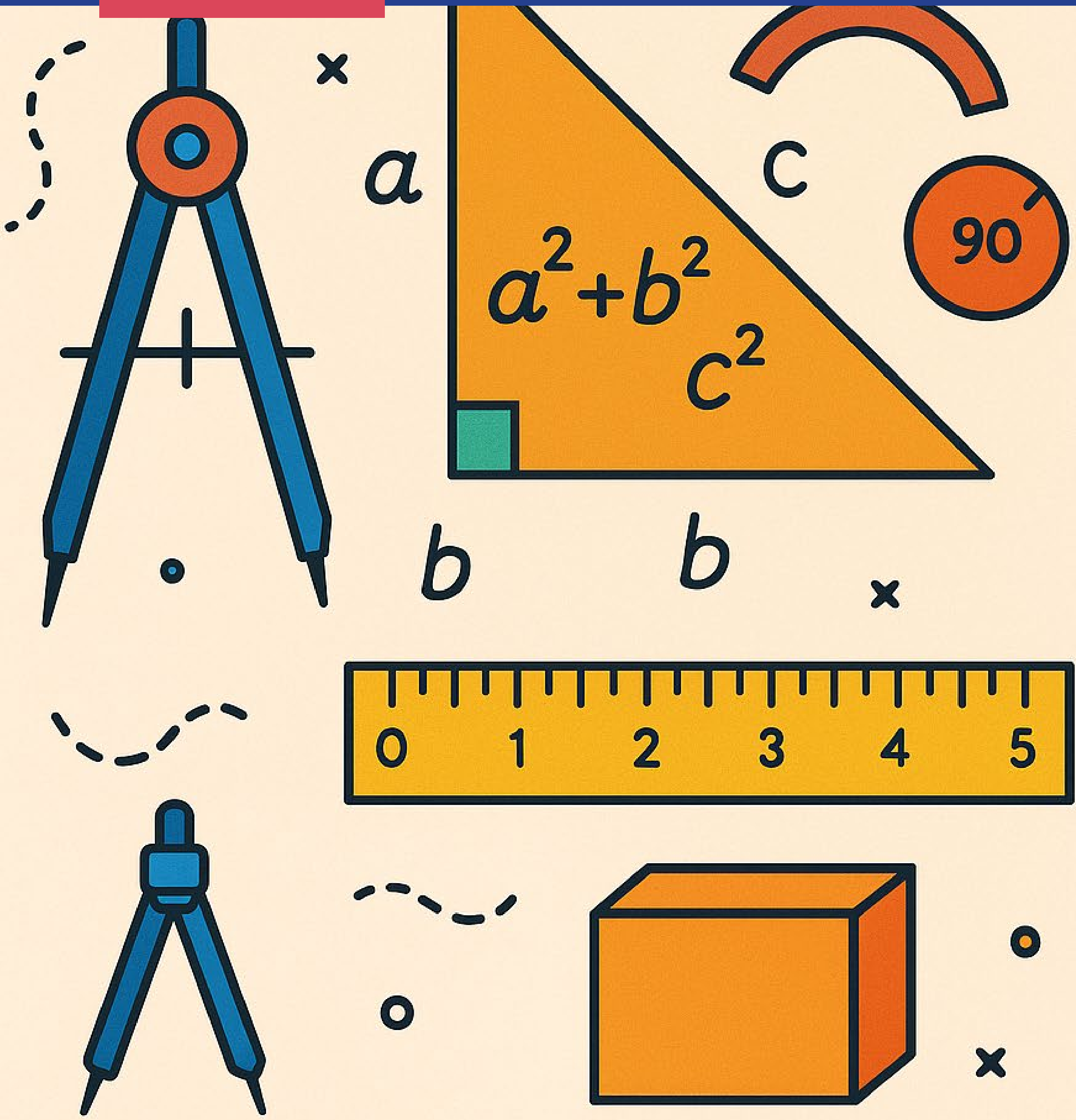
SECTION

4 GEOMETRICAL REASONING AND MEASUREMENT



GEOMETRY AND MEASUREMENT

Shape and Space/Measurement

In this section, you will learn to;

1. *Identify and apply parallel, perpendicular, complementary, supplemnentary angles, vertical and parallel lines cut by transversals in real-life contexts.*
2. *Develop and apply strategies for determining the perimeter and area of plane figures.*
3. *Develop and apply strategies for determining the surface area and volume of prisms.*

SECTION INTRODUCTION

In this section, you will learn to identify and apply parallel, perpendicular, complementary, supplementary angles and vertical and parallel lines cut by a transversal in real-life contexts, such as designing buildings, creating art or solving problems related to roads and bridges. Understanding these concepts will help you analyse and interpret angles and lines in everyday situations, improving your spatial awareness and problem-solving skills.

Next, you will develop and apply strategies to determine the perimeter and area of plane figures, which is crucial for tasks like planning a garden layout, designing a room or calculating the materials needed for a project. These skills will allow you to solve practical problems efficiently.

Finally, you will explore strategies for determining the surface area and volume of prisms, which are essential in real-life contexts such as packaging design, construction and understanding the capacity of various containers. Mastering these concepts will enable you to apply mathematical reasoning to everyday tasks, enhancing your ability to make informed decisions and solve problems.

ANGLES & ITS APPLICATIONS

Focal Area: Angles

Imagine you are a construction worker, building a house. You have to ensure that the walls are perfectly vertical, the ceiling is parallel to the floor, and the corners of the rooms are perfectly square. If any of these angles are incorrect, the walls could lean, the ceiling might not fit properly, and the room could end up misshaped. Understanding angles is crucial in this situation because they help you measure and construct objects accurately.

Angles are all around us, not just in construction but in everyday life. When you open a door, play a sport or even fold a piece of paper, you are working with angles. For example, when playing soccer, the angle at which you kick the ball can determine its path and whether it reaches the goal.

Reinforcement Activities

Angle Hunt

Purpose: This activity will help you recognise and understand different types of angles in your environment before learning about the concept of angles in detail.

Materials Needed:

- A piece of paper and a pen or pencil
- A protractor (if available)
- A camera (optional, if you want to take pictures)

Instructions:

Divide into Groups: Form small groups of 3-4 students. Each group will work together to find and identify angles around the classroom or school.

Angle Hunt: Your task is to find examples of angles in everyday objects around you. Look for objects like doors, windows, books, tables, chairs or even the clock on the wall. For each object, identify where an angle is formed. For example, the corner of a book forms an angle or the hands of the clock form an angle.

Classify the Angles:

- o After finding an angle, decide if it's an acute angle (less than 90°), a right angle (exactly 90°) or an obtuse angle (greater than 90° but less than 180°).
- o Write down the name of the object and the type of angle you identified.

Recording Your Findings:

- List at least 5 different objects and the angles you found in each.
- If you have a protractor, try to measure the angles and record the measurements next to your observations.
- (Optional) Take pictures of the objects and angles you found and label the angles directly on the pictures.

Share with the Class:

- After 15 minutes, come back together as a class.
- Each group will share one or two of the most interesting angles they found, describing the object, the type of angle and, if measured, the angle measurement.

Discussion:

- As a class, discuss the different types of angles found and how they are part of everyday objects and structures.
- Consider why understanding angles is important in designing and using these objects.

What Are Angles?

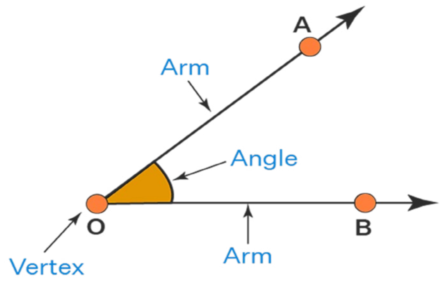
An angle is like a twist or a turn. It is the amount of rotation between two lines that meet at a common point, which we call the vertex. Angles are everywhere in our daily lives. These help us to describe shapes, navigate our environment and understand the relationships between objects. Angles help in building a house or flying a kite. They play a crucial role in how things move and interact.

Now that we have a basic understanding of what angles are and why they matter, let us dive deeper into our exploration to help identify and understand angles better.

Exploring angles and examples of angles in the environment using visual aids, interactive activities and concrete examples.

Imagine holding two straight lines, or rays, with one end connected at a point and then rotating one of the lines around that point. The space between the lines as they rotate is what we call an angle.

Parts of an Angle



From the above diagram, we can conclude that an angle is formed when two rays have a common endpoint called the vertex**.** The rays are known as the sides (or arms) of the angle. It is a measure. Angles are measured in degrees (°).

In the diagram, if ray 1 is A, the vertex is O and ray 2 is B then the angle form at the vertex or O is mathematically written as ∠AOB or ∠O. The tool used to measure angles is a protractor.

Parts of Angles

An angle has three parts

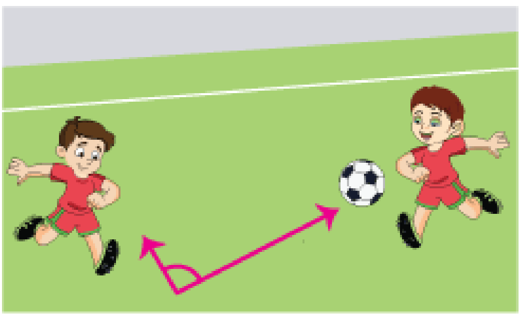
i. **Vertex:** the common endpoint where the two rays meet.

ii. **Sides or arms:** the two rays that form the angle.

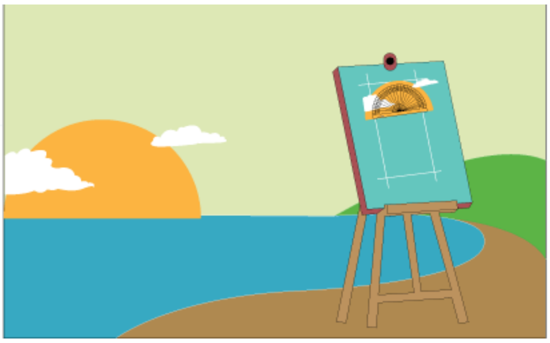
iii. **Measure:** the amount of rotation between the two rays**.**

Why Do We Need Angles?

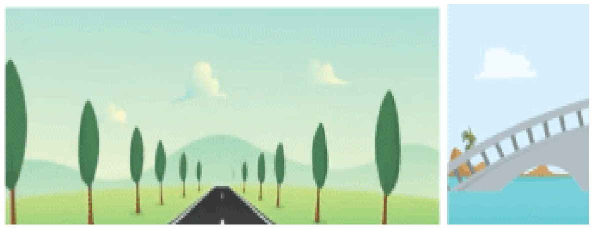
Our houses are full of angles, for example, the pitch of the roof to make the rain run off.



Even to pass the ball to the next player in soccer, we use angles.



Artists use angles every day to draw realistic pictures.



Architects can't do without them while designing roads and bridges

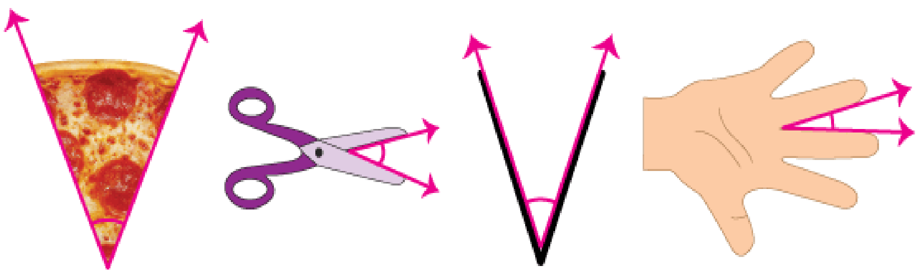
What are the Types of Angles?

Angles can be classified into different types based on their measures.

An Acute Angle

An acute angle is an angle which measures greater than 0° and less than 90°.

Acute Angles in Real Life

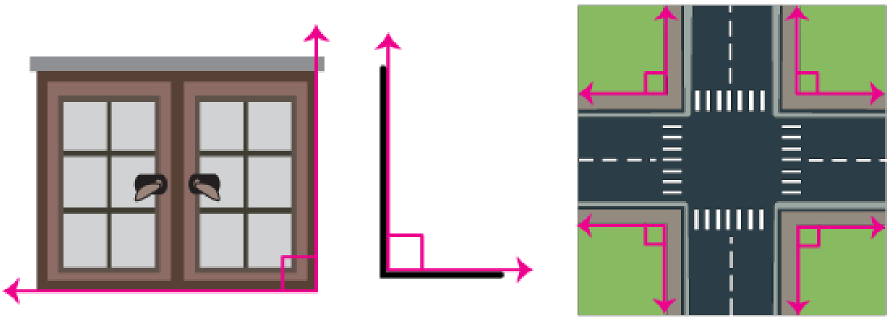


Just walk around, and you are bound to make a rich list of examples of acute angles.

A Right Angle

This is called a right angle (90 degrees) and is indicated with a little square rather than an arc.

Right Angles in Real Life

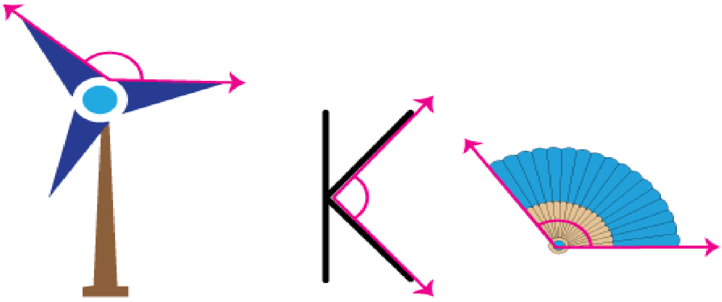


Just cast your eyes around a little bit, and you may find hundreds of examples to add on to your list of right angles.

An Obtuse Angle

This angle is larger than 90° but less than 180°.

Obtuse Angles in Real Life

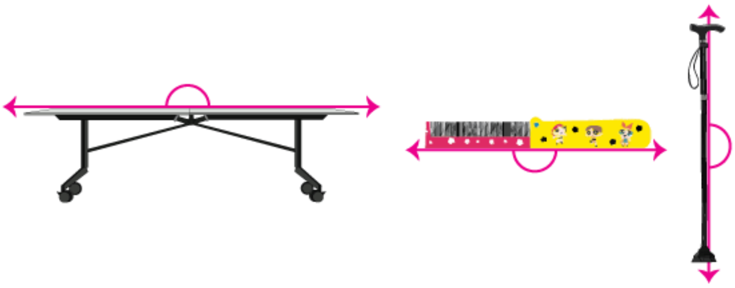


These are just a few instances from the tens of hundreds of obtuse angles that abound around you; look around for a good uptake of examples.

A Straight Angle

A straight angle measures 180°. A straight angle can be formed by adding two right angles.

Straight Angles in Real Life

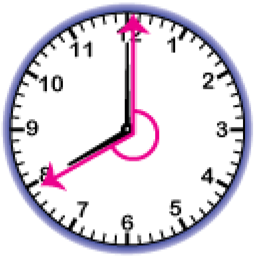


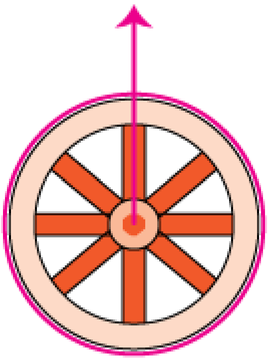
Reflex and Complete Angles

The angle formed when the ray moves past 180° and lies between 180° and 360° is called a reflex angle.

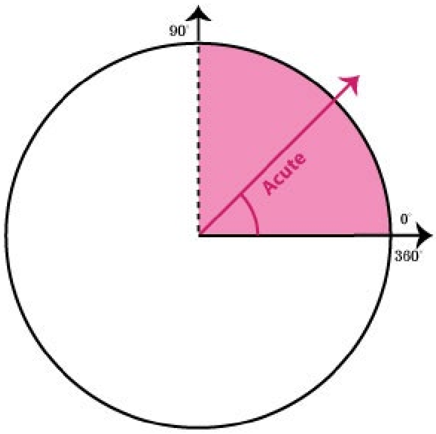
The angle formed when the ray completes one full rotation measures 360° and is called a full or complete angle.

Reflex and Complete Angles in Real Life





Bringing All Types of Angles Together!



In summary:

Acute Angle: An angle that is less than 90°.

Right Angle: An angle that is exactly 90°.

Obtuse Angle: An angle that is greater than 90° but less than 180°.

Straight Angle: An angle that is exactly 180°, forming a straight line.

Reflex Angle: An angle that is greater than 180° but less than 360°.

Full Rotation: An angle that is exactly 360°, representing a complete circle.

ACTIVITY 4.1: Individual/Pair/Group Work

Exploring Angles in Your Environment

Purpose: To identify and classify different types of angles in the environment and understand their practical applications.

Materials Needed:

- Protractor
- Ruler
- Notebook
- Pencil
- Camera or smartphone (optional for taking pictures)

Instructions:

1. **Angle Hunt**:
   - o Take a walk around your classroom, school or home. Look for examples of different types of angles. These could be in door frames, windows, corners of books, the clock or even the angle at which a tree branch meets the trunk.
   - o For each angle you find, use your protractor to measure the angle. Record the type of angle (acute, right, obtuse, straight, or reflex) and its measurement in your notebook.
   - o If possible, take a picture of each angle you find and attach it to your notes.
2. **Angle Investigation**:
   - o Choose five angles from your Angle Hunt. For each angle, draw a sketch of the object or place where you found the angle.
   - o Label the angles on your sketches and note whether each is acute, right, obtuse, straight or reflex.
3. **Group Discussion**:
   - o In small groups, share the angles you found and classified. Discuss how angles are used in different structures or objects you encountered. For example, how does the angle of a roof help in rainwater drainage or why are right angles important in building corners?

- Compare your findings with your group members. Did anyone find an angle type that others missed? Discuss why angles might be important in designing or building the objects you observed.

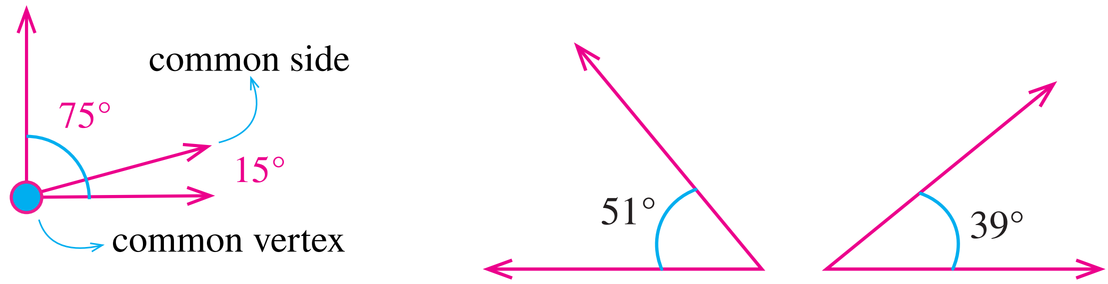
4. **Creative Angle Art**:
   - Use your knowledge of angles to create a piece of art or design on a piece of paper. Incorporate at least three different types of angles into your design.
   - Label each angle type in your artwork.
   - Share your artwork with the class, explaining where and why you used each angle.

5. **Reflection**:
   - Write a short reflection in your notebook about what you learned from this activity. Consider questions like: How do angles help in the construction of objects around you? Which angle did you find the most interesting and why?

Exploring Types of Angle Pairs

Angle pairs are two angles that have a specific relationship with each other. They are often based on their position relative to each other or to certain lines, such as parallel lines or a transversal. Here are some common types of angle pairs:

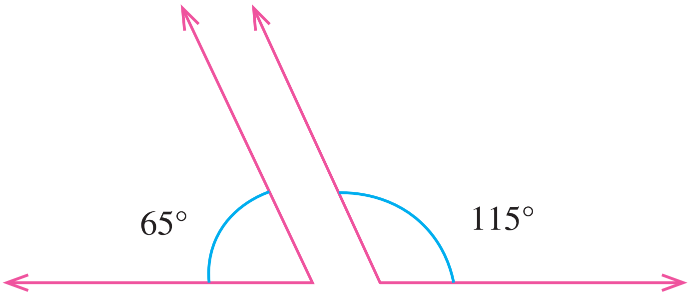
Complementary Angles: Two angles are complementary if their sum is exactly 90°.



Complementary angles do not have to be touching to be complementary.

Any two angles that sum to 90º are complementary angle pairs.

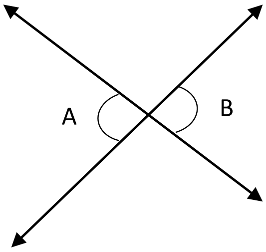
Supplementary Angles: Two angles are supplementary if their sum is exactly 180°.



Supplementary angles need not be linear pairs. They just have to add to 180°. They do not have to share a common side. They do not have to be adjacent angles:

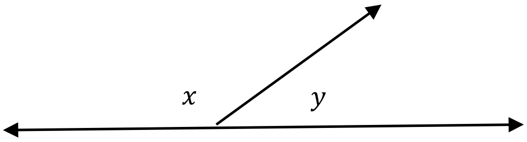
Any two right angles will always be supplementary and congruent (identical), whether they share a common vertex or common side.

Vertically opposite angles: vertically opposite angles are equal.



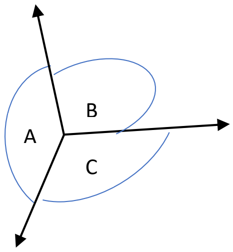
$\hat{A} = \hat{B}$

Adjacent angles: Adjacent angles in a straight line add up to 180°, ie, they are supplementary.



$x + y = 180$

Angles around a point: Angles around a point add up to 360°



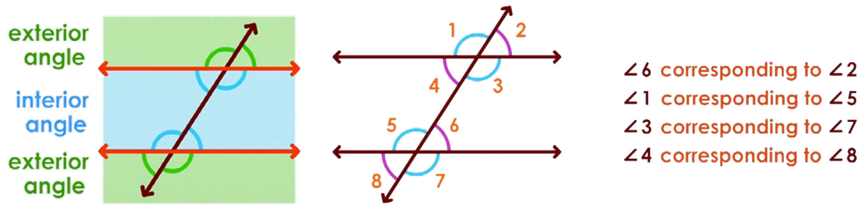
$\hat{A} + \hat{B} + \hat{C} = 360°$

Special pairs

If parallel lines are cut by a transversal, corresponding, alternate and co-interior angle are formed:

Corresponding angles

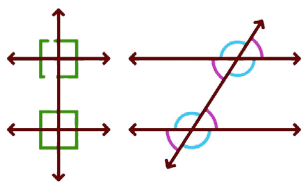
Two angles correspond or relate to each other by being on the same side of the transversal. One is an **exterior angle** (outside the parallel lines), and one is an **interior angle** (inside the parallel lines).



Did you notice **angle 6** corresponds to **angle 2**? They are a pair of corresponding angles. Did you find all four corresponding pairs of angles?

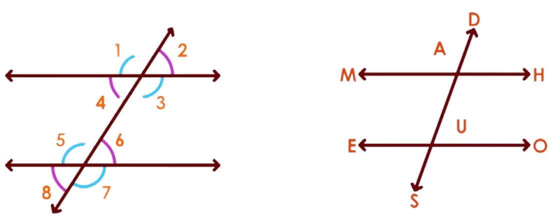
Because of the Corresponding Angles Theorem, you already know several things about the eight angles created by the three lines:

- If one is a right angle, all are right angles
- If one is acute, four are acute angles
- If one is obtuse, four are obtuse angles
- All eight angles can be classified as adjacent angles, vertically opposite angles and corresponding angles



Alternate angles

When the transversal intersects, it creates alternate angles. These are angles on opposite sides of the transversal line and have the same size.



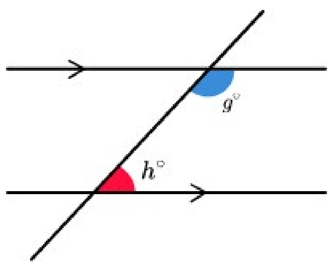
Can you find the alternate interior angles and alternate exterior angles?

- Alternate Interior Angles -- $\angle MAU$ and $\angle OUA$; $\angle HAU$ and $\angle EUA$, or angle 3 = angle 6, or angle 4 = angle 5
- Alternate Exterior Angles -- $\angle MAD$ and $\angle OUS$; $\angle HAD$ and $\angle SUE$, or angle 1= angle 8, or angle 2 = angle 7

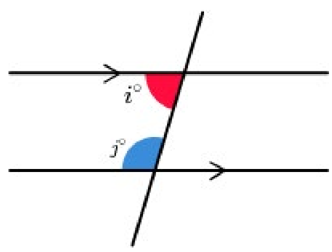
Co-interior angles

Co-interior angles occur in between two parallel lines when they are intersected by a transversal. The two angles that occur on the same side of the transversal always add up to 180º, ie, they are supplementary.

Co-interior angles add up to 180º



g + h = 1800

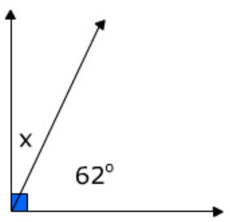


i + j = 1800

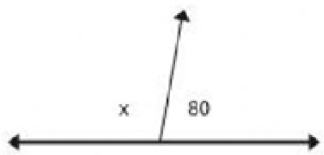
Example 1

Find the missing angles marked x

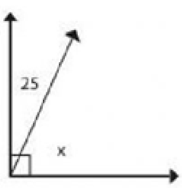
i.



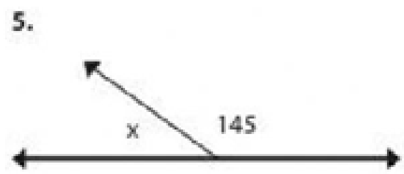
ii.



iii.



iv.



Solution

i. $x + 62 = 90$

$x + 62 - 62 = 90 - 62$

$x = 28^0$

ii. $x + 80 = 180$

$x + 80 - 80 = 180 - 80$

$x = 100^0$

iii. $25 + x = 90$

$25 - 25 + x = 90 - 25$

$x = 65^0$

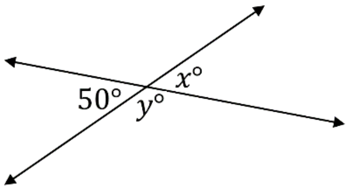
iv. $x + 145 = 180$

$x + 145 - 145 = 180 - 145$

$x = 35^0$

Example 2

Find the missing angles marked *x* and *y*.



Since vertically opposite angles are equal, we equate the two angles in order to find the value of *x*.

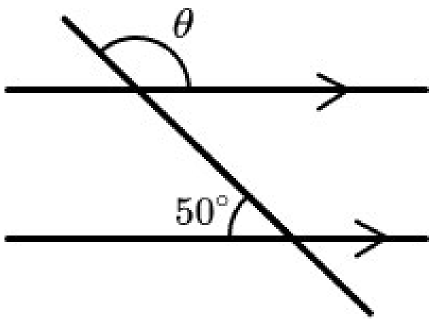
Therefore, the value of *x* is 50^0.

x and *y* are supplementary angles as they form a straight angle.

Therefore, *y* is $180 - 50 = 130^0$

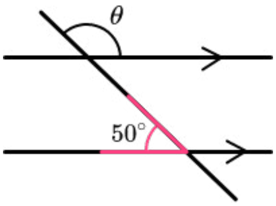
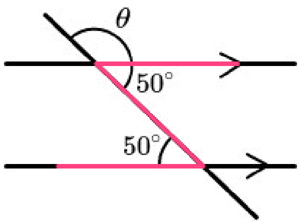
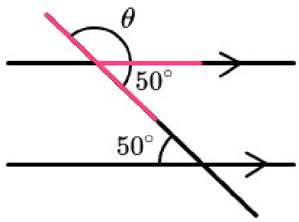
Example 3

Calculate the size of the missing angle θ. Give a geometrical reasons to justify your answer.



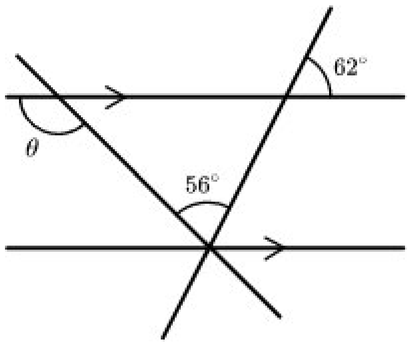
Solution

To solve this problem, let's follow these steps:

Step1: Highlight the angle(s) that you already know	**Step 2:** State the alternate angle, co-interior angle or corresponding angle fact to find a missing angle in the diagram.	**Step 3:** Use a basic angle fact to calculate the missing angle.
		
	Here we can label the **alternate angle** on the diagram as 50°.	Here as θ is on a straight line with 50° and these angles are **supplementary**. $\theta = 180° - 50°$ $\theta = 130°$

Example 4

Calculate the size of the missing angle θ. Give geometrical reasons to justify your answer.



Solution

To solve this problem, let's follow these steps

<table>
<tr>
<td>Step1: Highlight the angle(s) that you already know</td>
<td>Step 2: State the alternate angle, co-interior angle or corresponding angle fact to find a missing angle in the diagram.</td>
<td>Step 3: Use a basic angle fact to calculate the missing angle.</td>
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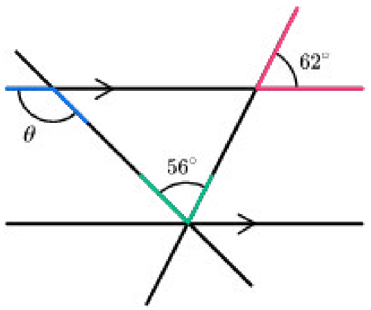
<td>
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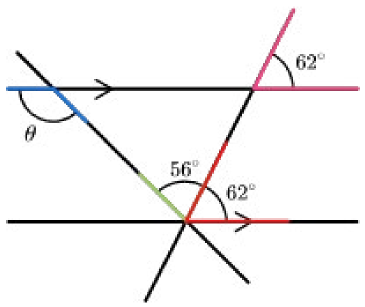
<td>
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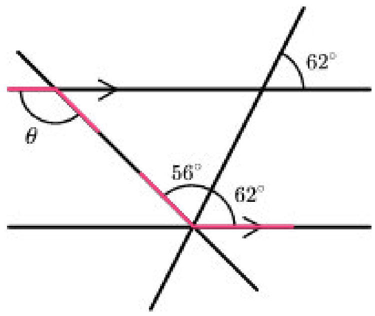
<td>
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<tr>
<td></td>
<td>Here we can state the angle 62° as it is corresponding to the original angle.</td>
<td>The angle θ is the alternate angle to the sum of 56° and 62° as shown in the diagram below.

$\theta = 56° + 62°$

$\theta = 118°$</td>
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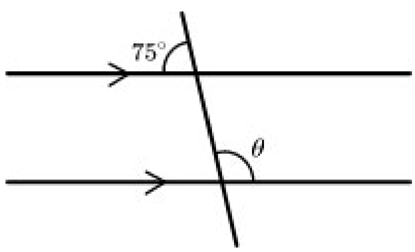
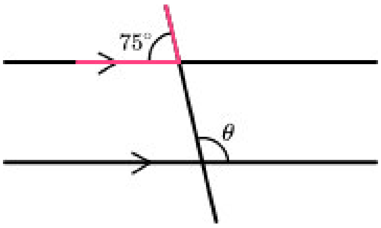
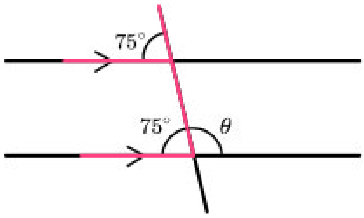
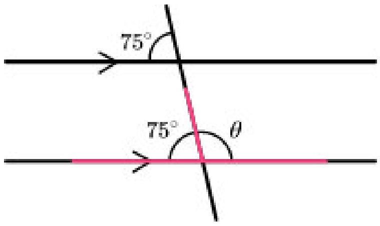
Example 5

Calculate the size of the missing angle θ. Give geometrical reasons to justify your answer.



Solution

To solve this problem, let's follow these steps

Step 1: Highlight the angle(s) that you already know	**Step 2:** Use corresponding angles to find a missing angle.	**Step 3:** Use a basic angle fact to calculate the missing angle..
		
	Here we can label the **corresponding** angle on the diagram as 75°.	Here θ is on a straight line with 75°, so the two angles are supplementary. $\theta = 180° - 75°$ $\theta = 105°$

ACTIVITY 4.2: Individual/Pair/Group Work

Finding Missing Angles

Purpose: Apply knowledge of angles and angle relationships to calculate the value of missing angles in various geometric figures.

Materials Needed:

- Protractor
- Ruler
- Pencil
- Paper with geometric diagrams (prepared by the teacher) [see a sample below]

Activity Instructions:

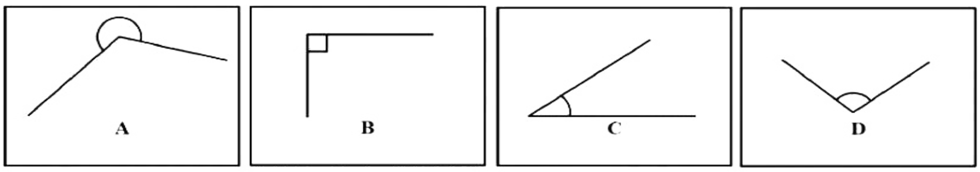
1. **Angle Hunt:**
   - o Your teacher will provide you with a sheet of paper (see below) that has several geometric diagrams. Each diagram will have one

or more angles labeled, but with some missing angles that need to be found.

2. **Step-by-Step Calculation:**
   - o **Identify the known angles:** Look at the diagram and identify all the given angles.
   - o **Use angle relationships:** Apply the appropriate angle relationship to calculate the missing angles.
   - o **Show your work:** Write down your calculations and explain your reasoning for each step.
3. **Measure to Verify:**
   - o After calculating the missing angles, use a protractor to measure them directly on the diagram. Compare your measurement with your calculated result. Are they the same or close? If not, check your work to find any mistakes.
4. **Group Discussion:**
   - o Once everyone has completed their calculations, discuss the answers as a group. Talk about any challenges you faced and how you overcame them. If there were differences in your answers, figure out why.
5. **Real-Life Connection:**
   - o Think about where you might need to calculate angles in real life. For example, when designing a piece of furniture or in architecture, why is it important to know how to find missing angles?

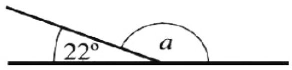
Exercise

1. Name the angle below



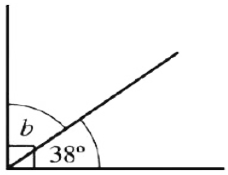
2. Find the missing angles in the questions below.

(a)



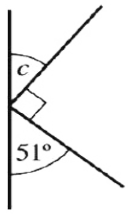
a=

(b)



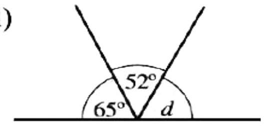
b=

(c)



c=

(d)

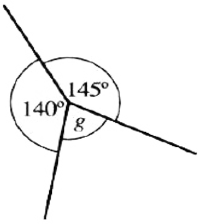


d=

(e)

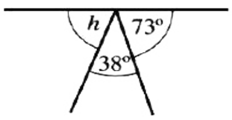
e= **f=**

(f)



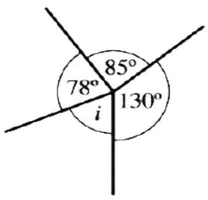
g=

g)



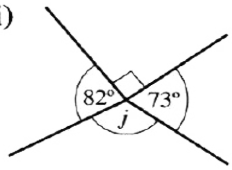
h=

(h)



i=

(i)



j=

Remember… Angles on a straight line add up to 180°
Angles around a point add up to 360°

MEASUREMENT OF PERIMETER

Focal Area 1: Perimeter of Plane Figures

Imagine you're helping your friend set up a fence around their garden. They have a beautiful rectangular plot of land and they want to know how much fencing material they need to buy. To figure this out, you need to know the total distance around the garden—this is where the concept of **perimeter** comes in.

The perimeter is the total length of the boundary of a two-dimensional shape. It's an important concept because it helps us measure the edges of any plane figure, whether it's a rectangle, triangle or any other shape. Knowing how to calculate the perimeter is essential in various real-life situations, such as fencing a garden, framing a picture or even designing the layout of a room. In this lesson, however, we will look at the perimeter of circular shapes.

REINFORCEMENT ACTIVITIES

Discovering the Concept of Pi

Purpose: Understand the relationship between the circumference and diameter of a circle, leading to the discovery of the concept of Pi (π).

Materials Needed:

- Several circular objects of different sizes (e.g., plates, lids, coins)
- String or flexible measuring tape
- Ruler
- Paper and pencil

Activity Instructions:

1. **Form Groups:**
   - o Get into small groups of 3-4 students. Each group will need one circular object, a piece of string, a ruler and a pencil.

2. **Measure the Diameter:**
   - o First, find the diameter of your circular object. Stretch the ruler across the centre of the circle, from one edge to the opposite edge, passing through the centre point. This is the longest distance across the circle.

     Write down this measurement in centimetres.
3. **Measure the Circumference:**
   - o Next, use the string to measure the circumference of the circle. Carefully wrap the string around the outer edge of the circle, keeping it tight. Mark where the string meets itself. Then, lay the string flat and measure the length with the ruler. Record this measurement in centimetres.
4. **Calculate the Ratio:**
   - o Now, divide the circumference by the diameter. Write down the result.
   - o Do this for each circular object your group has. Compare the ratios you find. What do you notice?
5. **Discussion:**
   - o As a class, compare the results from all groups. You should notice that the ratio of the circumference to the diameter is almost the same for all circles. This number is approximately 3.14, which we call Pi (π).
6. **Reflection:**
   - o Discuss why Pi is the same for all circles, no matter their size. How does this help us when we need to calculate the circumference or area of any circle?

Perimeter of a Circle (Circumference)

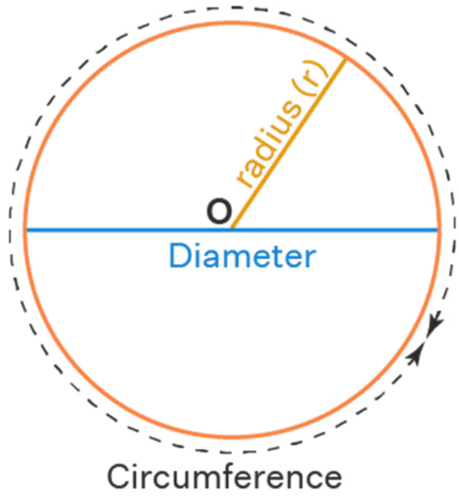
A circle is a two-dimensional shape made of points that are all the same distance from the centre.

Take a look at the diagram of the circle and identify the following:

Circumference: This is the distance round the circle.

Diameter: This is any straight line segment that passes through the centre of the circle. The diameter divides the circle into two equal halves.

Radius: This is the distance from the centre to the circle.



Establishing the Relationship Between Circumference and the Diameter

The relationship between the circumference (C) of a circle and its diameter (D) is one of the fundamental principles in geometry. This relationship is expressed through the mathematical constant known as **pi (π)**, which is approximately equal to 3.14159.

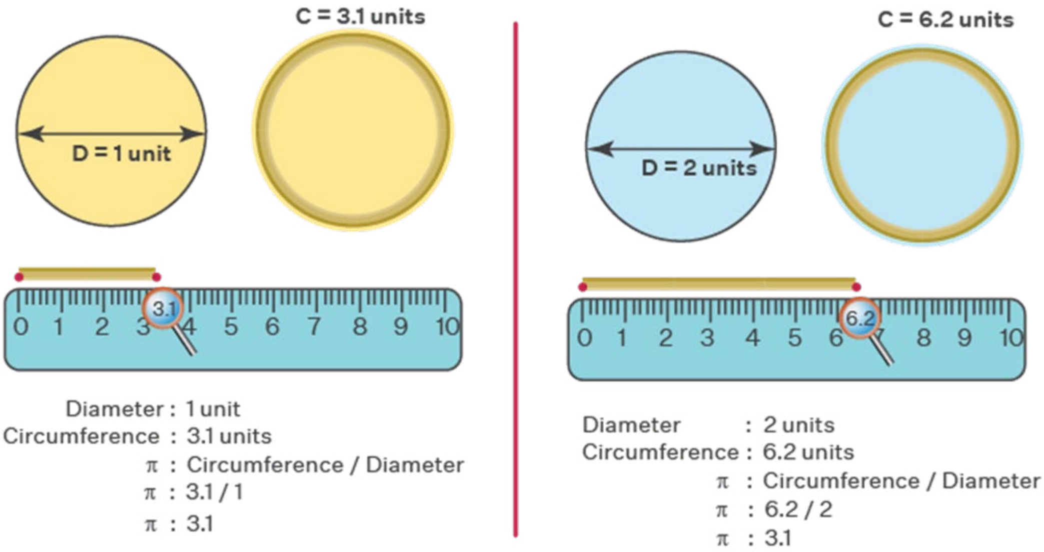
Key Concepts

1. **Circumference (C)**:
   - The circumference is the distance around the edge or boundary of a circle. It can be thought of as the perimeter of the circle.
2. **Diameter (D)**:
   - The diameter is the distance across the circle, passing through the centre. It is twice the length of the radius (r), where the radius is the distance from the centre of the circle to any point on its circumference. **Diameter = 2 × Radius**
3. **Pi (π)**:
   - Pi (π) is a special mathematical constant representing the ratio of a circle's circumference to its diameter. No matter the size of the circle, this ratio remains constant.

In our earlier activity, we demonstrated how to obtain the value of pi and why it is a constant. Apart from the practical activity that we performed, we can quickly go through these steps to confirm the value of pi **(π).**

The value of pi can be calculated with the help of a simple activity. Follow the steps given below to know why the value of pi is the ratio of the circumference to the diameter of the circle:

- **Step 1:** Draw a circle of diameter 1 unit.
- **Step 2:** After this step, take a thread and place it along the border of the circle (the circumference).
- **Step 3:** Now, place the thread on the ruler and note the length.



Repeat the process with diametres of 2 units, 3 units, 4 units, 5 units and record your observations in the table.

Diameter	Circumference	Circumference/Diameter
1 unit	3.1 units	3.1/1
2 units	6.2 units (approx.)	6.2/2 = 3.1
3 units	9.3 units (approx.)	9.3/3 = 3.1
4 units	12.4 units (approx.)	12.4/4 = 3.1
5 units	15.5 units (approx.)	15.5/5 = 3.1

We can observe that the ratio of circumference to diameter is always the same, which is 3.1.

Therefore we have generated the formula:

Circumference of a circle $= \pi \times \textit{diameter} = 2 \times \pi \times r$

As π is an irrational number we need to approximate a value for calculations.

We can use the π button on our calculators, if they have one, or use 3.14, or better, as a rounded value of π. The fraction $\frac{22}{7}$ is also a good approximation too.

Example 1

A circle has a radius of 3 units. What is the circumference (ie the perimeter) of the circle?

Take $\pi = \frac{22}{7}$

Solution

Circumference of a circle $= 2\pi r$

Circumference of the circle $= 2 \times \frac{22}{7} \times 3 = \frac{132}{7} = 18.8571$ *units*

Example 2

Find the circumference of the circle with a radius of 10 cm.

Solution

As we know:

Circumference (C) $= 2\pi r$, here r $= 10$ cm, $\pi = 3.141$

$= 2 \times 3.141 \times 10$

$= 62.82$ cm

Example 3

The circumference of a circle is 44 units. What is the diameter of the circle if the π value is taken as $\frac{22}{7}$?

Solution

The circumference of the given circle is 44 units; value of $\pi = \frac{22}{7}$.

Therefore, we will substitute the given values in the formula for the circumference of a circle, $2\pi r = 44$

$$2 \times \frac{22}{7} \times r = 44$$

$$44 \times r = 44 \times 7$$

$$r = \frac{44 \times 7}{44}$$

$$r = 7 \text{ units}$$

Diameter $= 2 \times$ radius
$= 2 \times 7$
$= 14$ units.

Therefore, the diameter of the circle $= 14$ units.

ACTIVITY 4.3: Individual/Pair/Group Work

Exploring Circumference and Radius of a Circle

Purpose: Apply knowledge of the circumference of a circle to calculate the circumference when the radius is given and find the radius when the circumference is given.

Materials Needed:

- String or flexible measuring tape
- Ruler
- Calculator
- Paper and pencil
- Several circular objects (e.g., lids, cups, coins)

Activity Instructions:

Measuring Circumference:

- o Choose a circular object (like a cup or lid) from the materials provided.
- o Use the string or flexible measuring tape to measure the circumference of the object. To do this, wrap the string around the widest part of the circle (the circumference) and mark where the string overlaps. Then, use a ruler to measure the length of the string.
- o Write down the circumference measurement.

Calculate the Radius:

- o Now, using the formula for the circumference of a circle, $C = 2\pi r$, calculate the radius r of the object.
- o Rearrange the formula to solve for r: $r = \frac{C}{2\pi}$
- o Use a calculator to compute the radius and write down your result.

Verify the Radius:

- o Use a ruler to directly measure the diameter of the circular object. Remember that this is the distance across the circle, going through the centre.
- o Halve your diameter to find the radius (the distance from the centre of the circle to the edge)
- o Compare this with the radius you calculated.

  Are the two values close? Discuss any differences and possible reasons why.

Explore with Different Objects:

- o Repeat steps 1-3 with at least two other circular objects. Measure their circumferences, calculate the radius from the diameter, and then compare your calculated radius with the direct measurements.

Finding Circumference from Radius:

- o Now, let's do the reverse. Choose one of the circular objects and directly measure its diameter using a ruler and from this the circle's radius.
- o Use the formula $C = 2\pi r$ to calculate the circumference based on the radius you measured.
- o Measure the circumference with the string or measuring tape and compare it with your calculated circumference.

  Are they similar?

Real-Life Connection:

- o Discuss situations where you might need to know the circumference, diameter or radius of a circle in real life. For example, measuring the size of a circular garden or finding the length of the border needed to edge a round table.

Focal Area 2: Area of Plane Figures (Triangle and Circle)

We have learnt in the past that area is the measure of the space enclosed by a 2D shape. Area is measured in square units. We learnt to calculate the area of squares and rectangles. Let us go through the following activities to remind ourselves of calculating the area of a square and rectangle.

REINFORCEMENT ACTIVITIES

Calculating the Area of Squares and Rectangles

Purpose: Apply knowledge of the area formulas to calculate the area of various squares and rectangles.

Materials Needed:

- Grid paper
- Ruler
- Pencil
- Calculator
- Several objects of rectangular or square shape (e.g., books, tiles, sheets of paper)

Activity Instructions:

Identifying Shapes:

- o Look around the classroom and identify different objects that are either square or rectangular in shape. Pick at least three objects, such as a textbook (rectangle), a floor tile (square) or a sheet of paper (rectangle).

Measuring Dimensions:

- o Use the ruler to measure the length and width of each rectangular object. For the square object, measure the length of one side. Write down these measurements on your grid paper.
- o Example: If you choose a textbook, measure its length and width in centimetres.

Calculating Area:

- o Now, use the following formulas to calculate the area:
  - **For Rectangles:** Area = Length × Width
  - **For Squares:** Area = Length x Length (L^2)
- o Perform the calculations and write down the area for each object.

Drawing on Grid Paper:

- o On your grid paper, draw a representation of each object using the measurements you took. Ensure that your drawing is to a scale of your choice but be ready to explain your scale to the class.

- Shade in the area of each object on the grid paper. Count the squares to visually confirm the area you calculated.

Compare and Discuss:

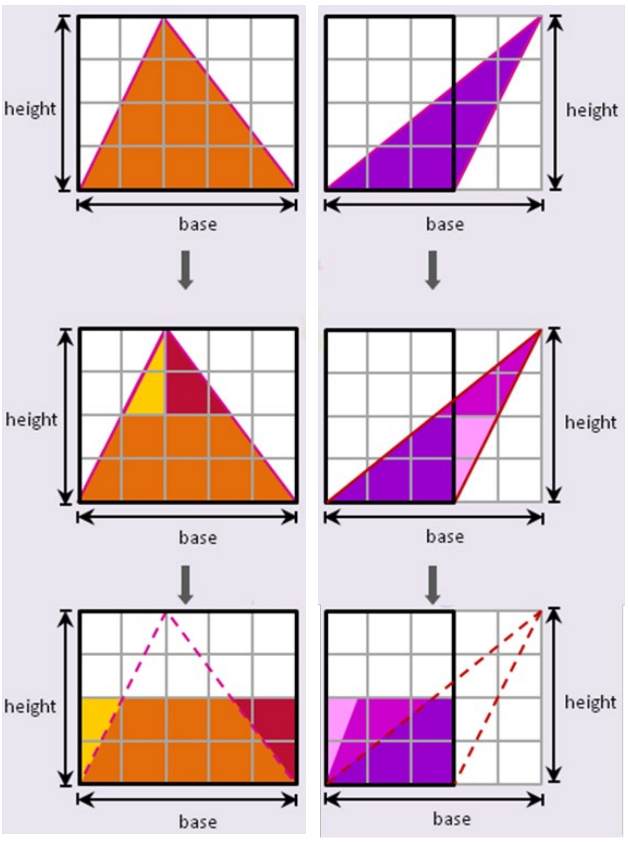
- Compare the areas of the objects you measured. Discuss which object has the largest area and which has the smallest. Consider why this might be the case.
- Talk about the real-life implications of area, such as determining how much material would be needed to cover a surface or how much space an object occupies.

Challenge Task:

- Imagine you need to cover a rectangular floor with square tiles. If the floor measures 5 metres by 4 metres, and each tile is 1 meter by 1 meter, how many tiles will you need? Calculate the area of the floor and the area of one tile to find the answer.

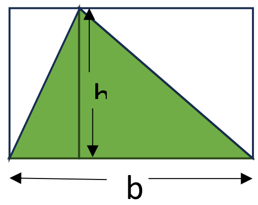
Area of a Triangle

Area of triangle is $A = \frac{1}{2}b \times h$. This can be interpreted as, the area of a triangle is half the area of a rectangle. Study the diagrams below and discuss it with a friend.



Example 1

Find the area of the triangle with base length = 10 cm and height = 8cm.



Solution

Area of triangle is $A = \frac{1}{2}b \times h$

Where b = base h is the height

Given b = 10 cm and h = 8cm

$A = \frac{1}{2}(10) \times 8$

$A = 5 \times 8$

$A = 40\ cm^2$

Example 2

Find the area of a triangle that has a base of 3 cm and a height of 6 cm.

Solution

Given b = 3 cm and h = 6 cm

$A = \frac{1}{2}(3) \times 6$

$A = \frac{1}{2} \times 18$

$A = 9\ cm^2$

Area of a Circle

To easily understand how to determine the area of a circle, let's use our knowledge of the area of a rectangle.

ACTIVITY 4.4: Individual/Pair/Group Work

Purpose: to determine the area of a circle using rectangles.

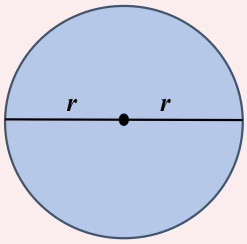
Materials Needed:

- Paper
- Ruler
- Pencil

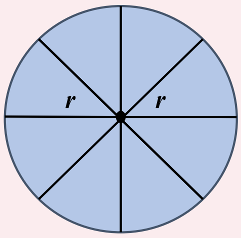
Working in Pairs: In Pairs, take time to go through these activity steps:

Activity Steps:

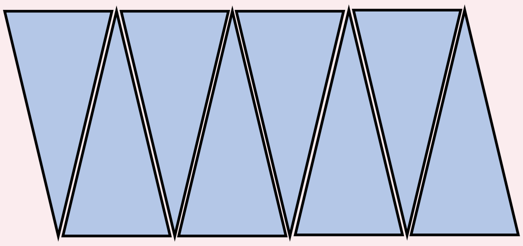
Step 1: Draw a circle on a card, of radius *r*, and divide it into two equal parts as shown below.



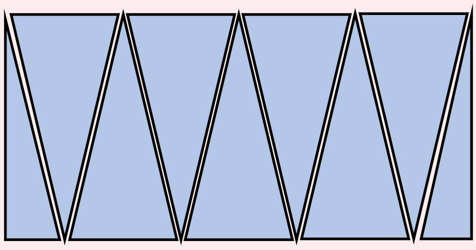
Step 2: Divide the circle into *n* equal sectors. The more sectors you use, the closer the approximation to the actual area. This activity makes use of 8 sectors



Step 3: Each sector will almost form a triangle. Then rearrange the 'triangles' so they look like this:

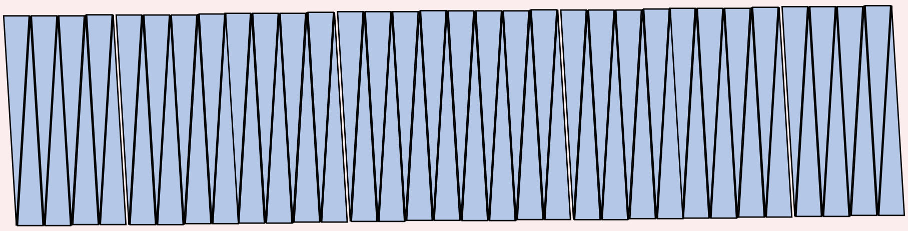


Step 4: The shape looks like a parallelogram. Shape it into a rectangle by cutting part of the end 'triangle' to fill the slant edge so it looks like this:



When dividing the circle into n equal sectors, each sector forms a triangle when rearranged. As n increases, the angle of each sector decreases, making the triangle's base (length of the sector's arc) approach very small values and height approaches the radius of the circle. As the number of sectors (n) increases, the approximation of the circle's area becomes more accurate. In the formula for the area of the circle, π represents the ratio of the circumference of the circle to its diameter.

As n approaches infinity, the sectors become very small, and the resulting shape closely resembles a rectangle. This concept illustrates the fundamental relationship between the circumference and area of a circle, providing a geometric interpretation of π



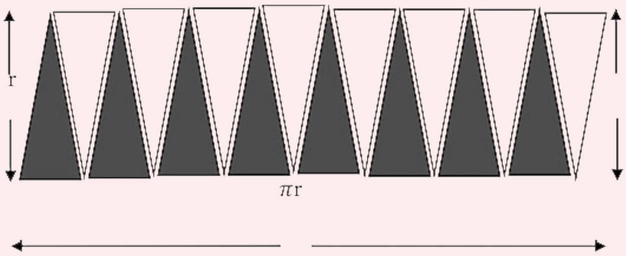
The role of π becomes apparent in this context. It represents the ratio of the circumference of a circle to its diameter, a fundamental constant in geometry. As such, when calculating the area of a circle using sectors to form a rectangle, π emerges naturally as a factor in the formula.

Step 5: The height of each triangle is the radius of the circle, and the base of the triangle is equal to the length of the sector's arc.

The area of each triangle is given by the formula $\frac{1}{2}\times\textbf{base}\times\textbf{height}$

The total area of all the triangles (sectors) will approximate the area of the circle.

Since the shape formed by the rearranged sectors is a rectangle, the area of the rectangle will be the same as the total area of the sectors.



Using the area of a rectangle, L x W as a reference, we have arranged the sectors into a rectangle.

With the *L* as πr and the width as *r*, the area of the circle can therefore be deduced as :

Area $= r \times \frac{1}{2}$ circumference

But circumference $= 2\pi r$

i.e. Area $= r \times \frac{1}{2} 2\pi r$

$\mathbf{A = \pi r^2}$

But d = 2r

$r = \frac{d}{2}$

Therefore, the area of a circle given the diameter, d

$A = \pi \left(\frac{d}{2}\right)^2$

$\mathbf{A = \frac{\pi d^2}{4}}$

Example 1

If the length of the radius of a circle is 4 cm. Calculate its area.

Solution

Radiusm(r) = 4 units (given)

Using the formula for the circle's area,

Area of a Circle $= \pi r^2$

Substitute in the values,

$A = \pi(4)^2$

$A = \pi \times 16$

$A = 16\pi = 50.27$ cm^2

Example 2

Find the circumference and the area of a circle whose radius is 14 cm. Take $\pi = \frac{22}{7}$.

Solution

Given: Radius of the circle = 14 cm

Circumference of the Circle = 2πr

$= 2 \times \frac{22}{7} \times 14$

$= 2 \times 22 \times 2$

= 88 cm

Using area of Circle formula = πr^2

$= \frac{22}{7} \times 14 \times 14$

$= 22 \times 2 \times 14$

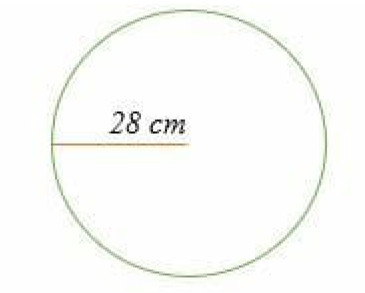
= 616 cm^2.

Area of the Circle = 616 cm^2.

Answer: Circumference = 88 cm and area = 616 cm^2.

Example 3

Calculate the area of the circle shown below.



Solution

$A = \pi r^2$ square units

$= (3.14 \text{ x } 28^2)\ cm^2$

$= (3.14 \times 28 \times 28)\ cm^2$

$= 2461.76\ cm^2$

MEASUREMENT OF SURFACE AREA AND VOLUME OF PRISMS

Focal Area: Surface Area and Volume of Prisms

Imagine you're helping a friend pack up their belongings to move to a new house. You have a collection of boxes in different shapes and sizes, and you need to figure out how much space each box can hold so you can decide what can go inside each one. This task involves understanding the **volume** of the boxes, which tells you how much space is inside each box. But to wrap these boxes properly for the move, you also need to know how much wrapping paper you'll need to cover the entire outside of each box. This is where understanding the **surface area** comes into play.

In real life, the concepts of surface area and volume are crucial in various situations, such as packaging, construction and manufacturing. When designing containers, engineers must calculate the volume to ensure the container holds the right amount of material and the surface area to determine the amount of material needed to create the container.

REINFORCEMENT ACTIVITIES

Exploring Surface Area and Volume of Prisms

Purpose: To help you understand the basic concepts of surface area and volume by exploring how much space an object occupies and how much material is needed to cover it.

Activity: Building and Wrapping Your Own Prisms

Materials Needed:

- A set of small cardboard boxes or different-sized rectangular prisms (like cereal boxes, shoe boxes, or gift boxes)
- Wrapping paper or coloured paper
- Scissors
- Tape or glue
- Ruler
- Measuring tape

Instructions:

Step 1: Choose a Box (Prism)

- Select one of the cardboard boxes provided.
- Take a moment to observe the shape. Notice that it has different sides, also known as faces.

Step 2: Measure the Box

- Use the ruler or measuring tape to measure the length, width and height of your box. Write down these measurements.
- Next, use these measurements to calculate the area of each face and then adding them together to get the total surface area of the box.

Step 3: Wrap the Box

- Using the wrapping paper or coloured paper, cut out pieces that will cover each face of your box. As you cut, keep track of how much paper you need to cover the entire box.
- Try wrapping the box with the paper pieces you've cut out. Does your paper fit the faces perfectly or do you need to make adjustments?

Step 4: Explore the Volume

- Now that your box is wrapped, think about what could fit inside it. How much space does it have?
- Imagine filling the box with small objects like cubes, and count how many cubes could fit inside it.

  This gives you an idea of the volume.

Step 5: Discuss and Reflect

- Discuss with your classmates: How did the surface area help you figure out how much wrapping paper you needed?
- How does the volume relate to how much space the box has inside?
- What would happen to the surface area and volume if the box were larger or smaller?

Surface Area of Prisms

A prism is a polyhedron in which all the faces are flat and the bases are parallel to each other. The sides other than the bases are called lateral faces. This prism is a solid object with its flat faces, identical ends and the same cross-section along with its length. The area and volume of prisms are fundamental concepts in geometry and are essential for understanding three-dimensional shapes. Let's take a look at the prisms that we will work with in this lesson.


Triangular Prism


Rectangular Prism


Square Prism

The area of a prism refers to the total surface area that covers all its faces, while the volume represents the space enclosed within the prism.

To calculate the surface area of a prism:

i. Find the area of each face (including the bases and lateral faces).

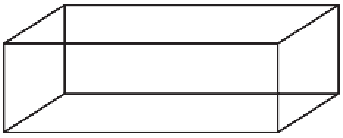
ii. Sum up the areas of all the faces to get the total surface area.

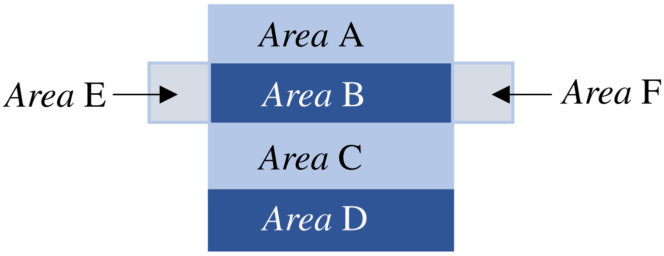
The formula for the total surface area of a prism depends on its shape.

Surface Area of a Rectangular Prism (Cuboid)

To determine the surface area of a rectangular prism (also known as a cuboid), the shapes that make up the rectangular prism must be known. If you look at the net of a rectangular prism, you will find that the shapes of the rectangular prism are six rectangles, with opposite sides of the boxes the same. Example:

A possible net of a rectangular prism looks like this:





Area of each rectangle

$= Length \times Width\ (l \times w)$

To determine the surface area of the rectangular prism (Area A = Area C, Area B = Area D, Area E = Area F), you need to determine the areas of all six rectangles. Since opposite sides are equal, you only have to calculate the area of three rectangles, double each area and add them.

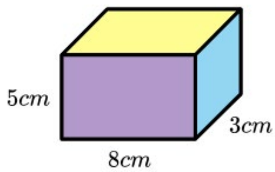
Surface Area of Rectangular Prism = 2(Area A) + 2(Area B) + 2(Area E)

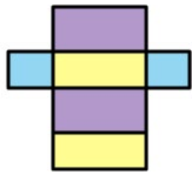
$= 2(L_A + W_A) + 2(L_B + W_B) + 2(L_E + W_E)$

Note: L_A is the length of rectangle A, while L_B is the length of rectangle B. The length of rectangle A may or may not be the same as the length of rectangle B. You need to be careful to use the correct dimensions to find each area. Don't panic, you are not expected to use the notation L_A.

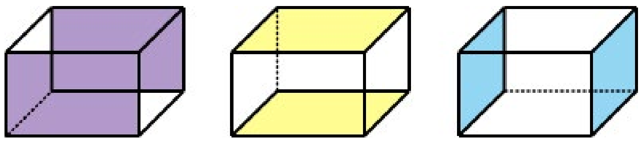
Worked Example

Determine the surface area of the box below.

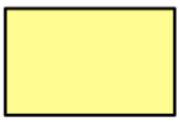
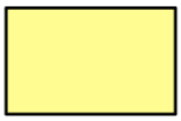




Notice that the box specifically has 3 pairs of congruent faces (not all prisms have this property) as the opposing faces are the same size.



To calculate the surface area of the cuboid, we need to calculate the area of each face, and then add them together.

Face	Area	Face	Area
5cm  8cm	$A = 5 \times 8 = 40cm^2$	5cm  8cm	$A = 5 \times 8 = 40cm^2$
3cm  8cm	$A = 3 \times 8 = 24cm^2$	3cm  8cm	$A = 3 \times 8 = 24cm^2$
5cm  3cm	$A = 5 \times 3 = 15cm^2$	5cm  3cm	$A = 5 \times 3 = 15cm^2$

Now that we know the area of each face, the surface area of the prism is the sum of these values.

$40 + 24 + 15 + 40 + 24 + 15 = 158.$

The surface area of the cuboid is equal to158cm^2.

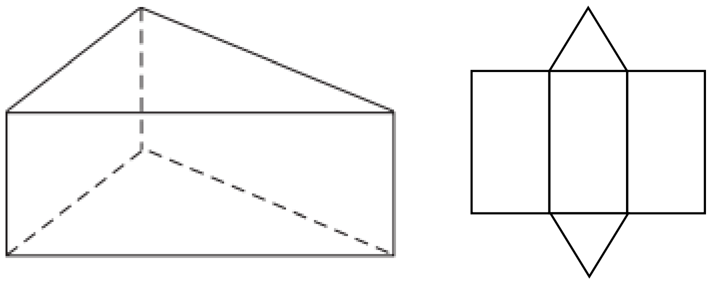
Note: Surface area is measured in square units (e.g. mm^2, cm^2, m^2 etc.).

Surface Area of a Triangular Prism

To determine the surface area of a triangular prism, the shapes that make up the triangular prism must be known. If you look at the net of a triangular prism, you will find that the shapes of the triangular prism are three rectangles and two triangles, with the opposite triangles being the same size.

A possible net of a triangular prism looks like this:

The rectangles may or may not be the same size, depending on the type of triangle the base is made from.



Area of rectangle $= l \times w$

Area of triangle $= \frac{b \times h}{2}$

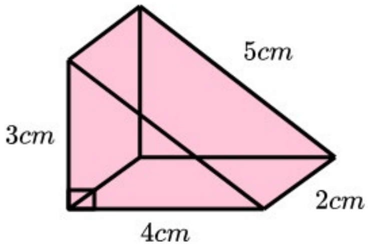
To determine the surface area of the triangular prism, you need to determine the area of the two triangles and the area of the three rectangles. You may be able to combine some areas if they contain the same measurements. A general formula for determining the surface area of a right triangular prism is as follows:

Surface Area of a Triangular Prism = (area of rectangle 1) + (area of rectangle 2) + (area of rectangle 3) + 2(area of triangle)

$= (l \times w) + (l \times w) + (l \times w) + \frac{b \times h}{2} + \frac{b \times h}{2}$

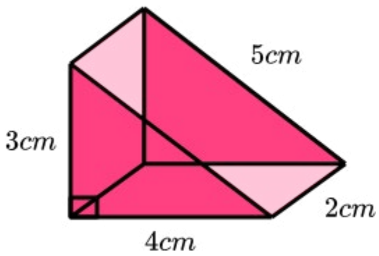
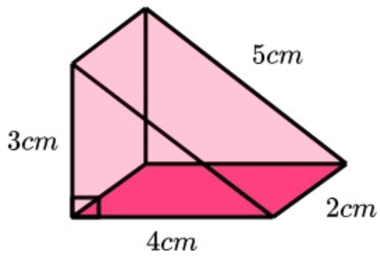
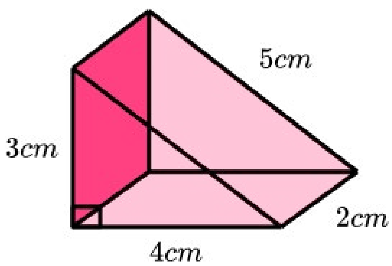
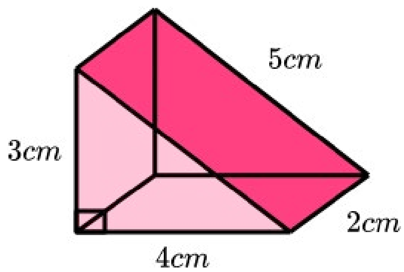
Example

Work out the surface area of the right triangular prism below.



Solution

Work out the area of each face.

Step 1: The area of the front of the prism is $\frac{1}{2} \times 4 \times 3 = 6\text{cm}^2$. The back face is the same as the front face so the area of the back face is also 6cm^2.	**Step 2:** The area of the base is $4 \times 2 = 8\text{cm}^2$.
	
Step 3: The area of the left side is $2 \times 3 = 6\text{cm}^2$.	**Step 4:** The area of the right side is $2 \times 5 = 10\text{cm}^2$.
	

It will make our working clearer if we use a table.

Face	Area
Front	$\frac{1}{2} \times 4 \times 3 = 6$
Back	6
Bottom	$4 \times 2 = 8$
Left side	$2 \times 3 = 6$
Top	$2 \times 5 = 10$

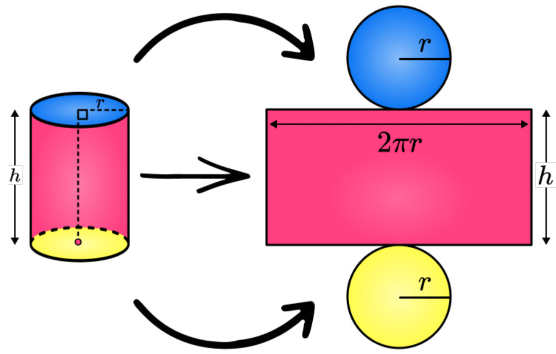
Add the area of each face together.

The total surface area (SA) = $6 + 6 + 8 + 6 + 10 = 36$ cm^2.

Surface Area of a Cylinder

To determine the surface area of a right cylinder, the shapes that make up the cylinder must be known. If you look at the net of a right cylinder, you will find that the shapes of the right cylinder are two circles (if there is a top and a bottom) and a rectangle.

A possible net of a right cylinder looks like this:



r **is the radius of the** circular ends of the cylinder and h **is the perpendicular height of** the cylinder.

The curved surface area of a cylinder is actually a rectangle.
The circumference of the circle is the length of the rectangle.
The height h **of the cylinder is the height of the rectangle.**

Total surface area $= 2\pi rh + \pi r^2 + \pi r^2$

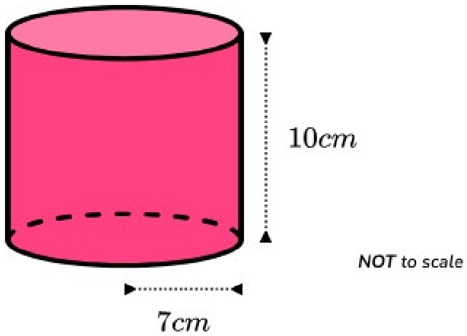
$= 2\pi rh + 2\pi r^2$

Surface area is measured in square units, written as cm², m², and so on.

Remember that the length of the rectangle is the same as the circumference of the circle.

Example

Find the total surface area of this cylinder with radius of the base 7cm and perpendicular height 10cm.



Solution

First, we need to find the curved surface area of the cylinder:

Curved surface area=$2\pi rh = 2 \times \pi \times 7 \times 10 = 140\pi$

We then need to find the area of the base of the cylinder:

Area of a circle $= \pi r^2 = \pi \times 7^2 = 49\pi$

The area of the top of the cylinder is the same as the area of the base so,

Total surface area:

$140\pi + 49\pi + 49\pi = 238\pi = 747.699905\ldots$

The surface area of the cylinder is 747.7cm^2 (1dp)

ACTIVITY 4.5: Individual/Pair/Group Work

Calculating the Total Surface Area of 3D Shapes

Purpose: To practise calculating the total surface area of different 3D shapes: cubes, cuboids, triangular prisms, and cylinders, using real-life objects and measurements.

Materials Needed:

- A set of models (or cut-out shapes) of a cube, cuboid, triangular prism, and cylinder
- Ruler or measuring tape
- Graph paper
- Calculator
- Scissors and paper (for creating models if needed)

Instructions:

Step 1: Measure the Dimensions

- o **Cube:** Measure the length of one side of the cube. All sides are equal.
- o **Cuboid:** Measure the length, width and height of the cuboid.
- o **Triangular Prism:** Measure the base, height and diagonal lengths of the triangular face and the length of the prism.
- o **Cylinder:** Measure the radius of the base and the height of the cylinder.

Step 2: Record Your Measurements

- o Write down the measurements for each shape on a piece of paper or in your notebook. Label each measurement clearly.

Step 3: Calculate the Surface Area

- o **Cube:** Use the formula $\textit{Surface Area} = 6 \times \textit{side}\ 2$.
    - Substitute the length of one side into the formula and calculate the total surface area.
- o **Cuboid:** Use the formula $\textit{Surface Area} = 2(lw + lh + wh)$.
    - Substitute the length, width and height into the formula and calculate the total surface area.
- o **Triangular Prism:** Use the formula $\textit{Surface Area} = 2 \times \textit{Base Area} + \textit{Perimeter of Base} \times \textit{Length of Prism}$.
    - First, calculate the area of the triangular base and the perimeter of the base, then use the formula to find the total surface area.
- o **Cylinder:** Use the formula $\textit{Surface Area} = 2\pi r(r + h)$.
    - Substitute the radius and height into the formula and calculate the total surface area.

Step 4: Create Models (Optional)

- o Use paper and scissors to create 3D models of each shape. Fold and glue the paper to construct the shapes based on your measurements.
- o Compare the models with your calculations to ensure accuracy.

Step 5: Draw and Calculate (If No Models)

- o On graph paper, draw the net (2D representation) of each shape and calculate the surface area by adding up the areas of all the faces.

Step 6: Reflect

- Reflect on how changing one dimension of the shape affects the total surface area. Write a short paragraph about your observations.

Volume of Prisms

To calculate the volume of a prism:

i. Find the area of the base (often a polygon) by using the appropriate formula.

ii. Multiply the area of the base by the height of the prism.

Mathematically, it is defined as the product of the area of the base and the perpendicular height.

Therefore,

$$\textit{The volume of a Prism} = \textit{Base Area} \times \textit{Perpendicular Height}$$

Note that previously we learnt how to determine the volume of cubes and cuboids. Hence, in this lesson, we will learn how to determine the volume of triangular prisms and cylinders.

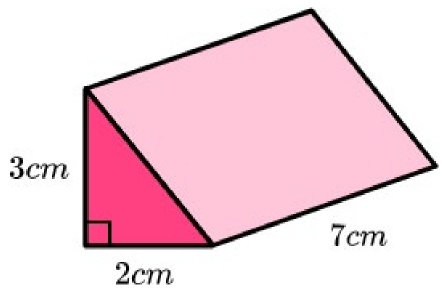
Remember that the units of volume are the length cubed, ie, cm^3 m^3, in^3 etc.

Volume of a Triangular Prism

The volume of a triangular prism is determined by multiplying the area of the base of the triangular prism by the height of the triangular prism.

Example 1

Work out the volume of this triangular prism.



Solution

Volume of a triangular prism = Area of triangular cross section x length

Calculate the area of the triangular cross-section and substitute the values.

The base of the triangle is 2cm and the height of the triangle is 3cm.

Area of triangle $= \frac{1}{2} \times b \times h$

Area of triangle $= \frac{1}{2} \times 2 \times 3$

Area of triangle $= 3\ cm^2$

The length of the prism is 7cm.

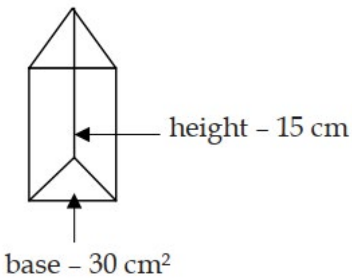
Volume of triangular prism = Area of triangular cross section x length

Volume of triangular prism $= 3 \times 7$

$\therefore$Volume of the triangular prism $= 21cm^3$

Example 2

Calculate the volume of the prism with base area of 30 cm^2 and a height of 15 cm.



Solution

Volume of a triangular prism = area of base $\times$ height.

$$= 30cm^2 \times 15cm$$

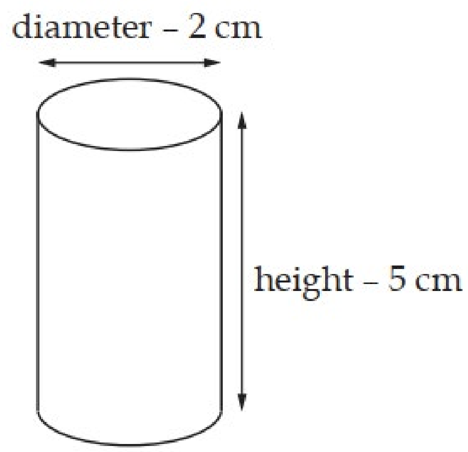
$$= 450cm^3$$

Volume of a Cylinder

The volume of a cylinder is determined by multiplying the area of the base by the height of the cylinder.

Example 1

Determine the volume of the cylinder drawn below;



Solution

Area of base = πr^2

Radius (r) = 1cm

Area = $\pi(1cm)^2$

Area =$3.14cm^2$

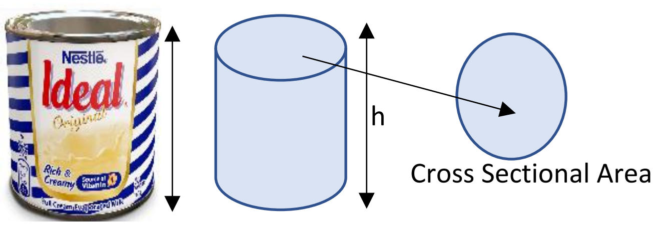
Volume = Area of base × height

= $3.14cm^2 \times 5cm$

= $15.7cm^3$

Example 2

The figure below is a milk tin with height 15cm and the diameter of the cross section is 10cm. Calculate the volume of the tin.



Solution

To find the volume of the milk tin, we use the formula for the volume of the prism (cylinder)

Volume = Base Area (cross section) × Height

Volume = $\pi r^2 \times h$

Where r is the radius of the circular cross-section and h is the height of the cylinder.

Given that the diameter is 10 cm, the radius r is half of that,

so $r = \frac{10}{2} = 5\text{cm}$

Substituting the given dimensions:

$V = \pi \times (5\,cm)^2 \times \mathbf{15}\,\text{cm}$

$V = \pi \times 25\,cm^2 \times \mathbf{15}\,\text{cm}$

$V = 375\pi\,cm^3$

But taking $\pi = \frac{22}{7}$

$V = 375 \times \frac{22}{7}\,cm^3$

So, the volume of the milk tin is $375\pi\,cm^3$, which is approximately $\mathbf{1178.6\,cm^3}$ when rounded to one decimal place.

ACTIVITY 4.6: Individual/Pair/Group Work

Calculating the Volume of Triangular Prisms and Cylinders

Purpose: To help you apply what you've learned about calculating the volume of triangular prisms and cylinders by measuring and calculating the volume of real objects.

Materials Needed:

- A set of triangular prisms (these could be made from cardboard or wooden blocks)
- Cylindrical objects (like cans or tubes)
- Ruler or measuring tape
- Calculator
- Graph paper

Instructions:

Step 1: Measure the Dimensions

- **For the Triangular Prism:** Measure the base and perpendicular height of the triangular face and then measure the length (height) of the prism.
- **For the Cylinder:** Measure the radius (half of the diameter) of the base and the height of the cylinder.

Step 2: Record Your Measurements

- Write down the measurements you've taken on a piece of paper or in your notebook. Make sure to label each measurement correctly.

Step 3: Calculate the Volume

- **Triangular Prism:** Use the formula Volume = $\frac{1}{2} \times$ Base $\times$ Height of triangle $\times$ Length of prism.
    - Substitute the values you measured into the formula and calculate the volume.
- **Cylinder:** Use the formula Volume = $\pi \times \text{radius}^2 \times$ Height of cylinder.
    - Substitute the values you measured into the formula and calculate the volume.

Step 4: Compare with Classmates

- Share your results with your classmates. Compare the volumes of different triangular prisms and cylinders.
- Discuss why some objects have larger or smaller volumes based on their dimensions.

Step 5: Visualise the Volumes

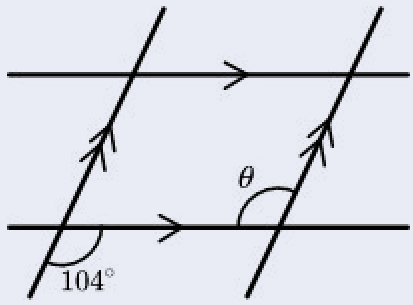
- On graph paper, draw a scale diagram of the cross-sections of your triangular prism and cylinder. This will help you better understand the relationship between the shape's dimensions and its volume.

Step 6: Reflect

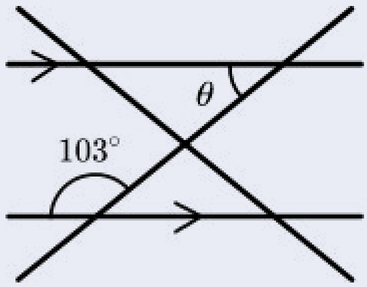
- Reflect on how changing one dimension of the prism or cylinder (e.g., making the height taller) would affect the volume. Write down your thoughts.

REVIEW QUESTIONS

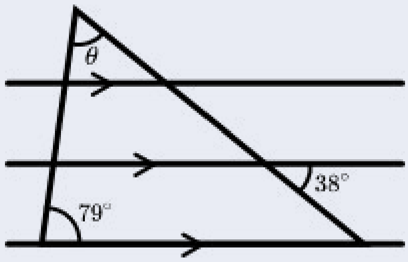
1. Calculate the size of the angle θ.



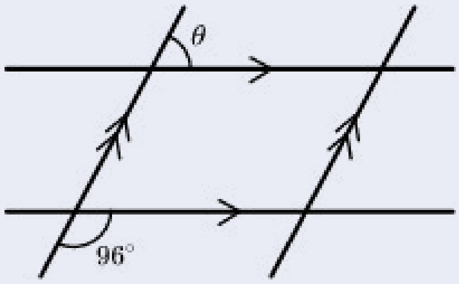
2. Calculate the size of angle θ.



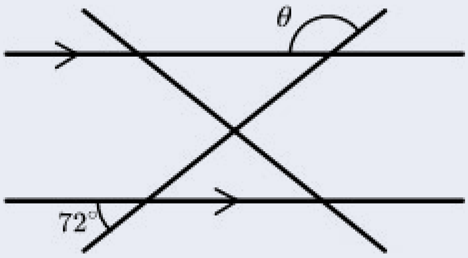
3. Find the value of θ



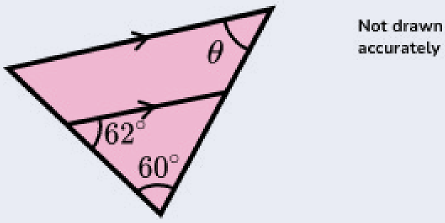
4. Calculate the size of angle θ



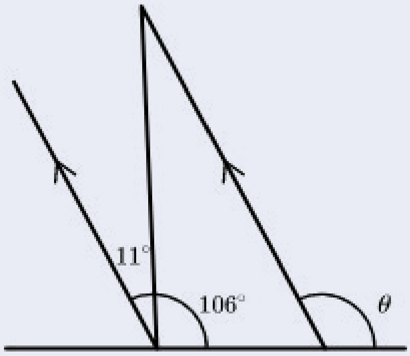
5. Calculate the value of θ.



6. Work out the size of angle θ



7. Calculate the value for angle θ



8. Find the circumference of a circle with a radius of 5 cm.

9. A circle has a diameter of 14 cm. What is the circumference?

10. Calculate the circumference of a circle with a radius of 8.5 cm.

11. What is the circumference of a circle with a diameter of 20 cm?

12. A circular garden has a radius of 10 metres. Find the circumference of the garden.

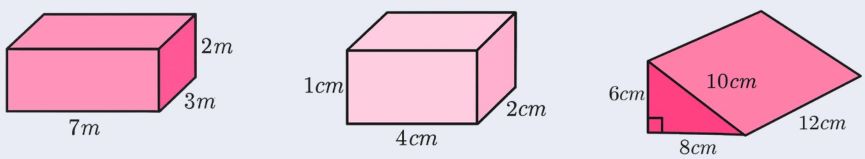
13. A triangle has a base of 8 cm and a height of 5 cm. What is the area of the triangle?

14. A triangle has a base of 10 metres and a height of 6 metres. What is the area of the triangle?

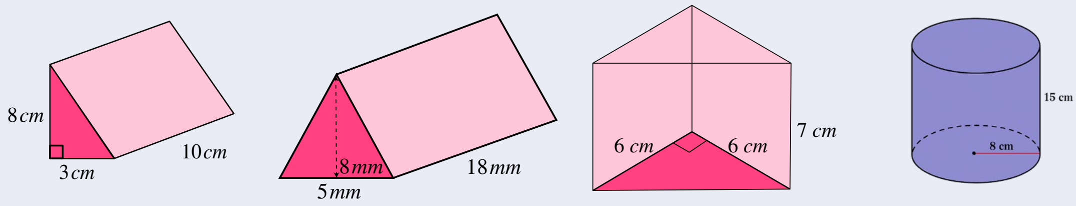
15. A triangle has a base of 15 inches and a height of 9 inches. What is the area of the triangle?
16. A triangle has a base of 7 cm and a height of 4 cm. What is the area of the triangle?
17. A triangle has a base of 12 metres and a height of 8 metres. What is the area of the triangle?
18. Find the area of a circle with a radius of 7 cm.
19. Find the area of a circle with a diameter of 10 cm.
20. Calculate the area of a circle with a radius of 3.5 cm.
21. What is the area of a circle with a diameter of 16 cm?
22. A circle has a radius of 12 cm. What is its area?
23. Find the area of a circle with a diameter of 24 cm.
24. What is the area of a circle with a radius of 8.5 cm?
25. A circle has a diameter of 6 cm. What is its area?
26. Find the area of a circle with a radius of 10.5 cm.
27. Calculate the area of a circle with a diameter of 14 cm.

Real-Life Problems on Finding the Area of a Circle

28. A pizza has a diameter of 14 inches. What is the area of the pizza?
29. A circular garden has a radius of 5 metres. How much area does the garden cover?
30. A clock face has a diameter of 20 cm. Find the area of the clock face.
31. A circular swimming pool has a radius of 7 metres. What is the area of the pool?
32. A round table has a diameter of 1.2 metres. What is the area of the tabletop?
33. Work out the surface area of the following prisms.



34. Workout the volume of the following prisms.



ANSWERS TO REVIEW QUESTIONS

1. $\theta = 104°$
2. Using alternate angles and angles on a straight line: $\theta = 180 - 103 = 77^0$
3. Using alternate angles, we can see that the angle in the bottom right vertex of the triangle is 38^0. We can then use angles in a triangle: $\theta = 180 - (79 + 38) = 63^0$
4. $\theta = 84°$
5. $\theta = 108°$
6. $\theta = 58°$
7. $\theta = 117°$
8. 31.4cm (to 1dp)
9. 44.0cm (to 1dp)
10. 53.4cm (to 1dp)
11. 62.8cm (to 1dp)
12. 62,8m (to 1dp)
13. $20cm^2$
14. $30m^2$
15. $67.5in^2$
16. $14cm^2$
17. $48m^2$
18. $153.9cm^2$ (to 1dp)
19. 78.5 cm^2 (to 1dp)
20. $38.5cm^2$ (to 1dp)
21. 201.1 cm^2 (to 1dp)
22. 452.4 cm^2 (to 1dp)
23. 452.4 cm^2 (to 1dp)
24. 227.0 cm^2 (to 1dp)
25. 28.3 cm^2 (to 1dp)

26. 346.4 cm^2 (to 1dp)

27. 153.9 cm^2 (to 1dp)

28. 153.9 in^2 (to 1dp)

29. 78.5 m^2 (to 1dp)

30. 314.2 cm^2 (to 1dp)

31. 153.9 m^2 (to 1dp)

32. 1.13 m^2 (to 2dp)

33. **i.** 82 m^2

ii. 28 cm^2

iii. 336 cm^2

iv. 226 mm^2

34. **i.** 120 cm^3

ii. 360 mm^3

iii. 126 cm^3

iv. 3 015.9 cm^3 (to 1dp)

ACKNOWLEDGEMENTS

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