Additional Mathematics **Year 1**

SECTION

6 VECTORS

GEOMETRIC REASONING AND MEASUREMENT

Spatial Reasoning

INTRODUCTION

Did you know that the Global Positioning System (GPS) used by Uber, Bolt and Yango drivers for pick-up and delivery is made possible by knowledge in Vectors? How about the paths in your favourite video games, did you know vectors are used for calculating those paths? Well, let's come to your football games. Vectors help to determine the direction of a ball and how far the ball moves when kicked by a player. As we go through this section, it is expected that you will learn about the types and forms of vectors as well as the algebraic and geometric operations of vectors. The concepts of vectors are applied in many fields such as physics, engineering and computer science.

At the end of this section, you will be able to:

- Recognise and explain various forms of vectors and apply the knowledge to find unit vectors.
- Perform algebraic and graphical operations (addition, subtraction, scalar multiplication) and their geometrical interpretation.
- Determine the resultant of vectors using triangle and parallelogram laws of addition.

Key Ideas:

- Vectors are mathematical quantities that represent both magnitude and direction. The magnitude of a vector is the length or distance of the vector.
- Types of vectors to be discussed are position, collinear, unit, free, negative, parallel, equal and co-initial.
- Vectors can be represented in column/component and magnitude and direction forms.

FORMS AND TYPES OF VECTORS

On a daily basis, specific quantities can be defined mathematically with a single number, which represents their magnitude or size. Mass, volume, distance and temperature are some examples of such quantities. Also, there are many other quantities that require both magnitude and direction to be fully described. These quantities are represented mathematically by *vectors*.

For, example, you can decide to throw a ball forward or backwards (direction) at a particular distance (magnitude). If you push a car forward, backwards or sideways, you will get different results. Thus, force, velocity and acceleration are examples of vector quantities. Vectors can come in various types and represented in different ways. The vector that represents the movement from point B to point A can be represented graphically with $\overrightarrow{BA}$ or i, while vector that represents the movement from point F to point E can be represented graphically with $\overrightarrow{FE}$ or h as shown in Figure 1.

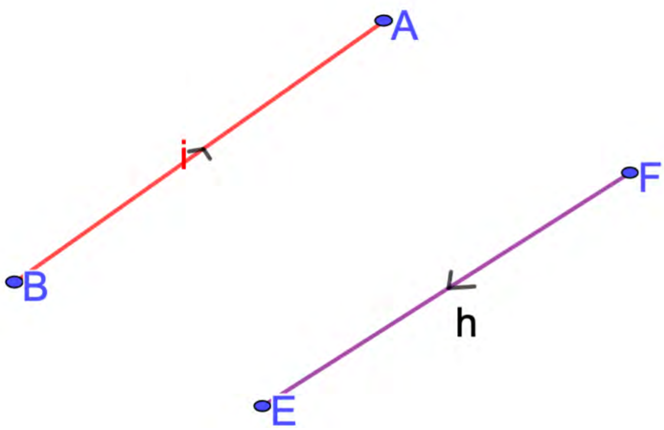


Figure 1: Vector Representation

Vectors are mathematical quantities that represent both magnitude and direction. The magnitude of a vector is the length or distance of the vector.

The Pythagorean theorem $x^2 + y^2 = z^2$ is used to calculate the magnitude of a vector.

Activity 6.1

i. On a graph paper, plot points O (0, 0), B (4, 5), Z (4, 0) and D (12, 10).

ii. Draw vectors **OB** and ZD.

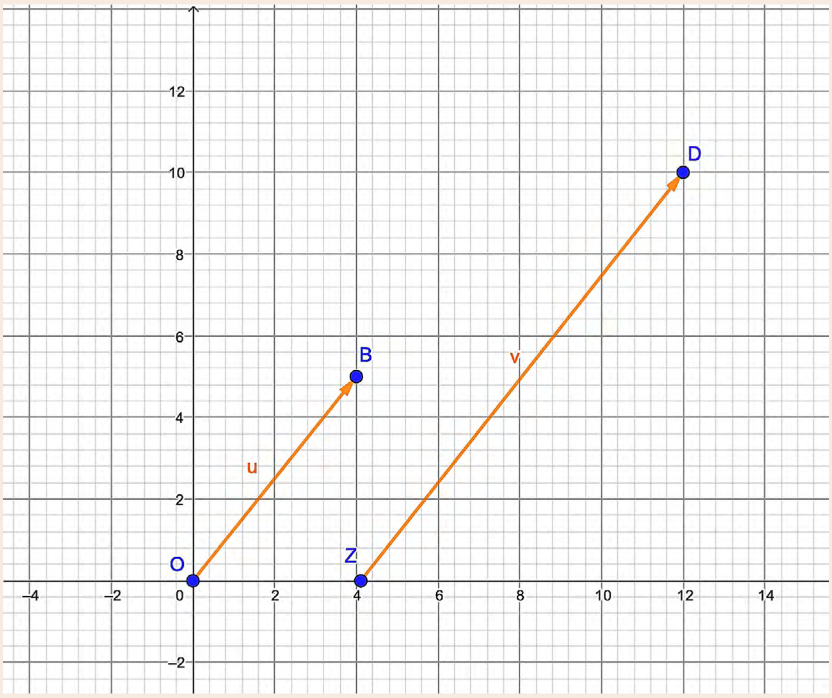


Figure 2: Collinear Vectors

iii. On graph paper, plot points A (2, 1), B (5, 5), C (6, 1), and D (9, 5).

iv. Draw vectors **AB** and **CD**.

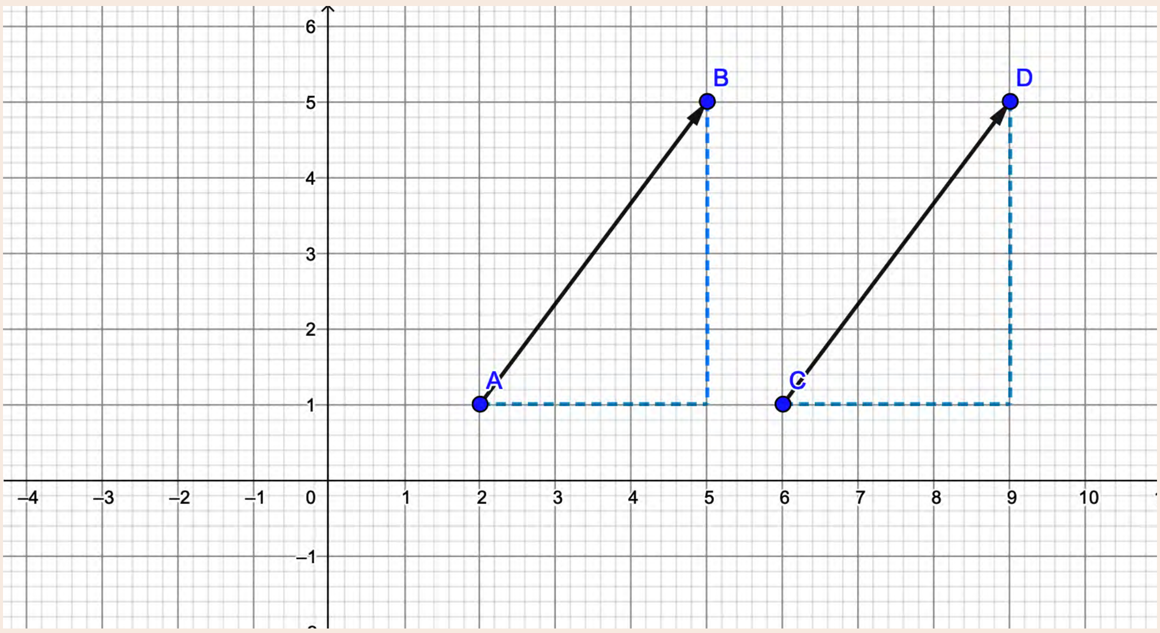


Figure 3: Parallel Vectors

v. Plot the point A (2, 2), B (5, 5), C (6, 4) and D (3, 6) on the graph.

vi. Draw vectors AB, AC and AD originating from A.

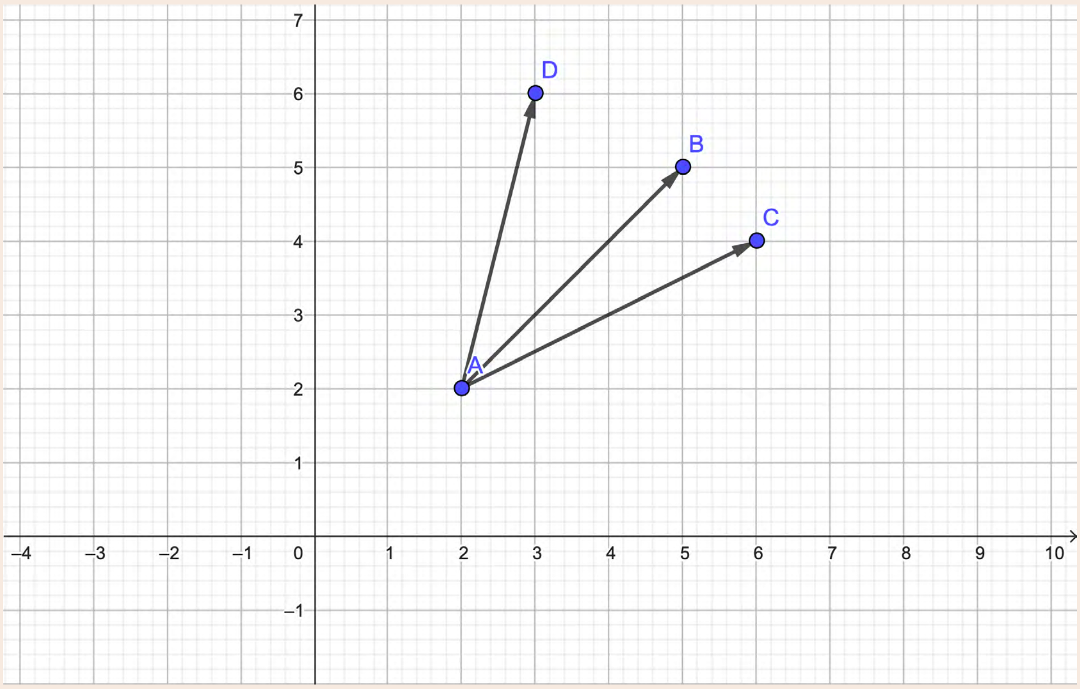
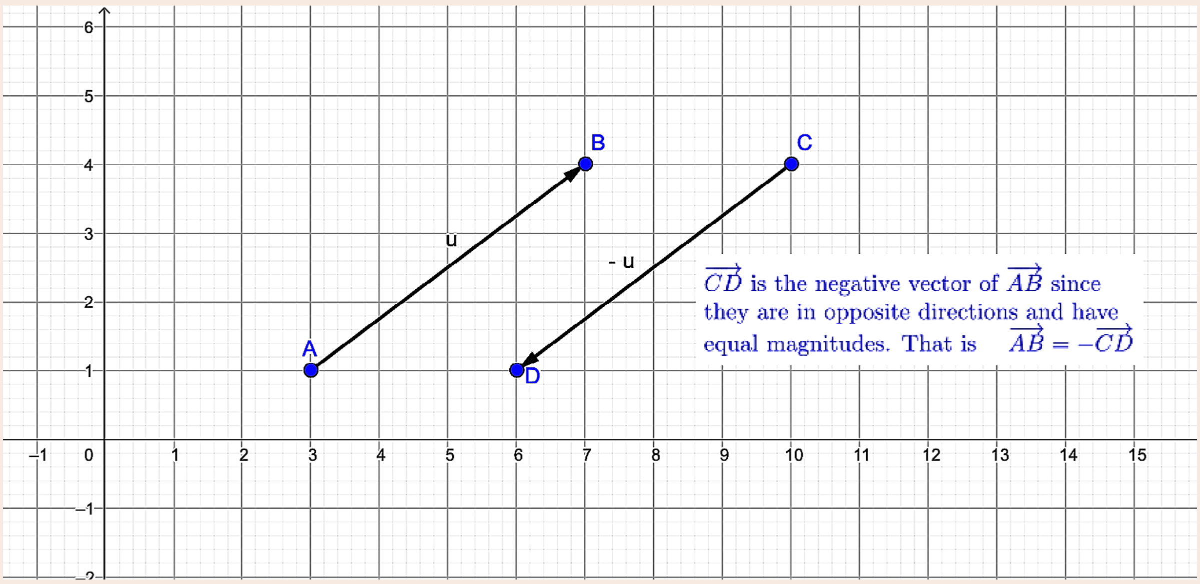


Figure 4: Co-initial vectors

vii. Plot points A (3, 1), B (7, 4), C (10, 4) and D (6, 1).

viii. Draw vectors **AB** and **CD**.



ix. Plot the points A (3, 1), B (7, 4), C (10, 4), and D (6, 1) on the graph.

x. Draw vectors **AB** and **DC**.

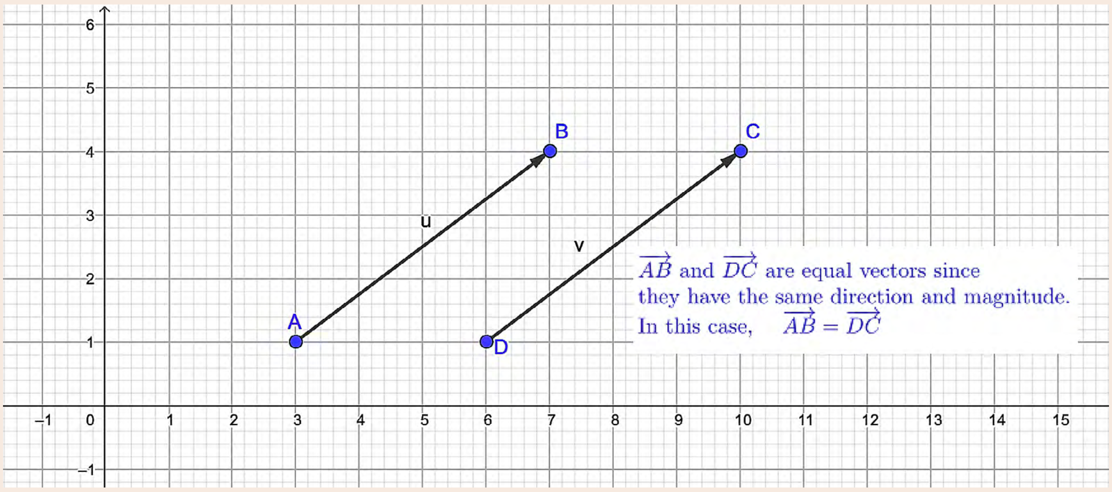


Figure 6: Equal vectors

xi. Plot points O (0, 0) and P (5, 4).

xii. Draw vector OP

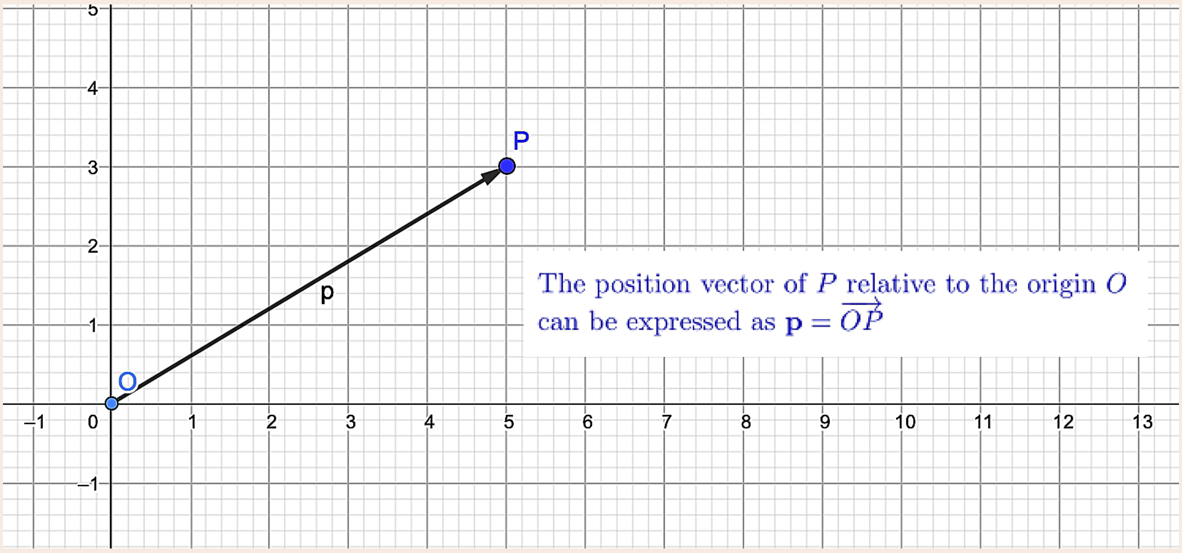


Figure 7: Position Vector

xiii. Plot points $A_1(4, 2)$, $A_2(3, 1)$, $B_1(9, 4)$, $B_2(8, 3)$, A(7, 2) and B(12, 4).

xiv. Draw vectors **AB,** $\boldsymbol{A_1B_1}$ and $\boldsymbol{A_2B_2}$.

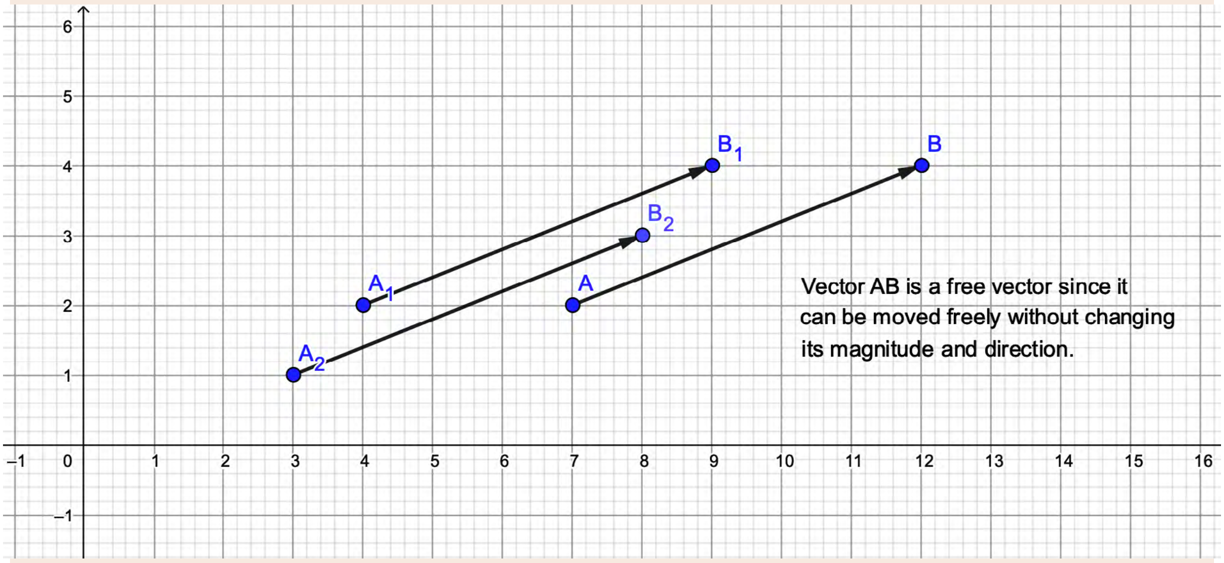


Figure 8: Free Vector

xv. Record your observations from the vectors drawn and discuss with a friend

Generalisations

i. Vectors that lie on the same straight line (with a common slope) are **collinear**.

ii. Vectors starting from the same point are **co-initial**.

iii. Vectors are parallel because they have the same direction and proportional components and are scalar multiples of each other.

- **Position vectors** represent the positions of points A, B, and C relative to the origin.
- **Free vectors** are positioned differently, they are identical in magnitude and direction, making them free vectors.
- **Negative Vectors** have the same magnitude but opposite direction.
- **Equal Vectors** have the same magnitude and direction even though they are placed at different locations.
- **Unit vector** has a magnitude of 1 unit ($\sqrt{x^2 + y^2} = 1$).

Forms of Vectors

The varied ways in which vectors can be written or represented are what we term *forms of vectors*. The forms we are going to focus on are:

- Column/component form $\binom{x}{y}$ where x represents the x (horizontal) direction and y the y (vertical) direction.
- Magnitude and direction form (r, θ) where r represents the magnitude (distance or length) and θ the direction.

Note: Algebraically, we can rewrite a vector as $xi + yj$.

Example 1

Given that Benyiwa and Ebo move from the same point (0, 0) towards a church building. If Benyiwa walks 5 units to the right and 2 units upward while Ebo moves 3 units to the left and 8 units upward, express their position as column vectors.

Solution

Step 1: *Let O represent point of origin*

O (0, 0)

Step 2: *Let B represent point of Benyiwa's movement*

B (5, 2)

Step 3: *Let E represent point of Ebo's movement*

E (-3, 8)

Step 4: *Write out column vector for Benyiwa*

$\overrightarrow{OB} = \binom{5}{2}$

Step 5: *Write out column vector for Ebo*

$\overrightarrow{OE} = \binom{-3}{8}$

A person positioned at O moving 5 units to the right denotes a positive displacement and 2 units upwards (positive displacement) to get to B. The vector that depicts this movement is $\overrightarrow{OB} = \binom{5}{2}$. Similarly, a movement of 3 units to the left and 8 units upward will move a point from O to E giving, $\overrightarrow{OE} = \binom{-3}{8}$.

Example 2

If the motion represented by $\overrightarrow{MN} = 6i - 3j$ is the translation of a particle on the $i - j$ plane from M to N. Represent $\overrightarrow{MN}$ as a component vector.

Solution

Step 1*:* Let the coefficient of i represent the x-coordinate for the column vector

Step 2*:* Let the coefficient of j represent the y-coordinate for the column vector

Thus, $\overrightarrow{MN} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$.

Example 3

Juliet's house is 15 metres away from her school and the bearing is 35°. Represent the location of Juliet's house in magnitude-direction vector form.

Solution

$(15m, 35^0)$

Example 4

Indicate the parallel vectors from the following given vectors;

$\boldsymbol{u} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}, \boldsymbol{v} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \boldsymbol{w} = \begin{pmatrix} -6 \\ 9 \end{pmatrix}, \boldsymbol{a} = \begin{pmatrix} -6 \\ -9 \end{pmatrix}.$

Solution

Step 1: Find the vectors that are scalar multiples of others.

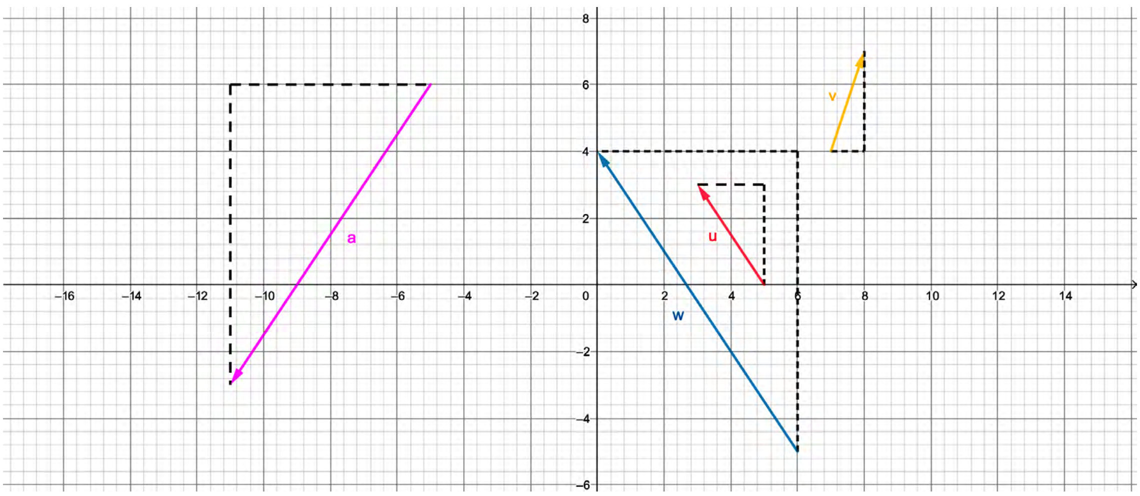


Figure 9: Parallel vector identification

$\begin{pmatrix}-2\\3\end{pmatrix}$, $\begin{pmatrix}-6\\9\end{pmatrix}$ the scalar multiple is 3.

Therefore, vectors $\boldsymbol{u}$ and $\boldsymbol{w}$ are parallel.

Example 5

Given that the vector $\overrightarrow{AB} = \begin{pmatrix}3\\-4\end{pmatrix}$, find the magnitude of $\overrightarrow{AB}$.

Solution

$$\left|\overrightarrow{AB}\right| = \sqrt{3^2 + (-4)^2}$$

$$= \sqrt{9+16}$$

$$= \sqrt{25}$$

$$= 5 \; units$$

Example 6

Find the unit vector for $\mathbf{g} = \begin{pmatrix}-5\\-6\end{pmatrix}$.

Solution

The unit vector can be determined by $\hat{\boldsymbol{g}} = \frac{\boldsymbol{g}}{|\boldsymbol{g}|}$.

Step 1: Find the magnitude of the vector

$$|g| = \sqrt{(-5)^2 + (-6)^2}$$

$$|g| = \sqrt{25+36}$$

$|g| = 7.810$ to three decimal places

Step 2: Substitute the g and $|g|$ in the unit vector formula.

$$\hat{g} = \frac{\begin{pmatrix}-5\\-6\end{pmatrix}}{\sqrt{61}}$$

$$\hat{g} = \begin{pmatrix}\frac{-5}{\sqrt{61}}\\ \frac{-6}{\sqrt{61}}\end{pmatrix}$$

ALGEBRAIC AND GEOMETRIC OPERATIONS ON VECTORS

The basic mathematic operations used are +, −, ÷, ×. For vectors, the operations that are applicable are addition, subtraction and scalar multiplication. We are going to explore how to operate on vectors both algebraically and geometrically.

Starting with the algebraical operations, let's discuss vector addition, subtraction and scalar multiplication!

In adding of vectors, sum the corresponding x−coordinates and corresponding y−coordinates. In the case of subtraction, subtract the corresponding x−coordinates and corresponding y−coordinates. In the case of scalar multiplication, the x and y coordinates are both multiplied by the scalar k.

Given that $\mathbf{a} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$, $\mathbf{a} + \mathbf{b} = \begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \end{pmatrix}$

Also given that $\mathbf{a} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$, $\mathbf{a} - \mathbf{b} = \begin{pmatrix} x_1 - x_2 \\ y_1 - y_2 \end{pmatrix}$

Given that a scalar multiplier k and $\mathbf{a} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$, $k\mathbf{a} = \begin{pmatrix} kx_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} kx_1 \\ ky_1 \end{pmatrix}$

Example 7

Given that $\mathbf{u} = \begin{pmatrix} 12 \\ 5 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, find $\mathbf{u} + \mathbf{v}$, $\mathbf{u} - \mathbf{v}$ and $5\mathbf{v}$.

Solution

Step 1: *Add the corresponding coordinates for vector u and v*

$$\mathbf{u} + \mathbf{v} = \begin{pmatrix} 12 + 2 \\ 5 + 3 \end{pmatrix}$$

$$= \begin{pmatrix} 14 \\ 8 \end{pmatrix}$$

Step 2: Subtract the corresponding coordinates for vector u and v

$$\mathbf{u} - \mathbf{v} = \begin{pmatrix} 12 - 2 \\ 5 - 3 \end{pmatrix}$$

$$= \begin{pmatrix} 10 \\ 2 \end{pmatrix}$$

Step 3: *Multiply the coordinates of vector* **v** *by the scalar multiplier 5.*

$5\mathbf{v} = 5\begin{pmatrix}2\\3\end{pmatrix}$

$= \begin{pmatrix}10\\15\end{pmatrix}$

Triangular Law of Vector Addition

Imagine that you walk from your classroom (point A) to the School Assembly Hall (point B), then to School Administration block (point C). The vectors, $\overrightarrow{AB}$ and $\overrightarrow{BC}$ can be used to represent the movement. It would be much simpler and straightforward to move from point A to point C. This movement can be represented by $\overrightarrow{AC}$ geometrically in Figure 10.

(Assuming a straight path connects the classroom to the Assembly Hall, Assembly Hall to Administration, and Classroom to Administration block).

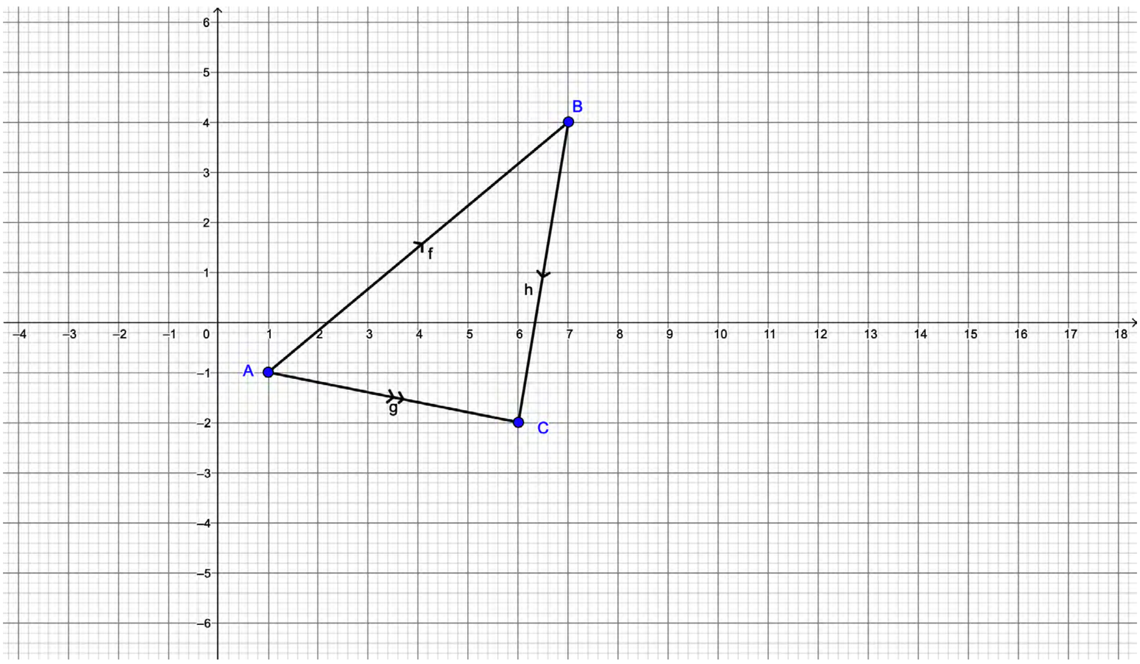


Figure 10: Triangular law of vector addition

Now $\overrightarrow{AC}$ is what we refer to as the resultant vector of $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$= \begin{pmatrix}6\\5\end{pmatrix} + \begin{pmatrix}-1\\-6\end{pmatrix}$

$= \begin{pmatrix}5\\-1\end{pmatrix}$

Note: The triangular law of vector addition can be applied to establish the relationship between a free vector, say $\overrightarrow{BC}$, and the position vectors, $\overrightarrow{OB}$ also written **b,** $\overrightarrow{OC}$ and written as **c**.

Recall that $\overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OC}$ from triangular law of vector addition.

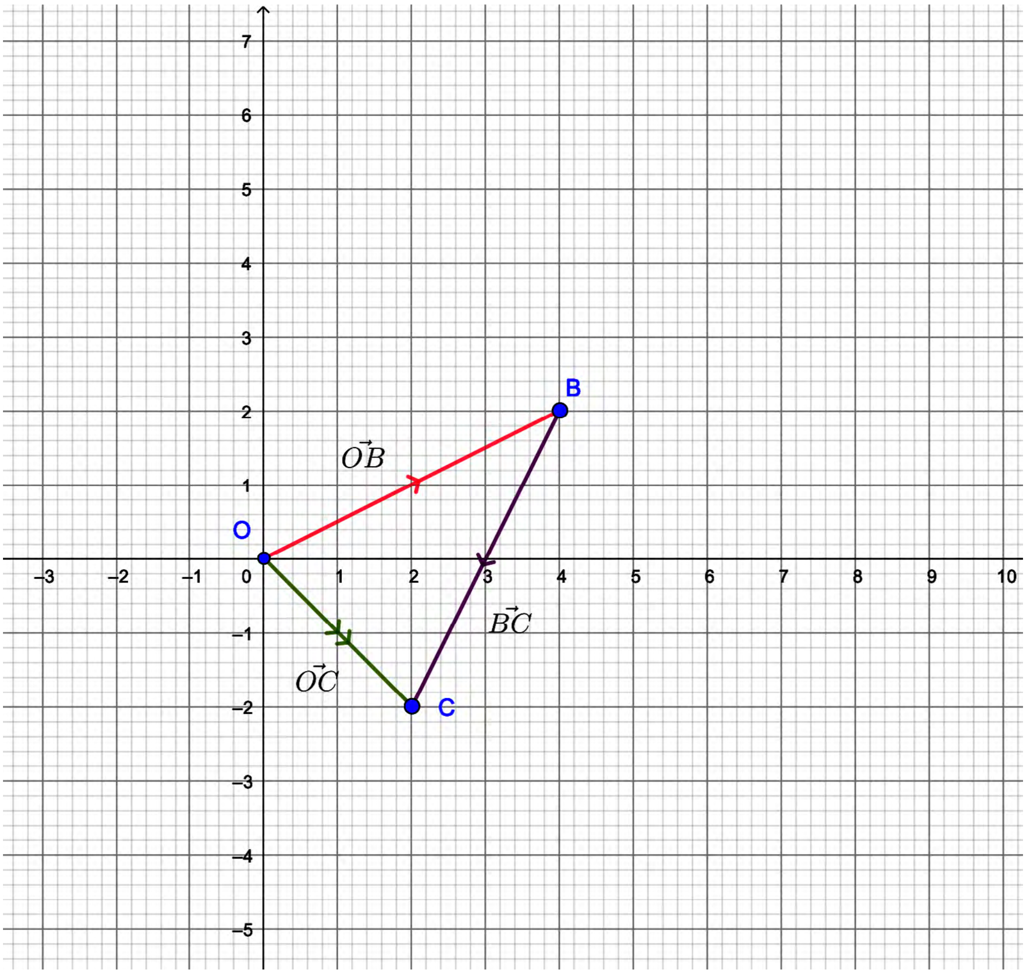


Figure 11: Triangular law of vector addition

Applying change of subject, $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

Now, $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$\overrightarrow{BC} = \mathbf{c} - \mathbf{b}$

Parallelogram Law of Vector Addition

Recall that a parallelogram is a four-sided figure with specific properties? Great! We are going to explore how vectors relate when they come together to form a parallelogram.

Analyse Figure 12 and share your observations with a classmate.

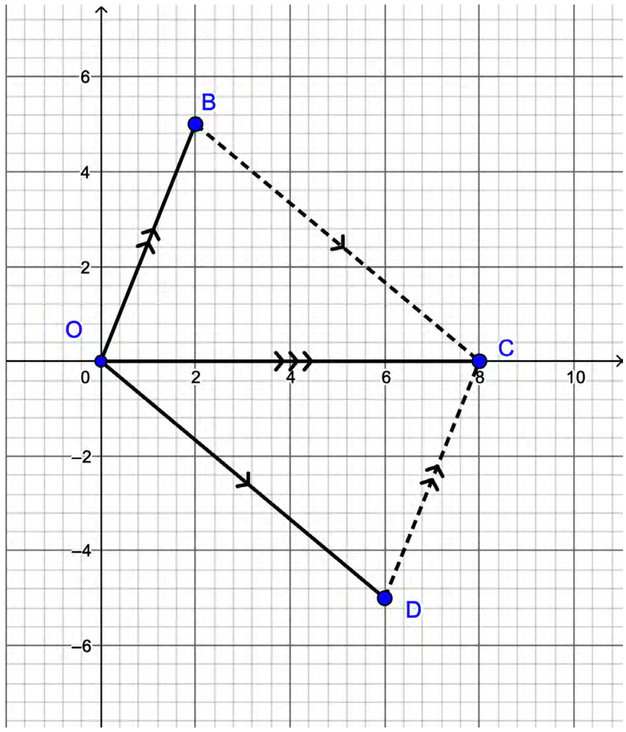


Figure 12: Parallelogram law of vector addition

Generalisation

- Given two vectors, $\overrightarrow{OB}$ and $\overrightarrow{OD}$, being co-initial vectors and representing adjacent sides of a parallelogram, OBCD, as in Figure 11, the resultant vector $\overrightarrow{OC}$ can be represented by the diagonal of the parallelogram passing through O.
- $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$ and $\overrightarrow{OC} = \overrightarrow{OD} + \overrightarrow{DC}$ by triangular law of vector addition
- $\overrightarrow{BC} = \overrightarrow{OD}$ and $\overrightarrow{DC} = \overrightarrow{OA}$, hence $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{OD}$

Example 8

The vertices of a parallelogram reservoir are at M (6, −1), N (5, 1), S (9, 3) and T (x, y).

a) Find the coordinates of T

b) Find $|\overrightarrow{MN}|$

Solution

a) $\overrightarrow{MT} = \overrightarrow{NS}$

$$\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 6 \\ -1 \end{pmatrix} = \begin{pmatrix} 9 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x - 6 \\ y + 1 \end{pmatrix} = \begin{pmatrix} 9 - 5 \\ 3 - 1 \end{pmatrix}$$

$$x - 6 = 4$$

$$x = 10$$

$$y + 1 = 2$$

$$y = 1$$

Hence **T (10, 1)**

b) $|\overrightarrow{MN}| = \sqrt{(5 - 6)^2 + (1 - (-1))^2}$

$= \sqrt{1 + 4}$

$= \sqrt{5}$ units

REVIEW QUESTIONS

1. Given that $\boldsymbol{a} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$, find:

   i. $5\boldsymbol{a} + 2\boldsymbol{b}$

   ii. $3\boldsymbol{a} - \frac{1}{2}\boldsymbol{b}$

2. Given that $\boldsymbol{a} = \begin{pmatrix} 4 \\ -8 \end{pmatrix}$ *and* $\boldsymbol{b} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ find $\boldsymbol{r}$ such that $\frac{1}{4}\boldsymbol{a} - \boldsymbol{b} + \boldsymbol{r} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

3. A hiker travels 4 km north, then turns and walks 3 km west. The hiker then walks directly back to the starting point.

   Represent the hiker's journey using vectors and apply the triangular law of vector addition to determine the resultant vector. How far is the hiker from the starting point after the first two legs of the journey?

4. Ama and Kojo are preparing to go to the market. Ama walks 3 km due east to reach the main road. From there, she walks 4 km due north to reach the market. Kojo, on the other hand, starts from a different point and walks 5 km due west and then 2 km due south to reach the same market.

   i. Represent Ama's and Kojo's displacements as vectors.

   ii. Determine the resultant vector of Ama's displacement.

   iii. If Ama and Kojo were to return home directly, calculate the vector representing their journey back.

5. A delivery van needs to transport goods from Kumasi to Nsawam. The driver first travels 120 km southeast to Konongo and then 80 km due east to Nsawam.

   i. Represent the two parts of the journey as vectors.

   ii. Determine the total displacement of the van from Kumasi to Nsawam.

   iii. If the driver takes an alternative route that goes directly from Kumasi to Nsawam, calculate the vector of this direct route and compare it with the total displacement vector.

6. A cyclist rides 6 km due east from Madina to Legon, then 8 km due north to Achimota.
   i. Represent the cyclist's journey as vectors.
   ii. Calculate the resultant vector representing the cyclist's total displacement.
   iii. Determine the vector representing the cyclist's journey if they return directly to Madina from Achimota.
7. Two fishing boats are on Lake Volta. Boat A moves 4 km due east from its initial position, while Boat B moves 8 km due east from a point 2 km north of Boat A's starting position.
   i. Are the displacement vectors of the two boats parallel? Explain your reasoning.
   ii. If the boats were to return to their starting points, would the return vectors be parallel? Justify your answer.
8. A delivery van travels 10 km due north from Adabraka to Achimota. Another delivery van starts from Kaneshie and travels 20 km due north to Kwabenya.
   i. Represent the displacement vectors of the two vans.
   ii. Determine if the vectors are parallel.
   iii. If the second van's route was due south instead of north, would the vectors still be parallel?
9. Two Minibuses operate in Kejetia. The first one travels 5 km due west from Kejetia to Santasi, while the second one travels 15 km due west from Kejetia to Abuakwa.
   i. Are the routes of the two Minibuses represented by parallel vectors?
   ii. If a third Minibuses traveled 5 km due east from Kejetia, would its vector be parallel to the first two?
10. A construction worker in Ghana is tasked with marking a 20-meter segment of a new road, which is oriented in the direction of the vector $v = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$ meters.
   i. Find the unit vector in the direction of **v**.
   ii. If the worker needs to mark a 20-meter segment in the same direction, determine the vector that represents this segment.

11. Kofi and Kwame are paddling a canoe on Lake Bosomtwe. Kofi paddles 4 km directly north, while Kwame paddles 3 km directly east.
    - i. Represent Kofi and Kwame's movements as vectors.
    - ii. Use the triangular law of vector addition to determine Kwame's position relative to Kofi's position.

iii. Calculate the magnitude and direction of this vector.

12. Two tugboats are towing a cargo ship off the coast of Tema. The first tugboat exerts a force of 10 kN in a direction due east, and the second tugboat exerts a force of 6 kN in a direction 60° north of east.
    - i. Represent the forces exerted by the tugboats as vectors.
    - ii. Use the parallelogram law of vector addition to determine the resultant force acting on the cargo ship.
    - iii. Calculate the magnitude and direction of the resultant force.

ANSWERS TO REVIEW QUESTIONS

1. i. $\begin{pmatrix}11\\13\end{pmatrix}$:

   ii. $\begin{pmatrix}10\\1\end{pmatrix}$

2. $r = \begin{pmatrix}-2\\7\end{pmatrix}$

3. $\begin{pmatrix}0\\4\end{pmatrix} + \begin{pmatrix}-3\\0\end{pmatrix} + \begin{pmatrix}3\\-4\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$

   After the first two legs of the hike the hiker is 5km from the starting point.

4. i. Ama's displacement: $\vec{a_1} = \begin{pmatrix}3\\0\end{pmatrix}$, $\vec{a_2} = \begin{pmatrix}0\\4\end{pmatrix}$

   Kojo's displacement: $\vec{b_1} = \begin{pmatrix}-5\\0\end{pmatrix}$, $\vec{b_2} = \begin{pmatrix}0\\-2\end{pmatrix}$

   ii. Ama's resultant displacement: $\begin{pmatrix}3\\4\end{pmatrix}$

   iii. Ama's return vector: $\begin{pmatrix}-3\\-4\end{pmatrix}$ and Kojo's return vector: $\begin{pmatrix}5\\2\end{pmatrix}$

5. i. First part of the journey: $\begin{pmatrix}84.85\\-84.85\end{pmatrix}$, Second part of the journey: $\begin{pmatrix}80\\0\end{pmatrix}$

   ii. Total displacement vector: $\begin{pmatrix}164.85\\-84.85\end{pmatrix}$

   Magnitude of the displacement =185.405 km to 3 decimal places

   iii. Direct route vector: $\begin{pmatrix}164.85\\-84.85\end{pmatrix}$ which is 185.405km, compared with his current route which is 200km, so is about 15km shorter.

6. i. First part of the journey: $\begin{pmatrix}6\\0\end{pmatrix}$, Second part of the journey: $\begin{pmatrix}0\\8\end{pmatrix}$

   ii. Resultant displacement vector: $\begin{pmatrix}6\\8\end{pmatrix}$

   iii. Return vector: $\begin{pmatrix}-6\\-8\end{pmatrix}$

7. **i.** Yes, the displacement vectors of the two boats are parallel because both boats are moving in the same direction (due east).

Parallel vectors have the same or exact opposite direction.

ii. Yes, the return vectors would also be parallel because both boats would be traveling in the same direction back to their starting points (due west), maintaining parallelism.

8. **i.** The displacement vector of the first van is $v_1 = \begin{pmatrix} 0 \\ 10 \end{pmatrix}$ and the displacement vector of the second van is $v_2 = \begin{pmatrix} 0 \\ 20 \end{pmatrix}$

ii. Yes, the vectors are parallel because both vectors have the same direction (due north) as $v_2 = 2v_1$

iii. If the second van's route was due south, the vectors would still be parallel, as they are in the exact opposite direction, $v_2 = \begin{pmatrix} 0 \\ -20 \end{pmatrix} = -2v_1$. They still have a scalar constant so they remain parallel.

9. **i.** Yes, the routes of the two minibuses are represented by parallel vectors because both travel due west, maintaining the same direction.

ii. Yes, the third Minibus vector would be parallel to the first two as it is in the exact opposite direction (due east rather than due west).

10. **i.** The unit vector **u** in the direction of **v** is given by: $\boldsymbol{u} = \begin{pmatrix} \frac{5}{13} \\ \frac{12}{13} \end{pmatrix}$

ii. The vector representing the 20-meter segment in the same direction is:

$$\mathbf{v'} = 20 \times \mathbf{u} = v = \begin{pmatrix} \frac{100}{13} \\ \frac{240}{13} \end{pmatrix}$$

11. **i.** Kofi's movement: a$= \begin{pmatrix} 0 \\ 4 \end{pmatrix}$ km

Kwame's movement: **b**$= \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ km

ii. Kwame's position, relative to Kofi's $= \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

iii. The magnitude of is: 5 km. The direction $(\theta) \approx 53.13°$ south of east.

12. **i.** First tugboat: $\boldsymbol{F}_1 = \begin{pmatrix} 10 \\ 0 \end{pmatrix} kN$ (due east)

Second tugboat: $\boldsymbol{F}_2 = \begin{pmatrix} 3 \\ 3\sqrt{3} \end{pmatrix}$ kN

ii. the resultant vector $\mathbf{R} = \begin{pmatrix} 13 \\ 3\sqrt{3} \end{pmatrix}$

iii. The magnitude of the resultant force **R**= 14 kN. The direction $(\theta) \approx 22^\circ$ north of east.

GLOSSARY

- Collinear vectors are vectors that lie along the same line or along parallel lines. These vectors can have the same or opposite directions. Mathematically, if two vectors **a and b** are collinear, then there exists a scalar k such that **a** = k**b**.
- Co-initial vectors are vectors that have the same starting point (initial point). Even though they may point in different directions or have different magnitudes, they all originate from the same location.
- Parallel vectors are vectors that have the same or exactly opposite direction. They may differ in magnitude but lie along lines that are parallel to each other.
- The triangular law of vector addition states that if two vectors are represented as two sides of a triangle in sequence, then the third side of the triangle (taken in the reverse order) represents the resultant vector.
- The parallelogram law of vector addition states that if two vectors are represented by adjacent sides of a parallelogram, then the resultant vector is represented by the diagonal of the parallelogram that starts from the same point.

EXTENDED READING

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ACKNOWLEDGEMENTS

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