Mathematics

Year 1

SECTION

3

ALGEBRAIC EXPRESSIONS AND FACTORISATION

ALGEBRAIC REASONING

Applications of Expressions, Equations and Inequalities

INTRODUCTION

Hello learner! In this section, we will delve into the world of number patterns, where we will discover hidden structures and relationships between numbers. We will learn how to represent these patterns using algebraic expressions, and how to perform operations such as simplification, addition, subtraction, multiplication, and division. We will again delve into perfect squares; learning how to identify and use them in solving equations and inequalities, including difference of two squares of binomials. Perfect squares have unique properties and play a significant role in various mathematical operations such as factorising expressions, simplifying expressions and solving equations. Finally, we shall explore algebraic fractions by performing operations on simple fractions with monomial and binomial denominators and finally, investigate the conditions under which these fractions become zero or undefined. This will help us in mastering the art of analysing and simplifying expressions. These also have real-world applications in fields like geometry, trigonometry and calculus.

At the end of this section, you will be able to:

- Use number patterns and variables to formulate mathematical expressions and apply the algebraic order of the four operations to solve.
- Factorisation of algebraic expressions.
- Recognise perfect squares and apply the idea to solve problems including the difference of two squares of binomials.
- Analyse and apply operations on simple algebraic fractions including monomial and binomial denominators and determine the conditions under which algebraic fraction is zero or undefined.

Key ideas

- **Number Patterns**: These are sequences or patterns that help in recognising relationships and predicting outcomes. For example, recognising that the sequence 2, 4, 6, 8 follows a pattern of increasing by 2.
- **Variables**: These are unknown quantities that are represented by letters or symbols.
- **Factorisation of algebraic expressions**: Algebraic expressions can be broken down into factors. For example, $x^2 - 4$ can be factorised as $(x - 2)(x + 2)$.
- A perfect square is an expression that can be written in the form $(x)^2$ or x^2, where x is a variable or an expression. For example, 4, 9, and 16 are perfect squares because they can be written as 2^2, 3^2, and 4^2 respectively.
- The difference of two squares formula is: $a^2 - b^2 = (a + b)(a - b)$. This formula can be used to factorise expressions like $x^2 - 4$ *or* $9 - y^2$.
- Algebraic Fractions: An algebraic fraction is an expression in the form $\frac{a}{b}$, where a and b are algebraic expressions. To perform operations on algebraic fractions, we can follow these rules:
- An algebraic fraction is zero if the numerator (a) is zero, and it is undefined if the denominator (b) is zero.

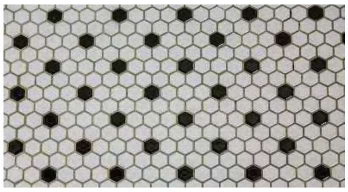
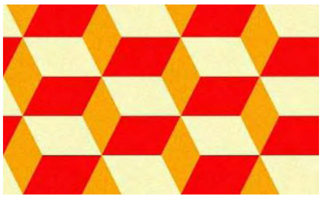
NUMBER PATTERNS, ALGEBRAIC EXPRESSIONS AND ITS OPERATIONS

Number patterns are sequences of numbers that follow a certain rule or formula. These patterns can be found all around us, from the natural world to man-made structures. Understanding number patterns helps us make predictions, solve problems, and uncover the underlying order in seemingly random sets of numbers.

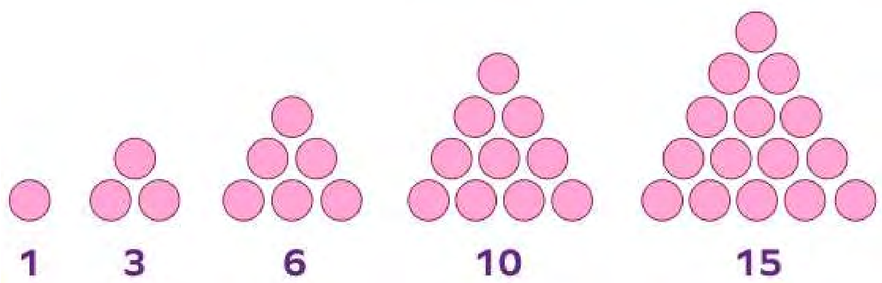
Let us take a look at some examples of patterns in real life.

From the pictures below, we realise that patterns can be found in all aspect of our lives. From our clothes, home decorations, building designs and even animals on both land and sea, we see various beautiful patterns. Can you identify some patterns around you? Share with a friend.



In Junior High School, we also learnt about patterns from shapes and numbers. Let us take a look at some examples.



Take a look at the pattern above, it is made up of circles. Can you determine how each successive group of circles came about? We realise that each next pattern is generated by adding the next counting number to the preceding pattern, right? But assuming we are to determine the number of circles in the 100^{th} term. Could that be easy? I'm sure you agree with me that it will be a challenging task. So, we will need to find a smart way to do that. This brings us to the need to find the rule or nth term rule. This will help us to be able to find any term in any given pattern.

Now in this lesson our focus will be on writing algebraic expressions for the rules of the patterns that we will explore. Then in SHS 2 we will turn our attention to exploring patterns and sequences including arithmetic and geometric patterns.

Dear Learner,

Now that we have explored Number Patterns, Algebraic Expressions, and their Operations, let's apply what we've learned through a series of engaging activities. Please follow the instructions below for each activity.

Activity 1

Let's look at this pattern:

Pattern: 2, 5, 8, 11, 14, ...

Materials: Bottle tops.

Rule:

- Use bottle tops to create the above pattern.
- We start with 2 bottle tops.
- Each time, we add 3 bottle tops. This means that to go from 'term to term' to find the next term we + 3

Algebraic Expression: Let a_n represent nth term in the pattern. The expression can be written as:

$a_n = 3n - 1$

Where n is the position of the term in the pattern. Hence, the algebraic expression is $3n - 1$.

Activity 2

The pattern below is made up of squares, made from lines, as shown below. Investigate the pattern, find the nth term rule and write an algebraic expression for the number of lines required to make the pattern, if the pattern continues.

Fig.1 **Fig.2** **Fig.3**

From the pattern, we realise that,

Fig 1	Fig 2	Fig 3	N^{th}
4	7	10	?

From the table let represent each term with n. Now, if we multiply n by 3 and add 1 we obtain the number of lines needed to create the shape in each term.

Therefore, our rule for the pattern is $a_n = 3n + 1$. Remember, $3n + 1$ is an algebraic expression.

Activity 3

Ama is collecting seashells on the beach. On her first day, she collected 1 seashell. Each day after that she collected 2 more seashells than the previous day. How many seashells will she have on the fifth day? Let's generate a pattern from the above and use it to formulate mathematical expression.

Step 1: Identify the pattern

Day 1: 1 seashell

Day 2: $1 + 2 = 3$ seashells

Day 3: $3 + 2 = 5$ seashells

Day 4: $5 + 2 = 7$ seashells

Day 5: $7 + 2 = 9$ seashells

Step 2: Formulate the mathematical expression

The pattern shows that each day, the number of seashells increases by 2. Therefore, the mathematical expression is:

$S = 1 + 2(n - 1) = 1 + 2n - 2 = 2n - 1$

where S is the total number of seashells and n is the day number.

Step 3: Use the expression to find the number of seashells on the fifth day

Substitute $n = 5$ into the expression:

$S = 2(5) - 1$

$S = 10 - 1$

$S = 9$

Therefore, Ama will have 9 seashells on the fifth day.

*Have you ever heard of the term **"Algebraic expression"**? if you do that's very good, if you do not let's now look at the meaning of Algebraic expression.*

An Algebraic expression in mathematics is an expression which is made up of variables, constants and arithmetic operations. (addition, subtraction, multiplication and division) Some examples include:

$3x - 5$, $n + 8$, x^2, $2x^2 + 3x - 5$ and $2x + 5xy + 3$.

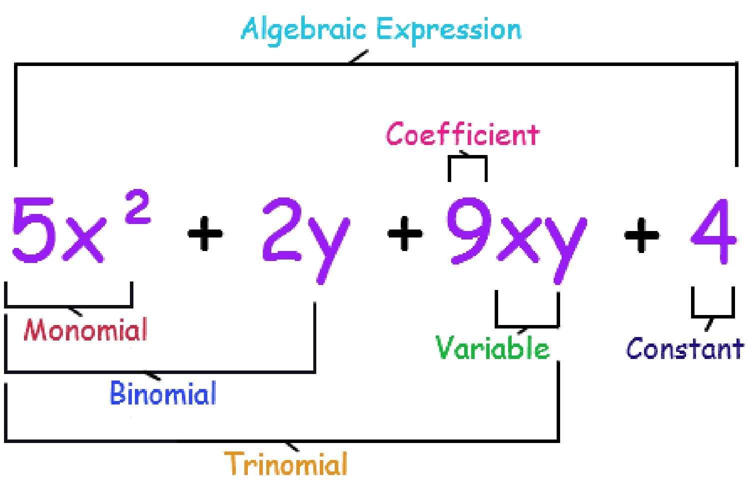
Definition of Key Concepts

1. **Term:** is the part of the expression, e.g. $x^2 + 5x - 4$ (three terms)
2. **Variable:** is a symbol (usually a letter), used to represent one or more numbers in algebraic expression (e.g. from above, x, n, y are the variables)
3. **Monomials:** an algebraic expression with only one term (e.g.: y, $2x^2$, $-5a$ etc.)
4. **Binomial:** an algebraic expression with two terms (e.g.: $7x^2 + 3$, $y - 2$, $4x + 9$, $n - m$ etc.)
5. **Trinomial:** algebraic expressions with three terms (e.g.: $2x^2 + 5x - 3$, $a - 5b + 2c$, $8x + 5y - z$)
6. **Coefficient:** is a number attached to a variable in an algebraic expression. Consider the expressions below and their corresponding coefficients

   $x + 5x - 3$ $9a - 3b + 2$

   1   5   9   -3
7. **Constant term:** is a number or symbol which is not attached to any variable in an expression. From the above expressions the constant terms are -3 and 2.

The diagram below gives a summary of the terms explained.



Formulation of Algebraic Expressions

Algebraic expressions can be created using models and variables.

Application of concepts in real world activities

1. **a)** Nine more than a certain number (x)

Solution: $x + 9$

b) Two less than one-third of a certain number (x) is

Solution: $\frac{1}{3}x - 2$

c) Twice the square of a number (y) minus the cube of another number (n).

Solution: $2y^2 - n^3$

d) The age of Mr. Mensah is thrice his son's age (a) plus ten.

Solution: $3a + 10$

e) There are 25 oranges in a bag. Write the algebraic expression for the number of oranges in x number of bags.

Solution: $25x$

Rules for the Use of the Operations of Algebraic Expression

2. Addition and Subtraction

In algebra, you can only add or subtract like terms, example $3x + 5x = 8x$, $2y - 5y = -3y$, $7a + 5b = 7a + 5b$

i. **Like terms:** Two or more terms are like terms if they have the same variables with the same exponent irrespective of their numerical coefficients. They can be added or subtracted to get a single term. For example: $3x^2$ and $7x^2$, $4xy$ and $2xy$, $5a^3b$ and $-7a^3b$

ii. **Unlike terms**: are terms that cannot be added or subtracted in an expression to get a single term. Example: $3x^2$ and a^2, $5y$ and $4xy$

3. Multiplication and Division

Both like terms as well as unlike terms can be multiplied and divided. In multiplying, we make use of distributive property depending on the expression given. To divide, look for factors that are common to both numerator and denominator and this can be divided or cancelled. Factors that are common to all terms of an expression can be factored out.

Simplifying Algebraic Expression

Example 1

Simplify the following expressions.

a. $x + 2y + 5x - y$

b. $5p - c - 9c$

c. $4x \times 2y$

d. $x^2 (x^3 - 2y)$

e. $10b \div 2b$

Solutions

a. $6x - y$

b. $5p - 10c$

c. $8xy$

d. $x^5 - 2x^2y$

e. $5b$

Activity 4

Expand $(2x + 3)(x - 4)$

Step 1: Multiply the two binomials using the distributive property:

$(2x + 3)(x - 4) = 2x(x) - 2x(4) + 3(x) - 3(4)$

Step 2: Multiply the terms:

$2x \times (x) = 2x^2$

$2x \times (-4) = -8x$

$3 \times (x) = 3x$

$3(-4) = -12$

Step 3: Combine like terms:

$2x^2 - 8x + 3x - 12$

Step 4: Simplify the expression by combining like terms:

$2x^2 - 5x - 12$

Therefore, the expanded form of $(2x + 3)(x - 4)$ is $2x^2 - 5x - 12$

We've explored number patterns, learned how to identify and describe the rules that govern them and also how to formulate algebraic expression from the number patterns. Let's now look at factorisation of algebraic expressions.

FACTORISATION OF ALGEBRAIC EXPRESSIONS

In factorising algebraic expressions, we look for like terms, group them and find common factors. Factorising is the reverse process of expanding brackets in algebraic expressions. Algebraic expressions could be factorised in many ways depending on the given expression(s) and it includes the common factor approach, algebraic tiles, regrouping of terms approach, standard identity approach, splitting-the-middle-term approach and using the quadratic formula. For instance;

Factorising

$$3x + 6 \equiv 3(x + 2)$$

Expanding brackets

Factorising

$$x^2 + 6x + 5 \equiv (x + 5)(x + 1)$$

Expanding brackets

Worked Examples

Factorise the following

1. $4x + 4by$
2. $ac + bc + ad + bd$

Solutions

1. Look for the common factor(s)

   Factor the common term out (Highest common factor)

   $4x + 4by = 4(x + by)$
2. First put the four terms in the expression into two groups of two and find the common factors from each. i.e., $(ac + bc) + (ad + bd) = c(a + b) + d(a + b)$ and add the outside terms and take one of the common terms to be the final answer. $(a + b)(c + d)$.

There are many ways to factorise algebraic expressions. Factorisation is an important skill in algebra that helps us solve equations. We'll look at different methods to factorise expressions, and each method is useful in different situations. By learning these methods, you'll become good at factorising expressions and solving equations.

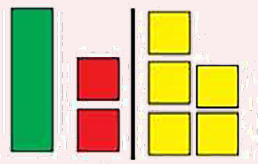
Factorising and Solving Equations Using Algebraic Tiles

Activity 5

Solve $x + (-2) = 5$ using the algebraic tiles

Solution

Step 1

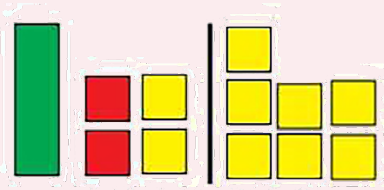


x + (-2) = 5

Let's represent the equation with our algebraic tiles

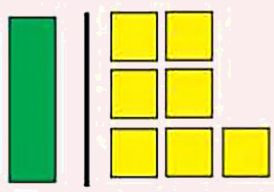
Step 2:

Transpose -2 by adding 2 to both sides (yellow squares are positive while red squares are negative numbers)



x + (-2) + 2 = 5 + 2

Step 3:



x = 7

I hope you understand factorisation by using algebraic tiles, Let's explore another method of factorisation, called Factorisation by Regrouping of Terms.

Factorisation by Regrouping of Terms Approach

In some algebraic expressions, not every term may have a common factor. For instance, consider the algebraic expression, $12a + n - na - 12$. The terms of this expression do not have a particular factor in common but the first and last term has a common factor of '12'. Similarly, the second and third terms have n as a common factor. So, the terms can be regrouped as:

$\Rightarrow 12a + n - na - 12 = 12a - 12 + n - an$

$\Rightarrow 12a - 12 - an + n = 12(a - 1) - n(a - 1)$

After regrouping, it can be seen that $(a - 1)$ is a common factor in each term,

$\Rightarrow 12a + n - na - 12 = (a - 1)\,(12 - n)$

Thus, by regrouping terms we can factorise algebraic expressions.

Let's now engage in an activity using the factorisation by regrouping of terms approach:

Activity 6

Example: Factorise the expression: $x^2 + 5x + 6$

Follow these steps below:

Steps:

Step 1: Write down the expression: $x^2 + 5x + 6$

Step 2: Look for two numbers whose product is 6 (the constant term) and whose sum is 5 (the coefficient of the linear (or x) term). These numbers are 2 and 3.

Step 3: Rewrite the expression by regrouping the terms: $x^2 + 2x + 3x + 6$

Step 4: Factor out the common term from each group: $x(x + 2) + 3(x + 2)$

Step 5: Factor out the common binomial term: $(x + 3)(x + 2)$

Therefore, the factorised form of the expression is $(x + 3)(x + 2)$.

I hope you're familiar with factorisation through regrouping terms. Now, let's dive into another exciting method: factorising expressions using standard identities.

Factorising expressions using standard identities

An equality relation is one which holds true for all the values of variables in mathematics. It is known as an identity and has this sign, $\equiv$, rather than the standard $=$ sign. Consider the following identities:

$(a + b)^2 \equiv a^2 + b^2 + 2ab$

$(a - b)^2 \equiv a^2 + b^2 - 2ab$

$a^2 - b^2 \equiv (a + b)(a - b)$

On substituting any value of a and b, both sides of the given equations remain the same. Therefore, these equations are called identities.

Hello learner! Let's engage in this activity by applying some standard identities which suits our example.

Activity 7

Example: $x^2 - 4$

Follow these steps of Activities

Step 1: Identify the Expression:

Given the expression $x^2 - 4$

Step 2: Recognise the Pattern:

The expression fits the pattern of the difference of two squares: $a^2 - b^2$

Step 3: Rewrite the Expression:

Rewrite 4 as 2^2 so the expression becomes $x^2 - 2^2$

Step 4: Apply the Identity:

- Using the identity $a^2 - b^2 \equiv (a + b)(a - b)$
- Here, $a = x$, $b = 2$

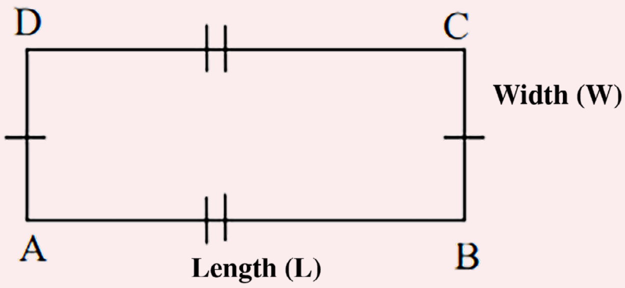
Step 5: Simplify and Combine:

Apply the identity: $x^2 - 2^2 \equiv (x + 2)(x - 2)$

Thus, the factorised form of $x^2 - 4$ is $(x + 2)(x - 2)$

Rectangle

A rectangle is a quadrilateral with opposite pair of sides equal



The side AB = DC = Length(L)

The side BC = AD = Width(W)

The opposite sides are parallel and equal

Area of a rectangle = $Length(L) \times Width(W)$

Area of a rectangle = $L \times W$

Note that area is expressed in square units

Let's try these two examples on area of a rectangle

Example 2

The length of a rectangle is 4 cm and its width (breadth) is 3 cm. Find its area.

Solution

Area of a rectangle = Length × Width

$= 4\ cm \times 3\ cm = 12\ cm^2$

Therefore, the area of the rectangle is $12cm^2$

Example 3

The picture below is Mr. Bluba's rectangular garden, where he grows cabbage and Kontomire, measures 8m in length and 3m in width. What is the total area of the garden?

Solution

Area of the garden = Length of the garden × Width of the garden

$= 8m \times 3\ m = 24m^2$

Thus, the area of the rectangular garden is $24m^2$

Think of it like this! If you have a room with a length of 10 feet and a width of 5 feet, the area would be 10 x 5 = 50 square feet. That's the amount of space you'd need to cover with flooring, paint, or carpet!

Now that we've learned about the area of a rectangle, let's use this knowledge to explore how to factorise quadratic trinomials using algebraic tiles.

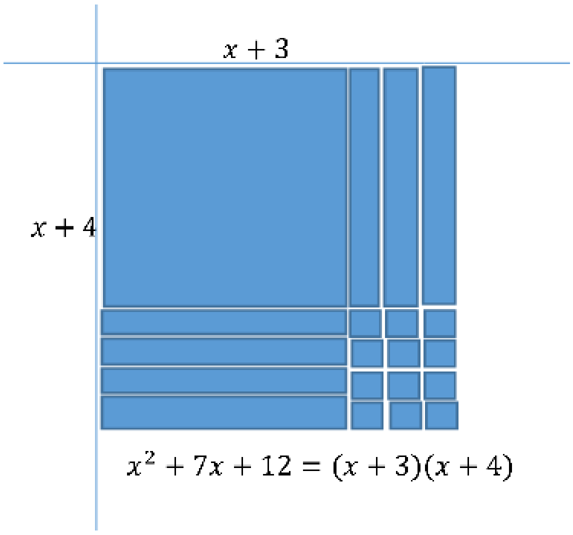
Factorise Quadratic Trinomials Using Algebraic Tiles

Example 4

Factorise $x^2 + 7x + 12$ using algebraic tiles

Solution

Use algebraic tiles



Let's undertake this activity on how to factorise quadratic trinomials using algebraic tiles

Activity 8

Let's factorise $x^2 + x - 6$ using trinomial algebraic tiles:

Step 1: Represent the expression as tiles:

$x^2 = (x \times x)$

$x = (1 \times x)$

$-6 = (-6 \times 1)$

Step 2: Arrange the tiles to form a rectangle:

$(x \times x) + (3 \times x) + (-2 \times x) + (-6 \times 1)$

Step 3: Identify the common factors:

$(x \times x) + (3 \times x) = x(x + 3)$

$(-2 \times x) + (-6 \times 1) = -2(x + 3)$

Step 4: Combine the factors:

$x(x + 3) - 2(x + 3)$

Step 5: Factor out the common binomial term:

$(x - 2)(x + 3)$

Factorising Quadratic Trinomials Using the Standard Identities

Example 5

Factorise $x^2 + 6x + 9$

Solution

$x^2 + 6x + 9 = 0$

The identity is $a^2 + 2ab + b^2 \equiv (a + b)^2$

The LHS is of the form $a^2 + 2ab + b^2$,

$\Rightarrow (x + 3)^2$

Or $(x + 3)\ (x + 3)$

Factors are $(x + 3)$ and $(x + 3)$

Let's now look at how to factorise using quadratic equations using formula approach.

Factorising Quadratic Equation Using Formula Approach

This method is very similar to the method of splitting the middle term.

Step 1: Consider the quadratic equation $ax^2 + bx + c = 0$

Step 2: Now, find two numbers such that their product is equal to ac and sum equals to b.

$(number\ 1)\ (number\ 2) = ac$

$(number\ 1) + (number\ 2) = b$

Step 3: Substitute these two numbers in the formula given below:

$(1/a)\ [ax + (number\ 1)]\ [ax + (number\ 2)] = 0$

Step 4: Finally simplify the equation

Example 6

Solve $3x^2 + 7x + 4 = 0$

Follow the steps carefully

Solution

$3x^2 + 7x + 4 = 0$

Here, $a = 3$, $b = 7$, $c = 4$

$ac = (3)(4) = 12$

Let's identify two numbers such that their sum is 7 and the product is 12.

Factors of 12: 1, 2, 3, 4, 6, 12

Sum of two factors = 7

Product of those two factors = 12

Number 1 = 3 and number 2

Now, substitute these two numbers in the formula (1/a) [ax + (number 1)] [ax + (number 2)] = 0.

$(\frac{1}{3})\ (3x + 3)\ (3x + 4) = 0$

$(3x + 3)\ (3x + 4) = 0$

$\frac{1}{3} \times 3\ (x + 1)\ (3x + 4) = 0$

$(x + 1)(3x + 4) = 0$

Thus, $(x + 1)$ and $(3x + 4)$ are the factors of the given quadratic equation.

Learners, now that we've mastered factoring quadratic equation using the formula approach. Let's explore another fascinating method of factorisation of quadratic equation by splitting the middle term

Get ready to dive into this exciting new technique.

Factorisation of Quadratic Equations by Splitting the Middle Term

Step 1: Consider the quadratic equation $ax^2 + bx + c = 0$

Step 2: Now, find two numbers such that their product is equal to ac and sum equals to b.

$(number\ 1)(number\ 2) = ac$

$(number\ 1) + (number\ 2) = b$

Step 3: Now, split the middle term using these two numbers,

$ax^2 + (number\ 1)\ x + (number\ 2)\ x + c = 0$

Step 4: Take the common factors out and simplify.

Let's have a look at the example problem given below:

Example 7

Factorise and then solve the quadratic equation $x^2 + 7x + 10 = 0$ by splitting the middle term

Given, $x^2 + 7x + 10 = 0$

Here, $a = 1, b = 7, c = 10$

$ac = (1)\ (10) = 10$

Factors of 10: 1, 2, 5, 10

Let's identify two of these factors such that their sum is 7 and their product is 10.

Sum of two factors $= 7 = 2 + 5$

Product of these two factors $= (2)(5) = 10$

Now, split the middle term.

$x^2 + 2x + 5x + 10 = 0$

Take the common terms and simplify.

$x(x + 2) + 5(x + 2) = 0$

$(x + 5)(x + 2) = 0$

Thus, $(x + 2)$ and $(x + 5)$ are the factors of the given quadratic equation.

$x + 2 = 0 \quad x = 0 - 2 \quad x = -2$

$x + 5 = 0 \quad x = 0 - 5 \quad x = -5$

Solving these two linear factors, we get $x = -2, -5$ as the roots.

Factorising Quadratic Equation Using Quadratic Formula

In quadratic formula, to get the roots of quadratic equation $ax^2 + bx + c = 0$ is given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Substituting the values of a, b, c and simplifying the expression, we get the values of roots.

Great job, everyone! Now that we've covered how to factor quadratic equations using the quadratic formula, let's put what we've learned into practice with a fun activity.

Activity 9

Given:

$x^2 + 4x - 21 = 0$

Here, $a = 1$, $b = 4$, $c = -21$

$b^2 - 4ac = (4)^2 - 4(1)(-21) = 16 + 84 = 100$

Substituting these values in the quadratic formula, we get

$$x = \frac{-4 + 10}{2(1)}$$

$$= \frac{-4 \pm 10}{2}$$

$$x = \frac{-4 + 10}{2}, \text{ or } x = \frac{-4 - 10}{2}$$

$$x = \frac{6}{2}, \text{ or } x = -\frac{14}{2}$$

$$x = 3, \text{ or } x = -7$$

Therefore, factors of the given quadratic equation are $(x - 3)$ and $(x + 7)$.

PERFECT SQUARES

Can you think of integers that can be expressed as the square of another integer? I hope you have written numbers such as $1 = 1^2$, $4 = 2^2$, $16 = 4^2$ and many others. These numbers; 1, 4, 16, ... are what we call 'Perfect Squares'. How would you describe 'perfect squares' in your own words?

What are Perfect Squares?

A perfect square is an integer that can be expressed as the square of another integer. E.g. 4 is a perfect square of 2, because it can be expressed as $2^2 = 2 \times 2 = 4$. Other examples such as 1, 9, and 16 are perfect squares because they can be written as 1^2, 3^2 and 4^2 respectively.

It is important to note the following;

- any number that cannot be expressed as the square of another number (integer) is **not** a perfect square. For example; 2 is not a perfect square because it cannot be expressed as the square of another integer.
- negative numbers are not considered as perfect squares. This is because negative numbers cannot be expressed as the square of another integer. For example, $-49 \neq 7^2 \ or \ (-7)^2$

Let us go through the activity below on how to generate some perfect squares by adding consecutive odd numbers.

Activity 10

List the first 5 consecutive odd numbers.

Add consecutive odd numbers to generate perfect squares

1, 2, **3**, 4, **5**, 6, **7**, 8, **9**, 10…

$1 = 1 = 1^2$

$1 + 3 = 4 = 2^2$

$1 + 3 + 5 = 9 = 3^2$

$1 + 3 + 5 + 7 = 16 = 4^2$

$1 + 3 + 5 + 7 + 9 = 25 = 5^2$

1 hope you have observed the pattern and can follow through to generate more of these perfect squares. Write them down and compare it to that of your friend(s).

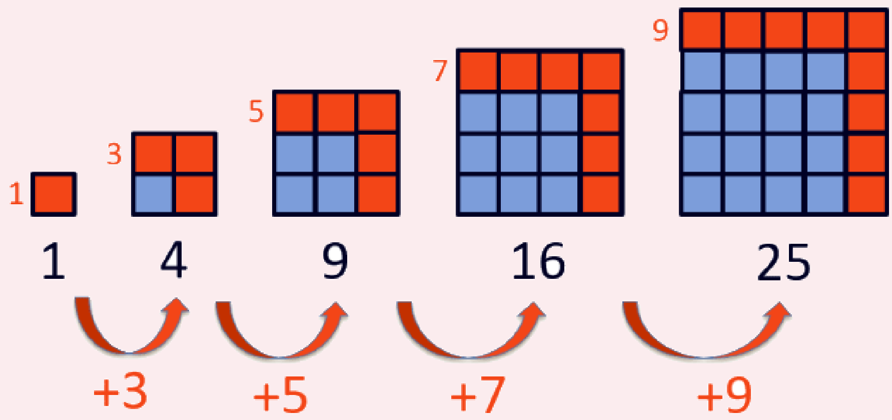
Let us now look us how to use the graph or squared paper to generate square numbers in the activity below.

Activity 11

1. Take a graph or squared paper and draw a square on it with a side length of 1 unit (centimetres will fit better than inches).
2. Draw another square to the right of the first one with a side length of 1 unit and extend it into a new square.
3. Shade the area of the new square obtained.
4. Count the number of grid squares in the shaded area.
5. Record the results:

   - Side length of outer square
   - Number of grid squares in shaded area (area of inner square)

6. Repeat steps 2-5 for different side lengths.
7. Look for patterns and relationships between the side lengths and areas.



Hello learner! I hope you have performed the activity, observed the pattern and can continue with that of 36, 49, 64, 81, ... etc and determine their respective differences. Very Good!

However, discuss your observations on the following with a classmate;

- How does the area of the larger square change as the side length increases?
- Can you predict the area of a larger square based on its side length?
- What is the pattern between the perfect squares and their areas?

Let us now investigate the difference between two perfect squares. I hope you have an idea.

Perfect squares can be used to illustrate the concept of difference of two squares. Here's how to go about it:

Activity 12

Let's consider two perfect squares, say:

$a^2 = \mathrm{a} \times \mathrm{a}$ (a squared)

$b^2 = b \times b$ (b squared)

Now, let's find the difference between these two squares:

$a^2 - b^2$

We can rewrite this expression as:

$(a \times a)$ - $(b \times b)$

Next, we can factorise the expression by grouping the terms:

$(a \times a)$ - $(b \times b) = (a + b)(a - b)$

Very Good! We've arrived at the difference of two squares formula:

$\boldsymbol{a^2 - b^2 = (a + b)(a - b)}$

This formula shows that the difference between two squares can be factorised into the sum and difference of the same two terms. This can be a powerful tool for simplifying and evaluating expressions and solving equations.

Also, click on the link to watch the video on the connection between Perfect squares and Difference of two squares [..\Downloads\videoplayback.mp4]

Perfect squares and difference of two squares (square with expression as the sides and some taken out and the area of the rest)

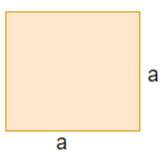


Fig. 1

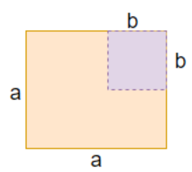
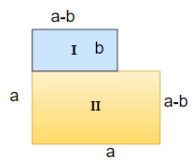


Fig.



2 Fig. 3

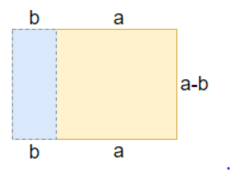


Fig. 4

Figures are not drawn to scale.

We already know that to find the area of square = side(s) × side(s)

Mathematically, Area of a Square = s^2

1. Fig. 1 above shows a square of side **a** and its Area= $\boldsymbol{a} \times \boldsymbol{a} = \boldsymbol{a}^2$

   $\boldsymbol{a}^2$ is the perfect square of $\boldsymbol{a}$

2. Fig. 2 shows the result of taking a square of side **b** from one of the four corners of Fig. 1.

   The result from Fig. 2 gives Fig. 3.

   The Area of Fig. 3 = Area of region I + Area of region II

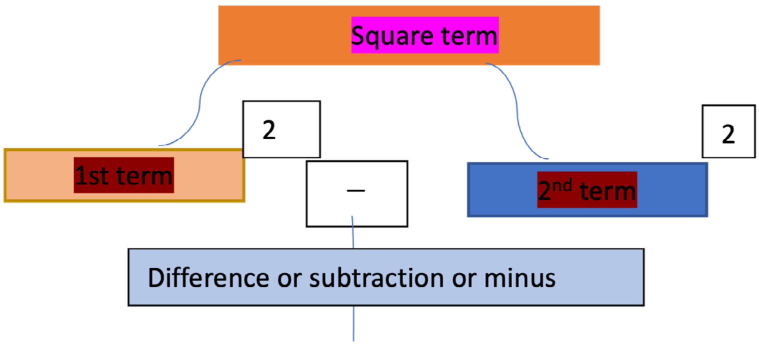
   $((a - b) \times b) + (a \times (a - b)) = a^2 - b^2$

   Area of Fig. 4 $(a+b)(a-b)$ = Length x breadth

   Area of Fig. 3 = area of Fig. 4

   $(a + b)(a - b) = a^2 - b^2$

Below is a pictorial representation on how to expand difference of two squares.



For example, $x^2 - 4$, $y^2 - 1$, $a^2 - b^2$ etc

$x^2 - 4 = (x + 2)(x - 2)$, $y^2 - 1 = (y - 1)(y + 1)$, $a^2 - b^2 = (a - b)(a + b)$

NOTE: *Anytime there is a binomial with each term being squared (i.e. having an exponent of 2) and subtraction (minus) as the middle sign then you are guaranteed to have the case of difference of two square.*

Again, the exponents of each term must be a multiple of two (2) and the coefficients should be a perfect square.

Caution: In the sum of two squares, it is not possible to factor a sum of two squares using real numbers.

Thus; $x^2 + 9 \neq (x + 3)(x + 3)$ or $(x - 3)(x + 3)$

May I take you back to the previous lesson on expansion of binomials and consider the product,

$(a + b)(a - b)$.

Using the distributive property to expand gives us;

$(a + b)(a - b) = a(a - b) + b(a - b)$

$= a^2 - ab + ab - b^2$

$= a^2 - b^2$

$\Rightarrow (a + b)(a - b) = a^2 - b^2$

Let us now explore how to factorise difference of two squares.

Example 8

Factorise $x^2 - 49$

Solution

Since the first term (x^2) has already been expressed as a perfect square, express 49 as a square of another number and that is $7 \times 7 = 7^2$

$x^2 - 49 = x^2 - 7^2$

$= (x + 7)(x - 7)$

Example 9

Factorise $25a^2 - 16$

Solution

$25a^2 - 16 = 5^2a^2 - 4^2$

$= 5(a)^2 - 4^2$

$= (5a - 4)(5a + a)$

Example 10

Factorise completely $p^4 - 81q^4$

Solution

$p^4 - 81q^4 = (p^2)^2 - 9^2(q^2)^2$

$= (p^2)^2 - (9q^2)^2$

$= (p^2 - 9q^2)(p^2 + 9q^2)$

I hope you have observed that $(p^2 - 9q^2)$ is also a difference of two squares and can be factorised as well. You can proceed to factorise the expression as this;

$\Longrightarrow (p^2 - 3^2q^2)(p^2 + 9q^2)$

$= (p - 3)(p + 3)(p^2 + 9q^2)$

Therefore, $p^4 - 81q^4 = (p - 3)(p + 3)(p^2 + 9q^2)$

Can you now try this example?

Example 11

Factorise the expression $(m^2 - n^2) - k(m + n)$ completely

Solution

Since $m^2 - n^2$ is a difference of two squares,

$\Longrightarrow (m^2 - n^2) - k(m + n) = (m - n)(m + n) - k(m + n)$

It can also be observed that $\boldsymbol{m + n}$ is common to both terms so we can factorise it out as;

$= (m + n)((m - n) - k)$

$= (m + n)((m - n - k)$

Let us now apply the concept of the difference of two squares in solving real life problems.

Example 12

If the length of a rectangular mirror is $(x + 2)$ units and the width is $(x - 2)$ units, find the area of the mirror?

Solution

Area of a rectangle $= L \times B$

$= (x + 2) \times (x - 2)$

$= (x^2 - 4)$ *square units*

Example 13

Without using calculator, evaluate $21^2 - 11^2$ completely.

Solution

$21^2 - 11^2 = (21 - 11)(21 + 11)$

$= (10)(32)$

$= 320$

Example 14

Without using calculator, evaluate $9R + 48 = 75$ hence, find the value(s) of R

Solution

$9R = 75 - 48$

$9R = 3(25 - 16)$

$9R = 3(5^2 - 4^2)$

$9R = 3(5 - 4)(5 + 4)$

$9R = 3(1)(9)$

$\frac{9R}{9} = \frac{27}{9}$

$R = 3$

Do look for more examples, try your hands on them together with your classmates and compare your answers.

I hope we are making positive progress. Very Good!

Let us now consider how to operate on fractions that contain variable(s) either as the numerator or denominator or even both.

ALGEBRAIC FRACTIONS

An algebraic fraction is a fraction that contains at least one variable. For example,

$\frac{x}{2}$, *x (the variable) is the numerator.*

$\frac{3}{x+1}$, *the denominator is an expression in terms of x,*

$\frac{x+5}{2x}$, *both the numerator and the denominator contain an x (variable) term*

Just as ordinary fractions, you can add, subtract, multiply and divide them.

Application of Concepts

To simplify algebraic fractions, we do it the same way as numerical fractions. We cancel common factors from the numerator and denominator until no common factors remain.

Note the following:

1. Algebraic fractions can be added or subtracted if they have the same denominators. (a common denominator).
   $\frac{4}{3x} + \frac{1}{3x}$ *or* $\frac{2x}{5} - \frac{3y}{5}$ *or* $\frac{x}{x-1} + \frac{5}{x-1}$ and use the Lowest Common Multiple (LCM) approach when the denominators are not the same.
2. When the fraction is multiplying, such as
   $\frac{m}{n} \times \frac{a}{b} = \frac{ma}{nb}$ where there are common factors, then cancel out if not, multiply the numerators and the denominators separately.
3. When a fraction is divided by another, multiply the first fraction by the reciprocal of the second fraction. Thus $\frac{m}{n} \div \frac{a}{b} = \frac{m}{n} \times \frac{b}{a} = \frac{mb}{na}$ etc.

Operations on Algebraic Fractions Including Monomial and Binomial Denominators

Hello Learner! In week 9, you have looked at monomials, binomials and trinomial algebraic expressions. Here, you are going to explore how to perform the four basic operations $(+, -, \times, \div)$ on Algebraic fractions involving monomial, binomial denominators. Let us begin with;

Addition and Subtraction of Algebraic Fractions with Monomial Denominators

Can you do a quick recall of how to add and subtract simple fractions? Perfect.

Just as you learnt how to add and subtract ordinary fractions in Week 6, addition and subtraction of algebraic fractions also follow the same procedure. This means that, you need to first identify the denominators of the algebraic fractions and find their lowest common multiple (LCM). Use the LCM. as their common denominator and express the fractions as a single fraction.

Let us go through an activity to express $\frac{2}{a} + \frac{3}{b}$ as a single fraction. Here are the steps:

Activity 13

Step 1: Identify the denominators of the two fractions. They are **a** and **b**.

Step 2: Find the LCM of a and b. This will be the common denominator. The LCM is *ab*

Step 3: Rewrite each fraction with the LCM as the denominator:

$\frac{2}{a} = \frac{2 \times b}{ab}$ (b is the multiplier to make the denominator ab)

$\frac{3}{b} = \frac{3 \times a}{ab}$ (a is the multiplier to make the denominator ab)

Step 4: Add the fractions and simplify if possible

$\Rightarrow \frac{2b}{ab} + \frac{3a}{ab} = \frac{(2b + 3a)}{ab}$

So, the final answer is: $\frac{2}{a} + \frac{3}{b} = \frac{(2b + 3a)}{ab}$

I hope this activity has been helpful. Now take your jotter, try this example and compare your answers to that of your friend.

Example 15

Express $\frac{1}{x} - \frac{3}{x^2}$ as a single fraction

Solution

LCM $= x^2$

$\frac{1}{x} - \frac{3}{x^2} = \frac{1(x)}{x^2} - \frac{3}{x^2} = \frac{x - 3}{x^2}$

Example 16

Express $\frac{4}{3m} + \frac{2}{m^2} - \frac{1}{nm}$ as a single fraction

Solution

The denominators are 3m, m^2 and nm. Their LCM is $3m^2n$

$\frac{4}{3m} + \frac{2}{m^2} - \frac{1}{nm} = \frac{4(mn)}{3m^2n} + \frac{2(3n)}{3m^2n} - \frac{1(3m)}{3m^2n} = \frac{4mn + 6n - 3m}{3m^2n}$

Please look for more examples, try them and show your workings to a classmate.

Addition And Subtraction of Algebraic Fractions with Binomial Denominators

Addition and subtraction of Algebraic Fractions with binomial denominators also go through the same steps as those with monomial denominators. Let us perform an activity with this example.

Activity 14

Work through the following examples with a classmate to make sure you fully understand what is happening.

Example 17

Simplify $\frac{10}{x-4}+\frac{2}{x+1}$ as a single fraction.

Solution

To express $\frac{10}{x-4}+\frac{2}{x+1}$ as a single fraction, we need to find a common denominator. Here are some steps to guide you;

Step 1: Identify the denominators of the two fractions: $x - 4$ and $x + 1$.

Step 2: Find the LCM of $x - 4$ and $x + 1$. To do this, we need to find the LCM of the two expressions by multiplying both expressions together because they do not have common factors:

LCM = $(x - 4)(x + 1)$ which can also be expressed as $x^2 - 3x - 4$

Step 3: Rewrite each fraction with the LCM as the denominator:

$$\frac{10}{x-4}=\frac{10(x+1)}{(x-4)(x+1)}=\frac{10x+10}{(x-4)(x+1)}$$

$$\frac{2}{x+1}=\frac{2\times(x-4)}{(x+1)(x-4)}=\frac{2x-8}{(x-4)(x+1)}$$

Step 4: Add the two fractions:

$$\frac{10x+10}{(x-4)(x+1)}+\frac{2x-8}{(x-4)(x+1)}=\frac{10x+10+2x-8}{(x-4)(x+1)}$$

Step 5: Simplify the fraction, if possible:

Simplifying this gives us; $\frac{12x+2}{(x-4)(x+1)}$

So, the final answer is:

$$\frac{10}{x-4}+\frac{2}{x+1}=\frac{12x+2}{(x-4)(x+1)} \text{ or } \frac{12x+2}{x^2-3x-4}$$

There you have it!

I hope this activity has also been helpful. Please try the examples that follow and compare your answers after.

Example 18

Simplify the expression $\frac{x}{x+1} - \frac{2x}{x+2}$ as a single fraction

Solution

LCM= $(x + 1)(x + 2)$

$$\frac{x}{x+1} - \frac{2x}{x+2} = \frac{x(x+2) - 2x(x+1)}{(x+1)(x+2)}$$

$$= \frac{x^2 + 2x - 2x^2 - 2x}{(x+1)(x+2)} = \frac{-x^2}{(x+1)(x+2)}$$

Example 19

Express $\frac{5b^2}{a^2 - b^2} + \frac{2a}{a-b} - \frac{3b}{a+b}$ as a single fraction

Solution

The denominators are $a^2 - b^2$, $a - b$, $a + b$

Their LCM is $(a - b)(a + b) = a^2 - b^2$ – remember this is a difference of two squares.

$$\frac{5}{a^2 - b^2} + \frac{2a}{a-b}\ \frac{3b}{a+b} = \frac{5b^2 + 2a(a+b) - 3b(a-b)}{(a-b)(a+b)}$$

$$= \frac{5b^2 + 2a^2 + 2ab - 3ab + 3b^2}{(a-b)(a+b)}$$

$$= \frac{8b^2 + 2a^2 - ab}{(a-b)(a+b)}$$

Example 20

At a youth club there were k people present. Of those, $\frac{2}{5}$ were playing football and $\frac{1}{4}$ were playing other games.

i. How many people were playing games?

ii. How many were not playing games?

Solution

i. Those who were playing football= $\frac{2}{5}k$

Those playing other games = $\frac{1}{4}k$

Total number of those present who were playing games = $\frac{2}{5}k + \frac{1}{4}k$

$$= \frac{4(2k) + 5(k)}{20} = \frac{13}{20}k$$

Therefore, $\frac{13}{20}k$ of those present were playing games.

ii. Number who were not playing games

$\Longrightarrow$ total number of those present (k)– total number of those playing games $\left(\frac{13}{20}k\right)$

$= k - \frac{13}{20}k = \frac{20(k) - 13k}{20} = \frac{7}{20}k$

Can you also recall how to multiply or divide ordinary fractions? Good. Multiplication and division of algebraic fractions are also done in the same manner.

So, let us now look at how to multiply or divide Algebraic Fractions with monomial or binomial denominators.

Multiplication and Division of Algebraic Fractions

When multiplying algebraic fractions, first of all check whether there are common factors, cancel them out and when there are none, multiply the numerators and the denominators separately. For example; $\boldsymbol{\frac{m}{n} \times \frac{a}{b} = \frac{m \times a}{n \times b} = \frac{ma}{nb}}$

Also, when an algebraic fraction is divided by another, multiply the first fraction by the reciprocal of the second fraction. Thus $\frac{m}{n} \div \frac{a}{b} = \frac{m}{n} \times \frac{b}{a} = \frac{mb}{na}$ etc.

Note: Always ensure that you leave your answer in its simplest form**.**

Now, let us consider an activity on how to perform these.

Activity 15

Simplify the expression $\frac{12xy}{7} \times \frac{14x}{20}$ in its simplest form.

Here are the steps:

Method 1 – with simplification happening at the end

Step 1: Multiply the numerators; $12xy \times 14x = 168x^2y$

Step 2: Multiply the denominators; $7 \times 20 = 140$

Step 3: Write the product as a fraction: $\frac{168x^2y}{140}$

Step 4: Simplify the fraction by dividing both the numerator and denominator by their common factor(s)

$\Rightarrow \frac{168x^2y}{140} = \frac{6x^2y}{5}$

Step 5: Write the final answer in its simplest form:

$$\Rightarrow \frac{12xy}{7} \times \frac{14x}{20} = \frac{6x^2y}{5}$$

Method 2 – with simplification happening at the beginning

To simplify the expression $\frac{12xy}{7} \times \frac{14x}{20}$ in its simplest form,

Step 1: Identify the common factors of the numerators and denominators and cancel them out. For instance, since 7 is a factor of 14, it can cancel out. Again,12 and 20 have common factors that can also cancel out. This gives us

$$\frac{12xy}{7} \times \frac{14x}{20} = \frac{3xy}{1} \times \frac{2x}{5}$$

Step 2: Multiply the numerators; $3xy \times 2x = 6x^2y$

Step 3: Multiply the denominators; $1 \times 5 = 5$

Step 4: Write the product as a fraction: $\frac{6x^2y}{5}$

Step 5: Write the final answer in its simplest form:

$$\Rightarrow \frac{12xy}{7} \times \frac{14x}{20} = \frac{6x^2y}{5}$$

There you have it!

Let us consider another example.

Example 21

Simplify the expression $\frac{6x+8}{4} \div \frac{x^2+3}{5x^2}$ as a single fraction.

Solution

Step 1: Simplify the numerator and denominator of the first fraction, $\frac{6x+8}{4}$

$$\frac{6x+8}{4} = \frac{2(3x+4)}{4} = \frac{2(3x+4)}{2 \times 2} = \frac{3x+4}{2}$$

Step 2: Simplify the second fraction, $\frac{x^2+3}{5x^2}$

Since the numerator and denominator cannot be simplified further, we leave it as is.

Now, the expression becomes; $\frac{3x+4}{2} \div \frac{x^2+3}{5x^2}$

Step 3: To divide fractions, we multiply by the reciprocal of the second fraction:

$$= \frac{3x+4}{2} \times \frac{5x^2}{x^2+3}$$

Step 4: Multiply the numerators and denominators;

$$= \frac{(3x+4) \times 5x^2}{2 \times (x^2+3)}$$

Step 5: Leave the answer in its simplest form.

$\Rightarrow \frac{6x+8}{4} \div \frac{x^2+3}{5x^2} = \frac{15x^3+20x^2}{2x^2+6}$

Example 22

Simplify $\frac{2m^2}{3} \div \frac{4m}{9}$

Solution

$\frac{2m^2}{3} \div \frac{4m}{9}$

Multiply the first fraction by the reciprocal of the second fraction

$\Rightarrow \frac{2m^2}{3} \times \frac{9}{4m}$

Cancel out those with common factors or multiply the numerators separately and the denominators also separately. Note, you can only cancel opposite numerators and denominators when we are at the multiplication stage, not while it is still in the division stage.

$\Rightarrow \frac{18m^2}{12m}$, here 6m divides both the numerator and the denominator to reduce it to; $\frac{3m}{2}$

$\Rightarrow \frac{2m^2}{3} \div \frac{4m}{9} = \frac{3m}{2}$

I encourage you to try your hands on more examples with your classmates and compare your answers.

Does it make sense when asked to divide an object into zero (0) equal parts and share it among a number of people? Obviously no!

What about dividing an object into equal number of parts and sharing them among nobody? Does this make a meaning? Of course, Yes!

This implies that when a number is divided by zero (0), we say the fraction is **not defined** or it is **undefined.** Also, when the numerator of a fraction is zero, the entire fraction is zero (0).

Therefore, an Algebraic Expression is said to be undefined or not defined if the variable at the denominator assumes a value (takes on a value) that makes the denominator zero.

Zero or Undefined Algebraic fractions

Algebraic fractions are said to be undefined or have no meaning if the denominator is equal to zero.

For example; the fraction $\frac{2}{x}$ is said to be undefined when $x = 0$

Likewise, an algebraic fraction is said to be zero if the numerator is equal to zero.

Let us do this activity to ascertain why a fraction is said to be undefined or zero.

Activity 16

Note:

$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient}$

$\Rightarrow$ Quotient $\times$ Divisor $=$ Dividend

Therefore, if $\frac{16}{2} = 8$

This implies $8 \times 2 = 16$

Again, $\frac{0}{13} = 0$. This implies $13 \times 0 = 0$

How would you determine that a fraction is undefined? I hope you have an idea. Below are some guidelines that help you

$\frac{5}{0} \neq 0$, This implies $0 \times 0 \neq 5$ (It cannot be defined)

Have you now seen the reason why we say a fraction is undefined or not defined when the denominator is zero? Very well.

Let us now explore some conditions under which an algebraic fraction is considered undefined or zero.

Conditions Under Which an Algebraic Expression is Said to be Zero or Undefined

To find the value(s) for which an algebraic fraction is undefined, here are some steps to guide you.

Step 1: Equate the denominator of the expression to zero

Step 2: Solve for the value(s) of the involving variable

Step 3: The value(s) of the variable makes the expression undefined.

This implies the variable at the denominator can assume or take all values except the value that makes the expression undefined. Let us consider the following examples.

Example 23

Find the value of x which makes the expressions undefined:

i. $\frac{3}{x-1}$

Solution

For the expression $\frac{3}{x-1}$ to be undefined, the denominator $x - 1$ must be equal to zero: $x - 1 = 0$ Solving for x:

$x = 1$

Therefore, the expression is undefined at $x = 1$

ii. $\frac{(2x-1)(x-4)}{4x^2-1}$

Solution

For the expression $\frac{(2x-1)(x-4)}{4x^2-1}$ to be undefined, the denominator $4x^2 - 1$ must be equal to zero:

$4x^2 - 1 = 0$

This can be factorised as a difference of two squares:

$(2x+1)(2x-1) = 0$

Set each factor to zero:

$2x + 1 = 0$ or $2x - 1 = 0$

Solve for x in each case:

$x = -\frac{1}{2}\ or\ x = \frac{1}{2}$

Therefore, the expression is undefined at $x = -\frac{1}{2}\ and\ x = \frac{1}{2}$.

Also, to find the value(s) for which an algebraic fraction is zero, here are some steps to guide you.

Step 1: Equate the fraction to zero.

Step 2: Solve for the value(s) of the involving variable.

Step 3: This value(s) of the variable makes the expression zero and is called the zeros of the expression. This means that, when the variable takes that value, the outcome of the entire expression is zero.

Let us consider an example.

Example 24

Determine the condition under which $3a$ _ $a - 4$ is undefined and determine the value of a and do the same for when the whole fraction is zero.

Condition: $a - 4 = 0$

$a = 4$, Therefore, when $a = 4$ the fraction is undefined.

Condition: under which the fraction is zero, put the whole fraction to zero.

$\frac{3a}{a-4} = 0$, $3a = 0$, $a = 0$, Therefore, when a is 0 the fraction is zero.

Example 25

For what values of x is the expression zero.

Solution

$\frac{x^2 + x - 6}{x}$

Equate the fraction to zero

$\frac{x^2 + x - 6}{x} = 0$

$x^2 + x - 6 = 0$

Solve the quadratic

$x^2 + 3x - 2x - 6 = 0$

$x(x + 3) - 2(x + 3) = 0$

$(x - 2)(x + 3) = 0$

$x - 2 = 0$ or $x + 3 = 0$

$x = 2$ or $x = -3$

This implies that the expression $\frac{x^2 + x - 6}{x}$ is zero when $x = 2$ or $x = -3$.

I encourage you to try more examples from the extended reading materials and discuss your

findings with a classmate.

REVIEW QUESTIONS

Review Questions 3.1

1. Identify patterns from the following sets of numbers and create a mathematical expression for each sequence

   a. 1, 3, 5, 7, 9…

   b. -1, 2, 5, 8, 11…

2. In a math competition, participants are asked to solve a series of problems. In the first round, each contestant solves 1 problem. In each subsequent round, they solve 2 more problems than in the previous round. How many problems will they need to solve in the 7th round?

3. Expand the algebraic expressions.

   a. $(x + 5)(x + 3)$

   b. $(x + 2)(x - 4)$

   c. $(y + 7)(y + 1)$

   d. $(z + 2)(z - 5)$

4. Factorise completely the following

   a. $xy + 2xz$

   b. $4(4n - 12x)$

   c. $3x^2 - 9xy$

   d. $6ab - 4pb - 2pq + 3aq$

5. Factorise completely the following quadratic expressions

   a. $x^2 + 2x - 3$

   b. $6 - 5x - x^2$

   c. $3x^2 - 17x + 10$

   d. $p^2 - 1$

6. The length of a rectangular field is 5 meters more than its width. Write down an expression for the perimeter of the field.

7. The length of a rectangular garden is 4 meters longer than its width. Write an expression to find the area of the garden.

Review Questions 3.2

1. Express the following numbers as perfect squares

a. 121

b. 81

c. 64

d. 169

e. 100

2. Simplify the following as single fractions

a. $\frac{x}{x^2 - 5x + 6} + \frac{1}{x - 2} + \frac{3}{x - 3}$

b. $\frac{2x - 1}{3} - \frac{x + 3}{2}$

c. $\frac{x - 2}{3} + \frac{x + 3}{5}$

d. $\frac{5x}{3} \times \frac{x + 3}{2}$

e. $\frac{2x - 1}{2x} \div \frac{4x^2 - 1}{6}$

3. Identify with reasons which of the following fractions have monomial or binomial denominators.

a. $\frac{2}{5x}$

b. $\frac{y}{x + 1}$

c. $\frac{x - 3}{x + 6}$

d. $\frac{7y - 1}{x^2 - 4}$,

e. $\frac{3z + 4}{z^2}$

4. Given that $p = \frac{2x}{1 - x^2}$ and $q = \frac{2x}{1 + x}$, simplify 3p – 2q

5. Simplify the following and state the value of the variable that makes the expression undefined or zero.

i. $\dfrac{4m^2 - 9}{(m-1)^2} \div \dfrac{2m+3}{m^2 - m}$

i. $\dfrac{6x^2 + 12x}{x^2 + x} \times \dfrac{x^2 - 1}{x^2 + 10x + 16}$

ii. $\dfrac{y^2 - 7y + 12}{y^3 - 9y}$

6. Fully factorise the following:

i. $4x^2 - 9$

ii. $16y^2$ -$25a^2$

iii. a^2b^2 -121

iv. $4m^2 - 100$

7. Find the value(s) for the unknown using the idea of perfect squares.

a. $3z + 128 = 162$

ANSWERS TO REVIEW QUESTIONS

Review Questions 3.1

1. **a.** 1, 3, 5, 7, 9…

Pattern: Adding 2 to the previous term

Algebraic expression: $a_n = 2n - 1$ (where n is the term number)

b. -1, 2, 5, 8, 11…

Pattern: Adding 3 to the previous term

Algebraic expression: $a_n = 3n - 4$

2. Therefore, in the 7th round, contestants will need to solve 13 problems.

3. **a.** $(x + 5)(x + 3) = x^2 + 5x + 3x + 15 = x^2 + 8x + 15$

b. $(x + 2)(x - 4) = x^2 + 2x - 4x - 8 = x^2 - 2x - 8$

c. $(y + 7)(y + 1) = y^2 + 7y + y + 7 = y^2 + 8y + 7$

d. $(z + 2)(z - 5) = z^2 + 2z - 5z - 10 = z^2 - 3z - 10$

4. **a.** $xy + 2xz = x(y + 2z)$

b. $4(4n - 12x) = 4(4(n - 3x)) = 16(n - 3x)$

c. $3x^2 - 9xy = 3x(x - 3y)$

d. $6ab - 4pb - 2pq + 3aq = (3a - 2p)(2b + q)$

5. **a.** $x^2 + 2x - 3 = (x + 3)(x - 1)$

b. $6 - 5x - x^2 = -(x^2 + 5x - 6) = -(x + 6)(x - 1)$

c. $3x^2 - 17x + 10 = 3x^2 - 15x - 2x + 10 = 3x(x - 5) - 2(x - 5) = (3x - 2)(x - 5)$

d. $p^2 - 1 = (p + 1)(p - 1)$

6. Let w = the width of the field, so the expression for the perimeter of the field is: w + w + w + 5 + w + 5 = 4w + 10 = 2(2w + 5).

OR Let l = length of the field, so the expression for the perimeter of the field is: $l + l + l - 5 + l - 5 = 4l - 10 = 2(2l - 5)$

7. Let the width of the garden = w, so the expression for the area of the garden is w(w + 4).

OR Let the length of the garden = l, so the expression for the area of the garden is $l(l - 4)$

Review Questions 3.2

1. **a)** 11^2

b) 9^2

c) 8^2

d) 13^2

e) 10^2

2. **a.** $\frac{5x-9}{(x-3)(x-2)}$

b. $\frac{x-11}{6}$

c. $\frac{8x-1}{15}$

d. $\frac{5x^2+15x}{6}$

e. $\frac{3}{2x^2-x}$

3. Monomial denominators; a, e

Binomial denominators; b, c and d

4. $\frac{2x+4x^2}{(1-x)(1+x)}$

5. **i.** $\frac{2m^2-3m}{m-1}$ $\therefore$ undefined when $m = 1$ and zero when $m = 0$ *or* $m = \frac{3}{2}$

ii. $\frac{6(x-1)}{x+8}$ $\therefore$ undefined when $x = -8$ and it is zero when $x = 1$

iii. $\frac{y-4}{y(y+3)}$ $\therefore$ undefined when $y = 0$ or $y = -3$ and zero when $y = 4$

6. **i.** $(2x-3)(2x+3)$

ii. $(4y-5a)(4y+5a)$

iii. $(ab-11)(ab+11)$

iv. $(2m-10)(2m+10)$

7. $z = \frac{34}{3}$

EXTENDED READING

- Akrong Series: Core mathematics for Senior High Schools New International Edition (Pages 51 – 70)
- Aki – Ola series : Core Mathematics for Senior High Schools in West Africa, Millennium edition 5 (Pages 58 -70) on sets
- Baffour – Ba Series: Core Maths for Schools and Colleges, (Pages 61 - 76)

REFERENCES

- Asiedu, P. (2016). *Mathematics for Senior High Schools*
- S. Coleman, K. A. Benson, H. A. Baah – Yeboah (Eds.). Aki - Ola Publications.
- Andrzej Trybulec. Enumerated sets. *Formalized Mathematics*, 1, 1990.
- Asiedu, P. (Millennium Edition 5). Core Mathematics for Senior High Schools in West Africa. Aki - Ola Publications.
- Akrong Series: Core mathematics for Senior High Schools New International Edition

ACKNOWLEDGEMENTS

Ghana Education Service (GES)

List of Contributors

NAME	INSTITUTION
Janet Waponade	OLA SHS
Yamusah Issahaku	Kumbungu SHS
Joseph Bakpil Nagbija	Nchumuruman SHS, Chinderi
Modzaka Godfred	Keta SHTS, Keta