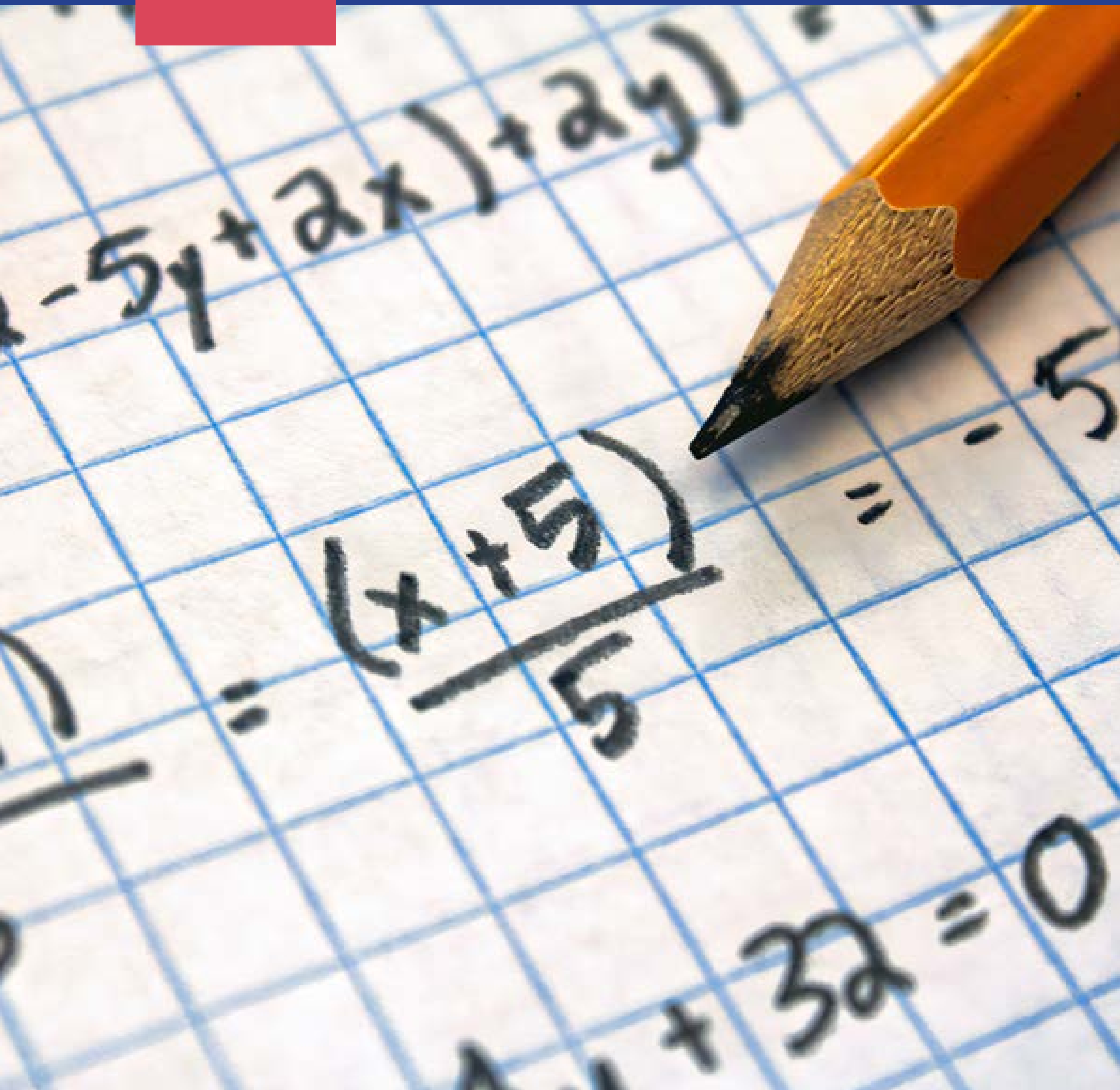
SECTION

3

REASONING WITH ALGEBRA



ALGEBRAIC REASONING

EXPRESSIONS AND EQUATIONS

In this section, you will learn to;

1. *Identify and apply the methods of factorising algebraic expressions.*
2. *Use the inverse property to rearrange a formula in one or two steps to change the subject.*

SECTION INTRODUCTION

In this section, you will identify and apply various methods of factorising algebraic expressions, which is essential in solving problems related to finances, construction and engineering. Factorisation helps you break down complex problems into simpler parts, making them easier to manage. Additionally, you will learn one or two steps to change the subject, which is crucial for interpreting and manipulating formulas in science, technology and daily life. These skills will help you solve equations, understand relationships between variables and make informed decisions in real-world situations.

FACTORISATION

FOCAL AREA: FACTORISATION OF ALGEBRAIC EXPRESSIONS

Imagine you're organising a large school event, such as a sports day. You need to divide all the learners into equal-sized teams for different activities. To do this efficiently, you have to find a way to group learners in a way that is fair and balanced, ensuring that no group is too large or too small. This process is similar to factorisation in algebra.

In algebra, when you factorise an expression, you are breaking it down into simpler parts (or factors) that can be multiplied together to give the original expression. Just like grouping learners into teams helps manage the event better, factorising algebraic expressions helps simplify complex problems, making them easier to solve. Whether you're working with numbers, shapes or real-life situations like organising teams, understanding how to factorise is a valuable skill.

Before we delve more into factorisation, let's go through this activity.

REINFORCEMENT ACTIVITIES

Factor Match-Up

Purpose: To prepare you for learning factorisation we will practise finding common factors.

Instructions:

1. **Group Work**: Form small groups of 3-4 classmates.
2. **Materials**: You will need a set of number cards (with numbers like 12, 18, 24, 36, 48) and factor cards (with numbers like 2, 3, 4, 6, 8, 12).
3. **How to Play**:
    - o Spread the number cards face up on the table.
    - o Shuffle and place the factor cards in a pile face down.
    - o One group member picks a factor card from the pile and reads it out loud.
    - o The group must quickly identify all the number cards that can be divided by this factor.
    - o For example, if the factor card is 6, you should select 12, 18, 24, 36 and 48.
    - o Discuss in your group why these numbers were chosen.
4. **Winning the Game**: The group that correctly matches the most number cards with their factors wins.

What is Factorisation?

Factorisation is the process of breaking down a mathematical expression or a number into a product of its factors. It is the reverse process of expansion. We find the common terms among the expression and express it as a product of these common factors.

Algebraic expressions can be factorised using the common factor method, regrouping like terms together and also by using algebraic identities.

Common Factor:

a. Identify if there is a common factor that can be factored out from all the terms of the expression.

b. Divide each term by the common factor.

c. Rewrite the expression as the common factor multiplied by the remaining terms.

Let's go through the step-by-step approach of using the "Common Factor Method" to factorise the algebraic expression $6x^2 + 9x$.

Step 1: Identify the Common Factor

- Look at each term in the expression and identify the common factor. The common factor is the largest expression that can evenly divide each term.
- For $6x^2$ and $9x$, the common factor is $3x$ because:
  - o 3 is the highest common factor (HCF) of 6 and 9.
  - o x is the common variable with the smallest power between x^2 and x.

Step 2: Factor Out the Common Factor

- Write the common factor outside the parentheses, and then divide each term by the common factor and place the result inside the parentheses.

  $6x^2 + 9x = 3x(2x + 3)$
  - o Here, $6x^2 \div 3x = 2x$
  - o $9x \div 3x = 3$

Step 3: Check Your Work

- To ensure the factorisation is correct, expand the factorised form by multiplying the terms inside the parentheses by the common factor.

  $3x(2x + 3) = 3x \times 2x + 3x \times 3 = 6x^2 + 9x$
  - o If you get the original expression back, the factorisation is correct.

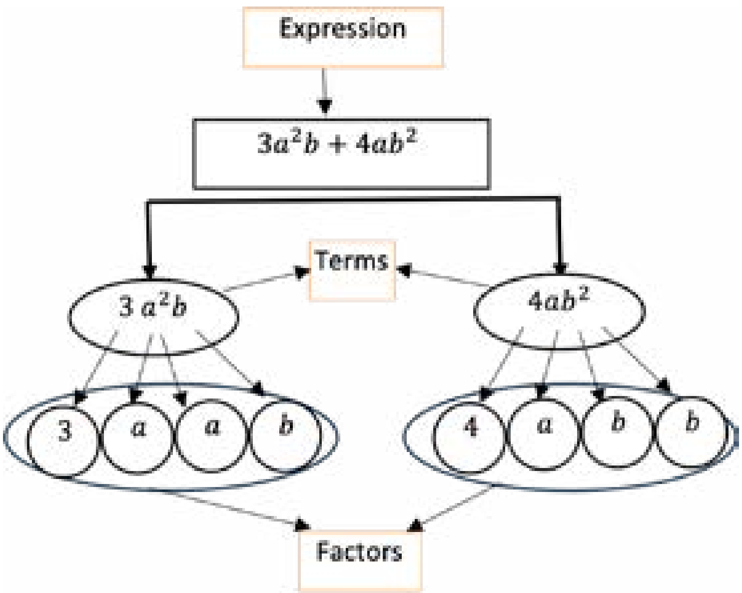
Final Factorised Expression:

The expression $6x^2 + 9x$ is factorised as $3x(2x + 3)$ using the Common Factor Method.

Example 1:

Find the factors of $3a^2b + 4ab^2$

Solution



That is; $(3 \times a \times a \times b) + (4 \times a \times b \times b)$

Pick out the common factors by pairing them off

$(3 \times \boldsymbol{a} \times a \times \boldsymbol{b}) + (4 \times \boldsymbol{a} \times \boldsymbol{b} \times b)$

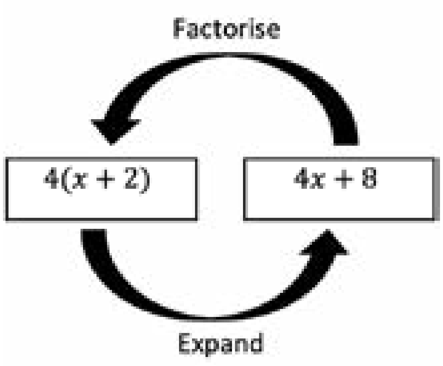
The common factors are ***a*** and ***b*.**

Therefore, $3a^2b + 4ab^2$ can be factorise to get $ab(3a + 4b)$.

Example 2:

Factorise the expression $4x + 8$.

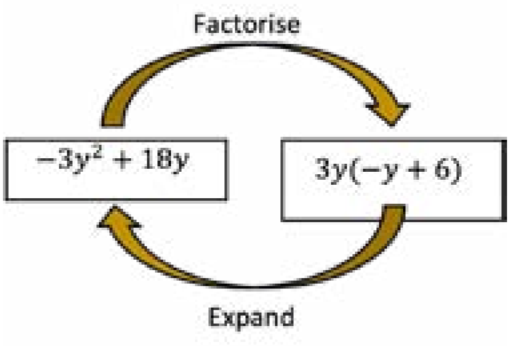
Solution



Example 3:

Factorise $-3y^2 + 18y$

Solution



Example 4:

Factorise $4a + 12b$

Solution

$4a + 12b = 4(a + 3b)$

Difference of two squares

a^2 and b^2 are perfect squares and so $(a^2 - b^2)$ is called a difference of two squares.

An expression in the form $(a + b)(a - b)$

can be expanded as $a(a - b) + b(a - b)$

$= a^2 - ab + ab - b^2$

$= a^2 - b^2$

Let's go through the step-by-step approach to factorise an algebraic expression using the "Difference of Two Squares" method.

We'll use the expression $9x^2 - 16$ as an example.

Step 1: Identify if the Expression Fits the Difference of Two Squares Pattern

- The difference of two squares has the general form $a^2 - b^2$, where both a^2 and b^2 are perfect squares.
- The expression $9x^2 - 16$ can be identified as a difference of two squares because:

- o $9x^2$ is a perfect square $(3x)^2$.
- o 16 is a perfect square $(4)^2$.
- o There is a subtraction (difference) between them.

Step 2: Rewrite the Expression in the Form $a^2 - b^2$

- Express each term as a square:
  $9x^2 - 16 = (3x)^2 - 4^2$

Step 3: Apply the Difference of Two Squares Formula

- The difference of two squares formula is:
  $a^2 - b^2 = (a - b)(a + b)$
- Apply the formula to the expression:
  $(3x)^2 - 4^2 = (3x - 4)(3x + 4)$

Step 4: Verify the Factorisation

- To ensure the factorisation is correct, expand the factorised expression:
  $(3x - 4)(3x + 4) = 3x \times 3x + 3x \times 4 - 4 \times 3x - 4 \times 4$
  Simplify the expression:
  $= 9x^2 + 12x - 12x - 16 = 9x^2 - 16$
- Since you get the original expression back, the factorisation is correct.

Final Factorised Expression:

The expression $9x^2 - 16$ is factorised as $(3x - 4)(3x + 4)$ using the Difference of Two Squares method.

Example 2:

Factorise $x^2 - 25$.

Solution

Note that $x^2 - 25$ is the difference of two squares because 25 is a square number $(25 = 5^2)$.

So we need to factorise $x^2 - 5^2$.

$x^2 - 5^2 = (x - 5)(x + 5)$

Example 3:

Simplify $6^2 - 5^2$

Solution

$6^2 - 5^2$can be simplified as $36 - 25 = 11$

Since $6^2 - 5^2$ is a difference of two squares, this implies that:

$6^2 - 5^2$ can be factored as $(6 + 5)(6 - 5)$

can be expanded as $6(6 - 5) + 5(6 - 5)$

$= 36 - 30 + 30 - 25$

$= 36 - 25$

$= 11$

ACTIVITY 3.1: Individual/Pair/Group Work

Factorisation Practice Using the Common Factor Method and the Difference of Two Squares Method

Purpose: You will apply the methods of factorisation you have learned—using the common factor and the difference of two squares—to solve problems and explore how these methods work in real-life scenarios.

Materials Needed:

- Pen and paper
- A set of algebraic expressions (provided by the teacher)
- Coloured markers
- A calculator (optional)

Instructions:

Warm-Up: Identifying Common Factors

- o Begin by identifying common factors in the following expressions.

Use coloured markers to highlight the common factors.

- o **Expressions:**

  1. $12x + 8y$
  2. $5x^2y - 15xy$

3. $24a^2b + 36ab^2$

- **Task:** Write the factorised form of each expression by factorising out the common factor.

Group Work: Factorising Using the Common Factor Method

- In groups, solve the following expressions by factorising out the greatest common factor:

1. $18xy + 27xz$
2. $6p^2q - 9pq^2$
3. $14m^2n + 21mn^2$

- **Task:** After factorising, check your work by expanding the factorised form to ensure it equals the original expression.

Exploring the Difference of Two Squares

- Review the formula for the difference of two squares: $a^2 - b^2 = (a - b)(a + b)$.
- **Example:** Factor the following expressions using the difference of two squares method:

1. $x^2 - 9$
2. $4y^2 - 25$
3. $16z^2 - 49$

- **Task:** Write the factorised form of each expression and explain how you recognised it as a difference of two squares.

Real-Life Application: Factoring in Action

- Your school is planning to create two square gardens of equal size, each with an area of 100 square metres. To plan the fencing, you need to calculate the perimeter of each garden by first factorising the expression for the area.
- **Expression:** $100m^2 - 0$
- **Task:** Factorise the expression using the difference of two squares method, then calculate the perimeter of each garden using the factorised form.

Reflection:

- After completing the activities, discuss how each method of factorisation was used.

- Which method did you find easier or more challenging?
- Why is it important to be able to factorise expressions in algebra?

Perfect Square Trinomial

Example 1:

Expand the expression $(a + b)^2$

Solution

$= (a + b)\,(a + b)$

$= a\,(a + b) + b\,(a + b)$

$= a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

Therefore, $a^2 + 2ab + b^2 = (a + b)^2$

Example 2:

Expand the expression $(a - b)^2$

Solution

$= (a - b)\,(a - b)$

$= a\,(a - b) - b\,(a - b)$

$= a^2 - ab - ab + b^2$

$= a^2 - 2ab + b^2$

Therefore, $a^2 - 2ab + b^2 = (a - b)^2$

This implies that,

An expression in the form of $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$, can be factored as $(a + b)^2$ or $(a - b)^2$ respectively.

Be guided by the steps used in the perfect square expansion:

First, square the first term add twice the product of the first and the last terms and add to the square of the last term.

ACTIVITY 3.2: Individual/Pair/Group Work

Expanding Perfect Square Trinomials

Purpose: Practise expanding and simplifying expressions like $(a + b)^2$ and understand the pattern of perfect square trinomials.

Instructions:

Group Work:

- o Form small groups of 3-4 learners.
- o Each group will receive a set of algebra tiles or coloured paper cutouts representing the terms in $(a + b)^2$.

Expanding $(a + b)^2$:

- o Begin by writing the expression $(a + b)^2$ on a large sheet of paper.
- o Use the algebra tiles or paper cutouts to visually represent the expression:
  - Let one colour represent ***a*** and another colour represent ***b*.**
  - Lay out the tiles or cutouts to show $(a + b)(a + b)$.
- o Arrange the tiles or cutouts to cover a square, showing the areas a^2, ab, ab and b^2

Identify the Trinomial:

- o Observe and identify that the arrangement creates a perfect square with the area: $a^2 + 2ab + b^2$
- o Write down the expanded form on your paper.

Pattern Recognition:

- o Discuss in your groups the pattern you notice when expanding $(a + b)^2$.
- o What happens to the middle term? How does it relate to the original expression $(a + b)$?

Practice with Different Variables:

- o Repeat the process for different expressions like $(x + y)^2$, $(p + q)^2$, and $(m + n)^2$.
- o Expand each one and verify that the pattern holds.

Class Discussion:

- o After the activity, come together as a class and discuss your findings.

- Share how the algebra tiles helped you understand the expansion.
- Discuss how this method can be used for more complex algebraic expressions.

Factorising By Grouping Method

For expressions with four terms, look for pairs of terms that have common factors. Factor out the common factors from each pair. Look for common factors in the resulting binomials and factor them out as well.

Worked Examples

Example 1:

Factorise $ab + ac + bd + cd$

Solution

$= a(b + c) + d(b + c)$ *factorising each pair separately*

$= (b + c)(a + d)$ *removing the common factor*

Many expressions with four terms cannot be factorised using this approach. There is therefore the need to reorder or regroup the terms first.

Example 2:

Factorise $6ax - 2y + 3ay - 4x$

Solution

$= 6ax + 3ay - 4x - 2y$ *Group like terms and remove the common factor*

$= (3a - 2)(2x + y)$

Example 3:

Factorise $3ab + d + 3ad + b$

Solution

$= 3ab + b + 3ad + d$

$= b(3a + 1) + d(3a + 1)$

$=(3a + 1)(b + d)$

ACTIVITY 3.3: Individual/Pair/Group Work

Factorising by Grouping Method

Purpose: Practise factorising algebraic expressions using the grouping method.

Instructions:

Group Work:

- o Form small groups of 3-4 learners.
- o Each group should have paper, pencils and algebra tiles or coloured paper cutouts to represent the terms in the algebraic expressions.

Start with an Example:

- o Write the following expression on your group's large sheet of paper: $3x^2 + 6x + 2x + 4$
- o Discuss how you might factorise this expression by grouping.

Identify Groups:

- o Divide the expression into two groups: $(3x^2 + 6x) + (2x + 4)$
- o Look at each group separately and identify the common factors in each:
  - For the first group $(3x^2 + 6x)$, the common factor is $3x$.
  - For the second group $(2x + 4)$, the common factor is 2.

Factorise the Common Factors:

- o Factor out the common factor from each group: $3x(x + 2) + 2(x + 2)$
- o Discuss what you notice about the expression after factorising.

Factorise the Entire Expression:

- o Observe that both terms now have a common binomial factor $(x + 2)$.
- o Factor out $(x + 2)$ from the entire expression: $(3x + 2)\,(x + 2)$.
- o Write the final factorised form on your group's paper.

Practice with Different Expressions:

- o Try factorising the following expressions using the grouping method:
  - $4y^2 + 8y + 3y + 6$
  - $5m^2 + 10m + 2m + 4$

- Follow the same steps as above: group, factorise out the common factors and then factorise the entire expression.

Class Discussion:

- After completing the activity, discuss as a class what made factorising by grouping easier.
- Share any challenges and how you overcame them.

Quadratic Trinomial

A quadratic trinomial is a type of algebraic expression with variables and constants. It is expressed in the form of $ax^2 + bx + c$, where x is the variable and a, b, and c are non-zero real numbers. The constant 'a' is known as a leading coefficient, 'b' is the linear coefficient, 'c' is the additive constant.

In factorisation of quadratic trinomials there are two forms:

(i) First form: $x^2 + px + q$

(ii) Second form:$ax^2 + bx + c$

However, under this focal area we will deal with only the first form.

Examples

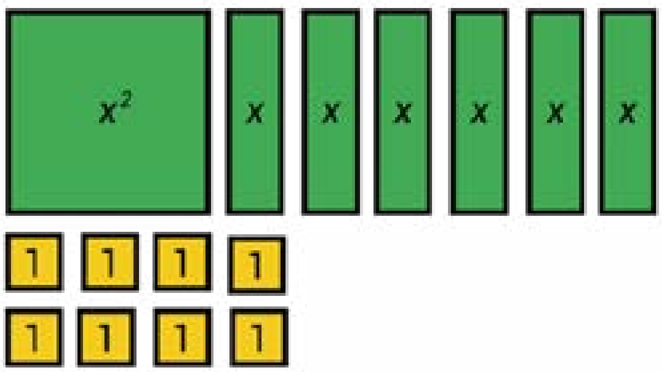
Using Algebra Tiles

Example 1:

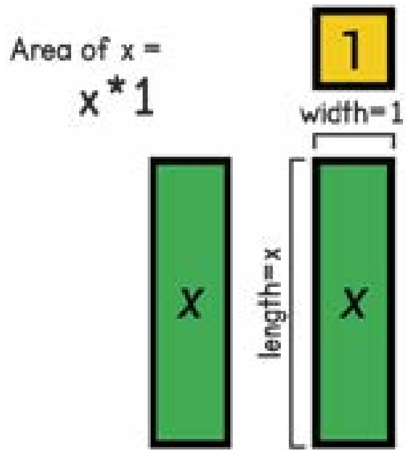
Factorise $x^2 + 6x + 8$ using algebra tiles.

Solution

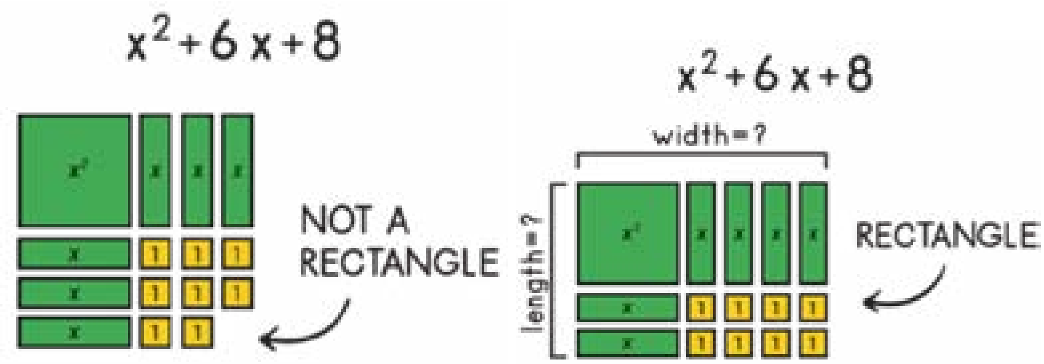
Firstly, let's represent the trinomial using Algebra tiles like this one below:



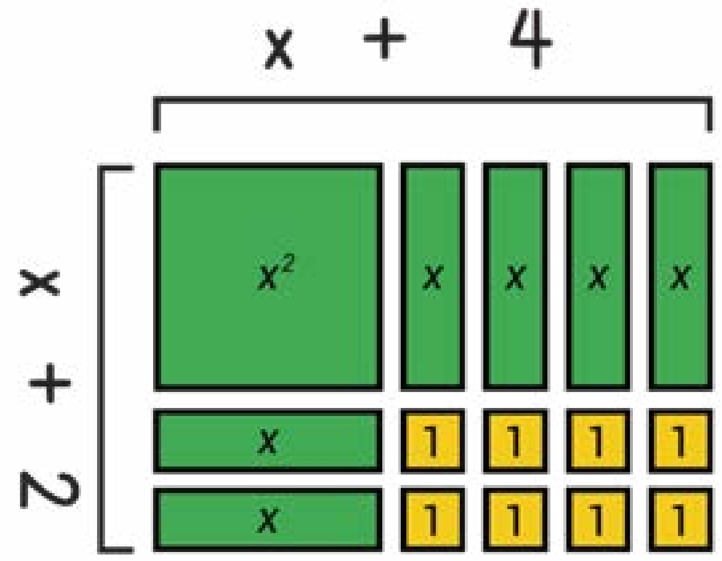
Before proceeding, know that the area of the x tile is $x \times 1$.



Now, we will arrange the tiles in a rectangle using all of the pieces. Sometimes your arrangement may not get you a rectangle so keep trying.



Now, we need to find out what the dimensions (the length and the width) of the rectangle would be. Here is the solution:



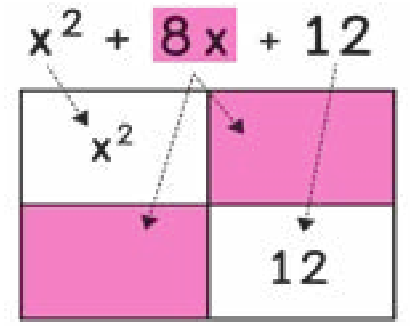
Therefore, the factorised form of $x^2 + 6x + 8$ is $(x + 2)\ (x + 4)$.

Using The Box Method

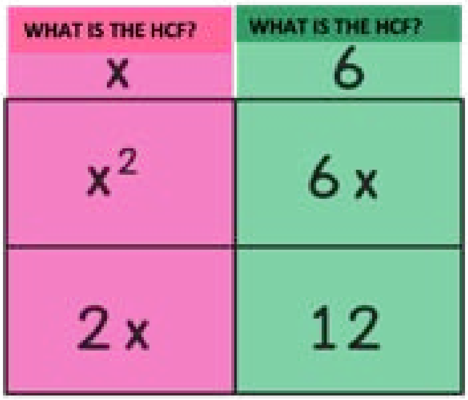
The box method is also a great tool to use to factorise trinomials. Let's use it to try this example.

Example 2:

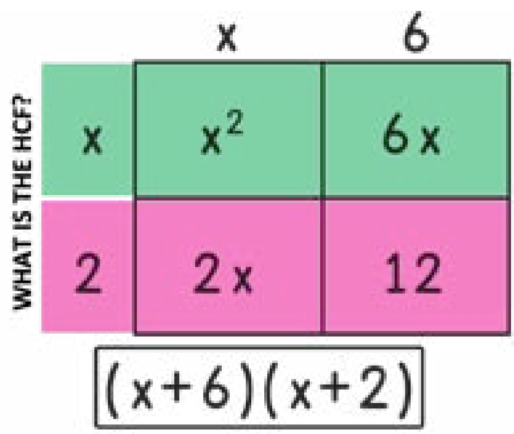
Factorise $x^2 + 8x + 12$.



Which two numbers multiply to get 12 and sum to get 8?



From our chart, we realise that 2 and 6 multiply to get 12 ($2 \times 6 = 12$) and sum to get 8 ($2 + 6 = 8$).



Therefore, the factorised form of $x^2 + 8x + 12$ is $(x + 6)\,(x + 2)$.

Example 3:

$x^2 - 7x + 12$

Solution:

The given expression is $x^2 - 7x + 12$

Find two numbers whose sum = -7 and product = 12

Clearly, such numbers are (-4) and (-3).

Therefore $x^2 - 7x + 12 = x^2 - 4x - 3x + 12$

$= x(x - 4) - 3(x - 4)$

$= (x - 4)(x - 3)$.

Example 4:

$x^2 + 2x - 15$

Solution:

The given expression is $x^2 + 2x - 15$

To factorise the given quadratic trinomial, we have to find two numbers a and b, such that $a + b = 2$ and $ab = -15$

Clearly, $5 + (-3) = 2$ and $5 \times (-3) = -15$

Therefore such numbers are 5 and -3

Now, splitting the middle term $2x$ of the given quadratic trinomial $x^2 + 2x - 15$, we get,

$x^2 + 5x - 3x - 15$

$= x(x + 5) - 3(x + 5)$

$= (x + 5)(x - 3)$

ACTIVITY 3.4: Individual/Pair/Group Work

Factorising Quadratic Trinomials

Purpose: Practise factorising quadratic trinomials using both algebra tiles and the standard method.

Materials Needed:

- Algebra tiles (or cut-out coloured paper to represent different terms)
- Graph paper or notebooks
- Pencils

Instructions:

Part 1: Factorising Using Algebra Tiles

Review the Quadratic Trinomial:

- Begin by writing a quadratic trinomial on your paper: $x^2 + 5x + 6$
- Identify the different parts: x^2 (the quadratic term), $5x$ (the linear term) and 6 (the constant term).

Model with Algebra Tiles:

- Use one large square tile to represent x^2.
- Use five rectangular tiles to represent $5x$.
- Use six small square tiles to represent the constant 6.

Arrange the Tiles:

- Arrange the tiles on your desk or paper to form a rectangle.
- Place the x^2 tile in one corner.
- Place the x tiles along two sides of the square tile to form the sides of the rectangle.
- Fill in the constant tiles to complete the rectangle.

Identify the Factors:

- Once the rectangle is formed, identify the length and width of the rectangle.
- These dimensions represent the factors of the quadratic trinomial.
- Write down the factorised form: $(x + 2)(x + 3)$

Practice with Another Example:

- Try factorising $x^2 + 7x + 10$ using the algebra tiles.
- Follow the same steps: model, arrange and identify the factors.

Part 2: Factorising Using the Standard Method

Start with the Trinomial:

- o Write the same quadratic trinomial from earlier: $x^2 + 5x + 6$

Find the Factors:

- o Identify two numbers that multiply to give the constant term (6) and add to give the coefficient of the linear term (5).
- o The numbers are 2 and 3.

Write the Factorised Form:

- o Express the trinomial as a product of two binomials: $(x + 2)(x + 3)$

Check Your Work:

- o Expand the binomials to ensure you get the original trinomial: $(x + 2)(x + 3) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6$

Practice with Another Example:

- o Factorise $x^2 + 7x + 10$ using the standard method.
- o Identify the factors and write down the factorised form.

Now, use any of the methods to solve the following questions;

i. $x^2 + 5x + 6$

ii. $x^2 + x - 6$

iii. $x^2 + 3x + 2$

SUBSTITUTION AND CHANGE OF SUBJECT

FOCAL AREA: CHANGE OF SUBJECT AND SUBSTITUTION

Imagine you're trying to rearrange your room. You want your desk to be the main focus so you move it to the centre. To make space, you have to move other items around until everything fits perfectly. In mathematics, the concept of "*Change of Subject*" is like rearranging your room, but instead of moving furniture, you're moving parts of an equation to focus on one particular variable.

Let's consider a real-life example: calculating speed.

If you know the distance you traveled and the time it took, you can find the speed using the formula:

$$\text{Speed} = \frac{Distance}{Time}$$

But what if you knew your speed and the time and wanted to find the distance instead? You'd need to rearrange the formula to make "Distance" the subject. This process is what we call "Change of Subject."

Example 1:

Given the formula $C = 2\pi r$

This formula represents the circumference C of a circle, where r is the radius, and π is a constant approximately equal to 3.14.

Change the subject of the formula to make r the subject.

Step 1: Identify the Target Variable

We want to make r the subject of the formula. This means that we need to isolate r on one side of the equation.

Step 2: Isolate the Variable

The given formula is:

$$C = 2\pi r$$

To isolate r, we need to remove the 2π that is multiplied by r. We can do this by doing the inverse of multiplying, so we divide both sides of the equation by 2π.

$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$$

Step 3: Simplify the Equation

On the right-hand side, 2π cancels out:

$$\frac{C}{2\pi} = r$$

Step 4: Write the Final Equation

Now that r is isolated, we can rewrite the equation with r as the subject:

$$\boldsymbol{r = \frac{C}{2\pi}}$$

Step 5: Verify the Solution

To check if our new equation is correct, we can substitute r back into the original formula to see if we get the same expression for C.

Substituting $r = \frac{C}{2\pi}$ into the original formula:

$$C = 2\pi \times \frac{C}{2\pi}$$

Simplifying:

$$C = C$$

Since the original equation is satisfied, the rearrangement is correct.

Example 2:

Make x the subject of the equation, $2x + 3 = 9$

Solution

By making x the subject, we are finding the value of x that satisfies the equation $2x + 3 = 9$.

$2x + 3 = 9$	*Start with the Original Equation*
$2x + 3 - 3 = 9 - 3$	*Isolate the Term with x* by subtracting 3 from both sides of the equation (subtracting being the inverse of addition) to move the constant term to the other side.
$2x = 6$	
$\frac{2x}{2} = \frac{6}{2}$	*Remove the Coefficient* by dividing both sides by the coefficient of x, which is 2, isolates x.

$$\boldsymbol{x = 3}$$

Example 3:

Make '***u***' the subject of the formula $v = u + at$

Solution

$v - at = u + \cancel{at} - \cancel{at}$ **[subtract "*at*" from both sides]**

$v - at = u$

or, $\boldsymbol{u = v - at}$

Example 4:

The volume of a rectangular shape (V), with length (l), width (w) and height (h) is given by the formula $V = lwh$.

Make h the subject.

Solution:

$V = lwh$

$$\frac{V}{l \times w} = \frac{l \times w \times h}{l \times w}$$

$$\boldsymbol{\frac{V}{l \times w} = h}$$

Example 5:

Write the formula for finding the area of the rectangle and indicate the subject in this formula. Also, make $\boldsymbol{l}$ as the subject. If A = 42 cm^2 and b = 6 cm, then find $\boldsymbol{l}$.

Solution:

If area is denoted by **A**, length by $\boldsymbol{l}$ and breadth by $\boldsymbol{b}$,

then area of the rectangle is given by $\boldsymbol{A = l \times b}$

In this formula, **A** is the subject.

When we change the subject, i.e., make $\boldsymbol{l}$ as the subject then the formula becomes $\boldsymbol{l = \frac{A}{b}}$

In order to find the value of $\boldsymbol{l}$, substituting the value of **A** and **b**,

we get $\boldsymbol{l} = \frac{42}{6}$ cm

Therefore, length (l) = 7 cm.

Example 6:

In the formula, $l = a + (n - 1)d$ make d as the subject.

Then find d when $l = 10$, $a = 2$, $n = 5$.

Solution:

$(n - 1)d = l - a$

$d = \frac{l - a}{n - 1}$ where d is the required subject

Now, substituting the values of l, a, n in the formula;

we get, $d = \frac{10 - 2}{5 - 1}$

$= \frac{8}{4}$

$= 2.$

Example 7:

i. Kwame put $GH¢$ 500.00 into his bank account. The interest rate was 5% per year. If he is paid simple interest, how much interest will he earn in two years. (Given $I = P \times R \times T$)

Solution:

Given $I = P \times R \times T$,

Principal, P = 500

Rate, R= 5% = 0.05

Time, T= 2

By substituting, Interest, $I = 500 \times 0.05 \times 2$

$I = GH¢\ 50.00$

ii. Kwame put an amount of money into his bank account. The simple interest rate was 20%. After two years he had earned 150 cedis as interest. What amount did he originally put in the bank?

Solution

If $I = PRT$, to find the original amount (Principal), make P the subject of the equation and find the value of P when $I = 150, R = 20\% = 0.20$ and $T = 2$

$\frac{I}{RT} = \frac{PRT}{RT}$, *Divide both sides of the equation by RT*

$\frac{I}{RT} = P$

By substituting, $P = \frac{150}{0.20 \times 2}$

$P = GH¢\ 375.00$

This implies that Kwame put $GH¢$ 375.00 into his bank account.

ACTIVITY 3.5: Individual/Pair/Group Work

Substitution and Change of Subject

Purpose: To practise substitution and changing the subject of a formula.

Materials Needed:

- Paper
- Pencil
- Calculator (optional)

Instructions:

Part 1: ***Substitution Practice***

Given Formula:

$A = \frac{1}{2} bh$

Where:

- A is the area of a triangle
- b is the base
- h is the height

Task 1:

Suppose you know that the base $b = 8$ cm and the height $h = 5$ cm.

Substitute these values into the formula to calculate the area A.

Task 2:

If the height $h = 12$ cm and the area $A = 48$ cm², make b the subject of the formula and substitute the values into the new formula to find the base b.

Part 2: ***Change of Subject Practice***

Given Formula:

$P = 2l + 2w$

Where:

- o P is the perimeter of a rectangle
- o l is the length
- o w is the width

Task 1:

Rearrange the formula to make l the subject. Write the new formula.

Task 2:

Using the new formula from Task 1, if $P = 50$ cm and $w = 10$ cm, calculate the length l.

Task 3:

Now, change the subject of the original formula to w. Write the new formula.

Task 4:

Using the formula you created in Task 3, find w when $P = 60$ cm and $l = 20$ cm.

Part 3: ***Combining Substitution and Change of Subject***

Given Formula:

$s = ut + \frac{1}{2}at^2$ Where:

- o S is the displacement
- o u is the initial velocity
- o t is the time
- o a is the acceleration

Task 1:

Rearrange the formula to make u the subject.

Task 2:

Substitute the following values into the original formula to calculate s:

- o $u = 10\ m/s$
- o $t = 5\ s$
- o $a = 2\ m/s^2$

Task 3:

Using the new formula from Task 1, calculate the initial velocity u if:

- o $S = 100\ m$
- o $t = 10\ s$
- o $a = 3\ m/s^2$

Reflection Questions:

- How does changing the subject of a formula help you solve problems more easily?
- How does substitution help you check your understanding of the relationship between variables?

REVIEW QUESTIONS

1. Factorise $6x^2 + 9x$
2. Factorise $12y^3 + 8y^2$
3. Factorise $15ab + 25a^2$
4. Factorise $x^2 - 16$
5. Factorise $9y^2 - 25$
6. Factorise $49m^2 - 36$
7. Expand $(x + 3)^2$
8. Expand $(2y - 5)^2$
9. Expand $(3a + 4)^2$
10. Factorise $ax + ay + bx + by$
11. Factorise $2x^2 + 4x + 3x + 6$
12. Factorise $6ab + 9a + 4b + 6$
13. Factorise $x^2 + 7x + 10$
14. Factorise $x^2 + 5x+6$
15. Factorise $x^2 + 8x + 15$
16. Make x the subject in the equation $y = 3x + 4$
17. Make t the subject in the equation $v = u + at$
18. Make h the subject in the equation $A = \frac{1}{2}bh$ and substitute $b = 8$ and $A = 24$ to find h.

ANSWERS TO REVIEW QUESTIONS

1. $3x(2x + 3)$
2. $4y^2(3y + 2)$
3. $5a(3b + 5a)$
4. $(x - 4)(x + 4)$
5. $(3y - 5)(3y + 5)$
6. $(7m - 6)(7m + 6)$
7. $x^2 + 6x + 9$
8. $4y^2 - 20y + 25$
9. $9a^2 + 24a + 16$
10. $(a + b)(x + y)$
11. $(2x + 3)(x + 2)$
12. $(3a + 2)(2b + 3)$
13. $(x + 5)(x + 2)$
14. $(x + 3)(x + 2)$
15. $(x + 5)(x + 3)$
16. $x = \frac{y - 4}{3}$
17. $t = \frac{v - u}{a}$
18. $h = \frac{2A}{b} = \frac{2 \times 24}{8} = 6$

Project 1: Exploring Operations on Sets and Their Applications

Objective: In this project, you will identify and apply the properties of operations on sets (such as union, intersection, and difference) to solve real-life problems.

Tasks:

1. **Understanding Set Operations:**
   - o Review the basic operations on sets: union, intersection and difference.
   - o Identify and list examples of these operations in everyday contexts, such as organising items in a store, categorising school subjects or grouping sports teams based on different attributes.
2. **Real-Life Application:**
   - o Imagine you are organising a community event where you need to create groups based on specific interests (e.g., music lovers, sports enthusiasts, and art lovers). Use the operations on sets to determine:
     - which participants belong to multiple groups.
     - how many participants are in each group.
     - which participants do not belong to any group.
3. **Presentation:**
   - o Create a visual representation (using Venn diagrams) to illustrate the sets and their operations.
   - o Write a brief report explaining how you applied set operations to organise the groups and the benefits of using this method.

Materials Needed:

- Graph paper
- Markers or colored pencils
- Ruler
- Calculator

Rubric for Assessment:

Criteria	Excellent (4)	Good (3)	Fair (2)	Needs Improvement (1)
Understanding of Set Operations	Clear and detailed explanation	Good understanding with minor gaps	Basic understanding, some confusion	Limited or incorrect understanding
Application to Real-Life Context	Highly relevant and creative	Relevant with some creativity	Somewhat relevant	Little to no relevance
Visual Representation	Clear, accurate, and well-organised	Mostly clear and accurate	Somewhat clear but disorganised	Unclear or inaccurate
Report Writing	Well-structured, clear, and concise	Clear but lacks structure	Basic but unclear	Unclear or incomplete

Project 2: Mastering Fractions and Decimals in Real-Life Contexts

Objective: In this project, you will perform operations on fractions with like and unlike denominators and approximate decimals, applying these concepts to solve real-life problems.

Tasks:

1. **Understanding Fraction Operations:**
   - Review addition, subtraction, multiplication, and division of fractions with like and unlike denominators.
   - Practise converting fractions to decimals and rounding decimals to a specified place value.
2. **Real-Life Application:**
   - Imagine you are helping in the kitchen to prepare a recipe that serves 4 people. Adjust the quantities of the ingredients if the recipe needs to be scaled up to serve 25 people. Perform the following:
     - Add fractions of different ingredients.
     - Multiply fractions to adjust the quantities.
     - Convert your results to decimals and round them to the nearest hundredth.
3. **Presentation:**
   - Create a poster showing the original recipe, the calculations you performed to scale it up and the final quantities of each ingredient.
   - Explain how understanding fractions and decimals helped you in adjusting the recipe.

Materials Needed:

- Recipe cards
- Calculator
- Poster board
- Markers or colored pencils

Rubric for Assessment:

Criteria	Excellent (4)	Good (3)	Fair (2)	Needs Improvement (1)
Understanding of Fraction Operations	Clear and accurate with detailed steps	Mostly accurate with minor errors	Basic understanding with errors	Limited understanding
Application to Real-Life Context	Highly relevant and creative	Relevant with some creativity	Somewhat relevant	Little to no relevance
Visual Representation	Clear, accurate, and visually appealing	Mostly clear and accurate	Somewhat clear but disorganised	Unclear or inaccurate
Explanation	Well-structured, clear, and concise	Clear but lacks structure	Basic but unclear	Unclear or incomplete

These projects encourage you to apply mathematical concepts to real-life situations, enhancing your understanding and making learning more meaningful.

Project 3: Comparing, Estimating Ratios, and Expressing Quantities as Percentages

Objective: In this activity, you will learn how to compare and estimate quantities in a given ratio and express one quantity as a percentage of another.

Tasks:

1. **Understanding Ratios:**
   - Begin by reviewing what a ratio is and how it is used to compare quantities.
   - Practise writing ratios for different real-life situations, such as comparing the number of boys to girls in your class or the number of apples to oranges in a basket.
2. **Estimating Quantities in Ratios:**
   - Given a scenario, estimate the quantities involved using a ratio. For example, if a recipe requires a ratio of 2 cups of flour to 1 cup of sugar, estimate how much flour and sugar you would need to double or halve the recipe.
3. **Expressing Ratios as Percentages:**
   - Convert the ratios you have worked with into percentages. For instance, if the ratio of boys to girls in your class is 3:2, calculate what percentage of the class are boys and what percentage are girls.
   - Practice expressing different quantities as percentages of one another in various contexts, such as determining the percentage of time spent on different activities during the day.
4. **Real-Life Application:**
   - Imagine you are organising a school event and need to prepare snacks for a group of students. You have a budget that allows you to buy a certain number of packets of chips and juice boxes. Use ratios to compare the quantities you can buy and express these quantities as percentages of your total budget. Estimate how much of each item you can purchase if you need to adjust your budget or the number of students attending.
5. **Presentation:**
   - Create a table or chart that shows the ratios and corresponding percentages for the different scenarios you worked on.

- Write a brief explanation of how understanding ratios and percentages helped you make decisions in your real-life application.

Materials Needed:

- Calculator
- Graph paper
- Markers or colored pencils
- Ruler

Rubric for Assessment:

Criteria	Excellent (4)	Good (3)	Fair (2)	Needs Improvement (1)
Understanding of Ratios	Clear and accurate comparisons	Mostly accurate comparisons	Basic understanding with minor errors	Limited or incorrect understanding
Estimation of Quantities	Accurate and well-reasoned estimates	Reasonable estimates with minor errors	Basic estimation with some errors	Poor estimation or lack of reasoning
Conversion to Percentages	Accurate and clearly explained	Mostly accurate with minor errors	Basic understanding with some errors	Limited or incorrect conversion
Real-Life Application	Highly relevant and creatively applied	Relevant with some creativity	Somewhat relevant	Little to no relevance
Presentation	Clear, well-organised, and visually appealing	Mostly clear and organised	Somewhat clear but disorganised	Unclear or poorly organised

Project 4: Factorising Algebraic Expressions and Rearranging Formulas

Objective: In this project, you will apply the methods of factorising algebraic expressions and use the inverse property to rearrange formulas, changing the subject in one or two steps.

Part 1: Factorising Algebraic Expressions

Task 1: Identifying Factorisation Methods

- Review different methods of factorisation, such as:
  - **Common Factor Method:** Factor out the highest common factor (HCF) from the terms.
  - **Difference of Squares:** Factor expressions like $a^2 - b^2$ into $(a - b)(a + b)$.
  - **Trinomial Factorisation:** Factor quadratic expressions like $ax^2 + bx + c$ into two binomials.

Task 2: Hands-on Practice

- Work on several algebraic expressions, applying the appropriate method to factorise them. For example:
  1. $6x^2 + 9x$
  2. $x^2 - 16$
  3. $x^2 + 5x + 6$
- Write down the factorised forms and explain the method you used for each.

Task 3: Real-Life Application

- Consider a scenario where you need to design a rectangular garden with an area expressed by the algebraic expression $6x^2 + 9x$. Factorise the expression to help you determine possible dimensions for the garden.

Part 2: Rearranging Formulas Using the Inverse Property

Task 1: Understanding the Inverse Property

- Learn about the inverse property and how it can be used to rearrange formulas. The inverse property states that performing the inverse operation on both sides of an equation will help isolate a variable.

Task 2: Practice Rearranging Formulas

- Given formulas, use the inverse property to rearrange them. For example:
  1. Rearrange the formula $P = 2L + 2W$ to solve for L.
  2. Rearrange the formula $V = l \times w \times h$ to solve for h.

Task 3: Real-Life Application

- Imagine you have a formula to calculate the cost of a project: $C = 2M + 3N$. If you know the total cost and the number of materials M, use the inverse property to rearrange the formula and solve for N, the number of items you need.

  Materials Needed:

  - Algebra worksheets
  - Graph paper
  - Calculator
  - Ruler
  - Markers or colored pencils

Rubric for Assessment:

Criteria	Excellent (4)	Good (3)	Fair (2)	Needs Improvement (1)
Factorisation of Expressions	Accurate and methodically correct	Mostly accurate with minor errors	Basic understanding with some errors	Limited or incorrect factorisation
Application of Factorisation	Creative and relevant application	Relevant with minor errors	Basic application	Little to no relevance
Rearranging Formulas	Accurate and efficiently done	Mostly accurate with minor errors	Basic understanding with some errors	Limited or incorrect rearrangement
Application of Inverse Property	Creative and relevant application	Relevant with minor errors	Basic application	Little to no relevance

Criteria	Excellent (4)	Good (3)	Fair (2)	Needs Improvement (1)
Presentation	Clear, well-organised, and visually appealing	Mostly clear and organised	Somewhat clear but disorganised	Unclear or poorly organised

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