Mathematics

Year 1

SECTION

9

PROBABILITY OF INDEPENDENT EVENTS

MAKING SENSE OF AND USING DATA

Probability/Chance

INTRODUCTION

Probability is about understanding and measuring how likely different things are to happen. It is used in many areas like statistics, economics, and finance. Simple probability experiments are things where every outcome is equally likely, like flipping a coin (heads or tails) or rolling a die (any number from 1 to 6). Compound probability experiments involve more steps or events. For example, drawing two cards from a deck without putting the first one back. This changes the chances for the second draw. Independent events are when the outcome of one event doesn't affect the outcome of another. For example, flipping a coin and rolling a die are independent because the result of one doesn't change the result of the other. Understanding these concepts helps you make better decisions when things are uncertain. It also helps with other subjects like physics and biology, where probability is used to understand random processes. Overall, learning about probability helps you improve your problem-solving and critical thinking skills.

At the end of this section, you will be able to:

- List the elements of the sample space from a simple or compound experiment involving two independent events.
- Determine the probabilities of independent events and express the results as fractions, decimals, percentages and/or ratios.
- Solve everyday life problems involving the probability of two-independent events

Key ideas

- Experiment: An experiment is a procedure or activity you perform to see what happens and to collect results.
- Trial: A trial is a single attempt or instance of an experiment.
- Outcomes: Outcomes are the possible results of a trial or experiment.

- Events: Events are specific results or sets of outcomes that you are interested in from an experiment.
- Sample space: is the set of all possible outcomes of an experiment

PROBABILITY OF INDEPENDENT EVENTS (SAMPLE SPACE)

Probability is a branch of mathematics that deals with the likelihood or chance of an event occurring. It is used to quantify uncertainty and predict the likelihood of different outcomes in situations where the outcome is uncertain. Probability is expressed as a number between 0 and 1, where 0 indicates that the event will never occur, and 1 indicates that the event will definitely occur.

Terminologies relating to the concept of probability

i. Experiment
ii. Random Experiment
iii. Trial
iv. Sample space
v. Event
vi. Equally Likely Events
vii. Exhaustive Events
viii. Favourable Events
ix. Additive Law of Probability

A random experiment is a mechanism that produces a definite outcome that cannot be predicted with certainty. The sample space associated with a random experiment is the set of all possible outcomes. An event is a subset of the sample space.

Independent experiments and list the sample space

Independent experiments are experiments where the outcome of one does not influence or change the outcome of another. In other words, the result of one experiment has no effect on the result of the other.

Two events, A and B, are independent if $P(A \cap B) = P(A) \times P(B)$.

Consider tossing a fair coin three times in a row. Since each of the throws is independent of the other two, we consider all 8 ($= 2^3$) possible outcomes as equally probable and assign each the probability of $\frac{1}{8}$. Here is the sample space of a sequence of three tosses, ie all the possible outcomes of tossing a coin 3 times:

{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}.

Let's consider the activity below

Activity 1

Determine whether the following experiments are independent or not and give reasons.

Experiment 1: Rolling a six-sided die and flipping a coin.

Experiment 2: Choosing a student from a class and then choosing another student from the same class without replacement.

Solution

Experiment 1: Independent

Reason: The outcome of rolling the die does not affect the result of flipping the coin. Each experiment has its own set of outcomes and does not influence the other

Experiment 2: Dependent

Reason: The result of the first choice affects the composition of the group for the second choice, as the number of students decreases and the probability changes.

Example 1

A fair die is rolled twice. List the sample space for the experiment.

Solution

Sample space for two dice (outcomes):

Table 1: Sample space

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Example 2

List the sample space of rolling a die and flipping a coin once

Solution

Let H be Head and T be Tail

{(1, H),(1,T),(2,H),(2,T),(3,H),(3,T),(4,H),(4,T),(5,H),(5,T),(6,H),(6,T)

Probabilities of independent events

Probability serves as a powerful tool in understanding and quantifying uncertainty. It enables us to make informed decisions, assess risks, and analyse the likelihood of various outcomes. We will now delve into independent events, where the occurrence or outcome of one event does not affect the occurrence or outcome of another event.

Example 3

Let's suppose there are ten balls in a box. Four balls are Green (G) and six balls are Red (R). If we draw two balls, one at a time, with replacement, find the probability of the following events:

1. Both balls are green.
2. The first ball is red and the second is green.

3. At least one ball is red

Solution

Let G1 and R1 be the events that the first ball is Green/Red respectively.

Similarly, let G2 and R2 be the events that the second ball is Green/Red.

Since we are dealing with sampling with replacement this means that:

$P(G1) = P(G2) = \frac{4}{10} = \frac{2}{5}$ and $P(R1) = P(R2) = \frac{6}{10} = \frac{3}{5}$

And it also means that the events are independent.

1. P(both balls are green) = P(G1 and G2) = $P(G1 \cap G2)$.

Since the trials are independent, $P(G1 \cap G2) = P(G1) \times P(G2) = \frac{2}{5} \times \frac{2}{5} = \frac{4}{5}$

2. P(First Red and Second Green) = P(R1 and G2) = $P(R1 \cap G2)$. since the trials are independent, so $P(R1 \cap G2) = P(R1) \times P(G2) = \frac{2}{5} \times \frac{3}{5} = \frac{6}{5}$

3. We use the fact that P(at least one ball is Red) = 1 – P(both balls are Green). Hence, P(at least one ball is Red) $= 1 - \frac{4}{5} = \frac{21}{25}$

Example 4

A poll finds that 72% of Kumasi indigenes consider themselves football fans. If you randomly pick two people from the population, what is the probability that the first person is a football fan and the second is as well? That the first one is a fan and the second one is not?

Solution

One person being a football fan does not have an effect on whether the second randomly selected person is. Therefore, the events are independent and the probability can be found by multiplying the probabilities together:

First one and second are football fans:

$P(A \cap B) = P(A) \times P(B) = 0.72 \times 0.72 = 0.5184.$

First one is a football fan, the second one isn't:

$P(A \cap B) = P(A) \times P(B) = 0.72 \times (1 - 0.72) = 0.2016.$

In the second part, we multiplied by the complement. As the probability of being a football fan is 0.72, then the probability of not being a fan is 1 – 0.72, or 0.28.

Events A and B are independent if the equation $P(A \cap B) = P(A) \times P(B)$ holds true. You can use the equation to check if events are independent by multiplying the probabilities of the two events together to see if they equal the probability of them both happening together.

Example 5

You roll two fair six-sided dice. What is the probability that:

(a) Both dice land on an even number?

(b) The first die lands on a 2 and the second die lands on a 5?

(c) At least one die lands on a 6

Solution

(a) Probability that both dice land on an even number

Step 1: Identify the even numbers on a die.

The even numbers on a six-sided die are 2, 4 and 6.

So, the probability of rolling an even number on one die is $P(\text{even on 1 die}) = \frac{3}{6} = \frac{1}{2}$

Step 2: Calculate the probability for both dice landing on even numbers.

Since the rolls are independent, we multiply the probabilities for each die

$P(\text{both even}) = P(\text{even on 1st die}) \times P(\text{even on 2nd die}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Thus, the probability that both dice land on an even number is: $P(\text{both even}) = \frac{1}{4}$

(b) Probability that the first die lands on a 2 and the second die lands on a 5

Step 1: Identify the probability of each specific outcome.

The probability of rolling a 2 on the first die is $P(2 \text{ on 1st die}) = \frac{1}{6}$

The probability of rolling a 5 on the second die is $P(5 \text{ on 2nd die}) = \frac{1}{6}$

Step 2: Multiply the probabilities.

Since the rolls are independent, we multiply the probabilities for each die:

$P(2 \text{ on 1st}, 5 \text{ on 2nd}) = P(2 \text{ on 1st die}) \times P(5 \text{ on 2nd die}) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{6}$

Thus, the probability that the first die lands on a 2 and the second die lands on a 5 is $\frac{1}{6}$.

(c) Probability that at least one die lands on a 6

Step 1: Calculate the probability that **neither die** lands on a 6 (complement rule).

The probability that a die **does not** land on a 6 is 1 – P(die lands on a 6)

$= P(not\ 6\ on\ 1\ die) = 1 - \frac{1}{6} = \frac{5}{6}$

The probability that **neither die** lands on a 6 is the product of the probabilities for both dice not landing on 6

$P(neither\ 6) = P(not\ 6\ on\ 1st\ die) \times P(not\ 6\ on\ 2nd\ die) = \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

Step 2: Use the complement of this to find the probability that **at least one die lands on a 6**.

The complement of “neither die is a 6” is “at least one die is a 6.”

Thus, we subtract the probability of neither being 6 from 1:

$P(at\ least\ one\ 6) = 1 - P(neither\ 6) = 1 - \frac{25}{36} = \frac{11}{36}$

Example 6

A bag contains 3 blue marbles, 5 red marbles and 2 yellow marbles. You randomly select two marbles with replacement. What is the probability that:

(a) Both marbles are blue?

(b) The first marble is red and the second is yellow?

(c) At least one marble is red?

Solution

(a) Probability that both marbles are blue

Step 1: Calculate the probability of drawing a blue marble on the first draw.

$$P(\text{Blue}) = \frac{\textit{Number of blue marbles}}{\textit{Total number of marbles}}$$

$$= \frac{3}{(3+5+2)} = \frac{3}{10}$$

Step 2: Since the marbles are drawn with replacement, the probability remains the same for the second draw.

$$P(\text{Blue}) = \frac{3}{10}$$

Step 3: Calculate the probability of drawing two blue marbles in a row.

$$P(\text{Both Blue}) = P(\text{Blue}) \times P(\text{Blue}) = \frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$$

(b) Probability that the first marble is red and the second is yellow

Step 1: Calculate the probability of drawing a red marble on the first draw.

P(Red) = Number of red marbles / Total number of marbles

$$= \frac{5}{(3+5+2)} = \frac{5}{10} = \frac{1}{2}$$

Step 2: Calculate the probability of drawing a yellow marble on the second draw.

$$P(\text{Yellow}) = \frac{\textit{Number of yellow marbles}}{\textit{Total number of marbles}} = \frac{2}{(3+5+2)} = \frac{2}{10} = \frac{1}{5}$$

Step 3: Calculate the probability of drawing a red marble followed by a yellow marble.

$$P(Red\ then\ Yellow) = P(Red) \times P(Yellow) = \frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$$

(c) Probability that at least one marble is red

Step 1: Calculate the probability that neither marble is red (i.e., both are blue or yellow).

$$P(Not\ Red) = P(Blue\ or\ Yellow) = \frac{1}{2}$$

$$= P(Both\ Not\ Red) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Step 2: Calculate the probability that at least one marble is red.

$$P(At\ Least\ One\ Red) = 1 - P(No\ Red)$$

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

Example 7

You flip a fair coin twice. What is the probability that:

(a) Both flips result in heads?

(b) The first flip is heads, and the second flip is tails?

(c) At least one flip results in heads?

Solution

(a) Probability that both flips result in heads

Step 1: Define the probability of getting heads on a single flip.

$$P(Heads) = \frac{1}{2}\ (since\ the\ coin\ is\ fair)$$

Step 2: Calculate the probability of getting heads on both flips.

$$P(Both\ Heads) = P(Heads) \times P(Heads) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

(b) Probability that the first flip is heads and the second flip is tails

Step 1: Define the probability of getting heads on the first flip.

$$P(Heads) = \frac{1}{2}$$

Step 2: Define the probability of getting tails on the second flip.

$$P(Tails) = \frac{1}{2}$$

Step 3: Calculate the probability of getting heads on the first flip and tails on the second flip.

$$P(Heads\ then\ Tails) = P(Heads) \times P(Tails) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

(c) Probability that at least one flip results in heads

Step 1: Calculate the probability of getting tails on both flips.

$$P(Both\ Tails) = P(Tails) \times P(Tails) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Step 2: Calculate the probability of getting at least one head.

$$P(At\ Least\ One\ Head) = 1 - P(Both\ Tails) = 1 - \frac{1}{4} = \frac{3}{4}$$

Alternatively, you can enumerate all possible outcomes:

{HH, HT, TH, TT}

There are 4 possible outcomes, and 3 of them have at least one head:

{HH, HT, TH }

So, P(At Least One Head) $= \dfrac{Number\ of\ favorable\ outcomes}{Total\ outcomes} = \dfrac{3}{4}$

Tree diagrams

A **tree diagram** in probability is a visual representation used to map out all possible outcomes of an event or series of events. It helps break down the probabilities step-by-step by showing all possible paths (branches) that can occur in each stage of an experiment.

A tree diagram is a useful tool to solve probability problems and they make things easier.

Watch the video on Probability - Tree Diagrams by clicking on the link below:

https://youtu.be/mkDzmI7YOx0

Example 8

A bag has 9 discs. 4 discs are red and 5 are blue. A disc is chosen at random, its colour noted and then replaced in the bag. Another disc is then chosen.

This information is represented on the tree diagram below. Can you see how it works?

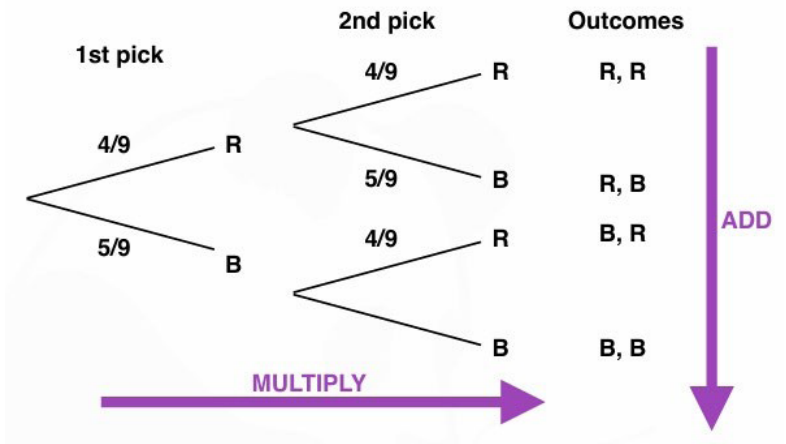

Figure 1: Tree diagram

Key ideas

- Probabilities go on the branches and outcomes at the end of each branch.
- The sum of the probabilities across all the branches must be 1.
- When going across the tree diagram, we multiply probabilities.

  E.g. $P\ (2\ red\ discs) = P(R, R)$

  $= P(R) \times P(R) = \frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$
- When going down the tree diagram, we add the probabilities.

  E.g. P ($same\ colour$) $= P(R, R) + P\ (B, B)$

  $= P(R) \times P(R) + P(B) \times P(B)$

  $= \frac{4}{9} \times \frac{4}{9} + \frac{5}{9} \times \frac{5}{9} = \frac{16}{81} + \frac{25}{81} = \frac{41}{81}$

Two common questions:

1. What is the probability of getting ***only one*** red disc?

Solution

To get one red disc, we could choose a red disc first and then a blue disc or we could choose a blue disc and then a red disc.

$= P(only\ 1\ red\ disc) = P(R, B) + P(B, R)$

$= P(R) \times P(B) + P(B) \times P(R)$

$= \frac{4}{9} \times \frac{5}{9} + \frac{5}{9} \times \frac{4}{9} = \frac{20}{81} + \frac{20}{81} = \frac{40}{81}$

N.B. This is the same as finding the probability of getting different colours.

2. What is the probability of getting at least one blue disc?

Solution

"At least one blue disc" is the complementary event of "no blue discs". "No blue discs" means that two red discs have been chosen

$P(at\ least\ 1\ blue\ disc) = 1 - \text{P(no blue discs)} = 1 - \text{P(2 red discs)} = 1 - \text{P(R, R)}$

$= 1 - \frac{4}{9} \times \frac{4}{9} = 1 - \frac{16}{81} = \frac{65}{81}$

N.B. You could also do P(at least 1 blue disc) = P(1 blue disc) + P(2 blue discs)

Real-life Problems of Probabilities of independent events

Example 9

A message is transmitted from Node-A to Node-B through three independent, intermediate nodes. The message will be successfully transmitted only if all the intermediate nodes are working. The probability that an intermediate node will fail is 1%. All nodes are independent of each other. What is the probability that the message will not be successfully transmitted?

Solution

We first find the probability that the message will be successfully transmitted. For successful transmission, we need all nodes to be working. The probability that a node will not fail is P(Node does not fail) = 1– P(Node fails) = 1 – 0.01 = 0.99.

Since all the nodes are independent, the probability that node 1 AND node 2 AND node 3 do not fail = 0.99 × 0.99 × 0.99 = 0.970299

Accordingly, P(message is not successful) = 1 – P(message is successful)

= 1– 0.970299 = 0.029701 ≈ **3%.**

Example 10

A retail store sells two types of products: electronics and clothing. The store manager wants to analyse the sales data to understand the relationship between the sales of these two product categories.

The store's sales data for the last month shows the following:

- The probability of a customer buying an electronic product is 0.3.
- The probability of a customer buying a clothing product is 0.4.

Assume that the purchases of electronic and clothing products are independent events.

Question: What is the probability that a randomly selected customer will buy both an electronic product and a clothing product?

Solution

To solve this problem, we need to use the concept of the probability of independent events.

The probability of two independent events occurring together is the product of their individual probabilities.

Let's define the events:

- E: A customer buys an electronic product
- C: A customer buys a clothing product

Given information:

- P(E) = 0.3 (the probability of buying an electronic product)
- P(C) = 0.4 (the probability of buying a clothing product)

We want to find the probability of a customer buying both an electronic product and a clothing product, which is P(E and C).

Since the purchases of electronic and clothing products are independent events, we can use the multiplication rule for independent events:

P(E and C) = P(E) × P(C) P(E and C) = 0.3 × 0.4

∴ P(E and C) = 0.12

Therefore, the probability that a randomly selected customer will buy both an electronic product and a clothing product is 0.12 or **12%**.

Example 11

A patient undergoes two independent medical tests, test A and test B:

- Test A has a 90% accuracy rate.
- Test B has an 85% accuracy rate.

What is the probability that:

(a) Both tests yield accurate results?

(b) At least one test yields an accurate result?

Solution

(a) Probability that both tests yield accurate results

Step 1: Define the probability of accurate results for each test.

$$\text{P(Accurate Test A)} = 90\% = \frac{90}{100} = \frac{9}{10}$$

$$\text{P(Accurate Test B)} = 85\% = \frac{85}{100} = \frac{17}{20}$$

Step 2: Since the tests are independent, multiply the probabilities.

$$P(Both\ Accurate) = P(Accurate\ Test\ A) \times P(Accurate\ Test\ B) = \frac{9}{10} \times \frac{17}{20}$$

$$= \frac{153}{200}$$

(b) Probability that at least one test yields an accurate result

Method 1: Direct Calculation

Step 1: Calculate the probability of inaccurate results for each test.

$$P(Inaccurate\ Test\ A) = 1 - P(Accurate\ Test\ A) = 1 - \frac{9}{10} = \frac{1}{10}$$

$$P(Inaccurate\ Test\ B) = 1 - P(Accurate\ Test\ B) = 1 - \frac{17}{20} = \frac{3}{20}$$

Step 2: Calculate the probability that both tests yield inaccurate results.

$$\text{P(Both Inaccurate)} = \text{P(Inaccurate Test A)} \times \text{P(Inaccurate Test B)}$$

$$= \frac{1}{10} \times \frac{3}{20} = \frac{3}{200}$$

Step 3: Calculate the probability that at least one test yields an accurate result.

$$P(At\ Least\ One\ Accurate) = 1 - P(Both\ Inaccurate) = 1 - \frac{3}{200} = \frac{197}{200}$$

Method 2: Using Complementary Probability

Step 1: Calculate the probability that both tests yield inaccurate results.

$$P(\text{Both Inaccurate}) = (1 - \frac{9}{10}) \times (1 - \frac{17}{20}) = \frac{1}{10} \times \frac{3}{20} = \frac{3}{200}$$

Step 2: Calculate the probability that at least one test yields an accurate result.

$$P(\text{At Least One Accurate}) = 1 - P(\text{Both Inaccurate}) = 1 - \frac{3}{200} = \frac{197}{200}$$

Example 12

Two flights, one from Korkorse to Chinderi and another from Chinderi to Boare are independently scheduled.

The probability of a delay for the first flight is 20% and the probability of delay for the second flight is 15%.

What is the probability that:

(a) Both flights are delayed?

(b) At least one flight is delayed?

Solution

(a) Probability that both flights are delayed:

Step 1: Define the probability of a delay for each flight.

$$P(\text{Delayed Flight 1}) = 20\% = \frac{20}{100} = \frac{1}{5}$$

$$P(\text{Delayed Flight 2}) = 15\% = \frac{15}{100} = \frac{3}{20}$$

Step 2: Since the flights are independent, multiply the probabilities.

$$P(\textit{Both Delayed}) = P(\textit{Delayed Flight 1}) \times P(\textit{Delayed Flight 2}) = \frac{1}{5} \times \frac{3}{20}$$

$$= \frac{3}{100}$$

(b) Probability that at least one flight is delayed:

Step 1: Calculate the probability of no delay for each flight.

$$P(\textit{On-Time Flight 1}) = 1 - P(\textit{Delayed Flight 1}) = 1 - \frac{1}{5} = \frac{4}{5}$$

$$P(\text{On-Time Flight 2}) = 1 - P(\text{Delayed Flight 2}) = 1 - \frac{3}{20} = \frac{17}{20}$$

Step 2: Calculate the probability that both flights are on time.

$$P(\text{Both On-Time}) = P(\text{On-Time Flight 1}) \times P(\text{On-Time Flight 2})$$

$$= \frac{4}{5} \times \frac{17}{20} = \frac{68}{100}$$

Step 3: Calculate the probability that at least one flight is delayed.

$$P(\text{At Least One Delayed}) = 1 - P(\text{Both On-Time}) = 1 - \frac{68}{100} = \frac{32}{100}$$

REVIEW QUESTIONS

1. Determine which of the following are examples of independent events.
   a. Rolling a 5 on one die and rolling a 5 on a second die.
   b. Randomly picking a cookie from the cookie jar and picking a jack from a deck of cards.
   c. Winning a hockey game and scoring a goal.
2. Determine which of the following are examples of independent events.
   a. Choosing an 8 from a deck of cards, replacing it, and choosing a face card.
   b. Going to the beach and bringing an umbrella.
   c. Getting gasoline for your car and getting diesel fuel for your car.
3. A fair die is rolled once. List the sample space.
4. Two coins are tossed once. List the sample space for the experiment.
5. A dice and a coin are tossed once. List the sample space for the experiment.
6. Explain, with relevant examples, the meaning of probability of independent events.
7. Two cards are chosen from a deck of cards. The first card is replaced before choosing the second card. What is the probability that they both will be face cards?
8. If the probability of receiving at least 1 piece of mail on any particular day is 22%, what is the probability of not receiving any mail for 3 days in a row?
9. Johnathan is rolling 2 dice and needs to roll a total of exactly 11 to win the game he is playing. What is the probability that Johnathan wins the game?
10. While driving to work, Sarah passes through two sets of traffic lights. The probability the first set is green is 0.3 and the probability the second is green is 0.4.

    Draw a tree diagram to represent this.

    Find the probability that:

    i. neither set of lights is green

ii. only one set of traffic lights is green

iii. at least one set of traffic lights is green

11. In Tania's homeroom class, 9% of the students were born in March and 40% of the students have a blood type of O+. What is the probability of a student chosen at random from Tania's homeroom class being born in March and having a blood type of O+ if the two events are independent events?

12. What is the probability of tossing 2 coins in any order one after the other and getting 1 head and 1 tail?

13. Joseph and David are playing with cards in a pack of 52 cards. Joseph draws a card at random then replaces it, and draws another card. Then he asks David what is the probability of drawing a queen followed by a king?

14. A juggler has seven red, five green and four blue balls. During his performance, he accidentally drops a ball and then picks it up. As he continues, another ball falls. What is the probability that the first ball that was dropped is blue, and the second ball is green?

15. In a survey, a company found that 6 out of 10 people eat pizza. If three people are chosen at random with replacement, what is the probability that all 3 people eat pizza?

GLOSSARY

- **Probability:** The likelihood or chance of an event happening.
- **Independent Events:** Two or more events where the outcome of one does not affect the outcome of the other(s).
- **Event:** A possible outcome or occurrence in a probability experiment.
- **Sample Space:** The set of all possible outcomes in a probability experiment.
- **Mutually Exclusive Events:** Events that cannot happen at the same time.
- **Complementary Events:** Two events where one must happen if the other does not.
- **Probability Formula for Independent Events:**
  $P(A \text{ and } B) = P(A) \times P(B)$ where A and B are independent events.
- **Outcome:** The result of a single trial of an experiment.
- **Trial:** A single occurrence in a probability experiment.
- **Experiment:** A process that leads to the occurrence of one or more outcomes.
- **Theoretical Probability:** Probability based on reasoning or calculations (e.g., flipping a fair coin).
- **Experimental Probability:** Probability based on actual experiment results or trials.
- **Intersection of Events:** The event that occurs if both events happen simultaneously.

EXTENDED READING

- Akrong Series: Core mathematics for Senior High Schools New International Edition (Pages 612 – 641)
- Aki – Ola series : Core Mathematics for Senior High Schools in West Africa, Millennium edition 5 (Pages 259– 266)
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ACKNOWLEDGEMENTS

Ghana Education Service (GES)

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