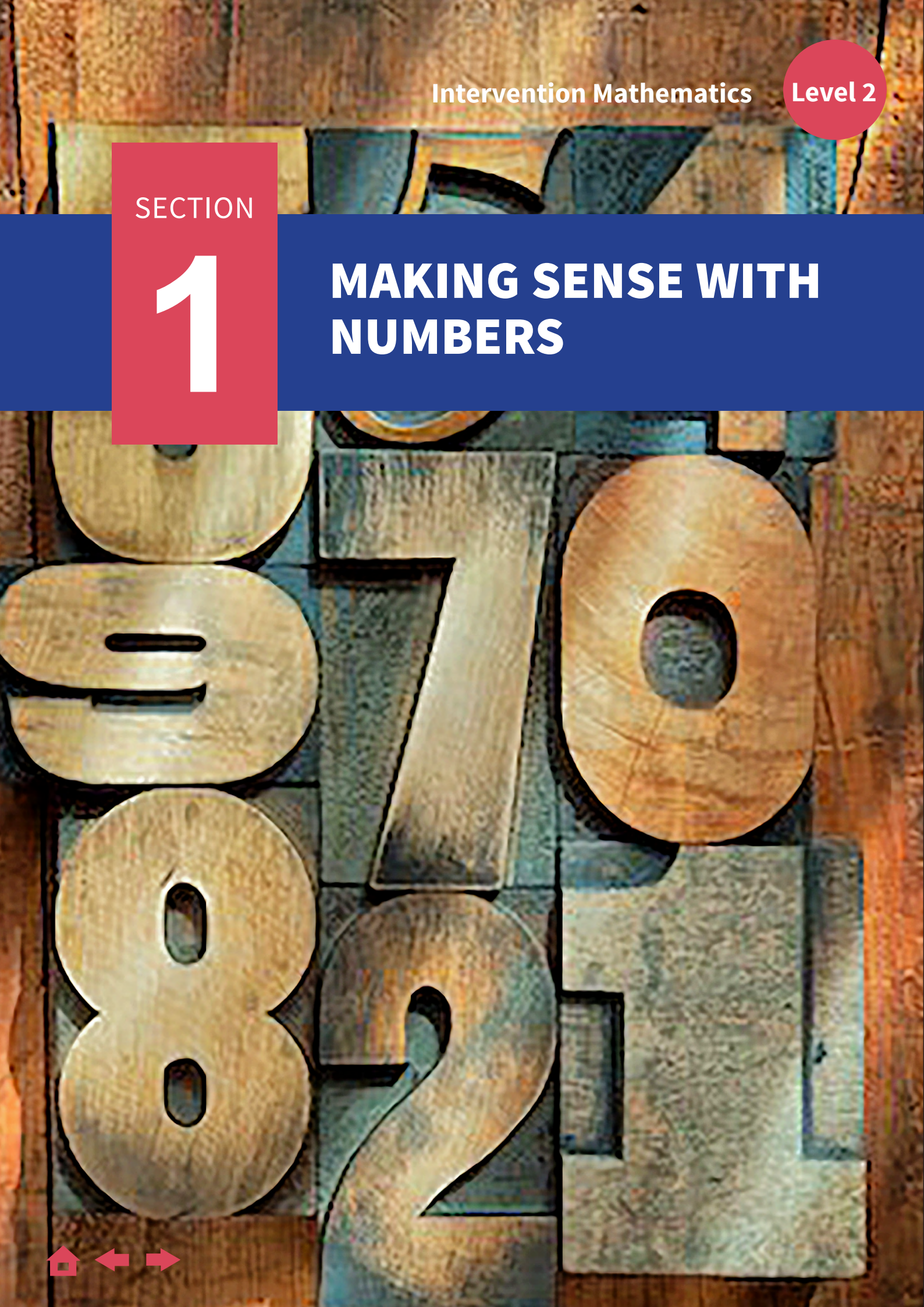
SECTION

1

MAKING SENSE WITH NUMBERS



NUMBERS FOR EVERYDAY LIFE

REAL NUMBER AND NUMERATION SYSTEM

In this section, you will learn to;

1. *Categorise real numbers as natural/counting numbers, whole numbers, integers and rational and irrational numbers*
2. *Explore the various subsets of counting numbers (even and odd, prime and composite)*
3. *Perform operations on the set of real numbers using real-life contexts.*

SECTION INTRODUCTION

You will learn to categorise real numbers into different groups, such as natural or counting numbers, whole numbers, integers, rational numbers and irrational numbers. This will help you understand how numbers are organised and related to each other. You'll also explore the various subsets of counting numbers, including even and odd numbers, as well as prime and composite numbers, gaining a deeper understanding of the different types of numbers and their characteristics. This knowledge will enhance your ability to work with numbers in various mathematical contexts.

REAL NUMBER AND NUMERATION SYSTEM

FOCAL AREA 1: THE REAL NUMBER SYSTEM

Imagine you're planning a road trip with your family. You need to calculate the total distance you'll drive, estimate how much fuel you'll need and determine the cost of fuel for the entire journey. You also want to track your speed, time and the miles left to your destination. To make these calculations accurately, you'll need to understand different types of numbers and how they work together. This is where the concept of the **real number system** becomes essential.

The real number system includes all the numbers we use in everyday life: the whole numbers when counting the miles on the road, the fractions or decimals when measuring fuel consumption, and even the negative numbers when calculating changes in elevation or loss in distance when you take a wrong turn.

Understanding the real number system allows you to make sense of all these different kinds of numbers and how they can be used together to solve problems. Before we go further into the real number system, let's have fun with this activity!

Reinforcement Activities

Exploring Everyday Numbers

Purpose: To help you recall and categorise different types of numbers you encounter in everyday life, preparing you for understanding the real number system.

Materials Needed:

- Index cards or small pieces of paper
- Markers or pens
- A large chart or board with four columns labeled: **Whole Numbers**, **Fractions/Decimals**, **Positive/Negative Numbers**, **Special Numbers (e.g., π, $\sqrt{2}$)**

Instructions:

1. **Group Formation:** Make yourselves into small groups of 3-4 students.
2. **Identifying Numbers:** Each group will receive a set of index cards or pieces of paper. On each card, you will write down different types of numbers you encounter in real-life situations. Think of examples such as:
    - **Whole Numbers:** The number of people in a room, the number of books on a shelf.
    - **Fractions/Decimals:** The amount of a half-eaten loaf of bread, the price of an item in cedis and pesewas.
    - **Positive/Negative Numbers:** A temperature above or below zero, a bank account balance.
    - **Special Numbers:** The value of π in calculations, the square root of 2 when measuring diagonal lengths.
3. **Categorising the Numbers:** After writing down your examples, each group will come up to the large chart or board and place their cards under the appropriate category: **Whole Numbers**, **Fractions/Decimals**, **Positive/Negative Numbers**, or **Special Numbers**.

4. **Class Discussion:** Once all the cards are placed, you can have a class discussion about the different types of numbers.

   Ask yourselves questions like:

   - What are the similarities and differences between the numbers in each category?
   - Why do we need different types of numbers for different situations?
   - How do these different types of numbers help us in our daily lives?

5. **Transition to the Real Number System:**

Note that all these different types of numbers are part of a larger system called the **real number system**.

THE REAL NUMBER SYSTEM

The **Real Number System** is a way of categorising and understanding all the numbers we use in everyday life. It includes different sets of numbers, each with its unique properties. The real number system is made up of:

1. **Natural Numbers (or Counting Numbers) (N):** These are the numbers we use for counting things. They start from 1 and go up without end. **Example:** Counting the number of students in a classroom (e.g., 1, 2, 3, ...).
2. **Whole Numbers (W):** These are the natural numbers, plus zero. **Example:** Counting the number of apples in a basket, including the possibility of having none (e.g., 0, 1, 2, 3, ...).
3. **Positive and negative whole numbers [Integers]:** These include all whole numbers and their negatives.

   Examples

   a. The temperature in Aburi yesterday was 20°C. In the evening the temperature dropped by 23°C. This means the current temperature will be below 0°C that is −3°C.

   b. If a student is to pay a fee of GH¢ 475.00 but paid GH¢ 470.00 instead, how much is left to be paid? The student now owes GH¢ 5.00. This amount can be written as GH¢ (–5.00). These numbers (–3) and (–5) are called negative numbers.

      Positive and negative whole numbers and zero are together called Integers (Z). Example: {…, –4. –3. –2. –1, 0, 1, 2, 3, 4, …}

Real world examples

Negative Numbers	Positive Numbers
Owing	Owning
Price decrease	Price increase
Temperature below freezing point (0^oC)	Temperature above freezing point (0^oC)

4. **Rational Numbers (Q):** These are numbers that can be expressed as a fraction or ratio of two integers, where the denominator is not zero. Rational numbers include integers, fractions and terminating or repeating decimals.

   Examples:

   - **Fractions:** Splitting bread into 4 equal parts and taking 3 pieces, which is $\frac{3}{4}$ of the bread.
   - **Decimals:** The price of an item, like GHc2.50, which can also be written as $\frac{5}{2}$.
   - **A set of rational numbers could include** $\left\{ -4, 3, -2.25, 1, 0, 15.2, \frac{2}{3}, 73 \right\}$

5. **Irrational Numbers (Q^1):** These are numbers that cannot be expressed as a fraction of two integers. Their decimal form goes on forever without repeating.

   Example:

   - $\sqrt{2}$ **(Square Root of 2):** The diagonal length of a square with sides of 1 unit.
   - π **(Pi):** The ratio of the circumference of a circle to its diameter, often used in measuring circles.

Note This!

Any number that does not terminate or recur is called an irrational number.

Any natural number under square root with the exception of perfect squares (1, 4, 9, 16, 25, etc.) are irrational numbers.

Example

$\sqrt{2} = 1.414213562373095048801688724209 7\ldots$

$\sqrt{8} = 2.828427124746190097603377448419 4\ldots$

The real value of pi is an irrational number. That is,

$\pi = 3.141592653589793238462643383279 5\ldots$

6. **Real Numbers:** This is the set that includes all the numbers mentioned above—natural numbers, whole numbers, integers, rational numbers and irrational numbers. Real numbers can be represented on a number line.

   Example: Any number you can think of in everyday life is a real number, such as 7, -3, $\frac{2}{5}$, 0.333..., $\sqrt{5}$, or π.

Real-Life Examples of Each Set:

- **Natural Numbers:** Counting the number of steps you take from your house to school.
- **Whole Numbers:** The number of pages in a book, including the possibility of zero pages if the book hasn't been written yet.
- **Integers:** Tracking temperature changes throughout the day, where temperatures can go below freezing (negative numbers) or above (positive numbers).
- **Rational Numbers:** Measuring ingredients for a recipe, such as $\frac{1}{2}$ cup of sugar.
- **Irrational Numbers:** Calculating the distance around a circular garden using π, or determining the diagonal of a square plot of land using $\sqrt{2}$.
- **Real Numbers:** Calculating your total spending in a week, which might include positive amounts (money earned) and negative amounts (money spent).

ACTIVITY 1.1: Individual/Pair/Group Work

Creating Models of the Real Number System

Purpose: In this activity, you will develop your own models to represent the different sets of numbers in the real number system. By the end, you will have a visual understanding of how these sets of numbers are related.

Materials Needed:

- Poster paper or large sheets of paper
- Markers, pencils, and coloured pencils
- Rulers
- Scissors
- Glue or tape
- Index cards or small pieces of paper

Instructions:

1. **Form Groups:**
   - Get into small groups of 3-4 students.
2. **Introduction to the Real Number System:**
   - On a sheet of paper, write down the five main sets in the real number system: Natural Numbers, Whole Numbers, Integers, Rational Numbers, and Irrational Numbers.
3. **Brainstorm and Plan:**
   - Discuss in your group how you can represent each set visually. Think about using shapes, diagrams, or even a layered model.
   - Consider how these sets relate to each other. For example, how are natural numbers a part of whole numbers? How do rational and irrational numbers together form real numbers?
4. **Create Your Model:**
   - Use the poster paper to draw and create your model.
   - Label each section clearly with the type of number it represents.
   - Add examples of numbers in each section (e.g., 1, 2, 3 for natural numbers; 0 for whole numbers; -5 for integers; $\frac{1}{2}$ for rational numbers; π for irrational numbers).

- Use different colours to represent each set of numbers to make your model clear and easy to understand.

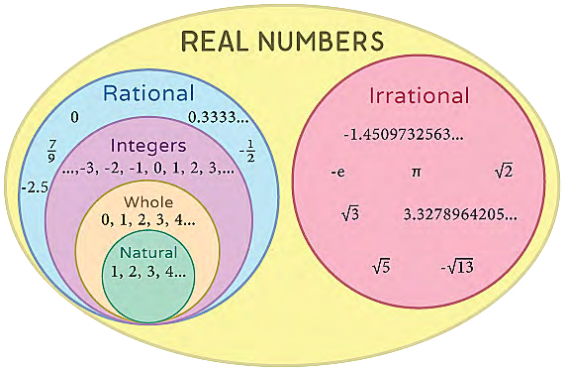
5. **Add Real-Life Examples:**
   - On index cards or small pieces of paper, write down real-life examples of where you might encounter each type of number. For example, you might write "counting money" for natural numbers or "temperature in winter" for integers.
   - Attach these examples to your model in the appropriate sections.
6. **Present Your Model:**
   - Once your model is complete, each group will present their model to the class.
   - Explain why you chose your design and how it helps to understand the relationships between the different sets of numbers.
7. **Class Discussion:**
   - After all the presentations, discuss, as a class, how each model helped you understand the real number system. What similarities did you notice across the different models? Were there any unique approaches that stood out?
8. **Reflection:**
   - Individually, write a short paragraph reflecting on what you learnt about the real number system through this activity. How does visualising these concepts help you understand them better?

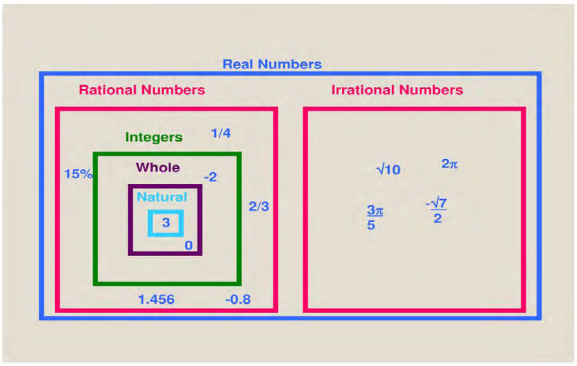
Extension Activity

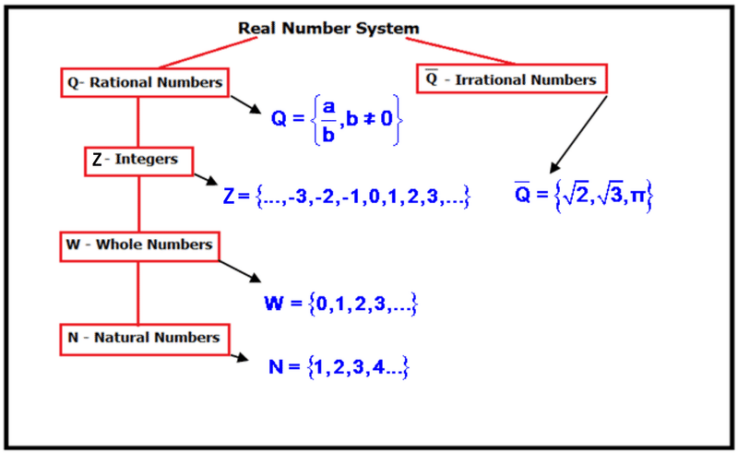
- As a group, think of another creative way to represent the real number system. This could be a 3D model, a digital presentation, or even a story that explains the relationships between the numbers.

CATEGORIES OF THE REAL NUMBER SYSTEM USING MODELS

We can use different models to show or represent the relationships between and among the subsets, of real numbers. Examples of models include Venn diagrams, number lines, etc. Below are some examples:







All these models can be used to represent the real number system.

Note This!

When listing the members of the subsets, be careful not to miss any of them. For example, when listing the members of integers, remember to include all the numbers under whole numbers and natural numbers (counting numbers). This is because both whole numbers and natural numbers are subsets of integers.

FOCAL AREA 2: EXPLORING SUBSETS OF COUNTING NUMBERS (EVEN AND ODD, PRIME AND COMPOSITE)

Imagine you're helping your parents organise a party. You need to set up tables and chairs for all the guests. Each table should have an even number of chairs so that everyone can easily find a seat. But as you're counting the chairs, you realize something: some numbers of chairs work perfectly for this, while others don't. You start to wonder why this happens.

This situation introduces the idea of **even and odd numbers**. Even numbers can be perfectly divided into two equal groups without any leftovers, making them ideal for setting up chairs evenly around a table. On the other hand, odd numbers leave one chair out, which might cause problems when everyone tries to sit down.

Later, as you're planning the games for the party, you realize that some games work best with certain numbers of players. For instance, some games require exactly 2, 3, or 5 players to work perfectly. This leads you to think about **prime numbers**, which are numbers that can only be divided by 1 and themselves. These numbers are unique because they can't be evenly divided into smaller groups, just like some games can't be played with any number of players.

Then there are numbers that can be split into smaller groups in several different ways. These are **composite numbers**—numbers that have more than two factors. Understanding these can help you figure out how to divide resources or organise teams efficiently, not just for games but for any situation where grouping is necessary.

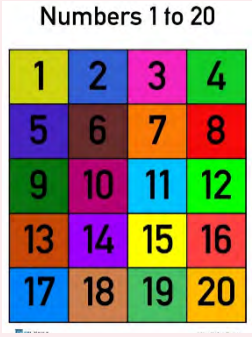
Reinforcement Activities

Exploring Divisibility

Purpose: To help you understand the concept of divisibility, which will lead to a better understand of even, odd, prime, and composite numbers.

Materials Needed:

- A set of counters, buttons, or small objects (at least 30 per group)
- Paper and pencils
- A chart with the numbers 1-20 written on it



Instructions:

1. **Grouping Activity:**
   - Get yourselves into small groups and each group will need a set of 30 counters.
   - Within your group pick any number between 1 and 20 from the chart.
   - Once you have chosen your number, try and divide the number of counters you have picked into two equal groups.
2. **Discussion Questions:**
   - Was it possible to divide your number into two equal groups? (This should remind you about the concept of even and odd numbers.)
   - Now, try dividing your number into three equal groups. Was it possible?
   - Continue the exercise with dividing into four, five, and six equal groups.
3. **Recording Results:**
   - Record the numbers that could be evenly divided into two equal groups (even numbers) and those that could not (odd numbers).
   - Then record the numbers which could only be divided by 1 and the number itself without leaving any objects leftover. (These form a special set of numbers called prime numbers.)

- Lastly, record the numbers that could be divided into more than two different group sizes.

(These form the set of numbers called composite numbers.)

4. Class Discussion:

- Come together as a class to discuss your findings.
- Share which numbers could be divided into equal groups and which could not.
- Discuss your idea that even numbers can always be divided into two equal groups, while odd numbers cannot.
- Discuss the special numbers, *prime numbers*, which have only two factors (1 and the number itself), while *composite numbers* have more than two factors.

SUBSETS OF COUNTING NUMBERS

1. **Even numbers** are whole numbers which are divisible by 2 without a remainder.

   Example; {0, 2, 4, 6, 8, 10, …}

2. **Odd Numbers** are whole numbers which give a remainder when divided by 2.

   Example; {1, 3, 5, 7, 9, …}

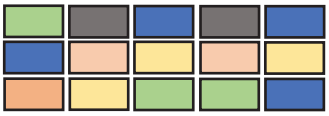
3. **Composite Numbers**

   Examples

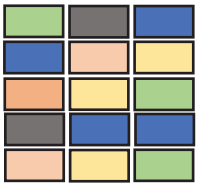
   a. Suppose you have a packet containing 12 toffees, and you intend to distribute them among your friends. You can divide these toffees evenly among 2, 3, 4, or 6 friends because 12 can be divided by these numbers without any remainder. Therefore, 12 is termed a composite number since it can be evenly divided into equal groups by numbers other than 1 and, 12, itself.

   b. Again, consider building blocks used to construct different structures. If you have 15 blocks, you can arrange them into rectangles of 1x15, 3x5, 5x3 or 15x1. These arrangements show that 15 is divisible by 3 and 5, besides 1 and 15, itself.

1 × 15



3 × 5



5 × 3

Therefore, 15 is a composite number because it has more than two factors.

Therefore, 12 and 15 are composite numbers.

Composite numbers are numbers with more than two factors. Example {4, 6, 8, 9, 10, 12, 14, 15, …}

4. Prime Numbers

You have a basket of apples, and you want to arrange them into rows or column. If you have 5 apples, you can only arrange them in a single row or column. You cannot form equal rows or columns other than 1 and 5 apples per row or column. This implies that 5 is a prime number because it has exactly two factors: 1 and itself, 5.

1 row, 5 columns

5 rows, 1 column

A prime Number is a number that has only two factors, that is 1 and itself.

Example {2, 3, 5, 7, 11…}

ACTIVITY 1.2: Individual/Pair/Group Work

Exploring Prime and Composite Numbers

Purpose: You will learn how to identify prime and composite numbers by exploring their factors.

Materials Needed:

- A list of numbers from 1 to 50
- A grid or chart (provided by your teacher) with numbers 1 to 50
- Coloured pencils or markers
- Paper and pencils

Number Chart 1-50

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Instructions:

1. **Understanding Factors:**
   - A *factor* is a number that divides into another number exactly (without leaving a remainder).
   - A *prime number* has exactly two factors: 1 and itself.
   - A *composite number* has more than two factors.
2. **Marking Prime and Composite Numbers:**
   - Look at the list of numbers from 1 to 50.
   - For each number, list all the factors that can divide it exactly. For example:
     - For 6, the factors are 1, 2, 3, and 6.
     - For 7, the factors are 1 and 7.
   - Use a red pencil to circle numbers that have exactly two factors (these are prime numbers).
   - Use a blue pencil to circle numbers that have more than two factors (these are composite numbers).
3. **Colouring the Grid:**
   - On your grid/chart, colour in all the prime numbers in red.
   - Colour in all the composite numbers in blue.
   - Leave the number 1 uncoloured, as it is neither prime nor composite.

4. **Classifying Numbers:**
   - On a sheet of paper, create two columns: "Prime Numbers" and "Composite Numbers."
   - Write down all the prime numbers in the first column and all the composite numbers in the second column.
5. **Class Discussion:**
   - Compare your grid with your classmates.
   - Discuss any differences in your charts and why some numbers are prime or composite.
   - Share your findings with the class and explain how you identified the prime and composite numbers.
6. **Challenge Question:**
   - Is the number 51 prime or composite? Try to list its factors and determine its classification.

INTEGERS AND OPERATIONS ON INTEGERS (REVISION)

FOCAL AREA 1: ADDITION AND SUBTRACTION OF INTEGERS

Introduction

In this focal area we will explore the basic arithmetic processes of addition and subtraction involving integers. We will also delve into the rules and techniques in solving real life problems.

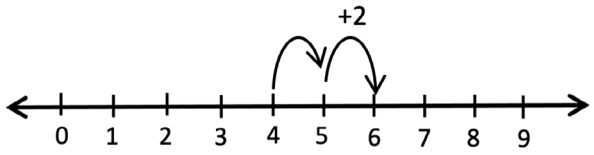
The result of adding two or more numbers is a **sum,** and the result of subtracting a number from another is the **difference.**

Using the number line to performs operations on integers

Addition represents moving towards the right. Subtraction represents moving towards the left.

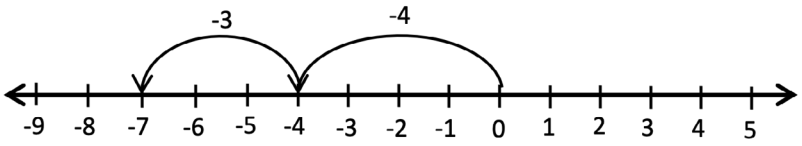
Examples

a. Kwame moved +4 steps from the starting point at 0 and then proceeded to take an additional +2 steps. What is the total number of steps he took?



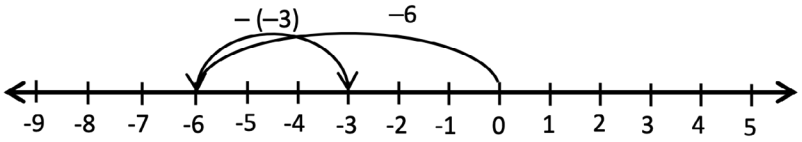
$$4 + 2 = 6$$

b. Alima borrowed *GH¢* 4.00 from her friend to buy food. She borrowed an extra *GH¢* 3.00 later, how much does she owe?



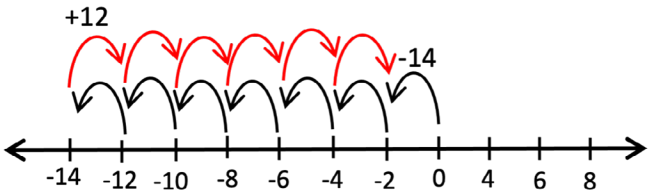
$$-4 + -3 = -7$$

c. Mawuli borrowed GH¢ 6.00 from Esi, and later returned GH¢ 3.00. The expression (–6) – (–3) which is (–6) + (3) models this situation.



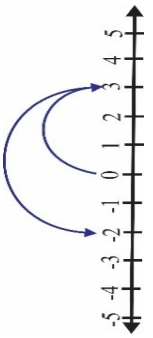
$$(–6) – (– 3) = – 3$$

d. Mensima has a jar of toffees, initially, she had 14 fewer toffees than she needed for a party. Kwesi gives her an additional 12. The expression (– 14) + 12 models this situation.



$$(–14) + 12 = (– 2)$$

e. Find $3 + (-5)$

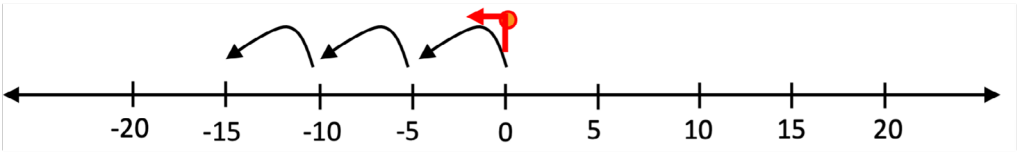


$3 + (-5) = -2$

FOCAL AREA 2: MULTIPLICATION AND DIVISION OF INTEGERS

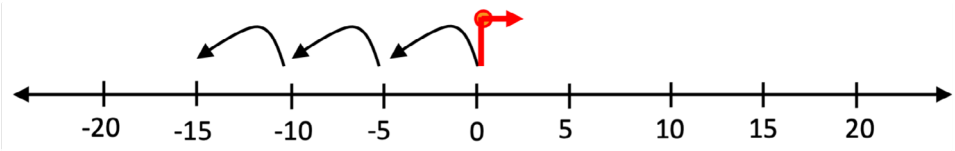
Example:

1. Evaluate -5×3



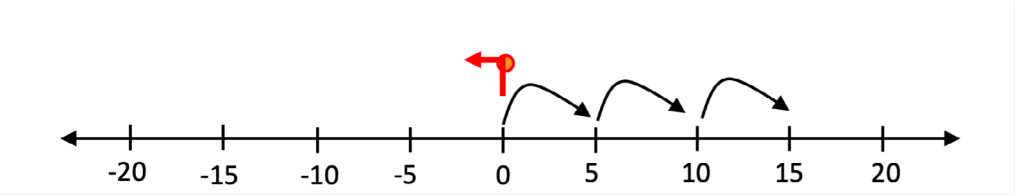
The negative attached to the number 5 indicates the direction you will face at the origin as shown in the diagram above. The 3 indicates the size of the movement from the origin in that direction. This implies that, there is a jump of each one of 5, three times, from the origin. Therefore, $-5 \times 3 = -15$.

2. Evaluate 5×-3



In the second example, the number 5 is positive and it indicates the direction you will face at the origin as shown in the diagram above. The number 3 is now negative, and it indicates the direction of the movement from the origin. This implies that, there is a jump of each one of 5 backward (negative direction) from the origin. Therefore, $5 \times -3 = -15$.

3. Evaluate -5×-3



In the third example, the number 5 is negative and it indicates the direction you will face at the origin as shown in the diagram above. The number 3 is also negative, and it indicates the direction of the movement from the origin. This implies that, there is a jump of each one of 5 backwards (negative direction) from the origin. Therefore, $-5 \times -3 = 15$.

4. If the cost of a notebook is GH¢ 13.00. How much would you pay for, if you buy 16 of them?

Solution

$13 \times 16 = 208$

From the above illustrations, the following rules of multiplication can be deduced.

Rules of multiplication

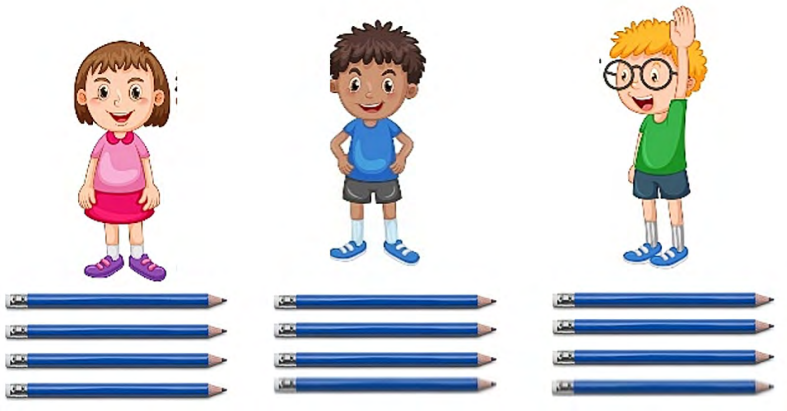
In multiplication, the sign of the product depends on the signs of the numbers being multiplied.

- Positive x Positive = Positive (e.g., $3 \times 4 = 12$)
- Negative x Negative = Positive (e.g., $-5 \times -2 = 10$)
- Positive x Negative = Negative (e.g., $2 \times -7 = -14$)
- Negative x Positive = Negative (e.g., $-2 \times 7 = -14$)

Division of Integers

Remember that division is the process of sharing or distributing quantities into smaller groups.

a. Kofi is to share 12 pencils among his three friends. How many pencils will each receive?



From the illustration, each friend receives 4 pencils. Therefore, $12 \div 3 = 4$

b. Your class is going on a field trip and the bus can hold 36 passengers There are 108 students in your class. How many buses will be needed to transport all the students?

Solution

This is a division problem: $108 \div 36 = 3$

c. A household owes electricity bill of GH¢450.00. This amount was shared among three members equally, how much will each tenant pay?

Solution

Since this is a debt, the amount owe will be written as – 450. Therefore, each member will pay an amount of $-450 \div 3 = -150$, which is a debt and can be written as – 150.

d. Again, you owe a friend GH¢30, and you owe another friend GH¢10. "What happens if you divide your total debt (–GH¢40) equally between these two friends?" To divide the total debt (–GH¢40) by the number of friends (–2) gives a positive result, indicating each friend receives a share of the debt.

Mathematically, $-40 \div -2 = 20$.

Rules of division

In division, the sign of the division depends on the signs of the numbers being divided.

- Positive ÷ Positive = Positive (e.g., 12 ÷ 4 = 3)
- Negative ÷ Negative = Positive (e.g., –20 ÷–2 = 10)
- Positive ÷ Negative = Negative (e.g., 28 ÷ –2 = –14)
- Negative ÷ Positive = Negative (e.g., −28 ÷ 2 = –14)

REVIEW QUESTIONS 1.1

1. Classify the following numbers as Natural, Whole, Integers, Rational, or Irrational (some numbers may belong to more than one category):

a) 7

b) 0

c) –3

d) 2.5

e) $\sqrt{2}$

2. Determine whether each number is Rational or Irrational:

a) $\frac{2}{5}$

b) 3.14159...

c) $\sqrt{25}$

d) 1.414213...

e) -7

3. Which of the following numbers are Irrational?

a) 5.25

b) $\sqrt{3}$

c) π

d) $\frac{3}{7}$

e) $-\sqrt{2}$

4. Identify the smallest set under the Real Number System to which each number belongs:

a) 15

b) – 8

c) $\frac{1}{4}$

d) $-\sqrt{5}$

e) 6

5. True or False:

a) All Natural numbers are Whole numbers.

b) All Irrational numbers are Real numbers.

c) Every Integer is a Rational number.

d) Some Rational numbers are Irrational.

e) The number 0 is a Natural number.

6. Identify whether each number is Prime or Composite:

a) 11

b) 15

c) 29

d) 36

e) 47

7. Identify whether each number is Even or Odd:

a) 22

b) 37

c) 44

d) 53

e) 68

8. Classify the following numbers as Odd, Even, Prime, or Composite (each number will have more than one classification):

a) 7

b) 16

c) 21

d) 23

e) 24

9. List all the Even numbers between 10 and 30, inclusive.

10. List all the Prime numbers between 20 and 50.

11. List all the Odd numbers between 1 and 15, inclusive.

ANSWERS TO REVIEW QUESTIONS 1.1

1. a) Natural, Whole, Integer, Rational
   b) Whole, Integer, Rational
   c) Integer, Rational
   d) Rational
   e) Irrational
2. a) Rational
   b) Irrational (π)
   c) Rational
   d) Irrational ($\sqrt{2}$)
   e) Rational
3. The Irrational numbers are $\sqrt{3}$, π, $-\sqrt{2}$)
4. a) Natural
   b) Integer
   c) Rational
   d) Irrational
   e) Natural
5. a) True
   b) True
   c) True
   d) False
   e) False
6. a) Prime
   b) Composite
   c) Prime
   d) Composite
   e) Prime

7. **a)** Even

b) Odd

c) Even

d) Odd

e) Even

8. **a)** Odd, Prime

b) Even, Composite

c) Odd, Composite

d) Odd, Prime

e) Even, Composite

9. 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30

10. 23, 29, 31, 37, 41, 43, 47

11. 1,3,5,7,9,11,13,15

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