Intervention Mathematics Level 2



# FRACTIONS AND OPERATIONS ON FRACTIONS



# NUMBERS FOR EVERYDAY LIFE Number Operations

#### In this section, you will learn to;

- **1.** Name, compare and order numbers expressed as a quotient of two integers where the denominator is not equal to zero.
- **2.** Recognise and name equivalent fractions using pictorial representations and number lines.
- **3.** Compare and order fractions with like denominators by using pictorial representations and >, < and =.
- 4. Solve problems on fractions involving the four basic operations.

# **SECTION INTRODUCTION**

In this section, you will learn to name, compare and order numbers expressed as a quotient of two integers (fractions), where the denominator is not zero. You will also recognise and name equivalent fractions using pictorial representations and a number line, helping you see how different fractions can represent the same value. Additionally, you will compare and order fractions with like denominators using pictorial representations and the symbols >, <, and =. This will enhance your understanding of fractions and improve your ability to work with them in different situations.

# **FRACTIONS AND ITS APPLICATIONS**

# **FOCAL AREA: CONCEPT OF FRACTIONS**

We estimate halves and quarters in everyday life. Sometimes we use them to compare or to describe something. For example,



Imagine you and three of your friends are sharing a large loaf of bread. The bread is cut into 8 equal slices. After everyone has taken a piece, there are still some pieces left. Now, you want to figure out how much of the bread is left and how much each person has eaten. This situation requires knowledge of **fractions**.

A fraction represents a part of a whole. In this case, the bread is the whole, and each piece represents a fraction of that whole. If you ate 2 pieces out of 8, you can say you ate  $\frac{2}{8}$  of the bread. Understanding fractions helps you to divide and share things equally, make decisions about portions and even compare amounts in different situations. Where else have you used fractions in everyday life?

Before we explore fractions further, let's enjoy this activity!

#### **REINFORCEMENT ACTIVITIES**

#### "Pizza Party Sharing"

**Objective:** Get ready to learn about fractions by sharing and dividing.

#### **Materials Needed:**

- Paper plates (one per group)
- Coloured pencils or markers
- Scissors
- A ruler

#### **Instructions:**



- Get yourselves into small groups. Each group will receive a paper plate. The plate to represent a pizza, or you can draw a large circle on a piece of paper to represent the pizza.
- Use a ruler to divide the plate/circle into 8 equal slices. These are your pizza slices.

#### 2. Colour the Slices:

• Colour each slice in a different colour. This will help you visualise each piece of the pizza.

#### 3. Sharing the Pizza:

- Imagine you and 3 friends (4 people in total) are going to share this pizza equally. Use the slices you drew to figure out how many slices each person would get.
- Write down how many slices each person will receive.

#### 4. Record Your Observations:

- On a sheet of paper, write down:
  - How many slices you have in total.
  - How many slices each person gets.
  - What fraction of the pizza each person gets?

#### 5. Discuss:

• Share your answers with the rest of the class. Discuss how the pizza was divided and what it means to share it equally.

# **The Concept of Fractions**

Assuming you fold a rectangular piece of paper into two equal parts. How many parts and folds are there? What do we call each part? How many halves are there in a whole?

If you fold the paper into two equal pieces; there will be 2 equal parts and 1 fold. Each part created by folding the paper is called "half" because it represents one of two equal parts. There are two halves in a whole.



If you now fold the halves of the paper strip into equal parts, how many parts will there be? What do we call each part? Show me two quarters. What is the other name for two quarters? Which is larger: one-half or one-quarter? How do you know, explain your answer?



If you now fold the halves of the paper strip into half: you will have four equal parts. Each part created by folding the paper is called a "quarter" because it represents one of four equal parts. There are two halves in a whole. To show two quarters, you would fold one half into 2 equal parts and then fold the other half into 2 equal parts. Therefore, two quarters would be represented by two of these resulting parts.



Another name for two quarters is "one half." This is because two quarters equals one half. You can see this by comparing the sizes: visually, two quarters will be the same size as one half when laid side by side. In this case,  $\frac{2}{4}$ (two quarters) is equivalent to  $\frac{1}{2}$  (one half). Therefore, one half is larger than one-quarter.

Assuming you have a chocolate bar with 8 equal pieces and you ate 1 piece, this means you have eaten one-eighth  $\left(\frac{1}{8}\right)$ .



Therefore, we can say that **fractions** represent part of a whole. Whenever we talk about fractions, we are talking about dividing a whole into smaller, equal parts.

# **Components of fractions**



Numerator represents the number of parts we have out of the whole.

**Denominator** represents the total number of equal parts into which the whole is divided.

# **NAMING FRACTIONS**

Based on the visual representations and the real-life activities above, learners should be able to use the language of fractions in naming and writing them correctly. The table below shows the names and visual representations of some fractions.

Fraction	Name of fraction	Visual representation of a fraction
$\frac{1}{2}$	One half	
$\frac{1}{3}$	One-third	
$\frac{1}{4}$	One-quarter or one-fourth	

Fraction	Name of fraction	Visual representation of a fraction
$\frac{3}{4}$	Three-quarters	
$\frac{2}{5}$	Two-fifths	
$2\frac{1}{4}$	Two and one- fourth	$\bigcirc \bigcirc \bigoplus$

### ACTIVITY 2.1: Individual/Pair/Group Work

### **Exploring Fractions with Paper Folding**

**Purpose:** Understand and identify fractions by folding paper and naming the fractions.

#### Materials Needed:

- Sheets of plain paper (two per student)
- Scissors
- Rulers
- Coloured pencils or markers

#### **Instructions:**

#### 1. Fold the Paper:

- Take a sheet of plain paper. Fold it in half.
  - **Fraction Name:** One-half  $(\frac{1}{2})$ .
- Unfold the paper and fold it again into quarters (fold it in half and then fold each half in half again).
  - **Fraction Name:** One-quarter  $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$  for each piece.
- Take a fresh piece of paper and fold it into thirds (fold the paper into three equal parts, ensuring each part is the same width).
  - **Fraction Name:** One-third  $(\frac{1}{3})$  for each piece.
- 2. Cut and Label:
  - Using scissors, carefully cut along the folds you made.

- Colour and label each fraction piece:
  - Label one half of the paper as  $\frac{1}{2}$ .
  - Label each quarter as  $\frac{1}{4}$ .
  - Label each third as  $\frac{1}{3}$ .

### 3. Combine Fractions:

- Take two quarters and put them together to see what fraction they form.
  - Fraction Name: Two-quarters or one-half  $(\frac{2}{4} = \frac{1}{2})$ .
- Take three quarters and combine them to see what fraction they form.
  - Fraction Name: Three-quarters  $(\frac{3}{4})$ .

#### 4. Create More Fractions:

- Take another piece of paper and fold it into fifths (fold the paper into five equal parts).
  - Fraction Name: One-fifth  $(\frac{1}{5})$  for each piece.
- Colour and label each fifth piece.

### 5. Construct Mixed Numbers:

- Using the pieces you created, combine whole pieces and fractions to form mixed numbers:
  - Example: Combine 2 whole pieces and <sup>1</sup>/<sub>4</sub> of a piece to represent 2 and <sup>1</sup>/<sub>4</sub>.

### 6. Discuss and Reflect:

- Share your folded and labeled paper pieces with the class.
- Discuss how the different folds represent different fractions and how combining pieces can form mixed numbers.

# **TYPES OF FRACTIONS**

# What Is Unit Fraction?

A fraction with the numerator 1  $(\frac{1}{2}, \frac{1}{5}, \frac{1}{13}, \text{etc.})$  is known as a unit fraction.

### **Example:**



All the fractions above have the numerator 1, so all of these are unit fractions.

# What Is Proper Fraction?

A fraction with its numerator smaller than its denominator is known as a proper fraction  $(\frac{1}{2}, \frac{3}{10}, \text{ etc.})$ .

### **Example:**



Here, 5 < 8, 3 < 8, 2 < 6

Each numerator is smaller than its denominator. Hence, all of these are proper fractions.

# What Is an Improper Fraction?

A fraction with its numerator greater than or equal to its denominator is known as an Improper fraction  $(\frac{5}{2}, \frac{7}{5}, \text{ etc.})$ .

### **Example:**



Each circle is divided into 8 parts, in which 11 parts are coloured.

Here, 11 > 8, so its numerator is greater than its denominator, making it an improper fraction.



Each hexagon is divided into 6 parts, in which 12 parts are coloured.

Here, 12 > 6, so its numerator is greater than its denominator, so it is another example of an improper fraction.

#### Note this!

All whole numbers are improper fractions. We can write all whole numbers in the form of a fraction like;  $4 = \frac{4}{1}, 8 = \frac{8}{1}, 1 = \frac{1}{1} (Taking denominator as 1)$ So, each denominator is either greater than 1 or equal to 1. Therefore, all whole numbers are improper fractions

# What Is a Mixed Fraction?

The sum of a whole number and a proper fraction is known as a mixed fraction.

#### **Example 1:**



2 is whole number and  $\frac{1}{8}$  is proper fraction.

In the above example, each circle is divided into 8 parts and there 17 parts are coloured. So, we can write above example in the form of an Improper fraction as:



#### Example 2:

Hilda's mum is sharing slices of 3 oranges for her and her siblings. She slices each orange into 4 equal parts.



If her siblings first had four slices of one orange and then she had one slice of the same size as the second orange. What is the total fraction of slices of oranges eaten?



In this case, the total oranges eaten are 5 slices of the orange which is one whole and 1 out 4 slices.  $(1 \ 1 \ 4)$  or  $(5 \ 4)$ .  $1 \ 1 \ 4$  is the mixed fraction or number and  $5 \ 4$  is its improper fraction.

# What Is a Like Fraction?

Fractions that have the same denominator are called like fractions.

### **Example:**



All fractions have the same denominator, 6. So, all of these are like fractions.

# **Unlike Fraction Definition:**

Fractions that have different denominators are called unlike fractions

# Example:



All fractions have different denominators. So, these are unlike fractions.

# What Is an Equivalent Fraction?

Fractions that have different numerators and denominators but their simplest form is the same are called equivalent fractions.

# Example:



All the above fractions are equivalent fractions as they represent half of the circle.

# Let's Summarise our ideas

Take a look at the picture below



# ACTIVITY 2.2: Individual/Pair/Group Work

### **Exploring Types of Fractions**

**Purpose:** Identify and classify different types of fractions: like, unlike, unit, proper, improper, and mixed fractions.

### **Materials Needed:**

- Paper plates or large circles cut from paper
- Markers or coloured pencils
- Rulers (optional)
- Scissors
- Glue or tape

#### **Instructions:**

- 1. Create Fraction Circles:
  - Take paper plates or a large circles cut from paper.
  - Divide them into equal parts using a ruler and marker. You can divide them into halves, thirds, quarters, fifths, etc.
  - Colour and label each section of your circle to represent different fractions (e.g.,  $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{3}{4}$ ).



#### 2. Classify the Fractions:

- Unit Fractions: Identify fractions where the numerator is 1 (e.g.,  $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ ). Colour and label these fractions.
- **Proper Fractions:** Identify fractions where the numerator is less than the denominator (e.g.,  $\frac{2}{3}, \frac{3}{4}$ ). Colour and label these fractions.
- Improper Fractions: Identify fractions where the numerator is equal to or greater than the denominator (e.g.,  $\frac{5}{4}, \frac{7}{4}$ ). Colour and label these fractions.
- Mixed Numbers: Represent mixed numbers by combining whole numbers with fractions (e.g.,  $1\frac{1}{2}$ ,  $2\frac{3}{4}$ ). Draw or use fraction circles to show these.

#### 3. Compare Fractions:

- Like Fractions: Find fractions with the same denominator and compare them (e.g.,  $\frac{1}{4}$  and  $\frac{2}{4}$ ). Use the circles to show that they have the same type of parts.
- Unlike Fractions: Find fractions with different denominators and compare them (e.g.,  $\frac{1}{3}$  and  $\frac{1}{4}$ ). Use your circles to show that these fractions represent different parts of a whole.

### 4. Create Your Own Fractions:

• Use the circles to create your own fractions. For example, divide a circle into 6 parts and colour 4 parts. Write the fraction it represents and identify its type (proper, improper, unit, etc.).

#### 5. Classify Your Fractions:

• Write down your created fractions and classify them into the different types you've learned about (like, unlike, unit, proper, improper, mixed).

#### 6. Presentation and Reflection:

• Share your fractions and classifications with the class. Discuss how different types of fractions can be represented and compared using the circles.

# **FOCAL AREA 2: EQUIVALENT FRACTIONS**

Imagine you are baking cookies for a big family gathering. The recipe you are using requires  $\frac{1}{2}$  cup of sugar, but you only have a  $\frac{1}{4}$  cup measuring spoon. To get the right amount of sugar, you'll need to use the  $\frac{1}{4}$  cup measuring spoon twice. This means  $\frac{1}{4}$  cup +  $\frac{1}{4}$  cup equals  $\frac{1}{2}$  cup. In other words,  $\frac{1}{2}$  cup and  $\frac{2}{4}$  cup are equivalent fractions—they represent the same quantity, just expressed in different ways.

# Why It's Important:

Understanding equivalent fractions is crucial in many real-life situations, especially when dealing with measurements in cooking, crafting, or sharing items equally. Knowing how to recognize and create equivalent fractions helps you:

- **1. Compare Fractions**: Determine if two fractions represent the same amount, which is essential for accurate measurements and comparisons.
- 2. Simplify Fractions: Convert fractions to their simplest form to make them easier to work with.
- **3. Perform Operations**: Add, subtract, multiply, or divide fractions accurately by ensuring they have the same denominator.

### **REINFORCEMENT ACTIVITIES**

### **Exploring Fraction Equivalence**

**Purpose:** To understand how different fractions can represent the same value by using visual aids.

### **Activity Instructions:**

- 1. Materials Needed:
  - Paper strips or cards (with different colours)
  - Scissors
  - Markers or coloured pens
  - Ruler

### 2. Step-by-Step Instructions:

- a. Create Fractions:
  - Working in small groups, each group will receive paper strips. Divide each strip into different equal parts (e.g., 2, 4, 6, and 8 parts).

• Use the ruler to ensure the divisions are equal. For example, if you have a strip and divide it into 4 equal parts, label them as  $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}$  and  $\frac{4}{4}$ .

### **b.** Colour and Cut:

- Colour different sections of each strip to represent fractions. For example, colour 2 out of 4 parts on one strip to represent  $\frac{2}{4}$ , and colour 1 out of 2 parts on another strip to represent  $\frac{1}{2}$ .
- Cut out these sections carefully.

#### c. Compare and Match:

• Match the coloured sections of different strips to see if they represent the same amount. For example, place the  $\frac{2}{4}$  strip next to the  $\frac{1}{2}$  strip and observe if they cover the same length.

#### d. Discussion:

- Discuss with your group whether the coloured sections you matched are equal in size.
- Write down your observations: Did you find that some fractions are equivalent? How can you tell?

### 3. Reflection:

- Present your findings to the class.
- As a class, discuss how different fractions can represent the same amount and why this is useful in comparing and simplifying fractions.

# **Showing Equivalent Fractions**

Let's take time to go through this scenario.



There are three friends Kofi, Afi and Safia. Each of them are given a pizza. Kofi cuts his pizza into 2 slices only and took 1 of them. So, he has taken  $\frac{1}{2}$  of his pizza. Afi cut her pizza into 4 slices and took 2 of them. So, she has taken  $\frac{2}{4}$  of her pizza.

And Safia cut her pizza into 6 slices and took 3 of them. So, she has taken  $\frac{3}{6}$  of her pizza.

Now let's check who has taken more quantity of pizza? On observing this situation, we can say that everyone has taken an equal number of slices. This shows that the three fractions  $\frac{1}{2}$ ,  $\frac{2}{4}$  and  $\frac{3}{6}$  appear different, but they have the same value. So, we can conclude that these fractions must be equivalent.

Now, we also know that two or more numbers are said to be equivalent if they have the same numerical value. Similarly, if two or more fractions that show the same part of a whole, are called *equivalent fractions*.



In the boxes drawn above we can see that, in the first box, out of three equal boxes, two are shaded which gives the fraction as  $\frac{2}{3}$ . In the next box, 4 out of 6 equal boxes are shaded, which gives us the fraction  $\frac{4}{6}$ .

It is evident from the above diagram that the fraction  $\frac{4}{6}$  is the same as  $\frac{2}{3}$  as they show the same shaded region. Similarly, all the fractions  $\frac{2}{3}$ ,  $\frac{4}{6}$ ,  $\frac{8}{12}$ ,  $\frac{16}{24}$  are equivalent because they all show the same shaded region.



# **Equivalent Fractions on a Number Line**

Equivalent fractions lie on the same positions on the number line.

Let us take an example of two equivalent fractions  $\frac{3}{4}$  and  $\frac{6}{8}$ . If we plot  $\frac{3}{4}$  on the number line, we get the following result:



Now, let us plot  $\frac{6}{8}$  on the number line then,



We can verify from the two number lines that equivalent fractions lie on the same position on the number line.

### ACTIVITY 2.3: Individual/Pair/Group Work

#### **Discovering Equivalent Fractions**

**Purpose:** To explore and understand equivalent fractions by using paper strips and visual comparisons.

#### **Materials Needed:**

- Coloured paper strips or construction paper
- Scissors
- Markers or coloured pens
- Ruler
- Glue or tape
- Large sheet of paper or poster board

#### **Instructions:**

- 1. Step 1: Prepare Your Fractions
  - Take the coloured paper strips and use the ruler to divide each strip into equal parts. For example, one strip could be divided into 2 equal parts, another into 4 equal parts and another into 8 equal parts.
  - Label each section of the strips with fractions. For example, if your strip is divided into 4 equal parts, label each part as  $\frac{1}{4}$ .

#### 2. Step 2: Colour and Cut

 Choose one fraction from each strip. For example, colour one of the <sup>2</sup>/<sub>4</sub> sections and one of the <sup>1</sup>/<sub>2</sub> sections. • Carefully cut out the coloured sections of each strip.

#### 3. Step 3: Compare Fractions

- Place the cut-out sections next to each other on a large sheet of paper or poster board. Compare them to see if they cover the same amount of space.
- Are any of the fractions equal in size? For example, do  $\frac{2}{4}$  and  $\frac{1}{2}$  look the same?

#### 4. Step 4: Create Your Equivalent Fractions Chart

- Glue or tape the cut-out fractions that are equal in size onto the poster board. Write down the fractions underneath, showing that they are equivalent. For example, you might write " $\frac{2}{4} = \frac{1}{2}$ ."
- Continue this process with other fractions. Try to find as many equivalent fractions as possible.

#### 5. Step 5: Present Your Findings

- Share your equivalent fractions chart with the class. Explain how you determined which fractions were equivalent and why they are important in maths.
- 6. Reflection:
  - Think about why understanding equivalent fractions is useful. How might you use this knowledge when solving maths problems, especially when comparing or simplifying fractions?

# **Comparing Fractions Having the Same Denominator** (Comparison of Like Fractions)

We can compare the fractions of the same <u>denominator</u> easily by comparing the numerators of the fractions. If the fractions have the same denominator, then the fraction having a bigger numerator is greater.

Let us understand this better by considering two fractions,  $\frac{3}{8}$  and  $\frac{7}{8}$ .

Let's try to compare them. We can observe that both these fractions have the same denominator. This shows that the whole is divided into the same number of equal parts.

Now let us see how many equal parts are taken by observing the numerators. The numerator that is greater shows a greater number of equal parts, and hence, it is the greater fraction.



Clearly,  $\frac{3}{8}$  is smaller than  $\frac{7}{8}$ . And this is represented as,  $\frac{3}{8} < \frac{7}{8}$ 

# **Compare Fraction of same Numerator**

Let us think of two identical pizzas. You have cut the first one into 4 slices, and the second one into 8 slices. You have taken 3 slices from each pizza. Now, from which pizza have you taken more?



By observing you can say that you got more pizza from the first one. Now, let us analyse the same situation using fractions.

Fraction of pizza taken from the first pizza =  $\frac{3(Number of equal parts taken)}{4(Number of equal parts)}$ Fraction of pizza taken from the second pizza =  $\frac{3(Number of equal parts taken)}{8(Number of equal parts)}$ 

Clearly,  $\frac{3}{4} > \frac{3}{8}$ 

When the numerators are the same, we look at the denominators of the fractions.

Look at these two fractions,  $\frac{3}{4}$  and  $\frac{3}{8}$ .

When we compare such fractions, the greater the denominator, the smaller the fraction. This is because, the more the number of parts the whole is divided into, the smaller the parts are. Therefore, the smaller the number of parts makes the fraction bigger. Hence, the fraction with the greater denominator is the smaller fraction.

For example, if we compare  $\frac{3}{7}$  and  $\frac{3}{5}$ . Both fractions have the same numerator, 3. Three times  $\frac{1}{7}$  is  $\frac{3}{7}$ , and three times  $\frac{1}{5}$  is  $\frac{3}{5}$ . So, on comparing the fractions we can see that the denominator of  $\frac{3}{7}$  is 7 which is greater than the denominator of  $\frac{3}{5}$  i.e. 5. Since we already know that a fraction with the greater denominator is the smaller fraction, this results in  $\frac{3}{7}$  being smaller than  $\frac{3}{5}$ , or,  $\frac{3}{7} < \frac{3}{5}$ .

# **Solving Problems Involving Equivalent Fractions**

#### Example 1:

Order the fractions  $\frac{7}{8}$ ,  $\frac{1}{8}$  and  $\frac{5}{8}$  from least to the greatest.

#### Solution

We can find the order of the fractions using a number line.

Plot the fractions on the number line.

All the 3 fractions have the same denominator, which is 8.

 $\frac{1}{8}$  is farthest to the left.  $\frac{7}{8}$  is farthest to the right, and  $\frac{5}{8}$  is between the other two fractions.

So, the order from least to greatest is  $\frac{1}{8}, \frac{5}{8}, \frac{7}{8}$ . Hence, we can state this as,  $\frac{1}{8} < \frac{5}{8} < \frac{7}{8}$ .

#### Example 2

Compare fractions on the number line  $\frac{3}{4}$  and  $\frac{6}{8}$ .

#### Solution

First draw the fractions on the number line.

Draw a number line through 0 and 1. Then divide that into 8 equal parts. Mark the fractions on the number line as shown:



We can see that both the numbers are at the same point or position on the number line. Hence, we can conclude that the given fractions are equivalent.

$$\operatorname{Or} \frac{3}{4} = \frac{6}{8}.$$

#### ACTIVITY 2.4- Individual/Pair/Group Work

#### **Comparing Fractions Using Equivalent Fractions**

**Purpose:** To compare different fractions by converting them into equivalent fractions with the same denominator.

#### **Materials Needed:**

- Fraction strips or fraction circles
- Rulers
- Coloured pencils or markers
- Paper and pencil
- A worksheet with fractions to compare

#### **Instructions:**

#### **Step 1: Understand the Task**

• Today, you will learn how to compare fractions by converting them into equivalent fractions with the same denominator. This will help you see which fraction is larger or smaller.

#### **Step 2: Visualise with Fraction Strips**

• Start by using the fraction strips or fraction circles. Choose two fractions that you want to compare.

For example, let's compare  $\frac{2}{3}$  and  $\frac{3}{4}$ .

• Place the fraction strips or circles next to each other. Notice how they cover different amounts of space.

#### Step 3: Find a Common Denominator

- To compare fractions easily, you need to find a common denominator. This is the same number in the bottom part of both fractions.
- For  $\frac{2}{3}$  and  $\frac{3}{4}$ , the common denominator could be 12 because both 3 and 4 can divide exactly into 12.

#### **Step 4: Create Equivalent Fractions**

- Convert each fraction to an equivalent fraction with the denominator of 12
  - $\frac{2}{3}$  becomes  $\frac{8}{12}$  (because  $2 \times 4 = 8$  and  $3 \times 4 = 12$ ).
  - $\frac{3}{4}$  becomes  $\frac{9}{12}$  (because  $3 \times 3 = 9$  and  $4 \times 3 = 12$ ).
- Write down the new fractions:  $\frac{8}{12}$  and  $\frac{9}{12}$ .

#### **Step 5: Compare the Fractions**

- Now that both fractions have the same denominator we can compare the numerators (the top numbers). Which is larger,  $\frac{8}{12}$  or  $\frac{9}{12}$ ? • Since  $\frac{9}{12}$  is larger than  $\frac{8}{12}$ , you can conclude that  $\frac{3}{4}$  is larger than  $\frac{2}{3}$ .

### **Step 6: Practice with More Examples**

- Work through several more examples on your worksheet. Use the same method: find a common denominator, create equivalent fractions, and compare.
- For example:
  - Compare  $\frac{5}{6}$  and  $\frac{7}{8}$ .
  - Compare  $\frac{1}{4}$  and  $\frac{2}{5}$ .
- Show your work by writing down the equivalent fractions and then • circle the larger one.

#### **Step 7: Reflect on the Process**

Think about why converting to equivalent fractions makes • comparing easier. How does this method help you understand the size of fractions better?

#### **Step 8: Challenge Yourself**

• Try comparing three fractions at once. Use the same steps to find a common denominator, create equivalent fractions, and then determine the order from smallest to largest.

# **OPERATIONS ON FRACTIONS**

# FOCAL AREA 1: SOLVE PROBLEMS ON FRACTIONS INVOLVING THE FOUR BASIC OPERATIONS

Imagine you're baking a cake for a family gathering. The recipe you're using calls for specific measurements of ingredients like flour, sugar and butter, but they aren't all in whole numbers. For example, the recipe might ask for  $\frac{3}{4}$  cup of flour,  $\frac{1}{2}$  cup of sugar and  $\frac{1}{3}$  cup of butter. What if you want to make a bigger cake and need to double the recipe? Or what if you want to share the cake equally with friends and need to divide the ingredients? These situations require you to understand how to perform operations on fractions—adding, subtracting, multiplying, and dividing them—to ensure the cake turns out just right.

Knowing how to operate with fractions is not only important for cooking but also for many real-life scenarios such as dividing resources, sharing costs, and adjusting quantities in measurements. Without this knowledge, tasks that involve fractions could become confusing and lead to mistakes. Learning to perform operations on fractions helps you solve practical problems with precision and confidence.

### **REINFORCEMENT ACTIVITIES**

#### **Exploring Fractions in Everyday Situations**

#### **Activity:** Fraction Hunt

**Purpose:** Before we dive into learning about operations on fractions, let's explore how fractions are part of our everyday life.

#### **Materials Needed:**

- A notebook or paper
- A pen or pencil

#### **Step 1: Exploring Your Surroundings**

- Look around your home, school or any other place you are familiar with.
- Find at least **three** examples of where you see fractions being used in everyday situations. These could be in a recipe book, a clock, a ruler or even in a game you play.



• For each example, write down what fraction is being used and describe how it is applied. For example, "In my recipe book, I found a recipe that uses  $\frac{1}{2}$  cup of sugar."

### **Step 2: Sharing and Discussing**

- Once you've identified your three examples, we'll come together as a class to share what we found.
- Be ready to explain how each fraction is used in your examples. For instance, "The clock shows that  $\frac{1}{4}$  of an hour is 15 minutes."

#### **Step 3: Reflecting on the Importance**

- After discussing our findings, think about why understanding fractions might be important in these situations.
- Consider questions like: What might happen if you didn't know how to work with fractions in these examples? How do fractions help you in real life?

### **Conclusion:**

• This activity will help you realise how often we use fractions in daily life and why it's important to know how to perform operations on them. Get ready to learn how to add, subtract, multiply, and divide fractions with confidence!



Therefore,  $\frac{1}{3} + \frac{5}{6} = 1\frac{1}{6}$ 



# Example 3

Sarah has  $\frac{2}{3}$  of a cup of sugar and needs  $\frac{1}{4}$  more for her recipe. How much sugar does she need in total?



### Solution

Find a common denominator:

The denominators of the fractions are 3 and 4. The lowest common multiple (LCM) of 3 and 4 is 12. Therefore, we will convert both fractions to have a denominator of 12.

For  $\frac{2}{3}$ ;  $\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$ 

For  $\frac{1}{4}$ ;

 $\frac{1}{4} = \frac{1 \times 3}{4 \times 3} = \frac{3}{2}$ 

Now that the fractions have a common denominator, we can add them and simplify (if necessary):

 $\frac{8}{12} + \frac{3}{12} = \frac{8+3}{12} = \frac{11}{12}$ 

The fraction  $\frac{11}{12}$  is already in its simplest form, therefore, Sarah needs  $\frac{11}{12}$  cup of sugar in total.

# **Subtraction of Fractions**



Therefore,  $\frac{3}{4} - \frac{5}{8} = \frac{6}{8} - \frac{5}{8} = \frac{1}{8}$ 



# Example 3

John had  $\frac{5}{8}$  of a pizza. He gave  $\frac{1}{4}$  of it to his friend. How much pizza does he have left?

# Solution

Draw a circle on the paper plate to represent the pizza.

Divide the pizza into 8 equal slices to represent  $\frac{8}{8}$ . You can do this by drawing lines through the center of the circle.



Shade 5 out of the 8 slices to represent  $\frac{5}{8}$  of the pizza that John has.



To represent  $\frac{1}{4}$  of the pizza, you need to convert it to a fraction with the same denominator (8). So,  $\frac{1}{4} = \frac{2}{8}$ 

Shade 2 out of the 8 slices with a different colour or pattern to show the part that John is giving away.



Count the remaining slices. After removing the 2 slices that John gave away, there should be 3 shaded slices left. These 3 remaining slices represent  $\frac{3}{8}$  of the pizza.

To solve this without drawing the pizzas we must find a common denominator:

The denominators of the fractions are 8 and 4. The lowest common multiple (LCM) of 8 and 4 is 8 Therefore, we will convert both fractions to have a denominator of 8.

For  $\frac{5}{8}$ ;  $\frac{5}{8}$  is already in the desired form with denominator 8.

For  $\frac{1}{4}$ ;  $\frac{1}{4} = \frac{1 \times 2}{4 \times 2} = \frac{2}{8}$ 

Now that the fractions have a common denominator, we can subtract them and simplify (if necessary):

$$\frac{5}{8} - \frac{2}{8} = \frac{5-2}{8} = \frac{3}{8}$$

The fraction  $\frac{3}{8}$  is already in its simplest form, therefore, John has  $\frac{3}{8}$  of pizza left.

### ACTIVITY 2.5- Individual/Pair/Group Work

### **Addition and Subtraction of Fractions**

**Objective:** In this activity, you will learn how to add and subtract fractions with like and unlike denominators through hands-on practice. This will help you understand how to combine and compare fractional parts in real-life situations.

#### **Materials Needed:**

- Fraction strips or fraction circles
- Grid paper
- Pencil and paper
- A calculator (optional)

#### Steps:

#### Warm-Up: Adding Fractions with Like Denominators

• Start by reviewing how to add fractions with the same denominator. Remember, you simply add the numerators (top numbers) and keep the denominator (bottom number) the same.

• For example 
$$\frac{2}{8} + \frac{3}{8} = \frac{5}{8}$$
.

• Try it yourself: Add  $\frac{1}{6} + \frac{4}{6}$ . Write your answer and check it using fraction strips or by drawing it on your grid paper.

### Step 1: Real-Life Addition Problem (Like Denominators)

- Imagine you have  $\frac{3}{8}$  of a pizza and your friend gives you another  $\frac{2}{8}$  of a pizza. How much pizza do you have in total?
- Add the fractions:  $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$ .
- Use fraction strips or draw a diagram to represent this on your grid paper.

#### Step 2: Adding Fractions with Unlike Denominators

- To add fractions with different denominators, first find a common denominator, then add the numerators.
- For example,  $\frac{1}{4} + \frac{1}{6}$  can be added by converting them to  $\frac{3}{12} + \frac{2}{12} = \frac{5}{12}$ .
- Now, try this: Add  $\frac{2}{3} + \frac{1}{4}$ . Find a common denominator, add the fractions, and simplify your answer if needed.



#### **Step 3: Real-Life Addition Problem (Unlike Denominators)**

• Imagine you're making a fruit salad. You have  $\frac{2}{3}$  of a cup of apples and  $\frac{1}{4}$  of a cup of oranges.

How many cups of fruit do you have in total?

• Add the fractions:  $\frac{2}{3} + \frac{1}{4}$ .

Convert them to have the same denominator, add them and find out how much fruit you have.

• Draw a diagram on your grid paper to show how you combined the two amounts.

#### **Step 4: Subtracting Fractions with Like Denominators**

- Subtracting fractions with the same denominator is simple: just subtract the numerators and keep the denominator the same.
- For example,  $\frac{5}{8} \frac{2}{8} = \frac{3}{8}$ .
- Try it yourself: Subtract  $\frac{7}{10} \frac{4}{10}$ . Write your answer and check it with a fraction strip or by drawing it.

#### Step 5: Real-Life Subtraction Problem (Like Denominators)

• Imagine you have  $\frac{7}{10}$  of a chocolate bar, and you eat  $\frac{4}{10}$  of it. How much do you have left?

• Subtract the fractions:  $\frac{7}{10} - \frac{4}{10} = \frac{3}{10}$ .

• Use a fraction strip or draw this on your grid paper to visualise what's left.

#### **Step 6: Subtracting Fractions with Unlike Denominators**

- For fractions with different denominators, find a common denominator, subtract the numerators, and simplify if needed.
- For example,  $\frac{5}{6} \frac{1}{4}$  can be subtracted by converting them to  $\frac{10}{12} \frac{3}{12}$ =  $\frac{7}{12}$ .
- Now, try this: Subtract  $\frac{3}{5} \frac{1}{3}$  Find the common denominator and simplify your answer.

#### Step 7: Real-Life Subtraction Problem (Unlike Denominators)

• Imagine you poured  $\frac{3}{5}$  of a cup of milk into a recipe, but you only needed  $\frac{1}{3}$  of a cup.

How much extra milk did you pour?

- Subtract the fractions:  $\frac{3}{5} \frac{1}{3}$ . Convert them to have the same denominator, subtract them, and figure out how much extra milk you poured.
- Draw a diagram on your grid paper to represent the difference.

#### **Step 8: Practice with More Examples**

- Solve these problems:
  - **1.** Add  $\frac{3}{7} + \frac{2}{5}$  and explain how you found the common denominator.
  - 2. Subtract  $\frac{4}{9} \frac{1}{6}$  and draw a diagram to show your work.

#### **Step 9: Reflect and Discuss**

- Think about how adding and subtracting fractions can help in cooking, measuring, or dividing items.
- Discuss with your group: Why is it important to know how to work with fractions in real life?
- Write down your thoughts and be ready to share with the class.

# **Multiplication of Fractions**

# Multiplying a fraction by a whole number

## Example 1

Kwame used  $\frac{1}{4}$  of an apple to make one apple pie. He made 3 pies. Let's find out how many apples he used in total.

# Solution

Draw a circle to represent an apple and divide it into 4 equal parts and shade  $\frac{1}{4}$  of the apple for one pie.

Since he made 3 pies, repeat this process 3 times.



Count all the shaded parts to see that  $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$  of an apple is used.

```
Alternative method:
```

```
Multiply \frac{1}{4} by 3.
\frac{1}{4} \times 3
```

Represent the whole number 3 as a fraction:  $3 = \frac{3}{1}$ 

Multiply the numerators (top numbers) together and the denominators (bottom numbers) together.

 $\frac{1}{4} \times \frac{3}{1} = \frac{1 \times 3}{4 \times 1} = \frac{3}{4}$ 

Simplify the Fraction (if necessary):

The fraction  $\frac{3}{4}$  is already in its simplest form.

Kwame used  $\frac{3}{4}$  of an apple in total to make 3 apple pies.

### Now, let's try this;

i. If Kwame had to make 5 pies, how many apples would he need?



Find  $\frac{1}{4}$  of 8

# Solution

Draw 8 rectangles, each divided into 4 equal parts:



Shade one part in each of the 8 rectangles:



Count the shaded parts:

You have a total of 8 shaded parts out of the total parts.

If you combine all the shaded parts, you see that you have a total of  $\frac{1}{4} \times 8 = \frac{1}{4} \times \frac{8}{1}$ 

$$=\frac{1\times8}{4\times1}=\frac{8}{4}=2.$$

So, when you multiply  $\frac{1}{4}$  by 8, you end up with 2 whole units.

#### **Alternative solution**



Distribute the marbles equally in 4 groups





# Multiplying a fraction by a fraction

 $\therefore \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ 

# Example 2

How much is  $\frac{1}{4} \times \frac{1}{2}$ ?

This represents finding a quarter of a half



The fraction which is vertical and shaded in red shows the  $\frac{1}{4}$  and the horizontal which is  $\frac{1}{2}$ . The overlapping area is shaded orange and shows a quarter of the half, which is one-eighth.

# Example 3:

Multiply  $\frac{2}{3}$  by  $\frac{4}{5}$ 

### Solution

Multiply the Numerators:  $2 \times 4 = 8$ Multiply the Denominators:  $3 \times 5 = 15$ Form the New Fraction: 2 = 4

 $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$ 

Simplify the Fraction (if necessary):

In this case,  $\frac{8}{15}$  already in its simplest form because 8 and 15 have no common factors other than 1.

$$\therefore \frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$$

# **Division of Fractions**

### **Example 1**

Suppose you have 3 apples, each cut in half. How many people can you distribute these 3 apples if each gets half a piece?



How many groups of  $\frac{1}{2}$  will make 3 wholes?

$$3 \div \frac{1}{4} = 6$$

Ask learners the number of people who can be served if each person is served a quarter piece?



12 people can be served with 3 apples if each person gets  $\frac{1}{4}$  each. 3  $\div \frac{1}{4} = 12$  How many people will be served if each person is given three quarters  $\frac{3}{4}$  of the apple



Four (4) people will be served if each person is served three out of four,  $\frac{3}{4}$ , of the apples.

Example 2:
Find $3 \div \frac{1}{3}$
Solution

This can be interpreted as how many groups of  $\frac{1}{3}$  will make 3 wholes.

1	who	le	1	who	le	1	who	le
1/3	1/3	1/3	1/3	1/3	1/3	1/3	1/3	1/3
Ţ	2	3	4	5	6	7	8	9

The answer is 9. Therefore,  $3 \div \frac{1}{3} = 9$ 

**Example 3:** Find  $\frac{1}{2} \div 3$ 



Therefore,  $\frac{2}{3} \div \frac{4}{5} = \frac{5}{6}$ 

## ACTIVITY 2.6- Individual/Pair/Group Work

### **Multiplication and Division of Fractions**

**Purpose:** In this activity, you will explore how to multiply and divide fractions by working through practical examples. This will help you understand how these operations affect the value of fractions and how they are applied in real-life situations.

#### **Materials Needed:**

- Fraction strips or fraction circles
- Grid paper or a number line
- Pencil and paper
- A calculator (optional)

#### **Steps:**

#### Warm-Up: Multiplying Fractions

• Start by reviewing how to multiply fractions. Remember, to multiply fractions, you multiply the numerators (top numbers) together and the denominators (bottom numbers) together.

• For example, 
$$\frac{1}{2} \times \frac{1}{3} = \frac{1 \times 1}{2 \times 3} = \frac{1}{6}$$

• Try it yourself: Multiply  $\frac{3}{4} \times \frac{2}{5}$ . Write your answer and check with a fraction strip or grid paper to visualise it.

### Step 1: Real-Life Multiplication Problem

- Imagine you have a garden that is  $\frac{3}{4}$  of an acre and you plan to plant flowers on  $\frac{2}{3}$  of that area.
- To find out how much of the entire garden will be covered with flowers, multiply the fractions:

$$\frac{3}{4} \times \frac{2}{3} = \frac{3 \times 2}{4 \times 3} = \frac{6}{12} = \frac{1}{2}.$$

• This means half of your garden will be covered in flowers. Draw a diagram to represent this on your grid paper.

### **Step 2: Dividing Fractions**

- Review how to divide fractions by flipping the second fraction (taking its reciprocal) and then multiplying.
- For example, to divide  $\frac{1}{2} \div \frac{1}{3}$  you flip  $\frac{1}{3}$  to  $\frac{3}{1}$  and multiply:  $\frac{1}{2} \times \frac{3}{1} = \frac{3}{2}$ =  $1\frac{1}{2}$ .



• Now, try this:  $\frac{5}{6} \div \frac{2}{3}$ . Write your answer and explain the steps you took.

### Step 3: Real-Life Division Problem

- Imagine you have  $\frac{5}{8}$  of a chocolate bar and you want to share it equally between two friends.
- To find out how much each friend will get, divide  $\frac{5}{8}$  by 2:

$$\frac{5}{8} \div \frac{2}{1} = \frac{5}{8} \times \frac{1}{2} = \frac{5}{16}.$$

• This means each friend will get  $\frac{5}{16}$  of the chocolate bar. Use a fraction strip or draw this on your grid paper to see the division.

### **Step 4: Practice with More Examples**

- Now, solve these problems:
  - 1. Multiply  $\frac{4}{5} \times \frac{3}{7}$  to find out what fraction of  $\frac{4}{5}$  of a cake is given to your friends when you give each of them  $\frac{3}{7}$  of the  $\frac{4}{5}$ .
  - **2.** Divide  $\frac{7}{8} \div \frac{3}{4}$  to find out how many  $\frac{3}{4}$  cups of sugar are in  $\frac{7}{8}$  of a cup.

### **Step 5: Reflect and Discuss**

- After solving these problems, think about how multiplying and dividing fractions can change their size.
- Discuss with your partner or group: How might these operations be useful when cooking, sharing, or dividing resources?
- Write down your thoughts and be prepared to share with the class.

# **REVIEW QUESTIONS**

# **Review Questions 2.1**

- 1. If a pizza is divided into 8 equal slices and you eat 3 slices, what fraction of the pizza have you eaten?
- Draw a circle and divide it into 5 equal parts. Shade 2 of the parts.
   Write the fraction that represents the shaded area.
- 3. Convert the mixed number  $3\frac{1}{4}$  into an improper fraction.
- 4. Which of the following is a unit fraction:  $\frac{2}{3}, \frac{5}{6}, \frac{1}{4}, \frac{7}{8}$ ?
- 5. Determine if the following fractions are proper or improper:  $\frac{3}{5}, \frac{7}{4}, \frac{6}{6}, \frac{9}{8}$ .
- 6. Convert the improper fraction  $\frac{15}{4}$  into a mixed number.
- 7. Are  $\frac{2}{5}$  and  $\frac{4}{5}$  like or unlike fractions?
- 8. Are  $\frac{1}{3}$  and  $\frac{1}{4}$  like or unlike fractions?
- 9. Classify the following fractions as unit, proper, improper, or mixed:

 $\frac{7}{10}, \frac{11}{8}, 3\frac{2}{5}, \frac{1}{2}.$ 

# **Review Questions 2.2**

- **1.** Write two equivalent fractions for each of the following:
  - a)  $\frac{1}{2}$  b)  $\frac{3}{4}$ c)  $\frac{2}{3}$  d)  $\frac{5}{6}$ e)  $\frac{7}{8}$
- 2. Fill in the missing numerator or denominator to make the fractions equivalent:

**a**) 
$$\frac{3}{5} = \frac{?}{10}$$
 **b**)  $\frac{4}{9} = \frac{12}{?}$ 

**c**) 
$$\frac{2}{7} = \frac{6}{?}$$
 **d**)  $\frac{5}{8} = \frac{?}{24}$ 

- **e**)  $\frac{1}{3} = \frac{?}{12}$
- **3.** Simplify the following fractions to put them in their simplest form:
  - a)  $\frac{8}{12}$  b)  $\frac{15}{20}$ c)  $\frac{18}{24}$  d)  $\frac{14}{28}$ e)  $\frac{9}{27}$
- **4.** Use equivalent fractions to compare the following pairs of fractions. Write "<", ">", or "=" between the fractions:
  - **a)**  $\frac{2}{3}$  and  $\frac{3}{4}$  **b)**  $\frac{5}{6}$  and  $\frac{15}{18}$  **c)**  $\frac{4}{10}$  and  $\frac{3}{8}$  **d)**  $\frac{4}{5}$  and  $\frac{6}{7}$ **e)**  $\frac{7}{12}$  and  $\frac{5}{9}$
- **5.** Arrange the following sets of fractions in ascending order (from smallest to largest) by finding equivalent fractions:
  - a)  $\frac{2}{5}, \frac{3}{10}, \frac{4}{7}$ b)  $\frac{5}{8}, \frac{3}{4}, \frac{7}{12}$ c)  $\frac{3}{6}, \frac{2}{4}, \frac{4}{8}$ d)  $\frac{5}{9}, \frac{4}{7}, \frac{6}{11}$ e)  $\frac{1}{2}, \frac{2}{3}, \frac{3}{5}$
- 6. Serwaa and Tsatsu are comparing how much of their homework they've finished. Serwaa has completed  $\frac{3}{4}$  of her homework and Tsatsu has finished  $\frac{4}{5}$  of his. Who has completed more of their homework? Use equivalent fractions to compare.
- 7. A recipe calls for  $\frac{2}{3}$  cup of sugar. You have a  $\frac{3}{4}$  cup measure of sugar. Do you have enough sugar for the recipe?

Use equivalent fractions to determine if  $\frac{2}{3}$  cup of sugar is more or less than  $\frac{3}{4}$  cup.

8. Solve the following questions, giving your answers in their simplest form:

**a.** 
$$\frac{2}{5} + \frac{3}{5}$$
**b.**  $\frac{1}{4} + \frac{3}{8}$ **c.**  $\frac{2}{7} + \frac{4}{9}$ **d.**  $\frac{5}{12} + \frac{1}{3}$ **e.**  $\frac{3}{10} + \frac{7}{15}$ **f.**  $\frac{5}{6} - \frac{1}{6}$ **g.**  $\frac{7}{8} - \frac{1}{4}$ **h.**  $\frac{9}{10} - \frac{3}{5}$ 

i.	$\frac{11}{12} - \frac{7}{12}$	j.	$\frac{4}{9} - \frac{1}{3}$
k.	$\frac{3}{4} \times \frac{2}{5}$	l.	$\frac{7}{8} \times \frac{3}{4}$
m.	$\frac{5}{9} \times \frac{6}{7}$	n.	$\frac{2}{3} \times \frac{4}{5}$
0.	$\frac{3}{10} \times \frac{5}{12}$	р.	$\frac{5}{6} \div \frac{2}{3}$
q.	$\frac{7}{8} \div \frac{3}{4}$	r.	$\frac{9}{10} \div \frac{3}{5}$
S.	$\frac{2}{5} \div \frac{1}{4}$	t.	$\frac{3}{7} \div \frac{2}{9}$

- 9. You have  $\frac{2}{3}$  of a cup of sugar, and you add  $\frac{1}{4}$  of a cup more. How much sugar do you have in total?
- 10. You pour  $\frac{5}{8}$  of a litre of juice into a glass. The glass already had  $\frac{1}{4}$  of a litre. How much juice is in the glass now?
- 11. A recipe calls for  $\frac{3}{4}$  of a cup of flour. If you make half of the recipe, how much flour will you need?
- 12. You have  $\frac{5}{6}$  of a pie. You want to divide it equally among 3 friends. How much pie will each friend get?

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