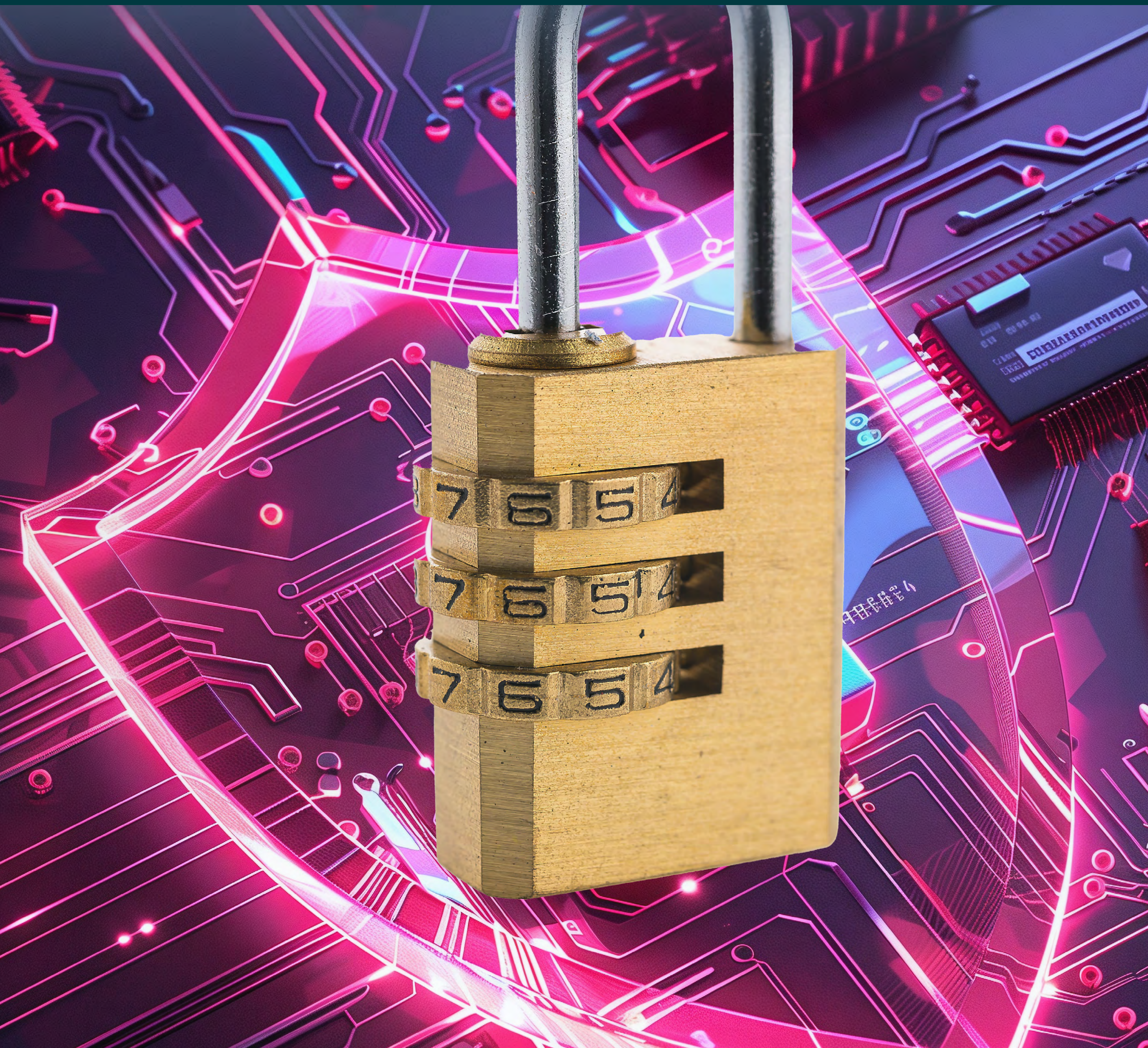
Additional Mathematics Year 1

SECTION

10 COMBINATIONS, PERMUTATIONS AND PROBABILITY



HANDLING DATA

Making Predictions with Data

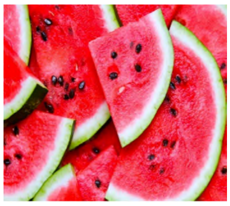
INTRODUCTION

Permutations, Combinations and Probability are three fundamental concepts in mathematics. They have a wide range of practical applications in everyday life from organising events to predicting outcomes. These help us make informed decisions and solve complex problems. These topics, with roots tracing back to the 17th-century mathematicians Pierre de Fermat and Blaise Pascal, provide a powerful language for understanding and analysing chance and possibility in everyday life.

Have you ever wondered how many different ways you can arrange books on a shelf or make a selection from different fruits to eat?

Books on a shelve

Banana



Water melon

Pawpaw

Figure 1: Items displayed to show different ways of arrangements or selection

What about selecting a team from a group of friends or determining the chances of winning a lottery? These are all scenarios where permutations, combinations and probability come into play. In this section, we will explore how to calculate these values and understand their significance through engaging examples. By the end, you will be able to apply these concepts to real-life problems, enhancing both your mathematical reasoning and decision-making skills.

At the end of this section, you will be able to:

- Use the fundamental counting principle to identify and determine the number of ways an event can occur
- Understand the concept of permutation and combination and use it to solve related problems.

- Distinguish between the concepts of permutation and combination and establish the relation between them.
- Simplify permutation and combination expressions and solve related problems
- Recall the basic terminologies such as experiment, events, outcome, trial, sample space etc. as used in the context of probability and give examples
- Work collaboratively on an experiment (e.g. tossing of coins or dice) to determine the relative frequencies of events and interpret them

Key Ideas

- **Permutations** help us understand how many different ways we can arrange items when the order is important.
- **Combinations** allow us to find how many ways we can choose items when the order does not matter.
- **Probability** gives us the tools to measure how likely an event is to occur.
- **Experiment:** An operation or process performed to observe a result or an outcome or to obtain measurements like flipping a coin (= the experiment) to see if it lands on heads or tails (= the outcome). The outcome of an experiment is not certain and thus cannot be definitive. Examples include:
  i. Tossing a coin
  ii. Rolling a die
  iii. Drawing a card from a set of playing cards
  iv. Drawing items from a bag or a bowl.
- **Random experiment:** An experiment whose outcome or results cannot be predetermined. Each outcome has an equal chance of occurring. Imagine drawing a bead from a bag full of different coloured beads (= the random experiment). You don't know which colour (= the outcome) you will get beforehand.
- **Trial:** A single performance of an experiment. Each kick in the penalty shootout is a separate trial, or selecting a single fruit from the market stall is one trial

- **Event (E)**: A specific set of outcomes you are interested in. In a football penalty shootout (experiment), scoring a goal (outcome) is an event you might be interested in.
- **Outcome:** The specific result of a single trial in an experiment. Each kick in the penalty shootout (experiment) has a specific outcome - goal (success) or miss (failure).
- **Sample Space (S)**: The collection of all possible outcomes for an experiment. For tossing a coin, the sample space is {heads, tails}. At a market stall selling apples, oranges and pineapples (experiment), the sample space is {apple, orange, pineapple}.
- **Mutually exclusive events:** Two events A and B are said to be mutually exclusive if event A and event B cannot happen simultaneously
- **Independent events:** Event A is said to be independent of event B if the occurrence of A does not affect the probability of the B

FUNDAMENTAL COUNTING PRINCIPLE (MULTIPLICATION RULE)

Figure 2: Athletes running a race

Look at the picture, how were the athletes arranged in the lanes fairly for the competition? This and many other related questions on how arrangement and selections are done will be the focus. The concept is applied in:

1. Menu and recipe planning
2. scheduling and time management
3. voting systems
4. password and security
5. genetics

Application of fundamental principles of counting Ordered and unordered arrangements

How do you arrange your books on your shelf? How do you arrange your clothes in your wardrobe? The answer lies in counting rules.

Consider the Table 1 below, which represents the tossing of coin (H/T) and die (1, 2, 3, 4, 5, 6) once.

Table 1: Probability Experiment-Tossing a coin and a die

		1	2	3	4	5	6
	H	H,1	H,2	H,3	H,4	H,5	H,6
	T	T,1	T,2	T,3	T,4	T,5	T,6

How many ways can both events occur? The table shows us that there are 12 different outcomes, so 12 different ways.

Let us look at another scenario

This time we have 4 cards with the letters A, B, C and D and 7 cards with the numbers 1 to 7.

Table 2: Probability Experiment-Drawing a card or a number

	1	2	3	4	5	6	7
A	A1	A2	A3	A4	A5	A6	A7
B	B1	B2	B3	B4	B5	B6	B7
C	C1	C2	C3	C4	C5	C6	C7
D	D1	D2	D3	D4	D5	D6	D7

How many ways can this event occur? Table 2 shows us that there are 28 different ways.

Drawing tables can be time consuming, so we need to find a quicker method. In the top example there are $2 \times 6 = 12$ outcomes and in the bottom example, $4 \times 7 = 28$ outcomes. We can generalise this:

Mathematically, if event A can occur in $\boldsymbol{m}$ different ways and event B can occur in $\boldsymbol{n}$ different ways, then the total number of ways for both events to happen together is to multiply the two different ways, i.e. $\boldsymbol{m} \times \boldsymbol{n}$**.**

If the same elements occurs in different ways then m=n and we have $\boldsymbol{n} \times \boldsymbol{n} = \boldsymbol{n}^2$.

For example, if a die (which has 6 elements) is thrown twice, the total number of ways will be $6 \times 6 = 6^2 = 36$

Example 1

How many possible outcomes are there if a coin (heads/tails) is flipped and a card, numbered from 1 to 10 is selected.

Solution

2 (coin) × 10 (numbered cards) = 20 combinations.

Example 2

Three SHS1 students, Naa, Afiba and Malik, are to be photographed. How many different ways are there to arrange them on 3 chairs?

Solution

Let Naa be represented by N, Afiba by A and Malik by M.

Table 3: Probability Experiment-Arranging chairs

Arrangements	First Chair	Second Chair	Third Chair
1	N	A	M
2	N	M	A
3	A	M	N
4	A	N	M
5	M	A	N
6	M	N	A

There are 6 possible arrangements.

Note, that in an *arrangement* order matters. If you were simply *selecting* them for the photo then there is only 1 way as order does not matter.

Example 3

You took three books Mathematics (M), English Language (E) and Home Economics (H) for prep. In how many ways can the subjects be:

a. arranged

b. be selected

Solution

a. for arrangement, it means *order matters*.

MEH
MHE
EMH
EHM
HME
HEM

The books can be arranged in 6 ways.

b. for selection *order is not important.*

MEH

The books can be selected in 1 way.

In the above examples, where the order of events was important, we use permutations and where order was not important we use combinations. This gives us two forms of arrangements: ordered arrangement and unordered arrangements.

Ordered Arrangements

When the order of events matter, we use **permutations**. Imagine arranging four friends in a line for a photo. Each person has different positions they can occupy. The number of arrangements, considering order, is calculated using factorials ($n!$). For *four*(4) friends, there would be $4! = 24$ unique orderings. We can also use the permutation button on our calculators $^{\mathbf{n}}\mathbf{P}_{\mathbf{r}}$.

Unordered Arrangements

If the **order** doesn't matter, we use **combinations**. For example, choosing a fruit salad from a variety of fruits. Whether you pick an apple first or a banana doesn't change the combination. Combinations are calculated using a formula involving factorials and selections and we can also use the combination button on our calculators, ^{n}Cr.

Activity 10.1: Exploring Arrangements with Coloured Cards

This activity is to introduce the concept of arrangements and counting by exploring how to arrange coloured cards in different positions.

Materials Needed: 3 coloured cards (e.g., Red, Blue, Blue); Paper, pen and pencils for recording results

Arranging the three colours at a time

i. In your small groups or individually;

ii. Using the three coloured cards, arrange them in every possible order so that each colour occupies every position (1st, 2nd, and 3rd).

iii. Record each arrangement you create.

iv. After arranging the cards, count the total number of unique arrangements you found.

v. Share your results with the entire class. How many different arrangements did you find?

vi. Discuss the method you used to ensure all arrangements were counted.

Arranging Two Colours at a Time

i. Now, choose any two colours from the three coloured cards.

ii. Arrange these two colours in the 1st and 2nd positions while leaving the 3rd position empty.

iii. Record all possible arrangements for your chosen two colours.

iv. Count the total number of unique arrangements for the two colours.

v. Share the total outcomes with the class. How many arrangements did you find when using only two colours?

vi. Discuss how this compares to the previous activity with all three colours.

Let us go through the following examples to explore how objects are arranged.

Example 4

Four athletes, A, B, C, D are to be arranged in the four lanes (1, 2, 3, 4) for a competition.

Figure 4: A Running Track with 4 lanes

In how many ways can this be done?

Solution

If A goes first, he or she has 4 lanes available

B will have 3 lanes because A has occupied one already

C will have 2 lanes available

D will have 1 lane available

Multiplying all the lanes available to them, we obtain

$4 \times 3 \times 2 \times 1 = 4! = 24$

Example 5

A girl has three different skirts (P, V, R) and 5 different tops (B, W, I, Y, L). If she can wear each skirt with each top, how many different outfits could she make?

Solution

One way is to use table to do the pairing

Table 4: Probability Experiment-Making different outfits from skirts and tops

	B	W	I	Y	L
P	P,B	P,W	P,I	P,Y	P,L
V	V,B	V,W	V,I	V,Y	V,L
R	R,B	R,W	R,I	R,Y	R,L

This shows us that there are 15 ways

Alternatively, we could multiply the number of skirts by the number of tops.

$3 \times 5 = 15$

Example 6

In how many ways can you select 3 books from 7 different books.

Solution

Step 1: Out of 7 books, the first book can be selected in 7 ways

Step 2: The second book can be selected 6 ways for the remaining 6 books

Step 3: The third book can be selected 5 ways for the remaining 5 books

Therefore, selecting 3 books out of the 7 books can be done in $7 \times 6 \times 5 = 210$ different ways.

Example 7

How many numbers of 3 different digits can be formed with no repetition by choosing from the digits 1, 2, 3, 4 and 5?

Solution

Step 1: choosing the hundred's digit: we can do this in 5 ways

Step 2: choosing the ten's digit: we can do this in 4 ways since one digit is fixed as a hundred digit.

Step 3: choosing the one's digit: this can be done in 3 ways because two digits are fixed as hundred and ten digits respectively

Therefore, this can be done in $5 \times 4 \times 3 = 60$ ways.

Example 8

Kwesi is setting a combination lock with 3 dials. Each dial has 5 digits (0-4). How many unique 3-digit code combinations are possible if repetition of the numbers is allowed?

Solution

Since each dial has 5 options, the total number of unique codes is achieved by multiplying the number of options for each dial:

$5\ (choices\ for\ dial\ 1) \times 5\ (choices\ for\ dial\ 2) \times 5\ (choices\ for\ dial\ 3) = 125\ possible\ codes.$

PERMUTATIONS AND COMBINATIONS

Permutations come into play when order matters. Imagine selecting your starting line-up for a crucial inter-school football match. The order you choose your players – striker first, midfielder next – significantly impacts your team's strategy. Permutations help us calculate the total number of unique ways to arrange players based on their positions. For instance, AB is not the same as BA and APC differs from PAC. Combinations, on the other hand, focus on situations where order doesn't matter. In the traditional kente scenario given earlier, the final design of the kente is not affected by picking the colour gold first or the colour blue second and vice versa. Combinations help us determine the total number of unique colour combinations possible for the kente, regardless of the order you choose the colours. The concept is applied in:

1. game theory
2. password generation
3. cryptography
4. committee formation

Permutations

The general rule for the number of permutations of n different items taking all at once is $n!$ Before we solve more examples on finding permutation of objects, let us revise the concept of factorial, $n!$. We were introduced to the concept of factorial notation under binomial theorem where it was observed that $n! = n(n-1)(n-2)(n-3)\ldots\ldots$

Don't forget that $\mathbf{0}! = 1$

Example 9

Evaluate the following and verify your answers using your calculator

a. $3!$

b. $6!$

c. $5!$

d. $8!$ _ $6!$

e. $7!$ _ $9!$

Solution

a. $3! = 3(3-1)(3-2) = 3 \times 2 \times 1 = 6$

b. $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

c. $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

d. $\frac{8!}{6!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = 8 \times 7 = 56$

e. $\frac{7!}{9!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{1}{72}$

Example 10

Evaluate:

a. $\frac{n!}{(n-1)!}$ **b.** $\frac{(n+1)!}{(n-1)!}$ **c.** $\frac{n!}{(n+2)!}$ **d.** $\frac{(n-1)!}{(n-2)!}$

Solution

a. $\frac{n!}{(n-1)!} = \frac{n(n-1)(n-2)\ldots}{(n-1)(n-2)\ldots} = n$

b. $\frac{(n+1)!}{(n-1)!} = \frac{(n+1)n(n-1)(n-2)\ldots}{(n-1)(n-2)\ldots} = (n+1)n$

c. $\frac{n!}{(n+2)!} = \frac{n(n-1)(n-2)\ldots}{(n+2)(n+1)n(n-1)(n-2)\ldots} = \frac{1}{(n+2\_(n+1)}$

d. $\frac{(n-1)!}{(n-2)!} = \frac{(n-1)(n-2)(n-3)\ldots}{(n-2)(n-3)\ldots} = n - 1$

Now let us use this in permutations:

Example 11

1. Find the number of permutations or unique arrangements of the letters in the following words

a. AYO

b. OWARE

c. CANOE

Solution

a. AYO $= 3! = 3 \times 2 \times 1 = 6$

b. OWARE $= 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

c. CANOES $= 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

Example 12

Kweku has 4 different mathematic books on his shelf. Find the number of ways in which the 4 books can be arranged on the shelf.

Solution

There will be 4 choices in the first slot

There will be 3 choices in the first slot

There will be 2 choices in the first slot

There will be 1choices in the first slot

$\therefore$ There are $4 \times 3 \times 2 \times 1 = 4! = 24$ permutations

We can use our calculators for finding permutations, with the $^{\mathbf{n}}\mathbf{P}_{\mathbf{r}}$ button. Use your calculator to evaluate $^6P_2$. Hopefully you got 30. What about $^8P_3$? The answer is 336.

If you don't have a calculator, we can compute this manually.

The number of ways in which r items can be selected out of a set n items in which order of arrangement matters is given by the formula:

$$^nP_r = P(n,r) = \frac{n!}{(n-r)!}$$

Now if we were selecting n items from a set n item in which order matters, then:

$${}^{n}P_{n} = P(n,n) = \frac{n!}{n-n!} = \frac{n!}{n-n!} = \frac{n!}{0!} = n! \text{ since } 0! = 1$$

Example 13

Use ${}^{n}P_{r} = \frac{n!}{(n-r)!}$, to simplify:

a. ${}^{6}P_{4}$

b. ${}^{10}P_{2}$

c. ${}^{7}P_{5}$

d. ${}^{5}P_{5}$

Solution

a. ${}^{6}P_{4} = \frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{6\times5\times4\times3\times2\times1}{2\times1} = 360$

b. ${}^{10}P_{2} = \frac{10!}{(10-2)!} = \frac{10!}{8!} = \frac{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{8\times7\times6\times5\times4\times3\times2\times1} = 90$

c. ${}^{7}P_{5} = \frac{7!}{(7-5)!} = \frac{7!}{2!} = \frac{7\times6\times5\times4\times3\times2\times1}{2\times1} = 2\,520$

d. ${}^{5}P_{5} = \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{5\times4\times3\times2\times1}{1} = 120$

Example 14

We can use permutations to explore how letters in words can be arranged where no letter is repeating and we are arranging all the letters in the word.

How many ways can the letters in the word **LEARN** be arranged?

Solution

Step 1: there are 5 letters in the given word and we are arranging all of them

Step 2: this can be written in the form as ${}^{5}P_{5}$

Step 3: we know that ${}^{n}P_{n} = n!$

$5! = 5\times4\times3\times2\times1 = 120$

Example 15

Find the number of permutations (unique arrangements) of the letters in the following words

a. HOME

b. ADOMI

c. CASTLE

d. STUDENT

e. CHEMISTRY

Solution

a. HOME = $^4P_4 = 4! = 4 \times 3 \times 2 \times 1 = 24$ ways

b. ADOMI = $^5P_5 = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ ways

c. CASTLE = $^6P_6 = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ ways

d. STUDENT = $^7P_7 = 5! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$ ways

e. CHEMISTRY = $^9P_9 = 9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 362880$ ways

Example 16

How many ways can 6 books be arranged out of 15 books?

Solution

$$^{15}P_6 = \frac{15!}{(15-6)!} = \frac{15!}{9!}$$

$$= \frac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$= 3\ 603\ 600 \text{ ways}$$

Permutations with identical/repeated objects

What happens if we want to find permutations where some object or letter is repeated? For example, in the word BOOKS, the O is repeated. In these cases we find the permutation and divide it by the product of the number of identical objects:

$$\frac{n!}{(r_1!.r_2!.r_3!.....r_k!)}$$

Example 17

In how many ways can the letters of the word **STATISTICS** be arranged?

Solution

The word **STATISTICS** has 10 letters with 3 **S**s**,** 3 **T**s and 2 **I**s

Thus the numbers ways for arranging the letters in the word STATISTICS is

$\frac{10!}{3! \times 3! \times 2!} = 50\ 400$ ways

Example 18

How many ways can you arrange the letters in the following words

a. COMMISSION

b. SYLLABUS

c. CURRICULUM

d. BUSINESS

Solution

a. COMMISSION = 10 letters with 2Os, 2Ms, 2Is, 2Ss

$\frac{10!}{2! \times 2! \times 2! \times 2!} = 226\ 800$ ways

b. SYLLABUS = 8 letters with 2Ss, 2Ls

$\frac{8!}{2! \times 2!} = 10\ 080$ ways

c. CURRICULUM = 10 letters with 2Cs , 3Us, 2Rs

$\frac{10!}{2! \times 3! \times 2!} = 151\ 200$ ways

d. BUSINESS = 8 letters with 3Ss

$\frac{8!}{3!} = 6\ 720$ ways

Arrangement of words with restrictions on letters

Conditions can be given with the arrangement of letters in words.

Activity 10.2: Arranging SHAVE with restrictions (work in groups)

Let us discover how many arrangements are there in the letters in SHAVE if the vowels in the word must come together.

Step 1: Rewrite the word SHAVE to SHV(AE) with the AE now considered to be 1 letter.

Step 2: Calculate the number of arrangements of the letters in the new word:

4 'letter' word without repetition $= 4! = 24$

Step 3: Calculate the number of arrangements of the letters that must come together (AE)

AE is 2 letters $= 2! = 2$

Step 4: Multiply these answers

$24 \times 2 = 48$

The number of ways of arranging SHAVE when the vowels must come together is 48.

Example 19

How many arrangements are there in of the letters in SHAVE if the vowels in the word must be separated?

Step 1: First find the number of ways of arranging the letters in the word SHAVE.

SHAVE has 5 letters, $5! = 120$

Step 2: Subtract the number of ways in which the vowels are together (example 1 above)

$120 - 48 = 72$ ways

Example 20

In how many ways can the letters in the word **LOOK** be arranged if

a. The Os must come together.

b. The Os must be separated.

Solution

a. If the O's must come together, then the word may be LK(OO)

LK(OO) = 3 letters = 3! = 6 arrangements

Number of arrangements of the Os (OO), 2 letters with 2 letters repeating

$\frac{2!}{2!} = 1$

Multiplying the two results, $6 \times 1 = 6$ arrangements

b. The O's must be separated.

Find the numbers of ways of arranging LOOK = 4 letters with 2 letters repeating

$\frac{4!}{2!} = 12$

Subtract the result of when the Os came together from the arrangement of the word LOOK

$12 - 6 = 6$ arrangements

Example 21

In how many ways can the letters in the word **BAILIFF** be arranged if

i. The Fs must come together.

ii. The Fs must be separated.

Solution

i. If the F's must come together, then the word may be BAIIL(FF)

BAIIL(FF) = 6 letters with the Is repeating = $\frac{6!}{2!} = 360$ arrangements

Arranging (FF) which has 2 letters with F's repeating: $\frac{2!}{2!} = 1$

Hence if the F's must come together, then $360 \times 1 = 360$ arrangements.

ii. The word BAILIFF has 7 letters with 2Is and 2Fs

$\frac{7!}{2! \times 2!} = 1260$ arrangements.

With the Fs separated = $1260 - 360 = 900$ arrangements

Cyclic Permutations

Figure 5: Circular table with chairs

What happens if we want to arrange four people around a circular table? The circular table has no end, so one person will sit at a place (fixed) leaving 3 people for the arrangement on the three remaining seats. This can be written as $1 \times (4 - 1)! = 1 \times 3 = 3$ ways.

What if we have 7 chairs around the circular table? 1 person will be fixed and the rest be arranged. i.e. $1 \times (7 - 1)! = 1 \times 6! = 720$ ways.

So, in general, given n objects to arranged in a circle is given as

$1 \times (\boldsymbol{n} - 1)! = (\boldsymbol{n} - 1)!$

Example 22

Six prefects attend a meeting in a conference room with a round table. How many ways can they arrange themselves for the meeting?

Solution

$(6 - 1)! = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 720$ ways

Example 23

10 girls formed a circle to play ampe. How ways can they arrange themselves?

$(10 - 1)! = 9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 362\ 880$ ways

Combinations

In section 1, we used combinations to help us generate coefficients. Remember that a combination is denoted by:

$$^{n}C_{r} = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Example 24

Evaluate:

1. $^{6}C_{2}$
2. $^{10}C_{4}$
3. $^{7}C_{5}$
4. $^{5}C_{5}$

Solution

1. $^{6}C_{2} = \frac{6!}{(6-2)!2!} = \frac{6!}{4! \times 2!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1 \times 2 \times 1} = 15$
2. $^{10}C_{4} = \frac{10!}{(10-4)!4!} = \frac{10!}{6! \times 4!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 4 \times 3 \times 2 \times 1} = 210$
3. $^{7}C_{5} = \frac{7!}{(7-5)!5!} = \frac{7!}{2! \times 5!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 5 \times 4 \times 3 \times 2 \times 1} = 21$
4. $^{5}C_{5} = \frac{5!}{(5-5)!5!} = \frac{5!}{0! \times 5!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1} = 1$

Example 25

How many ways can a form teacher choose 2 students from 9 students to represent the class in a meeting?

Solution

Here n = 9 and r = 2

Using, $n_{C_r} = \frac{n!}{(n-r)!r!}$

$= 9_{C_2} = \frac{9!}{(9-2)!2!}$

$= \frac{9!}{7! \times 2!}$

$= \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 2 \times 1}$

$= 36$ ways

Example 26

In a mathematics examination students select 12 questions out of 15 questions. How many ways can students to do this?

Solution

Here n = 15 and r = 12

Using, $^{n}C_{r} = \dfrac{n!}{(n-r)!r!}$

$= {}^{15}C_{12}$

$= \dfrac{15!}{3! \times 12!}$

$= \dfrac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$

$= 455$ ways

THE RELATIONSHIP BETWEEN PERMUTATIONS AND COMBINATIONS

Permutations are used when order matters. For example, when arranging trophies on a shelf, where one place each trophy (1st, 2nd, etc.) matters. Combinations, on the other hand, focus on selections or arrangements when order does not matter. The order in which you choose ingredients for a dish does not affect the final recipe.

Is there relationship between the two concepts mathematically? Let's see:

Permutations are given by $^{n}P_{r} = \dfrac{n!}{(n-r)!}$

Combinations are given by $^{n}C_{r} = \dfrac{n!}{r!(n-r)!}$

But if $^{n}P_{r} = \dfrac{n!}{(n-r)!}$ and we substitute this into the combinations formula, we have:

$$^{n}C_{r} = \frac{n!}{r!(n-r)!} = \frac{^{n}P_{r}}{r!}$$

So, permutations and combinations are related by $r!\,^{n}C_{r} = {}^{n}P_{r}$

Solving problems involving permutations and combinations

We can use the relationship between permutation and combinations to solve related problems.

Example 27

If $24\,{}^{n}C_{r} = {}^{n}P_{r}$, find the value of r.

Solution

Using, $r!\,{}^{n}C_{r} = {}^{n}P_{r}$

Comparing with the question:

$24\,{}^{n}C_{r} = {}^{n}P_{r}$

$\therefore$ r! = 24

Multiply counting numbers until you get 24.

$1 \times 2 \times 3 \times 4 = 24$ which means $4! = 24$

$r! = 4!$

Hence $\boldsymbol{r} = 4$

Example 28

If $\frac{n_{C_2}}{n_{P_3}} = \frac{1}{2}$, find n.

Solution

Simplifying the numerator, $n_{C_2} = \frac{n!}{(n-2)!2!}$

Simplifying the denominator, $n_{P_3} = n_{\frac{n!}{(n-3)!}}$

$$\frac{n_{C_2}}{n_{P_3}} = \frac{\frac{n!}{(n-2)!2!}}{\frac{n!}{(n-3)!}}$$

Simplify this expression: $\frac{n!}{(n-2)!2!} \times \frac{(n-3)!}{n!} = \frac{(n-3)!}{(n-2)! \times 2} = \frac{(n-3)!}{2 \times (n-2)(n-3)!}$

Solve the equation: $\frac{1}{(n-2)\times 2} = \frac{1}{2}$

$\therefore n - 2 = 1$

$\therefore \boldsymbol{n} = \mathbf{3}$

Example 29

If $n_{P_4} : n_{P_2} = 2$ find the value of n.

Solution

Express the ratio as a fraction: $\frac{n_{P_4}}{n_{P_2}}$

Using the formula for permutations: $\frac{n_{P_4}}{n_{P_2}} = \frac{\frac{n!}{(n-4)!}}{\frac{n!}{(n-2)!}}$

Therefore: $\frac{\cancel{n!}}{(n-4)!} \times \frac{(n-2)!}{\cancel{n!}} = \frac{(n-2)!}{(n-4)!}$

$\frac{(n-2)!}{(n-4)!} = \frac{(n-2)(n-3)\cancel{(n-4)!}}{\cancel{(n-4)!}} = 2$

$(n-2)(n-3) = 2$

$n^2 - 5n + 6 = 2$

$n^2 - 5n + 4 = 0$

$(n-4)(n-1) = 0$

$n = 4 \; or \; n = 1$

Hence $\boldsymbol{n} = 4$, because n > r but r <1, so $n = 1$ is rejected.

Example 30

If ${}^{3a}C_2 = 15$, find the value of a

Solution

${}^{3a}C_2 = \frac{3a!}{2!(3a-2)!} = 15$

$\frac{3a \times (3a-1) \times (3a-2) \times (3a-3) \times \ldots \times (3a-k)}{2 \times (3a-2) \times (3a-3) \times \ldots \times (3a-k)} = 15$

$\frac{3a \times (3a-1)}{2} = 15$

$3a \times (3a - 1) = 30$

$9a^2 - 3a - 30 = 0$

Solving quadratically, $a = 2$ or $a = -\frac{5}{3}$

Example 31

Using the digits 5, 6, 7, 8, 9, how many 3-digit numbers, without repetition, can be made?

Solution

$5 \times 4 \times 3 = 60$

Example 32

How many different ways can six volleyball players be arranged in a line?

Solution

$6_{P_6} = 6! = 720$

Example 33

How many different arrangements of the letters in the word KAKUM be written?

Solution

There are 5 letters with 2Ks.

$\frac{5!}{2!} = \frac{5 \times 4 \times 3 \times \cancel{2 \times 1}}{\cancel{2 \times 1}} = 60$

Example 34

A company wants to hire 3 people. 12 people were short listed. How many ways can the 3 people be chosen?

Solution

$12_{C_3} = \frac{12!}{(12-3)!3!}$

$= \frac{12!}{(9)!3!} = \frac{12 \times 11 \times 10 \times \cancel{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}}{\cancel{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} \times 3 \times 2 \times 1} = 440$ ways

Example 35

Mr. Ghartey has 8 different brands of water bottles. How many ways can the bottles of water be arranged?

Solution

$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40\,320$ ways

Example 36

In a mixed class of 15 students, 5 students are to be selected for an award. How many ways can this be done if any of the students qualifies for the award?

Solution

$$15_{C_5} = \frac{15!}{(15-5)!5!}$$

$$= \frac{15!}{(10)!5!}$$

$$= \frac{15 \times 14 \times 13 \times 12 \times 11 \times \cancel{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}}{\cancel{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} \times 5 \times 4 \times 3 \times 2 \times 1} = 3\,003 \text{ ways}$$

Example 37

7 girls and 5 boys applied for positions on a school council. If 2 boys and 3 girls are needed to fill the vacancies, in how any ways can this be done?

Solution

Step 1: Find the number of ways of selecting the 3 girls out of the 7 girls:

$7_{C_3} = 35$

Step 2: Find the number of ways of selecting the 2 boys out of the 5 boys:

$5_{C_2} = 10$

Step 3: The committee is for both the boys and the girls, so multiply the results in steps 1 and 2 to obtain number of ways of doing the selection

$35 \times 10 = 350$ ways.

Example 38

In a school there 6 members in the Reading club and 4 in the Cadets. The school is forming a committee for their speech and prize giving day comprising 4 members from the reading club and 3 from the Cadets. How many ways can the school form the committee?

Solution

Number of ways of selecting reading club members $= 6_{C_4} = 15$

Number of ways of selecting cadet members $= 4_{C_3} = 4$

Number of ways of forming the committee $= 15 \times 6 = 60$ **ways**

Example 39

In a class of 30 students, 8 study mathematics, 7 study history and 5 study Spanish. The headmaster was to select 3 mathematics, 4 History and a Spanish student for an award ceremony. In how many ways can this be done?

Solution

Number of ways of selecting mathematics students $= 8_{C_3} = 56$

Number of ways of selecting history students $= 7_{C_4} = 35$

Number of ways of selecting a Spanish student $= 5_{C_1} = 5$

Number of ways of forming the committee $= 56 \times 35 \times 5 = 9\ 800$ **ways**

PROBABILITY IN EVERYDAY LIFE

Probability is a branch of mathematics which measures the likelihood of events. For example, we can use it when predicting the outcome of a penalty shootout, predicting rain chances for a farmer's harvest or analysing the likelihood of drawing a specific card. By expressing chance as a numerical value (strictly between 0 and 1 inclusive), probability empowers you to make informed decisions and navigate the fascinating world of uncertainty. The concept of probability finds its application in numerous fields including weather forecasting, game theory, insurance and risk management, sports analysis and prediction etc.

Activity 10.3: Probability Spinner Challenge (Work in small group)

1. Create a spinner divided into 6 equal sections labelled with colours: Red, Blue, Green, Yellow, Purple, and Orange.

Figure 6: Sample of a Spinner

2. Note that each section has an equal chance of being selected.
3. Spin the spinner 10 times
4. Record the outcome for each spin in a tally chart.

Colour	Tally	Frequency
Red		
Blue		
Green		
Yellow		
Purple		
Orange		
Total	**10**	**10**

5. How many times did the spin fall on each colour out of the total?
6. Discuss the outcomes with your classmates.

Note: *Carrying out this activity is an experiment*

Example 40

An experiment is performed by tossing a coin. What is the sample space?

Solution

The sample space, $S = \{Head\ (H), Tail\ (T)\}$

Example 41

A die is tossed in an experiment. List the sample space and the following events.

a. Event A: An odd number is obtained

b. Event B: A number greater than 3 is observed

c. Event C: The outcome is a prime number

Solution

The sample space S obtained when a die is tossed is $S = \{1, 2, 3, 4, 5, 6\}$

a. $Event\ A = \{1, 3, 5\}$

b. $Event\ B = \{4, 5, 6\}$

c. $Event\ C = \{2, 3, 5\}$

Example 42

Ama is picking mangoes from her favorite mango tree. There are 3 large mangoes and 2 small mangoes on the tree. She reaches up and blindly picks one mango.

Identify the following from the statement:

a. The experiment

b. The outcome

c. Sample space

d. Event

Solution

a. Experiment: Picking a single mango from the tree.

b. Outcome: The size (large or small) of the mango Ama picks.

c. Sample Space: {Large, Small} (all possible sizes of the mango she can pick).

d. Event: Picking a large mango. (You are interested in this specific set of outcomes).

Example 43

State whether the following pairs of events are mutually exclusive events, independent events or neither.

a. Event A: It rains today.
Event B: The sun shines today.

b. Event A: You draw a red marble from a bag.
Event B: The marble you draw is larger than 1cm.

c. Event A: You roll a die and get an even number.
Event B: You roll a die and get a number greater than 3.

d. Event A: A coin toss lands on heads.
Event B: You win the lottery.

e. Event A: Today is Monday.
Event B: Tomorrow is Tuesday.

f. Event A: You roll a die and get a number greater than 5.
Event B: You roll a die and get a prime number

Solution

a. Neither mutually exclusive nor independent
b. Independent events
c. Neither mutually exclusive nor independent
d. Independent events
e. Neither mutually exclusive nor independent
f. Mutually exclusive events

PROBABILITY OF GIVEN EVENTS

Probability of events can be obtained in three ways, namely:

Relative frequencies of events

Relative frequency informs us of what occurs in an experiment. Experiments are performed using coins, dice, playing cards etc. and the findings are recorded.

The relative frequency of an event is defined as:

$$\textit{Relative Frequency} = \frac{\textit{frequency of the events}}{\textit{Total frequency}}$$

Example 44

The table shows the number of times a coin is tossed and the outcome of each trial.

Trial	1	2	3	4	5	6	7	8	9	10	11	12	13
Outcome	T	T	H	T	H	T	H	H	T	T	H	T	H

What is the relative frequency of (observing) heads after

a. each trial

b. the experiment

Solution

a. Relative frequency (RF) of heads after each trial

Trial	1	2	3	4	5	6	7	8	9	10	11	12	13
outcome	T	T	H	T	H	T	H	H	T	T	H	T	H
RF	0	0	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{2}{5}$	$\frac{2}{6}$	$\frac{3}{7}$	$\frac{4}{8}$	$\frac{4}{9}$	$\frac{4}{10}$	$\frac{5}{11}$	$\frac{5}{12}$	$\frac{6}{13}$

b. Relative frequency of heads after the experiment $= \frac{\textit{Number of heads } (H)}{\textit{Total number of trials}}$

$$= \frac{6}{13}$$

$$= 0.46$$

Example 45

The table shows the number of times a fair coin is tossed and the outcome of each trial.

Trial	1	2	3	4	5	6	7	8	9	10	11	12	13
Outcome	T	T	H	T	H	T	H	H	T	T	H	T	H
Trial	14	15	16	17	18	19	20	21	22	23	24	25	26
Outcome	T	H	T	T	H	H	T	H	H	T	T	T	H

What is the relative frequency of (obtaining)

a. tails after 10 trials?

b. tails after 20 trials?

c. tails after the experiment and how does it compare to the theoretical probability of obtaining tails?

Solutions

$$Relative\ frequency\ (RF) of\ tails = \frac{Number\ of\ tails}{Total\ number\ of\ trials}$$

a. Number of tails after 10 trials = 6, Number of trials = 10

RF $= \frac{6}{10} = 0.6$

b. Number of tails after 20 trials = 11, Number of trials = 20

RF $= \frac{11}{20} = 0.55$

c. Number of tails after the experiment = 14, Number of trials = 26

RF $= \frac{14}{26} = 0.54$

When a coin is tossed, sample space $S = \{Head\ (H), Tail(T)\}$, $n(S) = 2$, $n(T) = 1$

Theoretical probability of obtaining tails, $P(T) = \frac{n(T)}{n(S)} = \frac{1}{2} = 0.50$

Therefore, it is observed that as the number of trials increases, the relative frequencies get closer and closer to the theoretical probability.

Example 46

A football referee always uses a special coin to toss for ends. He noticed that out of the last twenty matches the coin has come down heads far more than tails. He wanted to know if the coin was fair, that is, if it was equally likely to come down heads as tails. He decided to do a sample experiment by tossing the coin lots of times. His results are shown below:

Number of trials	Number of heads	Relative frequency
100	40	0.4
200	90	0.45
300	142	0.47…
400	210	0.525
500	260	0.52
600	290	0.48…
700	345	0.49…
800	404	0.505
900	451	0.50…
100	499	0.499

The relative frequency$=\dfrac{\textit{number of successful trials}}{\textit{total number of trials}}$

After the large number of trials, did the coin appear to be fair?

Solution

The greater the number of trials the better the estimated probability or relative frequency is likely to be. The key idea is that increasing the number of trials gives a better estimate of the probability and the closer the result obtained by the experiment will be to that obtained by the calculation. Looking at the referees experiment the coin does appear to be fair.

Example 47

The following table shows the frequency distribution of the marks obtained by 50 pupils in a test.

Marks	0	1	2	3	4	5	6	7	8	9
Frequency	3	6	7	4	7	8	5	5	3	2

Find the probability that a randomly selected pupil scored:

i. 4

ii. less than 5

iii. greater 6

Solution

i. The number of students who scored 4 is 7.

The probability that a pupil selected scored 4 $= \frac{\textit{number of pupils who scored } 4}{\textit{total frequency}}$

The probability that a pupil selected scored 4 $= \mathbf{\frac{7}{50}}$

ii. The number of pupils who scored less than 5 $= 3 + 6 + 7 + 4 + 7 = 27$

The probability that a pupil selected scored less than 5 $= \mathbf{\frac{27}{50}}$

iii. The number of pupils who scored greater than 6 $= 5 + 3 + 2 = 10$

The probability that a pupil selected scored greater than 6 $= \frac{10}{50} = \mathbf{\frac{1}{5}}$

Theoretical probability of events

In an experiment, theoretical probability informs one of what should happen. Theoretically, the probability of an event is the likelihood of the event happening based on all the possible outcomes and is given by:

$$\textit{Probability of Event } E,\ P(E) = \frac{\textit{Number of favourable or desired outcomes } n(E)}{\textit{Number of possible outcomes } n(S)}$$

$$P(E) = \frac{n(E)}{n(S)}$$

Example 48

A die is rolled once. What is the probability of obtaining

a. an even number?

b. a number greater than 4?

Solution

Sample space, $S = \{1, 2, 3, 4, 5, 6\}$, $n(S) = 6$

a. Event of an even number, $E\,e = \{2, 4, 6\}$, $n(E\,e) = 3$

$$P(\textit{even number}) = \frac{n(E_e)}{n(S)} = \frac{3}{6} = \mathbf{0.5}$$

b. Numbers greater than $4 = \{5, 6\}$, $n(\textit{greater than } 4) = 2$

$$P(\textit{greater than } 4) = \frac{2}{6} = \frac{\mathbf{1}}{\mathbf{3}}$$

Example 49

What is the probability of drawing an ace from a standard deck of 52 cards?

Solution

$$\frac{\textit{Number of favorable outcomes}}{\textit{Total number possible outcomes}} = \frac{4(\textit{drawing an ace})}{52(\textit{drawing any card})} = \frac{4}{52} = \frac{\mathbf{1}}{\mathbf{13}}$$

Example 50

What is the probability of getting two heads when tossing two fair coins?

Solution

$$\frac{\textit{Number of favorable outcomes}}{\textit{total number possible outcomes}} = \frac{1(\textit{getting two heads})}{4(\textit{getting two heads, two tails or one of each})} = \frac{\mathbf{1}}{\mathbf{4}}$$

Example 51

What is the probability of landing on a specific color when spinning a spinner with 8 equal sections?

Solution

$$\frac{\textit{Number of favorable outcomes}}{\textit{total number possible outcomes}} = \frac{1(\textit{landing on a specific color})}{8(\textit{landing on any color})} = \frac{\mathbf{1}}{\mathbf{8}} = \mathbf{0.125}$$

Subjective probability of events

This is based on human opinion due to individual observations or available information. For example, one may observe the clouds and conclude that there is an 80% likelihood that it will rain. A person may say there is a 90% chance that Ghana would score against Nigeria in a friendly match. A farmer may assert she would have 100% crop yield. All these instances are subjective to individual opinions, which we might not be able to verify.

Example 52

1. **Event planner:** Assesses the probability of an outdoor event being cancelled due to weather conditions based on historical weather data and forecast.
   Subjective Probability: 0.4(40% chance of cancellation).
2. **Project Manager:** Estimates the probability of completing a project on time based on team performance and experiences.
   Subjective Probability: 0.8 (80% chance of completion).
3. **Marking Specialist:** Assesses the probability of a new product launch being successful based on market trends and company performance.
   Subjective probability: 0.7 (70% chance of increase)

REVIEW QUESTIONS

Review Questions 10.1

1. Evaluate the following

   a. 6! _ 4

   b. $\frac{(n-1)!}{(n+1)!}$

   c. $\frac{6_{P_3}}{8_{P_3}}$

   d. $\frac{(n+4)!}{(n+2)!}$

2. Find the different ways of arranging the letters in the word SYSTEMS if the S's

   a. Must be together

   b. Must be separated

3. WAEC core mathematics paper consisted of two parts:

   Section A made up of 5 questions (1-5) and section B made up of 8 questions (6-13). Candidates were required to answer all the questions in Section A and select 5 questions from section B. In how many ways can selection of the examination be done.

4. 5 Professors applied to fill a position of Vice Chancellor in a University. The search committee were to rank the applicants from the best to least.

   How many different outcomes are possible?

5. How many ways can a committee of 5 men and 6 women be formed from 9 men and 13 women.

6. How many football teams of 11 players can be formed from 22 players without regard to position being played.

7. How many ways can 10 students can be seated in a line?

8. If $^{n}P_5 = 60^{n-1}P_3$, the value of n is

9. A coin is tossed **n** times. What is the number of all the possible outcomes?

10. In how many ways can a team of 15 football players choose a captain, an assistant captain and a welfare officer?
11. How many ways can the letters in SOCIAL be arranged?
12. How many 4-digit numbers can be made using the digits 1, 2, 3, 4, and 5 without repetition?
13. How many unique 3 digit codes can be created from the 5 digits {1, 2, 3, 4, 5} if repeats are possible?
14. In how many ways can you create 2 letter words from the letters in the word VALUES?
15. How many different committees of 5 people can be chosen from 10 people?
16. Adwinsa is the Chairman of a committee. In how many ways can a committee of 5 be chosen from 10 people given that Adwinsa must be one of them?
17. In how many ways can you arrange the letters in the word LAPTOP if the vowels must come together.
18. An Ebusuapanyin had 5 kente clothes, 4 native sandals and 3 hand beads to select from. In how many ways can he dress if he is to dress with one of each of the items**?**
19. After a school won a competition, the Head of the school organised a dinner party for the team comprising 7 women and 5 men on a round table. How many ways can they be seated, including the Head?
20. If $n_{C_2} = 28$, find the value of n.
21. $n_{P_4} = 2\left(n_{P_2}\right)$, find the value of n.
22. A coach has available for selection 3 goalkeepers, 8 defenders, 7 midfielders and 4 strikers. He is planning to pick for the next game 1 goalkeeper, 4 defenders, 4 midfielders and 2 strikers.

    Find the number of possible ways he can select his team, assuming that all players are equally likely to be selected.
23. There are 5 boys and 6 girls in the student representative council (SRC) of a school. A committee of 7 people is to be selected from the members of this council to organise an SRC week.

    Find the number of different ways in which the committee can be selected if all the members are available.

24. If ${}^{n}C_{r} = 35$ and ${}^{n}P_{r} = 840$, find the value of r.

25. Show that ${}^{n}C_{r} = {}^{n}C_{n-r}$

REVIEW QUESTIONS 10.2

1. There is a group of 250 people in a hall. A girl calculates that the probability of randomly picking someone that she knows from the group is 0.032.

   Calculate the number of people in the group that the girl knows.

2. In a small town there are a number of sports clubs. The clubs have 750 members in total.

   The table below shows the types of sports clubs and the number of members each has.

	Tennis	Football	Golf	Hockey	Athletics
Men	30	110	40	15	10
Women	15	25	20	45	30
Boys	10	200	5	10	40
Girls	20	35	0	30	60

   A sports club member is chosen at random from the town. Calculate the probability that the member is:

   a) a man
   b) a girl
   c) a woman who does athletics
   d) a boy who plays football
   e) not a boy who plays football
   f) not a golf player
   g) a man who plays hockey.

3. Thirty students are asked to choose their favorite subject out of Math, English and Art.

The result are shown in the table below.

	Math	English	Arts
Girls	7	4	5
Boys	5	3	6

A student is chosen at random

i) What is the probability that it is a girl?

ii) what is the probability that it is a boy whose favourite subject is Art?

iii) what is the probability of not choosing a girl whose favourite subject is English

4. A bag contains 7 red counters, 5 blue, 3 green and 1 yellow. If one counter is drawn, what is the probability that it is:

a) yellow

b) red

c) blue or green

d) red, blue or green

e) not blue

5. A boy calculated that he has a probability of 0.004 of winning the first prize in a photography competition if the selection is made at random. If 500 photographs are entered into the competition, how many photographs did the boy enter?

6. The probability of getting any particular number on a spinner game is given as 0.04.

How many numbers are there on the spinner?

7. The probability of getting a bad egg in a batch of 400 is 0.035.

How many bad eggs are there likely to be in a batch?

8. The probability of drawing a red, blue or green marble from a bag containing 320 marble is red 0.5, blue 0.3, green 0.2.

How many marbles of each color are there?

9. A sports arena has 25000 seats some of which are VIP seats. For a charity event all the seats are allocated randomly. The probability of getting a VIP seat is 0.008.

How many VIP seats are there?

10. You flip a coin 100 times and it lands on heads 55 times.

What is the relative frequency of getting heads.

11. A teacher asks 30 students if they like Math and 18 students says yes.

What is the relative frequency students like Math.

12. The number of people admitted to the hospital each day is given below for a period of 100 days in 2005.

Number of Patients Admitted	Frequency
0	4
1	12
2	20
3	32
4	18
5	6
6	4
7	1
8	4
9	1
10	2

Estimate the following probabilities:

a) That no patient will be admitted in the hospital

b) That more than 4 patients will be admitted

c) That the modal number will be admitted.

ANSWERS TO REVIEW QUESTIONS

Review Questions 10.1

1.

- **a.** 180
- **b.** $\frac{1}{n(n+1)}$
- **c.** $\frac{5}{14}$
- **d.** $(n+4)(n+3)$

2.

- **a.** 120
- **b.** 720

3. 56

4. 120

5. 216 216

6. 705 432

7. 3 628 800

8. 10

9. n^n

10. 455

11. 720

12. 120

13. 125

14. 30

15. 252

16. 126

17. 120

18. 60

19. 479 001 600

20. 8

21. 4

22. 44 100

23. 330

24. 4

25. Use $n_{C_r} = \frac{n!}{(n-r)!r!}$, do same for $n_{C_{n-r}}$

Answers Review Questions 10.2

1. $250 \times 0.023 = 8$

2. **a)** $\frac{205}{750} = 0.27$

b) $\frac{145}{750} = 0.19$

c) $\frac{30}{750} = 0.04$

d) $\frac{200}{750} = 0.27$

e) $\frac{550}{750} = 0.73$

f) $\frac{685}{750} = 0.91$

g) $\frac{15}{750} = 0.02$

3. **i)** $\frac{16}{30} = \frac{8}{15}$

ii) $\frac{6}{30} = \frac{1}{5}$

iii) $\frac{26}{30} = \frac{13}{15}$

4. **a)** $P(Yellow) = \frac{1}{16}$

b) $P(red) = \frac{7}{16}$

c) $Pblue\ or\ green = \frac{1}{2}$

d) $Pred, blue\ or\ green = \frac{15}{16}$

e) $Pnot\ blue = \frac{11}{16}$

5. $0.004 \times 500 = 2$

6. $0.04 = \frac{1}{n} \Rightarrow n = 25$

7. $4\,00 \times 0.035 = 14$

8. **i)** $\textit{Red marbles}\ 320 \times 0.5 = 160$

ii) $\textit{Blue marbles}\ 320 \times 0.3 = 96$

iii) $\textit{Green marbles}\ 320 \times 0.2 = 64$

9. $25000 \times 0.005 = 200$

10. Relative frequency $= \frac{55}{100} = 0.55$

11. $\frac{18}{30} = 0.6$

12. **a)** $\frac{4}{100} = 4\%$

b) $\frac{18}{100} = 18\%$

c) $\textit{modal}\ \frac{32}{100} = 32\%$

GLOSSARY

- **Combination:** the selection of object from a set without any order.
- **Event (E)**: A specific set of outcomes you are interested in.
- **Experiment:** An operation or process performed to observe a result or an outcome or to obtain measurements like flipping a coin (experiment) to see if it lands on heads or tails (outcome).
- **Factorial**: Denoted by $n!$ is the product of all positive integers less than or equal to n.
- **Independent events:** Event A is said to be independent of event B if the occurrence of A does not affect the probability of event B.
- **Mutually exclusive events:** Two events A and B are said to be mutually exclusive if event A and event B cannot happen simultaneously.
- **Outcome:** The specific result of a single trial in an experiment.
- **Permutation** is the **arrangement** of any set of objects in a definite order.
- **Random experiment:** An experiment whose outcome or results cannot be predetermined.
- **Sample Space (S)**: The collection of all possible outcomes for an experiment.
- **Trial:** A single performance of an experiment.

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Ghana Education Service (GES)

List of Contributors

Name	Institution
Yaw Efa	Ghana National College
Benedicta Ama Yekua Etuaful	Ogyeedom SHTS
Isaac Buabeng	Ghana Education Service, Accra Metropolitan
Mpeniasah Kwasi Christopher	Three-Town SHS