SECTION

5 PROBABILITY OF EVENTS

COLLECTING AND HANDLING DATA

PROBABILITY

In this section, you will learn to;

1. *Classify the occurrence of everyday life situations as impossible, possible, or certain.*
2. *Design and conduct an experiment in which the likelihood of a single outcome occurring is impossible, possible (likely or unlikely), certain.*
3. *Conduct a given probability experiment a number of times, recording the outcomes and explaining the results.*

SECTION INTRODUCTION

In this section, you will classify everyday life situations based on their likelihood of occurring as impossible, possible or certain, helping you to better understand and predict outcomes in real-world scenarios, such as weather forecasts or game results. You will design and conduct experiments to explore the likelihood of different outcomes, determining whether they are impossible, possible (likely or unlikely), or certain. By conducting these probability experiments multiple times and recording the outcomes, you will gain insights into the patterns and probabilities that influence everyday events, enhancing your ability to make informed decisions.

FOCAL AREA 1: DESCRIBING THE LIKELIHOOD OF A SINGLE OUTCOME

There are everyday life situations which we cannot tell their occurrence with certainty. Clouds form and we may hope to have rain but sometimes it does not. At childbirth, the child which comes out may be a boy or a girl. In all these cases, we can only guess, or predict, which likely situations may occur. The value which we assign to the possibility of an event is the **probability.**

Reinforcement Activities

Probability Prediction Game

Purpose: To introduce the concept of probability through an interactive and fun game that helps you understand the likelihood of different outcomes.

Materials Needed:

- A coin
- A die (six-sided)
- A deck of cards (optional)
- A bag with different coloured marbles (e.g., 5 red, 3 blue, 2 green)
- Whiteboard or chalkboard
- Markers or chalk
- Notebook and pen for recording predictions and outcomes

Instructions:

Step 1: Group Discussion

- Within your class have a discussion about what you think the word "probability" means.

  Your teacher will write down your ideas on the board.
- Hopefully, you will come up with the idea that probability is a way of predicting how likely something is to happen.

Step 2: Coin Toss Activity

- Your teacher will flip a coin. "What do you think will happen if I flip this coin? Will it land on heads or tails?"
- Predict the outcome by raising your hands for "heads" or "tails."
- Your teacher will flip the coin and record the result on the board.
- Repeating this 10 times, recording the results each time.

Step 3: Dice Rolling Activity

- Next, your teacher will be rolling a die: "What number do you think will come up if I roll this die?"
- Predict the outcome by choosing a number between 1 and 6.
- Roll the die and record the result on the board.

- Repeating this 10 times, recording the results each time.
- Have a discussion about how each number had an equal chance of appearing, but some numbers may have appeared more often.

Step 4: Marble Drawing Activity

- This time your teacher has a bag with marbles of different colours. Contained within it are 5 red, 3 blue, and 2 green marbles.
- Answer this: "If I draw one marble from the bag, which colour do you think is most likely to come out? Which is least likely?"
- Predict by choosing a colour.
- Your teacher will draw a marble from the bag and record the result.
- Repeating this 10 times, recording the colour of each marble drawn.
- As a class, discuss the results: Did the most likely colour appear the most? Was there a surprise?

Step 5: Reflection and Discussion

- Now reflect on the activities:
  - Did the results match your predictions?
  - What did you notice about the likelihood of different outcomes?
  - How did the number of options (like 6 sides on a die or 3 colours of marbles) affect the chances?
- These activities help us understand how probability works in everyday life, from guessing outcomes in games to making decisions based on chances.

Step 6: Summary

- Hopefully you will have realised that probability is all about predicting the chances of something happening.
- In the next lesson, you will explore more about how probability works with different kinds of events and how to calculate it.

Conducting Experiments and Classifying Events as Likely or Unlikely or Certain

An activity whose outcome cannot be predicted with certainty can be described as an experiment. To start a game, a coin may be tossed to determine how the game is to start. The umpire/referee will be conducting an experiment. Similarly, during play, we throw a ludo die wishing for a certain number, but this is frequently not the number we roll. In all these activities, we can say that we are conducting an experiment.

During a ludo game, a single throw is termed a **trial**. Each trial can reveal; 1, 2, 3, 4, 5 or 6. These numbers form the **possible outcomes**. The successful outcome (the number that actually shows up) is our event. The set of all possible outcomes is termed **sample space.**

Example 1

A 20 pesewas coin is tossed once and the outcome observed as the "cocoa" turned up.

a. What are the possible outcomes?

b. Write the sample space.

c. What is the event?

d. Can both cocoa and coat of arms show up at the same time? Why?

e. Describe such an event.

Solution

a. Possible outcomes = cocoa, coat of arms

b. Sample space = {cocoa, coat of arms}

c. The event = {cocoa} – the side that turned up when the coin was flipped.

d. Both cocoa and coat of arms cannot show up at the same time because the face turns flat to show only one face at a time.

e. For Both cocoa and coat of arms to show up is an impossible situation.

Example 2

A game involves selecting pebbles numbered 0 to 10 and summing them to reach 50. One pebble is selected at a time in turns and observed. The first player to get a sum of 50, wins the game.

a. What is the sample space for a single selection?

b. Describe these events as certain, possible, impossible

i. A pebble numbered '7' will be selected

ii. A pebble numbered '12' will be selected

iii. A pebble numbered 1 to 10 will be selected

iv. A pebble numbered with prime number will be selected

v. A pebble numbered with a negative number will be selected

Solution

a. The sample space = **{0,1,2,3,4,5,6,7,8,9,10}**

b. Describing the events as sure, possible, impossible

i. A pebble numbered '7' will be selected = **possible event**

ii. A pebble numbered '12' will be selected = **impossible event**

iii. A pebble numbered 1 to 10 will be selected = **certain event**

iv. A pebble numbered with prime number will be selected = **possible event**

v. A pebble numbered with a negative number will be selected = **impossible event**

Example 3

A bag contains 12 red toffees, 8 green toffees and 16 yellow toffees. One toffee is selected at random. Give a reason why you think the event is impossible, certain or possible

a. The toffee is red or green or yellow.

b. The toffee is yellow.

c. The toffee will be both red and green.

d. The toffee is blue.

Solution

a. It is **certain** that the toffee being red or green or yellow because they are the only colours of toffees in the bag.

b. It is **possible** because there are yellow toffees in the bag.

c. **Impossible** because only one toffee was picked and there is no toffee which has two colours.

d. **Impossible** because there are no blue toffees in the bag.

ACTIVITY 5.1: Individual/Pair/Group Work

Classifying Events as Likely, Unlikely, or Certain"

Purpose: To apply the concept of probability by conducting simple experiments and classifying the outcomes as likely, unlikely or certain.

Materials Needed:

- A coin
- A die (six-sided)
- A deck of cards (optional)
- A bag with different coloured marbles (e.g., 5 red, 3 blue, 2 green)
- Probability event cards (with scenarios written on them)
- Notebook and pen for recording results

Instructions:

Step 1: Experiment Setup

o Divide yourselves into small groups (3-4 students per group).

- Each group will rotate through different experiment stations. The stations will have the following items:
  - Station 1: A coin
  - Station 2: A die
  - Station 3: A bag with different coloured marbles
  - Station 4: Probability event cards

Step 2: Conducting Experiments

- **Station 1 (Coin Toss):**
  - Your group will flip the coin 10 times.
  - For each flip, record whether the coin lands on heads or tails.
  - Discuss with your group: Is getting heads or tails a likely event, an unlikely event, or a certain event?
- **Station 2 (Die Roll):**
  - Your group will roll the die 10 times.
  - For each roll, record the number that comes up.
  - Discuss with your group: Is rolling a specific number (like a 6) likely, unlikely, or certain?
- **Station 3 (Marble Draw):**
  - Your group will draw a marble from the bag 10 times (with replacement).
  - Record the colour of each marble drawn.
  - Discuss with your group: Is drawing a red marble likely, unlikely, or certain?
- **Station 4 (Probability Event Cards):**
  - Your group will pick an event card, which might say something like "The sun will rise tomorrow," "It will rain today," or "You will roll a 7 on a die."
  - Discuss with your group whether the event is likely, unlikely or certain and explain why.

Step 3: Group Discussion

- After completing the experiments, come back together as a class for a discussion.

- Share your results and conclusions.
- Discuss how the outcomes varied and why some events were classified as likely, unlikely, or certain.

Step 4: Reflection

- Reflect in your notebooks:
  - Which events were surprising?
  - How did the number of possible outcomes affect the likelihood of an event?
  - What does it mean when something is certain or impossible in the context of probability?

Step 5: Summary

- Summarise the activity by explaining that probability helps us understand the chances of different events happening.
- Reinforce the concept that likely events have a good chance of happening, unlikely events have a small chance and certain events will definitely happen, whereas impossible events will never happen.

FOCAL AREA 2: CALCULATING PROBABILITIES

The chance that a baby being born will be a boy exist. It is a possible outcome. **How much** possibility exists for this to occur?

The "**How much"** is a measure or an amount. The chance of an event occurring can be given a numerical value.

By this we say probability is a measure assigned to the possibility that an event will occur.

Calculating probability of single experiment conducted once

To calculate probability of an event;

a. we identify the sample space count to identify the sample size as n(P)

b. we identify the event and count to determine the number of successful outcomes as n(E)

c. describe the chance /probability as the number of successful outcomes out of the total number of elements in the sample space

d. calculate the ***ratio of number of successful outcomes to the total number of elements in the sample space.***

That is $P(E) = \frac{n(E)}{n(S)}$

Example 1:

A ludo die is thrown once and the outcome observed. Find the probability of …

i. a '5' showing up.

ii. an even number showing up.

iii. a number greater than 4 showing up.

Solution

The sample space (S) = {1,2,3,4,5,6}

Sample size [n(S)] = 6

i. Event '5' showing up

n(E) = 1, ie. There is only one element in the sample space which is 5

$P(E) = \frac{n(E)}{n(S)}$

P(5 shows up) = $\frac{1}{6}$

ii. Event even number shows up = {2,4,6}

n(E) = 3

$P(E) = \frac{3}{6} = \frac{1}{2}$

iii. Event (number greater than 4 shows up) = {5,6}

n(E) = 2

$P(E) = \frac{2}{6} = \frac{1}{3}$

Example 2:

A random card is drawn from a deck of 52 cards. What is the probability that it is an ace?

Solution:

E = event of drawing an ace.

Total number of outcomes = 52

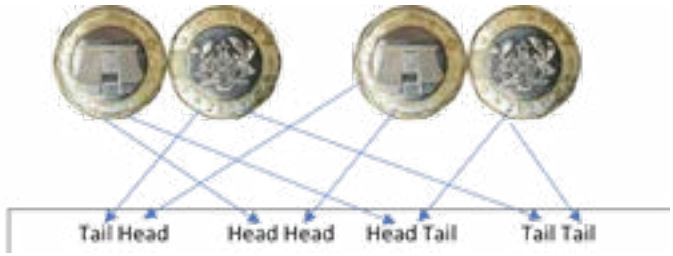
The favorable number of outcomes = 4 (there are 4 ace cards in a deck of cards. One belonging to each suit).

$P(E) = \frac{4}{52} = \frac{1}{13}$

Answer: P(E) = $\frac{1}{13}$

Calculating probability of experiments conducted multiple times

A 'Zim Zim' game is played with 2 similar coins or flat objects. The 2 coins are thrown at the same time and the faces that show are recorded. After 20 throws, the player with more similar faces showing up in each throw wins the game.



If we represent the faces of the coin as 'Head' (H) and 'Tail' (T), then the possible outcomes will be (HH), (HT), (TH) and (TT) .

Example 1:

If a coin is tossed 3 times what would be the event of getting at most two heads?

Solution

The sample space for tossing coin three times is

{(H, H, H), (T, H, H), (H, T, H), (H, H, T), (T, T, H), (H, T, T), (T, H, T), (T, T, T)} = 8 possible outcomes

E = {(T, H, H), (H, T, H), (H, H, T), (T, T, H), (H, T, T), (T, H, T), (H, H, H)} = 7 events which satisfy the condition

The event of getting at most two heads is {(T, H, H), (H, T, H), (H, H, T), (T, T, H), (H, T, T), (T, H, T), (H, H, H)}

Therefore, the probability of this occurring is $\frac{7}{8}$.

Example 2:

A coin is tossed twice and the outcome observed.

a. Write the sample space.

b. Calculate the probability that:

i. two Heads show up.

ii. different faces show up.

Solution

a. Sample Space = {(HH), (HT), (TH), (TT)}

b. **i.** sample size = 4

Event, 2 Heads show up = {HH}, Hence, n(E) = 1

$P(E) = \frac{1}{4}$

ii. Event, different faces show up = {(HT), (TH)}, Hence, n(E) = 2

$P(E) = \frac{2}{4} = \frac{1}{2}$

ACTIVITY 5.2: Individual/Pair/Group Work

Calculating the Probability of an Experiment

Purpose: To apply the concept of probability by calculating the probability of outcomes from a designed experiment.

Materials Needed:

- A bag containing 12 different coloured counters (e.g., 4 red, 3 blue, 2 green, 2 yellow, 1 white)
- A coin
- A six-sided die
- A spinner with 8 equal sections labeled 1 to 8
- Notebook and pen for recording results

Instructions:

Step 1: Design Your Experiment

- Divide yourselves into small groups of 3-4 students.
- Your group will select one of the following items to design their experiment:

- The bag of coloured counters
- A coin
- A six-sided die
- A spinner with 8 sections

Step 2: Conduct the Experiment – depending on which item your chose

o **Bag of Counters:**

- Each group will draw one counter randomly from the bag and record its colour.
- Replace the counter in the bag and repeat the draw 20 times.
- Record the results of each draw (e.g., how many times each colour was drawn).
- Calculate the experimental probability for drawing each colour (e.g., Probability of red $= \frac{Number\ of\ red\ draws}{20}$).

o **Coin Toss:**

- Each group will flip the coin 20 times and record whether it lands on heads or tails each time.
- Calculate the experimental probability of landing on heads and tails

  (e.g., Probability of heads $= \frac{Number\ of\ heads}{20}$).

o **Die Roll:**

- Each group will roll the die 30 times and record the outcome each time.
- Calculate the experimental probability of each number appearing

  (e.g., Probability of rolling a 4 $= \frac{Number\ of\ 4s\ rolled}{30}$).

o **Spinner:**

- Each group will spin the spinner 20 times and record the number it lands on each time.
- Calculate the experimental probability of landing on each number (e.g., Probability of landing on 3 $= \frac{Number\ of\ 3s\ spun}{20}$).

Step 3: Analyse the Results

- After conducting the experiment, each group will compare their experimental probabilities with the theoretical probabilities. For example, the theoretical probability of drawing a red counter from the bag might be $\frac{4}{12}$ while the experimental probability could differ based on the results.
- Discuss the differences between experimental and theoretical probability:
  - Did your results match the theoretical probabilities?
  - Why might there be a difference?
  - How does the number of trials affect the accuracy of your experimental probability?

Step 4: Class Discussion

- Each group will present their experiment and findings to the class.
- Discuss how probability helps in predicting outcomes and why it's important to understand the difference between experimental and theoretical probabilities.

Step 5: Reflection

- Reflect on how the results of the experiment demonstrate the principles of probability.
- Consider how probability could be used in real-life situations, such as predicting weather patterns, determining odds in games or making decisions based on likelihoods.

Step 6: Wrap-Up

- Do you think that probability is a way to quantify how likely an event is to happen and that more trials typically lead to experimental probabilities that are closer to theoretical probabilities?

REVIEW QUESTIONS

1. Write one situation in the school that can be considered "certain".
   a. Certain …….
   b. Impossible ……...
   c. Possible ……
2. A ludo dice with faces (1,2,3,4,5,6) is tossed once and the outcome observed.
   a. Write the sample space.
   b. What is the sample size?
   c. Write the event that a prime number shows up.
   d. Describe (in words) the probability that a prime number shows up.

3. A bag contains 5 red, 3 blue, and 2 green marbles. What is the probability of randomly picking a blue marble?
4. A fair six-sided die is rolled. What is the probability of rolling a 4?
5. A deck of 52 playing cards is shuffled and one card is drawn. What is the probability of drawing an Ace?
6. A fair spinner with 8 equal sections numbered 1 to 8 is spun. What is the probability of landing on an even number?
7. A coin is flipped. What is the probability that it lands on heads?

ANSWERS TO REVIEW QUESTIONS

1. For example:

(a) That we will go home at some point.

(b) That we will be forced to stay in school until we are old age pensioners.

(c) That we will all ace our maths tests.

2. **(a)** The sample space is {1,2,3,4,5,6}.

(b) The sample size is 6 as there are 6 possible outcomes when rolling a die.

(c) $P(\text{Prime}) = \frac{1}{2}$

(d) A prime number is equally likely to be rolled as a non-prime.

3. $P(\text{Blue marble}) = \frac{3}{10}$

4. $P(\text{Rolling a 4}) = \frac{1}{6}$

5. $P(\text{Drawing an Ace}) = \frac{4}{52} = \frac{1}{13}$

6. $P(\text{Even Number}) = \frac{4}{8} = \frac{1}{2}$

7. $P(\text{Heads}) = \frac{1}{2}$

PROJECT 1: EXPLORING ANGLES AND LINES IN ARCHITECTURE

Concepts Covered:

- Identifying and applying parallel, perpendicular, complementary, supplementary angles and vertical and parallel lines cut by a transversal in real-life contexts.

Project Description: You will explore building (structural) designs by identifying different types of angles and lines in the structures around you. Select a building or structure in your community (e.g., your school, a bridge or a local monument). Take pictures or make sketches of the structure, focusing on the different angles and lines you can observe.

1. **Identify and label** examples of parallel, perpendicular, complementary, supplementary angles and vertical and parallel lines cut by a transversal in your selected structure.
2. **Create a poster** or digital presentation that includes your labeled images or sketches, explaining where each type of angle or line is found and its significance in the design of the structure.
3. **Present your findings** to the class, explaining how the different angles and lines contribute to the stability and aesthetics of the structure.

Rubric for Project 1:

Criteria	Excellent (4)	Good (3)	Fair (2)	Needs Improvement (1)
Identification of Angles and Lines	All angles and lines correctly identified and labeled with clear examples	Most angles and lines correctly identified and labeled	Some angles and lines identified, but with errors or missing examples	Few or no angles and lines correctly identified

Criteria	Excellent (4)	Good (3)	Fair (2)	Needs Improvement (1)
Explanation and Analysis	Thorough and insightful explanation of the significance of each angle and line	Good explanation with minor gaps in analysis	Basic explanation with limited analysis	Incomplete or unclear explanation
Presentation Quality	Well-organised, clear, and visually appealing presentation	Clear presentation with minor issues in organisation or visuals	Somewhat clear, but lacks organisation or visual appeal	Disorganised or unclear presentation

Project 2: Park Layout and Measurement

Concepts Covered:

- Developing and applying strategies for determining the perimeter and area of plane figures.
- Developing and applying strategies for determining the surface area and volume of prisms.

Project Description: Imagine you are designing a new park for your community. Your park design will include different areas such as a playground, a garden, a picnic area and a fountain.

1. **Design the park layout** using geometric shapes on graph paper, where each shape represents different areas (e.g., rectangles, triangles, squares). Ensure that at least one area is designed as a prism (e.g., a cylindrical fountain).
2. **Calculate the perimeter and area** of each shape in your park design.
3. **Calculate the surface area and volume** of the prism (e.g., the fountain).
4. **Construct a 3D model** of your park layout using materials like cardboard or clay.
5. **Present your model and calculations** to the class, explaining how you determined the perimeter, area, surface area and volume.

Rubric for Project 2:

Criteria	Excellent (4)	Good (3)	Fair (2)	Needs Improvement (1)
Design and Layout	Well-organised layout with accurate representation of all areas	Clear layout with minor issues in representation	Somewhat clear layout with inaccuracies	Disorganised or unclear layout with many inaccuracies
Accuracy of Calculations	All calculations are accurate and correctly applied	Most calculations are accurate with minor errors	Several errors in calculations	Significant errors in calculations

Criteria	Excellent (4)	Good (3)	Fair (2)	Needs Improvement (1)
3D Model Construction	High-quality model with attention to detail and accurate representation	Good model with minor issues in detail or accuracy	Basic model with some inaccuracies	Poorly constructed model with many inaccuracies
Presentation	Clear, engaging, and well-organised	Mostly clear with minor organisational issues	Somewhat clear but lacking engagement or organisation	Disorganised or unclear presentation

PROJECT: EXPLORING PROBABILITY IN EVERYDAY LIFE

Project Description: In this project, you will become a probability detective, looking at how likely different events are to happen in your everyday life. You will classify events as impossible, possible (likely or unlikely) or certain and then conduct experiments to see how probability plays out in real situations.

Step 1: Classify Everyday Situations

- Think about different events that could happen in your day-to-day life, such as "It will rain tomorrow," "I will roll a 6 on a die," or "The sun will rise in the east tomorrow."
- **Classify these events** as impossible, possible (likely or unlikely), or certain.
- **Create a chart** with three columns labeled "Impossible," "Possible," and "Certain," and list each event in the appropriate column.

Step 2: Design a Probability Experiment

- Choose one event from your "Possible" list that you can test through an experiment (e.g., rolling a die to see if you get a 6).
- **Design the experiment** to test the probability of that event. Write down your prediction (e.g., "I think I will roll a 6 one out of six times").
- **Set up the experiment** by gathering any materials you need (e.g., a die, a coin).

Step 3: Conduct the Experiment

- **Conduct your experiment** a set number of times (e.g., roll the die 20 times).
- **Record the outcomes** each time you conduct the experiment.
- **Organise your results** in a table or chart showing each outcome.

Step 4: Explain the Results

- **Analyse your results** by comparing what actually happened to your original prediction.
- **Explain your findings** in a brief report, discuss whether the outcome was what you expected and why.
- **Present your findings** to the class, showing your chart, experiment setup and results.

Rubric for Assessment:

Criteria	Excellent (4)	Good (3)	Fair (2)	Needs Improvement (1)
Classification of Events	All events correctly classified	Most events correctly classified	Some events correctly classified	Few events correctly classified
Experiment Design	Well-designed with clear steps	Clear design with minor issues	Basic design with some unclear steps	Poorly designed or unclear
Data Collection and Recording	Accurate and well-organised data recording	Mostly accurate with minor errors	Some inaccuracies in recording data	Inaccurate or incomplete data
Explanation and Analysis	Thorough and insightful analysis	Good analysis with some details	Basic analysis with limited details	Incomplete or unclear analysis
Presentation	Engaging, clear, and well-organised	Clear with minor issues	Somewhat clear but lacks engagement	Disorganised or unclear presentation

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List of Contributors

Name	Institution
Gideon Kwame Ahiadziefe	OLA SHS
Evelyn Adjei	St. Vincent College of Education, Yendi
Adablah Mensah	St. Paul's Senior High School and Minor Seminary
Juliana Opare	St. Paul's Senior High School and Minor Seminary