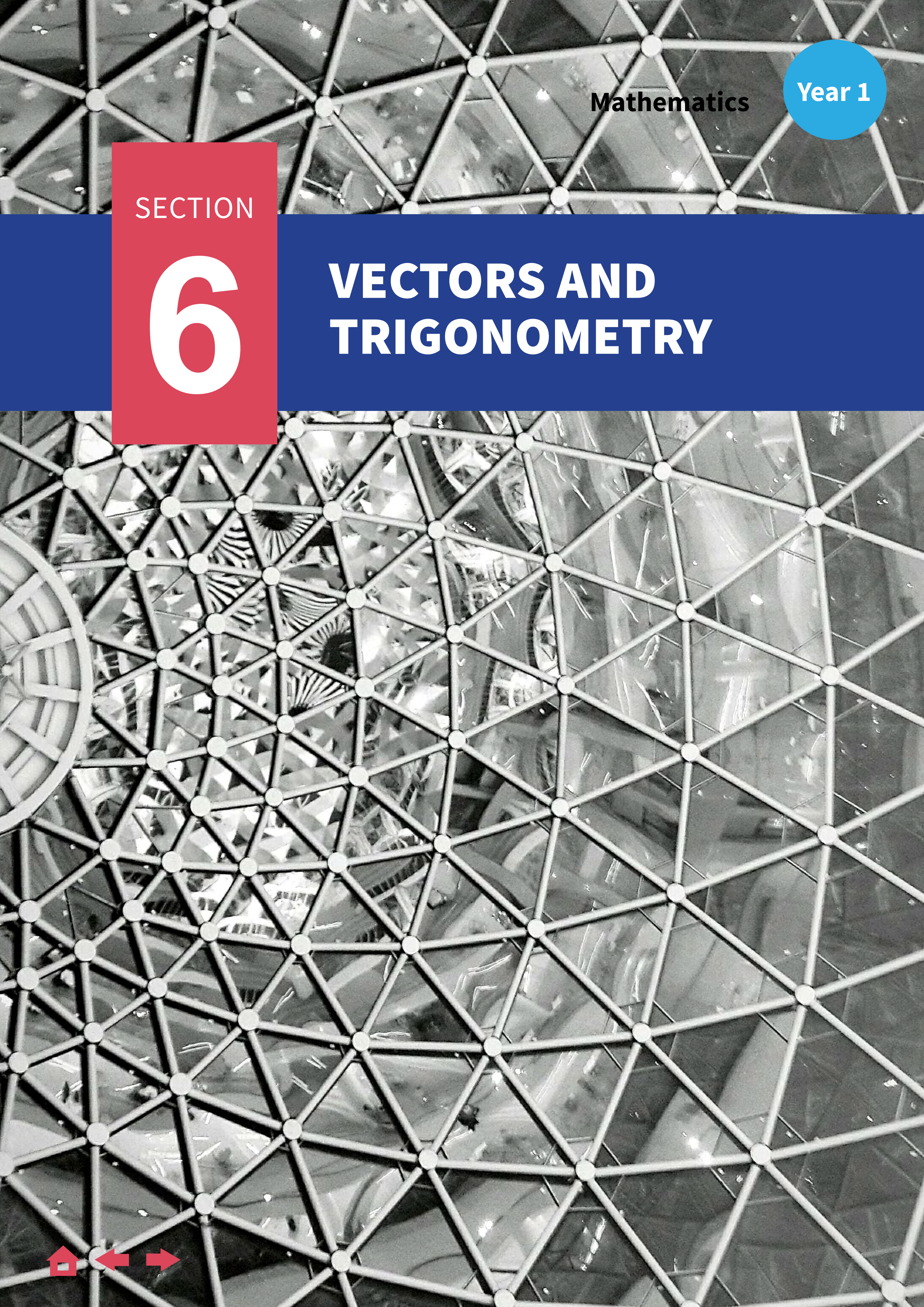
Mathematics

Year 1

SECTION

# 6

# VECTORS AND TRIGONOMETRY



# GEOMETRY AROUND US

## Measurement

## INTRODUCTION

Vectors and trigonometry are interconnected concepts that play a crucial role in mathematics, physics, engineering and various other fields. Vectors represent quantities that have both magnitude and direction and are essential for describing motion, forces and other physical quantities. Understanding vectors involves applying trigonometric ratios and functions, which relate the angles and sides of right-angle triangles. Trigonometry helps us calculate unknown angles and side lengths, serving as a tool for solving real-world problems involving distances, velocities, and forces. Learning about vectors and trigonometry enhances your spatial reasoning skills and prepares you for advanced studies in mathematics. These concepts are also closely linked to other subjects such as geography, where bearings are used to navigate and physics, where vectors are used to describe motion. Mastery of these concepts opens doors to a wide range of career paths in science, engineering, and technology.

**At the end of this section, you will be able to:**

- Recognise a vector as a quantity with both magnitude and direction, and identify, gather, and interpret information about real-world applications of vectors.
- Represent a vector in two-space geometrically as a directed line segment, with directions expressed in different ways (e.g., 320°, N40°W) and algebraically; then recognise vectors with the same magnitude and direction but different positions as equal vectors.
- Investigate the three basic trigonometric ratios (tangent, sine and cosine) of an acute angle in degrees.
- Find the trigonometric functions of special angles $30^0$, $45^0$ *and* $60^0$, including using the calculator to determine the values of sine, cosine and tangent of angles up to $360^0$.
- Solve problems, using the three primary trigonometric ratios for angles *from 0° to 360°* in standard position.

## Key Ideas

- **Vector:** It is a quantity that has both magnitude (length) and direction.
- **Magnitude:** In vectors, magnitude refers to its length or size.
- **Direction**: The direction of a vector refers to the line along which it points.
- Trigonometry is the study of the relation between the sides and angles of triangles, particularly right triangles.
- The three basic trigonometric ratios are: Sine (sin), Cosine (cos) and tangent (tan).
- Trigonometric functions of special angles are: $30^0$, $45^0$ and $90^0$.

In Junior High School, you were taught bearings and vectors. Now is the time to remind yourselves of true bearings, forward bearings and back-bearings and bearings with vectors. Have a look back at what you have done previously, or look in textbooks or go online to find out all.

# WHAT ARE VECTORS?

Vectors define the movement of objects from one point to another. Vectors carry a point A to point B. The length of the line between the two points A and B is called the magnitude of the vector and the direction of the displacement of point A to point B is called the direction of the vector AB.

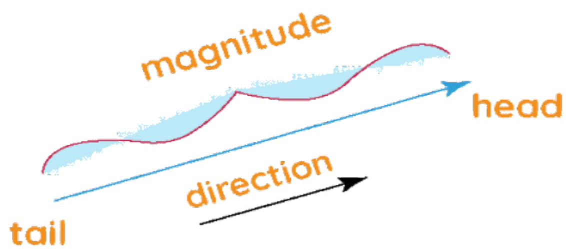



**Figure 1:** Representation of a vector

## Real life Applications of Vectors

Vectors play an important role in physics. For instance, velocity, displacement, acceleration and forces are all vector quantities that have a magnitude as well as a direction.

### Real-life uses of vectors

- Vectors can be used in finding the direction in which the force is applied to move an object.
- The concept of vectors aids in understanding how gravity uses a force of attraction on an object to work.
- Vectors can be used in obtaining the motion of a body which is confined to a plane.
- Vectors help in defining the force applied on a body simultaneously in the three dimensions.
- In the field of Engineering, for a structure not to collapse, vectors are used where the force is much stronger than the structure will sustain.
- Vectors are used in various oscillators.

## Types of vectors

| | |
|---|---|
| Zero Vectors | Negative Vector |
| Unit Vectors | Parallel Vectors |
| Position Vectors | Orthogonal Vectors |
| Equal Vectors | Co-initial Vectors |

### Parallel and Collinear Vectors

Parallel vectors are vectors that have the same or opposite direction. Collinear vectors are vectors that lie on the same line or have the same line of action. A vector parallel to another vector is also collinear with it because both vectors share the same direction (or opposite directions if one is negative), meaning they are aligned along the same line.

**Zero vector** does not have a defined direction because it does not point anywhere; it is essentially the point at the origin with no direction.

**Co-initial vectors:** Vectors that have the same initial point but different endpoints are called "co-initial vectors". Co-initial vectors are useful in various contexts, such as in physics to represent different forces or motions originating from the same point.

**Start Point (Initial Point):** The point where the vector begins.

**Endpoint (Terminal Point):** The point where the vector ends.

In vector notation, **v** is a vector with an initial point $(x_1, y_1)$ and a terminal point $(x_2, y_2)$, then the start point is $(x_1, y_1)$ and the endpoint is $(x_2, y_2)$.

## Vectors in 2-D

A 2D vector is a mathematical entity that has both a magnitude (length) and a direction in a two-dimensional space. It is represented as an ordered pair of numbers, typically (x, y).

I hope you are familiar with the standard $(x, y)$ Cartesian coordinate system in the plane. That is, each point P in the plane is identified with its $x$ and $y$ components: $P\ (p_1, p_2)$.

To determine the coordinates of a vector, **a,** in the plane, the first step is to translate the vector so that its tail is at the origin of the coordinate system. Then, the head of the vector will be at some point $(a_1, a_2)$ in the plane. We call $(a_1, a_2)$ the coordinates or the components of the vector **a**. We often write $a \in R^2$ to denote that it can be described by two real coordinates.

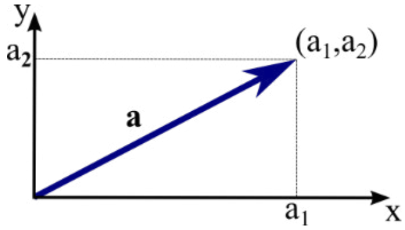



**Figure 2:** Vector in 2D

Using the Pythagorean Theorem, we can obtain an expression for the magnitude of a vector in terms of its components. Given a vector $a = (a_1, a_2)$, the vector is the hypotenuse of a right triangle whose legs are length $a_1$ and $a_2$. Hence, the length of the vector a is $|a| = \sqrt{(a_1)^2 + (a_2)^2}$

**Example 1**

Consider the vector ***a*** represented by the line segment which goes from the point (1, 2) to the point (4, 6). Calculate the components and the length of this vector?

**Solution**

To find the components, translate the line segment one unit left and two units down. Now, the line segment begins at the origin and ends at $(4-1, 6-2) = (3, 4)$.

Therefore, a = (3, 4). The length of a is

$$|a| = \sqrt{(3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \text{ units}$$

## Equal Vectors

Equal vectors in mathematics refer to vectors that have the same magnitude and direction. Here are some key points about equal vectors. That is, two vectors are equal if they have the same magnitude and the same direction. Equal vectors are important in various mathematical operations, such as vector addition, subtraction and comparison. When vectors have the same magnitude and direction, they exhibit similar characteristics and can be treated as equivalent in many mathematical contexts.

**Magnitude**: The magnitude of a vector refers to its length or size. If the magnitudes of two vectors are equal, they have the same length. The magnitude of a zero vector is 0, as it has no length.

**Direction**: The direction of a vector refers to the line along which it points. If two vectors have the same direction, they are parallel and point in the same line or path.

**Notation**: In mathematical notation, equal vectors are typically denoted by placing an arrow on top of the vector symbols. For example, if vector **a** and vector **b** are equal, it is written as $\vec{a} = \vec{b}$

**Example 2**

Consider two displacement vectors in a coordinate system. If vector A represents a displacement of 5 meters to the east, and vector B represents a displacement of 5 meters to the east, then vector A and vector B are equal. They have the same magnitude (5 meters) and the same direction (east).

**Example 3**

Imagine two velocity vectors of moving objects. If vector C represents a velocity of 30 kilometres per hour north, and vector D represents a velocity of 30 kilometres per hour north, then vector C and vector D are equal. They have the same magnitude (30 kilometres per hour) and the same direction (north).

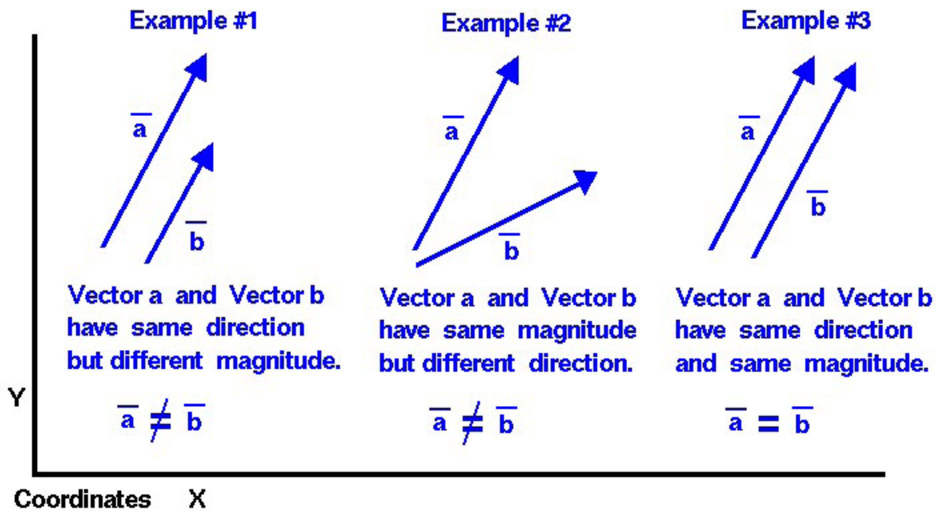



**Figure 3:** Demonstrating magnitude and direction of a vector

## Activity 1: Study the following scenario

**Scenario:** The local government in Ghana wants to establish a new market area in a region between two existing village, Village A and Village B. They need to determine a suitable location for the new market that is equidistant from both villages and strategically positioned.

### Given Information

- Village A is located at coordinates (2, 3) in kilometers.
- Village B is located at coordinates (7, 8) in kilometers.
- The new market area should be located such that it is equidistant from both Village A and Village B.

**Task:** Find the coordinates of the new market area that is equidistant from both Village A and Village B and the distance from each village to the market.

### Step-by-Step Solution

**Step One**: Calculate the Midpoint

**Step Two:** Use the distance formula to calculate the distance between two points, $(x_1, y_1)$ and $(x_2, y_2)$.

**Step Three:** locate the new market area

**Step Four**: Report the findings and coordinate with local authorities to finalise the location.

**Example 4**

A ship is sailing with a bearing of 060° at a speed of 20 km/h. What direction is it heading?

**Solution**

**Step One:** A bearing of 045° means the ship is heading 45° clockwise from True North.

**Step Two:** Interpret the direction:

- North is 0°
- East is 90°
- South is 180°
- West is 270°

∴ Bearing 045° means the ship is moving in a direction that is 45° clockwise from True North. This is exactly half way between North and East. So the ship is heading North-East

## Activity 2

Indicate the types of vectors in the following Scenario Card Examples:

- A car parked in front of the school?
- A direction arrow on a map?
- A student walking from the classroom to the library?
- A thrown ball flying through the air?
- A speeding car accelerating down the highway?

## Activity 3

Perform the following task in class.

- Take a task sheet with vectors written in component form (e.g., $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$)
- Identify equal vectors for each given vector.
- Work in pairs to match the vectors with their equal pairs.
- Discuss and explain your reasons to your friend.

I hope you are all feeling more confident now with your use of vectors. Remember that they are simply quantities which have direction as well as magnitude and they determine the position of one point in space, relative to another.

*It is now time to delve into the wonderful world of Trigonometry and all its applications.*

# TRIGONOMETRY AND ITS APPLICATIONS

## What is Trigonometry?

Trigonometry is the study of the relation between the sides and angles of triangles, particularly right-angled triangles. It thus helps in finding the measure of unknown dimensions of a triangle using formulas and identities based on this relationship.

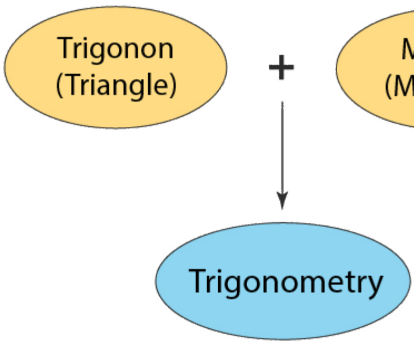



**Figure 4:** Trignometry

## Trigonometric Ratios

A ratio is a statement of a mathematical relationship between two objects, often represented as a fraction. If we consider two sides of a right-angled triangle with respect to a given internal angle, the ratio of the two sides has a special relationship to the angle. Take a look at the three sides of the triangle.

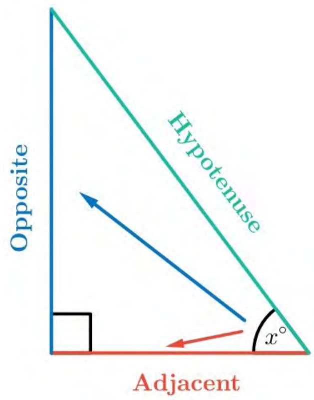



**Figure 5:** Right-angled triangle

In relation to the angle ($\theta$) using ideas from the concept of Pythagoras' theorem, we can deduce the basic ratios in trigonometry that help in establishing a relationship between the ratios of sides of a right-angled triangle with the angle.

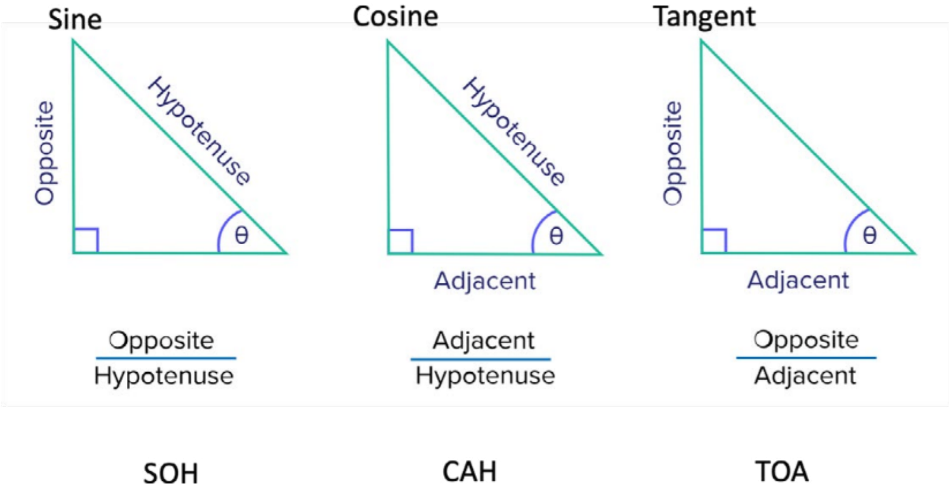



**Figure 6:** Relationship between the ratios of sides of a right-angled triangle with the angle

The three common trigonometric ratios we see above are Sine, Cosine and Tangent, shortened to become sin, cos and tan respectively. The trigonometric ratios enable us to determine the ratio of two sides of a right-angled triangle given an internal angle or find an angle given the ratio of two sides of a right-angled triangle. The three trigonometric ratios are defined as follows:

**For example,**

If $\theta$ is the angle in a right-angled triangle formed between the adjacent and hypotenuse, then

$$\sin\theta = \frac{Opposite}{Hypotenuse}$$

$$\cos\theta = \frac{Adjacent}{Hypotenuse}$$

$$\tan\theta = \frac{Opposite}{Adjacent}$$

*Let us now try some examples using these special ratios.*

**Example 5**

James is standing 31 metres away from the base of a Harbour Centre. He looks up to the top of the building at a 78° angle. How tall is the Harbour Centre?

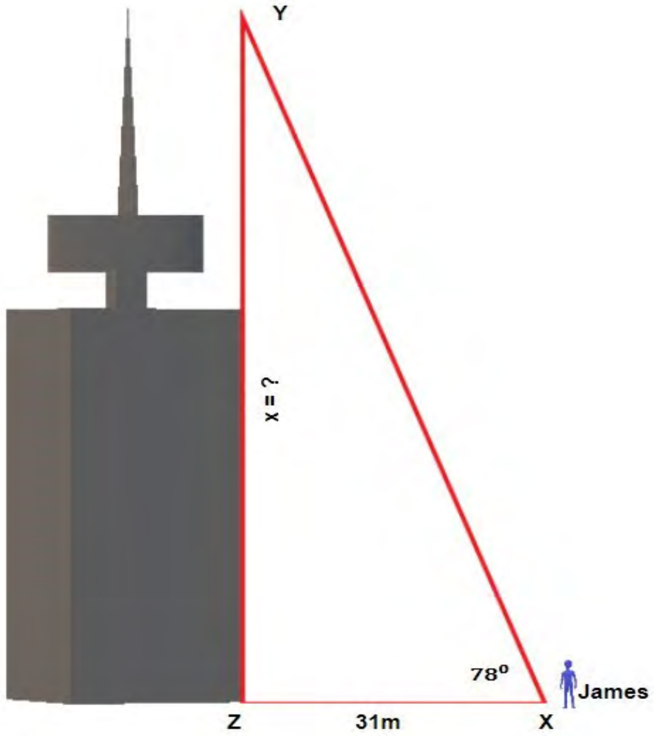



**Figure 7:** Diagram of the scenario

**Solution**

From the right-angled triangle XYZ, $\tan(\theta) = \frac{Opposite}{Adjacent}$

From the diagram, we can substitute the given side and angle.

Thus: $\tan(78^o) = \frac{\lfloor YZ \rfloor}{31}$

This implies, $\lfloor YZ \rfloor \;=\; 31 \times \tan(78^o)$

$\lfloor YZ \rfloor = 31 \times 4.704 \approx 145.824$

Therefore, the height of the harbour centre is approximately 146 m.

**Example 6**

Thomas is standing at the top of a building that is 45 metres high and looks at his friend who is standing on the ground, 22 metres from the base of the building. What is the angle of depression?

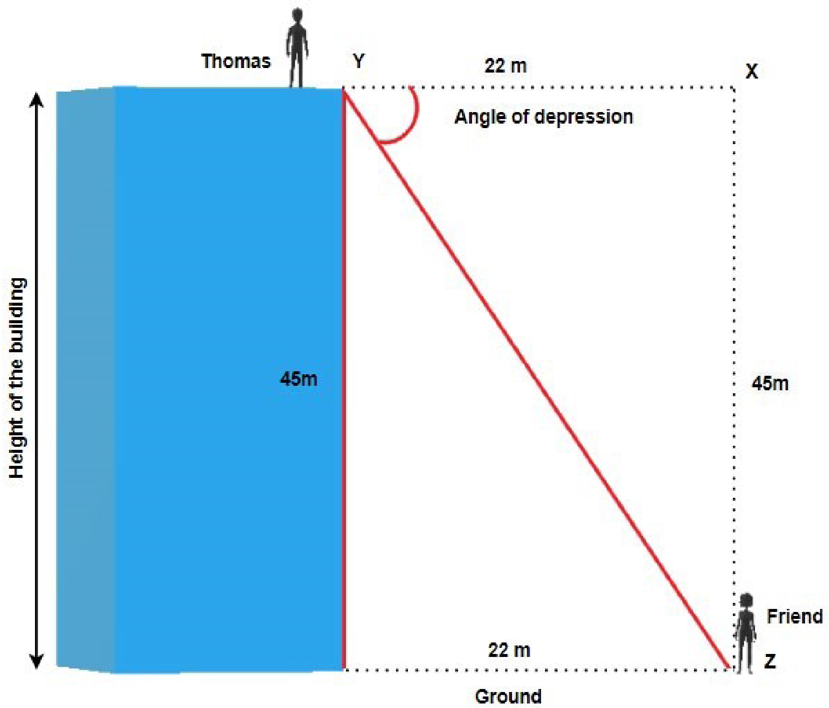



**Figure 8:** Diagram of the scenario

**Solution**

From right-angled triangle XYZ, $\tan(\theta) = \frac{Opposite}{Adjacent}$

From the diagram, we can substitute the given sides. Thus: $\tan(\theta) = \frac{45}{22}$

This implies, $\tan(\theta) = \mathbf{2.04545454545\ldots}$

$\theta = tan^{-1}(2.045454545\ldots) = 63.94650479\ldots$

Therefore, the angle of depression from Thomas to his friend is approximately 64°

# Trigonometric functions of special angles $30^0$, $45^0$ and $60^0$

Use a unit circle to calculate the values of basic trigonometric functions- sine, cosine and tangent.

The following diagram shows how trigonometric ratios sine and cosine can be represented in a unit circle.

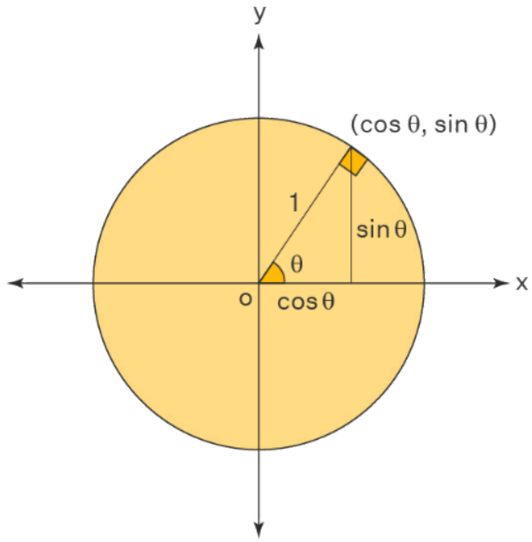
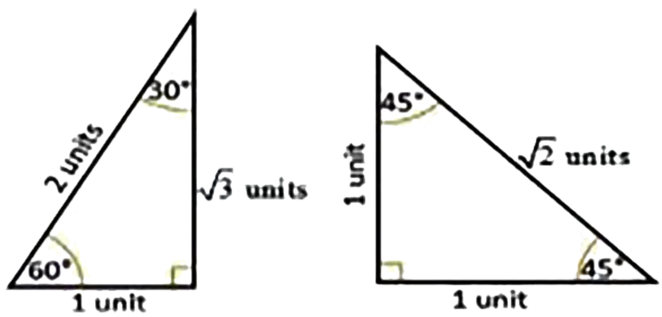



**Figure 9:** Unit circle

Derive the trigonometric ratios of 30°, 45° and 60° from the 30-60-90 and 45-45-90 special triangles.




| | 30° | 45° | 60° |
|---|---|---|---|
| sin | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ |
| cos | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ |
| tan | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ |

**Figure 10:** Trigonometric ratios

## Activity 4

Perform the following activities by considering the triangle in the figure. The solution is below – but have a go by yourself before looking!

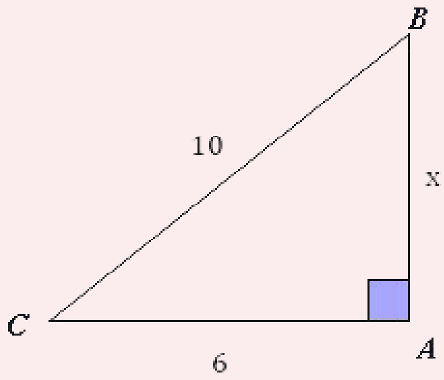



**Figure 11:** Right-angled triangle

If $\text{Cos}\theta = \frac{6}{10}$

**a.** Which angle is represented by $\theta$? How do you know?

**b.** Find the numerical value of sin $\theta$. Express your answer as a simplified fraction.

**c.** Find the numerical value of tan $\theta$. Express your answer as a simplified fraction.

### Solution

**a.** Since $Cos\ \theta = \frac{Adjacent}{Hypotenuse}$, then the angle $\theta$ is $BCA$.

**b.** $Sin\ \theta = \frac{Opposite}{Hypotenuse} = \frac{x}{10}$

Find the length of the adjacent side, $x$, by using the Pythagoras' theorem, $C^2 = A^2 + B^2$.

Substitute $C = 10$ and $A = 6$ into the Pythagoras' theorem, then square root the expression to find the value of $x$.

$$Sin\ \theta = \frac{\sqrt{10^2 - 6^2}}{10} = \frac{\sqrt{100 - 36}}{10} = \frac{\sqrt{64}}{10} = \frac{8}{10}$$

$$Sin\ \theta = \frac{4}{5}$$

**c.** In right-angled triangles, recall that the tan of an angle is equal to the ratio of the side length opposite that angle and the adjacent length.

That is, $tan\ \theta = \frac{Opposite}{Adjacent} = \frac{x}{6}$

We previously found that $x = 8$. Therefore, substitute the value into the trigonometric identity.

$tan\ \theta = \frac{8}{6}$

Simplify the fraction.

$tan\ \theta = \frac{4}{3}$

## Application of the three primary trigonometric ratios to solve Real-life Problems

The practical applications of these ratios are vast and diverse. Architects and engineers employ trigonometry to design structures, surveyors rely on it to measure land, astronomers utilise it to calculate distances between celestial bodies, and pilots use it for navigation purposes. Trigonometry can be used to measure the height of a building or mountains. Trigonometry truly lies at the heart of many scientific and technical fields.

**Trigonometry in video games:** Have you ever played the game, Mario? When you see him so smoothly glide over the road blocks, trigonometry helps Mario jump over these obstacles.

**Trigonometry in flight engineering:** Flight engineers have to take into account speed, distance and direction along with the speed and direction of the wind. The wind plays an important role in how and when a plane will arrive wherever needed. This is solved using vectors to create a triangle using trigonometry to solve. For example, if a plane is travelling at 234 mph, 45 degrees N of E, and there is a wind blowing due south at 20 mph. Trigonometry will help to solve for that third side of your triangle which will lead the plane in the right direction. The plane will travel with the force of wind added on to its course.

**Trigonometry in physics:** In physics, trigonometry is used to find the components of vectors, model the mechanics of waves (both physical and electromagnetic) and oscillations, sum the strength of fields and use dot and cross products. Even in projectile motion you have a lot of applications of trigonometry.

Other real-life Applications of Trigonometry are;

- archaeology
- criminology
- marine biology
- marine engineering
- navigation

## Activity 5:

Consider the following scenarios and answer the questions that follow. You will need a scientific calculator. Do not round your answers in the middle of the solution as this tends to lead to rounding errors. Try and keep using the full calculator display. Also, ensure that all measurements are in the same units before calculating. Write clear and concise solutions for each problem, including the sketch, steps taken and the final answer.

### Scenario 1

A boy is standing 30m from the base of a tree. To see the top of the tree he must look up at an angle of $45^0$. He looks up at the tree and wonders, how tall is the tree?

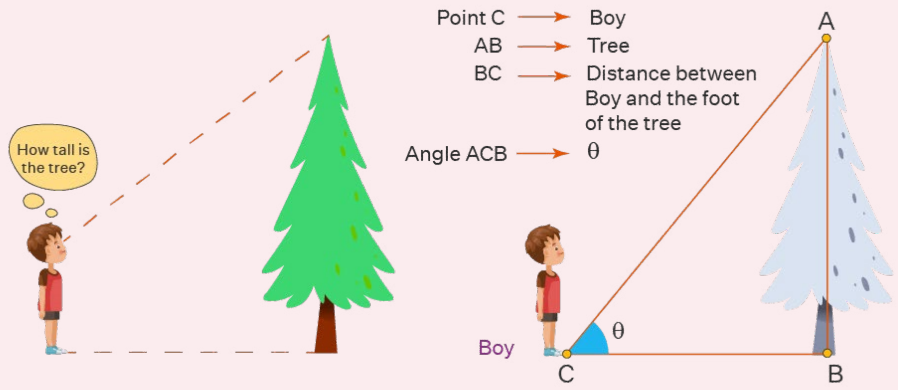



**Figure 12:** Diagram of the scenario

**Step One:** Identify which side forms an angle of $90^0$ with the horizontal.

**Step Two:** Based on step one, identify the triangle shown above. A quick sketch always helps in these scenarios.

**Step Three**: Identify which of the three basic ratios can be used to calculate the height of the tree.

**Step Four**: We know the distance to the base of the tree is 30m and the angle formed is 45 degrees, so we can calculate the height of the tree.

(Hint: Height = 30m, so if you found that you are doing brilliantly!)

### Scenario 2

You are standing at a point on the ground and looking up at the top of a building. The angle of elevation from where you are standing to the top of the building is 30°. You are standing 50 metres away from the base of the building.

**i)** Identify the right-angled triangle and do a quick sketch to represent the scenario.

**ii)** Which of the three basic trigonometric ratios can be used to calculate the height of the building?

**iii)** Use the ratio identified in (ii) to find the height of the building.

### Scenario 3

A person on top of a cliff looks down at a boat on the sea. The angle of depression from the top of the cliff to the boat is 25° and the horizontal distance from the base of the cliff to the boat is 200 meters.

**i)** Identify the right-angled triangle and do a quick sketch to represent the scenario.

**ii)** Which of the three basic trigonometric ratios can be used to calculate the height of the cliff?

**iii)** Use the ratio identified in (ii) to find the height of the cliff.

### Scenario 4

You are standing on one bank of a river and need to determine the width of the river. You know that the angle of depression from the top of a tall tree (20 meters high) to a point on the opposite bank directly across is 45°.

**i)** Sketch a right-angled triangle based on the scenario. Remember that a sketch always helps.

**ii)** Use the appropriate trigonometric ratio (sine, cosine, or tangent) based on the given information to find the width of the river.

# REVIEW QUESTIONS

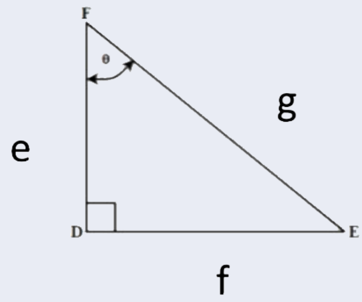
## Review Questions 6.1

1. The vector **u** has an initial point at (−2, 1) and an endpoint at (4, −2). What is the vector's length?
2. Determine if the vectors $\vec{a} = (3, -4)$ and $\vec{b} = (-6, 8)$ are equal.
3. Determine whether the two vectors are equal.
   - Vector P: Magnitude = 5, Direction = 30 degrees above the positive *x*-axis.
   - Vector Q: Magnitude = 5, Direction = 60 degrees above the positive *x*-axis.
4. Explain the following concepts as used in vectors:
   i) Why might a Parallel Vector also be considered Collinear Vectors?
   ii) What is the magnitude and direction of a Zero Vector?
   iii) Vectors that have the same initial point are known as…?
   iv) What is the start and endpoint of a vector called?
5. Find the length of the vector $a = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$.
6. Using the magnitude formula, find the magnitude of the vector with $u = \begin{pmatrix} -6 \\ 8 \end{pmatrix}$.
7. Given the vectors $\vec{u} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 4 \\ 10 \end{pmatrix}$, find a scalar $k$ such that $u = kv$.

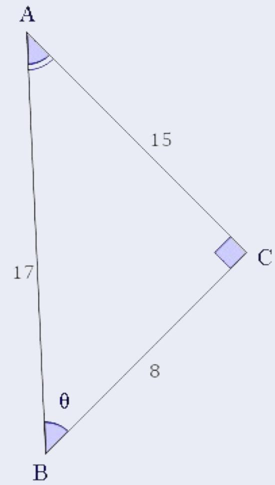
## Review Questions 6.2

1. Given a right-angle triangle ABC with angle A as the right-angle and side lengths as follows: AB = 5 cm, BC = 13 cm. Calculate the values of the trigonometric ratios for angle B (sine, cosine and tangent). Keep your answers as fractions.
2. State the exact values of the trigonometric functions (sine, cosine and tangent) for the special angles 30 degrees, 45 degrees, and 60 degrees.
3. Label the sides of the triangle and find the hypotenuse, opposite and adjacent with regards to $\theta$.

4. For the given triangle find the sine, cosine and tangent ratio.




5. For the given triangle find the sine, cosine and tangent ratios in relation to angle $\theta$.




6. Sakumonor Skating Club has a beginner ski slope which makes a $15^0$ angle with the ground. How far does a skier travel over a horizontal distance of 120 meters?
7. A ramp is to be built from the ground to the back of a semi-truck. The bed of the truck is 1.3 meters above the ground. The ramp makes a $22^0$ angle with the ground. How long will the ramp be?
8. The angle of elevation from a boat to the top of a 48-meter lighthouse is $25^0$. How far is the boat from the base of the lighthouse?
9 A prince is standing 100 meters from the base of a tower where a beautiful princess is being held captive. The princess sees the prince at a $30^0$ angle of depression. How high will the prince have to climb to rescue the princess?

# ANSWERS TO REVIEW QUESTIONS

## REVIEW QUESTIONS 6.1

1. The length of vector **u** is $3\sqrt{5}$
2. The vectors are not equal as they must have the same magnitude and direction. In this case: $-2\vec{a} = \vec{b}$. Therefore the have different magnitudes and directions, but they are parallel.
3. These vectors are not equal. Whilst they have the same magnitude they have different directions.
4. i) A vector parallel to another vector is also collinear because both vectors share the same (or opposite) direction, meaning they are aligned along the same line.

   ii) A zero vector has no magnitude of direction. It is essentially a point at the origin.

   iii) Vectors that have the same initial point but different end points are called co-initial vectors.

   iv) The start point of a vector is the *initial point* where the vector begins. The end point is the *terminal point* where the vector ends.
5. 5 units.
6. 10 units
7. The scalar $k = \frac{1}{2}$ makes the vectors $\vec{u}$ and $\vec{v}$ equal, since $\vec{u} = \frac{1}{2}\vec{v}$.

## Review Questions 6.2

1. Sine of angle B (sin B) $= \frac{12}{13}$

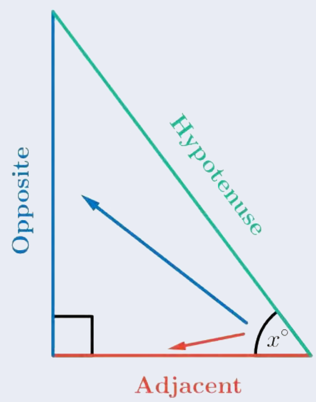
   Cosine of angle B (cos B) $= \frac{5}{13}$

   Tangent of angle B (tan B) $= \frac{12}{5}$

2.

| **Angle** | **$30^0$** | **$45^0$** | **$60^0$** |
|---|---|---|---|
| **Sine** | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ |
| **Cosine** | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ |
| **Tangent** | $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ | 1 | $\sqrt{3}$ |

3.




4. Sin a $= \frac{f}{g}$; Cos a $= \frac{e}{g}$ and tan a $= \frac{f}{e}$
5. Sin $\theta = \frac{15}{17}$; Cos $\theta = \frac{8}{17}$; tan $\theta = \frac{15}{8}$
6. The length of the slope is approximately 124.2 metres.
7. The length of the ramp is approximately 3.47 metres.
8. The boat is approximately 103 metres from the base of the lighthouse.
9. The prince will need to climb approximately 57.7 metres to reach the princess.

# EXTENDED READING

- Watch this video on Vectors and Bearings by clicking on the link below.
  https://youtu.be/m4iqjX1_LVM
- Watch this video on parallel and collinear vectors by clicking on the link below.

  https://youtu.be/itCLobhr-g4
- Watch this video on trigonometry by clicking on the link below.

  https://youtu.be/F21S9Wpi0y8
- Watch this video on trigonometry by clicking on the link below.

  https://youtu.be/sCyQ9DcDp2E

# REFERENCES


- Trigonometry Questions | Trigonometry Questions with Solutions (byjus.com)
- Sides and angles of right-angled triangles | Year 12 Maths | Australian Curriculum 12 Essential Mathematics - 2020 Edition | Mathspace
- Solve Applications: Sine, Cosine and Tangent Ratios. – Business/Technical Mathematics (opentextbc.ca)
- What is Tranversal | Angles formed between Transversal and Parallel Lines (byjus.com)
- Constructing Angle Bisectors - Construction using a compass, proof of angle bisector, examples. (cuemath.com)

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