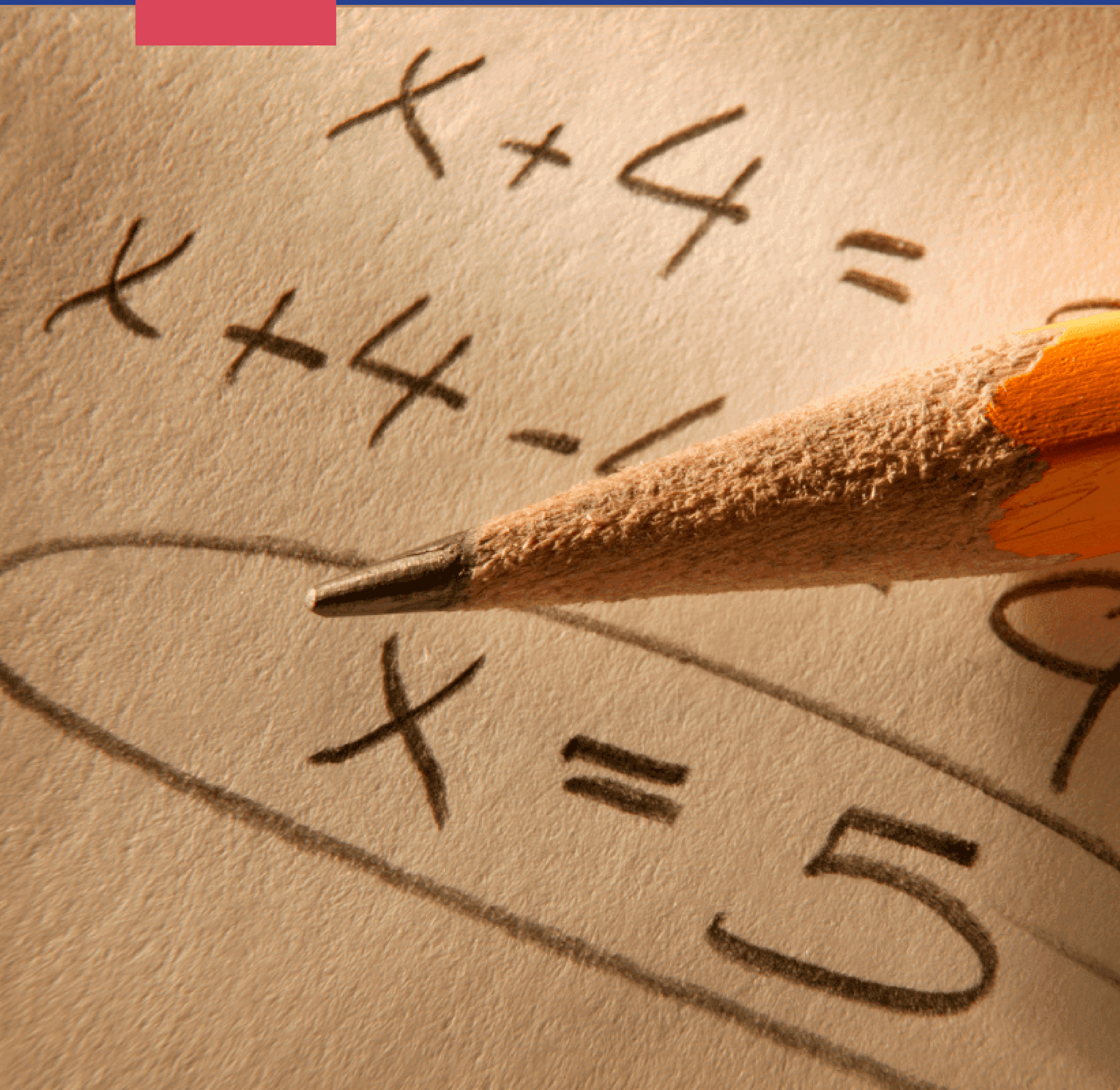
Intervention Mathematics Level 2

SECTION 3

REASONING WITH ALGEBRA



ALGEBRAIC REASONING

Algebraic Expressions, Equations and Inequalities

In this section, you will learn to;

1. *Model real-life situations into mathematical statements and perform operations on them.*
2. *Expand by removing brackets and simplifying algebraic expressions using the properties of operations.*
3. *Express a given problem as an equation representing the unknown by a letter to the variable and identify the unknown to solve the problem pictorially or symbolically.*
4. *Create a problem for a given equation.*

SECTION INTRODUCTION

In this section, you will learn to model real-life situations into mathematical statements and perform operations on them, allowing you to translate everyday scenarios into maths problems. You will also expand and simplify algebraic expressions by removing brackets, using the properties of operations to make complex expressions more manageable. Additionally, you will express a given problem as an equation, representing the unknown with a letter or variable and solve the problem either pictorially or symbolically. Finally, you will create your own problems for given equations, helping you deepen your understanding of how equations work in various situations.

ALGEBRAIC EXPRESSIONS

FOCAL AREA 1: CONCEPT OF ALGEBRAIC EXPRESSIONS

Imagine you're helping to plan a party and you need to decide how many snacks and drinks to buy. You know that each guest will need 3 snacks and 2 drinks, but you don't know exactly how many guests will come. How can you figure out how many snacks and drinks to buy?

This is where algebraic expressions come in handy. An algebraic expression allows us to represent situations like this using variables. In this case, we can use the variable "g" to represent the number of guests. Then, the total number of snacks can be written as $3g$ (3 snacks per guest multiplied by the number of guests) and the total number of drinks as $2g$ (2 drinks per guest multiplied by the number of guests).

Learning about algebraic expressions is important because it helps us to solve real-life problems where we don't have all the information right away. By using variables to represent unknown quantities, we can create expressions that help us understand and solve problems in a flexible and powerful way. Whether you're budgeting for a party, calculating the cost of supplies or planning a trip, algebraic expressions are a key tool for making informed decisions.

REINFORCEMENT ACTIVITIES

Understanding Variables Through Real-Life Contexts

Objective: To introduce the concept of variables and help you understand how they represent unknown values in real-life situations.

Activity:

Scenario: Planning a School Trip

- Imagine you are helping to plan a school trip. The cost of the trip includes transportation and lunch.
- The bus company charges a fixed amount of GH¢200 for the bus, plus GH¢5 for each student.
- The lunch costs GH¢8 per student.

Discussion Questions:

- How would you calculate the total cost if you knew the exact number of students going on the trip?
- What if you don't know how many students are going? How could you represent the total cost?

Introducing the Concept of a Variable:

- Let's call the number of students "s".
- How can we write an expression to represent the total transportation cost? (Hint: Use "s" to represent the number of students.)
- How can we write an expression for the total cost of lunch?

Writing Expressions:

- Write down the expression for the transportation cost: $200 + 5s$.
- Write down the expression for the lunch cost: $8s$.
- Combine these to write an expression for the total cost of the trip: $200 + 5s + 8s$.

Think and Share:

- Discuss with your group how you could use these expressions to estimate the total cost if you knew the number of students.
- Share your thoughts with the class.

Try these Scenarios

Scenario 1

You are in charge of organising a school fundraiser. You will be selling two items: T-shirts and snacks.

- Each T-shirt sells for GH¢12.
- Each snack sells for GH¢3.

Questions:

1. Let's call the number of T-shirts sold "t" and the number of snacks sold "n."
2. Write an expression to represent the total money raised from selling T-shirts.
3. Write an expression to represent the total money raised from selling snacks.
4. How can you write an expression for the total money raised from both items?

Scenario 2

You are organising a movie night for your class.

- The movie rental costs GH¢50.
- Each student attending will receive GH¢2 for snacks.

Questions:

1. Let's call the number of students attending "s."
2. Write an expression to represent the total amount given to the students for snacks.
3. Write an expression to represent the total cost of the event, including the movie rental and the money for the snacks.

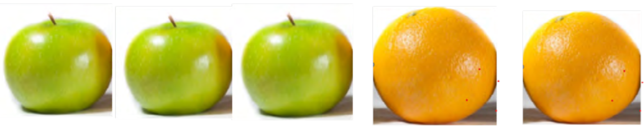
LIKE AND UNLIKE TERMS

Like and unlike terms are important concepts when learning algebraic expressions. We use the concept of like terms and unlike terms to represent terms that can be combined or simplified based on their similarity. Let's take a look at these examples;

Example 1

Consider the image below. How many fruits are there? How many apples and oranges can you identify?

There are five fruits; which includes 3 apples and 2 oranges.



In the absence of known quantities such as apples and oranges, we can represent these quantities with variables. That is a for apples and o for oranges.

This implies that there are $3a$ and $2o$.

ACTIVITY 3.1: Individual/Pair/Group Work

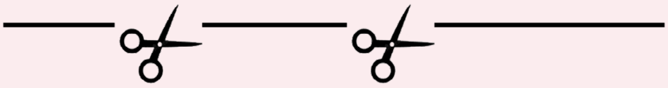
Let's try this activity

In pairs or small groups, obtain a piece of string and cut through it.

Now, how many pieces do you have now?

Answer = 2 pieces

Now, make another cut on one of the pieces. How many pieces are there now?



Answer= 3 pieces

Continue making more cuts (3, 4, 5) and ask the same question – how many pieces are there now?

Then we can leap to, how many pieces would there be if they made 57 cuts?

Answer = 58 cuts

Can you identify a rule that tells us how many pieces you will have after any number of cuts?

I guess you come up with something like; Pieces = Cuts plus one

Now, let's introduce symbols for Pieces (P) and cuts (C) i.e. **P = C+1**

ACTIVITY 3.2- Individual/Pair/Group Work

Let's try this activity

Step 1: Find the perimeter of a unit square with length 1cm.

Perimeter of a square is **L + L + L + L**

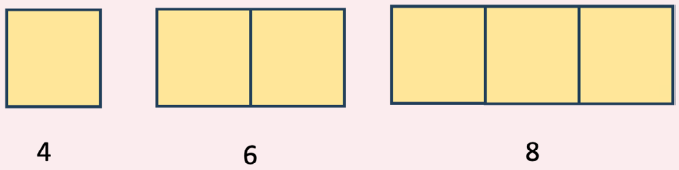
P = 1 + 1 + 1 + 1

P = 4 cm

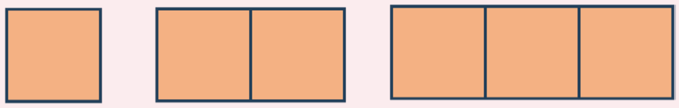
Step 2: Find the perimeter of the shapes made with the unit squares below

The length of the side of a unit square is 1. The perimeter, as indicated in **Step 1** above is 4.

Therefore, the perimeter of the squares will be 4, 6 and 8 respectively.



Step 3: Find the perimeter of the squares below with length unknown.

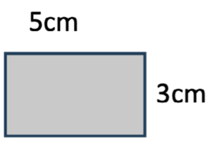


Since the length of the side is not known, let the variable **L** represent the length.

Therefore, the Perimeter will be **4L, 6L** and **8L** respectively.

Try This!

Step 1: Find the perimeter of a rectangle with length, 5cm and breadth 3cm.



Let the length be represented by L and breadth by B

Perimeter = L + L + B + B

P = 2L + 2B

Therefore, P = 2(5) + 2(3)

P = 10 + 6

P = 16 cm

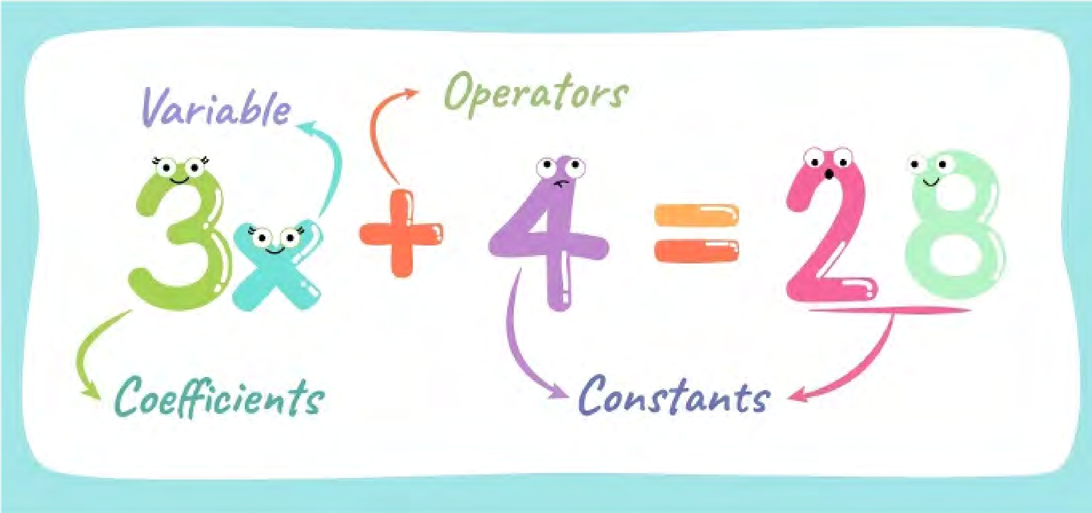
Step 2: Find the perimeter of the rectangles below with length 5 cm and breadth 3cm.

The perimeter of the first shape, as indicated in **Step 1** above is 16 cm.

Therefore, I'm sure you found the perimeter of all the rectangles to be: 16, 26 and 36 respectively. BRAVO!!!

DEFINITION OF TERMS

There are certain key terminologies that we use when dealing with algebraic expressions. Let's take the equation $3x + 4 = 28$.



Constant: A fixed numerical value that does not change. For example, in the expression $3x + 4 = 28$, the constant terms are 4 and 28.

Variable: A symbol that represents an unknown or changing quantity. In the expression $3x + 4 = 28$, x is the variable.

Coefficient: The numerical factor that multiplies a variable. In $3x + 4 = 28$, 3 is the coefficient of the variable x.

We can also talk about "Exponents" "Like terms" and "Unlike terms". Take a look at this example;

$y^2 + 2x - 3y^2 + s^2 = 5x + 2s$

Exponent: A number that indicates the power to which a variable is raised. For example, in y^2 , 2 is the exponent of y.

Like Terms: Terms that have the same variables and raised to the same powers. For instance, $2x$ and $5x$ are like terms, as well as y^2 and $3y^2$

Unlike Terms: Terms that have different variables or different powers of the same variable. For example, $2x$ and $2s$ are unlike terms. Also, s^2 and $2s$ are unlike terms.

Worked Examples

Example 1

Regroup the following according to like and unlike terms.

$4x^2$, $3xy, -2x^2, 5y, 7x, -3xy$

Solution

The like terms are, $4x^2$ and $-2x^2$, $3xy$ and $-3xy$ but $5y$ and $7x$ are unlike terms.

Example 2

4a, 2b, $3a^2$, $5b^2$, 7a, 6a

Solution

The like terms are 4a and 7a, and 6a.

The unlike terms are 2b, $3a^2$ and $5b^2$

Example 3

In the expression, $2x + 5y - 3x + 7y$, identify the coefficients of the terms

Solution

The coefficients of x, y, x and y are 2, 5, -3 and 7 respectively.

ACTIVITY 3.3- Individual/Pair/Group Work

Exploring Algebraic Expressions

Purpose: In this activity, you will identify and classify different components of algebraic expressions. You'll learn about constants, variables, coefficients, exponents, like terms, and unlike terms.

Instructions

1. **Identify Components**

   Look at the algebraic expressions provided below. For each expression, identify and label the following components:

   - Constant: A number on its own.
   - Variable: A letter or symbol representing a number.

- Coefficient: A number that multiplies the variable.
- Exponent: A small number that shows how many times a variable is used as a factor.
- Like Terms: Terms that have the same variable parts.
- Unlike Terms: Terms that have different variable parts.

2. **Complete the Table**

   Use the expressions provided to fill out the table below:

Expression	Constant	Variable	Coefficient	Exponent	Like Terms	Unlike Terms
$5x + 3$						
$2a^2 + 4a - 7$						
$6b - 3b^2 + 8$						
$9 - 4x + 2x^2$						

3. **Discuss and Compare**

   After filling out the table, discuss with your group how the different components of the expressions relate to each other.

FOCAL AREA 2: OPERATIONS ON ALGEBRAIC EXPRESSIONS

Let's revise our previous knowledge of algebraic expressions and use the idea to build and interpret simple algebraic expressions.

We have learnt that an algebraic expression consists of constants, variables and mathematical operations, such as addition, subtraction, multiplication, and division. Take a look at these examples:

1. 2n — 2 is the coefficient and n is the variable.
2. $2x + 6$ — 2 is the coefficient, 6 is the constant and x is the variable.
3. 2 (y + 4) etc. — 2 and 4 are the constants and y is the variable.

Write the mathematical expressions for the following statements:

a. **A number more than a given number**

Example:

This could be expressed in many ways, for example, a number plus 4, 4 added to a number, the sum of 4 and a number, 4 more than a number.

Algebraically it looks like this, $x + 4$, the number is x.

b. **A number less than a given number**

Example

Again it could be expressed in many ways, 17 minus a number, a number subtracted from 17, the difference of 17 and a number, a number less than 17.

Algebraically it would be this, $17 - b$, the number is b.

c. **Several times of a certain number**

Example

This could also be written as twice a certain number.

Algebraically it would be written as $2x$

Or we could have 5 times a number, the product of 5 and a number, etc.

Algebraically this would be written as $5x$.

d. **The quotient of a given number**

Example

16 divided by a number.

Algebraically this would be written as $\frac{16}{x}$

In pairs, discuss the following with their mathematical expressions.

Addition

Statement	Expression
The sum of x and y	$x + y$
7 more than m	$m + 7$
The total a, b and c	$a + b + c$
p increased by 6	$p + 6$

Subtraction

Statement	Expression
m subtracted from n	$n - m$
7 less than m	$m - 7$
The difference between a and b	$a - b$ or $b - a$
x decreased by 4	$x - 4$

Multiplication

Statement	Expression
The Product of x and y	$\boldsymbol{x} \times \boldsymbol{y} = xy$
8 of q	8 x q = 8q
Twice x	$x + x = 2x$
4 times y	4 x y = 4y

Addition and subtraction of algebraic expressions

We can only add or subtract expressions that have like terms. Let's take a look at some examples.

Example 1

Simplify $3x + 4 + 2x + 5$

Solution

1. **Combine Like Terms:**
   - The terms with x are $3x$ and $2x$.
   - The constant terms are 4 and 5.
2. **Add the Like Terms:**
   - $3x + 2x = 5x$
   - $4 + 5 = 9$
3. **Write the Simplified Expression:**
   - $5x + 9$

Final Answer: $3x + 4 + 2x + 5 = 5x + 9$

Example 2

Simplify $7a - 3 - 2a + 6$

Solution

1. **Combine Like Terms:**
   - The terms with a are $7a$ and $-2a$.
   - The constant terms are -3 and 6.
2. **Subtract the Like Terms:**
   - $7a - 2a = 5a$
   - $-3 + 6 = 3$
3. **Write the Simplified Expression:**
   - $5a + 3$

Final Answer: $7a - 3 - 2a + 6 = 5a + 3$

Example 3

Simplify**:** $4m + 7 - 2m + 3 - 5$

Solution

1. **Combine Like Terms:**
   - The terms with m are $4m$ and $-2m$.
   - The constant terms are 7, 3, and -5.
2. **Add and Subtract the Like Terms:**
   - For the m terms: $4m - 2m = 2m$
   - For the constant terms: $7 + 3 - 5 = 5$
3. **Write the Simplified Expression:**
   - $2m + 5$

Final Answer: $4m + 7 - 2m + 3 - 5 = 2m + 5$

Example 4

Kwaku has 3 plantains and 2 yams and his sister has 5 plantains and 4 yams. Express these using symbols and add the two expressions.

Solution

In this case, you can let p represent the plantains and y represent the yams.

Then Kwaku's expression is $3p + 2y$

Kwaku's sister's expression is $5p + 4y$.

To add these, we can only add the like terms together

$3p + 5p + 2y + 4y = 8p + 6y$

Together they have 8 plantains and 6 yams.

Example 5

There are two groups in a class, 8 learners offer mathematics and 5 offer English in the first group. In the second group, 3 students offer mathematics and 3 offer English.

i. Write mathematical expressions for both groups.

ii. How many more maths / English learners are there in the first group than the second group.

Solution

Let m represent mathematics and e represent English

Expression for the first group is $8m + 5e$

Expression for the second group is $3m + 3e$

$8m + 5e - (3m + 3e) = 8m + 5e - 3m - 3e$

$8m - 3m + 5e - 3e = \mathbf{5m + 2e}$

Multiplication of algebraic expressions

When multiplying algebraic expressions, we need to understand how exponents work.

Take a look at these examples;

1. $x \times x = x^2$
2. $x \times x \times x = x^3$
3. $x \times x^2 = x^3$
4. $x \times x \times x^2 = x^4$
5. $x \times y \times x \times y = x^2 \times y^2$

Worked Examples

Example 1

Simplify $3x \times 4$

Solution

1. **Multiply the coefficient and the constant:**
   - Coefficient of x is 3.
   - Constant is 4.
2. **Multiply:**
   - $3 \times 4 = 12$
3. **Write the Result:**
   - $12x$

Final Answer: $3x \times 4 = 12x$

Example 2

Simplify $2y \times 3y$

Solution

1. **Multiply the Coefficients:**
   - Coefficient of y in $2y$ is 2
   - Coefficient of y in $3y$ is 3.
   - $2 \times 3 = 6$

2. **Multiply the Variables:**
   - $y \times y = y^2$
3. **Write the Result:**
   - $6y^2$

Final Answer: $2y \times 3y = 6y^2$

Solving Algebraic Expressions Using Algebra Tiles

The tiles are as follows:

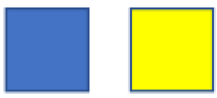
The smallest tile is a 1 by 1 unit representing the constant +1 or -1

The next tile is 1 by x, representing +x or -x.

The largest tile is x by x, representing $+x^2$ or $-x^2$

ZERO-SUM PAIR

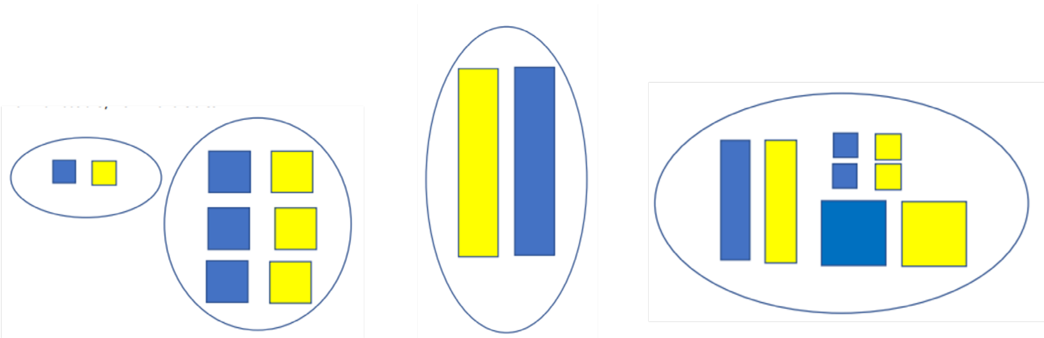
Before we use the tiles to solve some problems, the first thing you need to understand is the Zero-Sum Pair. Take a look at the two colours;



One represents a positive value and one a negative value of the same magnitude. When put together they make zero, just as

-1 + 1 = 0. This is called the **zero-principle** and the two tiles form a zero-sum pair.

Let's take a look at these examples



These are all examples of zero-sum pairs.

Modelling Integers

The first stage to using algebra tiles is to model different integers.

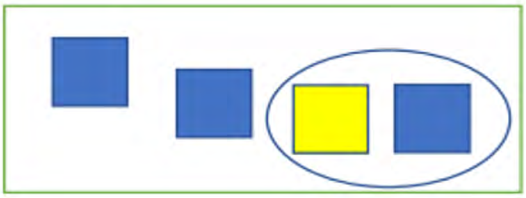
You need to familiarise yourself with the tiles, the meanings of each colour and the concept of zero-sum pairs.

e.g.

+2

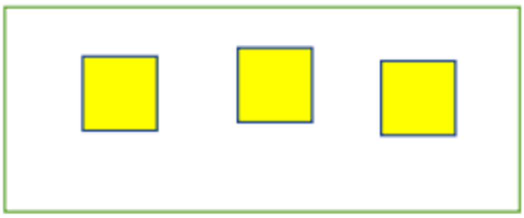
This has only blue tiles, hence it represents +2

(+2) + (-1 + 1) = +2



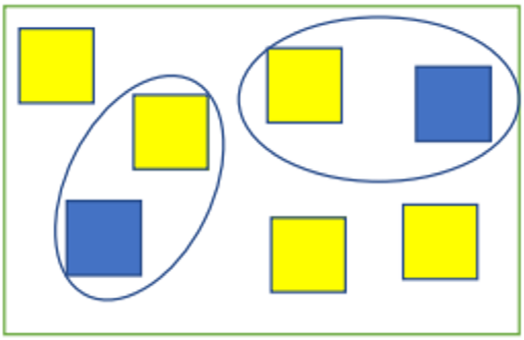
This has 3 blue tiles, and 1 yellow tile. This gives +3 -1 = +2

-3



This has only yellow tiles, hence it represents -3

(-3) + (-1 + 1) + (-1 + 1) = -3



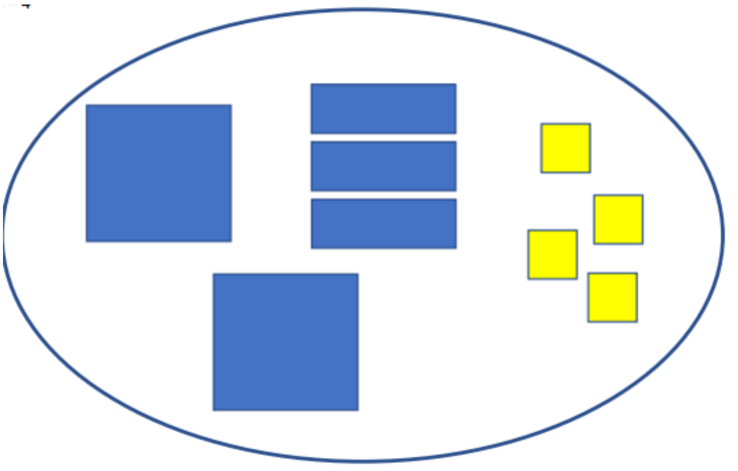
This has 2 blue tiles, and 5 yellow tiles. This gives +2 -5 = -3

Note: The tiles could be any colour representing the positive and negative number.

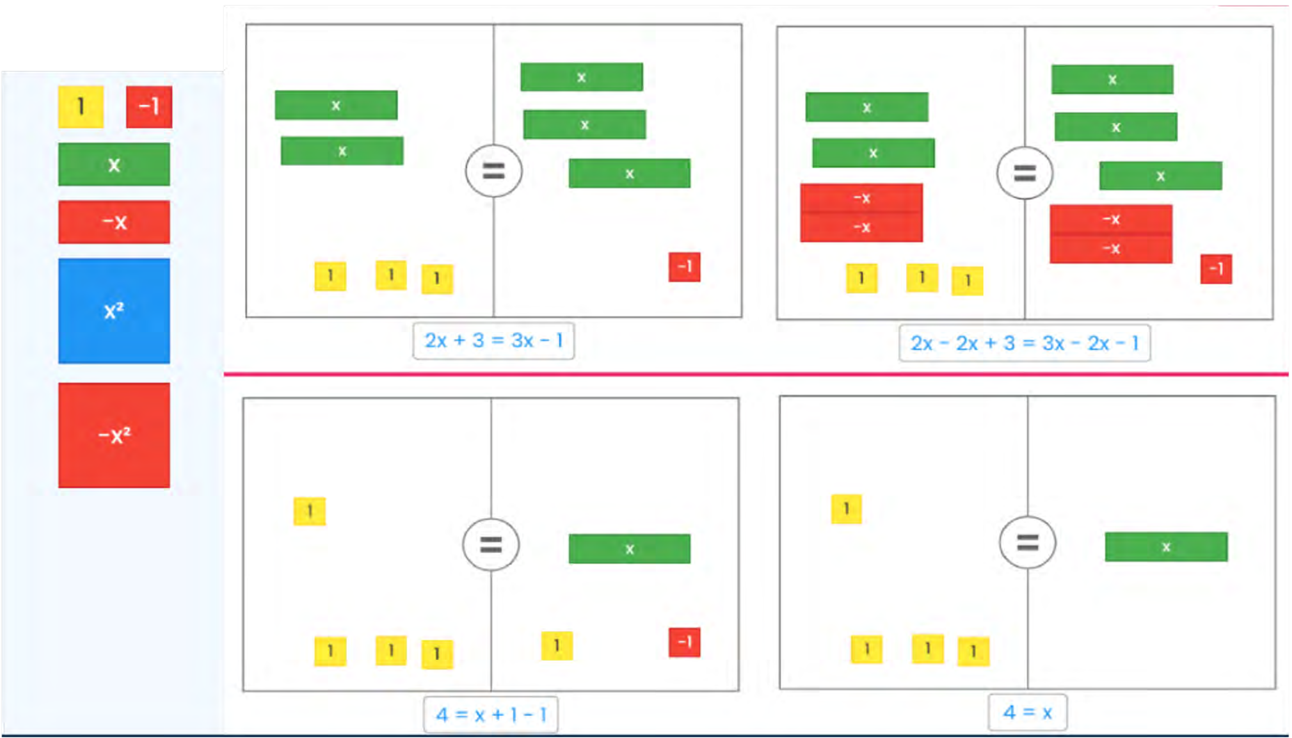
Forming algebraic expressions

We form algebraic expressions by taking the required number of algebra tiles and turning them to show the appropriate colour.

e.g. $2x^2 + 3x - 4$



Let's look at more examples:



Addition and Subtraction With Algebra Tiles

Example 1

Simplify $(2x^2 + 3x - 4) - (x^2 + x - 2)$

Solution

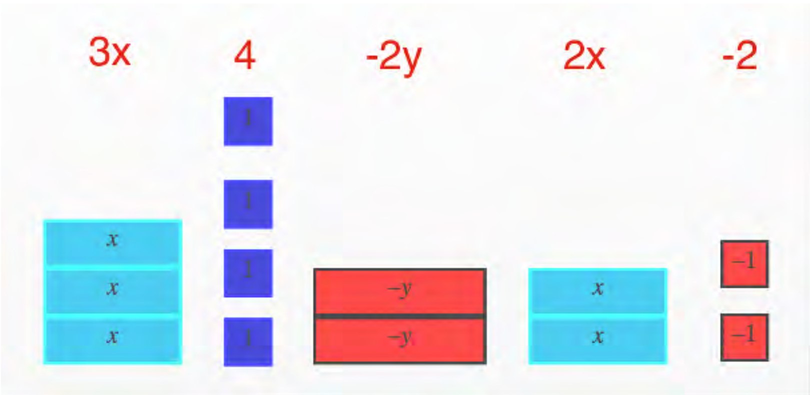
$(2x^2 + 3x - 4)$ $(x^2 + x - 2)$

Therefore, the results is $x^2 + 2x - 2$

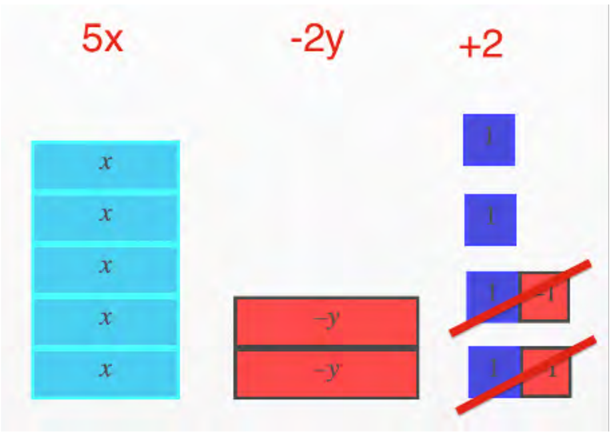
Example 2

Simplify $3x + 4 - 2y + 2x - 2$

Solution



Simply rearranging the tiles, or collecting like terms as we call it in algebra, can allow you to quickly see what we are dealing with:

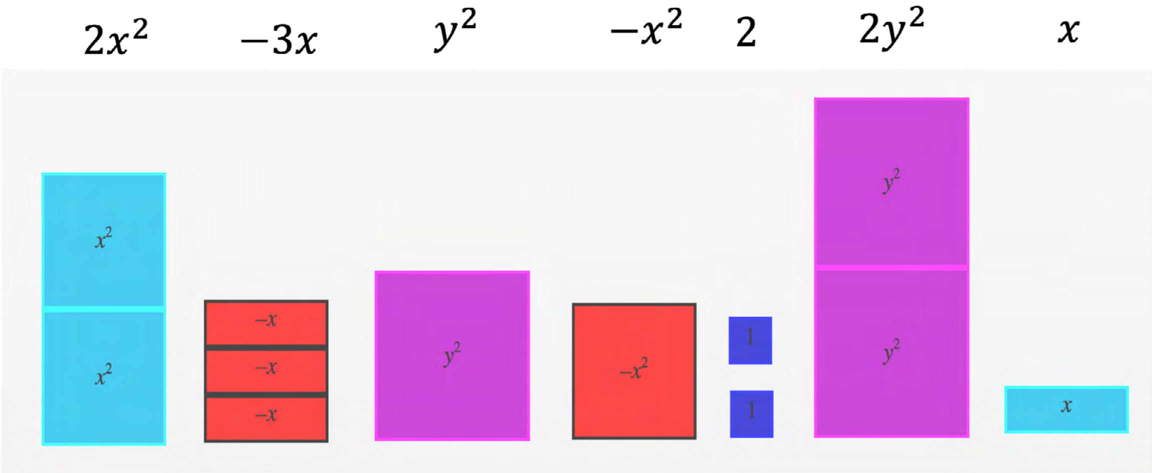


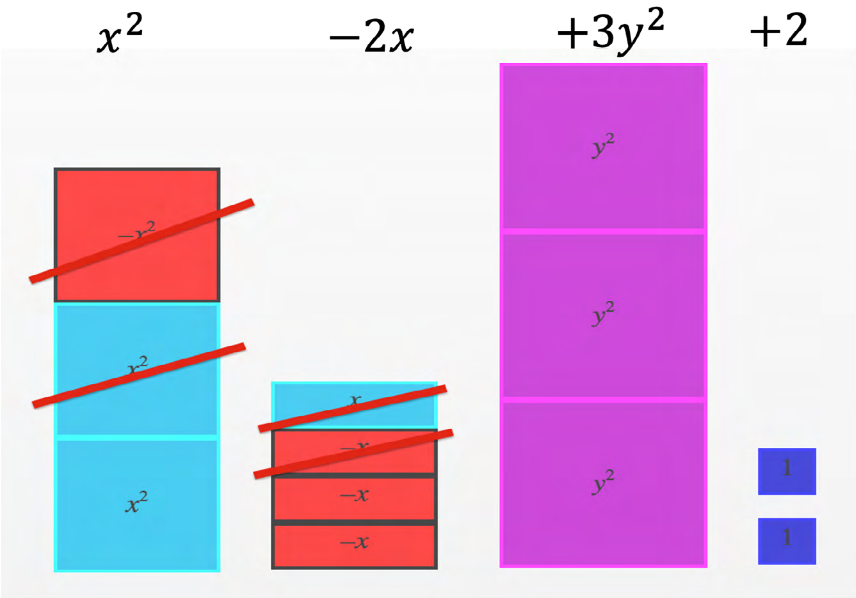
$\therefore 3x + 4 - 2y + 2x - 2 = 5x - 2y + 2$

Example 3

Simplify $2x^2 - 3x + y^2 - x^2 + 2 + 2y^2 + x$

Solution





$\therefore 2x^2 - 3x + y^2 - x^2 + 2 + 2y^2 + x = \boldsymbol{x}^2 - 2\boldsymbol{x} + 3\boldsymbol{y}^2 + 2$

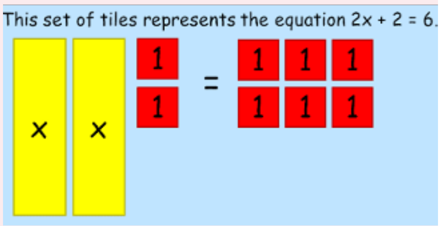
ACTIVITY 3.4: Individual/Pair/Group Work

Exploring Algebraic Expressions Using Algebra Tiles

Purpose: To reinforce understanding of algebraic expressions by using algebra tiles to model addition, subtraction, and multiplication of algebraic terms.

Materials Needed:

- Algebra tiles (variable tiles, unit tiles)
- Worksheet with algebraic expressions to model
- Grid paper or whiteboards for group work



Instructions:

Part 1: Understanding Algebra Tiles

1. Introduction to Tiles:
   - Each group will receive a set of algebra tiles.
   - Unit Tiles: Represent the number 1 (positive or negative).
   - Variable Tiles: Represent the variable x (positive or negative).

2. Discuss and Model:
   - Begin by discussing what each tile represents.
   - Your teacher will model a simple expression like $x + 3$ using the tiles:
     - Place one x-tile and three 1-tiles on the grid paper or board.

Part 2: Modeling Algebraic Expressions

1. Group Activity - Addition:
   - Task: Model the expression $2x + 3 + x + 2$ using the tiles.
   - Steps:
     - Each group places 2 x-tiles and 3 unit tiles on the left.
     - Then, add 1 x-tile and 2 unit tiles to the right.
     - Combine like tiles and write down the simplified expression.
   - Expected Result: The expression simplifies to $3x + 5$.

2. Group Activity - Subtraction:

- Task: Model the expression $3x + 4 - (x + 2)$ using the tiles.
- Steps:
  - Place 3 x-tiles and 4 unit tiles on the grid.
  - Show subtraction by removing 1 x-tile and 2 unit tiles from the model.
  - Write down the simplified expression after the subtraction.
- Expected Result: The expression simplifies to $2x + 2$.

3. Group Activity - Multiplication:

- Task: Model the expression $2x \times 3$ using the tiles.
- Steps:
  - Lay out 2 x-tiles horizontally and use 3 unit tiles vertically to create a rectangle.
  - The group should count the total number of tiles within the rectangle.
  - Write down the result of the multiplication.
- Expected Result: The expression simplifies to $6x$.

Part 3: Real-Life Application

1. Problem-Solving:

- Each group will be given a real-life problem to solve using algebra tiles. For example:
  - "Sarah has $2x$ apples, and she receives 3 more apples each day for a week.

    How many apples does Sarah have after a week?"
- Model the expression $2x + 7(3)$ and solve using tiles.
- Expected Result: The simplified expression is $2x + 21$.

Part 4: Sharing and Reflecting

1. Group Presentations:

- Each group will present one of their models to the class, explaining how they used the tiles to represent the expression.

2. Discussion:

- Discuss how the visual representation helped in understanding algebraic expressions.
- Reflect on any challenges faced during the activity and how they were overcome.

WORKSHEET

Practicing Algebraic Expressions with Algebra Tiles

1. $x + 2 + 3x + 5$
2. $4x + 3 + 2x + 7$
3. $2x + 4 + x + 6$
4. $5x + 8 + 3x + 2$
5. $3x + 5 + 4x + 1$
6. $5x + 7 - (2x + 3)$
7. $4x + 9 - (x + 4)$
8. $6x + 8 - (3x + 5)$
9. $7x + 10 - (4x + 6)$
10. $3x + 12 - (x + 9)$
11. $2x \times 3$
12. $x \times 4x$
13. $3x \times 2$
14. $5x \times 2$
15. $x \times 6x$
16. $2x + 3 - (x + 2)$
17. $3x + 4 + 2x - 5$
18. $5x - 2x + 7 - 3$
19. $4x + 5 - (2x + 1)$
20. $6x - 3 + x + 2$
21. Lydia has $3\boldsymbol{x} + 4$ candies, and she gives away $\boldsymbol{x} + 2$ candies to her friends. How many candies does Lydia have left?
22. John is saving $2\boldsymbol{x}$ dollars every month. After 5 months, how much money has John saved?
23. A rectangle has a length of $4\boldsymbol{x}$ and a width of $3\boldsymbol{x}$. What is the total perimeter of the rectangle?

24. If a garden has a length of $2x + 3$ metres and a width of $x + 5$ metres, what is the perimeter of the garden?

25. Sarah bought $2x + 6$ apples and $3x + 2$ oranges. How many fruits does she have in total?

SIMPLIFICATION OF ALGEBRAIC EXPRESSIONS

FOCAL AREA: EXPAND AND SIMPLIFY OF ALGEBRAIC EXPRESSIONS

Expansion is an essential operation in algebra that involves simplifying or multiplying out algebraic expressions. When an expression contains brackets, or parentheses, expansion allows us to remove them and write the expression as a sum or difference of terms. The process of expansion is also known as "distributing" or "applying the distributive property." Under this focal area, we will learn to expand algebraic expressions and simplify them as well as solve some real-life examples. Before we delve deeper into this concept, let's have fun with this activity.

REINFORCEMENT ACTIVITIES

Activity Title: Distributing and Grouping Objects

Purpose: To help you understand the distributive property, which is the foundation for expanding algebraic expressions.

Materials Needed:

- A collection of small objects (e.g., counters, buttons, or coins)
- Paper and pencil
- A partner

Instructions:

1. **Pair Up**: Find a partner and sit together with your materials.Distribute Objects:
   - Imagine you have 3 bags, and each bag contains 2 pencils and 3 erasers.
   - Use the small objects to represent the pencils and erasers.
   - Place the objects in front of you, grouping them by bag.
2. Visualise the Distributive Property:
   - On your paper, write down the expression that represents what you just did: $3 \times$ (2 pencils + 3 erasers).
   - Now, think about what this means. You have 3 bags and in each bag, there are 2 pencils and 3 erasers.

4. **Expand the Group:**
   - Now, take out all the objects from the bags and group them by type (all pencils together and all erasers together).
   - Count how many pencils and how many erasers you have in total.
   - Write down the new expression that represents the total number of each object: 3×2 pencils $+ 3 \times 3$ erasers.
5. **Discuss with Your Partner:**
   - What did you notice when you grouped the objects?
   - How does the expression change when you distribute the objects?
   - Can you see how the distributive property works in this situation?
6. **Connect to Algebra:**
   - Now, imagine instead of pencils and erasers, you have variables and numbers.
   - How would you write and expand an expression like $3(a + b)$ based on what you just did?
7. **Reflection**:

   After completing this activity, discuss with your partner how understanding the distributive property helps in expanding algebraic expressions. Write down your thoughts and be ready to share these with the class.

Expanding Brackets

Expanding brackets means multiplying each term inside the brackets by the terms outside the brackets. To successfully expand and simplify algebraic expressions one must be systematic and careful with the calculations. The properties of operations and the order of operations (PEMDAS/BODMAS) when working with multiple operations is of much importance. Let's have a quick activity on PEMDAS/BODMAS.

ACTIVITY 3.5: Individual/Pair/Group Work

Understanding the Concept of BODMAS/PEDMAS

Objective: To help you understand and apply the BODMAS/PEDMAS rule for solving mathematical expressions.

Materials Needed:

- Paper and pencils
- A set of number cards (0-9)
- Operation cards (+, -, ×, ÷)
- Parentheses cards
- A calculator (for checking answers)

Activity Instructions:

Step 1: Introduction to BODMAS/PEDMAS

- BODMAS/PEDMAS stands for Brackets (or Parentheses), Orders (or Exponents), Division and Multiplication, Addition and Subtraction.
- This rule helps you to remember the order in which you need to solve parts of a mathematical expression.

Step 2: Group Work - Creating Expressions

1. Form small groups of 3-4 students.
2. Each group must randomly select 4 number cards and 3 operation cards.
3. Arrange the numbers and operations to create a mathematical expression.
   - Example: If your group has the numbers 2, 3, 5, and 4, and the operations +, ×, and -, you might create the expression:
     $3 + (5 \times 4) - 2$.
4. Place parentheses around parts of the expression to change the order of operations if needed.

Step 3: Solving Expressions

1. Exchange your expression with another group.
2. Use the BODMAS/PEDMAS rule to solve the expression you received.
   - Remember:
     - Brackets first.
     - Orders (or exponents) next.

- Then Division and Multiplication (from left to right).
- Finally, Addition and Subtraction (from left to right).

3. Compare answers with the other group to see if they agree. If they don't, discuss the steps each group took to find where the difference occurred.

Step 4: Real-Life Application

1. Think of a real-life situation where you would use BODMAS/PEDMAS.
   - For example, if you were cooking and had to follow a recipe with multiple steps, you'd need to decide the order in which to mix ingredients.
2. Write down a real-life problem that involves using multiple operations and solve it using the BODMAS/PEDMAS rule.
   - Example: You have GH¢50. You spend GH¢20 on groceries, then find a GH¢5 coupon. How much do you have left if you also earned GH¢10 from a small job?

Step 5: Class Discussion

- After completing the activity, we'll come together as a class and discuss what we learned:
  - Why is the order of operations important?
  - What mistakes can happen if we don't follow BODMAS/PEDMAS?
  - Share the real-life problems you created and solved.

Step 6: Reflection

- Write a brief reflection on what you learned from this activity. How does BODMAS/PEDMAS help you in solving maths problems accurately?

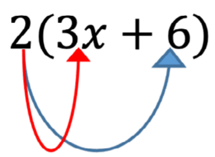
Let's solve some examples on expansion

Worked Examples

Example 1

Expand $2(3x + 6)$

Solution

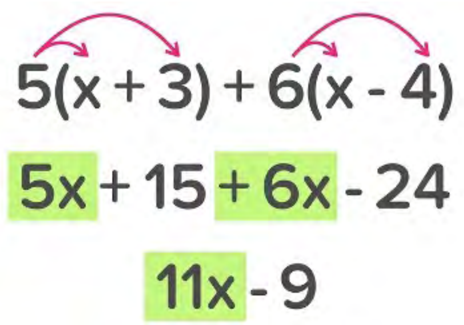


$(2 \times 3x) + (2 \times 6) = 6x + 12$

Example 2

Expand $5(x + 3) + 6(x - 4)$

Solution



Example 3

Expand the expression $3(x + 4)$

Solution

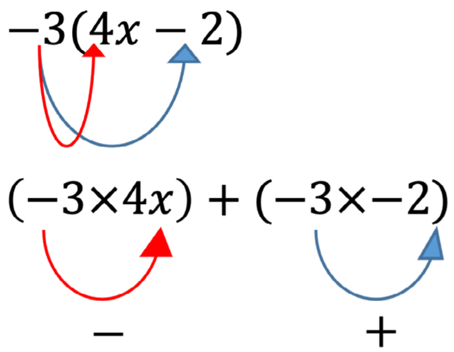
- Distribute the 3 to both terms inside the parentheses: $3(x) + 3(4)$
- Multiply each term: $3x + 12$
- **Final Answer:** $3(x + 4) = 3x + 12$

Let's take a look at what to do when negative values are involved

Example 4

Expand $-3(4x - 2)$

Solution



$-3(4x - 2) = -12x + 6$

Example 5

Ama sold a dress for m cedis and realised that she was making a loss, she decided to double the price to $2m$ cedis. Unfortunately, no one bought the dresses so she decided to reduce the new cost by 5 cedis.

Kofi sold a different dress for m cedis and realised he was making a loss. He then decided to sell it for four times the cost of m. He realised the patronage was encouraging so decided to increase it by 7 cedis.

If m =10, what will be the total cost of buying 3 of Ama's dresses and 2 of Kofi's.

Ama's dresses will be $(2m - 5)$

Kofi's dresses will be $(4m + 7)$

3 times Ama's and 2 times Kofi's will be: $3(2m - 5) + 2(4m + 7)$

$= 6m - 15 + 8m + 14 = 14m - 1$ Given $m = 10$, we have: $14 \times 10 - 1 = 139$,

Or we can think of it like this:

$3[2(10) - 5] + 2[4(10) + 7]$

$= 3(20 - 5) + 2(40 + 7)$

$= 3(15) + 2(47)$

$= 45 + 94$

$= 139$

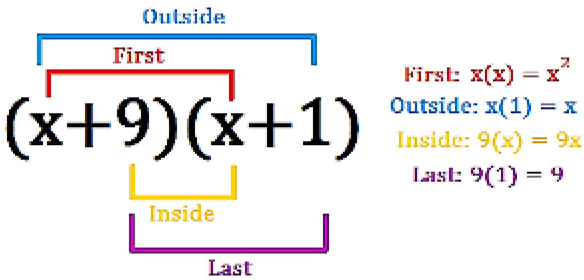
The total cost of buying 3 of Ama's dresses and 2 of Kofi's will be 139 cedis.

Expansion Involving two brackets

To do this, we will use the **FOIL** method. The **FOIL** is an abbreviation for **F**irst **O**utside **I**nside **L**ast.

Example 1

Expand $(x + 9)(x + 1)$ using the FOIL method as shown below;



So $(x + 9)(x + 1) = x^2 + x + 9x + 9 = x^2 + 10x + 9$

Example 2

Expand $(x + 2)(x + 3)$

Solution

Multiply the x outside the bracket over $(x + 3)$ and 2 over$(x + 3)$.

$= x(x) + x(3) + 2(x) + 2(3)$
$= x^2 + 3x + 2x + 6$
$= x^2 + 5x + 6$

Example 3

Expand $(x – 4)(2x + 5)$

Solution

$= x(2x + 5) - 4(2x + 5)$

$= 2x^2 + 5x - 8x - 20$

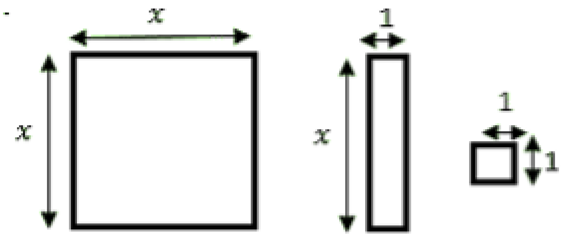
$= 2x^2 - 3x - 20$

Using algebraic manipulatives/tiles for expanding brackets

Expanding (single) brackets is good to model using algebra tiles, because you can physically get out and count however many lots of whatever is in the bracket and, hopefully, with enough of this going on, you'll remember to multiply all the terms inside the bracket by the value outside!

Study this carefully.

Take the algebraic tiles by size $(x \times x)(x \times 1)$ and (1×1) as shown below.



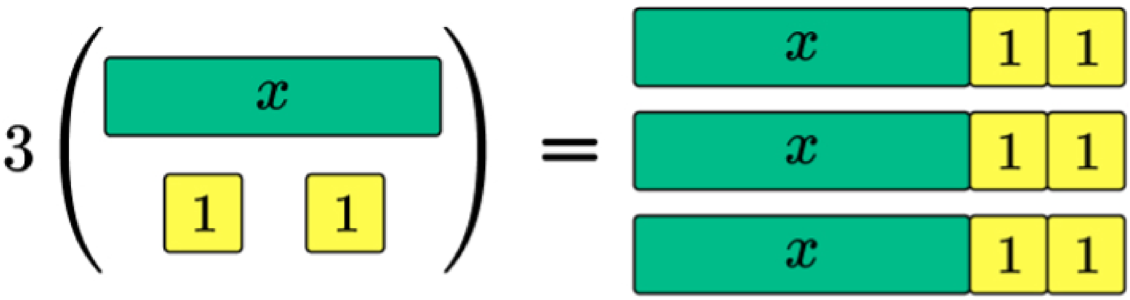
Area of big square $= x \times x = x^2$

Area of rectangle $= x \times 1 = x$

Area of small square $= 1 \times 1 = 1$

Example 1

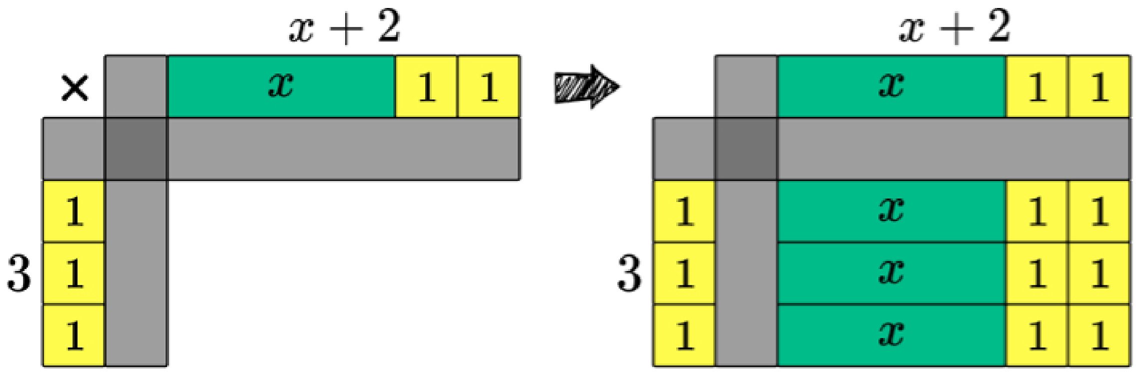
Expand $3(x + 2)$



$3(x + 2) = 3x + 6$

We can see that "3 lots of $x + 2$" gives us $3x + 6$.

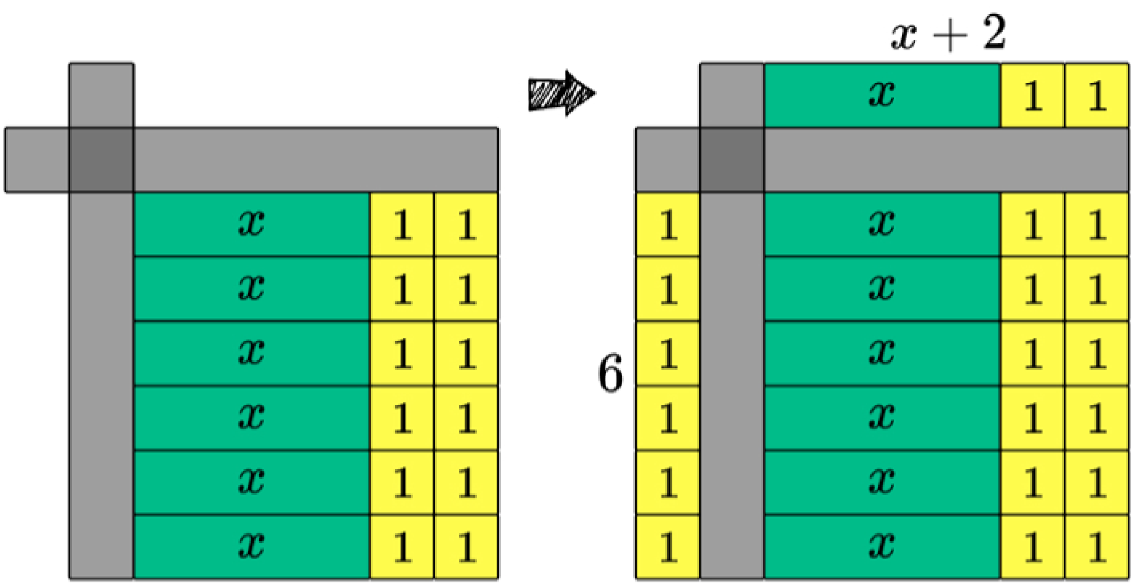
Now, once you understand it this way, we can begin to work towards this idea using a corner frame.



Example 2

Expand 6(x + 2)

Solution



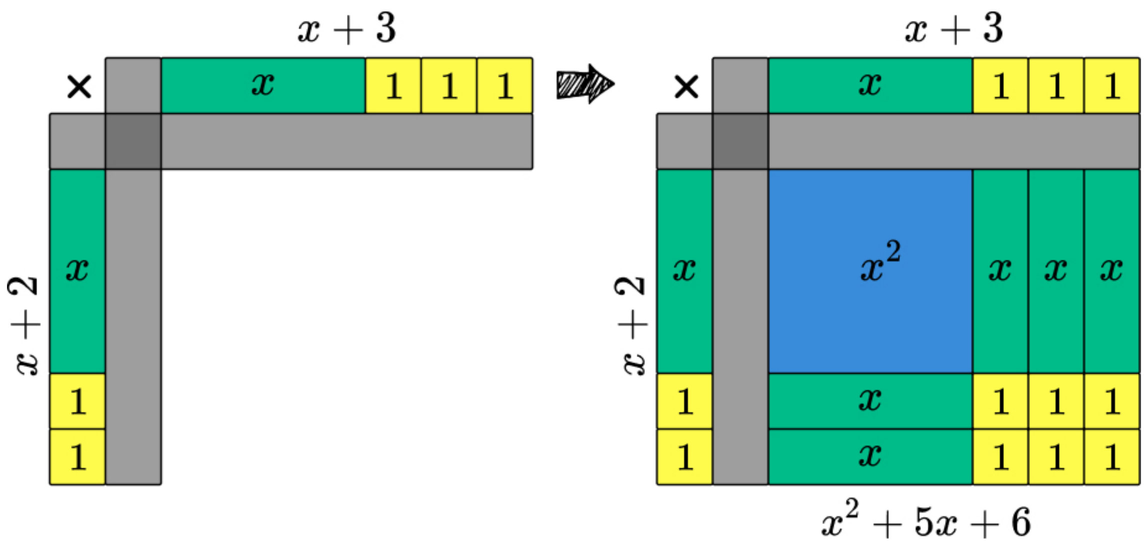
Therefore, the expansion is 6x + 12.

Example 3

Expand $(x + 2)(x + 3)$

Solution

We could build this expression with manipulative as:



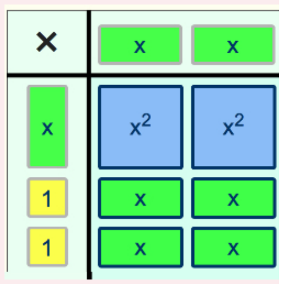
Therefore, the expanded form is $x^2 + 5x + 6$.

ACTIVITY 3.6: Individual/Pair/Group Work

Expanding Algebraic Expressions Using Algebra Tiles

Objective: Understand how to expand (remove brackets from) algebraic expressions using algebra tiles.

Materials Needed:



- Algebra tiles (or paper cut-outs representing different tiles)
    - **Large square tile**: Represents x^2
    - **Rectangle tile**: Represents x
    - **Small square tile**: Represents a constant (1)
- Worksheets or grid paper

Instructions for the Activity:

Introduction to Algebra Tiles:

- **Large square tile** represents $x = x^2$.
- **Rectangle tile** represents $x \times 1 = x$.

- **Small square tile** represents $1 \times 1 = 1$.

Warm-Up Exercise:

- Before starting with algebra tiles, let's do a quick review:
- Simplify $3(x + 2)$ on paper. Write down your answer but don't share it yet!

Step 1: Representing the Expression Using Algebra Tiles

- Let's take the expression $2(x + 3)$.
- Start by placing **two sets** of algebra tiles that represent $x + 3$.
  - **Rectangle tile** for x
  - **Three small square tiles** for the constant 3

Step 2: Expanding the Expression

- Now, let's expand $2(x + 3)$ by multiplying each part by 2.
- Take **two sets** of the algebra tiles and lay them out side by side:
  - Two **rectangle tiles** represent $2 \times x = 2x$
  - Six **small square tiles** represent $2 \times 3 = 6$

Step 3: Writing the Expanded Expression

- Now that you have laid out the tiles, write the expanded expression on your worksheet:
  - $2(x + 3) = 2x + 6$
- Notice how the expression is expanded to remove the brackets.

Step 4: Practice with Different Expressions

- Try the following expressions using your algebra tiles:
  - $3(x + 1)$
  - $2(2x + 4)$
  - $-4(x + 2)$
- For each expression:
  1. Use the tiles to represent the expression inside the brackets.
  2. Expand by multiplying with the number outside the brackets.
  3. Write down the expanded expression.

Step 5: Discuss and Reflect

- After you finish the practice, compare your answers with a partner.
- Discuss:

  - How did the algebra tiles help you visualise the expansion?
  - Was there any expression that was challenging? Why?

Step 6: Challenge Activity (Optional)

- Create your own expression and use the algebra tiles to expand it.
- Share your expression and its expansion with the class.

Conclusion:

- This activity helps you see how algebraic expressions are expanded using physical representations.
- Remember that the key idea in expansion is to distribute the number outside the bracket to each term inside the bracket.

Extension:

- Try expanding expressions with a negative sign, like $-2(x - 3)$. Use the tiles to see how subtraction is represented!

ALGEBRAIC EQUATIONS

FOCAL AREA: TRANSLATING WORD PROBLEMS INTO ALGEBRAIC EQUATIONS AND SOLVING FOR THE UNKNOWN PICTORIALLY AND SYMBOLICALLY

Understanding how to translate real-life problems into equations and solving for the unknown is a key skill in algebra. This process involves recognising patterns, identifying variables and using pictorial and symbolic methods to find solutions. Pictorial methods, like drawing diagrams, help you see the problem. Symbolic methods, like writing equations, provide a more abstract approach. By combining these methods, you will understand better how to turn real-life problems into algebraic equations and solve them.

Let's do this reinforcement activity before we begin.

REINFORCEMENT ACTIVITIES

Word Problem Scavenger Hunt

Purpose: This activity will help you get comfortable with identifying key information in word problems and translating them into algebraic expressions. By working together and using clues, you'll practice breaking down word problems into simple expressions.

Instructions:

1. **Divide into Teams:**

   You will be divided into small teams of 3-4 students. Each team will work together to solve the challenges.

2. **Scavenger Hunt Clues:**

   Around the classroom, you'll find envelopes with clues and word problems. Each envelope contains a word problem that you need to translate into an algebraic expression.

3. **Solving the Problem:**

   - Open the envelope and read the word problem together.
   - Discuss the problem with your team and identify the key information (like quantities, relationships, and the unknowns).

- Translate the problem into an algebraic expression by deciding what operation (addition, subtraction, multiplication, or division) is being described.
- Write down the expression on your answer sheet.

4. **Check Your Work:**

   After translating the problem into an expression, solve the expression to find the value of the unknown. Write your solution clearly on the answer sheet.

5. **Fun Twist - Clue to the Next Envelope:**

   After solving each problem, you will receive a hint or clue to find the next envelope. Use this clue to locate the next challenge in the classroom.

6. **Final Challenge:**

   Once you've solved all the word problems and translated them into expression, each team will use their expression to decode a final message. The first team to decode the message wins a small prize!

Example

"Sarah has 5 more apples than twice the number of apples John has. If John has x apples, how many apples does Sarah have?"

Step 1:

Identify key information:

- John's apples = x
- Sarah's apples = 5 more than twice John's apples.

Step 2:

Translate into an expression:

Sarah's apples = $2x + 5$

Step 3:

Write the expression on your answer sheet and solve it if needed.

Remember:

- Work together, communicate clearly, and help each other understand the problems.
- Have fun with the clues and enjoy the challenge!

Good luck, and let the scavenger hunt begin! 

Identifying The Unknown in Real-Life Scenarios and Translating It into Equations

In algebra, unknowns are variables that represent numbers we do not yet know. These variables are usually denoted by letters such as *x*, *y*, *or z*. An unknown is a placeholder for a value that can change or that we need to find. This reinforcement activity will help you identify the unknown in a real-life problem

Example 1

Maria has three times as many books as David. If Maria gives away 8 books, she will have 28 books left. Write an equation to represent the problem.

Solution

Step-by-Step Approach to Writing an Equation and Solving It:

Step 1: Understand the Problem

- **Read the problem carefully.** Identify the key information and what you are being asked to find.
- **Identify the quantities involved**:
  - The number of books Maria has initially.
  - The number of books David has.
  - The number of books Maria gives away.
  - The number of books Maria has left after giving some away.

Step 2: Define the Unknown

- **Let x be the number of books David has**.

  This is the quantity we need to find.

Step 3: Identify the Relationships and Operations

- **Relationship 1**: Maria has three times as many books as David.
- This can be written as:

  Maria's books = 3 × David's books = $3x$
- **Operation 1**: Maria gives away 8 books.
  - This changes the number of books Maria has to $3x - 8$.

- **Relationship 2**: After giving away 8 books, Maria has 28 books left.
  - This can be written as:
    $3\boldsymbol{x} - 8 = 28$

Example 2

Andam has some mangoes. If she buys 4 more, she will have 12 mangoes. Write an equation that represents this situation.

Solution

Identify the unknown

- Andam currently has an unknown number of mangoes. Let the unknown number of mangoes be x.
- Now, when Andam buys 4 more mangoes, she adds 4 mangoes to her current number of mangoes so she now has $x + 4$ mangoes.
- After buying 4 more mangoes, the total number of mangoes Andam will have is 12, represented by $x + 4 = 12$.
- Therefore, the equation is given by $\boldsymbol{x} + 4 = 12$

Example 3

Sam has some books. If he gets 5 more, he will have 17 books. Write an equation that represents this situation.

Solution

- Sam currently has an unknown number of books. Let's call this number p
- After getting 5 more books, the total number of books becomes $\boldsymbol{p} + 5$
- After getting 5 more books, the total is equal to 17, represented by $\boldsymbol{p} + 5 = 17$
- So, the equation is: $\boldsymbol{p} + 5 = 17$

Example 4

Bryan has some money. If, he spends GH¢10, he will still have GH¢30 left. Write an equation that represents this situation.

Solution

- Bryan currently has an unknown amount of money. Let's represent the unknown amount to be x.

- Bryan spends GH¢10, so we subtract 10 from his original amount to get $x - 10$
- After spending GH¢10, Bryan has GHÇ30 left, represented by $x - 10 = 30$
- So, the equation is: $x - 10 = 30$

ACTIVITY 3.7: Individual/Pair/Group Work

Translating Word Problems into Equations

Purpose: To practise and reinforce your ability to translate word problems into algebraic equations and solve them.

Instructions:

1. **Pair Up:**

   Find a partner to work with. You will be working together to solve word problems.

2. **Choose a Word Problem:**

   Each pair will be given a word problem to solve. Read the problem carefully and discuss it with your partner.

3. **Identify the Key Information:**
   - Identify the quantities given in the problem.
   - Determine what you are being asked to find (the unknown).
   - Identify the operations (addition, subtraction, multiplication, division) described in the problem.

4. **Translate the Word Problem into an Equation:**
   - Work together to write an algebraic equation that represents the problem.
   - Make sure to use a variable (like x or y) for the unknown quantity.

5. **Share Your Solution:**
   - Once you have written the equation, explain it to another pair or to the class.
   - Discuss how you translated the word problem into the equation and any challenges you faced.

6. **Create Your Own Word Problem:**
    - Create your own word problem for another pair to translate it into an equation.
    - Be creative! Think of a real-life situation that can be translated into an equation.
    - Write down the problem, translate it yourself, and then exchange it with another pair.

Extension

Solve a Peer's Problem:

- Solve the word problem created by another pair.
- Write the equation and solve it just like you did with the first problem.

Class Discussion:

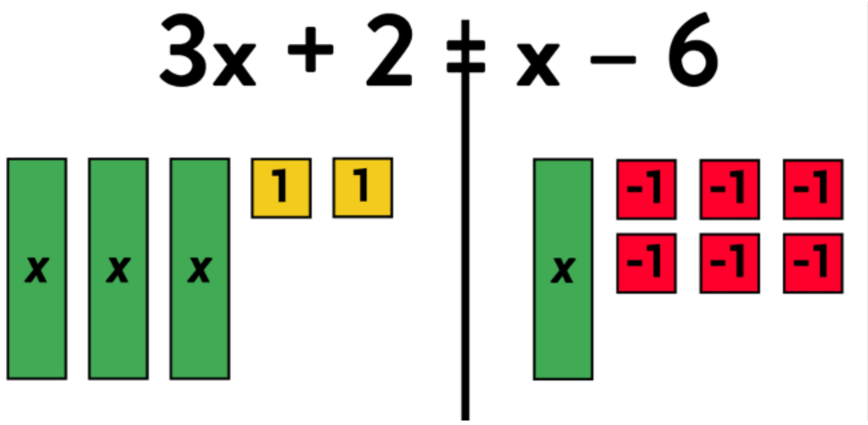
- After everyone has had a chance to solve their problems, we will come together as a class to discuss the different problems, equations, and solutions.
- Reflect on what you learned and share any tips or strategies that helped you.

Solve the Equations Using Pictorial and Symbolic Methods

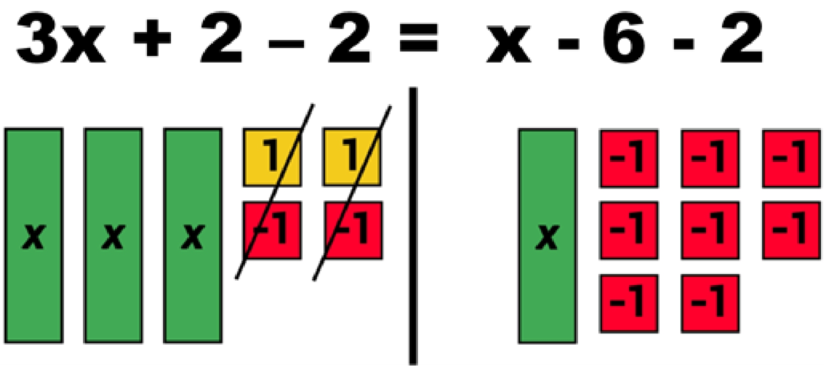
Example 1

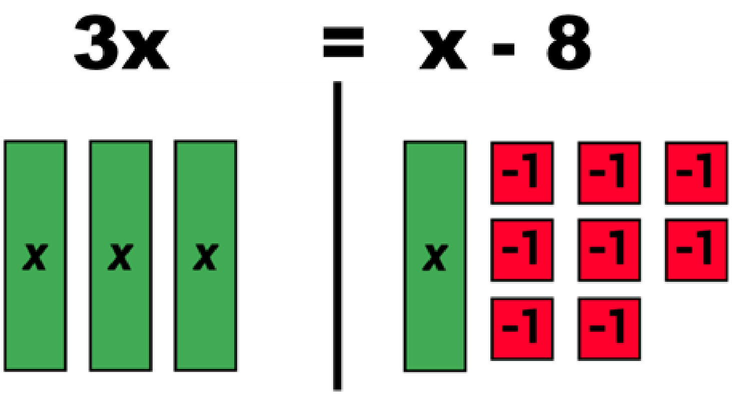
Solve $3x + 2 = x - 6$

Solution

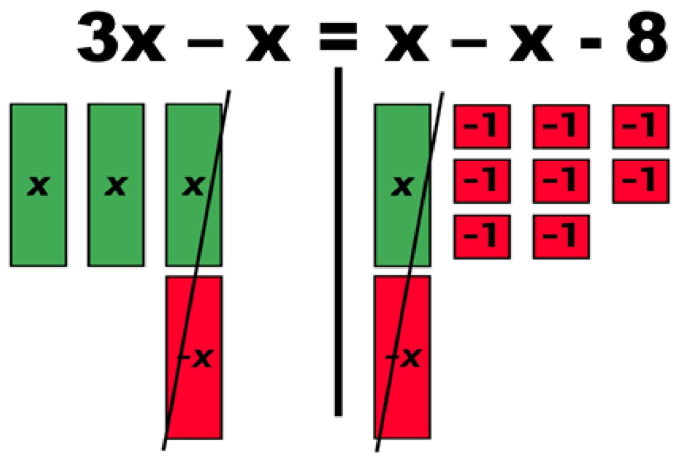


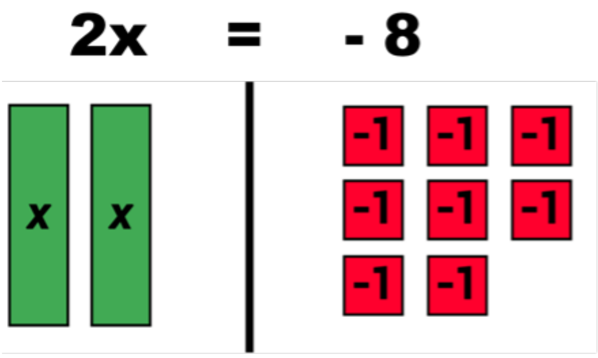
To isolate the variable, you need to get rid of positive 2. To do this, you need to add two negative tiles to make a zero pair. Adding negative 2 to one side would mean you would add to the other side to keep the equation balanced.



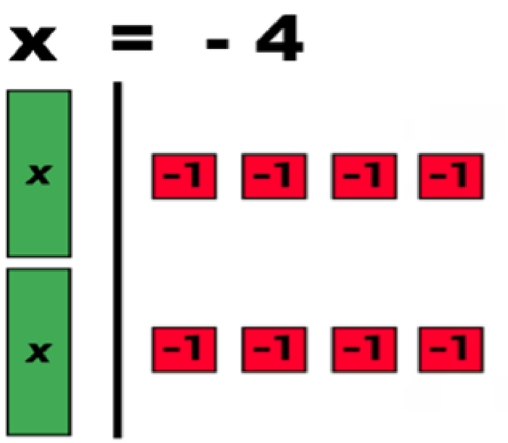


After you have removed the two zero pairs on the left side of the equation, remember we want all variables on one side, since we want to find out what one x is equal to. You would then add a negative *x* to both sides to create another zero pair.





Lastly, to get the solution for one *x*, students would divide the remaining red tiles among the two x tiles: $x = -4$.



Example 2

Solve $3x = x + 1$

Solution

Subtract x from both sides of the equation, to leave the x's on one side of the equation only.

$3x - x = x - x + 1$

$2x = 1$

Divide both sides of the equation by 2

$\frac{2x}{2} = \frac{1}{2}$

$\boldsymbol{x = \frac{1}{2}}$

Example 3

Solve $5x + 1 = 2x + 7$

Solution

Subtract $2x$ from both sides of the equation, to leave the x's on one side of the equation only.

$5x - 2x + 1 = 2x - 2x + 7$

$3x + 1 = 7$

Subtract 1 from both sides of the new equation to leave the x's on their own.

$3x + 1 - 1 = 7 - 1$

$3x = 6$

Divide both sides of the equation by 3 to solve for a single x.

$\frac{3x}{3} = \frac{6}{3}$

$\boldsymbol{x} = 2$

ACTIVITY 3.8: Individual/Pair/Group Work

Reinforcing Solving Equations Pictorially and Symbolically

Purpose: To reinforce your understanding of solving equations using both pictorial methods (balance method and algebra tiles) and symbolic methods.

Instructions:

1. **Divide into Groups:**
   - Form small groups of 3-4 students. Each group will work together to solve equations using both pictorial and symbolic methods.
2. **Algebra Tiles Practice:**
   - Each group will receive a set of algebra tiles.
   - Your teacher will give you an equation to solve using algebra tiles. For example: $2x + 3 = 7$.
   - Use the algebra tiles to represent the equation. Remember:
     - Use one colour or type of tile to represent the variable x.
     - Use another colour or type to represent the constant numbers.
   - Arrange the tiles to visually "balance" the equation on both sides of the equal sign.
   - Remove or add tiles to isolate the variable on one side of the equation, showing how you would solve it step by step.
   - Write down your steps as you solve the equation with the tiles.
3. **Symbolic Method Practice:**
   - Solve the equation symbolically, showing each step of the algebraic process.
   - Compare the symbolic solution to the solutions you found using the algebra tiles.
4. **Group Discussion:**
   - Once all groups have completed their equations, discuss how the pictorial and symbolic methods are related.
   - How do the algebra tiles help you understand what you are doing when you solve equations symbolically?

5. **Challenge Round:**
    - Your teacher will give you a more challenging equation, such as $3x - 2 = 4x + 1$.
    - Use the algebra tiles and symbolic method to solve this new equation.
    - See which method your group finds easiest and why.
6. **Create Your Own Equation:**
    - Each group will create a new equation for another group to solve.
    - Use a mix of addition, subtraction, multiplication, or even a combination of variables.
    - Solve your own equation first using both pictorial and symbolic methods, then pass it on to another group.
7. **Equation Swap:**
    - Exchange equations with another group and solve their equation using all two methods.
    - After solving, compare your results with the group that created the equation.
8. **Class Reflection:**
    - Come together as a class to reflect on what you've learned.
    - Discuss which method you found most helpful and why.
    - Share any tips or strategies that helped you understand how to solve equations.

FOCAL AREA 2: CREATING WORD PROBLEMS FROM A GIVEN EQUATION

Example 1

Write a word equation for the equation below.

Solution

From the above illustration, we assume that there is an unknown number of bell peppers in the bag that can be added to the 3 peppers to give a total of 7.

Now, let us use x to represent this unknown number of peppers in the bag.

This can be represented as:

$2x$ 3 7

Therefore, the equation $2x + 3 = 7$ can be written from the illustration above and then solved to find x.

We can also use words to explain the equation from the above illustrations:

Sarah had some bell peppers. Her friend gave her 3 more bell peppers. Sarah now has a total of 7 bell peppers.

How many bell peppers did Sarah have to begin with?

Example 2

Use words to explain this equation $x - 2 = 4$

Solution

There are many options here, but here is an example.

Jake had some marbles. He gave away 2 marbles to his friend, and now he has 4 marbles left. We could then solve the equation to find out how many marbles Jake had at the start.

Explanation:

In this problem, the equation $x - 2 = 4$ represents the situation:

- x is the unknown number of marbles Jake had initially (before giving any away)
- Jake gave away 2 marbles, which is represented by the "-2" in the equation.
- After giving away the marbles, Jake has 4 marbles left, which is represented by the "$= 4$" in the equation.
- So, to find out how many marbles Jake had initially, we need to determine the value of x in the equation $x - 2 = 4$

Example 3

Write a word equation to represent $5x - 2 = 4 + 2x$

Solution

For example, Emily is collecting stickers. She had x stickers at the beginning.

Now she has 2 fewer stickers than 5 times the number of stickers she started with.

This is the same as if she adds 4 stickers to her collection, plus 2 times the amount she started with.

How many stickers did Emily start with?

Explanation:

In this problem, the equation $5x - 2 = 4 + 2x$ represents the situation:

- x is the total number of stickers Emily started with.
- The term $5x - 2$ represents the number of stickers Emily has now, which is 2 fewer than 5 times the number she started with.
- The term $4 + 2x$ is equal to the number of stickers Emily has, with is 2 times her starting number plus 4.
- To find out how many stickers Emily started with, we need to solve the equation:

  $5x - 2 = 4 + 2x$.

ACTIVITY 3.9: Individual/Pair/Group Work

Writing Word Problems to Represent Given Equations

Activity Overview

In this activity, you will work in pairs or small groups to create your own word problems based on given algebraic equations. This activity will help you reinforce your understanding of how real-life situations can be translated into mathematical equations.

Step-by-Step Instructions

1. Pair Up or Form Small Groups:
   - Find a partner or form a group of 3-4 students.
   - Each group will be given a set of algebraic equations to work with.
2. Analyse the Equation:
   - Look at the given equation and discuss what it represents.
   - Think about real-life situations where this equation could be used. For example, if the equation is $3x + 2 = 11$, consider scenarios involving quantities, costs, or distances.
3. Create a Word Problem:
   - Together, create a word problem that fits the equation.
   - Ensure your problem includes a clear scenario, quantities and the question that needs to be answered.
   - Example: If the equation is $3x + 2 = 11$, a possible word problem could be: "Sarah bought 3 packs of markers and paid an additional GH¢2 for shipping. If her total cost was GH¢11, how much did each pack of markers cost?"
4. Write Down Your Word Problem:
   - Write your word problem clearly on a piece of paper or a whiteboard.
   - Double-check to make sure that when you solve the word problem, it matches the original equation.
5. Share and Swap:
   - Once your group has created a word problem, exchange your problem with another group.

- Each group will then try to solve the word problem they received and verify that the equation they derive matches the one originally given.

6. Discussion:
   - After solving the problems, discuss as a class how you came up with your word problems.
   - Talk about different ways the same equation could represent different real-life situations.

7. Reflect:
   - Think about what you learned from this activity.
   - Consider how writing word problems can help you better understand the relationship between algebraic equations and real-life scenarios.

Example Equations to Work With:

1. $2x + 3 = 9$
2. $5x - 4 = 16$
3. $4x + 7 = 19$
4. $3x - 5 = 10$
5. $6x + 2 = 20$

Remember, the goal is to have fun while learning how to create and solve word problems that match algebraic equations!

REVIEW QUESTIONS

A. Simplify the following expressions

1. $5x + 7 + 2x - 3$
2. $3a - 4b + 6a + 2b$
3. $8m - 4 + 5m + 9$
4. $4x - 5 + 2x + 10$
5. $7y - 3 - (2y - 5)$
6. $5p + 8 - (3p - 4)$
7. $6x - (4x - 2) + 3$
8. $9a - (2a + 6) + 7$
9. $3x \times 2 + 4x \times 3$
10. $5a \times 4 + 2b \times 6$
11. $7m \times 3 - 2m \times 4$
12. $8p \times 2 - 3p \times 5$

B. Expand the following expressions

1. $3(x + 4)$
2. $5(2x - 3)$
3. $-2(3x + 7)$
4. $4(x - 5)$
5. $6(2x + 1)$
6. $-3(2x - 4)$
7. $7(x + 2)$
8. $-5(x - 6)$
9. $2(3x + 5)$
10. $-4(2x - 3)$
11. $(x + 3)(x + 5)$
12. $(2x - 4)(x + 6)$
13. $(x - 7)(x - 2)$

14. $(3x + 2)(x - 3)$
15. $(x + 8)(2x - 5)$

C. Word Problems

1. In planning a birthday party, each bottle of Coca- Cola costs *GH¢*1.50 and $(2x + 5)$ bottles need to be bought. Form an algebraic expression with this statement to find the total cost of coca-cola needed for the party.
2. In a rectangular garden with dimensions length = $(x + 3)$ metres and width = $(2x - 1)$ metres, expand the expression using algebraic tiles or the FOIL method, to find the area of the garden.
3. You are setting up tables for a party. Each table can seat $x + 4$ guests. If you have 4 tables, how many guests can be seated in total?
4. Sarah has a rectangular garden with dimensions $2x + 6$ metres by $3x + 9$ metres. She wants to find the area of the garden. How should this be done?
5. Consider the expression $2x(3x + 4) - 4(2x + 4)$. Simplify the expression and explain the steps you took? What strategies did you use to manipulate the terms and arrive at the final result?
6. Sarah has 3 times as many apples as Tim. If Tim has 8 apples, how many apples does Sarah have?
7. John buys a notebook and a pencil. The notebook costs 2 dollars more than the pencil. If the pencil costs 5 dollars, how much does the notebook cost?
8. A movie theatre sells tickets for 12 dollars each and snacks for 5 dollars each. If a family spends a total of 68 dollars on tickets and snacks, and they bought 4 snacks, how many tickets did they buy?
9. Maria has twice as many red beads as blue beads. If she has a total of 24 beads, how many red beads does she have?
10. Alex buys 3 books and a pen. The total cost is 37 dollars. If each book costs 9 dollars, how much does the pen cost?

D. Solve the following equations

1. $3x + 5 = 20$
2. $2x - 4 = 10$
3. $4x + 3 = 15$

4. $6x - 8 = 16$
5. $5x + 2 = 17$
6. $3n - 2 = n$
7. $y + 5 = 12$
8. $5p - 9 = 2p + 6$
9. $2y - 7 = 13$

E. Write a word problem for the following equations

1. $2x + 5 = 15$
2. $4x - 3 = 9$
3. $3x + 7 = 19$
4. $5x - 8 = 12$
5. $6x + 4 = 22$
6. $7x - 2 = 26$
7. $8x + 9 = 33$
8. $3x + 6 = 24$
9. $10x + 3 = 43$
10. $4x + 11 = 27$

Project 1: Exploring and Operating on Real Numbers

Objective: In this project, you will categorise real numbers into different subsets, explore the various types of counting numbers and perform operations on these numbers using real-life contexts.

Part 1: Categorising Real Numbers

Task 1: Categorisation of Real Numbers

- Start by reviewing the types of real numbers:
  - **Natural/Counting Numbers:** $1,2,3,4,\ldots$
  - **Whole Numbers:** $0,1,2,3,\ldots$
  - **Integers:** $\ldots,-3,-2,-1,0,1,2,3,\ldots$
  - **Rational Numbers:** Numbers that can be expressed as a fraction $\frac{a}{b}$ where $b \neq 0$
  - **Irrational Numbers:** Numbers that cannot be expressed as a fraction, e.g., $\sqrt{2}, \pi$

Task 2: Sorting Activity

- Use a set of provided numbers and categorise them into natural, whole, integer, rational, and irrational numbers. Create a visual chart showing the classification.

Part 2: Exploring Subsets of Counting Numbers

Task 1: Subsets of Counting Numbers

- Explore the subsets of counting numbers:
  - **Even Numbers:** Numbers divisible by 2
  - **Odd Numbers:** Numbers not divisible by 2
  - **Prime Numbers:** Numbers greater than 1 with only two factors: 1 and itself
  - **Composite Numbers:** Numbers with more than two factors

Task 2: Interactive Sorting

- Sort a list of counting numbers into even, odd, prime, and composite categories. For each number, explain why it belongs to a particular subset.

Part 3: Performing Operations on Real Numbers

Task 1: Real-Life Contexts

- Apply operations on real numbers in the following real-life contexts:
  - **Budgeting:** Calculate your monthly expenses using positive and negative integers to represent income and expenses.
  - **Measurements:** Use rational and irrational numbers to measure distances, such as π in calculating the circumference of a circle.
  - **Temperature Changes:** Record daily temperatures and calculate the difference between the highest and lowest temperatures using integers.

Task 2: Practical Problem Solving

- Solve the following problems:
  1. You have $50 and spend $30. How much do you have left? Categorize the numbers used and the result.
  2. You need to measure the length of a rope that is approximately $\sqrt{2}$ meters. How would you categorise this number?
  3. You track your daily expenses as GH¢20, - GH¢15, and GH¢35. What is your total balance?

Materials Needed:

- Number cards or sticky notes
- Chart paper and markers
- Calculator
- Graph paper

Rubric for Assessment:

Criteria	Excellent (4)	Good (3)	Fair (2)	Needs Improvement (1)
Categorisation of Real Numbers	Accurately categorizes all numbers	Mostly accurate with minor errors	Somewhat accurate with several errors	Incorrect or incomplete categorization
Exploration of Counting Number Subsets	Accurately identifies and explains	Mostly accurate with minor errors	Basic understanding with some errors	Incorrect or incomplete identification
Application of Operations in Real-Life	Accurately solves all problems with clear explanation	Solves most problems correctly	Solves problems with some errors	Incorrect solutions with little explanation
Presentation and Organisation	Clear, well-organised, and visually appealing	Mostly clear and organised	Somewhat clear but disorganised	Unclear or poorly organised

Project 2: Mastering Fractions and Their Operations

Objective: In this project, you will explore fractions, compare and order them and solve real-life problems involving the four basic operations on fractions.

Part 1: Understanding Fractions as Quotients

Task 1: Naming and Comparing Fractions

- Start by expressing different numbers as fractions (quotients of two integers) where the denominator is not zero.
  For example, $\frac{3}{4}, -\frac{7}{2}$ and $\frac{5}{1}$.
- Compare and order these fractions from smallest to largest.

Task 2: Visual Representation

- Use a number line to plot these fractions and visually compare their sizes.
- Discuss how fractions like $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent.

Part 2: Recognising and Naming Equivalent Fractions

Task 1: Pictorial Representation

- Draw or use fraction circles or bars to represent fractions like $\frac{1}{2}, \frac{2}{4}$ *and* $\frac{4}{8}$.
- Recognise and name equivalent fractions by observing the pictorial representations.
  Discuss why these fractions are considered equivalent.

Task 2: Number Line Representation

- Place equivalent fractions on a number line to visualise their equality.
  For example, plot $\frac{1}{2}, \frac{2}{4}$ *and* $\frac{4}{8}$ on the same number line and observe how they align at the same point.

Part 3: Comparing and Ordering Fractions with Like Denominators

Task 1: Using Visuals to Compare Fractions

- Compare fractions with like denominators, such as $\frac{3}{8}$ *and* $\frac{5}{8}$, using fraction bars or circles.
- Order a set of fractions from smallest to largest using pictorial representations and symbols like >, <, and =.

Part 4: Solving Problems on Fractions Using the Four Basic Operations

Task 1: Real-Life Problem Solving

- Solve the following problems involving addition, subtraction, multiplication, and division of fractions:
  1. You have $\frac{3}{4}$ of a chocolate bar and eat $\frac{1}{4}$ of it. How much is left?
  2. If you buy $\frac{2}{3}$ kg of apples and $\frac{1}{3}$ kg of oranges, what is the total weight of the fruits?
  3. You want to share $\frac{5}{6}$ of a pizza equally among 3 friends. How much does each friend get?
  4. A recipe calls for $\frac{2}{5}$ cup of sugar, but you want to make half the recipe. How much sugar do you need?

Task 2: Application in Daily Life

- Create your own word problem involving fractions and solve it using one or more of the four operations.

  Present your problem and solution to the class.

Materials Needed:

- Fraction circles or bars
- Number line chart
- Graph paper
- Colored pencils
- Calculator

Rubric for Assessment:

Criteria	Excellent (4)	Good (3)	Fair (2)	Needs Improvement (1)
Understanding and Comparing Fractions	Accurately names and compares all fractions	Mostly accurate with minor errors	Somewhat accurate with several errors	Incorrect or incomplete comparisons
Equivalent Fractions Representation	Correctly identifies and represents all equivalent fractions	Mostly accurate with minor errors	Basic understanding with some errors	Incorrect or incomplete representation

Criteria	Excellent (4)	Good (3)	Fair (2)	Needs Improvement (1)
Comparing Fractions with Like Denominators	Accurately compares and orders all fractions	Mostly accurate with minor errors	Somewhat accurate with several errors	Incorrect comparisons and ordering
Solving Fraction Problems	Solves all problems correctly with clear explanations	Solves most problems correctly	Solves problems with some errors	Incorrect solutions with little explanation
Creativity and Presentation	Presents an original and creative word problem with a clear solution	Presents a good problem with some creativity	Presents a basic problem with a solution	Problem is unclear or solution is incorrect

This project will help you develop a strong understanding of fractions, enhance your problem-solving skills, and apply your knowledge to real-life situations.

Project 3: Mastering Algebra through Real-Life Applications

Objective: In this project, you will model real-life situations into mathematical statements, manipulate algebraic expressions, and solve problems by creating and working with equations.

Part 1: Modelling Real-Life Situations into Mathematical Statements

Task 1: Real-Life Scenario to Algebraic Expression

- Think of a real-life situation where you need to find the total cost of multiple items. For example, "You buy 3 pencils at x dollars each and 2 notebooks at y dollars each."
- Write a mathematical statement to represent the total cost. For example: $3x + 2y$.

Task 2: Performing Operations on the Expression

- Calculate the total cost if $x = 2$ dollars and $y = 5$ dollars.
- Use different values of x and y to see how the total cost changes.

Part 2: Expanding and Simplifying Algebraic Expressions

Task 1: Removing Brackets and Simplifying

- Start with an algebraic expression that includes brackets, such as $3(a + 4) -2(b - 3)$.
- Expand the expression by removing the brackets and simplifying it using the properties of operations (distributive property, combining like terms, etc.).
- For example, $3(a + 4) - 2(b - 3)$ becomes $3a + 12 - 2b + 6$, then simplifies to $3a - 2b + 18$.

Task 2: Applying Properties of Operations

- Simplify another expression using similar steps. For example, simplify $4(2x - 3) + 5(2x + 1)$.
- Explain each step as you expand and simplify the expression.

Part 3: Expressing Problems as Equations and Solving

Task 1: Translating Problems into Equations

- Think of a problem like: "The sum of twice a number and 5 is equal to 15. What is the number?"
- Express this problem as an equation: $2x + 5 = 15$.

- Solve the equation to find the unknown x. In this case, $x = 5$.

Task 2: Pictorial and Symbolic Representation

- Draw a simple diagram to represent the problem and solution. For example, draw a number line or use a visual model to illustrate how you solve the equation.

Part 4: Creating Your Own Problems

Task 1: Crafting an Algebraic Problem

- Create a word problem that can be solved using algebra. For example: "A rectangle's length is twice its width. If the perimeter is 24 units, what are the dimensions?"
- Express the problem as an equation and solve it. For example, if the width is w, the length is $2w$, and the equation for the perimeter is $2w + 2(2w) = 24$.

Task 2: Solving and Presenting

- Solve the equation to find the width and length of the rectangle. Present your problem, equation and solution to the class.

Materials Needed:

- Graph paper
- Calculator
- Ruler
- Colored pencils or markers

Rubric for Assessment:

Criteria	Excellent (4)	Good (3)	Fair (2)	Needs Improvement (1)
Modelling Real-Life Situations	Accurately models and interprets real-life scenarios	Mostly accurate with minor errors	Basic understanding with some errors	Incorrect or incomplete modelling
Expanding and Simplifying Expressions	Correctly expands and simplifies all expressions	Mostly accurate with minor errors	Somewhat accurate with several errors	Incorrect or incomplete simplification

Criteria	Excellent (4)	Good (3)	Fair (2)	Needs Improvement (1)
Translating Problems into Equations	Accurately translates and solves all problems	Mostly accurate with minor errors	Somewhat accurate with several errors	Incorrect translation or solution
Creating Algebraic Problems	Creates an original, challenging problem with a clear solution	Creates a good problem with minor errors	Creates a basic problem with a solution	Problem is unclear or solution is incorrect

This project will help you strengthen your algebraic thinking, apply mathematical operations to solve real-life problems, and gain confidence in working with equations and expressions.

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