SECTION

7 PERIMETER, AREA AND VOLUME

GEOMETRY AROUND US

Measurement

INTRODUCTION

Perimeter, area and volume are important concepts in geometry that help us measure and compare the sizes of 2D and 3D shapes. Understanding these ideas is useful for solving everyday problems, such as figuring out how much fencing is needed for a garden, how much paint is needed to cover a wall or how much liquid a container can hold. These concepts are connected and are the basis for learning more advanced topics in geometry and calculus. They are also important in subjects like physics and engineering, where measuring shapes and volumes is needed for designing and analysing structures

At the end of this section, you will be able to:

- Solve problems that involve identifying and comparing referents for SI and imperial area measurements of regular, composite and irregular 2-D shapes including decimal and fractional measurements and verify the solutions.
- Estimate the perimeter and area of a given regular, composite or irregular 2-D shapes.
- Solve a contextual problem that involves the perimeter and area of a regular, a composite or an irregular 2-D shape.
- Solve problems that involve SI and imperial units in volume of prisms.
- Solve real world problems that involves the volume of prisms.

Key ideas

Perimeter: refers to the total distance around the boundary of a two-dimensional shape.

Area: refers to the amount of space enclosed within the boundaries of a two-dimensional shape.

Volume: refers to the amount of space occupied by an object, such as a box, a swimming pool, or a water tank.

REFERENTS FOR MEASURING

Measurement is a key part of understanding the world and referents help by providing clear reference points for accurate measurement. Referents are standards or examples that we compare other quantities to, so everyone can use the same system to measure things. They help us make fair comparisons, understand results and keep measurements consistent in different situations. We will explore the importance of referents in measurement, looking at natural, man-made and relative referents. We will also discuss how choosing or changing referents can affect how we see things. By learning about referents, we can better understand how measurement works and why it is important.

Example 1

Investigate to validate the following referents in the tables.

Table 1: *Referents for Imperial Linear Measurement*

Imperial Measurement	Referent
Inch	Thumb length, from the tip to the first knuckle, or the thickness of a hockey puck
Foot	Standard floor tile in a classroom
Yard	Arm span from tip of nose, yard stick, length of a guitar
Mile	Distance walked in 20 minutes

Table 2: *Referents for SI Linear Measurement*

SI Measurement	Referent
Millimetres	Thickness of a dime, or a fingernail
Centimetres	Width of a fingernail, black keys on a piano, crayon, paper clip, or AA battery
Metre	Distance from a doorknob to the floor, width of a volleyball net, metre stick, waist height
Kilometre	Distance walked in 15 minutes

Table 3: *Referents for Area*

Measurement	Referent
≈1 ft^2	Area of a floor tile
≈1 in^2	Area of a postage stamp
≈1 cm^2	Area of a fingernail
≈ 2 m^2	Area of an exterior house door
≈ 93.5in^2 or 600cm^2	Area of exercise notebook
≈ 1500 m^2 or 17 000 ft^2	Area of an ice rink surface
≈ 32 ft^2 or 3 m^2	Area of a sheet of plywood

Project work

I would like you to do the project below by estimating the area of the referents in the table and submitting the final project to your teacher.

Table 4: *Referents for Area*

Referent	Measurement
Area of your television set	
Area of the marker board in the classroom	
Area of the classroom	
Area of the school field	
Area of the head teacher's office	

Perimeter and Area of 2-D Shapes

In Primary and Junior High school, you explored how to find the area and perimeter of basic shapes like squares, rectangles, triangles, and circles. We will now build on that knowledge by learning how to calculate the area and perimeter of more complex shapes—specifically, trapeziums, kites and parallelograms.

Before we dive into the new lesson, let's refresh our memory with a quick revision. Please watch the video below, which covers the methods we used to calculate the area and perimeter of squares, rectangles, triangles, and circles. This will help you recall the key concepts from the previous lesson and prepare you for the new material we'll cover this week.

A video on Area and Perimeter of Rectangle, Square, Triangle, Circle (https://youtu.be/9KvIQD2DjVg)

Perimeter and area of shapes

Perimeter is the total distance around a shape. To find the perimeter, you add up the lengths of all sides. Area is the measure of the space enclosed by a 2D shape. Area is measured in square units.

Investigating the area and perimeter of shapes using graphs/geodots

We can use graph sheets to investigate the perimeter and area of shapes. Look at the pictures below. A trapezium has been drawn on a graph sheet to help determine its area.

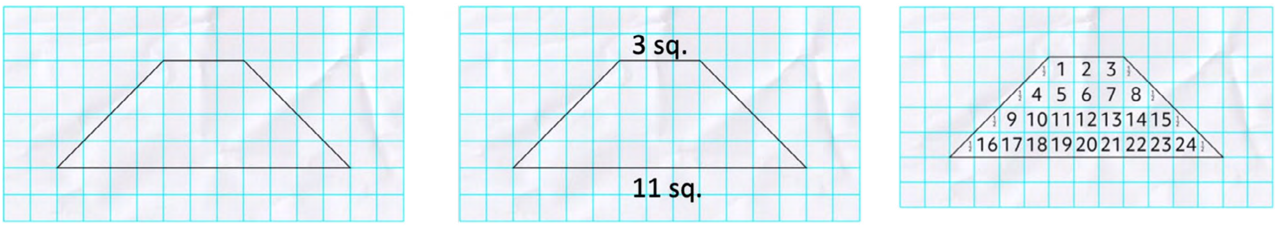


Figure 1: A trapezium on a grid

We know that area is a measure of how many square units will fit inside a shape. So, how many squares are inside our trapezium? There are 24 full squares plus eight half squares, which means the area of the trapezium is 28 square units.

Take a look at the rhombus drawn on the graph sheet below:

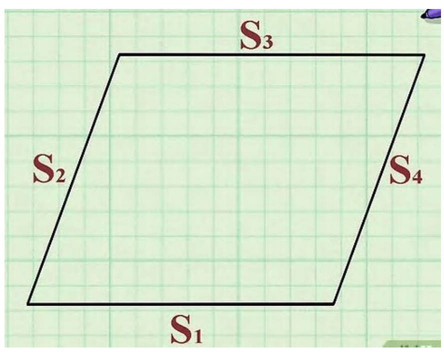


Figure 2: A rhombus on a graph

We can determine the perimeter of the shape by counting the number of squares covered by the line at S1 or S3. Each of these two sides cover approximately 12 squares. Now, since all the sides of a rhombus are equal, we can say that S2 and S4 are 12 squares as well. Assuming the side of a square on the graph is 1cm, then we can calculate the perimeter as $S_1 + S_2 + S_3 + S_4 = 12cm + 12cm + 12cm + 12cm = 48cm$.

Now, let us undertake activities to help us understand how to explore the perimeter and area of kites, parallelograms, rhombuses, and trapeziums using the graph sheet or geodot.

Understanding the Shapes

Kite: A quadrilateral with two pairs of adjacent sides that are equal. The diagonals intersect at right angles.

Parallelogram: A quadrilateral with opposite sides parallel and equal in length.

Rhombus: A parallelogram where all four sides are equal and diagonals bisect each other at right angles.

Activity 1

To help you explore the perimeter and area of kites, parallelograms, rhombuses and trapeziums using a geodot or graph sheet, here is a breakdown of hands-on activities to guide you.

Materials Needed:

- Geodot or graph sheets
- Rulers
- Pens or pencils
- Formula sheets for perimeter and area

Step-by-Step Process:

Kite:

1. Using the geodot or graph sheet, plot the vertices of a kite. Remember that adjacent sides are equal in length and the diagonals intersect at right angles.
2. Count the dots or use the graph scale to determine the lengths of the sides.

3. Calculate the perimeter by adding the lengths of all four sides.
4. To find the area, measure the lengths of the diagonals (from vertex to vertex) and use the formula:

   $\text{Area} = \frac{1}{2} \times \textit{diagonal } 1 \times \textit{diagonal } 2$

Parallelogram:

1. Draw a parallelogram on the geodot or sheet. Remember that the opposite sides are parallel. (Like a rectangle that has been sat on and pushed over.)
2. Measure the lengths of the base and height (perpendicular distance between the bases).
3. Perimeter is calculated as: $\text{Perimeter} = 2 \times (\textit{Base} + \textit{Side})$
4. Area is calculated using: $\text{Area} = \textit{Base} \times \textit{Perpendicular Height}$

Click on the links below to watch how to calculate the area of parallelogram using graph sheet:

https://youtu.be/nZcOo0HMSBg https://youtu.be/pfamBrBf8BQ

Rhombus:

1. Construct a rhombus on the graph sheet. Remember it is a shape with four equal length sides. (Like a square that has been sat on and pushed over.)
2. Measure the diagonals.
3. Calculate the perimeter by multiplying the side length by 4:

   $\text{Perimeter} = 4 \times \text{Side}$
4. For the area, use the diagonals:

   $\text{Area} = \frac{1}{2} \times \textit{diagonal } 1 \times \textit{diagonal } 2$

Click on the link below to watch how to calculate the area of rhombus using graph sheet: https://youtu.be/EM7b-IIbtTQ

Trapezium:

1. Draw a trapezium on graph paper. Remember it is a shape with only one pair of parallel sides of length
2. Measure and record the lengths of the sides and the perpendicular height between the parallel sides.

   Perimeter = sum of all sides

   Area using the formula:

   Area = $(\frac{1}{2})$ × (sum of parallel sides) × perpendicular height

Click on the link below to watch how to calculate the area of Trapezium using graph sheet: https://youtu.be/NLEiOsPPmwk

Calculating the perimeter and area of shapes using simple formulae

Take a look at the formulae for the following shapes:

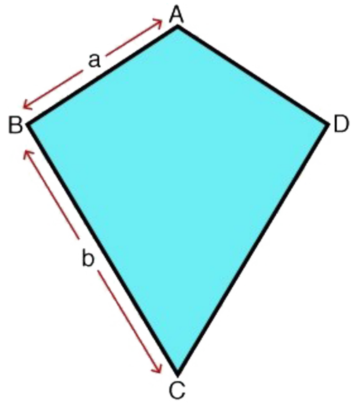


Figure 3: A kite

Formula

Perimeter (P) = 2(a + b)

here,

a and *b* are the 2 adjacent sides

In ◊ABCD,

a = AB = AD

b = BC = CD

Area of a kite = $\frac{1}{2} \times (d_1) \times (d_2)$ square units

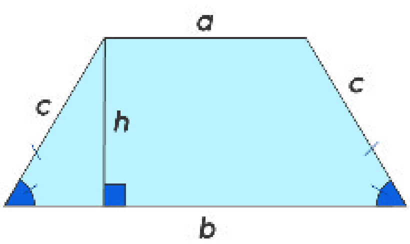


Figure 4: A trapezium

Perimeter = c +c + a + b

Area of an isosceles trapezoid

= $\frac{1}{2}$ (a + b) h square units

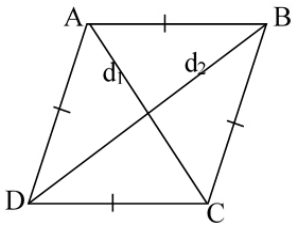


Figure 5: A rhombus

Perimeter = AB + AD + BC + CD

Area of a rhombus

$= \frac{1}{2} \times (d_1) \times (d_2)$ square units

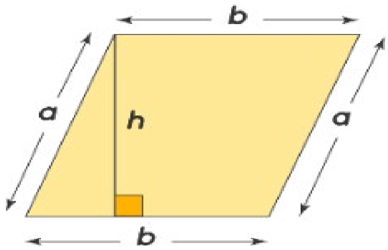


Figure 6: A parallelogram

Perimeter = 2(a + b) or (a + a + b + b)

Area of parallelogram

= base × perpendicular height

= b × h units^2

For **trapeziums**, investigate the different types:

- Isosceles Trapezium
- Scalene Trapezium
- Right Trapezium

Example 2

The perimeter and area for the following shapes can be determined as;

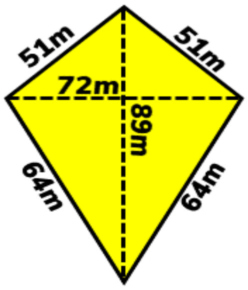


Figure 7: A kite.

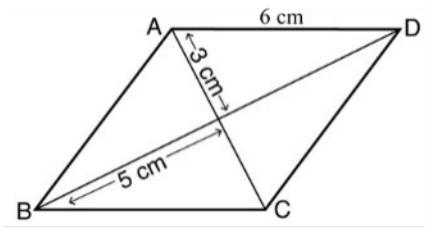


Figure 8: A rhombus

Perimeter (P) = 2(a + b),

$\therefore P = 2(51 + 64) = 230m$

Area $= \frac{1}{2} \times 72 \times 89$

$= 3204\, m^2$

Perimeter = AB + BC + CD + DA

$= 6 + 6 + 6 + 6 = 24cm$

Area of one triangle $= \frac{1}{2} \times 3 \times 5 = 7.5\, cm^2$

So, area of rhombus $= 4 \times 7.5 = 30\, cm^2$

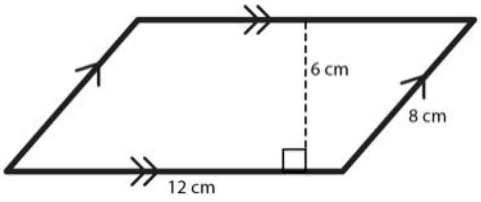


Figure 9: A parallelogram

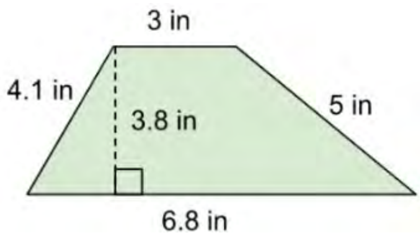


Figure 10: A trapezium

Perimeter $= 8 + 8 + 12 + 12 = 40\ cm$

Area $= 12 \times 6 = 72\, cm^2$

Perimeter $= 4.1 + 3 + 5 + 6.8 = 18.9\ in$

Area $= \frac{1}{2}(3 + 6.8) \times 3.8 = 18.62\ in^2$

Example 3

Find the perimeter of the shape below

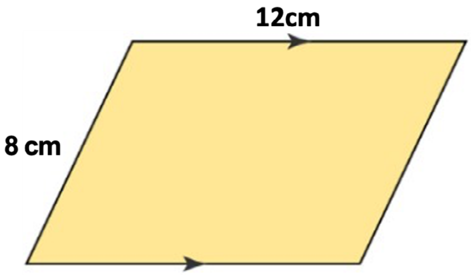


Figure 11: A parallelogram

Solution:

Step 1: Identify the shape if it is not given

In this case you will see that the shape is a parallelogram.

Step 2: Identify the lengths of the sides

A parallelogram has two pairs of equal sides.

In this case, you are given:

- Base $= 12\ cm$
- Side $= 8\ cm$

Step 3: Write the perimeter formula for a parallelogram

The perimeter is the total distance around the parallelogram, calculated as

Perimeter $= 2 \times (Base + Side)$

Step 4: Substitute the given values into the formula

$P = 2 \times (12\ cm + 8\ cm)$

Step 5: Add the length and width

Add the values inside the parentheses/Bracket:

$P = 2 \times 20\ cm$

Step 6: Multiply the result by 2

Finally, multiply by 2 to account for both pairs of sides: $P = 40\ cm$

The perimeter of the parallelogram is 40 cm

Example 4

Find the perimeter of the shape below

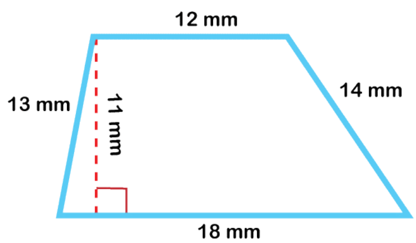


Figure 12: A trapezium

Solution:

Step 1: Identify the shape

This is a trapezium, which has four sides of varying lengths.

Step 2: Write the formula for the perimeter

The perimeter of a trapezium is the total distance around the shape, which is calculated by adding all four sides together:

P = side 1 + side 2 + side 3 + side 4

Step 3: Identify the side lengths

From the question, the given side lengths of the trapezium are:

- Side 1 = 12 mm
- Side 2 = 13 mm
- Side 3 = 14 mm
- Side 4 = 18 mm

Step 4: Substitute the values into the formula

$P = 12\text{ mm} + 13\text{ mm} + 14\text{ mm} + 18\text{ mm}$

Step 5: Add the side lengths

Add the values: $P=12+13+14+18=57\text{ mm}$

The perimeter of the trapezium is 57 mm.

Example 5

Calculate the area of the parallelogram below

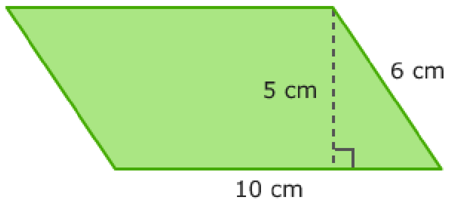


Figure 13: A parallelogram

Solution:

Step 1: Identify the base and perpendicular height

The base of the parallelogram is given as 10 cm.

The perpendicular height is the shortest distance from the base to the opposite side, which is given as 5 cm.

Step 2: Write the formula for the area of a parallelogram

The formula is: Area = base × perpendicular height

Step 3: Substitute the given values

Base = 10 cm

Perpendicular height = 5 cm

$\text{Area}=10\text{ cm} \times 5\text{ cm}$

Step 4: Multiply the base by the perpendicular height

$\text{Area} = 50cm^2$

The area of the parallelogram is 50 cm^2.

Solve real-life problems on perimeter and area of 2-D shapes

Example 6

The perimeter of a parallelogram is equal to 48cm. Two of its sides are 16cm each.

How long is each of the other side?

Solution:

We are given the Perimeter (P) = 48 cm and side a = 16 cm.

To find the length of the other side b of the parallelogram, we will use the formula of the perimeter of a parallelogram P = 2(a + b).

$P = 2(a + b)$

$\Rightarrow 48 = 2\ (16 + b)$

$\Rightarrow 16 + b = \frac{48}{2}$

$\Rightarrow b = 24 - 16$

$\Rightarrow b = 8$ cm

Answer: The other side of the parallelogram is 8 cm.

Example 7

A garden in the shape of a parallelogram has dimensions 12 metres and 9 metres and a perpendicular height of 8 metres. If the owner wants to put a fence around the garden, how much fencing material is needed? Also, what is the area of the garden?

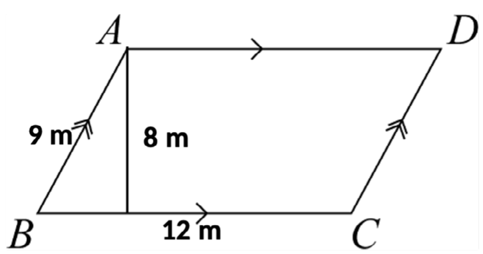


Figure 14: A parallelogram

Solution

To find the perimeter of the parallelogram, we need to add the lengths of all four sides. Since a parallelogram has opposite sides of equal length, the perimeter is:

Perimeter = 2(12 + 9) = 2(21) = 42 metres

To find the area of the parallelogram, we use the formula:

Area = base × perpendicular height = 12 × 8 = 96 square metres

Therefore, the owner needs 42 *m* of fencing material and the area of the garden is 96 m^2.

Example 8

Gina is designing a rectangular banner to promote her school's art exhibition. She wants to include a large rhombus-shaped logo in the middle of the banner. The diagonal dimensions of the logo rhombus are 96 cm and 60 cm. The banner dimensions are 1.5 *m* × 1 *m*.

Gina has a few design questions:

i. What is the height and the width of the rhombus logo?

ii. What is the area of the rhombus logo?

iii. Will her logo rhombus fit inside the 1.5 m × 1 m banner dimensions if she aligns the edges parallel?

iv. How much blank space will there be at the top/bottom or sides once the rhombus logo is centered aligned in the banner.

Solution

i. Height = Shorter diagonal = 60 cm

Width = Longer diagonal = 96 cm

ii. Area $= \frac{1}{2} \times Height \times Width$

$= \frac{1}{2} \times 60 \times 96$

$= 2\,880\, cm^2$

iii. Yes, aligned vertically or horizontally, the rhombus height 60 cm and width 96 cm will fit within the 150 *cm* × 100 *cm* banner with room to spare.

iv. Top/Bottom blank space $= \frac{(Banner\ height - Rhombus\ height)}{2} = \frac{150 - 60}{2} = 45cm$

Side blank space$= \frac{(Banner\ width - Rhombus\ width)}{2} = \frac{100 - 96}{2} = 2cm$

So, when centred, there would be 45 cm blank space above and below the rhombus logo in the banner and 2 cm on either side.

Example 9

Nagbija Isaac who is a builder is designing a window with a parallelogram-shaped glass pane. The base of the glass pane is 50 cm, and the perpendicular height is 40 cm. What is the area of the glass pane?

Solution:

To find the area of the parallelogram-shaped glass pane designed by Nagbija Isaac, we follow these steps:

Step 1: Write down the formula for the area of a parallelogram

The formula to calculate the area of a parallelogram is:

Area = base × perpendicular height

Step 2: Identify the values for the base and height

Base (b) = 50 cm

Perpendicular Height (h) = 40 cm

Step 3: Substitute the values into the formula

$\text{Area} = 50\,\text{cm} \times 40\,\text{cm}$

Step 4: Multiply the base by the height

$\text{Area} = 2000\,\text{cm}^2$

The area of the parallelogram-shaped glass pane is 2 000 cm^2.

Example 10

A park has a rhombus-shaped flower bed with diagonals measuring 12 metres and 8 metres. What is the area of the flower bed?

Solution

To find the area of the rhombus-shaped flower bed in the park follow these steps:

Step 1: Write down the formula for the area of a rhombus

The formula to calculate the area (A) of a rhombus using its diagonals is:

$A = \frac{1}{2} \times d_1 \times d_2$

Step 2: Identify the values for the diagonals

Diagonal 1 (d_1) = 12 metres

Diagonal 2 (d_2) = 8 metres

Step 3: Substitute the values into the formula

$$A = \frac{1}{2} \times 12\,m \times 8\,m$$

Step 4: Multiply the diagonals

$$A = \frac{1}{2} \times 96m^2$$

Step 5: Divide by 2 to get the final area

$$A = 48m^2$$

The area of the rhombus-shaped flower bed is 48 m^2.

Example 11

A contractor is designing a trapezium-shaped wheelchair ramp for Mr. Otonko. The parallel sides are 2 metres and 5 metres, and the perpendicular height of the ramp is 1.5 metres. What is the area of the ramp?

Solution

To find the area of the trapezium-shaped wheelchair ramp, follow these steps:

Step 1: Write down the formula for the area of a trapezium

The formula to calculate the area of a trapezium is:

$$\text{Area} = \frac{1}{2} \times (a + b) \times \text{perpendicular } h$$

Step 2: Identify the values for the parallel sides and height

Parallel side 1 (a) = 2 metres

Parallel side 2 (b) = 5 metres

Perpendicular Height (h) = 1.5 metres

Step 3: Substitute the values into the formula

$$\text{Area} = \frac{1}{2} \times (2\,m + 5\,m) \times 1.5\,m$$

Step 4: Add the lengths of the parallel sides

$$\text{Area} = \frac{1}{2} \times 7\,m \times 1.5\,m$$

Step 5: Complete the calculation to find the area

$$\text{Area} = 5.25\,m^2$$

The area of the trapezium-shaped wheelchair ramp is 5.25 m^2.

Example 12

You want to make a large decorative kite at a local Guinea Corn festival, celebrated by the Konkombas. The diagonals of the kite are 2.5 metres and 1.5 metres. How much material do you need to make the kite?

Solution

To determine how much material is needed to make the kite, we need to calculate the area of the kite. A kite's area can be found using its diagonals.

Step 1: Write down the formula for the area of a kite

The formula to calculate the area of a kite using its diagonals is:

$$A = \frac{1}{2} \times d_1 \times d_2$$

Step 2: Identify the values for the diagonals

Diagonal 1 (d_1) = 2.5 metres

Diagonal 2 (d_2) = 1.5 metres

Step 3: Substitute the values into the formula

$$\text{Area} = \frac{1}{2} \times 2.5\,\text{m} \times 1.5\,\text{m}$$

Step 4: Complete the calculation to find the area

$$\text{Area} = 1.875\,\text{m}^2$$

You will need 1.875m^2 of material to make the kite.

PRISMS

Let us now look into the world of prisms. We need to:

i. define prisms and identify the elements: bases, height/length, lateral faces.
ii. understand the appropriate formula for calculating the volume of prisms.
iii. calculate volume by substituting dimensions into formula.
iv. Solve problems involving prism volumes contextually.

Defining and identifying the elements: bases, height/length, lateral faces of prisms

Watch this video (5 minutes) by clicking on the link which will help you identify parts of prisms: https://youtu.be/SBJv3MqlGEA.

You will realise from the video that: **A prism** is a three-dimensional solid object having two identical and parallel shapes facing each other. Thus, a prism has a constant cross-section. The identical shapes are called the bases. The bases can have any shape of a polygon for example, triangles, squares, rectangles or a pentagon. The diagram below shows a triangular prism.

Parts

A prism has bases, lateral faces, edges and vertices.

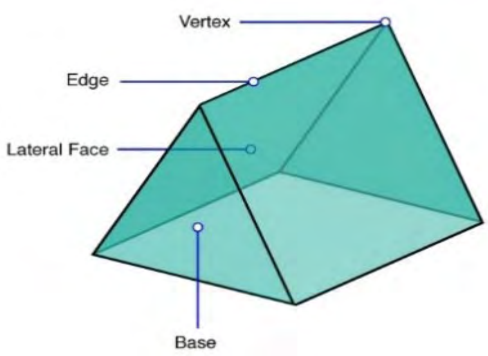


Figure 15: A prism

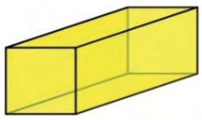
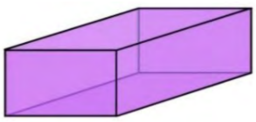
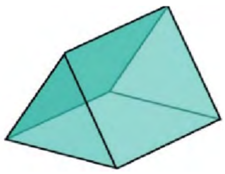
a. **Base** – The base is one of the **parallel faces** which makes the 2 ends of any prism. These are congruent. The base determines the cross-section of any prism and it remains uniform throughout the shape.

b. **Lateral faces** – The **non-parallel faces** which connects the 2 bases. In this example there are 3 lateral faces.

c. **Vertices** – The corners of the shape. In this example there are 6 vertices.

d. **Edges** – Where any 2 faces meet. In this example there are 9 edges.

Table 5: Examples of other prisms include

		
Square-faced cuboid • 6 faces (2 squares and 4 rectangular) • 12 edges • 8 vertices	**Rectangular-faced cuboid** • 6 faces (all rectangular) • 12 edges • 8 vertices	**Triangular** • 5 faces (2 triangular and 3 rectangular) • 9 edges • 6 vertices

Activity 2

Identify other prisms (in your immediate environment) and indicate the type of face, number of faces, edges and vertices. Indicate the properties and nets of the prisms (learned in Junior High school). Tabulate your results on a sheet of paper and show to your teacher and classmates.

Regular and Irregular Prisms

A prism can also be classified into **regular** or **irregular** based on the uniformity of its cross-section. It can be **right** or **oblique**, depending on the alignment of its bases. The diagram shows the difference between a regular and irregular triangular prism.

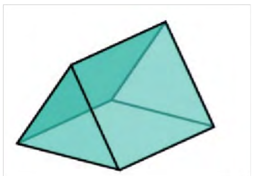


Figure 16a: Regular prism

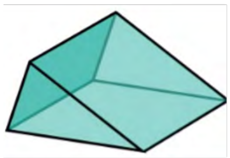


Figure 16b: Irregular prism

1. **Regular Prism** – It has a base which is a regular polygon with equal side lengths. Regular prisms have identical bases and identical lateral faces.
2. **Irregular Prism** – It has a base which is an irregular polygon with unequal side lengths. Irregular prisms have identical bases. However, the lateral faces are not identical.

Right and Oblique Prisms

The diagram shows the difference between a right and an oblique pentagonal prism.

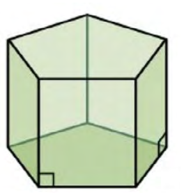


Figure 17a: Right prism

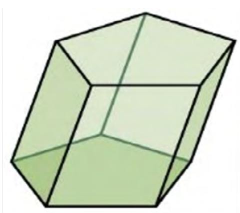


Figure 17b: Oblique prism

1. **Right Prism** – Its lateral faces are perpendicular to its bases. The 2 bases of a right prism are aligned perfectly over one another.

2. **Oblique Prism** – It is a slanted prism. This means that its lateral faces are not perpendicular to its bases. The 2 bases are not aligned perfectly over one another.

In this section we will be focusing on right prisms.

VOLUME OF PRISMS

In Junior High School, you learned about surface area and volume of solid shapes. Before we delve into today's lesson, let's refresh our minds on surface area and volume of solid shapes. Watch this video (17 minutes) by clicking on the link:

https://youtu.be/cS28z1cdAdY.

I hope you enjoyed the video. Let's now look at the following examples.

Example 13

Calculate the surface area of a cuboid consisting of a square base of length 4m where the height of the cuboid is 10m.

Solution

Cuboid

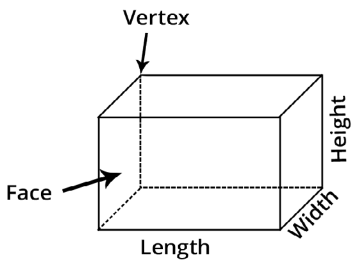


Figure 18: Cuboid

Net

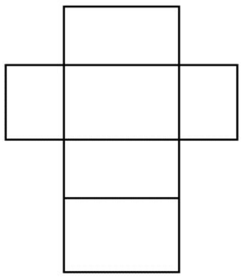


Figure 19: Net of a cuboid

To calculate the surface area of a cuboid with a square base, you need to calculate the area of all its faces.

Given:

- The side length of the square base L = 4m
- The height of the cuboid, H = 10m

The cuboid has:

- 2 square (base) faces (top and bottom) with area L^2
- 4 rectangular (lateral) faces, each with area L x H

Step 1: Calculate the area of the square (base) faces

Area of one square face $= L^2 = 4^2 = 16m^2$

Area of two square faces $= 2 \times 16 = 32m^2$

Step 2: Calculate the area of the rectangular (lateral) faces

Area of one rectangular face $= L \times H = 4 \times 10 = 40\ m^2$

Area of four rectangular faces $= 4 \times 40 = 160\ m^2$

Step 3: Calculate the total surface area

Total surface area = Area of square faces + Area of rectangular faces

Total surface area $= 32 + 160 = 192m^2$

Thus, the surface area of the cuboid is 192 m^2

If you are still unsure, watch this video (16 minutes) on Surface Area and volumes of solid figures by clicking on the link: https://youtu.be/eBAq_caikJ4.

Understanding volume helps quantify three-dimensional spaces which allows for practical applications like determining storage capacities, amounts that fit inside containers, calculating dosages and more.

Volume is a measure of the amount of three-dimensional space that an object occupies.

Some key points about volume:

1. Volume measures the space taken up by a 3D object. It quantifies how much a substance or solid material would fill a three-dimensional container.
2. Volume is measured in cubic units such as cubic metres (m^3), cubic centimetres (cm^3) and cubic inches (in^3).
3. The volume of a 3D shape depends on the dimensions of height, width and depth - unlike area which uses just height and width. The standard mathematical formula used is:

   Volume = Area of the base x Height

Some examples of volumes we encounter:

- Volume of liquid a bottle can contain
- Volume of concrete needed to lay a house's foundation
- Volume of medicines and dosage instructions based on that.

Understanding and calculating volumes has many practical real-world applications in fields from shipping and storage to calculating health or building metrics to determining carrying capacity of containers.

The basic volume formulas work for standard 3D geometrical shapes. However, for irregular shapes, volumes are measured by instruments or by estimation using simpler building blocks and their volume formulae.

Understanding the appropriate volume formula

Watch this video (4 minutes) by clicking on the link:

https://youtu.be/P72Jfnr66Ac and study Table 6.

Table 6: Formulae for calculating volume of various prisms

Shape	Base	Volume of Prism = Base area × height
Triangular Prism	Triangular	Volume of triangular prism = Area of triangle × height of the prism
Square Prism	Square	Volume of square prism = Area of square × height of the prism
Rectangular Prism	Rectangular	Volume of rectangular prism = Area of rectangle × height of the prism

Example 14

Find the volume of a triangular prism whose base area is 64 cm^2 and height is 7 cm.

Solution

As we know,

Volume (V) = Base Area × Height

$\therefore V = B \times H$, here $B = 64\ cm^2$, $H = 7$ cm

$= 64 \times 7$

$= 448\ cm^3$

Example 15

Find the volume of the prism shown below.

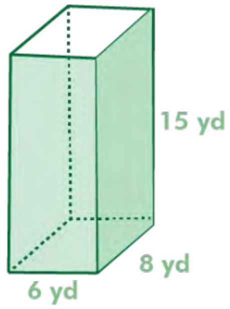


Figure 20: Prism

Solution

V = B × H [Volume of a prism formula]

For a rectanglular prism we know that the area of the base = Length × Width

V = L × W × H [Area of rectangle formula × Height]

V = 6 × 8 × 15 [Substitute values of L, W and H]

V = 720

The volume of the prism is 720 cubic yards.

Example 16

A box of popcorn holds 7000 cubic centimetres of popcorn. The length and width of the base of the box are 14cm and 20cm, respectively. Find the height of this box of popcorn.

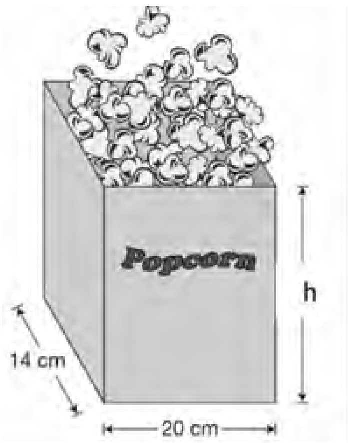


Figure 21: Popcorn box

Solution

$V = B \times H$ [Formula for the volume of a prism]

$7000 = L \times W \times H$ [Formula for the area of a rectangle; replace V with 7000]

$7000 = 14 \times 20 \times H$ [Substitute values of L and W]

$7000 = 280 \times H$ [Solve for H]

$25 = H$ [Divide each side by 280]

The height of the popcorn box is 25cm.

Example 17

What is the base area of the prism if the volume of the prism is 324 cubic units and the height of the prism is 9 units?

Solution

The given dimensions are the volume of the prism = 324 cubic units and the height of the prism = 9 units. Let the base area of the prism be "B".

Substituting the values in the volume of the prism formula:

Volume of prism $= V = B \times H = 324$ cubic units

$\Rightarrow 9B = 324$

$\Rightarrow B = 36$ square units

Therefore, the base area of the prism is 36 square units.

Activity 3

Look for solid objects in your immediate environment (for example, a Marker box, Money box) that are prisms and find the volume of each using the appropriate materials.

Record your results on a sheet of paper and show to your teacher or classmates.

Activity 4

Think about a cylinder. Is this a prism?

Can you determine the volume and surface area of a cylinder whose radius is 3m and height 6m?

Hint: i) Multiply the area of the base by the height to get the volume. ii) Use the net of the cylinder to determine the surface area.

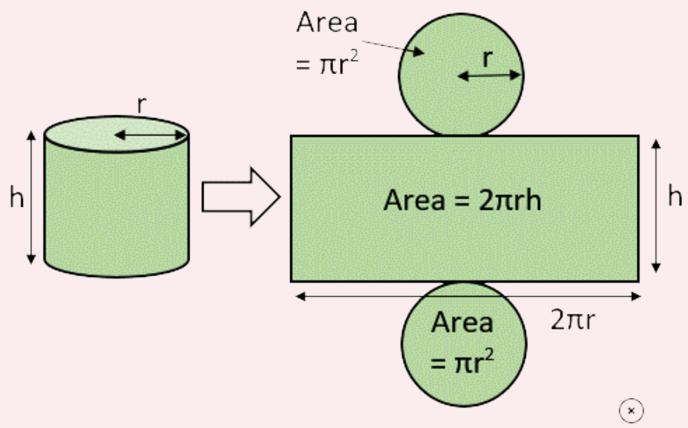
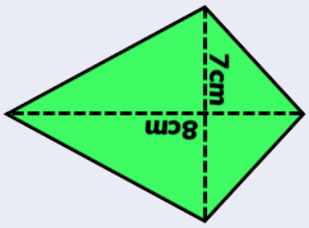
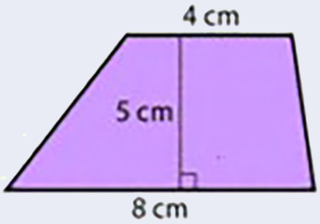


Figure 22: Net of a cylinder

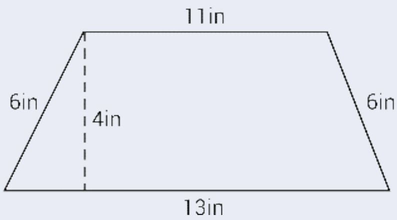
REVIEW QUESTIONS

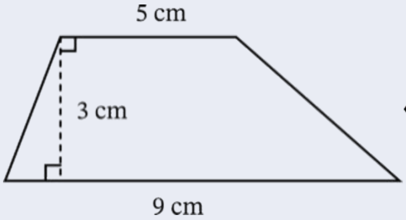
1. Calculate the area of the following shapes

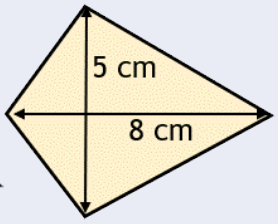




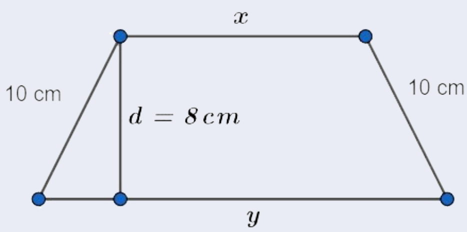
2. Find the area for the shapes below.





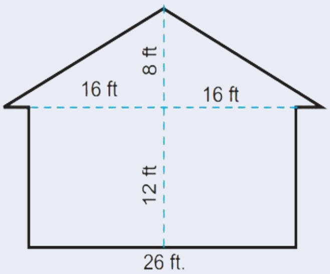


3. The perimeter of the given trapezium is 104 cm. Find its area.



4. Solve the following problems

i.

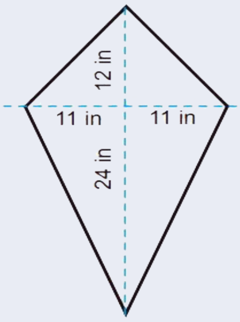


Miss. Tackie wants to paint the side of her house.

To buy paint, she must know the area.

What is the area of the side of Miss. Tackie's house?

ii.



Charles needs paper to cover his kite.

Find the area of the kite

5. A field in the shape of a rhombus has an area of 60 square metres. If one of its diagonals is 12 metres long, find the length of the other diagonal.

6. Jamila is building a deck outside her house with a rectangular area that has outer dimensions of 8m by 6m. She wants to have decorative floor tile patterns in a parallelogram shape inside the deck. Each parallelogram tile has one side measuring 1.2m and a perpendicular height of 0.8m against that side.

   (a) What is the area of each parallelogram tile?

   (b) If Jamila covers the entire inner deck area with these tiles, how many full tiles would she need?

7. A designer wants to use parallelogram-shaped tiles to cover a rectangular floor that is 6 metres long and 4 metres wide. If each tile has a base of 0.5 metres and a height of 0.3 metres, how many tiles will be needed to cover the entire floor?

8. An architect is designing a parallelogram-shaped roof for a house. The sides of the roof are 10 metres and 6 metres. If the perpendicular height of the roof is 4 metres, what is the area of the roof?

9. A construction company is laying out a rhombus-shaped foundation for a building. If one of the diagonals is 30 metres and the other diagonal is 24 metres, how much land will the foundation cover?

10. A farmer has a trapezium-shaped plot of land. The parallel sides measure 40 metres and 70 metres, and the shortest distance between the parallel sides is 30 metres. What is the area of the land?

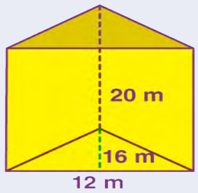
11. A farmer is designing a kite-shaped plot in their field, with diagonals measuring 10 metres and 6 metres. What is the area of the kite-shaped plot?

12. Find the height of the prism if the volume of the prism is 729 cubic units and the base area is 27 square units.

13. What is the base area of a prism if the volume of the prism is 300 cubic feet and the height of the prism is 6 feet?

14. The base area of a prism is 123 square yards. The height is 9 yards. Find the volume.

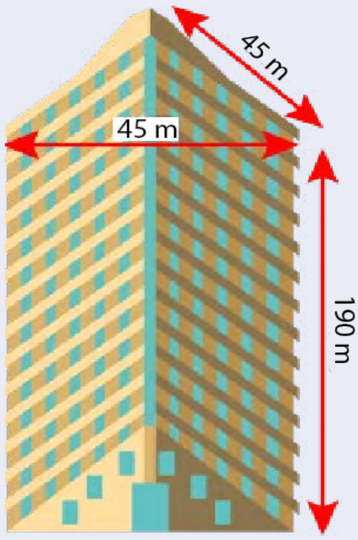
15. What is the volume of a triangular prism with dimensions of 12 m, 16 m and 20 m as shown in the figure below?



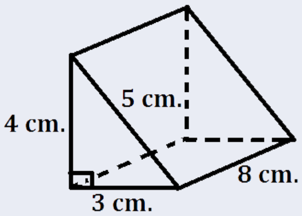
16. A swimming pool in the shape of a cuboid has a length of 10 metres, a width of 5 metres, and a depth of 2 metres. If the pool is filled with water, how much water does it hold?

17. The height of a square prism is 11 yards. The volume is 99 cubic yards. What is the length of the side of the square at the base of the prism?

18. A building is in the shape of a triangular prism. The height of the building is 190 metres. The base of the building is in the form of an equilateral triangle whose side length is 45 metres. Find the lateral surface area of the building.



19. Amina's doll's tent is in the shape of a triangular prism. How many cubic centimetres of space is in the tent?



ANSWERS TO REVIEW QUESTIONS

1. 28 cm^2, 30 cm^2
2. 48 in^2, 21 cm^2, 20 cm^2
3. 336 cm^2
4. (i) 464 ft^2
   (ii) 396 in^2
5. 10 metres
6. (a) 0.96 m^2
   (b) 50 tiles
7. 160 tiles
8. 40 m^2
9. 360 m^2
10. 1650 m^2
11. 30 m^2
12. 27 units
13. 50 square feet
14. 1107 cubic yards
15. 1920 m^3
16. 100 cubic metres of water
17. 3 yards
18. 25 650 m^2
19. 48 cm^3.

GLOSSARY

- **Perimeter**: The total distance around the boundary of a two-dimensional shape.
- **Area**: The measure of the amount of space enclosed within the boundaries of a two-dimensional shape.
- **Volume**: The amount of space occupied by a three-dimensional object or shape.
- **Circumference**: The perimeter of a circle, calculated using the formula $C=2\pi r$ where r is the radius of the circle.
- **Cuboid**: A three-dimensional shape with six rectangular faces.
- **Cube**: A special type of cuboid where all sides are equal in length, so all the faces are identical squares.
- **Parallelogram**: A four-sided polygon with the opposite sides parallel and equal in length.
- **Trapezium (Trapezoid)**: A four-sided polygon with only one pair of parallel sides.
- **Prism**: A three-dimensional solid with two parallel and congruent faces called bases and all other faces are rectangles.

EXTENDED READING

- Akrong Series: Core mathematics for Senior High Schools New International Edition (Pages 94 – 106)
- Aki – Ola series : Core Mathematics for Senior High Schools in West Africa, Millennium edition 5 (Pages 376 – 388)
- Baffour Asamoah, B. A. (2015). **Baffour BA series: Core mathematics**. Accra: Mega Heights, (Pages 165 - 200)
- Watch this video (30 minutes) on how to solve problems involving prisms by clicking on the link: https://youtu.be/x6QwX1D1akg.

REFERENCES

- Asiedu, P. (2016). *Mathematics for Senior High Schools 1*. S. Coleman, K. A. Benson, H. A. Baah – Yeboah (Eds.). Aki - Ola Publications.
- Andrzej Trybulec. Enumerated sets. *Formalized Mathematics*, 1, 1990.
- Asiedu, P. (Millennium Edition 5). Core Mathematics for Senior High Schools in West Africa. Aki - Ola Publications.
- Akrong Series: Core mathematics for Senior High Schools New International Edition
- Perimeter of Rhombus (math-salamanders.com)
- Prism - Definition, Shape, Types, Formulas, Examples & Diagrams (mathmonks.com)
- Volume of Prism - Formula, Derivation, Definition, Examples (cuemath.com).

ACKNOWLEDGEMENTS

Ghana Education Service (GES)

List of Contributors

NAME	INSTITUTION
Janet Waponade	OLA SHS
Yamusah Issahaku	Kumbungu SHS
Joseph Bakpil Nagbija	Nchumuruman SHS, Chinderi
Modzaka Godfred	Keta SHTS, Keta