25000(1.05) + 25000(1.05)^2 + \cdots + 25000(1.05)^{20-1}.$. $S_{20} = \frac{a(r^n - 1)}{r-1}$.$S_{20} = \frac{25000(1.05^{20}-1)}{1.05-1} = \text{GHC } 826{,}648.85$	Level 1 Recall Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** **Level 4 Extended critical thinking and reasoning**

	2.2.2.LI.5	2.2.2.AS.5
	Analyse, model, and solve real-life problems involving exponential growth. **Put learners into convenient groups and engage them in pairs** to analyse and model real-life problems of sequence involving growth and discuss your findings. Encourage learners to give fair criticisms to their colleagues' presentations. And in doing so, creating a sustainable discourse for learners to question norms, practices, and opinions; and to reflect on one's own values, perceptions and actions for decision-making as they engage in group and whole class activities on exponential growth. **Example**: An island has a population of 1500 and is growing at a rate of 3% per year. What will the population be after 7 years? **Solution** $start \rightarrow U_1$. $Year\ 1 \rightarrow U_2$. $Year\ 2 \rightarrow U_3$. $\downarrow$. $Year\ 7 \rightarrow U_8$.$U_n = ar^{n-1} \quad r = 1.03, \; n = 7 \quad a = 1500.$ $U_8 = 1500(1.03)^7$ $U_8 = 1844.81$	Level 1 Recall Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** **Level 4 Extended critical thinking and reasoning**
Teaching and Learning Resources	· Matchsticks · Cardboard	

Subject **MATHEMATICS**

Strand **3. GEOMETRY AROUND US**

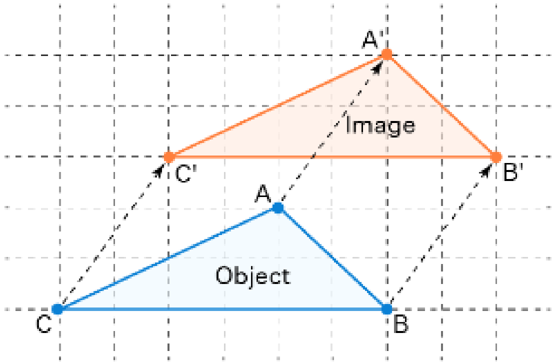
Sub-Strand **1. SPATIAL SENSE**

Learning Outcomes	**21st-century Skills and Competencies**	**GESI, SEL and Shared National Values**
2.3.1.LO.1		
Carry out a variety of transformations through translation, reflection, rotation and enlargement of plane shapes and identify scale drawing as an enlargement/reduction of a plane shape.	**Communication and Collaboration:** Learners communicate confidently and effectively to develop appropriate mathematics vocabulary for the concept of transformations through translation, reflection, rotation and enlargement. **Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on transformations through translation, reflection, rotation and enlargement using appropriate IT tools to boost their interest and desire to solve more problems on their own. **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to application of the concept of transformations through translation, reflection, rotation and enlargement to lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on transformations through translation, reflection, rotation and enlargement. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: Respect individuals of different backgrounds in their mathematics groups and beyond. · Embrace diversity and practise inclusion in the mathematics classroom and beyond. · Examine and dispel misconceptions/ myths about gender as they relate to the learning of mathematics, home management and human development. · Interrogate their stereotypes and biases about gender and the role members in a group play in the mathematics classroom and in home management. · Identify injustice, especially in recognition of the contributions of

	they engage in activities on transformations through translation, reflection, rotation and enlargement. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concept of transformations through translation, reflection, rotation and enlargement to lifelong learning.	different groups and individuals to the effective management and maintenance of the mathematics classroom and home. · Sensitive to the inter-relatedness of the various aspects of life even as they engage with others in the mathematics classroom and beyond. · Value and promote justice in the mathematics classroom, at home and in society. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the mathematics curriculum, the facilitator should apply the Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and finding confidence · Exhibiting motivation and SMART goal-setting · Managing emotions and conflicts · Showing empathy and cooperation These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-

		made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values: Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning of transformations through translation, reflection, rotation and enlargement. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination among learners. **National Core Values**: Develop tolerance, friendliness, open-mindedness, patience, hard work, and humility in learners as they interact with their peers in the mathematics classroom. **Truth and Integrity:** Reward truth and honesty as strong moral principles in the

		learning of mathematics, leading to responsible citizenship. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment. **Discipline and honesty:** Encourage learners to behave and work in a controlled way, which involves obeying mathematical rules, principles and standards, leading to self-directed learning.

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
2.3.1.CS.1	2.3.1.LI.1	2.3.2.AS.1
Demonstrate a conceptual understanding of spatial sense regarding changes and invariance achieved by performing a combination of successive transformations (reflection, translation, rotation) in a 2D shape.	**Identify and translate an object or point by a translating vector and describe the image.** **Think-pair share activities:** Learners in pairs research and make presentations on the concept of translation in math. Learners must endeavour to respect diversity among themselves as they share diverse views, leading to tolerance. **Example:** In Mathematics, a translation moves an object from one place to another. That is, it moves a shape left or right, up or down, and so every point on the object moves the same distance in the same direction. Shapes that are translated are congruent to each other. That is, they are of the same size as the original shape. When you transform a shape, the new shape is called the **image,** and the original shape is called the **preimage**. You then label the vertices of the preimage using uppercase letters. For example, ABCD, QRST, MNOP, etc. and the image is labelled with uppercase letters with a "prime" next to each. For example, Q'R'S'T', and is pronounced "Q-prime, R-prime, S-prime, T-prime". **Example**: In small groups, translate a given shape in a coordinate plane. In the figure, the preimage is ABC, and its image is A'B'C'. Moved up (vertically) by 3 units and then moved right (horizontally) by 2 units. 	Level 1 Recall **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

Remember, as discussed earlier, that when performing the translation of the triangle to the left/right/up/down, we move all the points of the triangle by an equal number of units in the same direction.

Using think (ink)-pair share activities: Task learners to research and make presentations on the translation rules. Learners must talk about the safe ways of using some technological tools as they make presentations of their work.

Example: Translation Rules

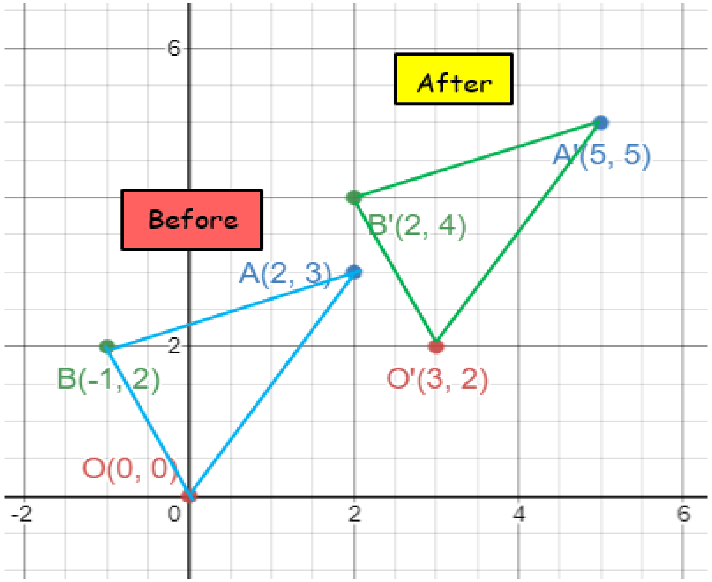
- If a shape moved towards the left by k units, x is replaced with x - k.
- If a shape is moved towards the right by k units, x is replaced with x + k.
- If a shape is moved up by k units, y is replaced with y + k.
- If a shape is moved down by k units, y is replaced with y - k.

In mixed groups, task learners to perform translation on a plane. Encourage the use of computer applications to perform the translation and discuss the appropriate use of technology in everyday life.

Example 1: Triangle OAB with vertices O (0, 0), A(2, 3) and B(-1, 2) is translated under the vector (3, 2). Find the image vertices and illustrate the object and the image.

Solution: To find vertices of the image after translation, we use the given vector; (3, 2)
$x' = x+3$ and $y' = y+2$

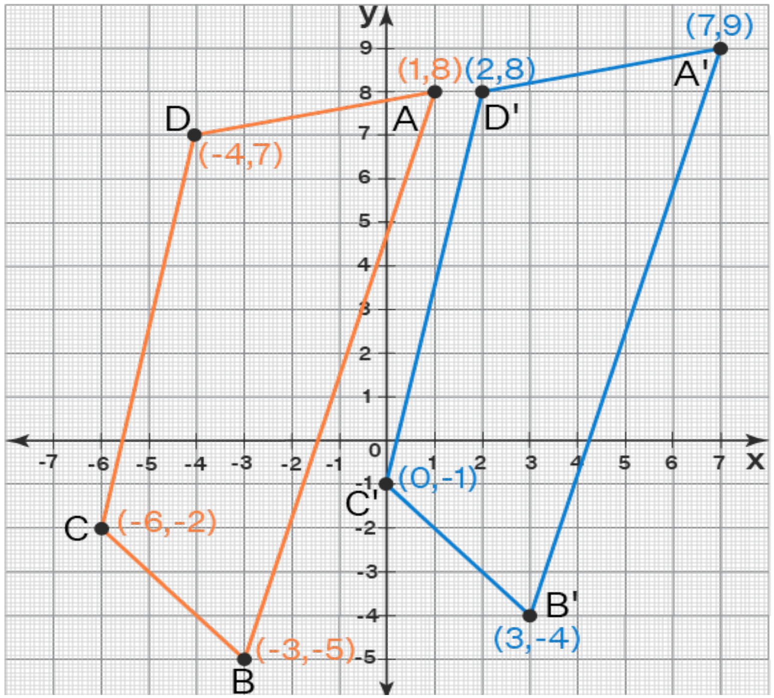
Before translation (preimage)	**After translation (Image)**
O(0, 0)	O'(0+3, 0+2) ==> O'(3, 2)
A(2, 3)	A'(2+3, 3+2) ==> A'(5, 5)
B(-1, 2)	B'(-1+3, 2+2) ==> B'(2, 4)



Example 2: A shape is formed with vertices A(1, 8), B(−3, −5), C(−4, 7), and D(−6, −2). Plot the image of this shape with respect to the translation (x, y) → (x + 6, y + 1).

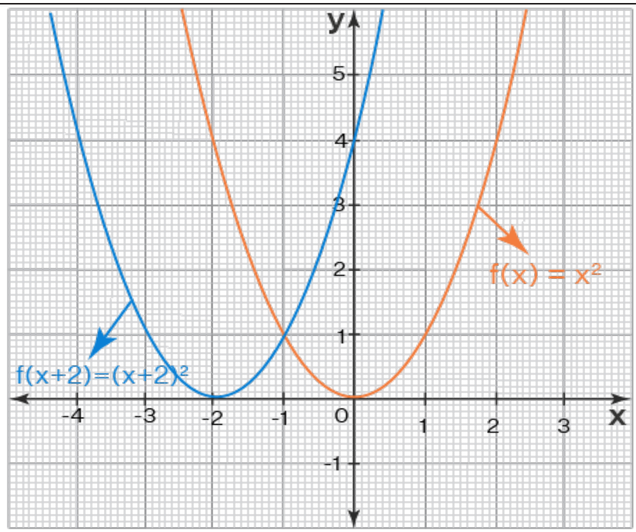
Solution: To find the vertices of the image after translation, we use the given vector (6, 1)

Before the Translation	**After the Translation**
A (1, 8)	(1 + 6, 8 + 1) = (7, 9) = A'
B (−3, −5)	(-3 + 6, -5 + 1) = (3, -4) = B'
C (−4, 7)	(-4 + 6, 7 + 1) = (2, 8) = C'
D (−6, −2)	(-6 + 6, -2 + 1) = (0, -1) = D'



Using a Collaborative Learning approach, learners discuss and establish the rules of translation of functions. As learners talk about rules of translation, encourage discussions on the need to obey rules and the consequences of doing otherwise in everyday life.

Example 1



From the graph, the image is f(x + 2), and the preimage is f(x). The f(x) has moved left by 2 units (instead of right by 2 units) to give f(x + 2).

Note that this is the case with horizontal translations of functions. However, this is not the case with vertical translations. Vertical translations work just like how they work with the translations of points on the coordinate plane.

Example 2: Find the image of $y = 2x^2$ under a translation with vector (3, -2).
Solution: Let us find any three points on the parabola.
Vertex of the parabola is (0, 0)
If x = -1, then y = 2
If x = 1, then y = 2
So, three points on the parabola are V(0, 0), A(-1, 2) and B(1, 2)
The image has to be translated along the vector <3, -2>.
(h, k) ==> (3,-2)
x' = x+3 and y' = y-2

<table>
<tr>
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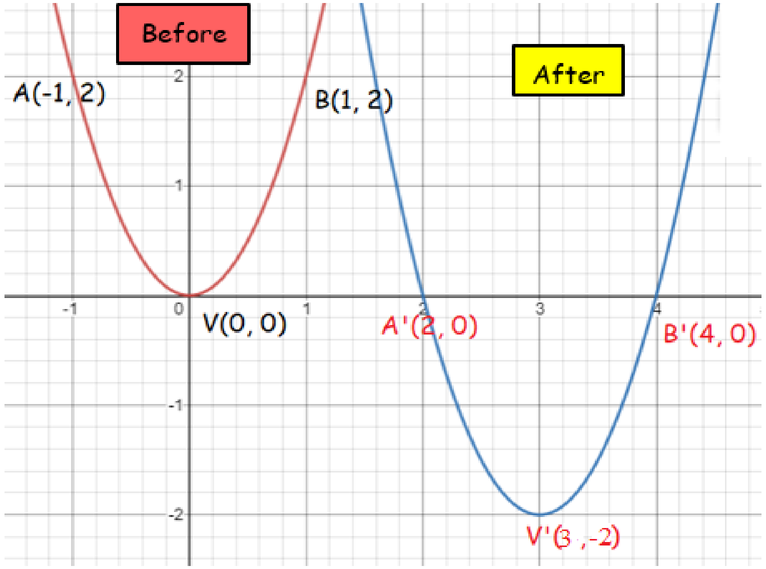
<table>
<tr><td>Before translation
V(0, 0)
A(-1, 2)
B(1, 2)</td><td>After translation
V'(0+3, 0-2) ==> V'(3, -2)
A'(-1+3, 2-2) ==> A'(2, 0)
B'(1+3, 2-2) ==> B'(4, 0)</td></tr>
</table>


(Note that the original parabola has been translated, and there is NO change in the shape. With the after picture, more of the parabola is revealed and makes it look bigger!)
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<td>2.3.1.LI.2</td>
<td>2.3.1.AS.2</td>
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<td>Identify and explain the reflection of an object in a mirror line and describe the image points of shapes in a reflection.

Think-pair share activities: Learners in pairs research and make presentations on the concept of reflection in Mathematics.</td>
<td>Level 1 Recall
Level 2 Skills of conceptual understanding
Level 3 Strategic reasoning</td>
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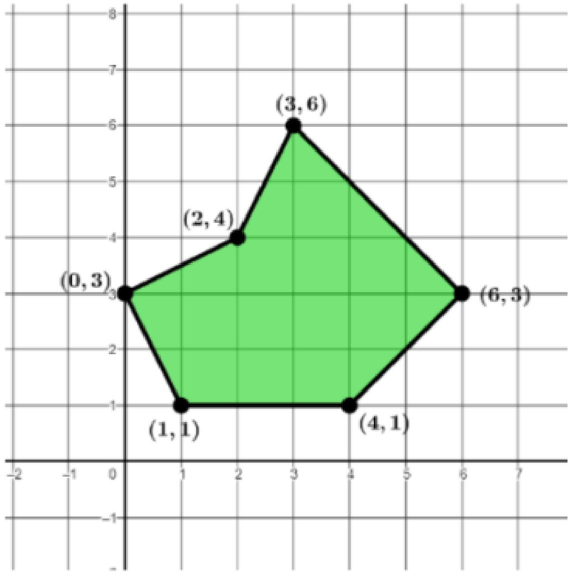
	Example · A reflection requires a mirror line. · Reflection is a type of transformation that flips a shape in a mirror line (also called a line of reflection) so that each point is the same distance from the mirror line as its reflected point. · A mirror line reflects the original image. All the vertices, or corners, of the shape, are always the same distance from the mirror line in the reflected image as they are in the original image. · Reflections may be shown on a grid, and the mirror line is given as an equation. · To find a mirror line, locate the mid-point between each set of vertices and draw the mirror line through these mid-points. **Think-pair share activities:** Learners in pairs research and make presentations on the reflection rules. As learners talk about rules of reflection, encourage discussions on the need to obey rules and the consequences of doing otherwise in everyday life. **Example: Reflection Rules** The reflection rules and reflection formulas are summarized in the table.	Level 4 Extended critical thinking and reasoning

Reflection	**Reflection Rule**	**In Words**	**What it looks like on the graph**
Over the x-axis	(x,y)-->(x,-y)	Negate the y coordinates	Image is directly above or below the original
Over the y-axis	(x,y)-->(-x,y)	Negate the x coordinates	Image is left or right of the original
Over y=x	(x,y)-->(y,x)	Swap the x and y coordinates	y=x passes through the plane at a 45-degree angle. Image

			is above or below this line from the original.
Origin	(x,y)-->(-x,-y)	Negate both the x and y coordinates	Image is rotated 180 degrees

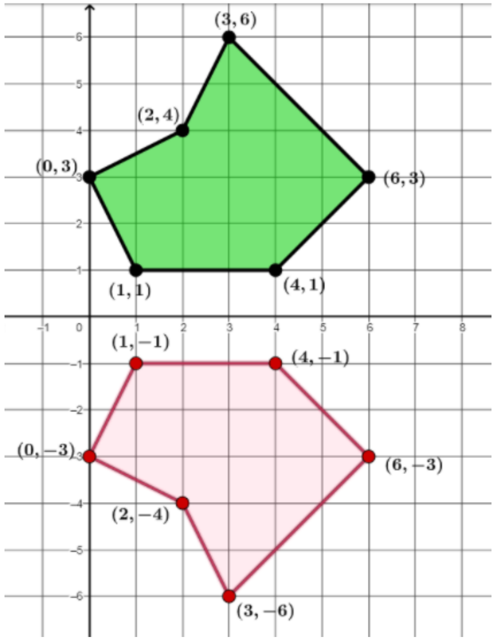
Experiential Learning: In small groups, task learners to perform reflection on a plane. Encourage learners to appreciate the importance of dwelling on their strength to embark on a given task to its successful conclusion.

E.g.,1: Reflect the polygon in Figure 1 over the x-axis.



	Solution: Only the y coordinates are affected since the reflection is over the x-axis, and this is a vertical reflection. Therefore, the y coordinates are affected. As a result, the sign on the y coordinate of each point is negated.	

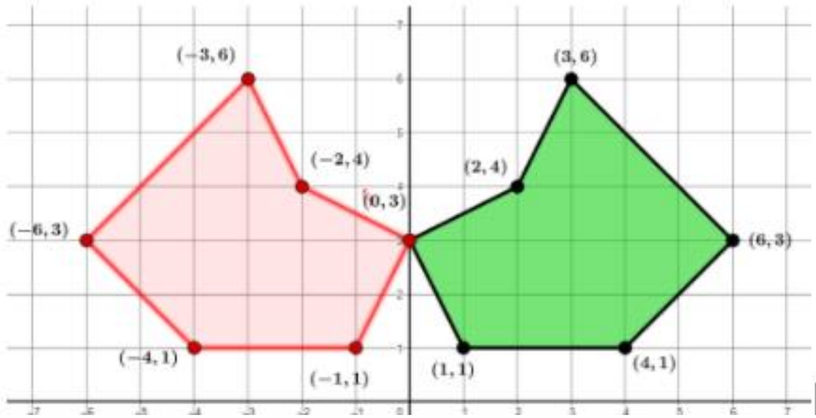
Before reflection		After reflection	
x	y	x	y
1	1	1	-1
0	3	0	-3
2	4	2	-4
3	6	3	-6
6	3	6	-3
4	1	4	-1



E.g.2: Let's reflect the polygon over the y-axis.

Solution: Only the x coordinates are affected since the reflection is over the y-axis, and this is a horizontal reflection. Therefore, the x coordinates are affected. As a result, the sign on the x coordinate of each point is negated.

Before reflection		After reflection	
x	y	x	y
1	1	-1	1
0	3	0	3
2	4	-2	4
3	6	-3	6
6	3	-6	3
4	1	-4	-1



2.3.1.LI.3

Identify shapes with rotational symmetry and show the image of an object (or point) after a rotation about the origin (or point).

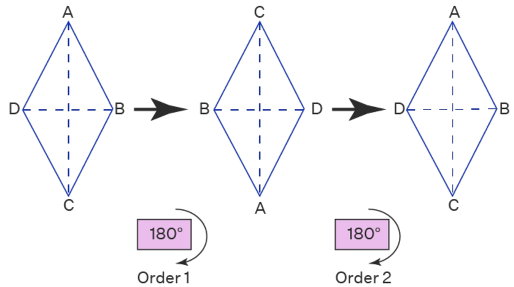
Think-pair share activities: Learners in pairs research and make presentations on the concept of rotational symmetry in Mathematics.

2.3.1.AS.3

Level 1 Recall
Level 2 Skills of conceptual understanding
Level 3 Strategic reasoning

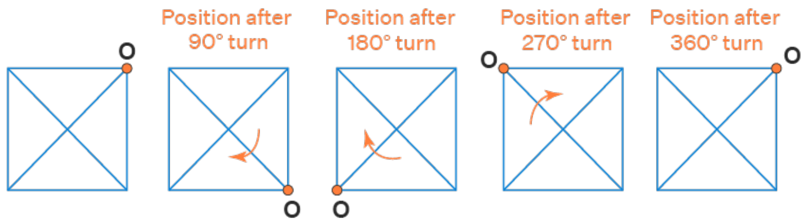
Example 1

An object is exactly similar to its original object when rotated in a particular direction. After turning a geometrical shape, the shape becomes identical to the origin, and this is known as rotational symmetry.



We observe from the images of the rhombus that it fits onto itself twice in one full rotation of 360°. Therefore, we can conclude that the order of rotational symmetry in a rhombus is 2, and the angle of rotation is 180°.

Example 2

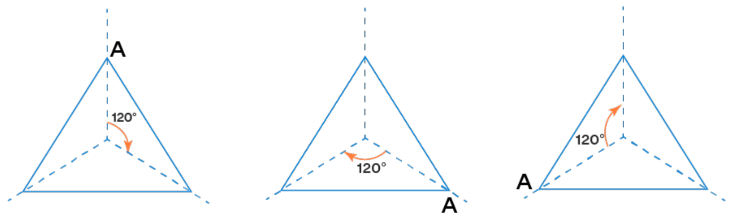


The angle of rotation is 90°. This is because from the above figure, we see that the order of rotational symmetry of a square is 4 as it fits into itself 4 times in a complete 360° rotation.

Example 3: Show the rotational symmetry of an equilateral triangle.

Solution: An equilateral triangle has 3 sides of equal measure and each internal angle measuring 60° each.

Level 4 Extended critical thinking and reasoning



The order of rotational symmetry of an equilateral triangle is 3, and its angle of rotation is 120°. This is because the equilateral triangle exactly fits into itself 3 times at every angle of 120°.

Think-pair share activities: Learners in pairs research and make presentations on the rotation rules. Encourage learners to volunteer to lead their groups and endeavour to ensure fair treatment of all group members. And, as learners talk about rules of rotation, encourage discussions on the need to obey rules and the consequences of doing otherwise in everyday life.

Example 1: Rotation Rules

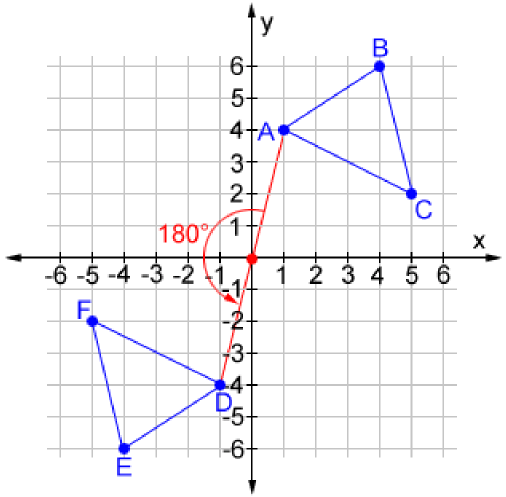
We can use the following rules to find the image after 90°, 180°, 270° clockwise and counterclockwise rotation.

Rotation	Preimage	Image
Clockwise Rotation of 90^0	(x, y)	(y, -x)
Anticlockwise Rotation of 90^0.	(x, y)	(-y, x)
Anti/Clockwise Rotation of 180^0	(x, y)	(-x, -y)
Clockwise Rotation of 270^0	(x, y)	(-y, x)
Anticlockwise Rotation of 270^0	(x, y)	(y, -x)

Example 2
a) Using a scale of 2cm to 2 units on both axes, draw two perpendicular axes, OX and OY, on a graph sheet.
b) On this graph sheet, mark the x-axis from –4 to 10 and the y-axis from –6 to 12
c) Plot on the same graph sheet the points A (1, 4), B (4, 6) and C (5,

2). Join the points to form a triangle ABC.
d) Draw the image of triangle ABC through 1800 anticlockwise rotation about the origin.
e) Label the vertices of the new triangle DEF.

Solution: A rotation of 180° (either clockwise or counterclockwise) around the origin changes the position of a point (x, y) such that it becomes (-x, -y).



Triangle ABC has vertices A (1, 4), B (4, 6) and C (5, 2). It is rotated 180° counterclockwise to land on DEF, which has vertices D (-1, -4), E (-4, -6), and F(-5, -2).

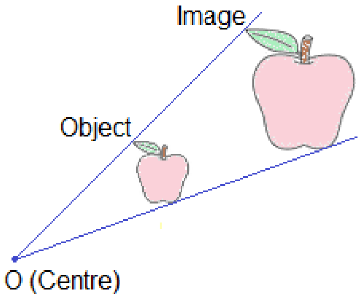
***Note**:* A clockwise rotation of 180° for triangle ABC also results in triangle DEF.

	2.3.1.LI.4	2.3.1.AS.4

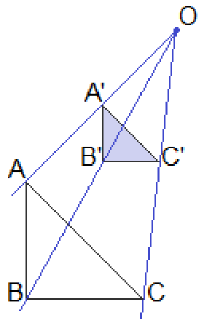
Carry out an enlargement of a plane shape given a scale factor.

Think-pair share activities: Learners in pairs research and make presentations on the concept of enlargement/reduction in transformations.

Example: In transformation, enlargement/reduction is when the size of an object is changed without changing its original shape. When the size of the object is increased, it is called an **enlargement**, and when the size of an object is decreased, it is called a **reduction**. Look at the images here.



Enlargement



Reduction

The enlargement is made with the help of a fixed point called the **centre of enlargement** and by the fixed ratio called **scale factor,** i.e., the ratio of the corresponding sides of the image and object.

Think-pair-share activities: Learners in pairs discuss the properties of enlargement/reduction. And, as learners talk about properties and rules of enlargement/reduction, encourage discussions on the need to obey rules and the consequences of doing otherwise in everyday life.

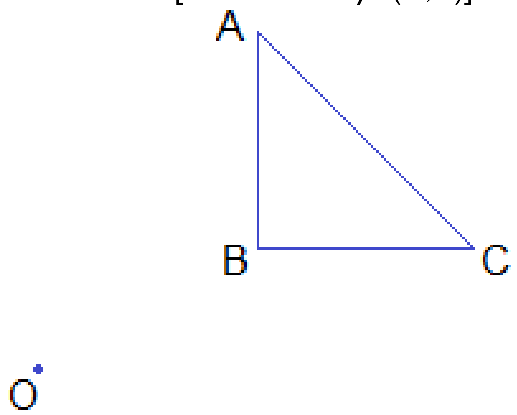
Example 1: Properties of enlargement/reduction
1. The object and the image under the enlargement are similar.
2. Scale factor (k) =

$$\frac{length\ of\ the\ side\ of\ the\ image\ figure}{lenght\ of\ the\ corresponding\ side\ of\ the\ object\ figure}$$

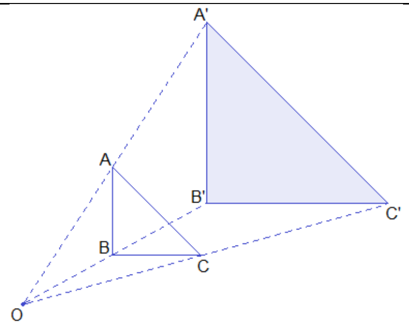
Level 1 Recall
Level 2 Skills of conceptual understanding
Level 3 Strategic reasoning
Level 4 Extended critical thinking and reasoning

3. If the scale factor $k>1$, then the transformation is called enlargement.
4. If the scale factor $0<k<1$, then the transformation is called reduction.
5. If the scale factor $k=1$, then the transformation is identity.
6. If the scale factor $k<0$, then the image will be on the opposite side of the object from the centre of enlargement.

Example 2: Find the image of ΔABC under the enlargement with the centre of enlargement O and scale factor 2. [It is denoted by E(O, 2)]



Solution: For the image ΔABC under the given enlargement, perform the following steps:
Step 1: Join OA, OB and OC.
Step 2: Produce OA up to A' such that OA' = 2OA.
Step 3: Produce OB up to B' such that OB' = 2OB.
Step 4: Produce OC up to C' such that OC' = 2OC.
Step 5: Join A'B', B'C' and C'A'.

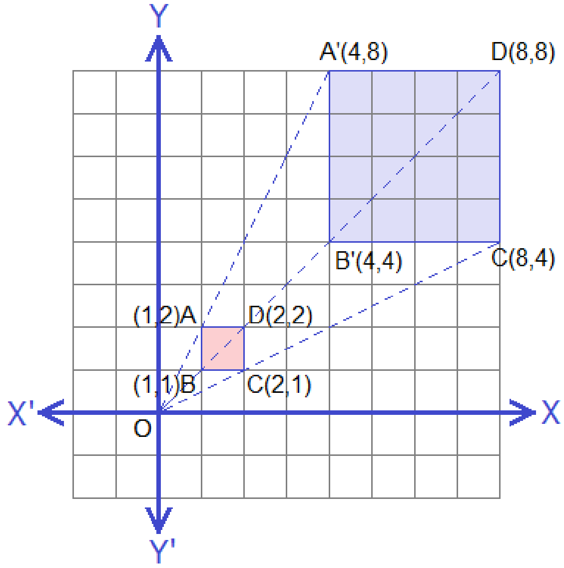


Hence, ΔA’B’C’ is the required image of ΔABC under the enlargement E(O, 2).

Example 3: Find the coordinates of the vertices of the image square ABCD and draw ABCD and its image on the same graph paper if a shape ABCD with vertices A(1, 2), B(1, 1), C(2, 1) and D(2, 2) is enlarged with the centre of enlargement O(0, 0) and scale factor 4.

Solution: First, obtain the coordinates for the image as:

Preimage	Image
A(1,2)	A^{I} (4, 8)
B(1,1)	B^{I} (4, 4)
C(2,1)	C^{I} (8, 4)
D(2,2)	D^{I} (8, 8)

			
Teaching and Learning Resources	• Mathematical sets. Graph sheet. • Technology tools such as computers, mobile phones, etc.	• Computer software applications like GeoGebra. • Compass	• clock face, etc.

Subject **MATHEMATICS**

Strand **3. GEOMETRY AROUND US**

Sub-Strand **2. MEASUREMENT**

Learning Outcomes	**21st-century Skills and Competencies**	**GESI, SEL and Shared National Values**
2.3.2.LO.1		
Carry out addition, subtraction and scalar multiplication of vectors and investigate with and without technology some properties (e.g., commutative, associative, and distributive properties) of the operations.	**Communication and Collaboration:** Learners communicate confidently and effectively to develop appropriate mathematics vocabulary for the concept of vectors. **Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on vectors using appropriate IT tools to boost their interest and desire to solve more problems on their own. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on vectors.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: Respect individuals of different backgrounds in their mathematics groups and beyond. · Embrace diversity and practise inclusion in the mathematics classroom and beyond. · Examine and dispel misconceptions/ myths about gender as they relate to the learning of mathematics, home management and human development. · Interrogate their stereotypes and biases about gender and the role members in a group play in the mathematics classroom and in home management. · Identify injustice, especially in recognition of the contributions of different groups and individuals to the effective management and maintenance of the mathematics classroom and home. · Sensitive to the inter-relatedness of the various aspects of life even as they engage with others in the mathematics classroom and beyond.

		· Value and promote justice in the mathematics classroom, at home and in society. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the mathematics curriculum, the facilitator should apply Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and finding confidence · Exhibiting motivation and SMART goal-setting · Managing emotions and conflicts · Showing empathy and cooperation These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; and respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning vectors.

		Diversity: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination among learners. **National Core Values**: Develop tolerance, friendliness, open-mindedness, patience, hard work, and humility in learners as they interact with their peers in the mathematics classroom. **Truth and Integrity:** Reward truth and honesty as strong moral principles in the learning of mathematics, leading to responsible citizenship. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment. **Discipline and honesty:** Encourage learners to behave and work in a controlled way, which involves obeying mathematical rules, principles and standards, leading to self-directed learning.
2.3.2.LO.2		
Determine the inverse of trigonometric ratios, calculate angles of elevation and depression in everyday life situations and apply the knowledge to calculate distances and heights.	**Communication and Collaboration:** Learners communicate confidently and effectively to develop appropriate mathematics vocabulary for the concept of trigonometry. **Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to:

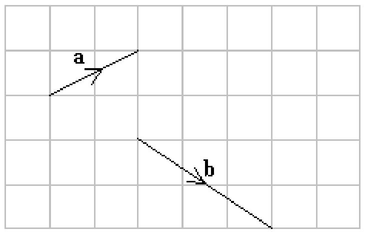
	trigonometry using appropriate IT tools to boost their interest and desire to solve more problems on their own. **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to the application of the concept of trigonometry to lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on their values, perceptions and actions for decision-making as they engage in group and individual activities on trigonometry. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on trigonometry. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concept of trigonometry to lifelong learning.	· Respect individuals of different backgrounds in their mathematics groups and beyond. · Embrace diversity and practise inclusion in the mathematics classroom and beyond. · Examine and dispel misconceptions/ myths about gender as they relate to the learning of mathematics, home management and human development. · Interrogate their stereotypes and biases about gender and the role members in a group play in the mathematics classroom and in home management. · Identify injustice, especially in recognition of the contributions of different groups and individuals to the effective management and maintenance of the mathematics classroom and home. · Sensitive to the inter-relatedness of the various aspects of life even as they engage with others in the mathematics classroom and beyond. · Value and promote justice in the mathematics classroom, at home and in society. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the mathematics curriculum, the facilitator should apply the Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and finding confidence · Exhibiting motivation and SMART goal-setting · Managing emotions and conflicts · Showing empathy and cooperation

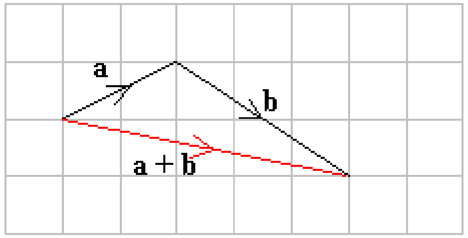
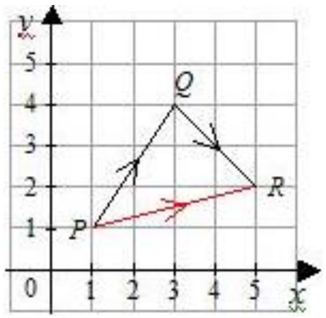
		These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; and respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning vectors. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination among learners. **National Core Values**: Develop tolerance, friendliness, open-mindedness, patience, hard work, and humility in learners as they interact with their peers in the mathematics classroom. **Truth and Integrity:** Reward truth and honesty as strong moral principles in the learning of mathematics, leading to responsible citizenship. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning

		through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment. **Discipline and honesty:** Encourage learners to behave and work in a controlled way, which involves obeying mathematical rules, principles and standards, leading to self-directed learning.
2.3.2.LO.3		
Determine the volume and capacity of solid shapes and Solve problems that involve SI and imperial units in surface area, volume and capacity measurements.	**Communication and Collaboration:** Learners communicate confidently and effectively to develop appropriate mathematics vocabulary for the concept of volume and capacity of solid shapes. **Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on volume and capacity of solid shapes using appropriate IT tools to boost their interest and desire to solve more problems on their own. **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to the application of the concept of volume and capacity of solid shapes to lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on their own values, perceptions and actions for decision-making as they engage in group and individual activities on volume and capacity of solid shapes. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: • Respect individuals of different backgrounds in their mathematics groups and beyond. • Embrace diversity and practise inclusion in the mathematics classroom and beyond. • Examine and dispel misconceptions/ myths about gender as they relate to the learning of mathematics, home management and human development. • Interrogate their stereotypes and biases about gender and the role members in a group play in the mathematics classroom and in home management. • Identify injustice, especially in recognition of the contributions of different groups and individuals to the effective management and maintenance of the mathematics classroom and home. • Sensitive to the inter-relatedness of the various aspects of life even as they engage with others in the mathematics classroom and beyond.

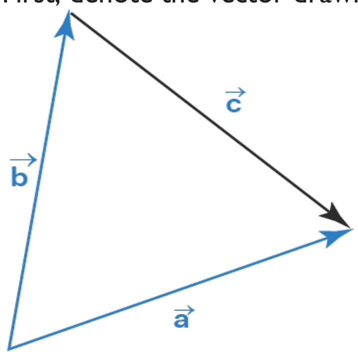
	develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on the volume and capacity of solid shapes. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concept of volume and capacity of solid shapes to lifelong learning.	· Value and promote justice in the mathematics classroom, at home and in society. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the mathematics curriculum, the facilitator should apply the Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and finding confidence · Exhibiting motivation and SMART goal-setting · Managing emotions and conflicts · Showing empathy and cooperation These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning the concepts of volume and capacity of solid shapes.

		Diversity: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination among learners. **National Core Values**: Develop tolerance, friendliness, open-mindedness, patience, hard work, and humility in learners as they interact with their peers in the mathematics classroom. **Truth and Integrity:** Reward truth and honesty as strong moral principles in the learning of mathematics, leading to responsible citizenship. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment. **Discipline and honesty:** Encourage learners to behave and work in a controlled way, which involves obeying mathematical rules, principles and standards, leading to self-directed learning.

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
2.3.2.CS.1	2.3.2.LI.1	2.3.2.AS.1
Demonstrate knowledge and understanding of measurement with respect to operations on bearings and vectors.	**Perform addition, subtraction, and scalar multiplication on vectors represented as directed line segments in two-space and in Cartesian form in two and three-space.** **Using Talk for Learning strategy,** review learners' previous knowledge of vectors and their types. **Group discussions:** In small groups, task learners to discuss the triangular law of addition. As learners talk about laws, encourage discussions on the need to obey laws and the consequences of doing otherwise in everyday life. **Example:** Vectors can be added using the 'nose-to-tail' method or "head-to-tail" method. Two vectors, a and b, represented by the line segments, can be added by joining the 'tail' of vector b to the 'nose' of vector a. Alternatively, the 'tail' of vector a can be joined to the 'nose' of vector b. **E.g.1**: Find the sum of the two given vectors, a and b. 	**Level 1 Recall** **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

	Solution: Draw the vector a. Draw the 'tail' of vector b joined to the 'nose' of vector a. The vector a + b is from the 'tail' of a to the 'nose' of b.  **E.g.2:** Given that, $\overrightarrow{PQ} = \binom{2}{3}$ *and* $\overrightarrow{QR} = \binom{2}{-2}$, find the sum of the vectors.  The sum of the vectors $\overrightarrow{PQ}$ *and* $\overrightarrow{QR}$ is the same as the vector $\overrightarrow{PR}$. That is $\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$. Therefore, we add the corresponding components of the vectors as $\overrightarrow{PQ} + \overrightarrow{QR} = \binom{2}{3} + \binom{2}{-2} = \binom{4}{1} = \overrightarrow{PR}$ **Group discussions:** In small groups, task learners to discuss how to use the triangular law of addition to doing subtraction of vectors. As learners discuss the triangular law, they come up with a discussion on the need for one to manage emotions and conflicts in order not to be on the wrong side of the law. **Example**: The vector subtraction of two vectors a and b is represented by a - b, and it is nothing but adding the negative of vector b to the vector a., i.e., a - b = a + (-b). Thus, subtraction of vectors involves the addition of vectors and the negative of a vector. **Interpreting the subtraction of vectors using the triangle law of vector addition.**	

First, denote the vector drawn from the endpoint of b to the endpoint of a by c.



$$\vec{b} + \vec{c} = \vec{a}$$
(or)
$$\vec{c} = \vec{a} - \vec{b}$$

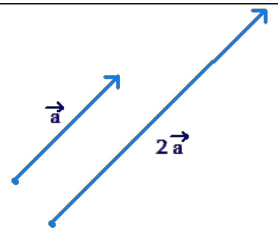
Note that b + c = a. Thus, c = a - b. In other words, the vector a - b is the vector drawn from the tip of b to the tip of a (if a and b are co-initial).

Group discussions: In small groups, task learners to discuss scalar multiplication on vectors.

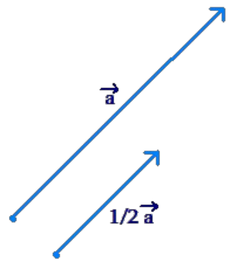
Example

- Multiplying a vector by a positive scalar (real number) preserves its direction and scales its length by the magnitude of the scalar.
- Multiplying a vector by a negative scalar reverses its direction and scales its length by the magnitude of the scalar.
- If the magnitude of the scalar is greater than 1, then the new vector is longer than the original vector; if it is less than 1, then the new vector is shorter; if the scalar is equal to 1, the new vector has the same length as the original.

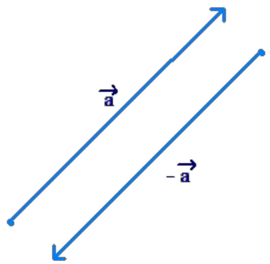
E.g., Consider a vector. $\vec{a}$. What happens if you multiply this vector by 2? What will the vector $2\vec{a}$ represent?



The vector $\frac{1}{2}\vec{a}$ will be a vector in the same direction as $\vec{a}$, but with a length equal to half of the length of $\vec{a}$:



We have seen how to interpret the vector $-\vec{a}$Given the vector $\vec{a}$:



Group discussions: In groups, task learners to solve some addition, subtraction and scalar multiplication of vectors in component form.

2.3.2.LI.2

Determine the properties (commutative, associative, distributive, etc.) of the operations on vectors through investigation with and without technology.

Group discussions: In small groups or pairs, learners discuss and solve examples of the various properties of vector addition. Initiate discussions about the need for each and everyone to examine and dispel misconceptions/myths about gender as they relate to the learning of mathematics.

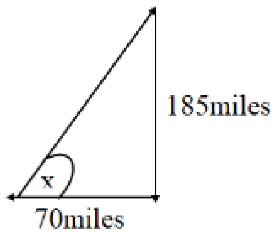
Example:

Property of Vector Addition	Explanation
Existence of identity	For any vector **v**, **v** + **0** = **v** Here, the **0** vector is the additive identity.
Existence of inverse	For any vector **v**, **v** + - **v** = **0** and thus, an additive inverse exists for every vector.
Commutativity	Addition is commutative; for any two arbitrary vectors **c** and **d**, **c** + **d** = **d** + **c**
Associativity	Addition is associative; for any three arbitrary vectors **i**, **j**, and **k**, **i** + **j** + **k** = **i** + **j** + **k** i.e., the order of addition does not matter.

Group discussions: In small groups or pairs, learners discuss and solve examples of the various properties of vector subtraction. Encourage learners to stay away from unwanted discrimination among themselves.

2.3.2.AS.2

Level 1 Recall
Level 2 Skills of conceptual understanding
Level 3 Strategic reasoning
Level 4 Extended critical thinking and reasoning

	Example: Properties of Vector Subtraction Here are some important properties of vector subtraction. • Any vector subtracted from itself results in a zero vector. i.e., **a** - **a** = **0**, for any vector **a**. • The subtraction of vectors is NOT commutative. i.e., **a** - **b** is not necessarily equal to **b** - **a.** • The vector subtraction is NOT associative. i.e., (**a** - **b**) - **c** does not need to be equal to **a** - (**b** - **c**). • $(\mathbf{a} - \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b}) = \lvert\mathbf{a}\rvert^2 - \lvert\mathbf{b}\rvert^2$. • $(\mathbf{a} - \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}) = \lvert\mathbf{a} - \mathbf{b}\rvert^2 = \lvert\mathbf{a}\rvert^2 + \lvert\mathbf{b}\rvert^2 - 2\,\mathbf{a} \cdot \mathbf{b}$.	
	2.3.2.LI.3	2.3.2.AS.3
	Solve problems involving the addition, subtraction, and scalar multiplication of vectors, including problems arising from real-world applications. **Think-pair-share activities:** In pairs, learners solve real-world problems on vectors. **Example 1**: The Antarctic expedition drives their snowmobiles 185 miles south from their camp. Then, they turn and drive 70 miles west. Now it is time to return to camp. In what direction must they drive, and will they make it on 190 miles worth of fuel? **Solution**: Vector Diagram **Note** that this is a special case where the resultant vector appears to point backwards. • This is because that is the path they will need to take back to their base. 	**Level 1 Recall** **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

	$c = \sqrt{185^2 + 70^2}$ $\sqrt{34225 + 4900}$ $\sqrt{39125}$ 198 Angle calculation $\tan^{-1}\frac{185}{70} = 2.64 = 69.30$ **Final answer** 20.8° and, no, they cannot make it on that quantity of fuel, so they will have to walk an additional 8 miles. **Example 2**: Michael is running some errands. • His first stop is 6 km to the east and 3 km to the south from his house. • His second stop is 2 km to the west and 1 km to the south from the first stop. • His third stop is 7 km to the west and 5 km to the north from the second stop. • What is Michael's direction relative to his starting point once he arrives at his third stop?	
Teaching and Learning Resources	• Mathematical sets. • Technology tools such as computers, mobile phones, etc.	• Computer software applications like

<table>
<tr><th>Content Standards</th><th>Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI</th><th>Assessment</th></tr>
<tr><td>2.3.2.CS.2</td><td>2.3.2.LI.1</td><td>2.3.2.AS.1</td></tr>
<tr><td>Demonstrate an understanding of the inverse of trigonometric ratios and angles of elevation/depression, and apply the knowledge to calculate distances and heights.</td><td>

Determine the inverse of trigonometric ratios (graphs excluded) and talk about their applications in the fields of astronomy, engineering, physics, geometry and navigation.

Group discussions: In convenient groups, task learners to discuss the inverse of trig. ratios. Learners should respect diverse views from friends in groups or class.

Example 1: Inverse Trigonometric Ratios Table

- $\sin^{-1}$ (Opposite)/(Hypotenuse) = θ
- $\cos^{-1}$ (Base)/(Hypotenuse) = θ
- $\tan^{-1}$ (Opposite)/(Base) = θ
- Cosec^{-1} (Hypotenuse)/(Opposite) = θ
- Sec^{-1} (Hypotenuse)/(Base) = θ
- Cot^{-1} (Base)/(Opposite) = θ

Example 2: The following table lists some examples of the sin−1 operation

<table>
<tr><th>Trigonometric Ratios</th><th>Inverse Trigonometric Ratios</th></tr>
<tr><td>$\sin 0 = 0$</td><td>$\sin^{-1} 0 = 0$</td></tr>
<tr><td>$\sin(\pi/6) = 1/2$</td><td>$\sin^{-1}(1/2) = \pi/6$</td></tr>
<tr><td>$\sin(\pi/4) = 1/\sqrt{2}$</td><td>$\sin^{-1}(1/\sqrt{2}) = \pi/4$</td></tr>
<tr><td>$\sin(\pi/3) = \sqrt{3}/2$</td><td>$\sin^{-1}(\sqrt{3}/2) = \pi/3$</td></tr>
<tr><td>$\sin(\pi/2) = 1$</td><td>$\sin^{-1} 1 = \pi/2$</td></tr>
</table>

</td><td>

Level 1 Recall
Level 2 Skills of conceptual understanding
Level 3 Strategic reasoning
Level 4 Extended critical thinking and reasoning

</td></tr>
</table>

Example 3: Here are some examples of the $\cos^{-1}$ operation:

Trigonometric Ratios	**Inverse Trigonometric Ratios**
$\cos 0 = 1$	$\cos^{-1} 1 = 0$
$\cos(\pi/6) = \sqrt{3}/2$	$\cos^{-1}(\sqrt{3}/2) = \pi/6$
$\cos(\pi/4) = 1/\sqrt{2}$	$\cos^{-1}(1/\sqrt{2}) = \pi/4$
$\cos(\pi/3)=1/2$	$\cos^{-1}(1/2) = \pi/3$
$\cos(\pi/2) = 0$	$\cos^{-1} 0 = \pi/2$

Example 4: Here are some examples of the tan−1 operation:

Trigonometric Ratios	**Inverse Trigonometric Ratios**
$\tan 0 = 0$	$\tan-1\ 0 = 0$
$\tan(\pi/6) = 1/\sqrt{3}$	$\tan-1(1/\sqrt{3}) = \pi/6$
$\tan(\pi/4) = 1$	$\tan-1(1) = \pi/4$
$\tan(\pi/3) = \sqrt{3}$	$\tan-1(\sqrt{3}) = \pi/3$

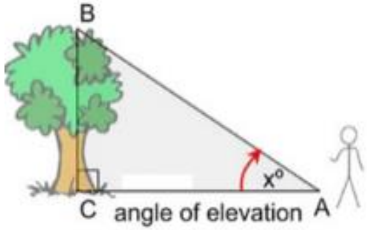
Example 5: Applications of Inverse trigonometric ratios are given below:

- Used to find the measure of the unknown angles of a right-angled triangle.
- Used in measuring the angle of depth or angle of inclination.
- Architects use it to calculate the angle of a bridge and the supports.
- Used by carpenters to create a desired cut angle.

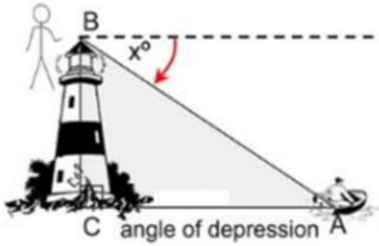
Example 6: Find the value of $\sin^{-1}(1/2) + \cos^{-1}(1/2)$ using the inverse trigonometric ratio formulas.
Solution: We have:
$\sin^{-1}(1/2) + \cos^{-1}(1/2) = \pi/6+\pi/3 = \pi/2$

Answer: The value of the given function is **$\pi/2$**.

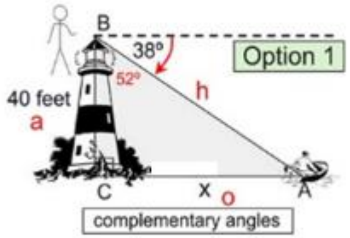
	Example 7: Find the value of $\text{Tan}^{-1}(\sqrt{3}) - \text{Cot}^{-1}(-\sqrt{3})$. **Solution**: $\text{Tan}^{-1}(\sqrt{3}) - \text{Cot}^{-1}(-\sqrt{3})$ $=\text{Tan}^{-1}(\sqrt{3}) - (\pi - \text{Cot}^{-1}(\sqrt{3}))$ $=\text{Tan}^{-1}(\sqrt{3}) - \pi + \text{Cot}^{-1}(\sqrt{3})$ $= \pi/3 - \pi + \pi/6$ $= \pi/2 - \pi$ $= -\pi/2$ **Answer:** Therefore, the answer is $-\pi/2$. **Example 8:** Find the value of $\text{Sin}\left(\frac{\pi}{3}+\text{Sin}^{-1}\left(\frac{1}{2}\right)\right)$. **Answer:** 1	
	2.3.2.LI.2	2.3.2.AS.2
	Solve real-life problems involving angles of elevation and depression and identify everyday life situations of these concepts. **Group discussions**: In convenient groups, task learners to discuss angles of elevation and depression. In their group task, learners to show respect for individual views, beliefs, religions, and cultures. **Example 1** **Angle of Elevation:**	Level 1 Recall **Level 2 Skills of conceptual understanding** Level 3 Strategic reasoning Level 4 Extended critical thinking and reasoning

 In this diagram, $x°$ marks the angle of elevation of the top of the tree as seen from a point on the ground.	The angle of elevation is always measured from the *ground up*. It is an upward angle from a horizontal line. It is always **inside** the triangle. You can think of the angle of elevation in relation to the movement of your eyes. You are looking straight ahead, and you must raise (*elevate*) your eyes to see the top of a tree. When trying to remember the meaning of an angle of elevation, think of an elevator that only goes up!

Example 2

Angle of Depression:	
 In this diagram, $x°$ marks the angle of depression of the boat at sea from the top of the lighthouse.	The angle of depression is always **OUTSIDE** the triangle. It is never inside the triangle. It is a downward angle from a horizontal line. You can think of the angle of depression in relation to the movement of your eyes. You are standing at the top of the lighthouse, and you are looking straight ahead. You must lower (*depress*) your eyes to see the boat in the water.

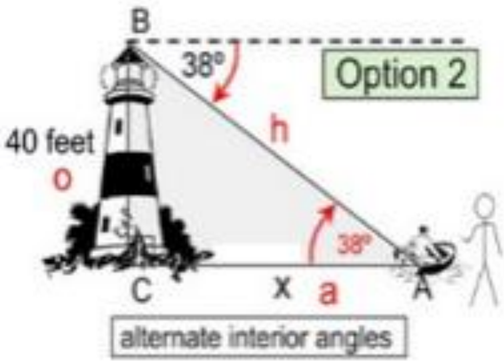
Other examples



When solving a problem with an angle of depression, you need to find the measure of an angle **INSIDE** the triangle. There are two options.

Option 1: Find the angle **inside** the triangle that is adjacent (next door) to the angle of depression. This adjacent angle will always be the complement of the angle of depression since the horizontal line and the vertical line are perpendicular (90°). In the diagram at the left, the adjacent angle is 52°.

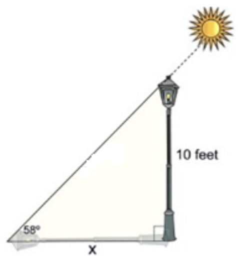
$$\tan 52 = \frac{oppossite}{adjacent} = \frac{x}{40}; 1.2799 = \frac{x}{40}; x = 51ft.$$

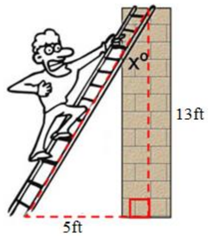


Option 2: Utilize the fact that the angle of depression = the angle of elevation and label $\angle BAC$ as 38° **inside** the triangle.

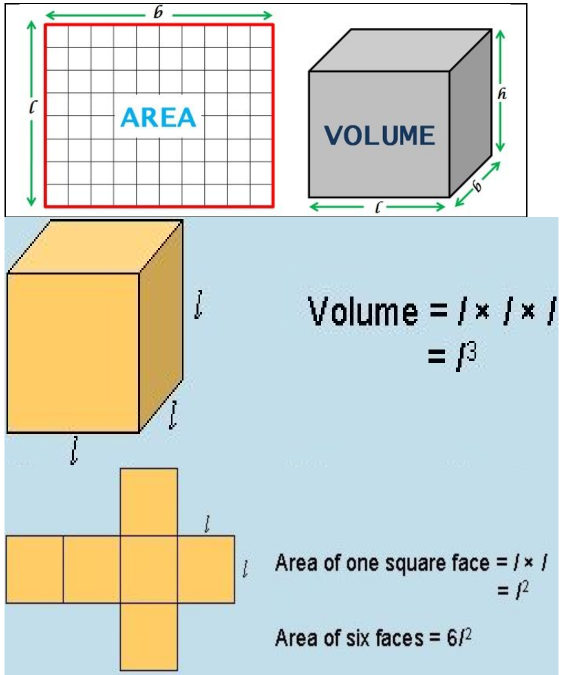
$$\tan 38 = \frac{opposite}{adjacent} = \frac{40}{x}; 0.7811286 = \frac{40}{x}; x = 51ft.$$

Notice that in both options, the answer is the same.

<table>
<tr>
<td>Find the shadow cast by a 10-foot lamp post when the angle of elevation of the sun is 58°. Find the length to the nearest tenth of a foot.

</td>
<td>Solution:
• Remember that the "angle of elevation" is from the horizontal ground line upward.
• It is assumed that the lamp post is vertical, making it perpendicular to the ground.
• Shadows are on the ground! If you place the "shadow" on the hypotenuse, you have created an apparition (a "ghost"), not a shadow!
• This solution deals with "opposite" and "adjacent", making it a tangent problem.
$$\tan 58 = \frac{10}{x}; 1.6003 = \frac{10}{x}; x = 6.2$$</td>
</tr>
</table>

<table>
<tr>
<td>A ladder leans against a brick wall. The foot of the ladder is 6 feet from the wall. The ladder reaches a height of 15 feet on the wall. Find to the nearest degree, the angle the ladder makes with the wall.

</td>
<td>Solution:
• In this problem, place x° where the ladder meets the wall. Do not assume that the angle will always be at the ground level.
• It is assumed that the wall is vertical, perpendicular to the ground.
• The foot of the ladder is the bottom of the ladder, where it hits the ground.
• This solution deals with "opposite" and "adjacent", making it a tangent problem.
$\tan x = \frac{6}{15} = 0.4;\ \tan^{-1}(0.4) = 22^0$</td>
</tr>
</table>

Teaching and Learning Resources	· Mathematical sets. · Technology tools such as computers, mobile phones, etc. · Computer software applications like GeoGebra

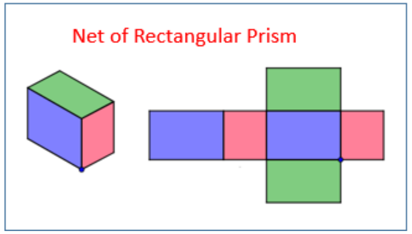
Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
2.3.2.CS.3	2.3.2.LI.1	2.3.2.AS.1
Demonstrate conceptual understanding of the measurement of surface area, volume and capacity of solid shapes.	**Solve problems that involve SI and imperial units in surface area measurements and verify the solutions.** **Experiential Learning:** In convenient groups, explain, showing practical examples, the difference between volume and surface area. Reward honesty as a strong moral principle as learners discuss their challenges solving problems on surface area measurements. **Example**: The area or region that an object's surface occupies is known as its surface area. Volume, on the other hand, refers to how much room an object has. Using examples, including nets, show the relationship between area and surface area.  	**Level 1 Recall** **Level 2 Skills of conceptual understanding** Level 3 Strategic reasoning Level 4 Extended critical thinking and reasoning

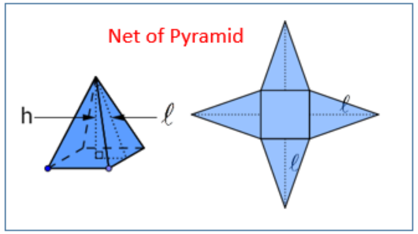
Experiential Learning: Explain how a referent can be used to estimate surface area.

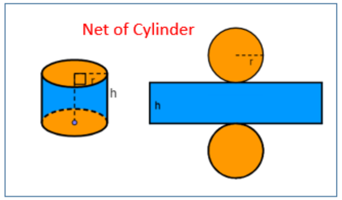
Example: Engage learners to determine the area of, say a rectangular box and use the knowledge of nets to estimate the surface area of the box using the area calculated as a reference point.

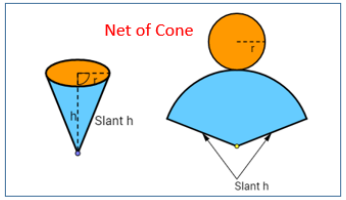
Experiential Learning: In pairs, learners draw nets of 3D objects (prisms, cones, pyramids, spheres, etc.), including the use of computer programmes like GeoGebra.

Example: Nets of 3D objects

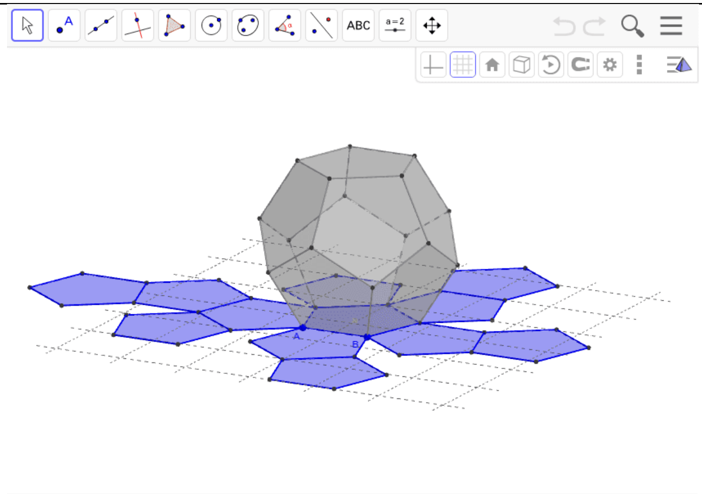
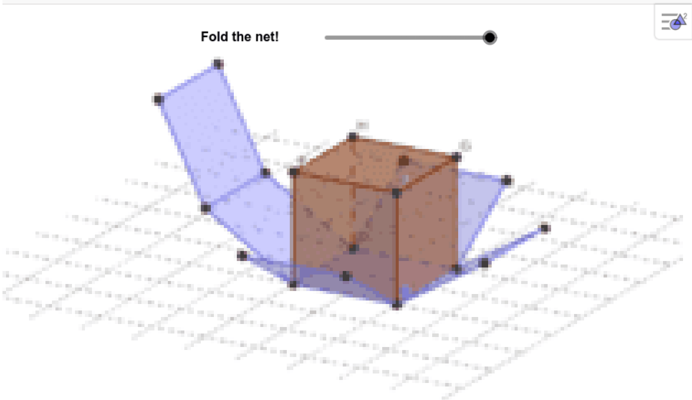








Example: Nets of 3D objects using GeoGebra

	  **Think-pair share activities**: In pairs, task learners to estimate the surface area of a 3D object (prisms, cones, pyramids, spheres, etc.). Encourage learners to see the need to embrace technology in solving problems in surface area measurements while appreciating the need to use them appropriately. Learners draw on their experiences in nets, area, perimeter, etc., to discuss how the various formulae can be generated.	

3D Shape	Total Surface Area (TSA)	Lateral Surface Area (LSA)/Curved Surface Area
Cube	$6a^2$	$4a^2$, where a is the length of each side
Cuboid	2 (lw + wh + lh)	2h (l + w), where l, w, and h are the length, width, and height of the cuboid
Cone	$\pi r(r + l)$	πrl, where r is the radius, and l is the slant height of the cone
Cylinder	$2\pi r(r + h)$	$2\pi rh$, where r is the radius, and h is the height of the cylinder
Sphere	$4\pi r^2$, where r is the radius of the sphere	Not applicable

Shape	Base	Surface Area of Prism = (2 × Base Area) + (Base perimeter × height)
Triangular Prism	**Triangle**	Surface area of triangular prism = bh + (s1 + s2 + b)H
Square Prism	**Square**	Surface area of square prism = 2a2 + 4ah
Rectangular Prism	**Rectangle**	Surface area of rectangular prism = 2(lw + wh + lh)

Trapezoidal Prism	**Trapezoid**	Surface area of trapezoidal prism = h (b + d) + l (a + b + c + d)
Pentagonal Prism	**Pentagon**	Surface area of pentagonal prism = 5ab + 5bh
Hexagonal Prism	**Hexagon**	Surface area of hexagonal prism = 6ah + 3√3a2
Octagonal Prism	**Octagon**	Surface area of octagonal prism = 4a2 (1 + √2) + 8aH

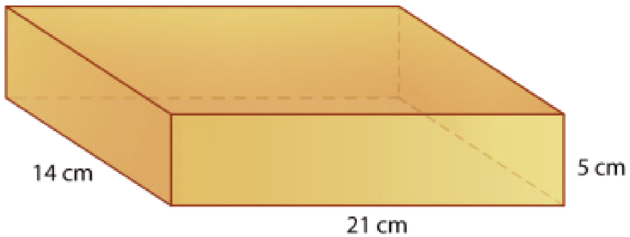
Example 1: Find the total surface area of a cylinder if its radius is 3.5 units and height is 6 units.

Solution: We know that the formula to find the total surface area of a cylinder = $2\pi r(r + h)$
$= 2 \times 22/7 \times 3.5 \times (3.5 + 6)$
$= 2 \times 22/7 \times 3.5 \times (9.5)$
= 209 unit^2
Therefore, the total surface area of the cylinder is 209 unit^2

Example 2: If the radius and slant height of an ice cream cone are 4 inches and 7 inches, respectively. What is its surface area?

Solution: Given: radius = 4 inches and slant height = 7 inches.
The surface area of cone = $\pi r(r + l)$
$= \pi \times 4(4 + 7)$
$= 3.14 \times 4 \times 11$
= 138.16 inches^2
∴ The surface area of the cone is 138.16 inches^2.

Example 3: The total surface area considers all the faces of the 3D shape, including the flat surfaces and the curved surfaces. Why?

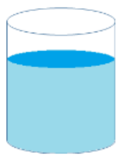
	Experiential Learning: In pairs, task learners to illustrate, using examples, the effect of dimensional changes on the surface area. Promote divergent views to ensure inclusivity in the learning environment. **Example**: The edge length of a small cube is increased by a scale factor of 5 to form a larger cube. How many times greater is the surface area of the large cube? x 5x 4) Hint: Plug in a number for x and find the surface area of both cubes. **Collaborative learning:** Learners discuss and solve contextual problems that involve the surface area of 3-D objects, including spheres, and that require the manipulation of formulas. **Examples**: i. If you decide to paint a building, you need to know the surface area of the building in order to buy the correct amount of paint. ii. The gift is in a box that has a length of 9 inches, a width of 12 inches and a height of 4 inches. Covering the box requires us to know the surface area. Now, using the surface area formula for a rectangular prism, we can determine. SA = 2lw + 2wh + 2lh SA = 2(9×12)+2(12×4)+2(9×4) SA = 216 + 96 + 72 SA = 384 cubic inches. iii. What is the surface area of the figure below?  All of the faces of this prism are rectangles, so you can use the formula for finding the surface area of a rectangular prism as follows. First, plug the values given above into the surface area formula and multiply the values together within each of the parentheses:	

<table>
<tr><td></td><td>SA = 2lw + 2wh + 2lh

Work out the answer: The answer is that the rectangular prism has a surface area of 938 square centimetres.</td><td></td></tr>
<tr><td></td><td>2.3.2.LI.2</td><td>2.3.2.AS.2</td></tr>
<tr><td></td><td>Solve problems that involve SI and imperial units in volume and capacity measurements.

Using Talk for Learning strategy, engage learners to explain, using examples, the difference between volume and capacity.
Bring learners’ attention to the fact that as differences exist between volume and capacity, which are two close concepts, it is the same among humans and, therefore, the need to appreciate these differences and respect each and everyone irrespective of their background.

Examples:
i. The total amount of any substance which is contained in a particular space is called the volume. The total potential amount of any substance that can be contained in a space is called the capacity of that space. Thus, the amount of space that a substance takes up is known as volume. The maximum amount of a substance that an object can contain is known as its capacity.

<table><tr><td></td><td>The container here holds some amount of liquid. The total amount of liquid in the container is the volume of the liquid, but the capacity of the container is the potential amount of liquid that can be contained in the entire space of the container.</td></tr></table></td><td>Level 1 Recall
Level 2 Skills of conceptual understanding
Level 3 Strategic reasoning
Level 4 Extended critical thinking and reasoning</td></tr>
</table>

ii.

Volume	Capacity
Volume indicates the total amount of space covered by an object in three-dimensional space.	Capacity refers to the ability of something (like a solid substance, gas or liquid) to hold, absorb or receive by an object.
Common units (units of measurement) – cm^3, m^3	Common units (units of measurement)- litre, gallons, pounds
Both solid and hollow objects have volume.	Only hollow objects have the capacity.
Example – Cube, Cuboid, Cone and Cylinder	Example – Cone, Cylinder, hollow hemisphere

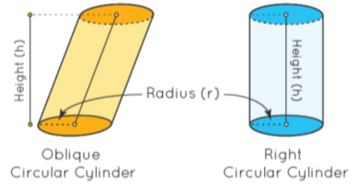
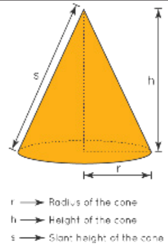
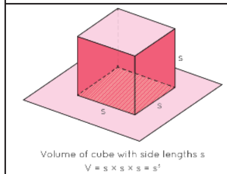
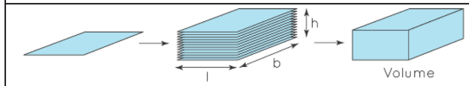
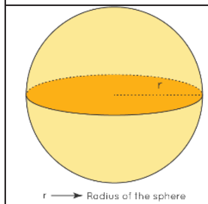
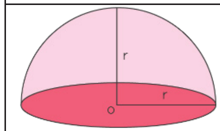
iii. Find the volume and capacity (in litres) of a cylinder whose radius is 7 cm and height is 20 cm.

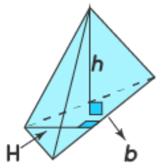
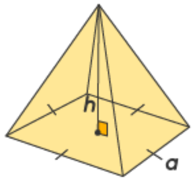
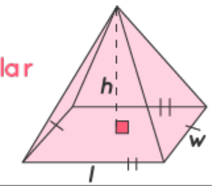
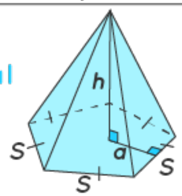
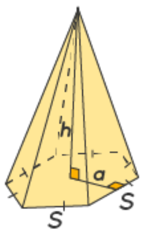
Solution: Given, r = 7 cm and h = 20 cm
We know that,
Volume of cylinder = $\pi r^2 h$
$= (22/7) \times 7 \times 7 \times 20$
$= 22 \times 7 \times 20$
$= 3080\ cm^3$
Capacity of the cylinder = 3080/1000 litres ($1000\ cm^3 = 1$ litre)
= 3.08 litres

Note: We can also represent capacity in terms of cm^3. Hence, in the above example, volume will be numerically equal to capacity.

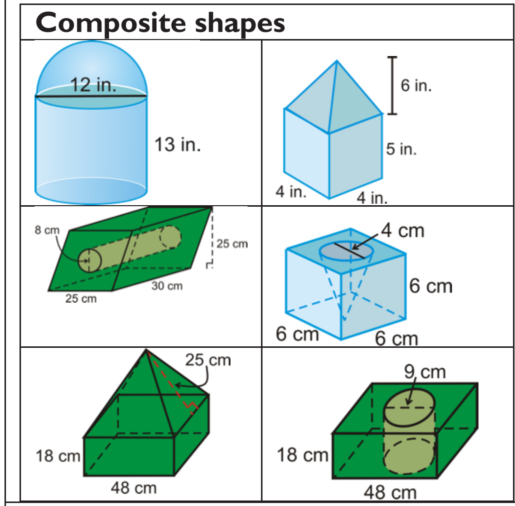
Experiential Learning: **In mixed-gender groups,** task learners to identify and compare referents, then estimate volume and capacity measurements in SI and imperial units.
Encourage learners to see the need to embrace technology in solving problems in surface area measurements while appreciating the need to use them appropriately.

	Examples: Practical Activities for Learners · How will you help Adzo use a 1-L milk carton to estimate 750 ml of water? · Estimate the number of cubic metres in the classroom. Explain how you determined this estimate. Find the actual measurement. Use this information to help you estimate the volume of another differently sized room, such as the library. · I need a box with a volume of 4000 cubic centimetres to hold a gift I have purchased. Describe what the box might look like. What is an example of a gift that would fit into this box? · A container holds 1.5 litres. Is it large enough to make a jug of orange juice if the concentrate is 355 ml and you have to use the concentrate can to add three full cans of water? Explain your thinking. · Investigate the capacities of various beverage containers to determine which size container is found most often. Record your findings in a graph or table and present this to the class. **Experiential Learning:** Engage learners in pairs to identify a situation where a given SI or imperial volume unit would be used. **Example**: I would measure water in a bath using L, but I would measure liquid in a baby-food bowl in ml. **Collaborative learning:** Learners discuss and solve problems that involve the volume of 3-D objects and composite 3-D objects using formulae. Then, write the volume measurement expressed in one SI/imperial unit cubed in another SI/imperial unit cubed. **Example 1**: Determine the volume of the following shapes (It should also include pyramids and all available solid shapes)	

3D Object	Formula
	$\pi r^2 h$
	$\frac{1}{3}\pi r^2 h$ cubic units.
	s^3
	Base Area × Height (lbh)
	$\frac{4}{3}\pi r^3$
	$\frac{2}{3}\pi r^3$

Pyramid	Volume
Triangular Pyramid 	$V = \frac{1}{3} \times B \times h$ $= \frac{1}{3} \times \frac{1}{2} \times bH \times h$ $\boxed{V = \frac{1}{6} BHh}$
Square Pyramid 	$V = \frac{1}{3} \times B \times h$ $\boxed{V = \frac{1}{3} a^2 h}$
Rectangular Pyramid 	$V = \frac{1}{3} \times B \times h$ $V = \frac{1}{3} \times lw \times h$ $\boxed{V = \frac{1}{3} lwh}$
Pentagonal Pyramid 	$V = \frac{1}{3} \times B \times h$ $= \frac{1}{3} \times \frac{5}{2} Sa \times h$ $\boxed{V = \frac{5}{6} Sah}$
Hexagonal Pyramid 	$V = \frac{1}{3} \times B \times h$ $V = \frac{1}{3} \times 3aS \times h$ $\boxed{V = aSh}$

Example 2: Determine the volume for the following composite shapes:



2.3.2.LI.3

2.3.2.AS.3

Solve real world problems that involve the volume/capacity of a 3-D object.

Collaborative learning: Learners discuss and solve real world problems on volume. While they experience this concept, model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping.

Example 1: Ewuradwoa wants to drink milk from a glass that is in the shape of a cylinder. The height of the glass is 15 units, and the radius of the base is 3 units. What is the quantity (volume) of milk that she requires to fill the glass completely?

Solution: Given that, the height of the glass = 15 units and the radius of the base = 3 units.
To find the volume of the glass, we need to use the formula for the volume of a cylinder, which is $\pi r^2 h$ cubic units.
The volume of the glass, $V = \pi r^2 h$
$V = \pi \times (3^2) \times 15$

Level 1 Recall
Level 2 Skills of conceptual understanding
Level 3 Strategic reasoning
Level 4 Extended critical thinking and reasoning

	$V = \pi \times 135$ $V = 423.9$ cubic units. Therefore, she needs approximately 424 cubic units of milk to fill her glass. **Example 2**: The pedestal on which a statue is raised is a rectangular concrete solid measuring 9 feet long, 9 feet wide and 6 inches high. How much is the cost of the concrete in the pedestal if concrete costs GH70 per cubic yard? **Solution:** We need to find the volume of the pedestal in cubic yards and then multiply it by the cost factor of GHC70 per cubic yard. Recall the general formula for computing the volume of a rectangular solid: $V = LWH$. In this case, $L = 9$ feet, $W = 9$ feet and $H = 6$ inches. Since we want to compute volume in cubic yards, we should convert all three measurements to yards before using the formula for volume. To convert from feet to yards, we divide by 3; to convert from inches to yards, we divide by 36. $L = 9$ feet = (9/3) yards = 3 yards $W = 9$ feet = (9/3) yards = 3 yards $H = 6$ inches = (6/36) yards = 1/6 yards Now we compute the volume: Volume = (3 yards) (3 yards)(1/6 yards) = 9/6 cubic yards = 1.5 cubic yards Finally, we multiply by the cost factor: Cost = (1.5 cu yd)(GH70 per cu yd) = GHC105 **Example 3:** Ampofo loves playing with building blocks. He has built a structure with 15 cubic blocks. If the edge of each cube is 3in, what would be the volume of his structure? **Solution**: Let's calculate the volume of one cube. The volume of cube = Edge × Edge × Edge = 3 in × 3 in × 3 in =27 in^3 There are 15 cubes in his structure. So, the volume of the structure is, Volume of structure =15 × Volume of one cube = 15 × 27 in^3 = 405 in^3 Therefore, the volume of the structure is 405 in^3. **Examples**: i. Ananga has a rectangular aquarium that is 12 inches long, 8 inches wide and 8 inches high, providing enough room to safely house 6 guppies. Assuming that the number of guppies that can	

	be safely housed depends upon the size of the aquarium, how many guppies can be safely housed in an aquarium that is 24 inches long, 16 inches wide and 16 inches high? ii. Adobe is digging a hole for a rectangular swimming pool measuring 38 feet long by 22 feet wide by 8 feet deep. How much water will the swimming pool hold, assuming that 1 cubic foot = 7.5 gallons iii. A cylindrical can that is four inches tall and has a radius of 1.5 inches can hold 10¢ worth of soda. Assuming that the value of the contents is proportional to the size (volume) of the can, what would be the value of the soda contained in a can that is 8 inches tall with a radius of 3 inches?	
Teaching and Learning Resources	· Mathematical sets. · Technology tools such as computers, mobile phones, etc.	· Computer software applications like GeoGebra

Subject **MATHEMATICS**

Strand **4. MAKING SENSE OF AND USING DATA**

Sub-Strand **1. STATISTICAL REASONING AND ITS APPLICATION IN REAL LIFE**

Learning Outcomes	**21st-Century Skills and Competencies**	**GESI, SEL and Shared National Values**
2.4.1.LO.1		
Design a data collection instrument and justify its appropriateness for collecting everyday life data to address a contextual issue.	**Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on data collection, analyses and presentation using appropriate IT tools to boost their interest and desire to solve more problems on their own. **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to the application of the concept of data collection, analyses and presentation to lifelong learning and further studies. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concepts of data collection, analyses and presentation for lifelong learning.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: • Respect individuals of different backgrounds in their mathematics groups and beyond. • Embrace diversity and practise inclusion in the mathematics classroom and beyond. • Examine and dispel misconceptions/ myths about gender as they relate to the learning of mathematics, home management and human development. • Interrogate their stereotypes and biases about gender and the role members in a group play in the mathematics classroom and in home management. • Identify injustice, especially in recognition of the contributions of different groups and individuals to the effective management and maintenance of the mathematics classroom and home. • Sensitive to the inter-relatedness of the various aspects of life even as they engage with others in the mathematics classroom and beyond.

		· Value and promote justice in the mathematics classroom, at home and in society. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the mathematics curriculum, the facilitator should apply the Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and finding confidence · Exhibiting motivation and SMART goal-setting · Managing emotions and conflicts · Showing empathy and cooperation These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning of data collection, analyses and presentation.

		Diversity: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination among learners. **National Core Values**: Develop tolerance, friendliness, open-mindedness, patience, hard work, and humility in learners as they interact with their peers in the mathematics classroom. **Truth and Integrity:** Reward truth and honesty as strong moral principles in the learning of mathematics, leading to responsible citizenship. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment. **Discipline and honesty:** Encourage learners to behave and work in a controlled way, which involves obeying mathematical rules, principles and standards, leading to self-directed learning.
2.4.1.LO.2		
Construct and interpret a variety of data presentation methods, including cumulative frequency curves (Ogive), waffle diagrams, etc. and describe the relationship between the measures of dispersion in data	**Communication and Collaboration:** Learners communicate confidently and effectively to develop appropriate mathematics vocabulary for the concepts of data construction and presentation methods. **Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on data	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to:

displays to solve and/or pose problems.	collection, analyses and presentation using appropriate IT tools to boost their interest and desire to solve more problems on their own. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on data construction and presentation methods. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on data construction and presentation methods.	· Respect individuals of different backgrounds in their mathematics groups and beyond. · Embrace diversity and practise inclusion in the mathematics classroom and beyond. · Examine and dispel misconceptions/ myths about gender as they relate to the learning of mathematics, home management and human development. · Interrogate their stereotypes and biases about gender and the role members in a group play in the mathematics classroom and in home management. · Identify injustice, especially in recognition of the contributions of different groups and individuals to the effective management and maintenance of the mathematics classroom and home. · Sensitive to the inter-relatedness of the various aspects of life even as they engage with others in the mathematics classroom and beyond. · Value and promote justice in the mathematics classroom, at home and in society. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the mathematics curriculum, the facilitator should apply the Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and finding confidence · Exhibiting motivation and SMART goal-setting · Managing emotions and conflicts

		· Showing empathy and cooperation These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; and respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning data construction and presentation methods. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination among learners. **National Core Values**: Develop tolerance, friendliness, open-mindedness, patience, hard work, and humility in learners as they interact with their peers in the mathematics classroom.

		Truth and Integrity: Reward truth and honesty as strong moral principles in the learning of mathematics, leading to responsible citizenship. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment. **Discipline and honesty:** Encourage learners to behave and work in a controlled way, which involves obeying mathematical rules, principles and standards, leading to self-directed learning.
2.4.1.LO.3		
Carry out mini-projects involving data handling (data collection, analysis and interpretation) of quantitative and qualitative data beyond the school environment.	**Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on data collection, analyses and presentation using appropriate IT tools to boost their interest and desire to solve more problems on their own. **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to the application of the concept of data collection, analyses and presentation to lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on data collection, analyses and presentation.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: · Respect individuals of different backgrounds in their mathematics groups and beyond. · Embrace diversity and practise inclusion in the mathematics classroom and beyond. · Examine and dispel misconceptions/ myths about gender as they relate to the learning of mathematics, home management and human development. · Interrogate their stereotypes and biases about gender and the role members in a group play in the mathematics classroom and in home management. · Identify injustice, especially in recognition of the contributions of different groups and individuals

	Innovation and Creativity: Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concept of data collection, analyses and presentation to lifelong learning.	to the effective management and maintenance of the mathematics classroom and home. · Sensitive to the inter-relatedness of the various aspects of life even as they engage with others in the mathematics classroom and beyond. · Value and promote justice in the mathematics classroom, at home and in society. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the mathematics curriculum, the facilitator should apply the Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and finding confidence · Exhibiting motivation and SMART goal-setting · Managing emotions and conflicts · Showing empathy and cooperation These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities.

		National Core Values: **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning of data collection, analyses and presentation. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination among learners. **National Core Values**: Develop tolerance, friendliness, open-mindedness, patience, hard work, and humility in learners as they interact with their peers in the mathematics classroom. **Truth and Integrity:** Reward truth and honesty as strong moral principles in the learning of mathematics, leading to responsible citizenship. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment. **Discipline and honesty:** Encourage learners to behave and work in a controlled way, which involves obeying mathematical rules, principles and standards, leading to self-directed learning.

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
2.4.1.CS.1	2.4.1.LI.1	2.4.1.AS.1
Demonstrate conceptual understanding of data handling in relation to designing and validating a variety of data collection methods.	**Design a data collection instrument (questionnaire, interview guide, checklist, observation guide, etc.) by employing a feasible digital technology (where available) and using it to collect real-life data.** **Experiential Learning:** In collaborative and mixed-gender/ability groups, engage learners to explore and design various data collection instruments around these types of student surveys: · Student Satisfaction Survey · Student perception survey · Student Course Evaluation Questions · Student Environmental Surveys/interviews · Student Harassment/Corporal Punishment Survey **Experiential Learning:** Using convenient groups, learners design their own survey questionnaires using questions like: Student perception survey questions about the class · Which activities in the classroom do you enjoy the most? · Given a chance, what is one change that you would like to see? · Do you have supportive classmates? · What motivates you to learn more? · Do you think that the school provides you with adequate sports facilities? · How many hours do you spend learning on your own? · Do you partake in any extracurricular activities? · How much time do you spend on homework and quizzes every day? **Using convenient groups,** task learners to use computer application software like MS Word, Wordpad, Excel, Apple TextEdit, Corel WordPerfect, Dropbox Paper, Google Docs, LibreOffice, etc., to design a data collection instrument. Encourage learners who demonstrate good behaviour and skills in working towards group goals.	Level 1 Recall **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

	Collaborative Learning: Engage learners to use the designed instrument to collect the appropriate data and present the report to the class, including the procedure adopted in collecting the data. Learners should be encouraged to use feasible technology to collect the data and talk about the appropriate ways of using the IT tools.	
	2.4.1.LI.2	2.4.1.AS.2
	Evaluate a given set of data and/or its instrument by identifying potential problems related to bias, use of language, gender, ethics, cost, time, privacy, cultural sensitivity, etc. **Using group discussions and think-pair-share activities**, engage learners to discuss and decide on a survey to undertake, consider what facts/contextual issues to take into consideration in designing the survey questionnaire, and choose a suitable data collection method that includes the cross-cutting considerations and how to collect the data. Encourage learners to extend their discussions on the need to be mindful of their choices in every situation and how to manage their emotions in such instances. **Example**: Recognise that a given question on a survey questionnaire contains bias. Suppose you tell your classmates that the response to the question in the Class Survey Question Form is to help you evaluate students' courses. The course was well organised in terms of the time frame, assessment, and access to materials. 1. Yes 2. No Such a question is directional and does not offer respondents the opportunity to express their opinion if they believe the organisation was done well to some extent. **A better question could be:**	**Level 1 Recall** **Level 2 Skills of conceptual understanding** Level 3 Strategic reasoning Level 4 Extended critical thinking and reasoning

	How organised was the course in terms of the time frame assessment and access to materials? · Very organised · Somewhat organised · Neutral · Disorganised · Highly disorganised **Examples**: i. Discuss the cost and time frame for collecting data on a particular area of interest and make recommendations on how to strategise to reduce the cost and time. ii. Discuss and decide on a survey to undertake, what facts/contextual issues to take into consideration in designing the survey questionnaire, and choose a suitable data collection method that includes the social considerations and how to collect the data.	
Teaching and Learning Resources	· Samples of questionnaires, interview guides, observation guides, etc. · Computer application software such as Excel, MS Word, Wordpad, etc. · Mathematical sets · Technology tools such as computers, mobile phones, etc	

<table>
<tr><th>Content Standards</th><th>Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI</th><th>Assessment</th></tr>
<tr><td>2.4.1.CS.2</td><td>2.4.1.LI.1</td><td>2.4.1.AS.1</td></tr>
<tr><td>Demonstrate an understanding of data presentations and analysis for grouped and ungrouped data and describe the relationship between the measures of dispersion in data displays.</td><td>

Organise and present data (grouped/ungrouped) by means of the ogive, waffle diagrams, box and whisker plots, etc., including generating 3D graphs and solving and/or posing problems.

Collaborative learning: In small groups, Organise a given data using a frequency curve (ogive) and interpret the graph. Use appropriate technology tools such as Microsoft Excel if available. Encourage learners to be fair and impartial towards other learners and help them to acknowledge that there is reward in being truthful and honest citizenship.

Example 1: Draw an ogive for the following distribution.

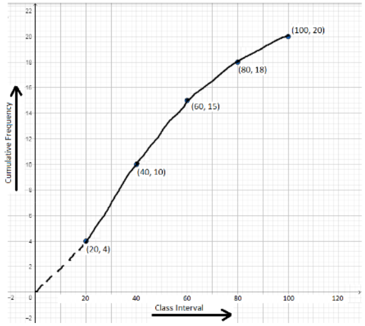
<table>
<tr><td>Class Interval</td><td>0 - 20</td><td>20 - 40</td><td>40 - 60</td><td>60 - 80</td><td>80 - 100</td></tr>
<tr><td>Frequency</td><td>4</td><td>6</td><td>5</td><td>3</td><td>2</td></tr>
</table>

Solution

<table>
<tr><td>Class Interval</td><td>0 - 20</td><td>20 - 40</td><td>40 - 60</td><td>60 - 80</td><td>80 - 100</td></tr>
<tr><td>Frequency</td><td>4</td><td>6</td><td>5</td><td>3</td><td>2</td></tr>
<tr><td>Cumulative Frequency</td><td>4</td><td>10 (4+6)</td><td>15 (4+6 +5)</td><td>18 (4+6 +5+3)</td><td>20 (4+6 +5+3+2)</td></tr>
</table>

Now, plot the points (20, 4), (40, 10), (60, 15), (80, 18) and (100, 20) following steps 3 and 4, and join the points following steps 5 and 6. We get the following ogive.

</td><td>Level 1 Recall
Level 2 Skills of conceptual understanding
Level 3 Strategic reasoning
Level 4 Extended critical thinking and reasoning</td></tr>
</table>



Scale: On the x-axis, 1 cm = width of interval.
On the y-axis, 2 mm = cumulative frequency 1.

Group Work/Collaborative Learning: Using think-pair-share activities, learners discuss and make presentations on the types of ogives ("less than" and "greater/more than" ogives).

Lesser Than Cumulative Frequency
Lesser than cumulative frequency is obtained by adding successively the frequencies of all the previous classes, including the class against which it is written. The cumulate starts from the lowest to the highest size.

Greater/more Than Cumulative Frequency
Greater than cumulative frequency is obtained by finding the cumulative total of frequencies starting from the highest to the lowest class.

Example: Graph the two ogives for the following frequency distribution of the weekly wages of the given number of workers at Serene Hotel. Hence, find the median.

Weekly wages	No. of workers
0-20	4
20-40	5
40-60	6
60-80	3

Solution

Weekly wages	No. of workers	C.F. (Less than)	C.F. (More than)
0-20	4	4	18 (total)
20-40	5	9 (4 + 5)	14 (18 - 4)
40-60	6	15 (9 + 6)	9 (14 - 5)
60-80	3	18 (15 + 3)	3 (9 - 6)

For plotting less than type curve, points (20,4), (40,9), (60,15), and (80,18) are plotted on the graph, and these are joined by freehand to obtain the less than ogive. For plotting greater than type curve, points (0,18), (20,14), (40,9), and (60,3) are plotted on the graph, and these are joined by freehand to obtain the greater than type ogive.

The less than and greater than ogives shown in the graph below.

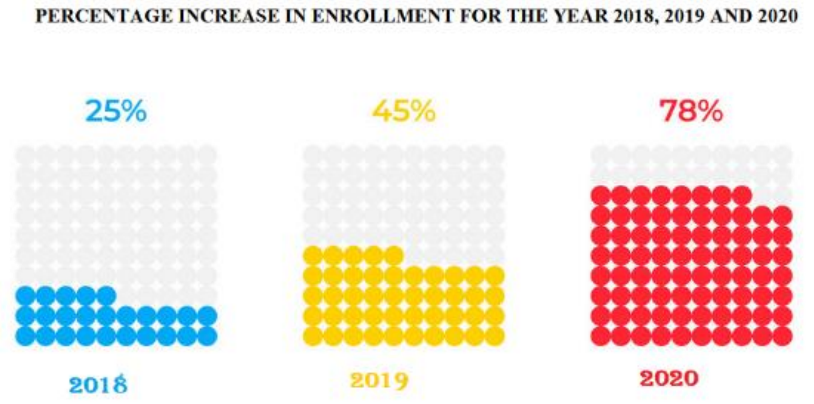
The median: A perpendicular line on the x-axis is drawn from the point of intersection of these curves. This perpendicular line meets the x-axis at a certain point. This determines the median. Here, the median is 40.

Experiential Learning: In small groups, organise a given data using a waffle diagram. Encourage learners to use the appropriate IT tools in designing waffle charts.

Example: The data below shows the percentage increase in enrollment in a Senior High School in Salaga over a period of three years. Represent the data using a waffle graph.

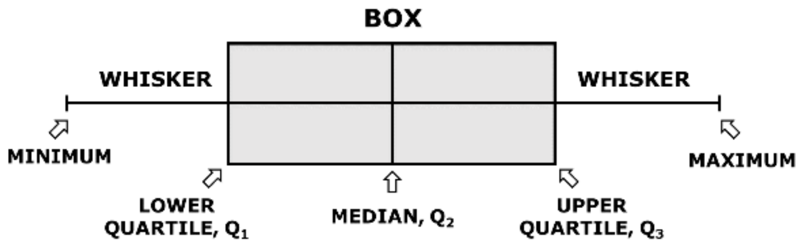
Year	Enrollment Increase(%)
2018	25
2019	45
2020	78

Solution



Think-pair-share: Discuss and make a presentation on the features box plot. Encourage learners to use the appropriate IT tools in designing box plots.

Example: A box and whisker plot (or box plot) is a graph that displays the data distribution by using five numbers. Those five numbers are the minimum, first (lower) quartile, median, third (upper) quartile and maximum.



In a box and whisker plot:

- The left and right sides of the box are the lower and upper quartiles. The box covers the interquartile interval, where 50% of the data is found.
- The vertical line that splits the box in two is the median. Sometimes, the mean is also indicated by a dot or a cross on the box plot.

- The whiskers are the two lines outside the box, which go from the minimum to the lower quartile (the start of the box) and then from the upper quartile (the end of the box) to the maximum.

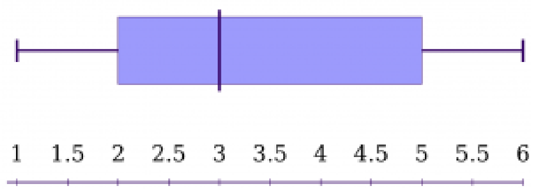
Experiential Learning: In small groups, Organise a given data using a box and whisker plot and interpret a given box plot. Encourage learners to use the appropriate IT tools in making a presentation on interpreting a box plot.

Example 1: Dziifa threw the dice 20 times and got these results: 6 3 3 6 3 5 6 1 4 6 3 5 5 2 2 2 2 3 2 3
Draw a box plot.

Solution: The first thing we need to do is to order the data from smallest to largest: 1 2 2 2 2 2 3 3 3 3 3 3 4 5 5 5 6 6 6 6
Furthermore, we need to calculate the median. Since the number of data points is even, we have.
$Me = \frac{x_{10+x_{11}}}{2} = \frac{3+3}{2} = 3$
After that, we have to calculate the quartiles.
The lower quartile is: $Q_1 = \frac{2+2}{2} = 2$, while the upper quartile is : $Q_1 = \frac{5+5}{2} = 5$.

Now, from the data, the minimum value is 1, and the maximum is 6.
The next step is to scale an appropriate axis for the obtained 5 numbers.
Then, we need to draw a box from the minimum value 1 to the value 3, which is the median, and put a vertical line through the median. Then, draw the box from the median to the lower and upper quartiles. Furthermore, we have to draw "whiskers". Those are the lines that extend parallel with the scale from the bo. In other words, the whisker goes from the lower quartile to the minimum and from the upper quartile to the maximum.

Finally, our box plot is:



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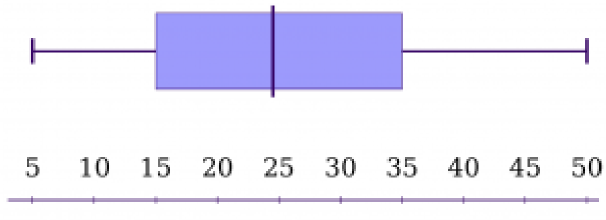
Example 2: (Interpreting box and whisker plots)
Find the range, the interquartile range and the median of the data in the box plot below.



Solution: Since the minimum value of the given data is 5 and the maximum is 50, the range is R = 50 – 5 = 45.
The lower quartile is 15, and the upper quartile is 35. Therefore, the interquartile range is = 35 – 15 = 20. Actually, the interquartile range represents the length of the bo.
The median is obviously 25.

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<td>2.4.1.LI.2</td>
<td>2.4.1.AS.2</td>
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<td></td>
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Analyse and interpret data using measures of dispersion and justify which of these measures best suits the data.

Using Talk for Learning strategies, learners brainstorm on the meaning of standard deviation.

Example: Standard deviation tells about the value and how much it has deviated from the mean value. If we get a low standard deviation, then it means that the values tend to be close to the mean, whereas a high standard deviation tells us that the values are far from the mean value. It is commonly abbreviated as SD and denoted by 'σ'.

Using think-pair-share activities, learners discuss the steps in determining standard deviation and deduce the formula for Standard deviation.

</td>
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Level 1 Recall
Level 2 Skills of conceptual understanding
Level 3 Strategic reasoning
Level 4 Extended critical thinking and reasoning

</td>
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</table>

Steps to Calculate Standard Deviation

- Find the mean, which is the arithmetic mean of the observations.
- Find the squared differences from the mean. (The data value – mean)^2
- Find the average of the squared differences. (Variance = The sum of squared differences ÷ the number of observations).
- Find the square root of variance. (Standard deviation = √Variance).

Standard Deviation Formulae: Two standard deviation formulas are used to find the standard deviation of sample data and the standard deviation of the given population.

Population	Sample
$\sigma = \sqrt{\frac{\sum(X - \mu)^2}{N}}$	$s = \sqrt{\frac{\sum(X - \bar{x})^2}{n - 1}}$
X - The Value in the data distribution μ - The population Mean N - Total Number of Observations	X - The Value in the data distribution $\bar{x}$ - The Sample Mean n - Total Number of Observations

Standard Deviation of Ungrouped Data: The calculations for standard deviation differ for different data. Distribution measures the deviation of data from its mean or average position. There are two methods to find the standard deviation.

Standard Deviation by the Actual Mean Method

$\sigma = \sqrt{(\sum x - \bar{x})2/n)}$

Example: Consider the data observations 3, 2, 5, 6. Here, the mean of these data points is 16/4 = 4.

The squared differences from mean = $(4-3)^2+(2-4)^2+(5-4)^2+(6-4)^2 = 10$

Variance = Squared differences from mean/ number of data points =10/4 =2.5

Standard deviation = √2.5 = **1.58.**

Standard deviation by Assumed Mean Method: When the x values are large, an arbitrary value (A) is chosen as the mean. The deviation from this assumed mean is calculated as d = x - A.

$\sigma = \sqrt{[(\sum(d)^2/n) - (\sum d/n)^2]}$

Using Talk for Learning: Learners brainstorm on the meaning of variance.

Example: Variance is a measure of dispersion. A measure of dispersion is a quantity that is used to check the variability of data about an average value. Data can be of two types - grouped and ungrouped. When data is expressed in the form of class intervals, it is known as grouped data. On the other hand, if data consists of individual data points, it is called ungrouped data. The sample and population variance can be determined for both kinds of data.

Using think-pair-share activities, learners discuss the steps in determining variance and deduce the formula for variance. Learners should be encouraged to use feasible technology to analyse data, determine the variance of the data, and talk about the appropriate ways of using the IT tools.

Example

- Find the mean of the observations.
- Subtract the mean from each observation.
- Square each of these values.
- Add all the values obtained in the previous step.
- Divide the value from step 4 by n (for population variance) or n - 1 (for sample variance).

	Population	Sample
Ungrouped	$\sigma^2 = \frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N}$	$\sigma^2 = \frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N-1}$
Grouped	$\sigma^2 = \sum_{i=1}^{N} \frac{f(M_i - \bar{X})^2}{N}$	$\sigma^2 = \sum_{i=1}^{N} \frac{f(M_i - \bar{X})^2}{N-1}$

Examples:

i. Suppose we have the data set {3, 5, 8, 1}, and we want to find the population variance. The mean is given as (3 + 5 + 8 + 1) / 4 = 4.25. Then by using the definition of variance we get $[(3 - 4.25)^2 + (5 - 4.25)^2 + (8 - 4.25)^2 + (1 - 4.25)^2] / 4 = 6.68$. Thus, variance = **6.68**.

ii. Find the sample variance of the data (3, 4, 7, 12, 14).

Solution:
n = 5
Mean = (3 + 4 + 7 + 12 + 14) / 5 = 8
Sample Variance = $\sum(Xi-\bar{X})^2$ /N−1
$[(3 - 8)^2 + (4 - 8)^2 + (7 - 8)^2 + (12 - 8)^2 + (14 - 8)^2) / 5 - 1 = 23.5$

Answer: Variance = 23.5

iii. Find the population variance of the data (1.2, 4.5, 6.7, 2.3).

Solution:
n = 4
Mean = (1.2 + 4.5 + 6.7 + 2.3) / 4 = 3.675
Population Variance = $\sum(Xi-X)^2/N$
$[(1.2 - 3.675)^2 + (4.5 - 3.675)^2 + (6.7 - 3.675)^2 + (2.3 - 3.675)^2] / 4 = 4.461$

Answer: Variance = 4.461

iv. Find the sample variance of

Class	**10 - 20**	**20 - 30**	**30 - 40**	**40 - 50**	**50 - 60**
Frequency	2	5	7	3	1

Solution:
The height of the interval is 10

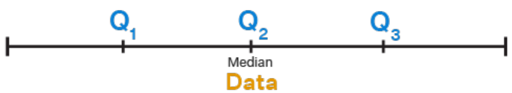
Class	F	Mi	d = (Mi - B) / 10	fd	d²f
10 - 20	2	15	-2	-4	8
20 - 30	5	25	-1	-5	5
30 - 40	7	35 = B	0	0	0
40 - 50	3	45	1	3	3
50 - 60	1	55	2	2	4

Mean = $\frac{\sum Mifi}{\sum fi}$ = 32.778.

Variance = $\frac{\sum fd^2 - \frac{(\sum fd)}{2n}}{n-1}.10^2$ = 112.4183. [This formula can be derived from ∑f(Mi−‾‾‾X)2N−1 to simplify calculations]
Answer: Variance = 112.4183.

Using Talk for Learning: Learners brainstorm on the meaning of Quartile Deviation.

Example: Quartile deviation is a statistic that measures the deviation in the middle of the data. Quartile deviation is also referred to as the semi-interquartile range and is half of the difference between the third quartile and the first quartile value. The formula for the quartile deviation of the data is



Quartile Deviation(Q.D.) = $\frac{Q_3 - Q_1}{2}$

	Examples: Find the quartile deviation for the following given data. 23, 8, 5, 16, 33, 7, 24, 5, 30, 33, 37, 30, 9, 11, 26, 32 **Solution**: The given data points are 23, 8, 5, 16, 33, 7, 24, 5, 30, 33, 37, 30, 9, 11, 26, 32 Let us arrange this data in the following ascending order. 5, 5, 7, 8, 9, 11, 16, 23, 24, 26, 30, 30, 32, 33, 33, 37 From the above data we have Q_1 = (8 + 9)/2 = 17/2 = 8.5, and Q_3 = (30 + 32)/2 = 62/2 = 31 **Quartile Deviation** = $\frac{Q3-Q1}{2}=\frac{31-8.5}{2}=\frac{22.5}{2}$ =**11.25.**	
	2.4.1.LI.3	2.4.1.AS.3
	Use mathematical arguments to support personal choices as well as incorporate views and perspectives of others to evaluate and make inferences from data presented in everyday life (including live debates on TV, radio, social media platforms, newspapers, magazines, etc.) **Using think-pair-share activities:** Task learners to make inferences from a given data and give reasons for their choices. Offer other learners the opportunity to make their own inferences as long as they are referring to the data. **Using Talk for Learning,** Learners should be engaged in a class debate where they support their argument with data and the use of appropriate mathematics terminology. **Project-based Learning:** Assign project works to learners (individually or in groups) to obtain current data about contemporary issues of interest, then make conclusions and give constructive criticisms. Encourage learners to communicate confidently and effectively to develop appropriate mathematics vocabulary for the data collected.	Level 1 Recall **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning
Teaching and Learning Resources	· Mathematical sets. · Technology tools such as computers, mobile phones, etc. · Computer software applications like GeoGebra · Graph sheets	· colour pens, etc. · Reports from analysed data · Worksheets · Posters · teaching presentations

	• computer with data organising software like MS Excel, MS PowerPoint, etc., • manila cards • flip charts • markers	• enquiry project-template • A4, A3 papers

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
2.4.1.CS.3	2.4.1.LI.1	2.4.1.AS.1
Demonstrate the ability to carry out a mini-project involving the collection, analysis and interpretation of quantitative and qualitative data beyond the school environment.	**Develop and implement a project plan for the collection, analysis and interpretation of data with useful conclusions and recommendations (including the use of appropriate computer applications, e.g., Excel) within and beyond the school environment.** **Project-based learning:** In convenient groups, obtain, for example, the COVID-19 data and analyse it by looking at the overall cases recorded for the various 16 regions, active cases, recovered cases and deaths. Learners should be encouraged to use feasible technology to analyse data and talk about the appropriate ways of using IT tools. **Project-based learning:** In convenient groups, obtain the cases of malaria in the local hospital in your area and analyse them. **Experiential Learning:** As part of the analysis, obtain the frequencies and percentages, then draw charts (box plots and ogive.) for the data. Also, from the analysis, make conclusions and give recommendations to the hospital. **Design a questionnaire/interview guide** on "Student Environmental Surveys/Interviews" or "Student Harassment/Corporal Punishment Survey" and use it to collect data from students in your school. Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of designing a questionnaire to collect data and apply the knowledge to lifelong learning. • As part of the project, decide on the sample and justify it appropriateness for generalisation to the entire population. • Analyse the data by obtaining the quartiles, standard deviations and variance and make conclusions and recommendations based on the results. • Make summaries of your results, conclusions and recommendations of your project and present them using a PowerPoint, infographic design, MS Word or handwritten to the class or at a mini forum in the school or the community where the data was obtained.	Level 1 Recall **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

	• Make oral presentations on the project by explaining the choice of project topic and its relevance, the choice of data collection method and the analysis, and talk about the challenges faced.	
	2.4.1.LI.2	2.4.1.AS.2
	Present a project report including the use of PowerPoint, infographics, etc. and publish it in school magazines, newspapers (Junior Graphic), local radio and TV stations, social media platforms, etc. **Experiential Learning,** • Summarize your results, conclusions and recommendations of your project and present them using a PowerPoint, infographic design, MS Word or handwritten to the class or at a mini forum (including school clubs) in the school. • Publish the summary of the findings and recommendations of the project in school magazines, newspapers (Junior Graphic), local radio and TV stations, social media platforms, etc.	Level 1 Recall **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning
Teaching and Learning Resources	• Mathematical sets. • Technology tools such as computers, mobile phones, etc. • Computer software applications like GeoGebra • Graph sheets • computer with data organising software like MS Excel, MS PowerPoint, etc., • manila cards • flip charts • markers	• colour pens, etc. • Reports from analysed data • Worksheets • Posters • teaching presentations • enquiry project-template • A4, A3 papers

Subject **MATHEMATICS**

Strand **4. MAKING SENSE OF AND USING DATA**

Sub-Strand **2. PROBABILITY/CHANCE**

Learning Outcomes	**21st-century Skills and Competencies**	**GESI, SEL and Shared National Values**
2.4.2.LO.1		
Demonstrate a conceptual understanding of simple and compound probability experiments involving two dependent events.	**Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on simple and compound probability experiments involving two dependent events. **Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on simple and compound probability experiments involving two dependent events using appropriate IT tools to boost their interest and desire to solve more problems on their own. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on simple and compound probability experiments involving two dependent events. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concept of simple and compound probability experiments involving two dependent events to lifelong learning.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: • Respect individuals of different backgrounds in their mathematics groups and beyond. • Embrace diversity and practise inclusion in the mathematics classroom and beyond. • Examine and dispel misconceptions/ myths about gender as they relate to the learning of mathematics, home management and human development. • Interrogate their stereotypes and biases about gender and the role members in a group play in the mathematics classroom and in home management. • Identify injustice, especially in recognition of the contributions of different groups and individuals to the effective management and maintenance of the mathematics classroom and home. • Sensitive to the inter-relatedness of the various aspects of life even as they engage

		with others in the mathematics classroom and beyond. · Value and promote justice in the mathematics classroom, at home and in society. **Social and Emotional Learning** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the mathematics curriculum, the facilitator should apply the Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and finding confidence · Exhibiting motivation and SMART goal-setting · Managing emotions and conflicts · Showing empathy and cooperation These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities.

		National Core Values: **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning simple and compound probability experiments involving two dependent events. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination among learners. **National Core Values**: Develop tolerance, friendliness, open-mindedness, patience, hard work, and humility in learners as they interact with their peers in the mathematics classroom. **Truth and Integrity:** Reward truth and honesty as strong moral principles in the learning of mathematics, leading to responsible citizenship. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment.

		Discipline and honesty: Encourage learners *to* behave and work in a controlled way, which involves obeying mathematical rules, principles and standards, leading to self-directed learning.

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
2.4.2.CS.1	2.4.2.LI.1	2.4.2.AS.1
Demonstrate a conceptual understanding of simple and compound probability experiments involving two dependent events.	**List the elements of the sample space from a simple or compound experiment involving two dependent events.** **Using Talk for Learning:** Engage learners to brainstorm and make presentations on the meaning of dependent events. Encourage learners to tolerate each other's views. **Example 1:** Events are dependent if the outcome of one event affects the outcome of another. For instance, if you draw two colored balls from a bag and the first ball is not replaced before you draw the second ball, then the outcome of the second draw will be affected by the outcome of the first draw. **E.g.** If A and B are dependent events, then the probability of A happening AND the probability of B happening, given A, is P(A) × P(B after A). P(A and B) = P(A) × P(B after A) P(B after A) can also be written as P(B | A) then P(A and B) = P(A) × P(B | A) **Example 2**: A purse contains four GHC5 notes, five GHC10 notes and three GHC20 notes. Two notes are selected without the first selection being replaced. Find P(GHC5, then GHC5). **Solution**: There are four GHC5 notes. There are a total of twelve notes. P(GCH5) = $\frac{4}{12}$. The result of the first draw affected the probability of the second draw. There are three GHC5 notes left. There are a total of eleven notes left. P(GHC5 after GHC5) = $\frac{3}{11}$. P(GHC5, then GHC5) = P(GHC5) · P(GHC5 after GHC5) = $\frac{4}{12} \times \frac{3}{11} = \frac{1}{11}$.	**Level 1 Recall** **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

	The probability of drawing a GHC5 bill and then a GHC5 bill is $\frac{1}{11}$. **Using think-pair-share** activities, discuss the differences between dependent and independent probability events. **Example**		

Dependent Events	Independent Events
1. The occurrence of one event affecting the probability of another event.	1. The occurrence of one event not affecting the probability of another event.
2. Examples include a power cut in case you don't pay your bill on time, winning the lottery after buying 10 lottery tickets (the more tickets bought, the greater the chance of winning)	2. Examples include riding a bike and watching your favourite movie on a laptop
3. Formula can be written as: P(A and B) = P(A) × P(B | A)	3. Formula can be written as: P(A and B) = P(A) × P(B)

Using think-pair-share activities, discuss the steps in determining whether a probability is dependent or independent. As learners pair-share ideas, promote respect for divergent views to ensure inclusivity in the mathematics learning environment.

Example: Steps to Check Whether the Probability Belongs to Dependent or Independent Events

- Step 1: Is it possible for the events to happen in any order? If yes, go to step 2; if no, go to step 3.
- Step 2: Does one event affect the outcome of the other event? If yes, go to step 4; if no, go to step 3.
- Step 3: The event is independent. Simply put the formula of independent event and get the answer.
- Step 4: The event is dependent. Simply put the formula of the dependent event and get the answer.

	2.4.2.LI.2	2.4.2.AS.2
	Solve everyday life problems involving the probability of two independent events. **Collaborative Learning:** In convenient groups, task learners to identify situations where dependent events are possible and create and solve problems on them. Engage learners in a discussion on the need not to misuse ideas in probability in unacceptable ways. **Example 1**: Entertainment centres A juggler has seven red, five green, and four blue balls. During his stunt, he accidentally drops a ball and doesn't pick it up. As he continues, another ball falls down. What is the probability that the first ball that was dropped is blue and the second ball is green? **Solution:** As we know, the first ball is not replaced by the juggler. So, after dropping the first ball, he is left with 15 balls. The probability that the first ball is blue or P(blue ball) = $\frac{4}{16}$ The probability that the second ball is green or P(green ball) = $\frac{5}{15}$ The probability that the first ball is blue and the second ball is green: P(blue than green) = P(blue) × P(green) $=\frac{4}{16}\times\frac{5}{15}=\frac{1}{12}$.	**Level 1 Recall** **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning
Teaching and Learning Resources	· Manipulative (dice, coins, spinners, playing cards, counters, digit cards), · Simple Probability Mazes (Printable & Digital), · Worksheets · Task Cards	

YEAR THREE

Subject	**MATHEMATICS**
Strand	**1. NUMBERS FOR EVERYDAY LIFE**
Sub-Strand	**2. PROPORTIONAL REASONING**

Learning Outcomes	**21st-century Skills and Competencies**	**GESI[5], SEL[6] and Shared National Values**
3.1.2.LO.1		
Establish the validity of logical arguments and use it to make relevant decisions in solving problems.	**Communication and Collaboration:** Learners communicate confidently and effectively to develop appropriate mathematics vocabulary for the concept of logical reasoning. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on logical reasoning. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on logical reasoning.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: · Respect individuals of different backgrounds in their mathematics classroom and beyond. · Embrace diversity and practise inclusion in the mathematics classroom and beyond. · Examine and dispel misconceptions/myths about gender as they relate to the mathematics classroom, home management and human development. · Interrogate their stereotypes and biases about gender and the role members in a group play in the mathematics classroom and in home management. · Identify injustice, especially in recognition of the contributions of different groups and individuals to the effective management and maintenance of the mathematics classroom and home.

[5] Gender Equality and Social Inclusion
[6] Socio-Emotional Learning

		• Sensitive to the inter-relatedness of the various aspects of life even as they engage with others in the mathematics classroom and beyond. • Value and promote justice in the mathematics classroom, at home and in society. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the mathematics curriculum, the facilitator should apply the Social Emotional Learning strategies to ensure that learners are: • Self-reflecting and finding confidence • Exhibiting motivation and SMART goal-setting • Managing emotions and conflicts • Showing empathy and cooperation These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning of logical reasoning.

		Equity: Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination among learners in all mathematics discourse, including the learning of logical reasoning. **Truth and Integrity:** Reward truth and honesty as strong moral principles in the learning of logical reasoning to help them become responsible citizens. **Discipline and honesty:** Encourage learners to behave and work in a controlled way, which involves obeying mathematical rules, principles and standards, leading to self-directed learning.
3.1.2.LO.2		
Analyse the impact of variations and conduct simple investigations in solving date-to-day problems.	**Communication and Collaboration:** Learners communicate confidently and effectively to develop appropriate mathematics vocabulary for the concept of variations. **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to the application of the concept of variations to lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's values, perceptions and actions for decision-making as they engage in group and individual activities on variations. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concept of variations to lifelong learning.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: Respect individuals of different backgrounds in their mathematics classroom and beyond. Embrace diversity and practise inclusion in the mathematics classroom and beyond. Examine and dispel misconceptions/myths about gender as they relate to the mathematics classroom, home management and human development. Interrogate their stereotypes and biases about gender and the role members in a group play in the mathematics classroom and in home management. Identify injustice, especially in recognition of the contributions of different groups and individuals to

		the effective management and maintenance of the mathematics classroom and home. Sensitive to the inter-relatedness of the various aspects of life even as they engage with others in the mathematics classroom and beyond. Value and promote justice in the mathematics classroom, at home and in society. **SEL:** Help learners to develop the ability to manage their emotions, thoughts, and behaviours as they interact in their mathematics classrooms and their groups in the learning of the concept of variations. **Relationship Skills:** Engage learners in the development of healthy and supportive relationships with their peers as they communicate with diverse individuals in the learning of the concept of variations in the everyday mathematics classroom and beyond. **Responsible Decisions:** Support learners to make responsible and caring choices to help improve their social interactions with others, make decisions and justify them in their mathematics classroom. **National Core Values:** **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment, including the learning of the concept of variations.

		Equity: Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination among learners in all mathematics discourse, including the learning of variations. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment.

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
3.1.2.CS.1	3.1.2.LI.1	3.1.2.AS.1
Demonstrate conceptual understanding of logical reasoning to solve real-life problems.	**Investigate to establish differences between variables in situational growth (symbolic logic) and apply it to real world situations.** **Through think-pair-share:** Establish the concept of logical reasoning, use it to compare statements and quantifiers, make intelligent guessing and judge the validity of logical arguments (including positive and negative statements), and ensure participation by all learners. **Example**: Make a declarative sentence and identify whether it is true or false. i.e. i. (***False***) ii. Accra is the capital city of Ghana. (***True***) iii. The earth is the third planet from the sun. (***True***) ***Positive Statement***: Rain fell in northern Ghana today. ***Negative Statement***: No rain fell in northern Ghana today. **In collaborative and gender-responsive groups,** investigate to establish truth tables, statements and connections using conjunction (**And)** or disjunction (**Or)**. i.e. *Logical connectives: like, and, or, not, and if . . . then, are examples of connectives.* i.e. i. Conjunct 1: It is raining heavily ii. Conjunct 2: The dogs are barking iii. Conjunction: It is raining heavily, and the dogs are barking	Level 1 Recall **Level 2 Skills of conceptual understanding** Level 3 Strategic reasoning Level 4 Extended critical thinking and reasoning
	3.1.2.LI.2	3.1.2.AS.2
	Make intelligent guessing to establish valid arguments and draw logical conclusions. **In collaborative and gender-responsive groups**, learners discuss to establish and analyse arguments using diagrams (truth tables) to carry out investigation on Logical reasoning, including conjunctions. Be conscious of and interpret norms, embrace diversity through collaborative small groups and self-directed	Level 1 Recall Level 2 Skills of conceptual understanding

or individualised work in discussion and evaluation, leading to truth, integrity, respect, tolerance, problem-solving skills and making effective judgments.

Example: Use Venn diagrams to represent each of the following.

a) A: All my brothers are farmers
b) B: Farmers are happy people.
c) C: Abu is a farmer but not a brother.
d) D: Baba is my brother
e) E: Kofi is not happy

U= {people}, H= {happy people}, F={farmers}, B={brothers}
$a = Abu, a \in F, \quad b = Brothers, b \in B,$
$k = Kofi, k \notin F,$

Using Problem-based Learning approaches, learners in mixed-ability groups investigate the situation where the logical connective "but ($p \wedge q$)" and "or ($p \vee q$)" is used in compound statements, ensuring equity at all stages.

Eg.1. 'She wants to go to school' but 'she wants to go to the party'.
Notice this time round, "but" is used in place of "and" to give a different sort of emphasis to the statement.
. Let p stand for "7 $>$ 5" and q stand for "3 $<$ 0." Find the truth value of $p \wedge q$.

Truth Table for the Conjunction and		
T	T	T
T	F	F
F	T	F
F	F	F

Fig. 1: Truth Table for the Conjunction p and q

Level 3 Strategic reasoning
Level 4 Extended critical thinking and reasoning

Truth Table for the disjunction or		
T	T	T
T	F	T
F	T	T
F	F	F

Fig. 2: Truth Table for the Conjunction p or q

Teaching and Learning Resources	· Technology tools: · Computer, GeoGebra, Google Search,	· YouTube videos, etc · Cardboards,	· measuring instruments

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
3.1.2.CS.2	3.1.2.LI.1	3.1.2.AS.1
Demonstrate a conceptual understanding of proportionality in relation to variation and use it to solve real-life problems.	**Use proportional reasoning to investigate the various types of variation (direct and inverse) and extend this to make generalisations.** **Using group/collaborative strategies**, engage learners to establish the concepts of variation (direct, inverse variations), relate them to scientific concepts and use them to solve real life problems. Be aware of instilling leadership, respect for others' opinions, commitment to excellence achievement, discipline and self-confidence and with supportive differentiated instruction to make appropriate generalisations through critical thinking and problem-solving strategies. **Examples** 1. **Direct Variation**: The growth of a variable results in the growth of another variable. **Eg.1** The area of a given circle is directly proportional to the square of its radius. $A \propto r^2 \Rightarrow A = \pi r^2$ **Eg.2** The more kilometres you travel, the more petrol you will have to use (direct variation). 2. **Inverse Variation**: The growth of a variable results in the degeneration of another variable. **E.g. 3** The longer the term of your subscription, the less you will have to pay per year. **E.g. 4** varies inversely as if there exists a real number such that. $t \propto \frac{1}{v} \Rightarrow t = \frac{k}{v}$	Level 1 Recall **Level 2 Skills of conceptual understanding** Level 3 Strategic reasoning Level 4 Extended critical thinking and reasoning
	3.1.2.LI.2	3.1.2.AS.2
	Use proportional reasoning to investigate the various types of variation (joint and partial) and extend this to make generalisations.	**Level 1 Recall**

	Using group/collaborative strategies, engage learners to discuss direct and inverse variations to establish joint and partial variations, relate them to scientific concepts and use them to solve real life problems. Take note of introducing leadership, respect for team members' views, commitment to excellence achievement, discipline and self-confidence. **Example 1** **1. Joint Variation** **E.g. 5** Establish the connection between a variable and other variables that are either directly or inversely related. i.e. $F \propto m_1 m_2, and, F \propto d^2$ $\Rightarrow F = G\frac{m_1 m_2}{d^2}$ **2. Partial Variation** Establish the equation of a straight line as an example of a variation of this form: $y = ax + c$ where the value y is partly constant and partly varies as the value x. **Example 2:** Investigate the possible areas of applications of variations in real life situations.	**Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning
	3.1.2.LI.3	3.1.2.AS.3
	Carry out mini real-life investigations to solve problems involving logical reasoning within their local community. **Using group/collaborative strategies**, engage learners to discuss and establish the connection between the language of Geometry and logic. i.e., establish the meaning of the following vocabulary as used in logical reasoning. 1. Statement 2. True value 3. Negation 4. Conjunction 5. Disjunction	**Level 1 Recall** **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

	6. Conditional statement 7. Hypothesis 8. Conclusion 9. All statements. **Example**: In a mixed-gender group, conduct a mini survey by visiting the following centres: 1. Physical health department/unit, 2. Accounts department/unit, 3. science laboratory, 4. Computer laboratory, 5. School kitchen (if any) to establish logical connections and draw valid arguments, including the use of diagrams (truth tables or sets). i.e., carry out an investigation on logical reasoning based on symptoms. E.g. i.The presence or absence of any symptoms implies the presence or absence of a certain disease. ii. If a solution changes litmus paper to red, then it is acidic. iii. If an athlete wins hits, he/she qualifies for finals. **Through think-pair-share,** carry out outreach investigation on the real-life applications of logical reasoning (i.e....... Assignments outside the school environment). E.g. At the banks, hospitals, markets, mechanics shops, law courts, artisans, etc. Be aware of inculcating leadership, respect for and tolerate team members' views, commitment to excellence achievement, discipline and self-confidence and with supportive differentiated classroom instruction to make appropriate generalisation through critical thinking and problem-solving	
Teaching and Learning Resources	· Teaching and Learning Resources · Manipulative (dice, coins, spinners, playing cards, counters, digit cards), · Simple Probability Mazes (Printable & Digital),	· Worksheets · Task Cards

Subject **MATHEMATICS**

Strand **2. ALGEBRAIC REASONING**

Sub-Strand **2. PATTERNS AND RELATIONS**

Learning Outcomes	**21st-century Skills and Competencies**	**GESI, SEL and Shared National Values**
3.2.2.LO.1		
Solve problems on quadratic functions and equations, including real-life problems.	**Communication and Collaboration:** Learners communicate confidently and effectively to develop appropriate mathematics vocabulary for the concept of quadratic functions and equations. **Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to the application of the concept of quadratic functions and equations to lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on quadratic functions and equations. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on circles.	**GESI:** Learners have experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: · Respect individuals of different backgrounds in their mathematics classroom and beyond. · Embrace diversity and practise inclusion in the mathematics classroom and beyond. · Examine and dispel misconceptions/myths about gender as they relate to the mathematics classroom, home management and human development. · Interrogate their stereotypes and biases about gender and the role members in a group play in the mathematics classroom and in home management. · Identify injustice, especially in recognition of the contributions of different groups and individuals to the effective management and maintenance of the mathematics classroom and home. · Sensitive to the inter-relatedness of the various aspects of life even as they engage with others in the mathematics classroom and beyond.

		· Value and promote justice in the mathematics classroom, at home and in society. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the mathematics curriculum, the facilitator should apply the Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and finding confidence · Exhibiting motivation and SMART goal-setting · Managing emotions and conflicts · Showing empathy and cooperation These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning quadratic functions and equations. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning

		environment, including the learning of the concept of quadratic functions and equations. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination among learners in all mathematics discourse, including the learning of quadratic functions and equations. **National Core Values**: Develop tolerance, friendliness, open-mindedness, patience, hard work, and humility in learners as they interact with their peers in the mathematics classroom. **Truth and Integrity:** Reward truth and honesty as strong moral principles in the learning of quadratic functions and equations to help them become responsible citizens. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment. **Discipline and honesty:** Encourage learners to behave and work in a controlled way, which involves obeying mathematical rules, principles and standards, leading to self-directed learning.

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
3.2.2.CS.1	3.2.2.LI.1	3.2.2.AS.1
Demonstrate understanding of the concept of quadratic functions and equations and solve real-life problems with them.	**Identify and solve quadratic equations.** **Using group Work/Collaborative Learning strategy:** review learners' knowledge of quadratic expressions, deal with their misconceptions about such concepts, and extend the ideas to quadratic equations and functions using GeoGebra. **Example:** A quadratic expression contains only a quadratic term, and it is in the form $ax^2 + bx + c$. Where $a., b., c$ are constants and $a \neq 0$ example $2x^2 + 2x - 6$ while a quadratic equation contains a quadratic expression that is equal to any other expression. It is also in the form $ax^2 + bx + c = 0$. Example $2x^2 + 2x - 6 = 6$. **Using group Work/Collaborative learning strategy: review learners' knowledge** on how to solve quadratic expressions using the factorization method. **Example**: Factorise $x^2 - 2x - 3$ completely. **Solution**: Find two numbers that, if you multiply, will give you -3, but if you add, will give you -2. $x^2 - 3x + x - 3$ $(x^2 - 3x)(x - 3)$ Factor the common factors out. $x(x - 3) + 1(x - 3)$ Add the terms outside and multiply by one of the common terms to give the final result. $(x + 1)(x - 3)$ **Collaborative Learning:** In convenient groups, engage learners to explore why $a \neq 0$ in a quadratic equation. **Example**: Learners should be encouraged to put the value of $a = 0$ into the standard form of a quadratic equation.	Level 1 Recall Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

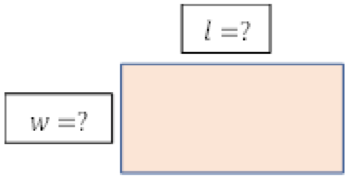
Think-pair-share activities: In pairs, task learners to solve a contextual problem that involves quadratic equations using the factorisation method.

Example: Solve the following equations.

1. $x^2 - 9x = 14$
2. $2x^2 - 5x = 3$
3. $(x - 2)(x + 6) = 0$
4. $x^2 - 16$

Example: Madam Alamisi has a rectangular board of area $14m^2$ and perimeter of 18 metres. Find the dimensions of her rectangular board.

Solution: Let $\mathrm{w = width,\ and\, l = length}$



Think:

$2l + 2w = 18 \ldots \ldots \ldots 1$

$\Rightarrow w = 9 - l \ldots \ldots \ldots 2$

$l \times w = 14 \ldots \ldots \ldots 3$

$l(9 - l) = 14$

$9l - l^2 = 14$

$l^2 - 9l + 14 = 0$

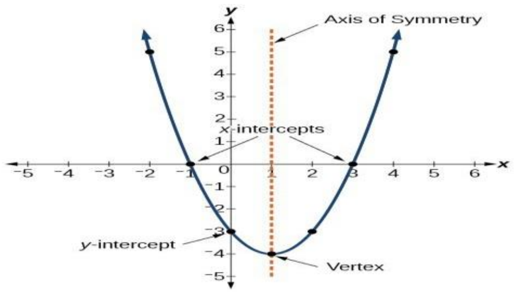
$(l - 2)(l - 7) = 0$

$\therefore l = 2\ and 7$

$w = 7, when\ l\ is\ 2$

$w = 2, when\ l\ is\ 7$

$\therefore 2 \times 7 = 14$ and $2(2) + 2(7) = 18$

	3.2.2.LI.2	3.2.2.AS.2
	Solve quadratic equations graphically and find the maximum and minimum points of quadratic graphs. **Using Talk for Learning,** Review, through a whole class discussion, learners' knowledge on how to draw graphs of a linear equation, deal with their misconceptions about such concepts, and extend the ideas to draw the graph of quadratic functions. **Experiential Learning:** In convenient groups, engage learners to draw and identify the properties of a quadratic graph. Offer positive support when students are having difficulties with self-regulation. **Example**: The graph of a quadratic function is a U-shaped curve called a parabola. It has an extreme point, or a turning point, called the verte. If the parabola opens up, the vertex or the point represents the lowest point on the graph or the minimum value of the quadratic function. If the parabola opens down, the vertex or the turning point represents the highest point on the graph, or the maximum value. The graph is also symmetric with a vertical line drawn through the vertex, called the axis of symmetry Graph of a parabola showing the x and y intercepts, vertex, and axis of symmetry. 	Level 1 Recall **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

The y-intercept is the point at which the parabola crosses the y-axis. The x-intercepts are the points at which the parabola crosses the x-axis. If they exist, the x-intercepts represent the zeros, or roots, of the quadratic function, the values of x at which y=0.

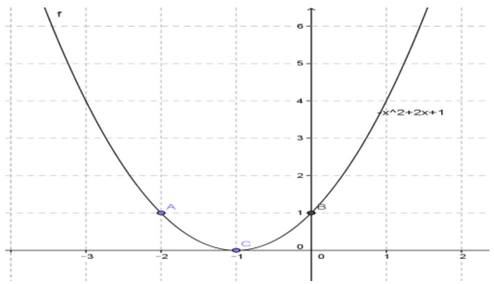
Think-pair share: Using pair activities, solve quadratic problems using the graphical method.

Examples:

i. Draw the graph of $y = x^2 + 2x + 1$ in an interval $-3 \leq x \leq 1$ investigate and justify the effect of the graph if
   a. The value of a changes
   b. The value of b changes
   c. The value of c changes.
   d. The value of $a < 0$ or negative

Solution: To draw the graph of a quadratic function, make a table with the values of x given to help you find the values of y, which will help you draw the graph of the quadratic functions given.

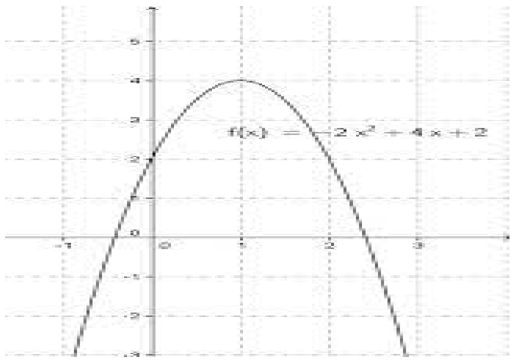
x	-3	-2	-1	0	1
y	4	1	0	1	4



ii. Make a table of values for the equation $y = -2x^2 + 4x + 2$ and determine the maximum point, maximum value, and the roots of the parabola.

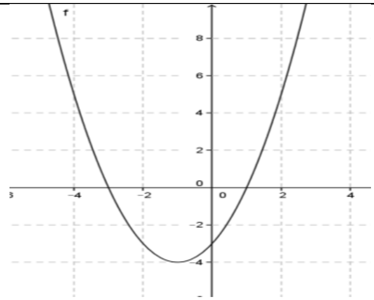
Solution

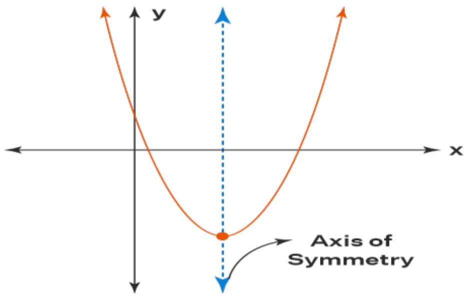
x	$y = -2x^2 + 2x + 2$
-1	-4
0	2
1	4
2	2
3	-4



The maximum point is (1,4)
The maximum value is 4
The roots are (-1.5, 2.5)

iii. If $y = x^2 + 2x - 3$, use the graph below to identify the roots, the y-intercept, and the turning point.

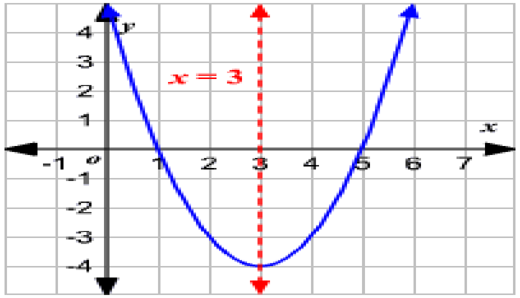
	 **Solution:** From the graph The roots are (-3,1) The y-intercept -2 The turning point (-1,4)	
	3.2.2.LI.3	3.2.2.AS.3
	Identify and explain the axis of symmetry, write its equation and solve linear and quadratic equations simultaneously using graphs. **In collaborative and gender-responsive groups**, use an ***integrated*** learning approach to establish, analyse and interpret quadratic graphs using diagrams to carry out investigation on quadratic equations concurrently through problem-solving approaches. **Using Problem-based Learning** in a whole class discussion, identify and brainstorm the meaning of axes of symmetry. **Example 1**: The axis or line of symmetry is a line that divides the graph into two equal parts.	**Level 1 Recall** **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning



Note that the line of symmetry is always on the x-axis, and so its equation is $x = \frac{a+b}{2}$ $(or\ x = \frac{-b}{2a})$

Example 2: Draw the graph of $y = (x-1)(x-5)$ and write the equation of the line of symmetry.

Solution



From the graph, the equation of the line of symmetry is $x = 3$

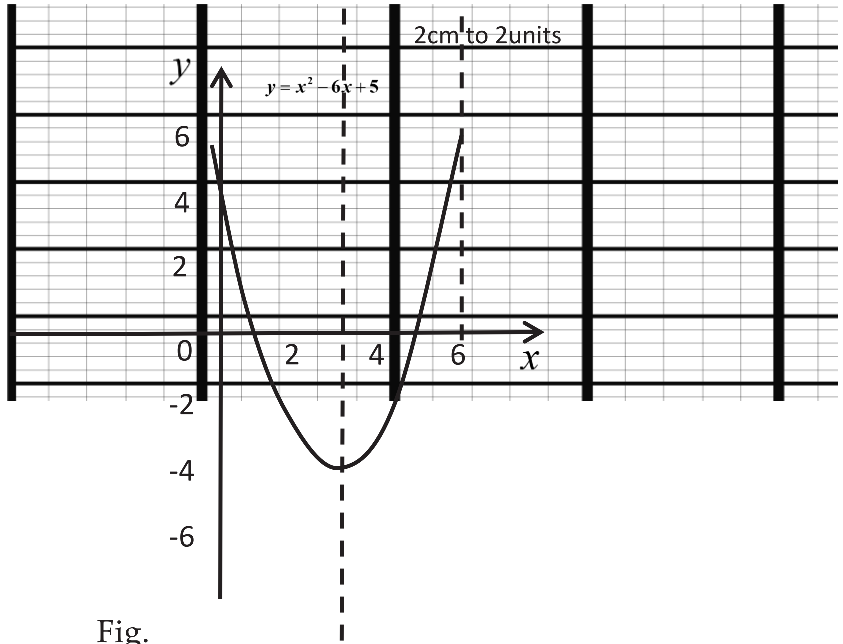
Using small gender-responsive grouping, identify and brainstorm the connection between straight line and quadratic equations and find their points of intersections.

Using a Problem-based Learning strategy, task learners to determine the solution set of equations.

	3.2.2.LI.4	3.2.2.AS.4
	Use quadratic graphs to solve related equations and solve real-life problems to find the range of values of (x) for which the other value (y) is increasing or decreasing and the range of values (x) for which the other value (y) is positive or negative. **Using Talk for Learning in a GESI-aware class discussion**, engage learners to investigate the range of values of x for which y increases or decreases. **Example 1**: Find the range of values of x for which y is increasing using the table of values of a quadratic equation.	Level 1 Recall Level 2 Skills of conceptual understanding Level 3 Strategic reasoning **Level 4 Extended critical thinking and reasoning**

x	0	1	2	3	4	5	6
x^2	0	1	4	9	16	25	36
-6x	0	-6	-12	-18	-24	-30	-36
5	5	5	5	5	5	5	5
y	5	0	-3	-4	-3	0	5

Example 2: Find the range of values of x for which y is increasing using the graph of a quadratic equation.

	See the illustration in the figure below.  Fig.	

Teaching and Learning Resources	· Teaching and Learning Resources · Manipulative (dice, coins, spinners, playing cards, counters, digit cards), · Simple Probability Mazes (Printable & Digital),	· Worksheets · Task Cards

Subject **MATHEMATICS**

Strand **3. GEOMETRY AROUND US**

Sub-Strand **1. SPATIAL SENSE**

Learning Outcomes	**21st-century Skills and Competencies**	**GESI, SEL and Shared National Values**
3.3.1.LO.1		
Draw circles for given radii and use the circle theorems; identify the tangent as perpendicular to the radius at the point of contact and verify that tangents drawn from an external point to the same circle are equal when measured from their point of contact.	**Strategic Competency:** Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to the application of the concept of circles to lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities about circles. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities in circles. **Innovation and Creativity:** Make conscious efforts to enable learners to develop and implement innovative and creative actions that reflect their level for application of the concept of circles to lifelong learning.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: • Respect individuals of different backgrounds in their mathematics groups and beyond. • Embrace diversity and practise inclusion in the mathematics classroom and beyond. • Examine and dispel misconceptions/ myths about gender as they relate to the learning of mathematics, home management and human development. • Interrogate their stereotypes and biases about gender and the role members in a group play in the mathematics classroom and in home management. • Identify injustice, especially in recognition of the contributions of different groups and individuals to the effective management and maintenance of the mathematics classroom and home. • Sensitive to the inter-relatedness of the various aspects of life even as they engage with others in the mathematics classroom and beyond. • Value and promote justice in the mathematics classroom, at home and in society.

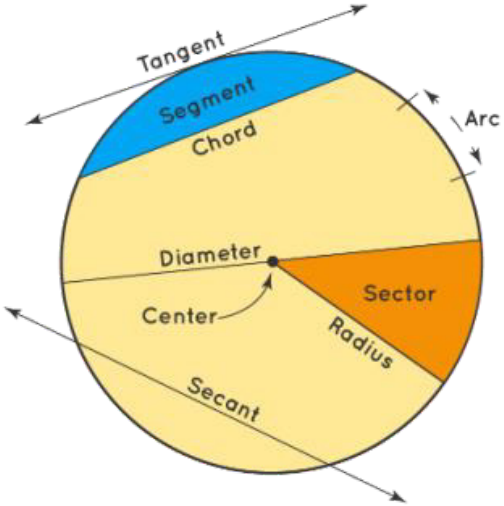
		SEL: Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the mathematics curriculum, the facilitator should apply Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and finding confidence · Exhibiting motivation and SMART goal-setting · Managing emotions and conflicts · Showing empathy and cooperation These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning about circles. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment.

		Equity: Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination among learners. **National Core Values**: Develop tolerance, friendliness, open-mindedness, patience, hard work, and humility in learners as they interact with their peers in the mathematics classroom. **Truth and Integrity:** Reward truth and honesty as strong moral principles in the learning of mathematics, leading to responsible citizenship. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment. **Discipline and honesty:** Encourage learners to behave and work in a controlled way, which involves obeying mathematical rules, principles and standards, leading to self-directed learning.
3.3.1.LO.2		
Perform geometric construction of quadrilaterals and given loci.	**Communication and Collaboration:** Learners communicate confidently and effectively to develop appropriate mathematics vocabulary for the concept of geometric construction of quadrilaterals. **Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on geometric construction of quadrilaterals using appropriate IT tools to boost their interest and desire to solve more problems on their own.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: • Respect individuals of different backgrounds in their mathematics groups and beyond. • Embrace diversity and practise inclusion in the mathematics classroom and beyond.

	Strategic Competency: Make conscious efforts to enable learners to collectively develop and implement innovative actions that promote sustainability at their level, leading to the application of the concept of geometric construction of quadrilaterals to lifelong learning and further studies. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on own one's values, perceptions and actions for decision-making as they engage in group and individual activities on geometric construction of quadrilaterals. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on the geometric construction of quadrilaterals. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concept of geometric construction of quadrilaterals to lifelong learning.	• Examine and dispel misconceptions/ myths about gender as they relate to the learning of mathematics, home management and human development. • Interrogate their stereotypes and biases about gender and the role members in a group play in the mathematics classroom and in home management. • Identify injustice, especially in recognition of the contributions of different groups and individuals to the effective management and maintenance of the mathematics classroom and home. • Sensitive to the inter-relatedness of the various aspects of life even as they engage with others in the mathematics classroom and beyond. • Value and promote justice in the mathematics classroom, at home and in society. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the mathematics curriculum, the facilitator should apply Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and finding confidence · Exhibiting motivation and SMART goal-setting · Managing emotions and conflicts · Showing empathy and cooperation These may be done by the facilitator through modelling emotional self-regulation and decision-

		making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork; respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work in the course of learning of geometric construction of quadrilaterals. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination among learners. **National Core Values**: Develop tolerance, friendliness, open-mindedness, patience, hard work, and humility in learners as they interact with their peers in the mathematics classroom. **Truth and Integrity:** Reward truth and honesty as strong moral principles in the learning of mathematics, leading to responsible citizenship. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment.

		Discipline and honesty: Encourage learners to behave and work in a controlled way, which involves obeying mathematical rules, principles and standards, leading to self-directed learning.

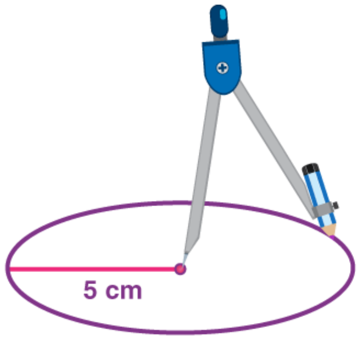
Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
3.3.1.CS.1	3.3.1.LI.1	3.3.1.AS.1
Demonstrate a conceptual understanding of spatial sense with respect to circles and their theorems and apply its properties to solve everyday life problems.	**Identify parts of a circle and draw circles for given radii and through points.** **Group discussions:** In small groups, task students to engage in discussions to recall the definition of circles and their properties. Provide opportunities for students to reflect on positive and negative choices in their discussions and the consequences of each choice. **Example**: A circle is a two-dimensional figure formed by a set of points that are at a constant or at a fixed distance (radius) from a fixed point (centre) on the plane. **<u>Parts of a Circle</u>**  **Centre:** The centre of the circle is the fixed point from which all points on the boundary of the circle are equidistant, often noted on diagrams as 'O'. **Radius:** The distance from the centre of a circle to the outside. The radius of the circle is half the diameter of the circle. The plural of radius is radii.	Level 1 Recall Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

	Diameter: The distance across the circle going through the centre. The diameter is twice the radius. **Circumference:** The distance once around the circle. **Arc:** A part of the circumference. **Major arc** – A major arc is greater than half the circumference. **Minor arc** – A minor arc is less than half the circumference. **Chord:** A line segment going from one point of the circumference to another but does not go through the centre. **Secant:** A line that goes through the circle at two points. **Tangent:** A straight line that touches the circle at a single point only. **Sector:** A section of the circle created by two radii. **Major sector** – A major sector has a central angle which is more than 180^0. **Minor sector** – A minor sector has a central angle which is less than 180^0. **Semi-circle:** Half of a circle. It could be considered a sector where the circle has been split by the diameter. **Quadrant:** A quarter of a circle created by two perpendicular radii. **Segment:** A section of the circle created by a chord. **Major segment** – a segment where the arc is greater than half the circumference. **Minor segment** – a segment where the arc is less than half the circumference. **Think-pair-share activities***:* In pairs, engage learners to draw circles given various radii.	

How to Draw a Circle of a Given Radius?

To draw a circle whose radius is given, we require a ruler and compasses. Given that the radius is 5 cm, the steps to be followed are:

- *Step 1*: Place the pointer of the compass at the initial point of the ruler (0 cm) and extend the other end of the pencil measuring 5 cm from the initial point (i.e., 5 cm)
- *Step 2:* Mark a point O on a piece of paper. This point is supposed to be the centre of the circle that you are about to construct.
- *Step 3:* Place the pointer of the compass at point O.
- *Step 4:* Turn the compass slowly through 360 degrees to draw a circle



Think-pair-share activities: In pairs, engage learners to draw circles through points.

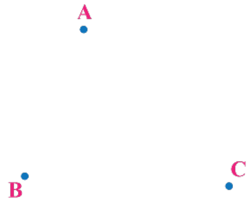
Example: **Drawing a circle through three points.**

Infinitely, many lines can pass through a single point in the plane. However, exactly one line can pass through two separate places in the plane. That is, a line can only be determined by two unique points. What occurs when circles are involved? How many points must be present for a circle to be identified as such?

It should be clear that an endless number of circles can travel past a single point.

There are infinitely many circles that can pass through two points.

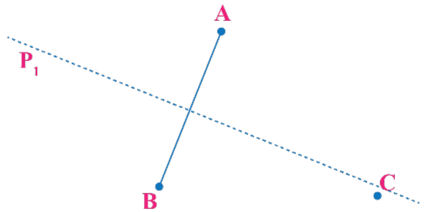
Let's now examine how to create a special circle that passes through three distinctive non-collinear locations. Three of these points—A, B, and C—are depicted in the following figure:



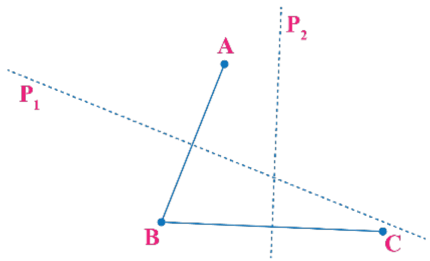
All three spots must be equidistant from the circle's centre.
This means that in order for OA = OB = OC, we must find the point in the plane designated as O.

Remember that O must be located on the perpendicular bisector of the segment connecting the two fixed points that it is equally far from. Consequently, we move forward as follows:

1. Join the points A and B and draw a perpendicular bisector P1 of AB. Any point on P1 will be equidistant from A and B:

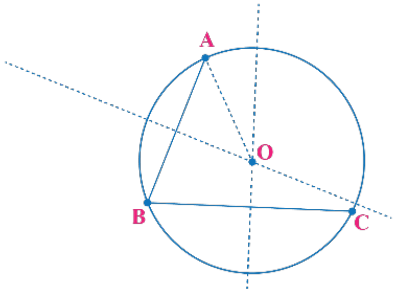


2. Join B and C and draw a perpendicular bisector P_2 of BC. Any point on P_2 will be equidistant from B and C:



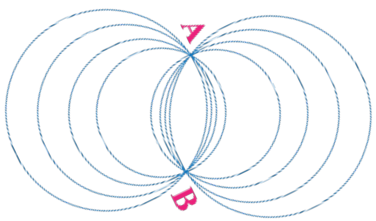
3. Indicate the point of intersection of P1 and P2 as O. This point O is equidistant from all of A, B, and C.
4. Let this be the centre of the circle.

5. We draw a circle with O as the centre, using the compass with the radius as the measure of length OA (or OB or OC).



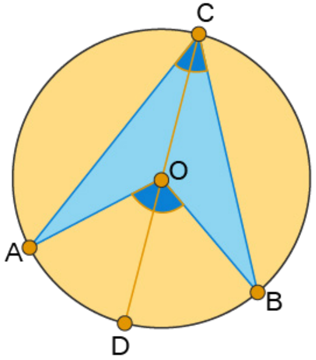
Example: Safia marked 2 points on a sheet of paper. She is trying to figure out the number of circles that could be constructed that will pass through the given two points. Can you help her?

Solution: If there are two points, we can consider them as the endpoints of the diameter to start with.
For the next circle that we try to draw, we let the distance between the two points as the chord to that circle.
By doing so, we will get infinite circles passing through the given two points.

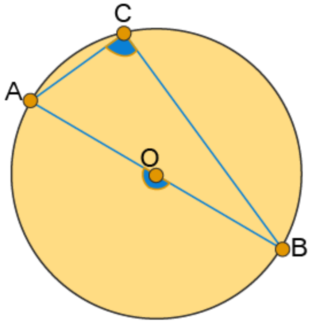


∴ infinite circles can be constructed if two points are given.

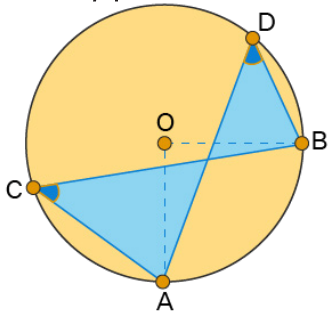
	3.3.1.LI.2	3.3.1.AS.2
	Discuss the circle theorems by identifying the statements, proofs, examples and applications. **Using think-pair-share activities,** learners discuss the various circle theorem statements. Encourage students to have a decision-making role related to classroom activities and rules. **Example: Circle Theorems Statements** • The angle subtended by a chord at the centre is twice the angle subtended by it at the circumference. • The angle subtended by the diameter at the circumference is a right angle. • The angles subtended at the circumference by the same arc are equal. • Two equal chords subtend equal angles at the centre of the circle. • If the angles subtended by two chords at the centre are equal, then the two chords are equal. • The opposite angles in a cyclic quadrilateral are supplementary. • The angle between the radius and the tangent at the point of contact is 90 degrees. **Think-pair-share activities**: In pairs, discuss the various circle theorem proofs. **Example: Circle Theorems Proofs** **Theorem 1:** The angle subtended by a chord at the centre is twice the angle subtended by it at the circumference. **Proof:** Consider the following circle, in which an arc (or segment) AB subtends $\angle AOB$ at the centre O and $\angle ACB$ at a point C on the circumference. We have to prove that $\angle AOB = 2 \times \angle ACB$. Draw a line segment through O and C, and let it intersect the circle again at point D, as shown.	**Level 1 Recall** **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning



	There are two triangles formed: ΔOAC and ΔOBC. So, we make the following observations. In ΔOAC, $\angle OAC = \angle OCA$ because $OA = OC$ (OA and OC being the radii. Angles opposite to equal sides are equal). In ΔOBC, $\angle OBC = \angle OCB$ because $OB = OC$ (OB and OC being the radii. Angles opposite to equal sides are equal). Hence, using the exterior angle theorem, we get, $\angle AOD = 2\times\angle ACO \cdots (1)$ $\angle DOB = 2\times\angle OCB \cdots (2)$ Add equations (1) and (2): $\angle AOD + \angle DOB = 2\times (\angle ACO + \angle OCB)$ $\Rightarrow \angle AOB = 2\times \angle ACB$ **Theorem 2:** The angle subtended by the diameter at the circumference is a right angle. **Proof:** Consider the figure below, where AB is the diameter of the circle. We need to prove that $\angle ACB = 90°$	



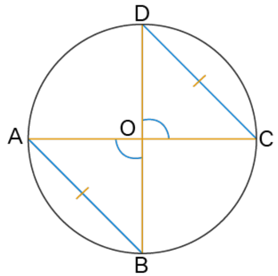
	Using theorem 1, 'The angle subtended by a chord at the centre is twice the angle subtended by it at the circumference.' We have $\angle AOB = 2 \times \angle ACB$. Now, $\angle AOB = 180°$ as AB is a straight line (diameter). So, $2 \times \angle ACB = 180°$ which implies $\angle ACB = 90°$. **Theorem 3:** The angles subtended at the circumference by the same arc are equal. **Proof:** Consider the following figure, which shows an arc AB subtending angles ACB and ADB at two arbitrary points, C and D, on the circumference. O is the centre of the circle.	



	We need to prove that $\angle ACB = \angle ADB$. Using the circle theorem, 'The angle subtended by a chord at the centre is twice the angle subtended by it at the circumference.' we have that $\angle ACB = 1/2 \times \angle AOB \cdots (1)$ $\angle ADB = 1/2 \times \angle AOB \cdots (2)$ From equations (1) and (2), we get $\angle ACB = \angle ADB$. Since angles ACB and ADB are arbitrary angles, therefore, the result is true for all angles subtended by the same arc.	

Theorem 4: Two equal chords subtend equal angles at the centre of the circle.

Proof: Consider a circle given below with centre O and two chords AB and CD, such that AB = CD. Now, we need to prove ∠AOB = ∠COD.

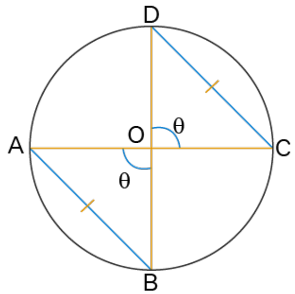


In triangles AOB and COD, we have
OA = OC (Radii)
OB = OD (Radii)
AB = CD (Given)

So, triangles AOB and COD are congruent by SSS congruence rule. So, we have ∠AOB = ∠COD (Corresponding parts of congruent triangles).

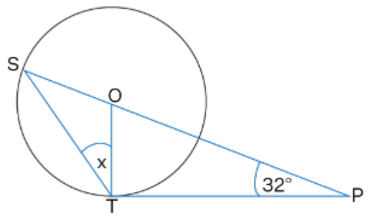
Theorem 5: If the angles subtended by two chords at the centre are equal, then the two chords are equal.

Proof: Consider a circle given below with centre O and two chords, AB and CD, such that ∠AOB = ∠COD. Now, we need to prove AB = CD.
In triangles AOB and COD, we have
OA = OC (Radii)
OB = OD (Radii)
∠AOB = ∠COD (Given)
So, triangles AOB and COD are congruent by SAS congruence rule. So, we have AB = CD (Corresponding parts of congruent triangles).



Group discussions: In groups, task learners to apply the various circle theorems to solve some examples. Encourage learners to show respect to individuals of different backgrounds in their groups as they solve real-life problems on circle theorems.

Example 1: Consider a circle with Centre O given below. Find the value of x using circle theorems.



Solution: We are given a circle with a centre O. Sine OS, and OT are radii, OS = OT. Using the circle theorem 'The angle between the radius and the tangent at the point of contact is 90 degrees', we have $\angle OTP = 90°$. In triangle OTP, using the angle sum theorem, we have

$\angle TOP + \angle OTP + \angle OPT = 180°$

$\Rightarrow \angle TOP + 90° + 32° = 180°$

$\Rightarrow \angle TOP = 180° - (90° + 32°)$

$= 58°$

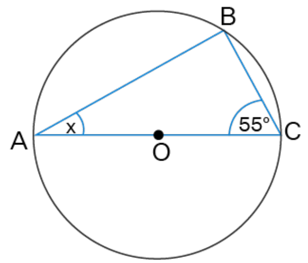
Since OS = OT $\Rightarrow \angle OSP = \angle OTP = x$ (because angles opposite to equal sides are equal).

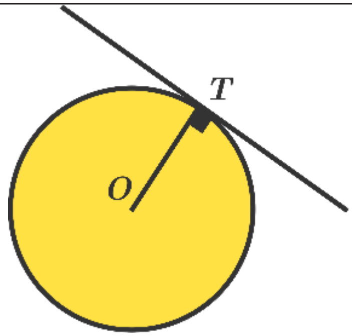
Using the exterior angle theorem, we have $\angle OSP + \angle OTP = \angle TOP$

$\Rightarrow x + x = 58°$

$\Rightarrow 2x = 58°$

$\Rightarrow x = 29°$

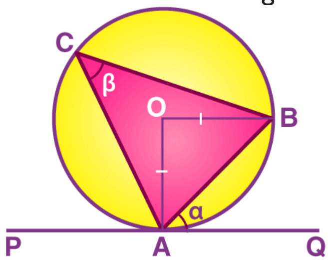
	Answer: x = 29° **Example 2:** Consider the circle given below with centre O. Find the angle x using the circle theorems.  **Solution:** Using the circle theorem 'The angle subtended by the diameter at the circumference is a right angle', we have ∠ABC = 90°. So, using the triangle sum theorem, ∠BAC + ∠ACB + ∠ABC = 180° ⇒ x + 55° + 90° = 180° ⇒ x + 145° = 180° ⇒ x = 180° - 145° = 35° **Answer:** x = 35	
	3.3.1.LI.3	3.3.1.AS.3
	Identify the tangent as perpendicular to the radius at the point of contact and verify the Alternate Segment Theorem. **Group discussions:** In convenient groups, learners discuss the concept of tangent and prove that the tangent at the point of contact with the circumference of the circle is perpendicular to the radius. **Example**: A **tangent** to a circle is a line intersecting the circle at exactly one point, the **point of tangency** or **tangency point**. An important result is that the radius from the centre of the circle to the point of tangency is perpendicular to the tangent line.	**Level 1 Recall** **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning



Proof: Let T be the point of tangency, O be the centre of the circle, and P be the foot of the altitude from O to the tangency line. Suppose that P and T are different points.
Since ∠OPT=90 and OT<OP, ∠OTP>∠OPT, ∠OTP>90. But then △OPT has an angle sum greater than 180∘, which is a contradiction. Thus, P and T must be the same point, so the radius from the centre of the circle to the point of tangency is perpendicular to the tangent line, as desired.

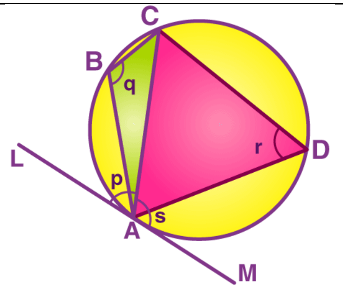
Group discussions: In convenient groups, learners discuss the alternate segment theorem and establish the proof. Help learners to dispel misconceptions/myths about gender as they relate to each other in their groups.

Example: Alternate Segment Theorem Statement
The alternate segment theorem is one of the circle theorems. The theorem states that "For any circle, the angle formed between the tangent and the chord through the point of contact of the tangent is equal to the angle formed by the chord in the alternate segment". The alternate segment theorem is also known as the tangent-chord theorem.



Let us assume that the tangent is drawn to the circle, such that the point of contact is A.

	Through A, a chord AB is drawn that should be inclined to the tangent at an angle "α". Suppose that AB subtends an angle β at point C anywhere on the surface of the circle, as shown in the figure. Assume that ∠ACB = ∠β is the alternate angle in the alternate segment for the angle between the tangent A and the chord AB. **Proof**: Let A be the point on the circumference of the circle, and "O" be the centre of the circle. Assume that PQ is the tangent of the circle that passes through point A. The tangent makes an angle α with the chord AB. Now, consider that ∠ACB = ∠β in the alternate segment. Now, we have to prove that ∠α =∠β. Thus, OA =OB (Both are the radii of the circle) Also, ∠OAB = ∠OBA (since the angles opposite to the equal sides are equal) Since, OAB is an isosceles triangle $\angle AOB = 180° - \angle OAB - \angle OBA$ $\angle AOB = 180° - 2\angle OAB$ …(1) Since the line segment, PQ is the tangent line, $\angle OAQ = 90°$ Therefore, $\alpha = 90° - \angle OAB$ …(2) From the equations (1) and (2), we can write $\angle AOB = 2\alpha$ We know that the angle at the centre of the circle is twice the angle at the circumference of the circle. $\angle AOB = 2\angle ACB$ $\angle ACB = (1/2)\angle AOB$ Now, substitute $\angle AOB = 2\alpha$ in the above equation, we get $\angle\beta = (1/2)\, 2\alpha$ $\angle\beta = \angle\alpha$ Thus, the alternate segment theorem is proved. **Alternate Segment Theorem Quadrilateral**	

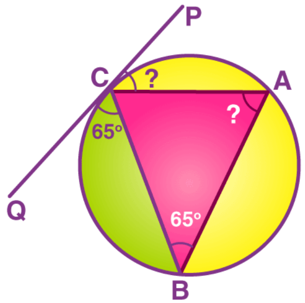


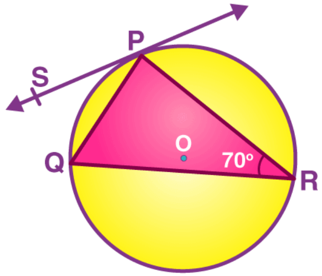
Considering the image given above, by using the alternate segment theorem, we can say that ∠p = ∠r
Now, we need to prove that ∠s = ∠q
As the tangent line, LM, is straight, we get
∠p + ∠s = 180° …(3)

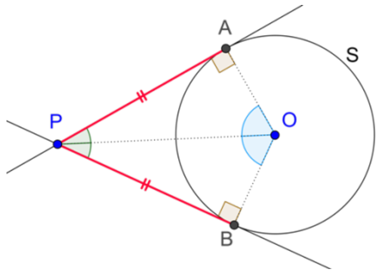
Since the angles ∠r and ∠q are the opposite angles in the cyclic quadrilateral, we can say that
∠q + ∠r = 180° …(4)

Now equating equations (3) and (4), we get
∠p + ∠s = ∠q + ∠r
Thus, ∠p = ∠r and ∠s = ∠q.
Hence, proved.

Example: Find the unknown angles in the figure, given that the chord BC makes the angles 65° with the tangent line PQ.



	Solution: Given that, ∠QCB = 65° By using the alternate segment theorem, we can say that ∠CAB = 65° Similarly, by using the angles in the alternate segment, ∠PCA = 65° Therefore, ∠CAB = 65° and ∠PCA = 65°. **Example**: Find the angle ∠QPS in the given figure.  **Solution:** Given that, ∠PRQ = 70°. By using the alternate segment theorem, ∠R= ∠P, (i.e.,) ∠QPS = ∠PRQ Hence, ∠QPS = 70°.	
	3.3.1.LI.4	3.3.1.AS.4
	Verify that tangents drawn from an external point to the same circle are equal when measured from their point of contact. **Group discussions:** In convenient groups, learners discuss the concept and prove that tangents drawn from an external point to the same circle are equal when measured from their point of contact. Use technology Literacy Skills, combined problem-solving competency, and critical thinking skills to enable creative and innovative techniques about circle theorems to verify tangents drawn from an external point to the same circle and equal when measured from their point of contact. **Examples** **Theorem:** Suppose that two tangents are drawn to a circle S from an exterior point P. Let the points of contact be A and B, as shown:	Level 1 Recall Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning



The theorem states that:

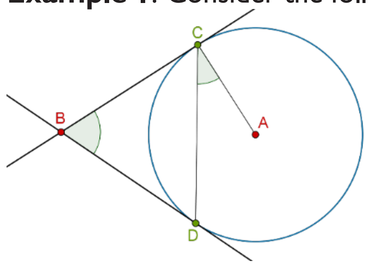
- The lengths of these two tangents will be equal, that is, PA = PB.
- They will also subtend equal angles at the centre, that is, ∠POA=∠POB
- The angle between them will be bisected by the line joining the exterior point and the centre, that is, ∠APO=∠BPO

Proof: All three parts will be proved if we show that ΔPAO is congruent to ΔPBO
Comparing the two triangles, we see that:

1. OA = OB (radii of the same circle)
2. OP = OP (common)
3. ∠OAP=∠OBP=90^0

Thus, by the RHS criterion, ΔPAO is congruent to ΔPBO, and the truth of all three assertions follows.

Group discussions: In convenient groups, learners solve examples.
Example 1: Consider the following figure, where BC and BD are tangents to the circle:



What is the relation between ∠DBC and ∠DCA?

Solution: Since BC is tangent to the circle at C, we note that ∠BCA=90^0
Thus, ∠BCD=∠BDC=90^0−∠DCA. Applying the angle sum property in ΔBCD
we have:

∠DBC+∠BCD+∠BDC=180^0

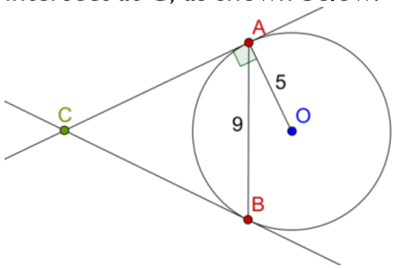
□∠DBC+(900−∠DCA)+(90^0−∠DCA)=180^0

□ ∠DBC−2∠DCA=0^0

□ ∠DBC=2∠DCA

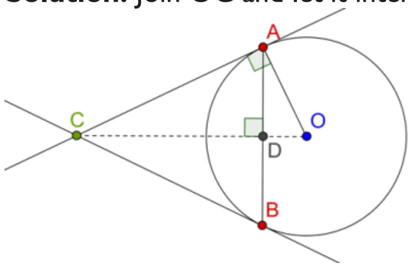
This is the required relation.

Example 2: Consider a chord AB of length 9 cm in a circle of radius 5 cm. Tangents at A and B intersect at C, as shown below:



What are the lengths of these tangents, that is, of CA and CB?

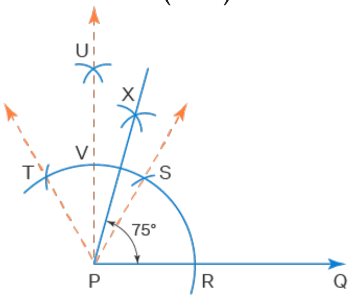
Solution: Join OC and let it intersect AB at D:

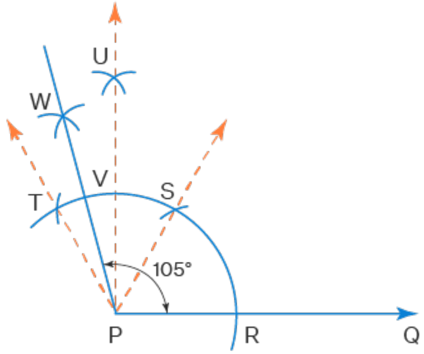
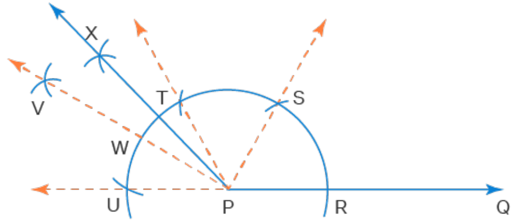


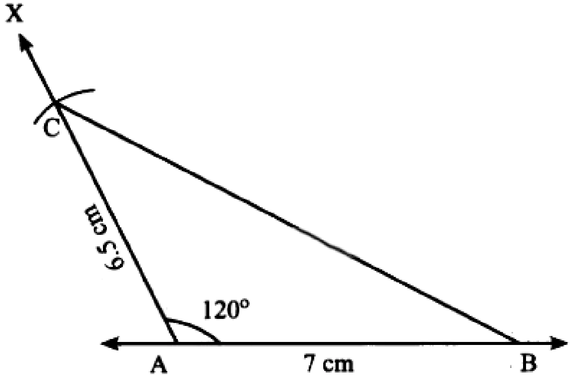
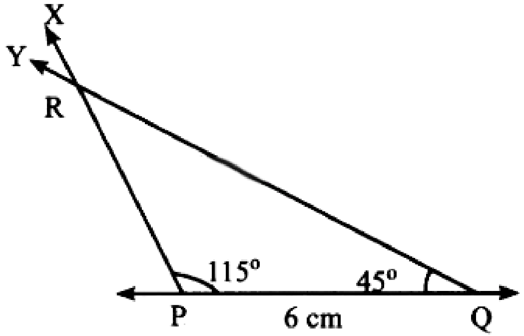
Note that ∠ADC=900. Now, compare ΔOAC with ΔODA:

1. ∠OAC=∠ODA=900
2. ∠DOA=∠COA(common)

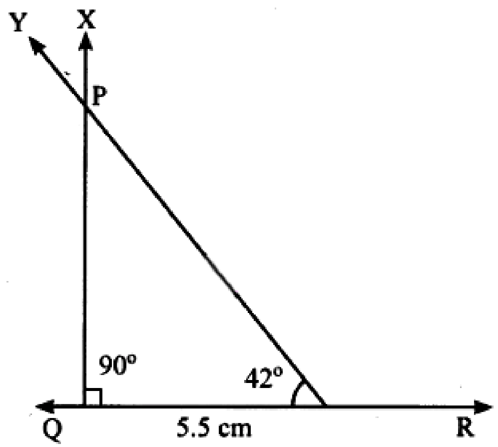
	Thus, the two triangles are similar by the AA similarity criterion. This means that OD:OA = AD:AC (make sure that you understand this). We know that OA = 5 cm, and AD is half of AB, which is 9 cm, so AD is 9/2 cm. We do not know the value of OD, but it can easily be calculated using the Pythagoras Theorem: OD2=OA2−AD2=52−(9/2)2=19/4 ☐ OD=√(19/4)=√(4.75)cm We plug this value into the similarity relation OD:OA = AD:AC to get: AC=(OA×AD)/OD=(5×9/2)/√(4.75)=10.3cm This is the (approximate) length of the two tangents CA and CB.	
Teaching and Learning Resources	• Mathematical sets. Graph sheet. • Technology tools such as computers, mobile phones, etc. • Computer software applications like GeoGebra.	

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
3.3.1.CS.2	3.3.1.LI.1	3.3.1.AS.1
Demonstrate knowledge and understanding of geometrical construction, use the knowledge to construct plane shapes and apply these in the world around them.	**Recall the construction of various angles such as 75°, 105°, 135° and 150°.** **Group discussions:** In convenient groups, learners discuss and draw various angles using the appropriate mathematical tools and/or technology. **Example**: Construct the following angles and verify by measuring them with a protractor: (i) 75° (ii) 105° (iii) 135° **Solution** **(i) Angle 75°** We need to construct two adjacent angles of 60°. The second angle should be bisected twice to get a 15° angle. $75° = 60° + 15°$ $15° = 30°/2 = (60/2) \div 2$  **(ii) Angle 105°** We need to construct two adjacent angles of 60°. In the second angle, we need to bisect it to get two 30° angles. The second 30° angle should be bisected again to get a 15° angle. Altogether, we can make an angle of 105°. $105° = 60° + 45°$ $105° = 60° + 30° + 15°$	Level 1 Recall **Level 2 Skills of conceptual understanding** Level 3 Strategic reasoning Level 4 Extended critical thinking and reasoning

	 (iii) **Angle 135°** We need to construct three adjacent angles of 60° each. The third angle should be bisected twice successively to get an angle of 15°. Altogether, we will get an angle of 135°. $135° = 15° + 60° + 60°$ $15° = (60°/2) \div 2$  	
	3.3.1.LI.2	3.3.1.AS.2
	Construct a triangle or quadrilateral under given conditions. **Group discussions:** In small groups, learners draw various triangles and quadrilaterals using the appropriate technology tools to enable creative and innovative techniques to construct and verify a triangle or a quadrilateral under given conditions. **Example 1**: Construct a triangle ABC with given conditions. (i) AB = 7 cm, AC = 6.5 cm and $\angle A = 120°$.	Level 1 Recall **Level 2 Skills of conceptual understanding** Level 3 Strategic reasoning Level 4 Extended critical thinking and reasoning

	Solution  **Example 2:** Construct triangle PQR such that ∠P = 115°, ∠Q = 40° and PQ = 6  **Construction steps:** • Step 1: Drawn a line. Marked P and Q on the line such that PQ = 6 cm. • Step 2: At P, draw O ray PX, making an angle of 115° with PQ. • Step 3: At Q, draw another ray QY, making an angle of 40° with PQ. Marked the point of intersection of the rays PX and Q Y as R. PQR is the required triangle.	

Example 3: Construct triangle PQR such that ∠Q = 90°, ∠R = 42° and QR = 5.5cm

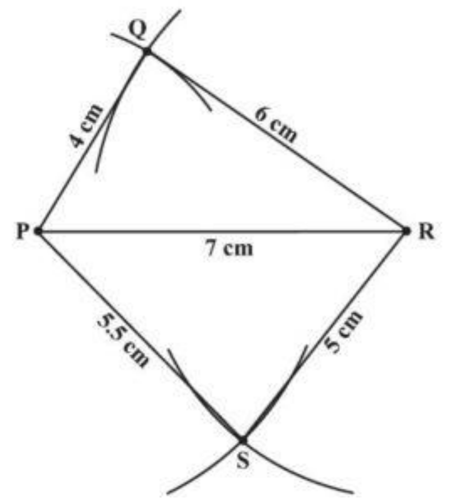


Construction steps:

- Step 1: Drawn a line. Marked Q and R on the line such that |QR| = 5.5 cm.
- Step 2: At Q, draw a ray |X, making an angle of 90° with QR.
- Step 3: At R, draw another ray RY, making an angle of 42° QR.

Marked the point of intersection of the rays QX and RY as P.
PQR is the required triangle.

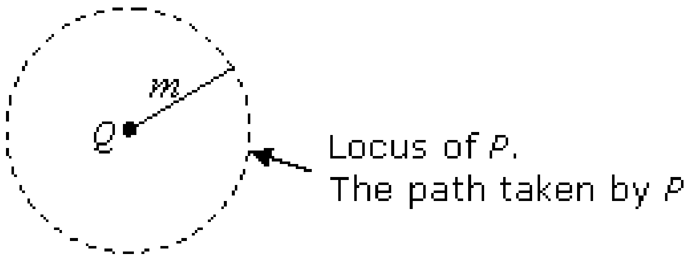
Example 4: Construct a quadrilateral PQRS where |PQ| = 4 cm, |OQ| = 6 cm, |RS| = 5 cm, |PS| = 5.5 cm and |PR|= 7 cm

	 **Construction steps:** • Step 1: Draw Δ PQR using SSS construction condition. • Step 2: With P as the centre, draw an arc of radius 5.5 cm. • Step 3: With R as the centre, draw an arc of radius 5 cm. • Step 4: S is the point of intersection of the two arcs. Also, mark S and complete PQRS. PQRS is the required quadrilateral.	
	3.3.1.LI.3	3.3.1.AS.3
	Construct a particular locus for a given condition. **Group discussions:** Learners discuss the concept of loci and draw various loci for triangles and quadrilaterals using the appropriate mathematical and IT tools to boost their interest and desire to solve more problems on their own. **Example 1: Locus Theorems** **Locus Theorem 1:** The locus of points at a fixed distance, d, from the point P is a circle with the given point P as its centre and d as its radius. **Locus Theorem 2:** The locus of the points at a fixed distance, d, from a line, l, is a pair of parallel lines d distance from l and on either side of l.	Level 1 Recall Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

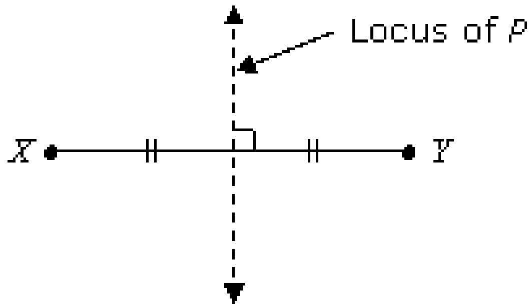
Locus Theorem 3: The locus of points equidistant from two points, P and Q, is the perpendicular bisector of the line segment determined by the two points.
Locus Theorem 4: The locus of points equidistant from two parallel lines, l_1 and l_2, is a line parallel to both l_1 and l_2 and midway between them.
Locus Theorem 5: The locus of points equidistant from two intersecting lines, l_1 and l_2, is a pair of bisectors that bisect the angles formed by l_1 and l_2.

Example 2: Drawing various Loci

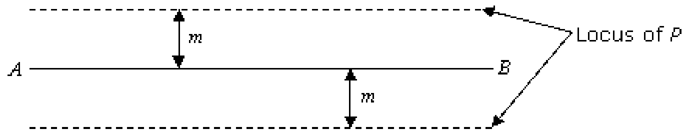
CONDITION 1: A point P moves such that it is always m units from the point Q.
Locus formed: A circle with centre Q and radius m.



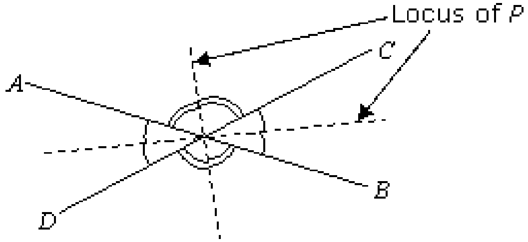
CONDITION 2: A point P moves such that it is equidistant from two fixed points, X and Y.
Locus formed: A perpendicular bisector of the line XY.



CONDITION 3: A point P moves so that it is always m units from a straight-line AB.
Locus formed: A pair of parallel lines m units from AB.

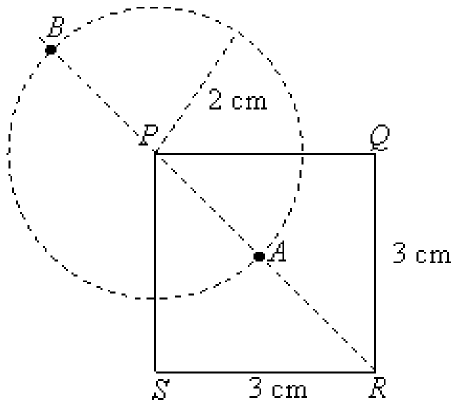


CONDITION 4: A point P moves so that it is always equidistant from two intersecting lines, AB and CD.
Locus formed: Angle bisectors of angles between lines AB and CD.



Group discussions: In convenient groups, solve problems involving Loci.
Example 1: Given a square PQRS with sides 3 cm. Construct the locus of a point that is 2 cm from P and equidistant from PQ and PS. Mark the points as A and B.

Solution: Construct a circle with centre P and radius of 2 cm. Since PQRS is a square, the diagonal PR would be the angle bisector of the angle formed by the lines PQ and PS. The diagonal, when extended, intersects the circle at points A and B.

	 **Examples**: i. A goat is on a lead tethered to a post in the corner of a garden. The lead is 5 m long. A horse is free to roam all parts of the garden but is not allowed within 3 m of the dog by its owner. Show the safe area where the horse can safely roam. ii. A treasure map shows a treasure hidden in a park near a tree and a statue. The map indicates that the tree and the stature are 10 feet apart. The treasure is buried 7 feet from the base of the tree and also 5 feet from the base of the stature. How many places are possible locations for the treasure to be buried? Draw a diagram of the treasure map and indicate with an X each possible location of the treasure. iii. Adobea's backyard has two trees that are 40 feet apart. She wants to place lampposts so that the posts are 30 feet from both of the trees. Draw a sketch to show where the lamp posts could be placed in relation to the trees. How many locations for the lamp posts are possible?	
Teaching and Learning Resources	• Mathematical sets, Graph sheet. • Technology tools such as computers, mobile phones, etc. • Computer software applications like GeoGebra.	

Subject **MATHEMATICS**

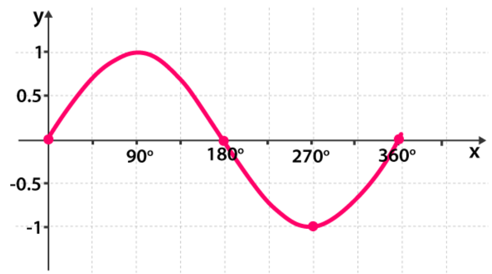
Strand **3. GEOMETRY AROUND US**

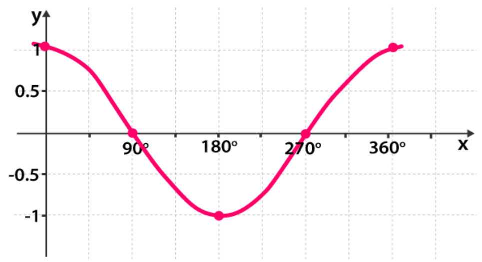
Sub-Strand **2. MEASUREMENT**

Learning Outcomes	21st-century Skills and Competencies	GESI, SEL and Shared National Values
3.3.2.LO.1		
Draw graphs of given trigonometric functions and use them to determine equations and solve related problems.	**Communication and Collaboration:** Learners communicate confidently and effectively to develop appropriate mathematics vocabulary for the concept of trigonometry. **Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on trigonometric functions using appropriate IT tools to boost their interest and desire to solve more problems on their own. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on trigonometry. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on trigonometry. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of the concept of trigonometry to lifelong learning.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: • Respect individuals of different backgrounds in their mathematics groups and beyond. • Embrace diversity and practise inclusion in the mathematics classroom and beyond. • Examine and dispel misconceptions/ myths about gender as they relate to the learning of mathematics, home management and human development. • Interrogate their stereotypes and biases about gender and the role members in a group play in mathematics and in home management. • Identify injustice, especially in recognition of the contributions of different groups and individuals to the effective management and maintenance of the mathematics classroom and home. • Value and promote justice in the mathematics classroom, at home and in society. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-*

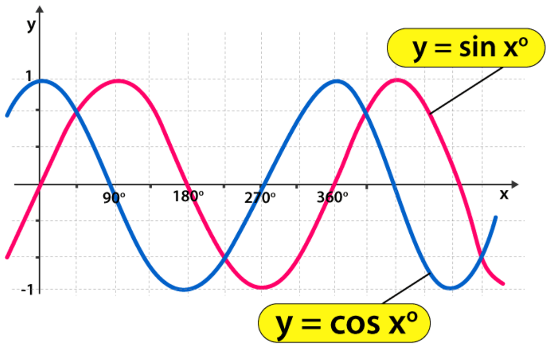
		Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the mathematics curriculum, the facilitator should apply Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and finding confidence · Exhibiting motivation and SMART goal-setting · Managing emotions and conflicts · Showing empathy and cooperation These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork, respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination among learners.

		National Core Values: Develop tolerance, friendliness, open-mindedness, patience, hard work, and humility in learners as they interact with their peers in the mathematics classroom. **Truth and Integrity:** Reward truth and honesty as strong moral principles in the learning of mathematics, leading to responsible citizenship. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment. **Discipline and Honesty:** Encourage learners to behave and work in a controlled way, which involves obeying mathematical rules, principles and standards, leading to self-directed learning.

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
3.3.2.CS.1	3.3.2.LI.1	3.3.2.AS.1
Demonstrate conceptual understanding of trigonometric graphs and use them to solve trigonometric equations.	**Draw graphs of given trigonometric functions and use them to solve related problems.** **Group discussion:** Learners discuss Sine, Cosine and Tangent graphs with examples. Encourage learners to behave and work in a controlled way, which involves obeying mathematical rules, principles and standards, leading to self-directed learning. **Example 1** **What is the plot of sin?** The Sine Function has this beautiful up-down curve (which repeats every 2π radians, or 360°). It starts at **0**, heads up to **1** by π/2 radians (90°) and then heads down to **−1**.  **What is the plot of cosine?** Cosine is just like Sine, but it starts at 1 and heads down until π radians (180°) and then heads up again.	Level 1 Recall **Level 2 Skills of conceptual understanding** Level 3 Strategic reasoning Level 4 Extended critical thinking and reasoning



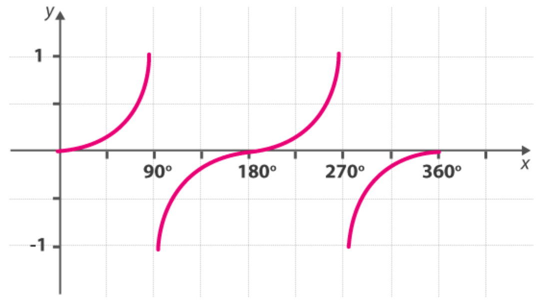
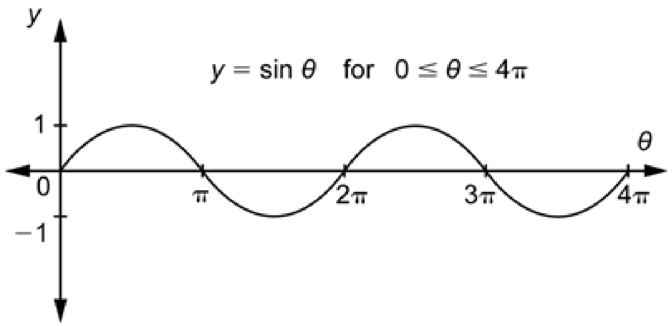
The combined graph of sine and cosine functions can be represented as follows.

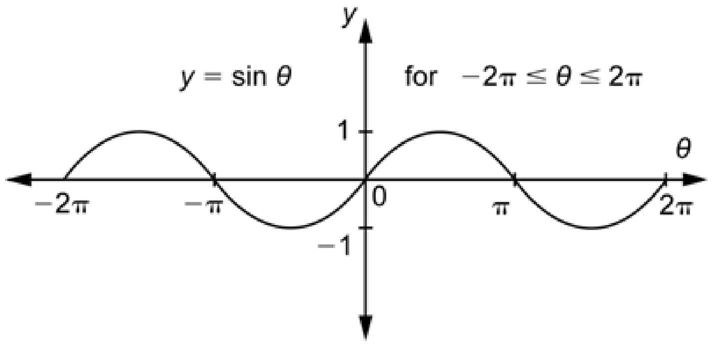


What is the plot of the tangent?

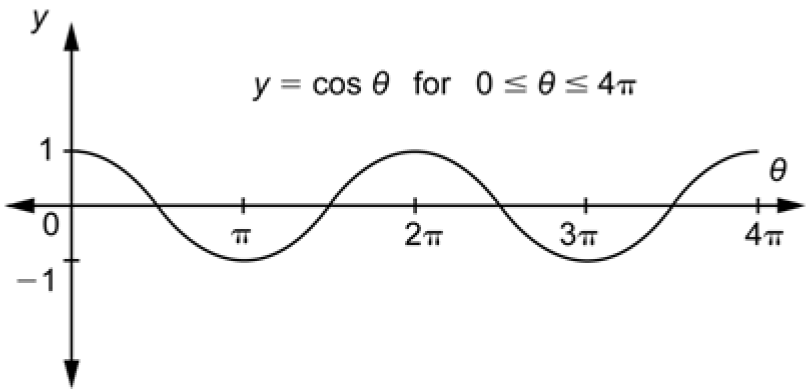
The Tangent function has a completely different shape ... it goes between negative and positive Infinity, crossing through 0 and at every π radian (180°), as shown on this plot.

At π/2 radians (90°), and at −π/2 (−90°), 3π/2 (270°), etc., the function is officially undefined because it could be positive Infinity or negative Infinity.

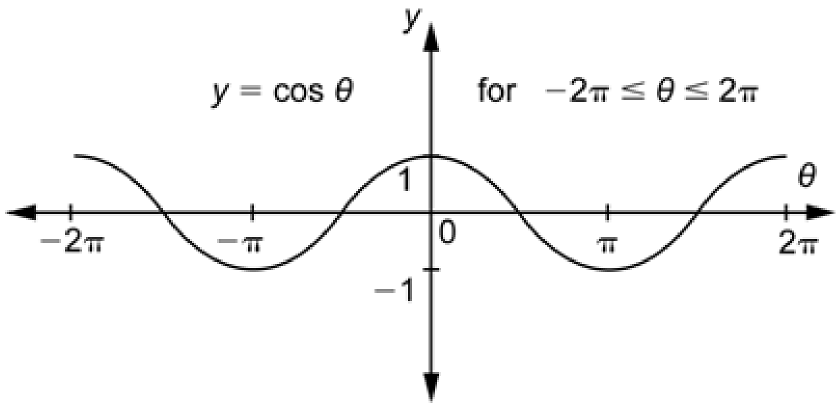
	 **Example 2**: i. Sketch the graph of the sine function on the interval $[\mathbf{0},\ \mathbf{4\pi}]$ and find the range.  The range of $y = \sin\theta$ is $-1 \leq y \leq 1$. ii. Sketch the graph of the sine function on the interval $(-\mathbf{2\pi}, \mathbf{2\pi})$.	



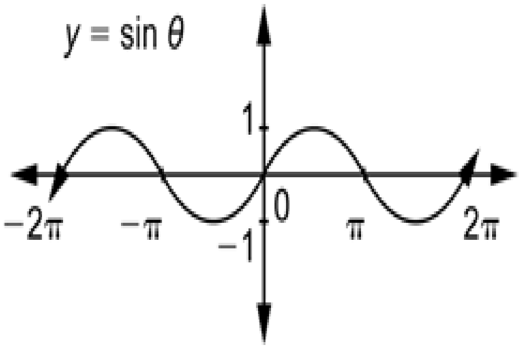
iii. Sketch the graph of the cosine function on the interval $(\mathbf{0}, \mathbf{4\pi})$.

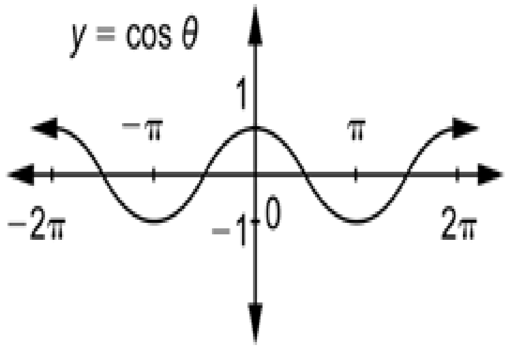


v. Sketch the graph of the cosine function on the interval $(-\mathbf{2\pi}, \mathbf{2\pi})$.



A Comparison of the Graphs of Sine and Cosine
The graphs of sine and cosine both have hills and valleys in a repeating pattern.
Since this repeating pattern can be extended indefinitely to the left and right, the domain for both functions is the real numbers. The range for both of them is the interval.





Group discussions: Learners draw sine, cosine and tangent graphs. Engage learners in the development of healthy and supportive relationships with their peers as they communicate with diverse individuals in their groups.

Plotting a cosine graph

Examples:

i. Sketch the graph of $Y = f(\theta) = \cos\theta$ [$0° \le \theta \le 360°$].

Use your calculator to complete the following table.

Choose an appropriate scale and plot the values of θ on the *x*-axis and $\cos\theta$ on the *y*-axis. Round your answers to 2 decimal places.

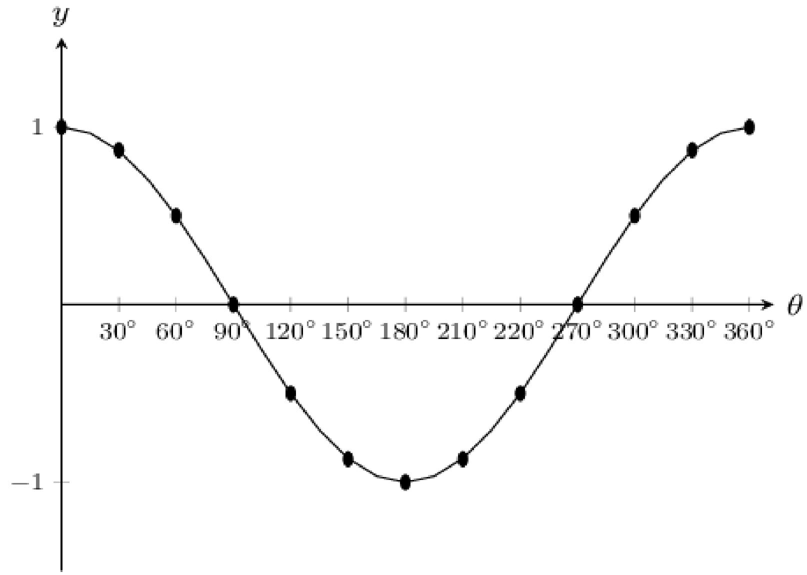
θ	0^0	30^0	60^0	90^0	120^0	150^0	180^0	210^0	270	300^0	330^0	360^0
$\cos\theta$												

Solution

Step 1: Substitute values for θ.

θ	0^0	30^0	60^0	90^0	120^0	150^0	180^0	210^0	240	270^0	300^0	330^0	360^0
$\cos\theta$	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0	0.5	0.87	1

Step 2: Plot the points and join with a smooth curve



Notice the similar wave shape of the graph. The period is also 360°, and the amplitude is 1. The maximum value of $y = \cos \theta$ is 1, and the minimum value is −1.

Domain: [0°; 360°]
Range: [−1; 1]
x-intercepts: (90°; 0), (270°; 0)
Y: (0°; 1)
Maximum turning points: (0°; 1), (360°; 1)
Minimum turning point: (180°; −1)

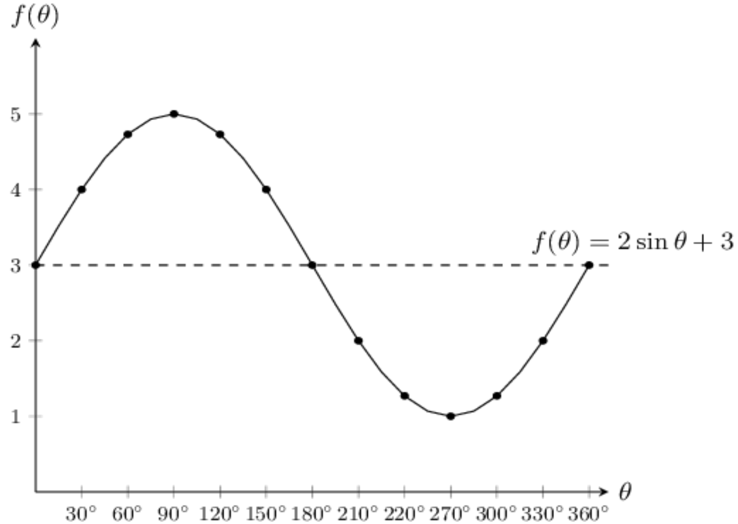
ii. Sketch the graph of $f(\theta) = 2 \sin \theta + 3$ for $\theta \in [0°; 360°]$.

Step 1: Examine the standard form of the equation:
From the equation, we see that $a > 1$, so the graph is stretched vertically. We also see that $q > 0$, so the graph is shifted vertically upwards by 3 units.
Step 2: Substitute values for θ:

θ	0^0	30^0	60^0	90^0	120^0	150^0	180^0	210^0	240^0	270^0	300^0	330^0	360^0

$f(\theta)$	3	4	4.73	5	4.73	4	3	2	1.27	1	1.27	2	3

Step 3: Plot the points and join with a smooth curve



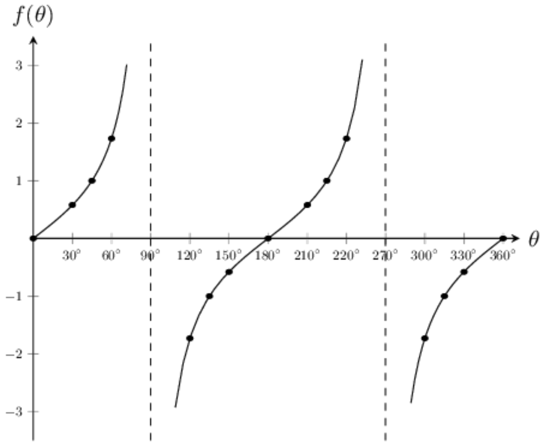
Domain: [0°;360°]
Range: [1;5]
x-intercepts: none
y-intercepts: (0°;3)
Maximum turning point: (90°;5)
Minimum turning point: (270°;1)

iii. Plotting a tangent graph

Sketch the graph of Y = f(θ) = tan θ [0° ≤ θ ≤ 360°]
Use your calculator to complete the following table.
Choose an appropriate scale and plot the values with θ on the x-axis and tan θ on the y-axis. Round your answers to 2 decimal places.

θ	0^0	30^0	45^0	60^0	90^0	120^0	135^0	150^0	180^0
$tan\,\theta$	0	0.58	1	1.73	undef	-1.73	-1	-0.58	0
θ	210^0	235^0	240^0	270^0	300^0	315^0	330^0	360^0	
$tan\,\theta$	0.58	1	1.73	undef	-1.73	-1	-0.58	0	

Plot the points and join them with a smooth curve.



There is an easy way to visualise the tangent graph. Consider our definitions of sin θ and cos θ for right-angled triangles:

So, for any value of θ:

Tan $\theta = \frac{sin\theta}{cos\theta}$

So, we know that for values of θ for which sin θ = 0, we must also have tan θ = 0. Also, if cos θ = 0, the value of tan θ is undefined as we cannot divide by 0. The dashed vertical lines are at the values of θ where tan θ is not defined and are called the asymptotes.

Asymptotes: the lines θ = 90° and θ = 270°

Period: 180°

	Domain: {θ: 0° ≤ θ ≤ 360°,θ ≠ 90°; 270°} Range: {f(θ) : f(θ) ∈R} x-intercepts: (0°;0), (180°;0), (360°;0) y-intercept: (0°;0)	
	3.3.2.LI.2	3.3.2.AS.1
	Use trigonometric graphs to determine equations. In convenient group discussions, demonstrate and task learners to determine equations using trigonometric graphs. Engage learners in the development of healthy and supportive relationships with their peers as they communicate with diverse individuals in their groups.	Level 1 Recall **Level 2 Skills of conceptual understanding** Level 3 Strategic reasoning Level 4 Extended critical thinking and reasoning.
Teaching and Learning Resources	• Mathematical sets, Graph sheet. • Technology tools such as computers, mobile phones, etc. • Computer software applications like GeoGebra.	

Subject **MATHEMATICS**

Strand **4. MAKING SENSE OF AND USING DATA**

Sub-Strand **1. STATISTICAL REASONING AND ITS APPLICATION IN REAL LIFE**

Learning Outcomes	**21st-century Skills and Competencies**	**GESI, SEL and Shared National Values**
3.4.1.LO.1		
Establish simple mathematical relationships between two variables in a given observational or experimental context; illustrate using scatter graphs and use them to solve and/or pose problems.	**Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on data handling involving simple mathematical relationships of bivariate data. **Integrated Problem-solving Competency:** Engage learners in different problem-solving processes to develop viable, inclusive, and equitable solution options that promote sustainable learning outcomes as they engage in activities on data handling involving simple mathematical relationships of bivariate data. **Innovation and Creativity:** Make conscious efforts to enable learners develop and implement innovative and creative actions that reflect their level for application of data handling involving simple mathematical relationships of bivariate data.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: • Respect individuals of different backgrounds in their groups. • Examine and dispel misconceptions/ myths about gender as they relate to the teaching and learning of mathematics. • Interrogate their stereotypes and biases about gender and the role members in a group play in mathematics and in home management. • Value and promote justice in the mathematics classroom, at home and in society. **SEL:** Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in
3.4.1.LO.2		
Compare different datasets and use mathematical vocabulary and language (concretely, pictorially, symbolically, spoken or written, etc.) to contribute		

to mathematical and everyday discussions using valid and reliable data to explain and justify the information produced from the data.		the mathematics curriculum, the facilitator should apply the Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and finding confidence · Exhibiting motivation and SMART goal-setting · Managing emotions and conflicts · Showing empathy and cooperation These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities.. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork, respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment. **Equity:** Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination among learners.

		National Core Values: Develop tolerance, friendliness, open-mindedness, patience, hard work, and humility in learners as they interact with their peers in the mathematics classroom. **Truth and Integrity:** Reward truth and honesty as strong moral principles in the learning of mathematics, leading to responsible citizenship. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment. **Discipline and honesty:** Encourage learners to behave and work in a controlled way, which involves obeying mathematical rules, principles and standards, leading to self-directed learning.

<table>
<tr><th>Content Standards</th><th>Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI</th><th>Assessment</th></tr>
<tr><td>3.4.1.CS.1</td><td>3.4.1.LI.1</td><td>3.4.1.AS.1</td></tr>
<tr><td>Demonstrate understanding of data handling involving simple mathematical relationships of bivariate data in observational and experimental contexts.</td><td>

Establish simple mathematical relationships between two variables in a given observational or experimental context; illustrate using scatter graphs and use them to solve and/or pose problems.

Encourage learners to use technology Literacy Skills, combined problem-solving competency and critical thinking skills to collect data from an observational study in which, for example, the interest is the relationship between weight and height to establish mathematical relationships of bivariate data in observational and experimental context and use them to solve related problems.

Project-based Learning: In convenient groups, task learners to collect data of interest within or outside the school community (including using social media platforms or any technological means for collecting data). Provide opportunities for students to evaluate various real-world scenarios and make decisions based on the data at hand.

Examples of data collected are presented in the tables below.

<table>
<tr><th>Meat (kg)</th><th>Price</th></tr>
<tr><td>1</td><td>14</td></tr>
<tr><td>2</td><td>28</td></tr>
<tr><td>3</td><td>32</td></tr>
<tr><td>5</td><td>70</td></tr>
</table>

<table>
<tr><th>Litres</th><th>Km Driven</th></tr>
<tr><td>20</td><td>160</td></tr>
<tr><td>30</td><td>240</td></tr>
<tr><td>45</td><td>360</td></tr>
<tr><td>50</td><td>400</td></tr>
</table>

<table>
<tr><th>Score</th><th>Frequency</th></tr>
<tr><td>10</td><td>2</td></tr>
<tr><td>20</td><td>6</td></tr>
<tr><td>35</td><td>4</td></tr>
<tr><td>50</td><td>3</td></tr>
</table>

i.
ii.
iii.
iv.
v.

i. Identify which table does **not** show bivariate data.
ii. Identify the independent and dependent variables in the tables that show bivariate data.
iii. What effect has the number of litres of fuel used on the number of kilometres driven? (Learners should note the relationship between the two variables.)

</td><td>Level 1 Recall
Level 2 Skills of conceptual understanding
Level 3 Strategic reasoning
Level 4 Extended critical thinking and reasoning</td></tr>
</table>

	iv. Can any comparison be made between Score and Frequency in Table C? [**Note** in this case, though the frequencies are not the same, there is (i) one variable – univariate and (ii) no relationship between Score and Frequency.] **Example**: The bivariate data presented in the table below shows the hours studied and the percentage scores of two variables—-independent and dependent – respectively obtained in a statistics course by 10 learners.	

Learner	Hours Studied(h)	Test Score(s)
Gifa	3	90
Kewo	1	86
Dauda	5	84
Ekow	4	92
Kapio	3	91
Alhassan	5	100
Serwaa	0	76
Ada	1	82
Fofio	2	85
Baaba	1	77

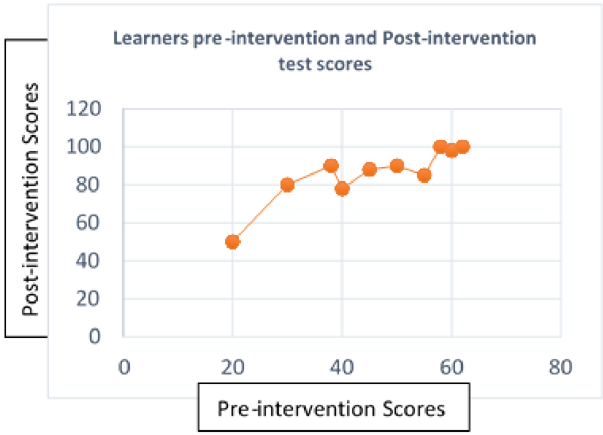
i. Place the information on a graph sheet (scatter plot) by plotting each learner as an ordered pair with Hours Studied on the x-axis and Test Score on the y-axis.
ii. Discuss the scatter plot, find the relationship between hours studied and test score, draw their conclusion and justify it.
iii. Pose questions based on the analyses.

<table>
<tr><td></td><td>3.4.1.LI.2</td><td>3.4.1.AS.2</td></tr>
<tr><td></td><td>

Collect data from an experimental study in which the interest is based on treatment and non-treatment (control) groups. Illustrate the data using scatter graphs and find the relationship between the variables, if any.

Experiential and Project-based Learning: In convenient groups, task learners to interpret data presented in graphs (scatter plots), draw conclusions and give justification for the conclusions.

Example 1: A reading test is given to 10 learners in Basic 6. They then participated in an extensive reading programme. After participating in the programme (group manipulated), they were retested. The data collected was organised and plotted as a scatterplot (the ordered pair of scores for each learner) as follows:



In small groups, study the scatterplot (using the skills for plotting and interpreting points on a graph sheet), find the relationship between Pre-intervention Reading Test Scores and Post-intervention Reading Test Scores, do a comparison, draw a conclusion and justify the conclusion.

Example 2: The blood sugar level of 10 learners is tested before and after an exercise session. The bivariate (i.e., two variables – independent and dependent) data collected are organised and presented in the table below:

</td><td>

Level 1 Recall
Level 2 Skills of conceptual understanding
Level 3 Strategic reasoning
Level 4 Extended critical thinking and reasoning

</td></tr>
</table>

Age	Sex	Blood Sugar Level before the Exercise (mmol/L)	Blood Sugar Level after the Exercise (mmol/L)
12	F	9.0	8.1
11	M	8.5	7.5
13	M	10	8.7
12	F	7.2	6.6
12	F	9.5	8.1
11	M	12.0	10.8
13	F	8.0	6.9
12	M	16.0	14.3
14	F	7.5	6.7
11	M	9.0	7.5

i. Do a scatterplot of the bivariate data (you may round off the blood sugar levels to the nearest whole numbers).

ii. What is the relationship between the Blood Sugar Level before and after the exercise sessions?

Teaching and Learning Resources

- Samples of bivariate data.
- Computer application software such as MS Excel, MS Word, Wordpad, etc.
- Mathematical sets.
- Technology tools such as computers, mobile phones, etc.
- Graph sheets
- A computer with data-managment software like MS Excel, MS PowerPoint, etc.,
- A4 and A3 papers, manila cards, flip charts, markers, colour pens, etc.

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
3.4.1.CS.2	3.4.1.LI.1	3.4.1.AS.1
Demonstrate the ability to compare different data sets and use appropriate vocabulary to contribute to mathematical and everyday discussions and make inferences about the information gathered.	**Compare the quality of different datasets using knowledge and skills acquired from data analysis techniques you learned in earlier statistics work.** **Experiential Learning:** In convenient groups, tasks learners to compare the quality of given data sets and draw conclusions on the appropriateness of the data in relation to the purpose of the data and its analysis techniques. **Example**: Data was taken on the heights of twelve SHS 3 learners in Joaboso Senior High School. The tables below show how two researchers recorded the same data (heights of the students in centimetres and in metres rounded to the nearest tenth).	Level 1 Recall Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** **Level 4 Extended critical thinking and reasoning**

A	
Student	Height (cm)
1	170.1
2	173.4
3	174.1
4	175.1
5	175.9
6	173.2
7	175.0
8	175.2
9	175.6
10	173.0
11	175.7

B	
Student	Height (m)
1	1.7
2	1.7
3	1.7
4	1.8
5	1.8
6	1.7
7	1.8
8	1.8
9	1.8
10	1.7
11	1.8

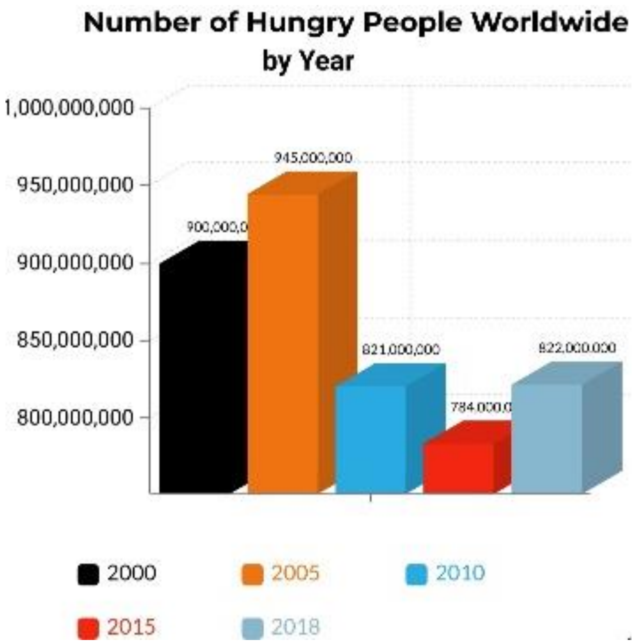
12	175.2	12	1.8

i. Which of the two data sets gives a better representation of the heights of the students?

ii. Which of the data sets will aid easy analysis and interpretations, and why?

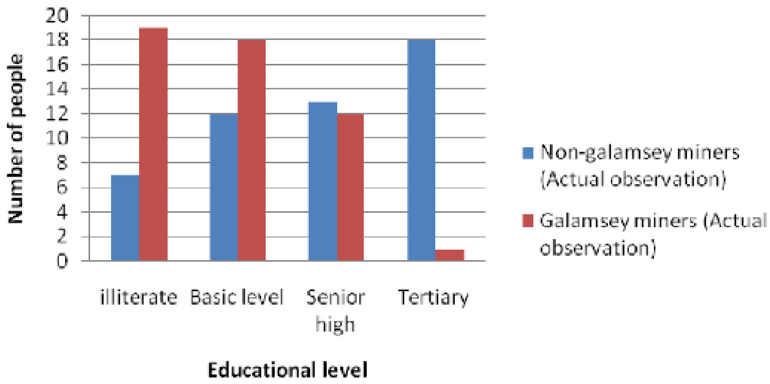
Using think-pair-share activities, task learners to make inferences from a given data and give reasons for their choices. Offer other learners the opportunity to make their own inferences as long as they are referring to the data.

Example: The chart below shows the number of hungry people in the world by year from 2000 to 2018.



	i. From the chart, what is your opinion on the trend of hunger across the world? ii. What will be your recommendations to world leaders on the strategies they can adopt to combat the menace? **Using Talk for Learning strategy,** engage learners in a class debate where they support their argument with data and the use of appropriate mathematics terminology. **Example**: As a speaker in a school debate on the topic: "The rise of HIV/AIDs cases in Ghana, a course for worry or a mere public scare", obtain data on HIV/AIDs cases from the Ghana AIDs Commission or from any reputable source (radio, TV, Newspaper, etc.), analyse the data (including using any appropriate technological tool) using charts/tables to support your arguments for or against the motion. **Project-based Learning:** Assign project works to learners (individually or in groups) to obtain current data about contemporary issues of interest, then make conclusions, give criticisms and make useful recommendations. As they embark on the project, encourage them to be tolerant, friendly, open-minded, patient, hardworking, etc., as they interact with their peers in their assigned project. **Example**: Using a mind map such as the one below, learners must think critically, design a mini-project, and execute it to its successful conclusion.	

	 	
	3.4.1.LI.2	3.4.1.AS.3
	Discuss data information published in local and/or international media platforms (local and international TV stations, newspapers, journals, etc.) by making useful inferences from the data to draw conclusions and/or to explain other phenomena. **Group discussions:** In small groups, learners discuss data published on a local media platform and draw conclusions with justifications. Encourage tolerance among learners as they work in groups by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment. **Example**: As part of a study to understand the impact and effect of illegal mining (galamsey) towards the socio-economic development of mining communities, a case study was carried out in Kenyasi in the Brong Ahafo Region. The chart below shows the educational background of the respondents interviewed for the study.	**Level 1 Recall** Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

	 Interpret the information from the chart and make some conclusions and recommendations.	
Teaching and Learning Resources	• Samples of bivariate data. • Computer application software such as MS Excel, MS Word, Wordpad, etc. • Mathematical sets. • Technology tools such as computers, mobile phones, etc. • Graph sheets • A computer with data-managment software like MS Excel, MS PowerPoint, etc., • A4 and A3 papers, manila cards, flip charts, markers, colour pens, etc.	

Subject **MATHEMATICS**

Strand **4. MAKING SENSE OF AND USING DATA**

Sub-Strand **2. PROBABILITY/CHANCE**

Learning Outcomes	**21st-century Skills and Competencies**	**GESI, SEL and Shared National Values**
3.4.2.LO.1		
Explain how a given probability from print and electronic media influences individual decisions; select and analyse real-life data to solve problems involving the probability of two dependent and independent events.	**Communication and Collaboration:** Learners communicate confidently and effectively to develop appropriate mathematics vocabulary for the probability of two dependent and independent events through teamwork. **Technology Literacy Skills:** Initiate mathematical thinking process to solve challenging problems on probability using appropriate IT tools to boost their interest and desire to solve more problems on their own. **Critical Thinking:** Create sustainable discourse for learners to question norms, practices, and opinions; to reflect on one's own values, perceptions and actions for decision-making as they engage in group and individual activities on probability concepts. **Innovation and Creativity:** Make conscious efforts to enable learners to develop and implement innovative and creative actions that reflect their level for application of the concept of probability to lifelong learning.	**GESI:** Learners having experienced a teaching approach that ensures gender equality and social inclusion, where they work with each other in an inclusive way; cross-sharing knowledge and understanding among groups and individuals lead them to: • Respect individuals of different backgrounds in their mathematics classroom and beyond. • Examine and dispel misconceptions/ myths about gender as they relate to their mathematics classroom, home management and human development. • Interrogate their stereotypes and biases about gender and the role men and women play in the mathematics classroom and home management. • Identify injustice, especially in recognition of the contributions of different groups and individuals to the effective management of their groups and the general mathematics classroom and maintenance of the home. • Sensitive to the interrelatedness of the various aspects of life through the application of the various mathematics concepts to real life. • Value and promote justice in the mathematics classroom, at home and in society.

		SEL: Creating opportunities for learners to build their Social Emotional Learning Competencies - *Self-Awareness, Self-Management, Social Awareness, Relationship Skills and Responsible Decisions* are integrated throughout all lessons to encourage inclusion. As part of achieving each learning outcome in the mathematics curriculum, the facilitator should apply Social Emotional Learning strategies to ensure that learners are: · Self-reflecting and finding confidence · Exhibiting motivation and SMART goal-setting · Managing emotions and conflicts · Showing empathy and cooperation These may be done by the facilitator through modelling emotional self-regulation and decision-making, promoting positive self-talk with self-made portraits, creating a vision board, creating respectful icebreakers for healthy debates, encouraging diversity presentations, and learners writing on the sequence of their activities. **National Core Values:** **Leadership and Respect for others' views:** Inculcate the habit of leadership through teamwork respect for individuals' views, beliefs, religions, and cultures through interactive and collaborative/group work. **Diversity**: Promote respect for divergent views to ensure inclusivity in the mathematics learning environment.

		Equity: Develop fair and impartial opportunities or resources for learners devoid of unwanted segregation or discrimination among learners. **National Core Values**: Develop tolerance, friendliness, open-mindedness, patience, hard work, and humility in learners as they interact with their peers in the mathematics classroom. **Truth and Integrity:** Reward truth and honesty as strong moral principles in the learning of mathematics, leading to responsible citizenship. **Tolerance:** Model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping within a differentiated mathematics classroom instruction and assessment. **Discipline and honesty:** Encourage learners to behave and work in a controlled way, which involves obeying mathematical rules, principles and standards, leading to self-directed learning.

Content Standards	Learning Indicators and Pedagogical Exemplars with 21st-century Skills and Competencies, and GESI	Assessment
3.4.2.CS.1	3.4.2.LI.1	3.4.2.AS.1
Demonstrate an understanding of the role of probability in society and apply probability reasoning to analyse and make useful predictions about everyday life events.	**Identify examples from print and electronic media, e.g., newspapers, television, and the Internet, where probability is used, and explain how the given probability influences individual decisions.** Encourage learners to use their communication skills, combined problem-solving competencies and critical thinking skills to discuss the role of probability in society by applying probability reasoning to investigate and make useful guesses about everyday life events and use them to make judgements and suitable decisions in their everyday activities. **Group discussions:** In small groups, task learners to list and present with an explanation, at plenary, some decisions that point to uncertainties/certainties of everyday life. **Examples**: i. Going out with or without an umbrella, the safety of crossing a road, the chance of dying in an accident on a particular stretch of a road/highway, and so on. ii. Before planning for a picnic, you check the weather forecast, and it says there is a 60% chance (probability) that rain may occur. Discuss and provide answers to the following questions: a) What does this probability mean? b) How was the 60% determined? c) What are the things taken for granted in determining the probability (assumptions) and/or anything that could change the forecast (limitations), if any? d) How will it influence your decision on the planned picnic? iii. Discuss and provide answers to the following questions. (Interpret and explain the answers, indicating the assumptions and limitations involved, if any. (Refer to E.g., 2) a) How do political analysts predict a certain political party to come into power? b) Flipping a coin is one of the most important events before the start of a football match. What is the chance or the probability of your team getting the desired outcome?	**Level 1 Recall** Level 2 Skills of conceptual understanding **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

	c) As an active smoker, the chances (probability) of getting lung disease are higher in you. Aware of this fact, which insurance scheme will you go for health, vehicle or house insurance? d) Nowadays, people are using mobile phones, and the chances of their mobile phones getting damaged or lost are high. Do you think they should consider getting insurance for their expensive mobile phones?	
	3.4.2.LI.2	3.4.2.AS.2
	Solve everyday life problems involving the probability of two dependent and independent events, including addition and multiplication laws. Encourage learners to use their communication skills, combined problem-solving competencies and critical thinking skills to discuss and solve everyday life problems involving the probability of two dependent and independent events using addition and multiplication laws. **Examples** i. A card is chosen at random from a standard deck of 52 playing cards. Without replacing it, a second card is chosen. What is the probability that the first card chosen is a queen and the second card chosen is a jack? ii. Mr. Mills needs two students to help him with a science demonstration for his class of 15 girls and 13 boys. He randomly chooses one student who comes to the front of the room. He then chooses a second student from those still seated. (Note that the sample space of the dependent event will change.) What is the probability that both students chosen are girls? iii. In a shipment of 20 computers, 3 are defective. Three computers are randomly selected and tested. What is the probability that all three are defective if the first and second ones are not replaced after being tested? iv. A school has 200 seniors, of whom 140 will be going to college next year. Forty will be going directly to work. The remainder are taking a gap year. Fifty of the seniors going to college play sports. Thirty of the seniors going directly to work play sports. Five of the seniors taking a gap year play sports. What is the probability that a senior is taking a gap year?	**Level 1 Recall** **Level 2 Skills of conceptual understanding** **Level 3 Strategic reasoning** Level 4 Extended critical thinking and reasoning

	v. Studies show that about one woman in seven (approximately 14.3%) who live to be 90 will develop breast cancer. Suppose that of those women who develop breast cancer, a test is negative 2% of the time. Also, suppose that in the general population of women, the test for breast cancer is negative about 85% of the time. Let B = woman develops breast cancer and let N = tests negative. Suppose one woman is selected at random. a) What is the probability that the woman develops breast cancer? What is the probability that the woman tests negative? b) Given that the woman has breast cancer, what is the probability that she tests negative? c) What is the probability that the woman has breast cancer AND tests negative? d) What is the probability that the woman has breast cancer or tests negative? e) Are having breast cancer and testing negative independent events? f) Are having breast cancer and testing negative mutually exclusive?	
Teaching and Learning Resources	• Manipulative (dice, coins, spinners, playing cards, counters, digit cards) • Simple Probability Mazes (Printable & Digital) • Worksheets • Task Cards • Mathematical sets	