MINISTRY OF EDUCATION

MATHEMATICS
for Senior High Schools

TEACHER MANUAL

Year 2

NATIONAL COUNCIL FOR CURRICULUM & ASSESSMENT OF MINISTRY OF EDUCATION

MINISTRY OF EDUCATION

REPUBLIC OF GHANA

Mathematics

for Senior High Schools

Teacher Manual

Year Two

NaCCA
NATIONAL COUNCIL FOR
CURRICULUM & ASSESSMENT
OF MINISTRY OF EDUCATION

MATHEMATICS TEACHER MANUAL

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Introduction

The National Council for Curriculum and Assessment (NaCCA) has developed a new Senior High School (SHS) curriculum which aims to ensure that all learners achieve their potential by equipping them with 21st Century skills, competencies, character qualities and shared Ghanaian values. This will prepare learners to live a responsible adult life, further their education and enter the world of work.

This is the first time that Ghana has developed an SHS Curriculum which focuses on national values, attempting to educate a generation of Ghanaian youth who are proud of our country and can contribute effectively to its development.

This Teacher Manual for Mathematics is a single reference document which covers all aspects of the content, pedagogy, teaching and learning resources and assessment required to effectively teach Year Two of the new curriculum. It contains information for all 24 weeks of Year Two including the nine key assessments required for the Student Transcript Portal (STP).

Thank you for your continued efforts in teaching our children to become responsible citizens.

It is our belief that, if implemented effectively, this new curriculum will go a long way to transforming our Senior High Schools and developing Ghana so that we become a proud, prosperous and values-driven nation where our people are our greatest national asset.

Acknowledgements

Special thanks to Professor Samuel Ofori Bekoe, Director-General of the National Council for Curriculum and Assessment (NaCCA) and all who contributed to the successful writing of the Teacher Manuals for the new Senior High School (SHS) curriculum.

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SECTION 1: SURDS, INDICES AND LOGARITHMS

Strand: Number for Everyday life

Sub-Strand: Real Number and Numeration System (RNNS)

Learning Outcomes

1. *Evaluate the relationships between laws and properties of surds, indices and logarithms and apply them to solve problems with radicands.*
2. *Apply the laws and properties of indices and logarithms to solve real-life problems.*

Content Standards

1. *Demonstrate knowledge and understanding of surds, indices and logarithms and establish their laws and properties.*
2. *Demonstrate knowledge and understanding of the laws and properties of indices and logarithms and their applications to solving real-life problems.*

Hint

Assign Portfolio Assessment *for the academic year by Week 4. Portfolio should be submitted by Week 22.* ***See Appendix A*** *of this Section and Teacher Assessment Manual and Toolkit pages 22-25 for more information on how to organise a portfolio assessment.*

INTRODUCTION AND SECTION SUMMARY

This section explores **surds, exponents, logarithms, and modulo arithmetic**, focusing on their properties, operations, and real-world applications. We begin with **surds**, examining irrational numbers, rationalisation, and problem-solving techniques. Next, we delve into **exponents and logarithms**, reinforcing exponent rules, linking them to logarithms, and solving equations using logarithmic properties. The relationship between **indices and logarithms** is then established, applying their properties to real-world modelling and common logarithm calculations. Finally, we explore **modulo arithmetic**, covering divisibility, notation, and integer relationships, while using modular operations to solve practical problems. Through this structured approach, we develop a deeper understanding of these essential mathematical concepts and their applications in solving real-world problems.

The following weeks with their respective focal areas are considered in Section 1:

Week 1

Focal Area 1: Introduction and Simplification of Surds

- *Concept, examples and visualization of surds*
- *Rationalizing monomial denominators*

Focal Area 2: Operations with Surds and Their Applications

- *Addition and Subtraction*
- *Multiplication and Division*
- *Conjugation*
- *Real-World Examples*

Week 2

Focal Area 1: Basic Concepts and Properties of Exponents

- *Review of Powers of integers (positive, negative, zero)*
- *Order of operations with exponents*
- *Simplifying expressions with exponents (product rule, quotient rule, power of a power rule)*

Focal Area 2: Concept of Logarithms

- *Definition of logarithms (inverse of exponentiation)*
- *The concept of logarithmic equations* ($\log_a b = x$, *means* $a^x = b$)
- *Laws of Logarithm*
- *Solve real-life problems involving Logarithm*

Week 3

Focal Area 1: Connection Between Indices and Logarithms

- *Connecting indices to logarithms*
- *Model real-life situation where indices and logarithms are applied*
- *Solve real-life problems using the properties of indices and logarithms*

Focal area 2: Common Logarithms and Applications

- *Introduction and Properties of common logarithms*
- *Operation on logarithms*
- *Model and solve real-life problems involving indices and logarithms*

Week 4

Focal Area 1: Introduction and Basic Operations with Modulo Arithmetic

- *The concept of modulo arithmetic using sorting and divisibility rules of integers*
- *Notation of modulo arithmetic (e.g. the notation "a mod b,")*
- *determine the connections of integers to modular arithmetic*
- *Model and solve simple situations involving modular arithmetic concepts*

Focal Area 2: Operations and Properties and Applications of Modulo Arithmetic

- *Operations of modulo arithmetic*
- *Properties of Modulo Arithmetic: (Associative Property; Identity Element; Inverse Element)*
- *Model and solve simple real-life problems using modulo technology*

SUMMARY OF PEDAGOGICAL EXEMPLARS

1. **Think-Pair-Share:** Learners engage in discussions and hands-on exploration of surds. They solve problems on addition and subtraction, as well as manipulate Geodot paper to understand multiplication and division, and also extend to rationalising denominators, conjugation, and real-life applications.
2. **Problem-Based Learning:** Learners collaborate to solve problems involving surds and exponents. They review powers and logarithm definitions, simplify exponential expressions, and explore logarithmic equations and real-life applications.
3. **Collaborative Exploration:** In mixed-ability groups, learners apply radicals to problem-solving. They connect indices to logarithms and review modular arithmetic rules, model real-life scenarios, and solve advanced problems involving indices, logarithms, and modular arithmetic.
4. **Interactive Learning & Technology:** Learners model real-world situations using modular arithmetic. They explore integer sorting and divisibility, study modulo notation and properties, and solve real-life problems with modular arithmetic and technology.

ASSESSMENT SUMMARY

Various forms of assessments should be carried out to ascertain learners' performance on the concepts that will be taught under this section. Teachers are entreated to administer these assessments and record them for onward submission into the Student Transcript Portal (STP). The following assessment would be conducted and recorded for each learner:

Week 2: *Group Project Work will be assigned and due at end of Section 3 or Week 5.*

Week 3: *Discussion and Presentation should be recorded as the class assessment for first semester.*

Week 4: *By this week, teachers should have assigned each learner their* ***portfolio task****.*

Note

For additional information on how to effectively administer these assessment modes, refer to the Appendices.

WEEK 1: SURDS

Learning Indicators

1. *Carry out basic operations on surds.*
2. *Rationalise monomial denominators.*

FOCAL AREA 1: INTRODUCTION AND SIMPLIFICATION OF SURDS

Surds, also known as radicals, are expressions containing a number under a radical symbol ($\sqrt{}$). They represent the non-perfect square root of a number and cannot be expressed as a simple fraction. This introductory session will explore surds, focusing on how to simplify them using Geodot paper, as a visual aid.

Generating surds from Geodot paper

Geodot paper is a specialized grid filled with evenly spaced dots. These dots represent unit squares, making it a fantastic tool for understanding and manipulating square roots.

Note

- **Unit Squares:** consider each small square on the grid as a unit square with an area of 1.
- **Visualising Square Roots:** By counting the unit squares that perfectly fit within a larger shape drawn on the grid, we determine its area. For example, a square that covers 9-unit squares has an area of 9 (the square root of 9 is 3).
- The radical symbol ($\sqrt{}$) applies only to the remaining unshaded area (non-perfect square part).

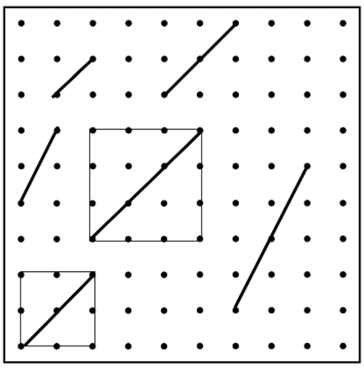


Figure 1.1: *Geodot*

Generating and simplifying surds

Geodot paper allows us to visually identify perfect squares within surds, aiding in generating and simplifying surds. Consider the step-by-step approach below:

On your geodot paper, sketch a square or rectangle (2×2 ; 2×3 ; 3×3 ; 4×3 ; *etc*.) to represent the number under the radical symbol ($\sqrt{}$). Example, $\sqrt{4}$, $\sqrt{6}$, $\sqrt{9}$, $\sqrt{12}$, etc.

Analyse these rectangles to establish if they represent perfect squares or not.

Worked Examples

Example 1.1

a) From the 3×3 square, (the shaded square) covers the whole rectangle. So, the area $A_1 = 9$ square units. This means $\sqrt{9} = 3$ (perfect square).

b) From the 2×2 square, (the /shaded square) covers the whole rectangle. So, the area , $A_2 = 4$ sqr units. This means $\sqrt{4} = 2$ (perfect square).

c) From 4×3, simplify $\sqrt{12}$ as shown below:

i. We visually decompose the square into a 4-unit square (perfect square) and a remaining a 3×1 rectangular area to simplify $\sqrt{12} = \sqrt{4} \times \sqrt{3}$

ii. Since 4 is a perfect square, $\sqrt{4} = 2$.

iii. The remaining rectangle is not a perfect square, so it remains under the radical symbol.

iv. Therefore, $\sqrt{12}$ simplifies to $2\sqrt{3}$.

Example 1.2

Simplify the following surds

1. $\sqrt{8}$: solution, $\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$

2. $\sqrt{18}$: solution, $\sqrt{18} = \sqrt{9} \times \sqrt{2} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$

3. $\sqrt{225}$ solution, $\sqrt{225} = \sqrt{25} \times \sqrt{25} = 5 \times 5 = 25$

4. $\sqrt{6^3}$ solution, $\sqrt{6^3} = \left(\sqrt{6} \times \sqrt{6}\right) \times \sqrt{6} = 6\sqrt{6}$

Determine square root using prime factorisation

The square root of a perfect square number is easy to calculate using the prime factorization method. As the numbers are perfect squares there are always an even number of equal prime factors. In order to square root, we halve the number of equal prime factors. See Table 1.1 for how this works in practice.

Table 1.1: *Using prime factorisation to generate square root*

Number	Prime factorization	Square root
16	$2 \times 2 \times 2 \times 2$	$\sqrt{16} = 2 \times 2 = 4$
144	$2 \times 2 \times 2 \times 2 \times 3 \times 3$	$\sqrt{144} = 2 \times 2 \times 3 = 12$
169	13×13	$\sqrt{169} = 13$
256	$256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$	$\sqrt{256} = (2 \times 2 \times 2 \times 2) = 16$
576	$576 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$	$\sqrt{576} = 2 \times 2 \times 2 \times 3 = 24$

Finding square roots using repeated subtraction method

This method is used if a number is a perfect square, then we can determine its square root by:

- Repeatedly subtracting consecutive odd numbers from it
- Subtract until the difference is zero
- Number of times we subtract is the required square root

Example 1.3

Find the square root of 25.

- $25 - 1 = 24$
- $24 - 3 = 21$
- $21 - 5 = 16$
- $16 - 7 = 9$
- $9 - 9 = 0$

Since the subtraction is done 5 times, the square root of 25 is 5.

Rationalising denominators

Here, we represent fractions as parts of a whole, units, or ratios between quantities where denominators contain surds, also known as square roots or cube roots.

Rationalising a denominator involves manipulating the fraction to eliminate the surds from the denominator. This often leads to a simpler and more manageable expression. i.e. $\frac{3}{\sqrt{2}}$ becomes $\frac{3\sqrt{2}}{2}$ after rationalising the denominator.

Strategies for rationalising denominators

When dealing with denominators (containing only a single term with a surd), we can employ the following strategy.

Conjugation (for square roots)

This method is applicable when the denominator contains a square root of a binomial (two terms combined). Here is how it works:

a) Identify the Binomial: Locate the binomial term under the radical in the denominator.

b) Find the Conjugate: The conjugate of the binomial is the same expression with the opposite sign between the terms.

- o The conjugate of $(2x + 5)$ is $(2x - 5)$,
- o Conjugate of $(5x - 3)$ is $(5x + 3)$
  - · This $(a^2 - b^2)$ often leads to a difference of squares in the denominator, which can be factorized using the following identity:

c. $a^2 - b^2 = (a + b)\,(a - b)$

c) Multiply by the Conjugate: Multiply both the numerator and denominator by the conjugate.

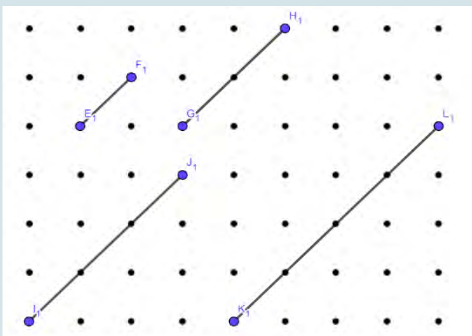
Multiply $\frac{(numerator) \times (conjugate)}{(\text{denominator}) \times (\text{conjugate})}$.

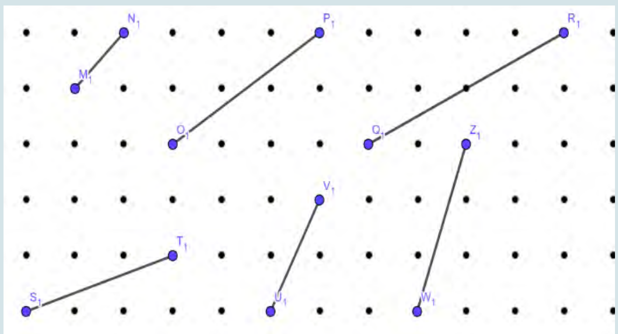
Using the example $(\frac{2}{\sqrt{x+3}})$, we multiply the fraction $\frac{2}{\sqrt{x+3}}$ (both numerator and denominator) by its conjugate $\sqrt{x} - 3$, the denominator becomes a difference of two squares, allowing us to eliminate the radical entirely. Thus:

$$(\frac{2}{\sqrt{x+3}}) \times (\frac{\sqrt{x}-3}{\sqrt{x}-3}) = \frac{2\sqrt{x}-6}{(\sqrt{x+3})(\sqrt{x}-3)} = \frac{2\sqrt{x}-6}{x+3\sqrt{x}-3\sqrt{x}-9} = \frac{2\sqrt{x}-6}{x-9}$$

Learning Task

In mixed groupings [gender/ability] learners complete each of the following straight lines as either a diagonal of a rectangle or a triangle and develop radicals for class discussions that lead to the formation of surds; model and solve problems involving surds.





Note

Use Pythagoras theorem where possible

PEDAGOGICAL EXEMPLARS

Collaborative Learning

1. In small groups, engage learners to discuss and generate surds from models.
2. In small groups, engage learners to simplify radicals to the lowest surd
3. In pairs, have learners model irrational fractions involving surds and rationalise them.

KEY ASSESSMENT: CLASS EXERCISE/GROUP WORK

Level 1: Recall

1. Express $\sqrt{48}$ in its simplest form.
2. Is the expression $2+\sqrt{(5)}$ a rational number? Give reasons for your answer.
3. Rationalise

   i. $\frac{2}{\sqrt{7}}$.

   ii. $\frac{\sqrt{2+5}}{\sqrt{3}}$.

Level 2: Conceptual Understanding

4. Find the conjugate of each of the following:

   i. $5x - 3$

   ii. $2\sqrt{x} - 5$

   iii. $\frac{\sqrt{2}}{2+\sqrt{3}}$

5. Rationalise

   i. $\frac{2}{\sqrt{x}-4}$

   ii. $\frac{\sqrt{2}}{2+\sqrt{3}}$

FOCAL AREA 2: OPERATIONS WITH SURDS AND THEIR APPLICATIONS

Here, we shall perform basic operations involving surds which is crucial for solving real-life problems involving irrational numbers.

Addition and Subtraction of Surds

Addition and subtraction of surds are possible with the following condition

Note

1. To add or subtract surds, they must have the same radical index (order of the root) and the same radicand (number under the root sign).
2. If the radicands are different, we cannot combine the surds directly.

For example

$\sqrt{3} + 2\sqrt{3}$ can be simplified to $3\sqrt{3}$ since they have the same radical index and radicand. i.e.$n\sqrt{a} \pm n\sqrt{b} = n(\sqrt{a} \pm \sqrt{b})$, where a and b are non-negative.

- $\sqrt{2} + \sqrt{8}$ can be simplified with further simplification on $\sqrt{8}$.

  Thus, R2 +√ R8 $= \sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$
- $\sqrt{3} + \sqrt{5}$ cannot be simplified further.

Steps

1. Simplify the surds, if possible, by factorising out any perfect squares or cubes.
2. Group-like terms (surds with the same radicand and index).
3. Add or subtract the coefficients of the like terms.

Worked Examples

Example 1.4

Simplify the expression $(3\sqrt{5} - 2\sqrt{5} + \sqrt{5})$.

Solution

$3\sqrt{5} - 2\sqrt{5} + \sqrt{5} = (3 - 2 + 1)\sqrt{5} = 2\sqrt{5}$

Example 1.5

Simplify the expression $(\sqrt{27} + 3\sqrt{3} - 2\sqrt{12})$.

Solution

$\sqrt{27} + 3\sqrt{3} - 2\sqrt{12} = 3\sqrt{3} + 3\sqrt{3} - 2\sqrt{(3 \times 2^2)}$

Simplifying the terms so they all have the common factor $\sqrt{3}$

$= 3\sqrt{3} + 3\sqrt{3} - 4\sqrt{3}$

$= 6\sqrt{3} - 4\sqrt{3}$

$= 2\sqrt{3}$

Example 1.6

Simplify the expression $(5\sqrt{8} - 2\sqrt{18} + 3\sqrt{2})$.

Solution

$5\sqrt{8} - 2\sqrt{18} + 3\sqrt{2} = 5\sqrt{(2^3)} - 2\sqrt{(2 \times 3^2)} + 3\sqrt{2}$

Working out perfect squares/cubes from the radicands. Thus,

$= 5(2\sqrt{2}) - 2(3\sqrt{2}) + 3\sqrt{2}$

$= 10\sqrt{2} - 6\sqrt{2} + 3\sqrt{2} = 7\sqrt{2}$

Example 1.7

A construction company is building a new park with a rectangular area for picnics. The length of the rectangular area is 18 meters, and the width is 12 meters. The company wants to install square-shaped tiling stones along the diagonal of the rectangular area to create a visually appealing path. Calculate the length of the diagonal path, leaving your answer in surd form.

Solution

Given information:

- Length (L) of the rectangular area = 18 meters
- Width (W) of the rectangular area = 12 meters

  Using the Pythagoras' theorem, the length of the diagonal path (hypotenuse) can be expressed as:

  $D^2 = L^2 + W^2$

  $D^2 = 18^2 + 12^2$

  $D^2 = 324 + 144$

  $D^2 = 468$

The length of the diagonal path is $\sqrt{468}$ meters or $6\sqrt{13}$ meters. The construction company needs to order enough square-shaped tiling stones to cover the diagonal path of length $6\sqrt{13}$ meters.

Multiplication of Surds

To be able to multiply surds, we follow the rules of exponents and radicals.

Steps:

1. Investigate the surds and factorize out any perfect squares or cubes.
2. Multiply the coefficients, (if available).
3. Multiply the radicands.
4. Add the indices (orders of the roots) if they are the same.

   Thus, $m\sqrt{a} \times n\sqrt{b} = mn\sqrt{ab}$; $m\sqrt{a} . n\sqrt{b} = mn\sqrt{ab}$, where a and b are non-negative

Example 1.8

Simplify the expression $(2\sqrt{3} \times 3\sqrt{5})$.

Solution

$2\sqrt{3} \times 3\sqrt{5} = 2 \times 3 \times \sqrt{3} \times 5) = 6\sqrt{15}$

Division of Surds

To divide surds, we follow the rules of exponents and radicals, similar to multiplication, but in reverse order.

Steps: We explore the steps involved in dividing surds with examples for better understanding.

1. Simplify before division

   Look for perfect squares (numbers that can be obtained by squaring an integer) or perfect cubes (numbers that can be obtained by cubing an integer) within the radicals in both the numerator and denominator. For example, simplify $\sqrt{(72)}$ to $6\sqrt{(2)}$ by factoring out a perfect square of 36 (6^2) from under the radical.

2. Perform the division

   Once we simplify both the numerator and denominator, we can proceed with the actual division. Consider the steps as follows: E.g. $\dfrac{6\sqrt{8}}{2\sqrt{12}}$

   a. *Divide the coefficients*

      Divide any numerical coefficients: $\dfrac{6\sqrt{8}}{2\sqrt{12}} = \dfrac{3\sqrt{48}}{\sqrt{12}}$

b. *Divide the Radicands:*

Divide the expressions under the radical symbols: $\frac{3\sqrt{48}}{\sqrt{12}} = 3\sqrt{\frac{48}{12}} = 3\sqrt{4} = 6$

Note

When the radicand in the denominator is a perfect power of the same base as the radicand in the numerator, you can simplify the expression further.

Worked Examples

Example 1.9

Simplify $\sqrt{27} \div \sqrt{3}$

Solution

$$\frac{\sqrt{(27)}}{\sqrt{(3)}} = \sqrt{\frac{27}{3}}$$

$$= \sqrt{9}$$

$$= 3$$

Example 1.10

Solve $\sqrt{72}$ divided by $\sqrt{(2)}$

Solution

$$\sqrt{72} \div \sqrt{2} = \frac{\sqrt{(72)}}{\sqrt{(2)}}$$

$$= \sqrt{\frac{72}{2}}$$

$$= \sqrt{36}$$

$$= 6$$

Example 1.11

Divide $\sqrt{48}$ by $\sqrt{8}$

Solution

$$\sqrt{48} \div \sqrt{8} = \frac{\sqrt{48}}{\sqrt{8}}$$

$$= \sqrt{\frac{48}{8}}$$

$$= \sqrt{6}$$

Example 1.12

Consider the following offsetting of roofs below. Determine:

a. *the slope lengths* $\boldsymbol{l} = |\boldsymbol{AB}|$ *and* $|\boldsymbol{FB}|$

b. *the actual slope of the roof (angle of elevation)*

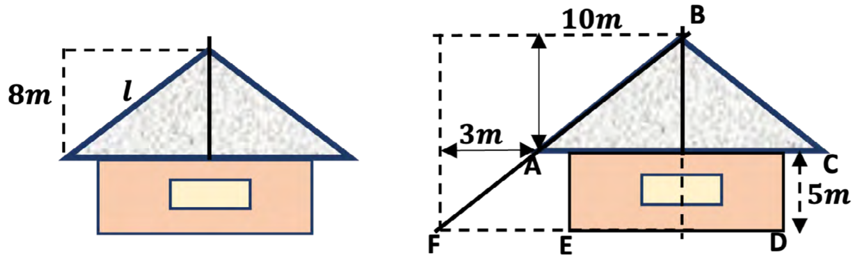


Figure 1.2: *Slope length of roofs*

Solution

- By Pythagoras theorem, we have $l = |AB| = \sqrt{8^2 + 7^2} = \sqrt{64 + 49} = \sqrt{113}$

  Similarly, |FB| = |FA| + |AB|

  But, $FA| = \sqrt{3^2 + 5^2} = \sqrt{9 + 25} = \sqrt{34}$

  Therefore, $|FB| = \sqrt{34} + \sqrt{113} = 16.4m$

  Alternatively, $|FB| = \sqrt{10^2 + 13^2} = \sqrt{100 + 169} = \sqrt{269} = 16.4m$

- Slope of the length $|FB|$is given by $tan(< BAC)$

  Or $tan(< BAC) = \frac{8}{7}$, implies, $\tan^{-1}\left(\frac{8}{7}\right) = 48.8^o$

  Or $sin(BAC) = \frac{8}{\sqrt{113}} = \frac{8\sqrt{113}}{113}$, implies, $\sin^{-1}\left(\frac{8\sqrt{113}}{113}\right) = 48.8^o$

Real-world Applications of Surds

Surds have numerous applications in various fields, including:

1. Physics: Surds are used in calculations involving vectors, forces, and other physical quantities.
2. Engineering: Surds arise in calculations related to construction, design, and structural analysis.
3. Geometry: Surds are used to represent irrational lengths, such as the diagonal of a square or the side of a regular pentagon.
4. Trigonometry: Surds appear in trigonometric ratios and identities involving angles that cannot be expressed as rational multiples of π.

5. Finance: Surds can be used in calculations involving interest rates, loan payments, and other financial computations.

Learning Task

In mixed groupings [gender/ability] learners explore the various rules for the operations of surds and investigate their real-world applications.

PEDAGOGICAL EXEMPLARS

1. **Think-pair-share**: Learners approaching proficiency grasp the core concept and operations (+ and –) of surds.
2. **Problem-Based Learning**: Learners in interactive and collaborative groups solve problems involving operations ($\times$ and $\div$) with respect to surds.
3. **Collaborative Learning:** In small groups, engage learners to discuss, generate, and solve *real-world problems involving surds*

KEY ASSESSMENT

Level 2: Conceptual Understanding

1. A rectangle has area 12^2cm and length $2 + \sqrt{7}$ cm. Find its width in the form $ab + \sqrt{7}$, where a and b are integers.
2. Write each of the following surd expressions as simple as possible.
   a. $\sqrt{24} + \sqrt{6}$
   b. $(2 + \sqrt{3})(4 - \sqrt{12})$
3. Write each of the following surd expressions as simple as possible.
   i. $\sqrt{48} - 6_{\sqrt{3}} + \sqrt{6} \times \sqrt{2}$
   ii. $(\sqrt{7} + 3)(2\sqrt{7} - 3)$
4. Write each of the following surd expressions as simple as possible.
   i. $(1 + \sqrt{2})^3$
   ii. $2\sqrt{75} + \frac{3 + \sqrt{3}}{3 - \sqrt{3}} - \sqrt{2} \times \sqrt{2}.$

WEEK 2: INDICES AND LOGARITHMS

Learning Indicators

1. *Explain the concepts of indices and logarithms with examples.*
2. *Compose and decompose logarithm laws and properties with exponents and apply the concepts to solve real-life problems.*

FOCAL AREA 1: BASIC CONCEPTS AND PROPERTIES OF EXPONENTS

Exponents and Operations

Exponents are a powerful tool in mathematics that allows us to express repeated multiplication concisely. Understanding exponents and their properties is essential for various applications in science, engineering, and mathematics. Here, we will delve into the core concepts of exponents and explore how to simplify expressions involving them. Consider the following key points for a better understanding of the concept of powers of numbers.

Note

- **Exponent:** The exponent, also called the index, is a small number written above and to the right of another number (the base). It tells us how many times the base number is multiplied by itself. For example, considering the following repeated factors, $3 \times 3 \times 3 \times 3 \times 3$, the exponent is 5, since 3 is multiplied by itself five times. Thus, $3^{5(\textit{exponent})}$
- **Base:** The base is the number being raised to the power of the exponent.

  For example, in the expression 3^5, the base is 3 and the exponent is 5. This can be read as "three to the exponent of five".
- **Power:** The power is both the number written above (exponent) and its base. For example, in the expression 3^5, both the base (3) with the exponent (5) are referred to.
- If $x^a = y^a$, then $x = y$
- If $x^a = x^b$, then $a = b$

Powers of Integers (Positive, Negative and Zero)

1. **Positive exponents:** A positive exponent (n) indicates that the base (a) is multiplied by itself n times. For example, the repeated factors $(2 \times 2 \times 2 \times 2)$ of 16 can be

expressed as a power of 2 as 2^4. Similarly, the repeated factors ($7 \times 7 \times 7 \times 7$) of 2401 can be expressed as a power of 7 as 7^4.

2. **Zero exponent:** Any number raised to the exponent of zero (a^0) equals one, as long as the base is not zero. For Example: $3^0 = 1$.
3. **Negative exponents:** A negative exponent (n) indicates the reciprocal of the base raised to the absolute value of the exponent. In simpler terms, it is like dividing one by the base raised to the positive exponent. Example: $3^{-2} = \frac{1}{3^2} = \frac{1}{3 \times 3} = \frac{1}{9}$.

Order of Operations with Exponents

When dealing with expressions that contain exponents along with other mathematical operations (addition, subtraction, multiplication, and division), it is crucial to follow the order of operations (PEMDAS/BEDMAS/BODMAS):

1. Parentheses, Exponents, Multiplication, and Division (from left to right)
2. Addition and Subtraction (from left to right)

For instance, in the expression $2 + 3^2 \times 5$, we first evaluate the exponent ($3^2 = 9$) and then perform the multiplication ($9 \times 5 = 45$). Finally, we add 2 to get the answer:

$2 + 3^2 \times 5 = 2 + 45 = 47$.

Simplify Expressions with Exponents

Rules of exponents

1. **Product rule:** When multiplying terms with the same base, add the exponents. Example: $a^m \times a^n = a^{(m+n)}$: Implies, $2^3 \times 2^4 = 2^{(3+4)} = 2^7$.
2. **Quotient rule:** When dividing terms with the same base, subtract the exponents. Example: $a^m \div a^n = a^{(m-n)}$: Implies, $2^3 \div 2^4 = 2^{(3-4)} = 2^{-1}$.
3. **Power of a power rule:** When raising a term with an exponent already present to another exponent, multiply the exponents. Example: Implies, $(a^m)^n = a^{mn}$: $(3^4)^{-5} = 3^{4(-5)} = 3^{-20}$.

Worked Examples

Example 1.13

Simplify $2^3 \times 2^2$.

Solution

$2^3 \times 2^2 = 2^{(3+2)} = 2^5 = 32$

(Since multiplying exponents with the same base means adding the powers).

Example 1.14

Simplify $(3^2)^4$.

Solution

$(3^2)^4 = 3^{2\times4} = 3^8$

(When the power of a number is raised to another exponent, we multiply the exponents).

Example 1.15

Simplify $x^5 \div x^2$.

Solution

$x^5 \div x^2 = x^{5-2} = x^3$ (Divide exponents with the same base means subtracting the powers).

Example 1.16

Simplify $(x^2y^3)^3$.

Solution

$(x^2y^3)^3 = (x^2)^3(y^3)^3 = x^{(2 \times 3)}y^{(3 \times 3)} = x^6y^9$ (Raise each term within the parentheses to the power).

Example 1.17

Compare 2^4 and 4^2.

Solution

First, let's calculate 2^4: $2^4 = 2 \times 2 \times 2 \times 2 = 16$

Now, let's calculate 4^2: $4^2 = 4 \times 4 = 16$

Comparing the results: $2^4 = 16$ and $4^2 = 16$

Therefore, 2^4 is equal to 4^2.

You could now investigate if this works for any other numbers. For example, is it true that $2^3 = 3^2$?

Example 1.18

Write 128 as a power of 2

Solution

Since we are looking for an exponent n such that $2^n = 128$.

We start with smaller powers of 2 and work our way up:

$2^1 = 2$; $2^2 = 4$; $2^3 = 8$; $2^4 = 16$; $2^5 = 32$; $2^6 = 64$; $2^7 = 128$

Therefore, 128 written as a power of 2 is 2^7.

Example 1.19

A bacteria population doubles every hour. If you start with 100 bacteria, how many will there be after 6 hours?

Solution

We can solve this problem by recognizing that the bacteria population doubles every hour, which means its growth is exponential. Here's how to find the number of bacteria after 6 hours:

Starting population: We know you begin with 100 bacteria ($B_0 = 100$).

Growth factor: The number of times the population doubles each hour is the growth factor ($a = 2$).

Number of hours (exponent): The number of times the population doubles correspond to the number of hours that have passed ($t = 6$).

Formula for exponential growth:

The formula for exponential growth is: $\boldsymbol{B(t) = B_0 \times a^t}$

where:

- $\boldsymbol{B(t)}$ is the number of bacteria at a specific time (t).
- $\boldsymbol{B_0}$ is the initial number of bacteria (100 in this case).
- $\boldsymbol{a}$ is the growth factor (2 for doubling).
- t is the number of hours passed (6 in this case).

Calculation:

Substitute the values into the formula:

$B(6) = 100\ bacteria \times 2^6$

Evaluate the equation:

$B(6) = 100\ bacteria \times 64$

$B(6) = 6{,}400\ bacteria$

Therefore, after 6 hours, there will be 6400 bacteria.

Example 1.20

A piece of paper is folded in half 8 times. How many layers of paper will there be?

Solution

Each fold doubles the number of layers.

We start with 1 layer and double it 8 times.

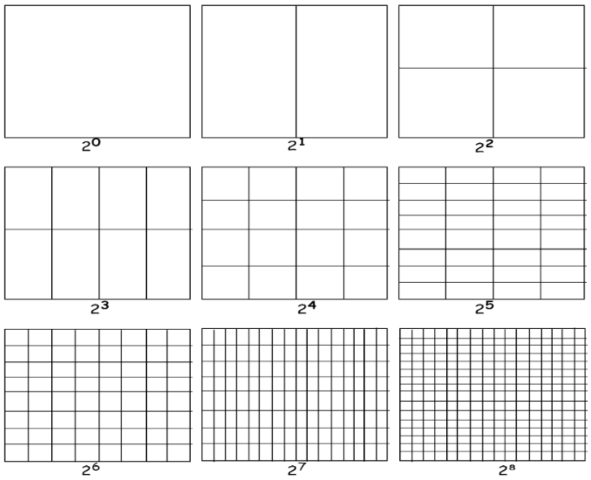


Figure 1.3: *Paper folding*

This can be summarised as follows:

$2^0 = 1$

$2^1 = 2$

$2^2 = 2 \times 2 = 4$

$2^3 = 2 \times 2 \times 2 = 8$

$2^4 = 2 \times 2 \times 2 \times 2 = 16$

$\vdots$

$2^8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$

Can you have a go at doing this in reality with a piece of paper? What do you discover? Is it possible to fold a piece of paper in half 8 times?

Example 1.21

An earthquake of magnitude 6 is 10 times more intense than one of magnitude 5. How many times more intense is an earthquake of magnitude 8 than one of magnitude 5?

Solution

The difference in magnitude is $8 - 5 = 3$.

Each increase of 1 in magnitude represents a 10-fold increase in intensity

So, we need to calculate 10^3

$10^3 = 1{,}000$.

An earthquake of magnitude 8 is 1,000 times more intense than one of magnitude 5.

Learning Task

In mixed groupings [gender/ability] learners explore the various rules for the operations of indices and investigate their real-world applications:

1. Addition-Product Law
2. Subtraction-Quotient Law
3. Index (Power) Law
4. product-index law
5. Quotient – Index law
6. Fractional Power (or Root) Law
7. Zero Exponent or Index
8. Negative Integer Exponents

PEDAGOGICAL EXEMPLARS

1. **Think-pair share:** Learners in pairs investigate and make presentations on the concept of indices and its applications.
2. **Problem-based learning**: Learners in interactive and collaborative groups solve problems involving operations *o*f indices and apply these to analyse and solve real-world problems involving indices.

FOCAL AREA 2: CONCEPT OF LOGARITHMS

Logarithms, often considered the inverse of exponentiation, is a powerful tool in mathematics. They allow us to solve problems involving exponents in a more efficient way. Let us introduce the learner to the essence of logarithms, their properties, and how they are used in various fields.

Note

1. **Definition:** if $b^x = a$, where b is a positive constant base ($b \neq 1$) and a is any positive number. The logarithm of a to base b, written as $\log_b a$ is the exponent (x) to which we raise the base (b) to get the number (a).
2. **Logarithmic equations:** The relationship between logarithms and exponentiation can be expressed as $\log_b \boldsymbol{a} = \boldsymbol{x}$ if and only if $\boldsymbol{b}^x = \boldsymbol{a}$. Understanding this connection is crucial for working with logarithms.

Laws of logarithm

1. *The Addition-Product Law:* $\log_k a + \log_k b = \log_k ab$. This rule states that the logarithm of a product with the same base as the individual factors is equal to the sum of the logarithms of those factors.
2. *The Subtraction-Quotient Law:* $\log_k a - \log_k b = \log_k \frac{a}{b}$. Similar to the product rule, the logarithm of a quotient with the same base is the difference of the logarithms of the numerator and denominator
3. *The exponent law:* $\log_a b^m = m\log_a b$. where m is any real number. This rule applies when raising a term to a power. The logarithm of a term raised to an exponent is equal to the exponent times the logarithm of the base.
4. *Same base law:* $\log_b b = 1$
5. *Log one;* $\log_b 1 = 0$. for any base b $\neq$ 1. This makes sense because raising any positive constant (except 1) to the power of zero results in 1
6. *Change of base law*: $\log_a b = \dfrac{\log_k b}{\log_k a}$.

Common logarithms

A special type of logarithm, the common logarithm (denoted by *log*), uses base 10. So, $log(a)$ is equivalent to $\log_{10}(a)$. Common logarithms were widely used before calculators due to the ease of working with base 10.

Properties of common logarithms

1. $log(1) = 0$ (same as any base)
2. Converting exponents to logarithms: a positive number can be expressed as a power of 10 using *log*: i.e. $10000 = 10^4$. Implies, $\log_{10}(10000) = 4$

Applications of logarithms

Logarithms are not just abstract mathematical concepts. They have numerous practical applications across various fields:

1. **Solving Exponential Equations:** Logarithms are instrumental in solving equations where the variable is in the exponent. By taking the logarithm of both sides, we can convert the equation into a simpler form to solve for the unknown variable.
2. **The pH Scale (Chemistry):** The pH scale, a measure of acidity or alkalinity, is based on the negative base-10 logarithm of the hydrogen ion concentration.
3. **The Richter Scale (Earthquakes):** The Richter scale, used to quantify the magnitude of earthquakes, is a logarithmic scale. This means that an increase of one unit on the Richter scale corresponds to a tenfold increase in ground motion amplitude.

4. **Signal Processing (Decibel Units):** Decibels (dB), a unit used to measure the intensity of sound or electrical signals, are based on logarithms. This allows us to express a wide range of signal strengths in a manageable way.

Worked Examples

Example 1.22

Solve for x: $\log_3(x + 1) = 2$.

Solution

$log_3(x + 1) = 2$

$3^2 = x + 1$ (*raising both sides as a power of* 3)

$9 = x + 1$

$x = 8$

Example 1.23

Simplify $\log_2(8 \times 2)$.

Solution

Or simply use the addition product law, log, base 2, (8×2) = Log, base 2 of 16 which is 4…

$log_2(8 \times 2) = log_2 8 + log_2 2$

$= log_2(2^3) + log_2 2$ (*since* $8 = 2^3$)

$= 3 + 1$ (*property of logarithms*)

$= 4$

Example 1.24

Express $\log_5 125$ in terms of $\log_5 5$.

Solution

$log_5 125$

$= log_5(5^3)$ (*since* $125 = 5^3$)

$= 3\ log_5 5$ (*property of logarithms*)

$= 3$ (*since* $log_5 5 = 1$)

Example 1.25

What power of 2 gives 32?

Solution

Using logarithm, we have

$log_2(2^x) = log_2 32$ *(taking log base 2 of both sides)*

$x\ log_2 2 = 5\ (since\ 32 = 2^5)$

$x \times 1 = 5\ (since\ log_2 2 = 1)$

$x = 5$

Example 1.26

If $\log_3 x = 4$ and $\log_3 y = 5$, find the value of $\log_3(xy)$.

Solution

$log_3 x = 4\ and\ log_3 y = 5$

$log_3(xy) = log_3 x + log_3 y\ (property\ of\ logarithms)$

$= 4 + 5$

$= 9$

Example 1.27

If a star of magnitude 1 is 100 times brighter than a star of magnitude 6. What is the factor by which brightness changes for each step, in magnitude?

Solution

Let x be the factor by which brightness changes for each step

There are 5 steps between magnitude 1 and 6

So, $x^5 = 100$

Taking the logarithm of both sides: $5\ log(x) = log(100)$

$5\ log(x) = 2$

$log(x) = \frac{2}{5} = 0.4$

$x = 10^{0.4} \approx 2.512$

Therefore, the brightness changes by a factor of approximately 2.512 for each step in magnitude.

PEDAGOGICAL EXEMPLARS

1. **Think-pair-share**: Through think-pair-share activities. Learners approaching proficiency grasp the core concept and laws of logarithms
2. **Problem-Based Learning**: Learners in interactive and collaborative groups solve problems involving logarithms.
3. **Collaborative Learning:** In small groups, engage learners to discuss and generate, and solve real-world problems involving logarithms

KEY ASSESSMENT
LEVEL 2: CONCEPTUAL UNDERSTANDING

1. As a group, solve the following problems and give an explanation to how you solved each of the questions.
   - **a.** Solve for x: $\log_2(x + 3) = 5$
   - **b.** Simplify: $\log_3 27 + \log_3 9$
   - **c.** Express $\log_4 64$ in terms of $\log_4 2$
   - **d.** Solve the equation: $3^{x+2} = 81$
   - **e.** If $\log_5 x = 3$ and $\log_5 y = 2$, find the value of $\log_5(x/y)$
   - **f.** Express $\log_8 128$ as a rational number
   - **g.** Solve the equation: $\log_2(x + 1) + \log_2(x - 1) = 3$
   - **h.** Solve the system of equations:
     - $\log_2 x + \log_2 y = 6$
     - $x - y = 48$

Level 3: Strategic Thinking

2. Parents/guardians of Abakrampa Senior High Technical School are constantly complaining about the poor performances of their wards in Mathematics. As a learner of the school, embark on project/research to investigate why learners are performing poorly.

Hint

- *The Recommended Mode of Assessment for Week 2 is* ***Project Work (group)****.*
- *The outline indicating a detailed scope and rubrics for learners to focus on in their research has been provided in the* ***Appendix B****.*

WEEK 3: INDICES AND LOGARITHMS

Learning Indicators

1. *Investigate real-life problems using laws and properties of indices and logarithms.*
2. *Use mathematical connections to explore the relevance of indices and logarithms and their applications to scientific concepts.*

FOCAL AREA 1: CONNECTION BETWEEN INDICES AND LOGARITHMS

In mathematics, indices (also known as exponents) and logarithms are closely related concepts.

Let's explore how they connect:

1. **Indices (Exponents)**
   a. An index (exponent) represents the number of times a base is multiplied by itself.
   b. For example, the expression , where is the base and is the exponent, means we multiply by itself times.
2. **Logarithms**
   a. Logarithms are the inverse operation of indices.
   b. The logarithm of a number with respect to a given base tells us what exponent we need to raise the base to in order to obtain that number.
3. **Key Relationship**
   a. The logarithm function "undoes" the effect of exponentiation. It allows us to find the unknown exponent when we know the base and the result.
   b. Logarithms are useful in solving exponential equations, analysing growth rates, and working with data that follows exponential patterns.
   c. Indices and logarithms are two sides of the same mathematical coin. They complement each other and play essential roles in various fields, including science, engineering, and finance.

Real-Life Applications

Learners can conduct short surveys of areas in which indices and logarithms are applied in our lives (the following practical scenarios will help):

1. Compound interest
   a. The compound interest formula involves exponential growth.
   b. The interest accrued over time can be modelled using indices.

   c. Logarithms help us find the time needed for an investment to reach a specific value.

2. Sound intensity and decibels

   a. Sound intensity follows an exponential scale.

   b. Decibels (dB) measure sound intensity using logarithms.

   c. The relationship between sound intensity and decibels is logarithmic.

3. Radioactive decay

   a. The decay of radioactive substances follows an exponential pattern.

   b. Half-life (the time for half of a substance to decay) is determined using logarithms.

4. Population growth

   a) Suppose a city's population doubles every years.

   b) Express this growth using indices and find the population after 30 years.

FOCAL AREA 2: APPLICATIONS OF COMMON LOGARITHMS

Worked examples

Example 1.28

A bacterial population doubles every 3 hours. If there are initially 1000 bacteria, how many will there be after 15 hours?

Solution

Using the formula $N(t) = N_o \times 2^{\frac{t}{3}}$, where $N_o = 1000$ and $t = 15$:

$N(15) = N_o \times 2^{\frac{t}{3}} = N(15) = 1000 \times 2^{\frac{15}{3}} = N(15) = 1000 \times 2^5 = 1000 \times 32 = 32{,}000$ bacteria

Example 1.29

If you invest GH¢5000.00 at 6% annual interest compounded monthly, how much will you have after 5 years?

Solution

Using $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where $P = 5000$, $r = 0.06$, $n = 12$, $t = 5$:

$A = 5000\left(1 + \frac{0.06}{12}\right)^{12 \times 5} = GH¢6744.25$

Example 1.30

If the sound intensity level increases by 20 decibels, by what factor does the sound intensity increase?

Solution

Using the formula $\Delta dB = 10\ log_{10}\left(\frac{I_2}{I_1}\right)$, we have:

$20 = 10\ log^{10}\left(\frac{I_2}{I_1}\right) = 10\ log\left(\frac{I_2}{I_1}\right),$

$2 = log\left(\frac{I_2}{I_1}\right)$

$10^2 = \frac{I_2}{I_1}$. The intensity increases by a factor of 100.

Example 1.31

A solution has a hydrogen ion concentration of $1.0 \times 10^{-5}\ mol/L$. What is its pH?

Solution

Using $pH = -log[\mathrm{H}^+]$:

$pH = -log(1.0 \times 10^{-5}) = log(1.0 \times 10^{5}\ ) = log10^5 = 5log10 = 5$

Example 1.32

An earthquake has 1000 times the energy of another earthquake. What is the difference in their Richter scale magnitudes?

Solution

Using the formula $\log\left(\frac{E_1}{E_2}\right) = 1.5(M_1 - M_2)$, where $\frac{E_1}{E_2} = 1000$:

$log\ (1000) = 1.55(M_1 - M_2)$

$3 = 1.55(M_1 - M_2)$

$2 = (M_1 - M_2)$

The difference in magnitude is 2 on the Richter scale.

KEY ASSESSMENT

Level 2: Conceptual Understanding

1. As a group discuss the following questions and prepare a presentation. Your presentation should include a detailed explanation of how you solved each of the following questions.
   a. A small business in Accra starts with 100 customers and grows by 15% each month. How many customers will it have after 6 months?
   b. If you deposit GHS 2000 in a savings account with 12% annual interest compounded quarterly, how much will you have after 3 years?
   c. The noise level at a concert in Kumasi is 100 dB. How many times more intense is this than a normal conversation at 60 dB?
   d. A solution has a pH of 3.5. What is its hydrogen ion concentration in mol/L?
   e. An earthquake in Ghana registers 4.5 on the Richter scale. How much more energy does it release compared to a quake measuring 3.0?
2. A small business in Accra starts with 100 customers and grows by 15% each month. How many customers will it have after 6 months?
3. If you deposit GHS 2000 in a savings account with 12% annual interest compounded quarterly, how much will you have after 3 years?
4. An earthquake in Ghana registers 4.5 on the Richter scale. How much more energy does it release compared to a quake measuring 3.0?
5. The population of a town in Ghana is 50,000 and decreases by 2% annually. What will be the population after 10 years?
6. The sound intensity of traffic in Accra is measured at 70 dB. What is the intensity level if the traffic doubles?
7. A lake in Ghana has a pH of 6.5. How many times more acidic is rain with a pH of 4.5?

Hint

- *The Recommended Mode of Assessment for Week 3 is **Discussion** and **Presentation.***
- *Refer to Question 1 of the Key Assessment above for sample discussion and presentation questions.*

WEEK 4: MODULAR ARITHMETIC

Learning Indicators

1. *Undertake a brief review on sorting and divisibility rules of integers and determine their connections to modular arithmetic using models such as number arrays.*
2. *Model simple situations involving modular arithmetic concepts, connect the ideas to real-world problems, and solve them using appropriate models and technology.*

FOCAL AREA 1: INTRODUCTION AND BASIC OPERATIONS WITH MODULO ARITHMETIC

Modulo arithmetic, often abbreviated as mod, is a branch of mathematics that deals with remainders after division. That is modulo (mod) as an operation that gives the remainder after dividing one number by another.

Notation: Explain the notation "a mod b," which represents the remainder when a is divided by b.

Basic operations with modulo

Addition: Explain how to add modulo a number. Focus on the concept of "wrapping around" when the sum exceeds the modulus.

For instance: $(5 \ mod \ 3) + (2 \ mod \ 3) = (7 \ mod \ 3) = 1$ (wrapping around)

Subtraction: Introduce modulo subtraction, emphasizing borrowing from the modulus if necessary.

For instance: $(7 \ mod \ 4) - (2 \ mod \ 4) = (5 \ mod \ 4) = 1$

Multiplication: Discuss modulo multiplication, explaining how to multiply terms and take the remainder modulo the chosen number.

For instance: $(3 \ mod \ 5) \times (2 \ mod \ 5) = (6 \ mod \ 5) = 1$

Worked Examples

Example 1.33

Sort integers into odd and even blocks (mod 2 and mod 3): i.e., For modulo 2, the integers are sorted into 2 distinct subsets as,

$\{\ldots, -4, -2, 0, 2, 4, 6, 8, \ldots\}$ and

$\{\ldots, -3, -1, 1, 3, 5, 7, \ldots\}$.

(Remember that the set of integers is used in its entirety).

Similarly, in modulo 3 the set of integers is sorted into 3 distinct subsets, such as
$\{\ldots, -6, -3, 0, 3, 6, 9, 12, \ldots\}$,
$\{\ldots, -5, -2, 1, 4, 7, 10, 13, \ldots\}$, *and*
$\{\ldots, -4, -1, 2, 5, 8, 11, 14, \ldots\}$.

Example 1.34

Use repeated subtraction/addition to determine the modulo of negative numbers. i.e., -8 $(mod\ 3)$

Solution

$$
\begin{aligned}
-8\ (mod\ 3) &= -8 + 3 = -5 \\
&= -5 + 3 = -2 \\
&= -2 + 3 = 1
\end{aligned}
$$

$\therefore -8\ (mod\ 3) = 1$

Uses of modulo in everyday activities

1. Traveling and Vehicle capacity,
2. Modelling time (using a 12-hour clock),
3. The market day arithmetic,
4. Restaurants and feeding capacity.
5. Modulo arithmetic in music (i.e., base 8 for octave).

The idea is that a and b are "equivalent" when they leave the same remainder upon division by a mod number. Create number arrays using spreadsheets.

-9	-8	-7	-6	-5
-4	-3	-2	-1	0
1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

Knowing that $9 \equiv 4(mod\ 5)$, and $7 \equiv 2(mod\ 5)$
Then, $9 + 7 \equiv 4 + 2 \equiv 1\ (mod\ 5)$

Application: if $9 \equiv \mathrm{x}(mod5)$, and $7 \equiv \mathrm{y}(mod5)$, then $x + y \equiv z\ (mod5)$. What is the value of z?
Knowing that $9 \equiv \mathrm{x}(mod5)$, and $7 \equiv \mathrm{y}(mod5)$
Then, $9 + 7 \equiv x + y \equiv 1\ (mod5) = z(mod5)$. Therefore, $z = 1$

Properties of modulo arithmetic

1. Commutative Property: Not applicable to modulo arithmetic ($x\ mod\ b \neq b\ mod\ x\ in\ general$).
2. Associative Property: Applicable to modulo arithmetic ($a\ mod\ b)\ mod\ c) = (a\ mod\ (b\ x\ c)$.
3. Identity Element: The identity element for modulo addition is ($a\ mod\ b) + 0 = a\ mod\ b$.

Commutativity: Addition and multiplication are commutative

$(a + b \equiv b + a\ (mod\ m)$ and $a \times b \equiv b \times a\ (mod\ m))$.

Justification

a. Addition in mod 6: $4 + 5 \equiv 5 + 4 \equiv 3\ (mod\ 6)$

b. Multiplication in mod 8: $3 \times 7 \equiv 7 \times 3 \equiv 5\ (mod\ 8)$

Associativity: Addition and multiplication are associative

$(a + b) + c \equiv a + (b + c)\ (mod\ \mathrm{m})$ and $(a \times b) \times c \equiv a \times (b \times c)\ (mod\ m))$.

Justification

a. Addition in mod 9: $(2 + 3) + 7 \equiv 2 + (3 + 7) \equiv 3\ (mod\ 9)$

b. Multiplication in mod 11: $(3 \times 4) \times 5 \equiv 3 \times (4 \times 5) \equiv 5\ (mod\ 11)$

Identity: 0 is the additive identity $a + 0 \equiv a\ (mod\ m)$.

Justification

a. mod 7: $0 + 4 \equiv 4\ (mod\ 7)$

b. mod 13: $0 + 12 \equiv 12\ (mod\ 13)$

Inverse: Not all elements have inverses (modular multiplicative inverses). We say a number b is an inverse of a (mod m) if $a \times b \equiv 1\ (mod\ m)$.

a. In mod 7: $3 \times 5 \equiv 1\ (mod\ 7)$, so 5 is the multiplicative inverse of 3 in mod 7.

b. In mod 11 : $4 \times 3 \equiv 1\ (mod\ 11)$, so 3 is the multiplicative inverse of 4 in mod 11.

Applications

1. **Cryptography:** Modular arithmetic plays a crucial role in public-key cryptography (e.g., Rivest–Shamir–Adleman-RSA encryption) for key generation and encryption/decryption processes.
2. **Error Detection and Correction:** It's used in checksums (adding digits and using the remainder for error detection) and cyclic redundancy checks (CRC) for data transmission reliability.

3. **Clock Arithmetic:** The 12-hour clock system is essentially modular arithmetic modulo 12 (e.g., 10 hours later than 3 PM is 1 AM).

4. **Hash Functions:** Modular arithmetic is employed in hash functions to map large data to smaller fixed-size values, used for data storage and retrieval.

5. **Computer Science:** It's widely applied in various computer science areas like pseudo-random number generation, finite field arithmetic, and generating permutations.

Learning Task

In mixed-groupings (gender/ability) learners solve examples of modulo arithmetic variety of TLRs

PEDAGOGICAL EXEMPLARS

In an interactive and differentiated learning environment, model simple situations involving modular arithmetic concepts, connect the ideas to real-world problems and solve them using appropriate models and technology. Employ differentiated assessment and ensure values such as tolerance, truth, honesty, respect for others' views, etc. among learners.

In GESI-responsive group activities, initiate reflective thinking (or discourse) on the application of modulo-arithmetic concepts in everyday business: Investigate the concept and existence of modulo arithmetic in learners' environment and introduce it as the arithmetic of remainders.

KEY ASSESSMENT
LEVEL 3: STRATEGIC THINKING

1. Create a visual presentation or poster explaining modular arithmetic with examples and real-world applications. Below are examples of real-life applications of modulo arithmetic. Refer to **Appendix C** on how to administer the project and poster.

Level 2: Conceptual Understanding

1. A lock requires a 4-digit code where the sum of the digits is congruent to 3 modulo 9. What are some possible valid codes?

2. In a 24-hour clock, what time will it be 100 hours from now?
3. You have 36 cookies and want to share them equally among your friends. If you have a remainder of 4 cookies after sharing, how many friends do you have?
4. An encryption code replaces each letter with a number ($A \equiv 1$). If the decrypted message "MATH" has a modulo sum of 5, what is the encrypted message?
5. Find the possible value for x in the equation $3x \equiv 7 \pmod{2}$.
6. In a 24-hour clock, what time will it be 100 hours from now?
7. Find the remainder when 100 is divided by 7.
8. Find the smallest positive integer n such that $2^n \equiv 1 \pmod{11}$.
9. What is the remainder when 123456789 is divided by 9?
10. Solve the congruence: $3x \equiv 4 \pmod{7}$
11. If today is Tuesday (represented by the number 3), what day will it be in 15 days (mod 7)?
12. Calculate $12^3 \pmod{7}$.
13. You have 19 cookies and want to share them equally among your friends. If you have a remainder of 4 cookies after sharing, how many friends do you have?

Hint

Assign Portfolio Assessment *for the academic year by Week 4. The portfolio should be submitted by Week 22.* ***See Appendix A*** *of this Section and Teacher Assessment Manual and Toolkit pages 22–25 for more information on how to organise a portfolio assessment.*

Section Review

Over the past four weeks, we have undertaken an in-depth study of surds, exponents, logarithms, and modulo arithmetic, building a solid mathematical foundation:

1. **Surds**: We learned about surds, rationalizing denominators, and applied operations like addition, subtraction, multiplication, division, and conjugation to solve real-world problems involving surds.
2. **Exponents and Logarithms**: We revisited integer exponents, simplified expressions using exponent rules, defined and applied logarithms (inverse of exponentiation), studied logarithmic equations and laws, and solved practical logarithm-based problems.
3. **Connecting Indices and Logarithms**: We bridged the connection between indices and logarithms, modelled real-life scenarios using both concepts,

solved complex problems using their properties, and worked with common logarithms.

4. **Modulo Arithmetic**: We understood modulo arithmetic through integer divisibility, learned its notation and number system connections, performed modulo operations, studied properties like associativity and inverse elements, and applied modulo arithmetic to real-world modelling and problem-solving.
5. With this comprehensive knowledge, learners are now equipped with powerful mathematical tools applicable across domains. Continue honing these skills through practice and exploring more advanced applications of these and related concepts.

Reminder

The following key assessment should be conducted and submitted for the transcript

1. *Class Exercise scores should be ready for submission to the student portal.*

APPENDIX A: SAMPLE PORTFOLIO ASSESSMENT

Task: Compile and submit a comprehensive portfolio that represents your work for the entire academic year. The portfolio should include a selection of exercises/assignments, project work, reflective pieces, and both mid-semester and end of semester examination papers.

Structure and Organisation of the Portfolio

As part of the structure of the portfolio assessment, make sure the following information has been provided:

1. **Cover Page with:**
   a. **learner's name**
   b. **class**
   c. **subject**
   d. **period/date, etc.**
2. **Table of Contents which has the list of items included with page numbers.**
3. **Brief description/background of items such as** background information for each included artefact, etc**.**

Learners' works to be included in the Portfolio

1. **Class Exercises/Assignments**
2. **Project works**
3. **Reflective Pieces**
4. **Mini-research work**
5. **Mid-semester examination papers**
6. **End of semester examination papers, etc.**

Mode of Administration for Portfolios

1. Clearly explain the purpose of the portfolio and its various components to the learners. Provide examples and templates for each section to guide them in their work.
2. Set up regular review sessions, every 4 weeks, to monitor learners' progress. During these checkpoints, they offer feedback and guidance to help them improve their portfolios.
3. Share the scoring rubrics with the learners and thoroughly explain how their work will be evaluated.

Set the final due date for portfolio submission in Week 22 of the academic calendar. Offer a grace period for learners to make revisions based on the final feedback they receive.

Mode of submission/presentation

1. Clearly inform all learners of the final deadline for portfolio submission to ensure that all work is completed and submitted on time.
2. Learners should organise their portfolios in a clear and logical manner, with each section clearly labelled and easy to access.
3. Learners may submit their portfolios either in physical form or via the school's online submission system.
4. For digital submissions, learners should upload their portfolios either as a single file or in well-organised folders within the online platform.
5. Ensure the portfolio contains all required components: assignments, projects, quizzes, tests, reflective pieces, mini-research work, as well as mid-semester and end of semester examination papers.

Feedback strategy

1. Schedule regular meetings to review learners' progress, set new goals, and make any necessary adjustments to their learning strategies.
2. Provide helpful comments throughout the learning process to support learners' development. Ensure that learners clearly understand how to use this feedback to continually improve their work and achieve better results.

Scoring rubric/ Marking scheme

Learner's pieces of work	*Items*	*Marks per Item*	*Total Marks*
Assignments/Exercises	*2*	*1 mark each*	*2 marks*
Projects works (Individual/ Group)	*2*	*2.5 marks each*	*5 marks*
Mini-project work (Week 21)	*1*	*2 marks*	*2 marks*
Reflective Piece (Week 13)	*1*	*2 marks*	*2 marks*
Mini-research Work (Week 10)	*1*	*2 marks*	*2 marks*
Mid-semester Examination Papers	*2*	*2 marks each*	*4 marks*
End of semester Examination Paper	*1*	*3 marks*	*3 marks*
Total Marks			*20 marks*

APPENDIX B: SAMPLE GROUP PROJECT WORK

Project Title: Investigating the causes of poor performance in Mathematics at Abakrampa Senior High Technical School

Sample mode of administration

1. Group learners into small, manageable groups based on ability or gender.
2. Assign each group the task of conducting a well-researched group project on the topic, creating a written report for presentation and grading.
3. Specify the required length of the project, for example, 2-3 pages.
4. Establish interim deadlines to track progress at different stages of the project. For instance, ask learners to submit a project proposal by *Week 3*, a draft of their model charts by *Week 6*, and the final project by *Week 9*.
5. Clearly outline the project's required content to the learners.

Presentation Guidelines

1. Each group is allocated 10-15 minutes for their presentation.
2. Presentations should be structured in a simple and clear format, ensuring the key findings are easy to follow.
3. Groups should deliver results that are both clear and engaging.
4. Visual aids such as charts, graphs, and slides must be used to effectively support and illustrate key points.
5. Include an interactive component, like a question-and-answer session, to actively involve the audience.

Feedback Strategy

1. Share the scoring rubrics with learners beforehand for transparency.
2. Offer immediate feedback, focusing on strengths, areas needing improvement, and recognising creativity and comprehension after presentations.
3. Encourage peer feedback after presentations to promote a collaborative learning experience.
4. Provide detailed feedback on written reports, addressing content accuracy, clarity, and organisation.
5. Recommend additional resources or exercises if necessary to help learners reinforce key concepts.

Sample scoring guide for the project work

Criteria	3 marks	2 marks	1 mark
Topic/Issue	*Identifies three causes: e.g.* *1. Lack of qualified Mathematics teachers.* *2. Insufficient teaching resources.* *3. Students' negative attitudes toward the subject.*	*Identifies any two causes: e.g.,* *1. Lack of qualified Mathematics teachers.* *2. Insufficient teaching resources.* *3. Students' negative attitudes toward the subject.*	*Identifies any one cause e.g.,* *1. Lack of qualified Mathematics teachers.* *2. Insufficient teaching resources.* *3. Students' negative attitudes toward the subject.*
Data Collection Method	*Uses two methods:* *1. surveys for students to capture their challenges.* *2. interviews with teachers.*	*Uses any one of the methods:* *1. surveys for students to capture their challenges.* *2. interviews with teachers.*	*Uses none of the two stated e.g.,* *informal observations only.*
Design of Data Collection Tool	*Survey includes three targeted questions: e.g.,* *1. What specific challenges do you face in Mathematics?* *2. Do you have access to a Mathematics textbook?* *3. How do you rate your teacher's teaching style?*	*Survey includes two general questions:* *e.g.,* *1. Do you like Mathematics?* *2. How often do you study?*	*Survey includes one question that is unclear:* *e.g.,* *1. Why do you fail Mathematics?*

Criteria	3 marks	2 marks	1 mark
Validation of the Tool	*Two Mathematics teachers and a school counsellor review the tool.*	*One Mathematics teacher and a school counsellor review the tool.*	*Tool is used without review.*
Data Collection	*Surveys distributed to 40-50 students and interviews conducted with 4-5 teachers.*	*Surveys distributed to 30-39 students, and 2-3 teachers are interviewed.*	*Surveys or interviews involve only one group, e.g., students only.*
Organisation of Data	*Data is categorised into three groups,* *1. Teacher-related factors.* *2. Student-related factors.* *3. Resource-related factors.*	*Data is categorised into any two groups,* *1. Teacher-related factors.* *2. Student-related factors.* *3. Resource-related factors.*	*Data is presented in one broad group.* *e.g., "general responses," without categorisation.*
Interpretation of Data	*Findings identify three key causes: e.g.,* *1. 65% of students lack textbooks.* *2. 50% of teachers feel underqualified.* *3. 70% of students dislike Mathematics due to perceived difficulty.*	*Findings identify two key causes: e.g.,* *1. 65% of students lack textbooks.* *2. 50% of teachers feel underqualified.* *3. 70% of students dislike Mathematics due to perceived difficulty.*	*Findings identify one cause: e.g.,* *1. 65% of students lack textbooks.* *2. 50% of teachers feel underqualified.* *3. 70% of students dislike Mathematics due to perceived difficulty.*

Criteria	3 marks	2 marks	1 mark
Presentation/Publication of Results	*Results presented in with three types of visuals:* *1. Bar graphs* *2. Pie charts* *3. Tables*	*Results presented with two types of visuals:* *1. Bar graphs* *2. Pie charts* *3. Tables*	*Results presented with one types of visuals:* *1. Bar graphs* *2. Pie charts* *3. Tables*

APPENDIX C: SAMPLE RUBRIC FOR SCORING THE PROJECT AND POSTER

Criteria	Score: 3 (Excellent)	Score: 2 (Good)	Score: 1 (Needs Improvement)
Definition	Modular arithmetic is a system of arithmetic where numbers "wrap around" after reaching a set value, called the modulus. It involves calculating remainders when dividing one number by another.	Modular arithmetic finds remainders after division, but the "wrap-around" concept is not explained.	Modular arithmetic deals with remainders but lacks proper context or detail.
Step-by-Step Explanation	1. Compute for instance,17 mod5. Divide 17 by 5. The quotient is 3, and the remainder is 2. So, 17mod 5=217. 2. Explains steps clearly: divide, find quotient, then remainder. 3. Uses accurate and logical calculations.	1. Compute for instance, 17mod 5: Divide 17 by 5 to get a remainder of 2. 2. The quotient is briefly mentioned but not fully explained. 3. Lacks full breakdown of calculations.	1. Mentions for instance, 17mod 5=2 without any explanation of steps. 2. Skips key details like quotient and remainder. 3. Includes errors or omissions.

Criteria	Score: 3 (Excellent)	Score: 2 (Good)	Score: 1 (Needs Improvement)
Examples	Give all three forms of examples; 1. $15 \bmod 12 = 3$ (time on a clock). 2. $17 \bmod 5 = 2$ (division remainders). 3. Cyclic patterns like repeating colours (1, 2, 3, 1, 2, 3).	Any two forms of the examples; 1. $15 \bmod 12 = 3$ (time on a clock). 2. $17 \bmod 5 = 2$ (division remainders). 3. Cyclic patterns like repeating colours (1, 2, 3, 1, 2, 3).	Any one of the forms of the examples; 1. $15 \bmod 12 = 3$ (time on a clock). 2. $17 \bmod 5 = 2$ (division remainders). 3. Cyclic patterns like repeating colours (1, 2, 3, 1, 2, 3).
Real-Life Connection	Mentions three forms of real-life use: 1. **Time**: A 12-hour clock resets after 12, so $15 \bmod 12 = 3$. 2. **Cryptography**: Modular arithmetic secures communication (e.g., RSA encryption). 3. **Scheduling**: Cycles of tasks like periodic train schedules.	States two forms of real-life use; 1. **Time**: A 12-hour clock resets after 12, so $15 \bmod 12 = 3$. 2. **Cryptography**: Modular arithmetic secures communication (e.g., RSA encryption). 3. **Scheduling**: Cycles of tasks like periodic train schedules.	Gives one form of the real-life use: 1. **Time**: A 12-hour clock resets after 12, so $15 \bmod 12 = 3$. 2. **Cryptography**: Modular arithmetic secures communication (e.g., RSA encryption). 3. **Scheduling**: Cycles of tasks like periodic train schedules.

SECTION 2: EQUATIONS AND INEQUALITIES

Strand: Algebraic Reasoning

Sub-Strand: Applications of Expressions, Equations and Inequalities

Learning Outcomes:

1. *Solve linear equations in two variables using the elimination, substitution and graphical methods.*
2. *Analyse, model, and solve simultaneous linear equations involving real-life problems.*

Content Standard: *Demonstrate knowledge and understanding of the concept of simultaneous equations involving two variables and apply it to solve every day-life problem.*

INTRODUCTION AND SECTION SUMMARY

Understanding simultaneous equations is fundamental in algebra and has practical applications in various fields. The concept and solution of simultaneous equations using the elimination method involves manipulating two equations to eliminate one variable, making it possible to solve for the remaining variable. Substitution can also be made to solve the problem. These methods are efficient and straightforward for solving linear equations in two variables. Additionally, solving linear equations graphically involves plotting the equations on a coordinate plane and identifying their point of intersection, which represents the solution. Interpreting these graphs helps visualize the relationship between variables and provides insights into their behavior. Real-life problems involving simultaneous equations are abundant; they can be used to determine the optimal combination of ingredients in a recipe, allocate resources efficiently in a project, or find the break-even point in a business scenario. Mastery of these techniques equips students with valuable problem-solving skills applicable to everyday situations and various professional fields.

The weeks covered by the section are:

Week 5:

1. *The concept and solution of simultaneous equations using the elimination and substitution methods.*

Week 6:

1. *Solving linear equations in two variables using the graphical method and interpreting graphs of linear equations in two variables.*
2. *Real-life problems involving Simultaneous equations.*

SUMMARY OF PEDAGOGICAL EXEMPLARS

Learners will benefit from a variety of teaching and learning strategies aimed at building a solid foundation in these concepts.

1. **Graphical Representation:** Use graphing software to graph linear equations in two variables. Encourage students to interpret the graphs and understand how the intersection points represent solutions to the equations.
2. **Real-life Applications:** Incorporate real-life examples of simultaneous equations to illustrate their relevance and practical applications. For example, demonstrate how simultaneous equations can be used to solve problems related to cost and revenue, or to analyze the intersection of supply and demand curves in economics.
3. **Collaborative Learning:** Assign group projects that require students to solve real-world problems using simultaneous equations. This promotes teamwork and enhances problem-solving skills.
4. **Problem-based Learning:** Present students with challenging problems that require them to apply simultaneous equations and linear equations in two variables. These problems should be relevant and meaningful, encouraging students to think critically and creatively.

ASSESSMENT SUMMARY

Different types of assessments should be conducted to evaluate learners' understanding of the concepts covered in this section. Teachers are encouraged to administer these assessments and document the results for submission to the Student Transcript Portal (STP). The following assessments will be carried out and recorded for each learner:

Week 6: *Mid-Semester Examination.*

Note

For additional information on how to effectively administer the mid-semester, please, refer to the Appendices at the end of section 2.

WEEK 5: SIMULTANEOUS EQUATIONS

Learning Indicator: *Analyse two linear equations in two variables and solve them using the elimination and substitution methods.*

Review of solutions of linear equations in one and two variables

Back in Year 1, we looked at linear equations in one variable. Take a look at the following examples;

Example 2.1

Solve: $4(x - 3) + 2x = 7x - 9$

Solution

$4(x - 3) + 2x = 7x - 9$

$4x + 4(-3) + 2x = 7x - 9$

$4x - 12 + 2x = 7x - 9$

$6x - 12 = 7x - 9.$

$-12 + 9 = 7x - 6x$

$-3 = x$

The solution is -3

Example 2.2

Solve $\frac{2x}{3} + \frac{1}{6} = 2$

Solution

$\frac{2x}{3} + \frac{1}{6} = 2$ *[multiply through by 6(LCM)]*

$6 \times 2\frac{x}{3} + \frac{1}{6} \times 6 = 2 \times 6$

$2 \times 2x + 1 = 12$

$4x = 11$ *[divide through by 4]*

$\frac{4x}{4} = \frac{11}{4}$

$x = 2\frac{3}{4}$

Learning Task

1. In pairs or mixed –ability groups, learners solve problems on linear equations in one variable.

2. In mixed-groupings [gender/ability] learners create simple linear equations in one variable and write the steps involved in solving the equation.

FOCAL AREA 1: THE CONCEPT AND SOLUTION OF SIMULTANEOUS EQUATIONS USING THE ELIMINATION METHOD

Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

Worked Examples

Example 2.3

Solve the simultaneous equations;

$3x + y = 5$ and $x + y = 1$

Solution

$$\begin{array}{rl} & 3x + y = 5 \\ - & x + y \ \ = 1 \\ \hline & 2x \qquad = 4 \end{array}$$ So $x = 2$	**1.** The y coefficients are the same, so we SUBTRACT the second equation from the first equation to eliminate the y term.
Using $x + y = 1$ $2 + y = 1$ So, $y = -1$	**2.** To find the value of y, substitute $x = 2$ into one of the original equations.
Check: equation 1: $3 \times 2 + (-1) = 5$ YES equation 2: $2 + (-1) = 1$ YES	**3.** Substitute the values of x and y into both equations to check your answers.

Example 2.4

Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

Solution

$x + 2y = 13$ $+ \quad 5x - 2y = 5$ $6x \quad = 18$ So $x = 3$	**1**	The y coefficients have the same magnitude, but different signs, so we ADD the two equations to eliminate the y term.
Using $x + 2y = 13$ $3 + 2y = 13$ So, $y = 5$	**2**	To find the value of y, substitute $x = 3$ into one of the original equations.
Check: equation 1: $3 + 2 \times 5 = 13$ YES equation 2: $5 \times 3 - 2 \times 5 = 5$ YES	**3**	Substitute the values of x and y into both equations to check your answers.

Example 2.5

Solve $2x + y = 10$ and $3x - 2y = 25$ simultaneously

$2x + y = 10$……… equu. 1 $3x - 2y = 29$ ……...equ. 2	**1**	Since none of the coefficients are the same we need to first make one of the equations have the same coefficient as in the other.
$4x + 2y = 20$………equ. 3 $+ \quad 3x - 2y = 25$……..equ. 2 $4x + 2y = 29$……..equ. 3	**2**	Multiply through equ. 1 by 2 to make the coefficient of y the same.
$7x = 49$ So, $x = 7$	**3**	The y coefficients have the same magnitude, but different signs, so we ADD the two equations to eliminate the y term.
Using $3(7) - 2y = 29$ $21 - 2y = 29$ $- 2y = 8$ So, $y = -4$	**4**	To find the value of y, substitute $x = 7$ into one of the original equations.

Check: equation 1: 2(7) - 4 = 10 YES equation 2: 3(7) – 2(-4) = 29 YES	**5.** Substitute the values of x and y into both equations to check your answers.

Learning Task

In mixed-groupings [gender/ability] learners solve examples of simultaneous equations using the elimination method.

PEDAGOGICAL EXEMPLARS

1. Using mixed-ability groupings, review learners' knowledge on linear equations, deal with their misconceptions about such concepts.
2. Using talk for learning strategy, engage learner in a whole class, to explain simultaneous equations, their solutions and the strategies for solving them. (i.e., graphical, elimination, and substitution method.)
3. Collaborative Learning: In small groups, engage learners to discuss and come out with the process of solving simultaneous equations using the elimination method.
4. Collaborative Learning: In small groups, engage learners to solve simultaneous equations using elimination method.

FOCAL AREA 2: SOLVING SIMULTANEOUS LINEAR EQUATIONS IN TWO VARIABLES USING THE SUBSTITUTION METHOD

Key points

- Solve one equation for one variable.
- Substitute the expression into the other equation.
- Simplify and solve the resulting equation for the second variable.
- Substitute back to find the first variable.
- Write the solution.
- Verify the solution (optional, but recommended).

Example 2.6

Solve the simultaneous equations $y = 2x + 1$ and $5x + 3y = 14$

Solution

$5x + 3(2x + 1) = 14$	**1**	Substitute $2x + 1$ for y into the second equation.
$5x + 6x + 3 = 14$ $11x + 3 = 14$ $11x = 11$	**2**	Expand the brackets and simplify.
So, $x = 1$	**3**	Work out the value of x.
Using $y = 2x + 1$ $y = 1 + 1 \times 2$ So, $y = 3$	**4**	To find the value of y, substitute $x = 1$ into one of the original equations.
Check: equation 1: $3 = 2 \times 1 + 1$ YES equation 2: $5 \times 1 + 3 \times 3 = 14$ YES	**5**	Substitute the values of x and y into both equations to check your answers.

Example 2.7

Solve $2x - y = 16$ and $4x + 3y = -3$ simultaneously.

Solution

$y = 2x - 16$	**1**	Rearrange the first equation to make y the subject.
$4x + 3(2x - 16) = -3$	**2**	Substitute $2x - 16$ for y into the second equation.
$4x + 6x - 48 = -3$	**3**	Expand the brackets and simplify.
$10x - 48 = -3$ $10x = 45$ So $x = 4\frac{1}{2}$	**4**	Work out the value of x.
Using $y = 2x - 16$ $y = 2 \times 4\frac{1}{2} - 16$ So $y = -7$	**5**	To find the value of y, substitute $x = 4\frac{1}{2}$ into one of the original equations.
Check: equation 1: $2 \times 4\frac{1}{2} - (-7) = 16$ YES equation 2: $4 \times 4\frac{1}{2} + 3 \times (-7) = -3$ YES	**6**	Substitute the values of x and y into both equations to check your answers.

Learning Task

In mixed-groupings [gender/ability] learners solve examples of simultaneous equations using the substitution method.

PEDAGOGICAL EXEMPLARS

1. Collaborative Learning: In small groups, engage learners to discuss and come out with the process of solving simultaneous equations using substitution method.
2. Collaborative Learning: In small groups, engage learners to solve simultaneous equations using substitution method.
3. In pairs, learners write their own examples of simultaneous equations for a partner to solve using either elimination or substitution method.

KEY ASSESSMENT

Solve these simultaneous equations using the elimination method.

1. $4x + y = 8$
   $3x + 2y = 5$
2. $3x + y = 7$
   $x + y = 5$
3. $4x + y = 3$
   $3x - y = 11$
4. $3x + 4y = 7$
   $x - 4y = 5$

Solve these simultaneous equations using the substitution method.

1. $y = x - 4$
   $2x + 5y = 43$
2. $y = 2x - 3$
   $5x - 3y = 11$
3. $2y = 4x + 5$
   $9x + 5y = 22$
4. $2x = y - 2$
   $8x - 5y = -11$

WEEK 6: LINEAR EQUATIONS IN TWO VARIABLES

Learning Indicators

1. *Solve simultaneous linear equations involving two variables using the graphical method and interpret them.*
2. *Analyse, model, and solve word problems of simultaneous linear equations involving numbers and age, etc.*

FOCAL AREA 1: SOLVING LINEAR EQUATIONS IN TWO VARIABLES USING THE GRAPHICAL METHOD AND INTERPRETING GRAPHS OF LINEAR EQUATIONS IN TWO VARIABLES

Key points

You can solve any pair of simultaneous equations by drawing the graph of both equations and finding the point/points of intersection.

Example 2.8

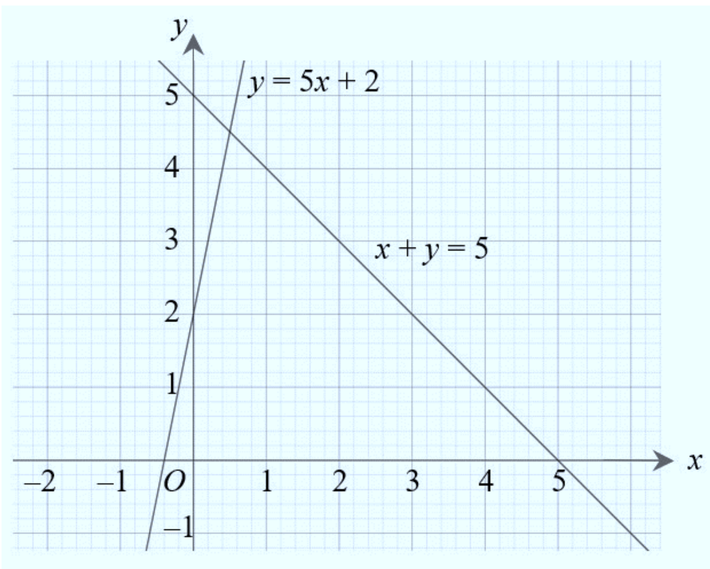
Solve the simultaneous equations y = 5x + 2 and x + y = 5 graphically.

$y = 5 - x$

1 Rearrange the equation $x + y = 5$ to make y the subject.

$y = 5 - x$ has gradient –1 and y-intercept 5. $y = 5x + 2$ has gradient 5 and y-intercept 2.

2 Plot both graphs on the same grid using the gradients and y-intercepts.



Lines intersect at

$x = 0.5, y = 4.5$

Check:

First equation $y = 5x + 2$:

$4.5 = 5 \times 0.5 + 2$ YES

Second equation $x + y = 5$:

$0.5 + 4.5 = 5$ YES

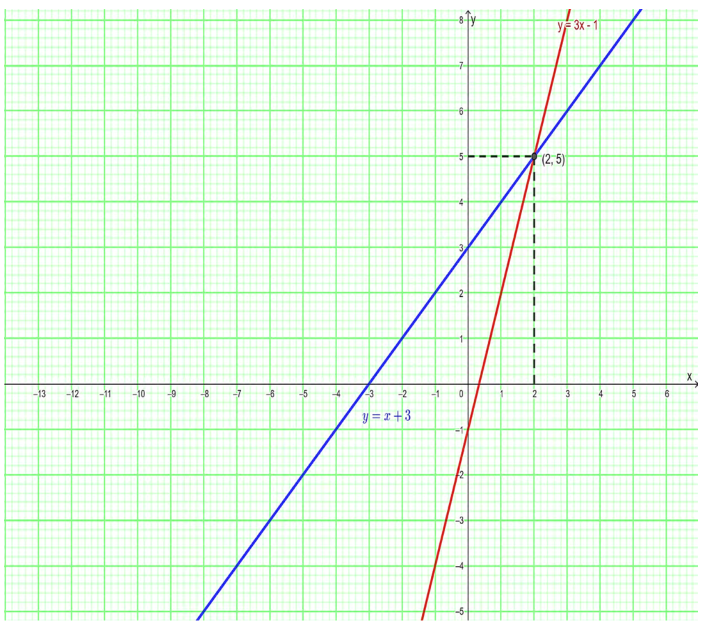
3 The solutions of the simultaneous equations are the point of intersection.

4 Check your solutions by substituting the values into both equations.

Example 2.9

Solve the simultaneous equations $y = 3x - 1$ and $y = x + 3$ graphically.

$y = 3x - 1$ has gradient 3 and y-intercept -1. $y = x + 3$ has gradient 1 and y-intercept 3.



1. Plot both graphs on the same grid using the gradients and y-intercepts.

Lines intersect at

$x = 2, y = 5$

Check:

First equation $y = 3x - 1$:

$5 = 3 \times 2 - 1$

$5 = 5$ YES

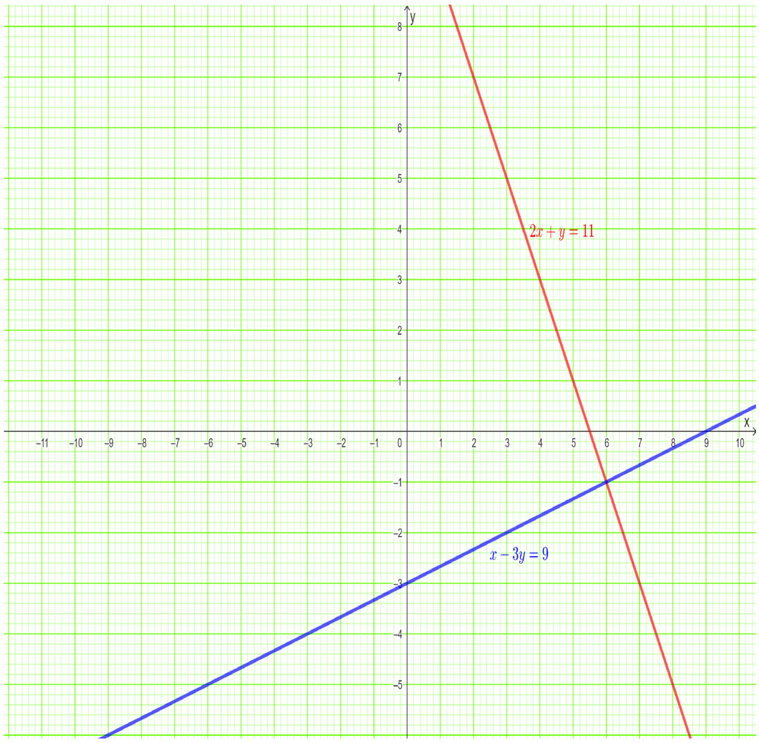
Second equation $y = x + 3$:

$5 = 2 + 3$ YES

2. The solutions of the simultaneous equations are the point of intersection.
3. Check your solutions by substituting the values into both equations.

Example 2.10

Study the graph carefully and use it answer the questions that follow.



a. What is the gradient of the line with the equation $x - 3y = 9$?

b. Which of the two lines has a negative slope?

c. What is the coordinate of the point of intersection of the two lines?

d. Show that the point of intersection is truly the solution to the two systems of equations.

Solution

a. $x - 3y = 9$

Re-write the equation in the form $y = mx + c$

$x - 9 = 3y$

$y = \frac{1}{3}x - 3$

Gradient $= \frac{1}{3}$.

b. The line $2x + y = 11$ has a negative gradient. Thus, $y = -2x + 11$

c. The coordinate of the point of intersection of the two lines is (6, -1)

d. Check your solutions by substituting the values into both equations

$(x, y) = (6, -1)$

$x - 3y = 9$

$6 - 3(-1) = 9$

$6 + 3 = 9$ ………… YES

$2x + y = 11$

$2(6) - 1 = 11$

$12 - 1 = 11$ ……… YES

FOCAL AREA 2: REAL-LIFE PROBLEMS INVOLVING SIMULTANEOUS EQUATIONS

In solving real-life problems, use the following strategy.

Step 1 - Read the problem and identify the unknown quantities.

Step 2 - Choose variables to represent these quantities.

Step 3 - Formulate the equation in terms of the variables to be determined.

Step 4 - Solve the system of equations using any of the methods learnt earlier.

Step 5 - Check the answer to make sure it makes sense.

Example 2.11

7 pairs of jeans and 4 shirts cost GH¢655. 5 pairs of jeans and 3 shirts cost GH¢475. Find the cost of each item.

Let the cost of a pair of jeans be GH¢x and that of a shirt be GH¢y. Then

$7x + 4y = 655$ and $5x + 3y = 475$

Solution

$7x + 4y = 655$………1

$5x + 3y = 475$………2 [make x the subject in equ. 2]

$x = \frac{75}{5} - \frac{3y}{5}$

$x = 95 - \frac{3y}{5}$ [substitute x into equ. 1]

$7(95 - 3y\ \_\ 5\ ) + 4y = 655$

$665 - \frac{21y}{5} + 4y = 655$ [multiply through by 5]

$3325 - 21y + 20y = 3275$

$3325 - 3275 = y$

$y = 50$ [substitute the value of y into equ. 1]

$7x + 4(50) = 655$

$7x + 200 = 655$

$7x = 655 - 200$

$7x = 455$

$\frac{7x}{7} = \frac{455}{7}$

$x = 65$

$\therefore$ The cost of a pair of jeans is GH¢65 and the cost of a shirt is GH¢50.

Example 2.12

Two friends, Alice and Bright, went to a stationery store. Alice bought 3 notebooks and 2 pens for GH¢16. Bright bought 2 notebooks and 5 pens for GH¢19. Find the price of one notebook and one pen.

Solution

Let x be the price of one notebook, and y be the price of one pen.

We can set up the following system of equations based on the information given:

$3x + 2y = 16$ ……Equ. 1

$2x + 5y = 19$……Equ. 2

Let's multiply Equ. 1 by 2 and Equ. 2 by 3. (This step is to make the coefficients of x the same in both equations)

$6x + 4y = 32$ ……Equ. 3

$6x + 15y = 57$……Equ. 4

Now, subtract Equ. 3 from Equ. 4

$11y = 25$

$So\ y = 2\frac{3}{11}\ or\ 2.27$

Substitute the value of y into any of the original equations and find the value of x.

$3x + 2y = 16$

$3x + 2\left(\frac{25}{11}\right) = 16$

$33x + 50 = 176$

$33x = 126$

$x = 3.82$

Therefore, the price of one notebook (x) is approximately GH¢3.82, and the price of one pen (y) is approximately GH¢2.27.

Learning Task

1. In mixed-groupings [gender/ability] learners solve examples of simultaneous equations using the graphing method.
2. In mixed-ability groups, learners interpret given graphs of simultaneous equations.

PEDAGOGICAL EXEMPLARS

1. Using talk for learning strategy, engage learner in a whole class to discuss and explain the graphical method of solving simultaneous equations using the slop-intercept form. i.e., y = mx + c where *m* is the slope, and *c* is the y- intercept.
2. Problem-Based Learning: In small groups, learners are tasked to solve and plot equations on graph sheets or books and submit for discussion.

KEY ASSESSMENT

1. Solve these pairs of simultaneous equations graphically.
   - **i.** $y = 3x - 1$ and $y = x + 3$
   - **ii.** $y = x - 5$ and $y = 7 - 5x$
   - **iii.** $y = 3x + 4$ and $y = 2 - x$
   - **iv.** $x + y = 0$ and $y = 2x + 6$
2. A farmer sold a goat and a cow for GH¢14800, thereby making a profit of 20% on the goat and 10% on the cow. By selling them for GH¢15100, he would have made a profit of 10% on the goat and 20% on the cow. Find the cost price of each.
3. A restaurant sells coffee and sandwiches. The first customer buys 3 cups of coffee and 4 sandwiches for GH¢27. The second customer buys 2 cups of coffee and 3 sandwiches for GH¢19. Find the cost of each item.

Hint

- *The Recommended Mode of Assessment for Week 6 is **Mid-semester Examination.***
- *Refer to Appendix D at the end of Section 2 for further information on how to go about the mid-semester examination.*

Section 2 Review

In this section, we learnt about the concept and solution of simultaneous equations using the elimination and substitution methods. We also solved linear equations in two variables using the graphical method and interpreted graphs of linear equations in two variables. We solved real-life problems involving simultaneous equations. We learnt the following key ideas about the concepts:

- **Simultaneous Equations Using Elimination Method:** Eliminate one variable by adding or subtracting equations. Solve the resulting single-variable equation and substitute back to find the other variable.
- **Simultaneous Equations Using Substitution Method:** Solve one equation for one variable and substitute this expression into the other equation. Solve the resulting single-variable equation, then substitute back to find the other variable.
- **Simultaneous Equations Using Graphical Method:** Graph both equations on the same coordinate plane. The point where the two lines intersect represents the solution to the simultaneous equations.

ADDITIONAL READING

· Graphical Simultaneous pdf (corbettmaths.com)	
· Microsoft Word - S2 Topic 3 Simultaneous Linear Equations. doc (edb.gov.hk)	

APPENDIX D

Sample of principles to consider when designing effective multiple-choice questions (MCQ's)

1. Write questions that are clear and straightforward so learners can easily understand them.
2. Make sure each question matches the learning goals and the topics that have been taught in the course.
3. Add incorrect answer choices (distractors) that seem believable and are closely related to the correct answer.
4. Ensure each question has only one clear and correct answer. Avoid using options like "all of the above" or "none of the above," which can sometimes confuse learners.
5. Create questions with different levels of difficulty to distinguish between learners with varying levels of understanding.
6. Make sure questions are free from cultural, language, or gender bias. Use language and examples that are fair and accessible to all learners, regardless of their background.

Sample of principles to consider in developing essay-type questions

1. Write questions that are clear and straightforward so learners can easily understand them.
2. Make sure each question matches the learning goals and the topics that have been taught in the course.
3. Add incorrect answer choices (distractors) that seem believable and are closely related to the correct answer.
4. Ensure each question has only one clear and correct answer. Avoid using options like "all of the above" or "none of the above," which can sometimes confuse learners.
5. Create questions with different levels of difficulty to distinguish between learners with varying levels of understanding.
6. Make sure questions are free from cultural, language, or gender bias. Use language and examples that are fair and accessible to all learners, regardless of their background.

Sample Table of Test Specifications

Weeks	Learning indicators	Type of Questions	DoK Levels				Total
			1	2	3	4	
1	a. Carry out basic operations on surds	Multiple Choice	3	2	1	-	6
	b. Rationalize monomial denominators	Essay type	1	-	-	-	1
2	a. Explain the concepts of indices and logarithms with examples.	Multiple Choice	2	2	2	-	6
	b. Compose and decompose logarithm laws and properties with exponents and apply the concepts to solve real-life problems.	Essay type	-	-	-	-	
3	a. Investigate real-life problems using laws and properties of indices and logarithms.	Multiple Choice	1	2	2	-	5
	b. Use mathematical connections to explore the relevance of indices and logarithms and their applications to scientific concepts.	Essay type	-	1	-	-	1

Weeks	Learning indicators	Type of Questions	DoK Levels				Total
			1	2	3	4	
4	a. Undertake a brief review on sorting and divisibility rules of integers and determine their connections to modular arithmetic using models such as number arrays. b. Model simple situations involving modular arithmetic concepts, connect the ideas to real-world problems, and solve them using appropriate models and technology.	Multiple Choice	1	2	2	-	5
		Essay type	-	1	-	-	1
5	Analyse two linear equations in two variables and solve them using the elimination and substitution methods.	Multiple Choice	1	2	1	-	4
		Essay type	-	1	-	-	1

<table>
<tr><th rowspan="2">Weeks</th><th rowspan="2">Learning indicators</th><th rowspan="2">Type of Questions</th><th colspan="4">DoK Levels</th><th rowspan="2">Total</th></tr>
<tr><th>1</th><th>2</th><th>3</th><th>4</th></tr>
<tr><td rowspan="2">6</td><td rowspan="2">a. Solve simultaneous linear equations involving two variables using the graphical method and interpret them.
b. Analyse, model, and solve word problems of simultaneous linear equations involving numbers and age, etc.</td><td>Multiple Choice</td><td>1</td><td>2</td><td>1</td><td>-</td><td>4</td></tr>
<tr><td>Essay type</td><td>-</td><td>-</td><td>1</td><td>-</td><td>1</td></tr>
<tr><td colspan="8">Total</td></tr>
<tr><td colspan="2">Multiple Choice</td><td></td><td>9 (30%)</td><td>12 (40%)</td><td>9 (30%)</td><td>-</td><td>30 (100%)</td></tr>
<tr><td colspan="3">Essay type</td><td>1</td><td>3</td><td>1</td><td></td><td>5</td></tr>
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SECTION 3: RIGID MOTION

Strand: Geometry around us

Sub-Strand: Spatial Sense

Learning Outcome: *Carry out a variety of transformations through translation, reflection, rotation and enlargement of plane shapes and identify scale drawing as an enlargement/reduction of a plane shapes.*

Content Standard: *Demonstrate conceptual understanding of spatial sense regarding changes and invariance achieved by performing a combination of successive transformations (reflection, translation, rotation) in 2D shape.*

INTRODUCTION AND SECTION SUMMARY

Rigid motions, including translation and reflection, are fundamental concepts in geometry and transformation. Translation involves moving an object without rotating or changing its size, while reflection is a transformation that flips an object over a line, creating a mirror image. These transformations are important in understanding symmetry and patterns in shapes and figures. Rotation is another type of rigid motion that involves turning an object around a fixed point called the centre of rotation. This concept is commonly used in geometry to describe the orientation of shapes and figures. Understanding rotations helps in analyzing the properties of shapes and predicting their positions after being rotated. Enlargement is a transformation that increases the size of a shape while maintaining its shape and proportions. This concept is useful in scaling objects in various applications, such as in architecture, engineering, and art. Understanding how to enlarge shapes allows for accurate representations and measurements in these fields.

The weeks covered by the section are:

Week 7: *Rigid Motion: Translation and Reflection*

Week 8: *Rotation of images and Enlargement of plane shape.*

SUMMARY OF PEDAGOGICAL EXEMPLARS

This section emphasizes the importance of hands-on activities and real-life examples to enhance understanding and application of concepts related to rigid motion, rotation, and enlargement of plane shapes. Learners will benefit from a variety of teaching and learning strategies aimed at building a solid foundation in these concepts.

1. **Hands-on Exploration:** Engage students in physical activities where they can physically move objects to demonstrate translation, reflection, and rotation.
2. **Visual Representations:** Utilize visual aids such as diagrams, animations, and interactive simulations to demonstrate the effects of translation, reflection, and rotation.
3. **Real-life Applications:** Incorporate real-life examples of rigid motion, rotation and enlargement to illustrate their relevance and practical applications.
4. **Collaborative Learning:** Encourage collaborative learning by assigning group projects that require students to apply rigid motion, rotation, and enlargement concepts to solve real-world problems. This promotes teamwork and enhances problem-solving skills.
5. **Problem-based Learning:** Present students with challenging problems that require them to apply rigid motion, rotation, and enlargement concepts.

ASSESSMENT SUMMARY

In covering these concepts, a variety of assessment strategies should be used, including:

1. **Practical Demonstrations:** Assess understanding of translation, reflection, rotation and enlargement through practical demonstrations where learners physically manipulate objects to demonstrate these concepts.
2. **Visual Representations:** Use visual aids such as diagrams, animations, and interactive simulations to assess learners' ability to visualize and understand the effects of translation, reflection, rotation, and enlargement.
3. **Real-life Applications:** Present learners with real-world scenarios that require them to apply concepts of rigid motion, rotation, and enlargement to solve problems. This assesses their ability to apply theoretical knowledge to practical situations.
4. **Problem-solving Tasks:** Provide learners with challenging problems that require them to apply concepts of rigid motion, rotation, and enlargement to find solutions.

WEEK 7: TRANSLATION & REFLECTION

Learning Indicators

1. *Identify and translate an object or point by a translating vector and describe the image.*
2. *Identify and explain the reflection of an object in a mirror line and describe the image points of shapes in a reflection.*

FOCAL AREA 1: TRANSLATION BY A TRANSLATION VECTOR

Rigid motion refers to a transformation that preserves the size and shape of a figure, meaning the figure does not change in any way except for its position. Translation specifically involves moving every point of a figure the same distance in the same direction, defined by a translation vector.

A translation vector, usually denoted as $\vec{v} = \binom{a}{b}$, indicates how far and in which direction each point of the figure will be moved. For instance, if $\vec{v} = \binom{3}{-2}$, each point of the figure is moved 3 units to the right and 2 units down. This results in the figure maintaining its orientation and proportions, effectively sliding to a new location on the plane without rotating or resizing.

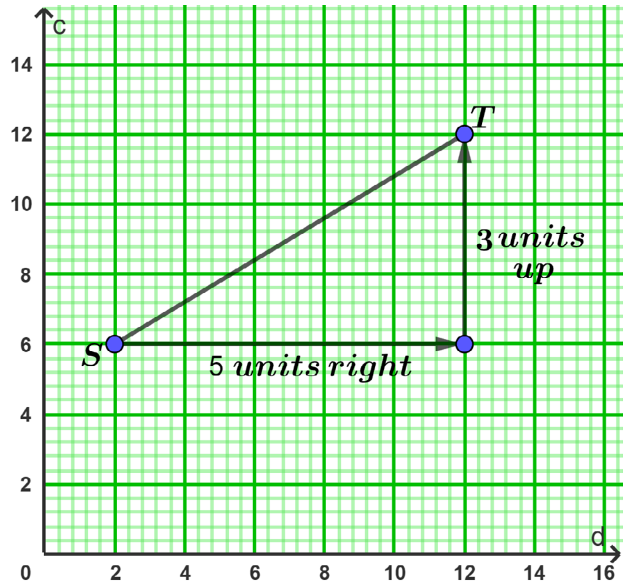
Hence, this lesson will require that learners apply their knowledge in vectors from JHS and SHS 1.

Performing Translations

A vector is a quantity that has both direction and magnitude, or size, and is represented in the coordinate plane by an arrow drawn from one point to another.

Vectors

The diagram shows a vector. The initial point, or starting point, of the vector is S, and the terminal point, or ending point, is T. The vector is named $\overrightarrow{ST}$, which is read as "vector $\overrightarrow{ST}$." The horizontal component of $\overrightarrow{ST}$ is 5, and the vertical component is 3. The component form of a vector combines the horizontal and vertical components. So, the component form of $\overrightarrow{ST}$ is $\binom{5}{3}$.



A **transformation** is a function that moves or changes a figure in some way to produce a new figure called an image. Another name for the original figure is the preimage. The points on the preimage are the inputs for the transformation, and the points on the image are the outputs.

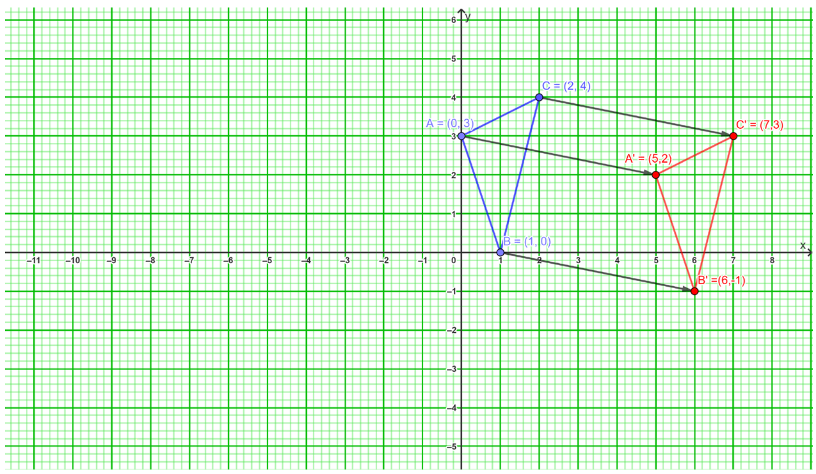
Translating a Figure Using a Vector

Example 3.1

The vertices of $\triangle ABC$ are A(0, 3), B(2, 4), and C(1, 0). Translate $\triangle ABC$ using the vector $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$.

Solution

First, graph $\triangle ABC$. Use $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$ to move each vertex 5 units right and 1 unit down. Label the image vertices. Draw $\triangle A'B'C'$. Notice that the vectors drawn from preimage vertices to image vertices are parallel.



You can also express a translation along the vector $\binom{a}{b}$ using a rule, which has the notation (x, y) → (x + a, y + b).

Writing a Translation Rule

Using our earlier example, write a rule for the translation of △ABC to △A′B′C′.

Solution

To go from A to A′, you move 5 units right and 1 unit down, so you move along the vector ⟨5, -1⟩. Again, to go from B to B', you move 5 units right and 1 unit down. The same applies to moving from C to C'. So, a rule for the translation is (x, y) → (x + 5, y - 1).

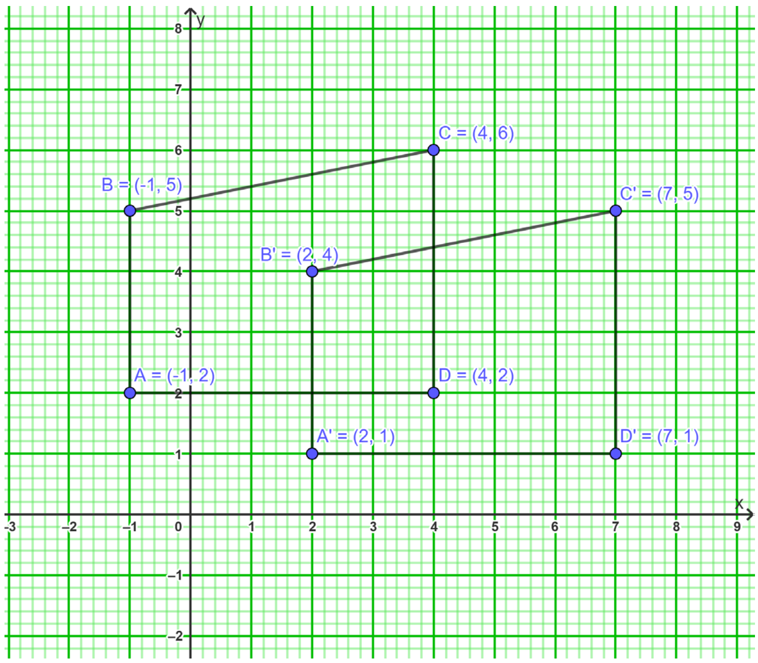
Example 3.2

Graph quadrilateral ABCD with vertices A (−1, 2), B (−1, 5), C (4, 6), and D (4, 2) and its image after the translation (x, y) → (x + 3, y − 1).

Solution

Graph quadrilateral ABCD. To find the coordinates of the vertices of the image, add 3 to the x-coordinates and subtract 1 from the y-coordinates of the vertices of the preimage. Then graph the image, as shown at the left.

(x, y) → (x + 3, y − 1)



A (−1, 2) → A′ (2, 1)

B (−1, 5) → B′ (2, 4)

C (4, 6) → C′ (7, 5)

D (4, 2) → D′ (7, 1)

Learning Task

1. Learners identify translation vectors of given graphs.
2. Learners translate given coordinates using translation vectors.

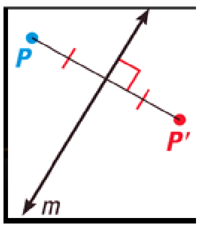
PEDAGOGICAL EXEMPLARS

1. Think-pair share activities: Learners in pairs research and make presentations on the concept of translation and its characteristics.
2. In mixed-ability groups, leaners discuss translation vectors and apply their use in translating given coordinates.
3. Collaborative Learning: In mixed-grouping (gender/ability) task learners to graph triangles and quadrilaterals by translation.

FOCAL AREA 2: REFLECTIONS OF IMAGES

Reflection is a transformation that uses a line like a "mirror" to reflect an image.

- That line is called **Line of Reflection.**
- P and P′ are the same distance from the line of reflection.



- The line connecting P and P′ is perpendicular to the line of reflection.

Rules of Reflection

To perform a reflection in a coordinate plane, you need to follow some basic rules that relate to a given line of reflection.

1. **Reflection Across the x-Axis**
   - **Rule:** $(x, y) \rightarrow (x, -y)$
   - **Description:** When a point is reflected across the *x*-axis, the *x*-coordinate remains the same, while the *y*-coordinate changes sign.
2. **Reflection Across the y-Axis**
   - **Rule:** $(x, y) \rightarrow (-x, y)$
   - **Description:** When a point is reflected across the *y*-axis, the *y*-coordinate remains the same, while the *x*-coordinate changes sign.

3. **Reflection Across the Line $y = x$**
   - **Rule:** $(x, y) \rightarrow (y, x)$
   - **Description:** When a point is reflected across the line $y = x$, the coordinates are swapped.

4. **Reflection Across the Line $y = -x$**
   - **Rule:** $(x, y) \rightarrow (-y, -x)$
   - **Description:** When a point is reflected across the line $y = -x$, the coordinates are swapped and their signs are changed.

5. **Reflection Across a Vertical Line $x = a$**
   - **Rule:** $(x, y) \rightarrow (2a-x, y)$
   - **Description:** When a point is reflected across a vertical line $x =$ a, the y-coordinate remains the same, while the x-coordinate is transformed by the formula $2a - x$.

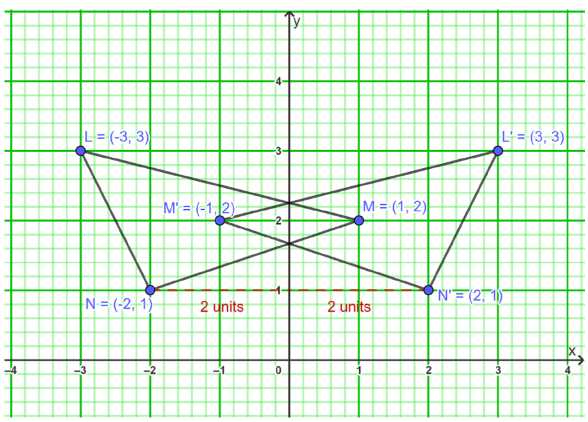
6. **Reflection Across a Horizontal Line y = b**
   - **Rule:** $(x, y) \rightarrow (x, 2b-y)$
   - **Description:** When a point is reflected across a horizontal line y = b, the x-coordinate remains the same, while the y-coordinate is transformed by the formula $2b - y$.

Worked Examples

Example 3.3

The vertices of ΔLMN are L(–3, 3), M(1, 2), and N(–2, 1). Plot the coordinates on a graph and join the points to form ΔLMN. Draw the image ΔL'M'N' of ΔLMN using the y-axis as the mirror line.

Solution



From the graph, we can identify the coordinates as;

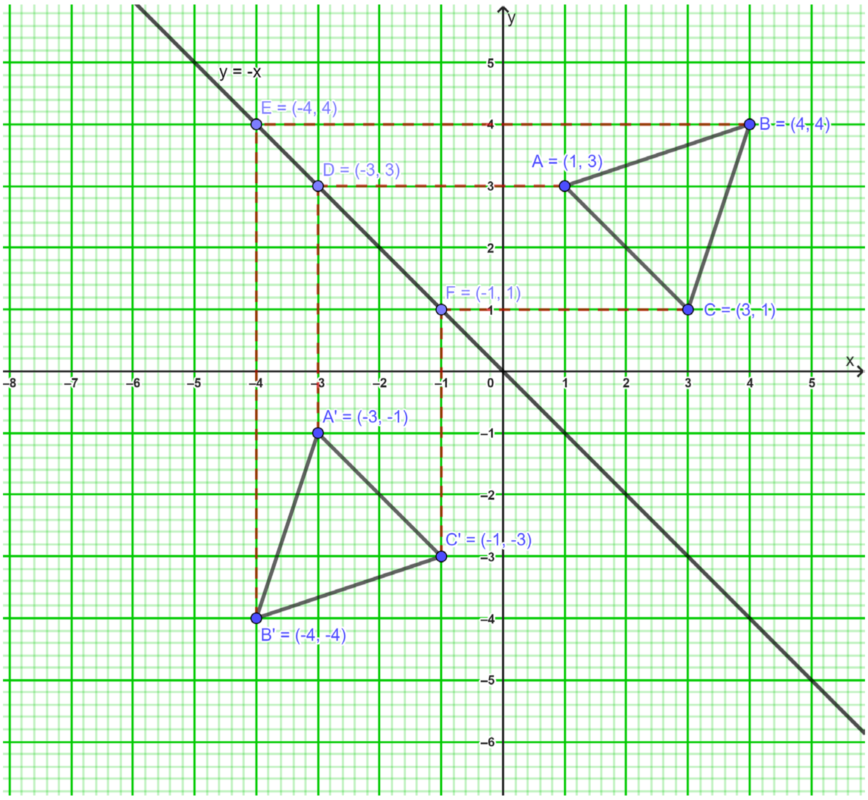
(a, b) → (–a, b)

L (–3, 3) → L′(3, 3)

M(1, 2) → M′(–1, 2)

N(–2, 1) → N′(2, 1)

Graph ΔABC with vertices A(1, 3), B(4, 4), and C(3, 1). Reflect ΔABC in the line y = −x.



From the graph, we can see how the object [ΔABC] is reflected in the mirror line $y = -x$. Study the dash lines carefully to understand how each point if reflected.

Learning Tasks

1. Learners identify and explain the reflection of an object in a mirror (reflection) line.
2. Learners describe the image points of shapes in a reflection.

PEDAGOGICAL EXEMPLARS

1. Think-pair share activities: Learners in pairs research and make presentations on the concept of reflection and its characteristics.
2. In mixed-ability groups, leaners discuss the rules of reflection and apply its use in reflecting given coordinates.
3. Collaborative Learning: In mixed-grouping (gender/ability) task learners to graph triangles and quadrilaterals and perform reflection on a plane.

KEY ASSESSMENT

1. Use the translation $(x, y) \rightarrow (x - 8, y + 4)$

i. What is the image of A (2, 6)?

ii. What is the image of B (–1, 5)?

iii. What is the preimage of C′ (–3, –10)?

iv. What is the preimage of D′ (4, –3)?

2. Graph △PQR with vertices P (–2, 3), Q (1, 2), and R (3, –1) and its image after the translation.

i. $(x, y) \rightarrow (x + 4, y + 6)$

ii. $(x, y) \rightarrow (x + 9, y - 2)$

iii. $(x, y) \rightarrow (x - 2, y - 5)$

iv. $(x, y) \rightarrow (x - 1, y + 3)$

3. Graph △XYZ with vertices X (2, 4), Y (6, 0), and Z (7, 2) and its image after the composition.

i. Translation: $(x, y) \rightarrow (x + 12, y + 4)$
Translation: $(x, y) \rightarrow (x - 5, y - 9)$

ii. Translation: $(x, y) \rightarrow (x - 6, y)$
Translation: $(x, y) \rightarrow (x + 2, y + 7)$

4. The vertices of △ABC are A (2, 2), B (4, 2), and C (3, 4). Graph the image of △ABC after the transformation $(x, y) \rightarrow (x + y, y)$. Is this transformation a translation? Explain your reasoning.

5. The vertices of ΔABC are A (3, 2), B (–1, 3), and C (1, 1). Find the image of ΔABC after the glide reflection

6. Graph △ABC with vertices A (1, 3), B (5, 2), and C (2, 1) and its image after the reflection described.

i. In the line n: $x = 3$ **ii.** In the line m: $y = 1$

7. Graph △ABC with vertices A(3, 2), B(6, 3), and C(7, 1) and its image after the glide reflection.

i. Translation: $(x, y) \rightarrow (x - 12, y)$

ii. Reflection: in the x-axis

> ***Reminder:*** *Learners' group project work assigned in Week 2 should be ready for submission by the end of Week 7.*

WEEK 8: ROTATION & ENLARGEMENT

Learning Indicators

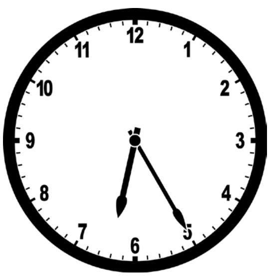
1. *Identify shapes with rotational symmetry and show the image of an object (or point) after a rotation about the origin (or point).*
2. *Carry out an enlargement of a plane shape given a scale factor.*

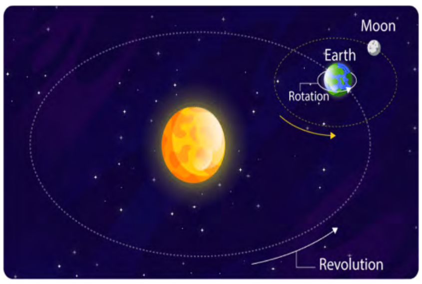
FOCAL AREA 1: ROTATION OF IMAGES OF PLANE SHAPE

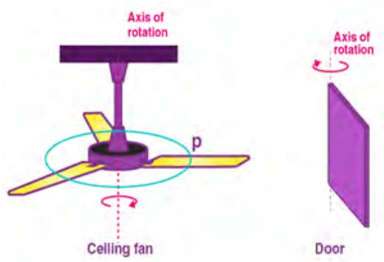
A rotation involves turning a figure around a fixed point, known as the centre of rotation, through a specified angle and direction, typically measured in degrees. This transformation preserves the shape and size of the figure, maintaining its proportions while altering its orientation.

In a rotation, each point of the figure moves along a circular arc centered at the point of rotation. The angle of rotation determines how far the figure is turned, while the direction of rotation (clockwise or counterclockwise) specifies the direction of this movement. Common rotations include 90°, 180°, and 270°, but any angle can be used.

Real-life Examples of Rotation



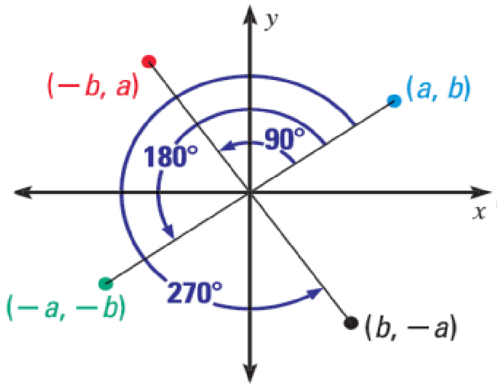




Learning Task

Learners investigate real-life examples of rotation and enlargement of plane shapes, such as triangles, rectangles, etc.

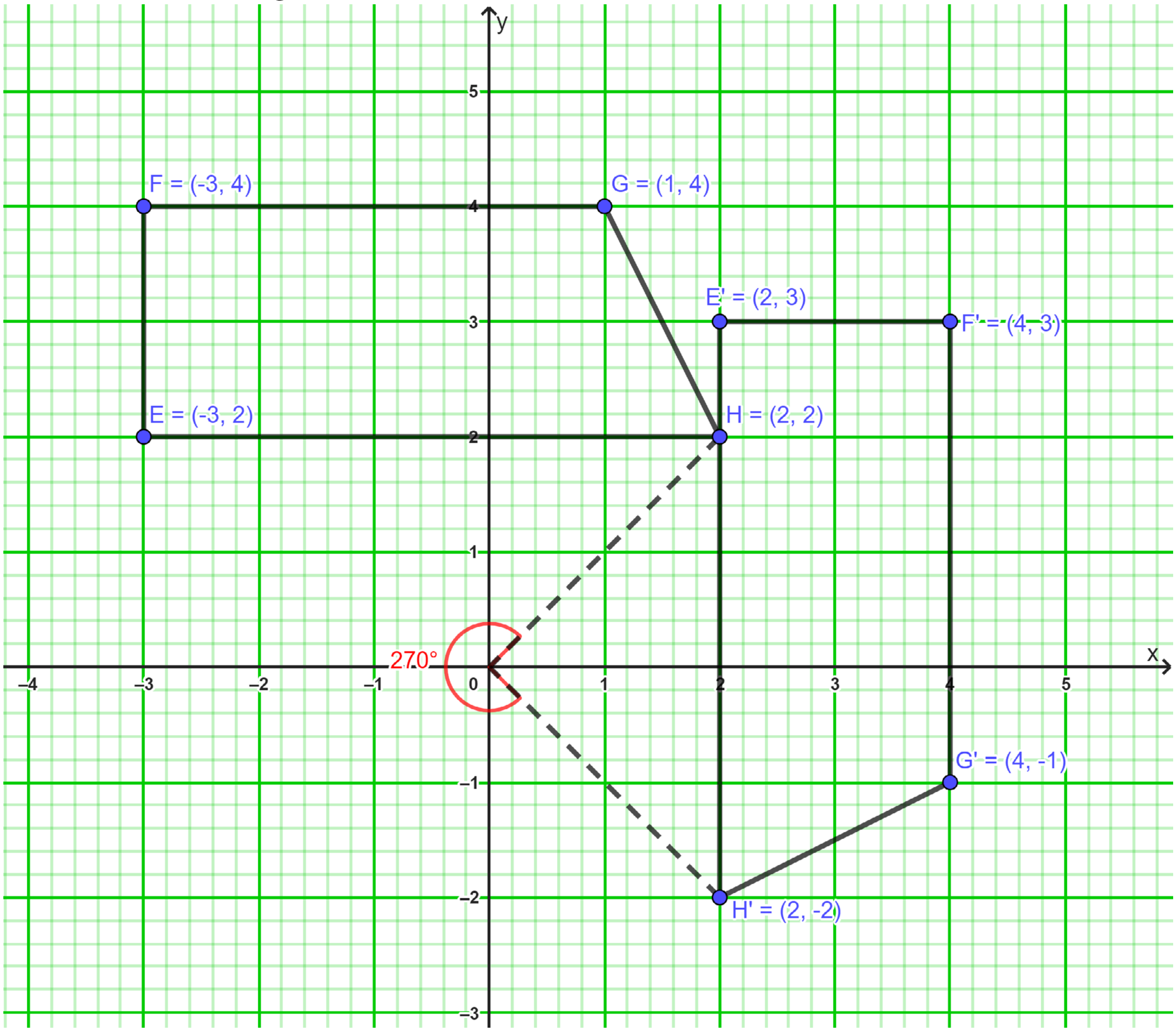
Coordinate Rules for Counterclockwise Rotations about the Origin



1. **90° Anticlockwise Rotation**
   - Rule: (x, y) → (−y, x)
   - Description: When a point is rotated 90° anticlockwise about the origin, the x-coordinate becomes the negative y-coordinate, and the y-coordinate becomes the x-coordinate.
2. **180° Rotation (Anticlockwise or Clockwise)**
   - Rule: (x, y) → (−x, −y)
   - Description: When a point is rotated 180° about the origin, both the x- and y-coordinates are negated.
3. **270° Anticlockwise Rotation (or 90° Clockwise Rotation)**
   - Rule: (x, y) → (y, −x)
   - Description: When a point is rotated 270° anticlockwise (or 90° clockwise) about the origin, the x-coordinate becomes the y-coordinate, and the y-coordinate becomes the negative x-coordinate.

Example 3.4

If E (–3, 2), F (–3, 4), G (1, 4), and H (2, 2). Find the image matrix for a 270° anticlockwise rotation about the origin.



$(a, b) \rightarrow (b, -a)$

$E(-3, 2) \rightarrow E'(2, 3)$

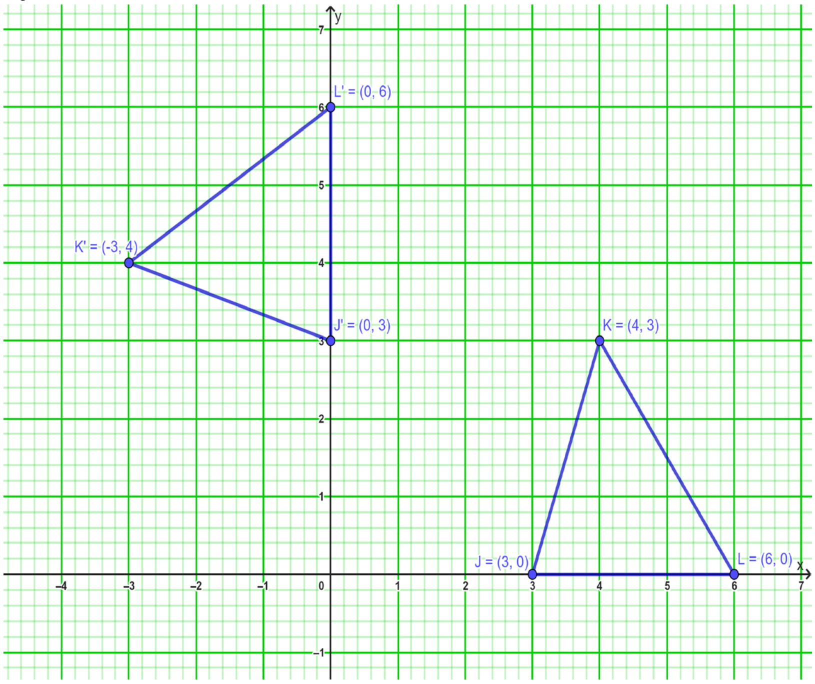
$F(-3, 4) \rightarrow F'(4, 3)$

$G(1, 4) \rightarrow G'(4, -1)$

$H(2, 2) \rightarrow H'(2, -2)$

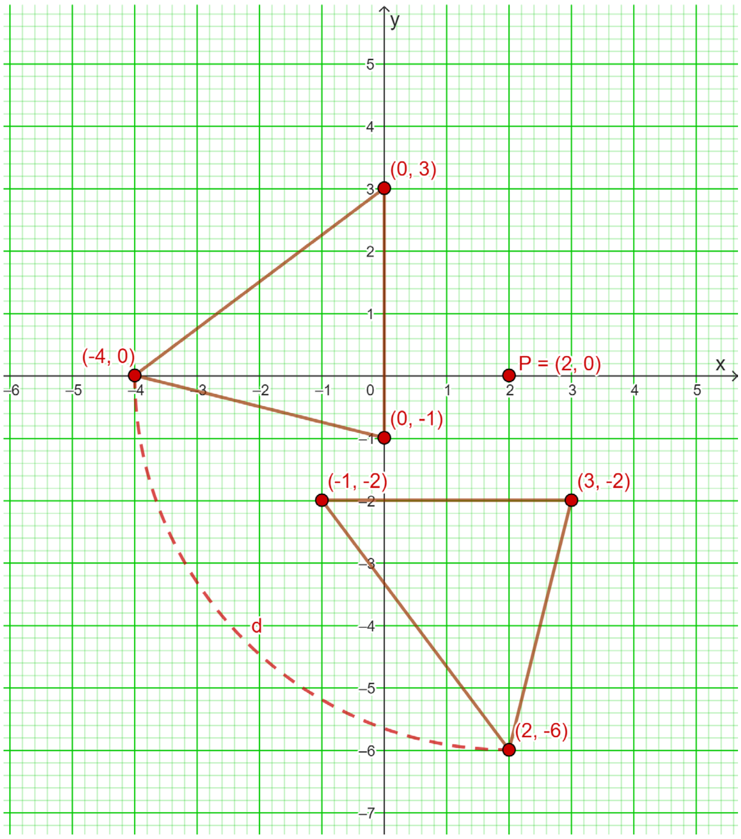
Example 3.5

Graph △JKL with vertices J(3, 0), K(4,3) L (6,0) and its image after a 90° rotation about the origin. Show your work!



Example 3.6

Graph △ABC with vertices A(0, 3), B(-4,0) and C (0, -1) and its image after a 270° clockwise rotation about the point P(2,0). Show your work!



Learning Tasks

Learners perform rotation of objects in a coordinate plane.

PEDAGOGICAL EXEMPLARS

1. Think-pair share activities: Learners in pairs research and make presentations on the concept of rotational symmetry in Mathematics including real-life examples.
2. Think-pair share activities: Learners in pairs research and make presentations on the rotation rules.
3. In pairs, learners are tasked to solve problems on rotation.

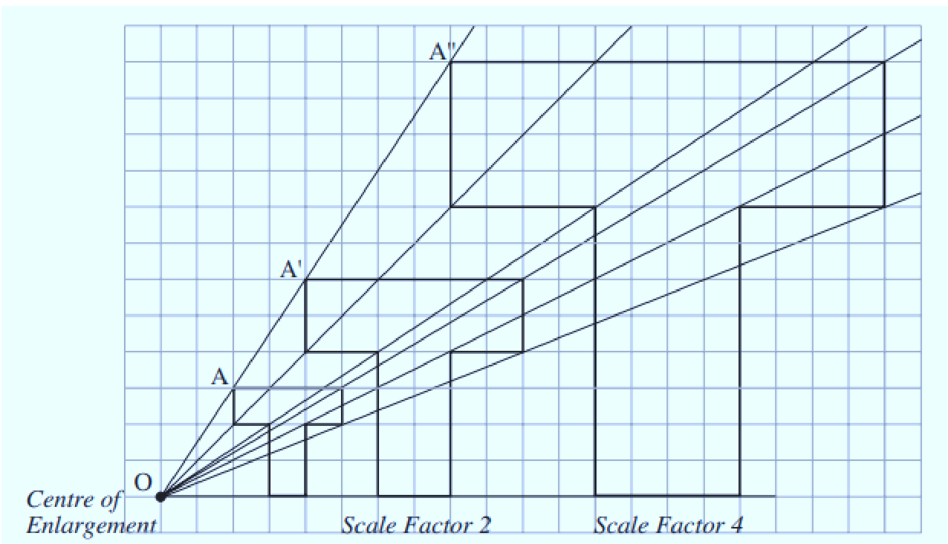
FOCAL AREA 2: ENLARGEMENT OF PLANE SHAPE

Enlargement, also known as dilation, is a transformation that alters the size of a plane shape while preserving its overall geometry. Unlike other rigid motions such as translation, rotation, and reflection, which maintain the size and shape of the figure, enlargement changes the size but keeps the shape similar.

In an enlargement, a plane shape is scaled by a specific factor relative to a fixed point called the centre of enlargement. This scale factor, often denoted as k, determines how much the shape is enlarged or reduced. If $k > 1$, the shape is enlarged; if $k < 1$, the shape is reduced in size. The transformation ensures that all points on the original shape (the pre-image) move radially outward or inward from the center of enlargement by a distance proportional to k.

The mathematical rules governing enlargement involve multiplying the coordinates of each point in the original shape by the scale factor k, resulting in a new shape (the image) that is similar to the original but proportionally larger or smaller.

Take a look at the diagram



The example shows how the original, A, was enlarged with scale factors 2 and 4. A line from the centre of enlargement passes through the corresponding vertex of each image.

Note

The distances, OA′ and OA″, are related to OA:

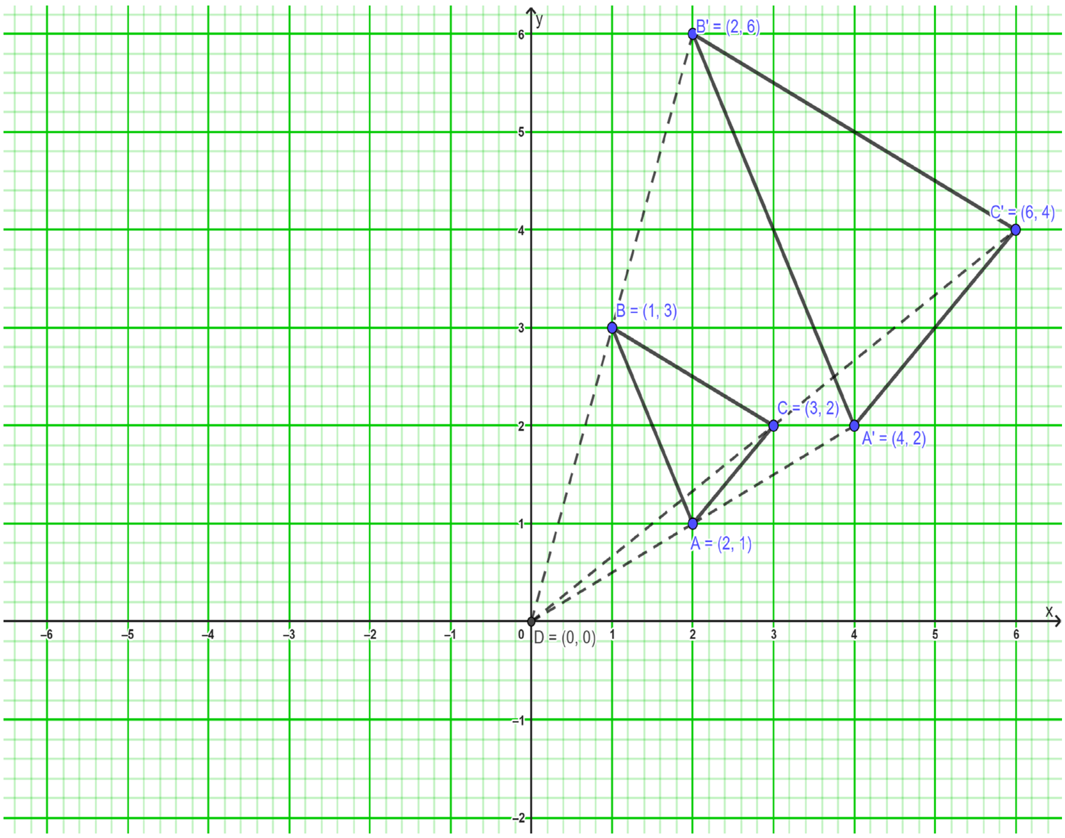
$OA' = 2 \times OA$

$OA'' = 4 \times OA$

The same is true of all the other distances between O and corresponding points on the images.

Example 3.7

Plot the coordinates A(2, 1) B(1, 3) C(3, 2) of $\triangle$ABC in a Coordinate Plane. Perform an enlargement of $\triangle$ABC using a scale factor of 2 and a centre of the enlargement at the origin to form $\triangle$A′B′C′. Compare the coordinates, side lengths, and angle measures of $\triangle$ABC and $\triangle$A′B′C′.



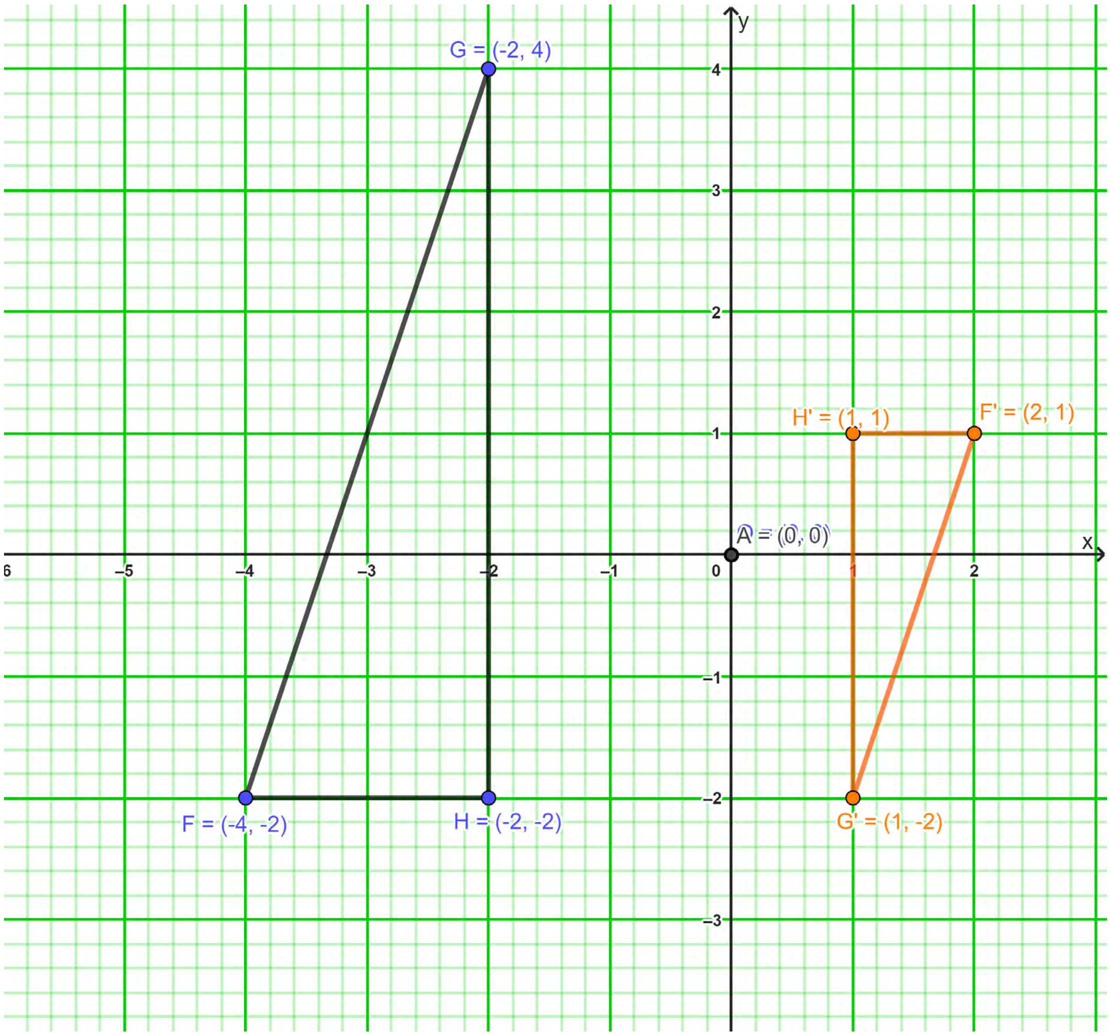
Sometimes, the scale factor is a negative number. When this happens, the figure rotates 180°. So, when $k > 0$, a dilation with a scale factor of $-k$ is the same as the composition of a dilation with a scale factor of k followed by a rotation of 180° about the centre of dilation. Using the coordinate rules for a dilation centred at (0, 0) and a rotation of 180°, you can think of the notation as $(x, y) \rightarrow (kx, ky) \rightarrow (-kx, -ky)$.

Example 3.8

Graph △FGH with vertices F(–4, –2), G(–2, 4), and H(–2, –2) and its image after a dilation centered at (0, 0) with a scale factor of $-\frac{1}{2}$.

Solution

Use the coordinate rule for a dilation with center (0, 0) and $k = -\frac{1}{2}$ to find the coordinates of the vertices of the image.



Then graph △FGH and its image.

$(x, y) \rightarrow (-\frac{1}{2}x, -\frac{1}{2}y)$

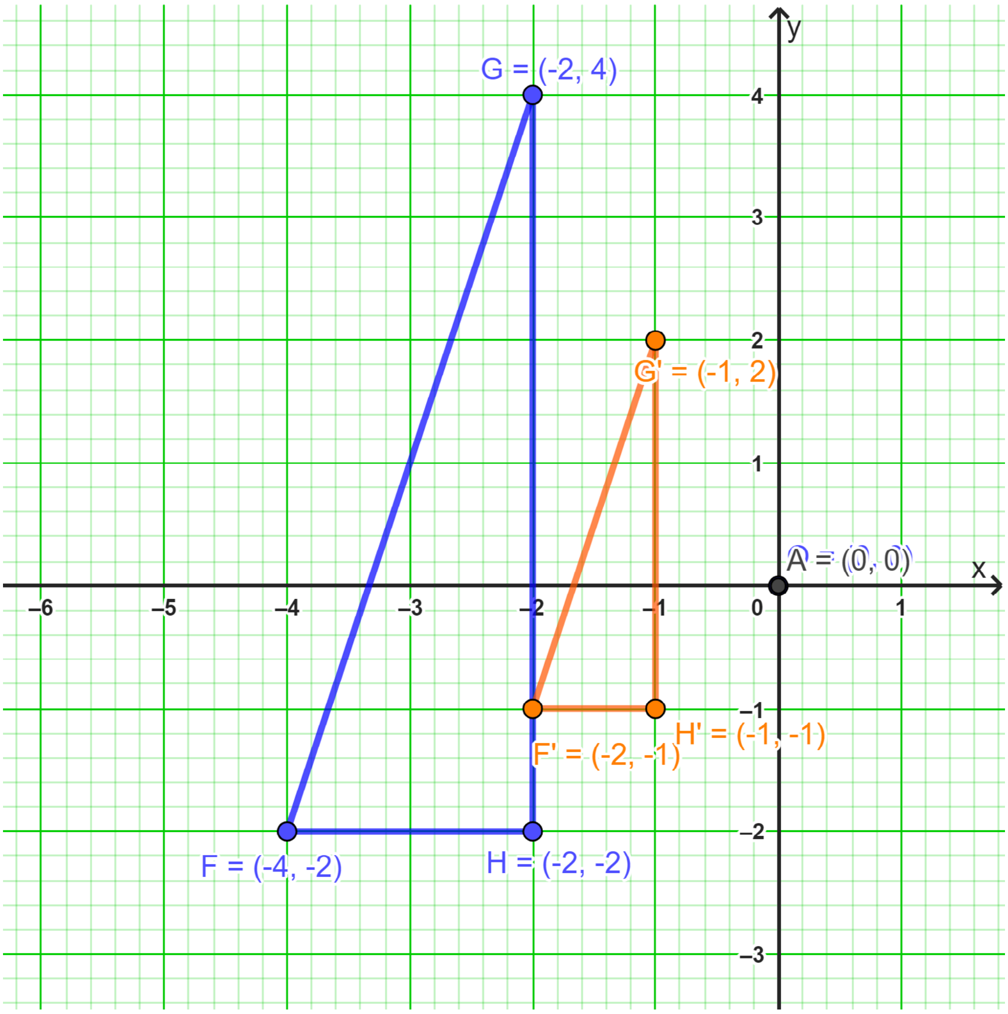
F(–4, –2) → F′(2, 1)

G(–2, 4) → G′(1, –2)

H(–2, –2) → H′(1, 1)

Example 3.9

Graph △FGH with vertices F(–4, –2), G(–2, 4), and H(–2, –2) and its image after a dilation centred at (0, 0) with a scale factor of $\frac{1}{2}$.



Then graph △FGH and its image.

$(x, y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$

F(–4, –2) → F′(-2, -1)

G(–2, 4) → G′(-1, 2)

H(–2, –2) → H′(-1,-1)

Looking at Example 2 and Example 3, we realise that the scale factor had different effects on the image of △FGH. The negative scale factor is affected differently from the positive scale factor.

Application

Example 3.10

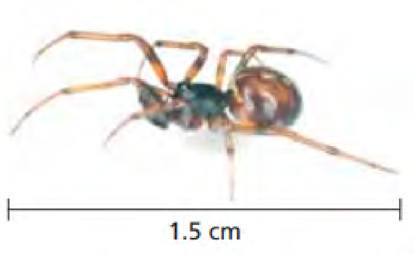
You are using a magnifying glass that shows the image of an object that is six times the object's actual size. Determine the length of the image of the spider seen through the magnifying glass.

Solution

$$\frac{\textit{image length}}{\textit{actual length}} = k$$

$x\ \_\ 1.5 = 6$

$x = 9$



So, the image length through the magnifying glass is 9 centimeters.

Learning Tasks

1. Learners perform enlargement/dilation of objects in a coordinate plane.
2. Learners create real-life problems on enlargement/dilation.

PEDAGOGICAL EXEMPLARS

1. Think-pair share activities: Learners in pairs research and make presentations on the concept of enlargement/dilation in transformations.
2. Think-pair share activities: learners in pairs discuss the properties of enlargement/ dilation.
3. In mixed-groups, learners discuss the effect of positive and negative scale factor on an image in enlargement.
4. In mixed-groups, learners solve problems, including real-life problems, on enlargement/dilation.

KEY ASSESSMENT

1. Graph the line RS with points R(1, –3) and S(2, –6). Rotate the line through180° about the origin. Then reflect RS in the y-axis.
   - **i.** Rotation: (a, b) → (-a, -b) R(1, –3) → R′(–1, 3) S(2, –6) → S′(–2, 6)
   - **ii.** Reflection: (a, b) → (-a, b) R′(–1, 3) → R″(1, 3) S′(–2, 6) → S″(2, 6)
2. Graph ΔABC with vertices A(–4, 4), B(–1, 7), and C(-1,4) and its image after a 270^0 rotation about the origin. Show your work!

3. Graph $\triangle PQR$ and its image after a dilation centered at C with scale factor k.

i. P(–2, –1), Q(–1, 0), R(0, –1); C(0, 0), k = 4

ii. P(5, –5), Q(10, –5), R(10, 5); C(0, 0), k = 0.4

4. The image of a spider seen through the magnifying glass is 4 times its actual length as shown below. Find the actual length of the spider.

Reminder:

Learners' scores on the mid-semester examination should be ready for onward submission to the STP.

Section 3 Review

This session looked at concepts on rigid motion: translation and reflection rotation of images and enlargement of plane shape. We learnt that:

1. Translation: Moving an object without rotating or changing its size.

2. Reflection: Flipping an object over a line to create a mirror image.

3. Rotation of images: Turning an object around a fixed point.

4. Enlargement/reduction: Changing the size of a shape while maintaining its shape and proportions.

ADDITIONAL READING

You may visit these sites for additional reading.

• Rigid Motion Transformations & Examples | What is Rigid Motion? - Lesson | Study.com	
• Introduction to Rigid Motions | CK-12 Foundation (ck12.org)	

SECTION 4 DATA COLLECTION, ORGANISATION AND REPRESENTATION

Strand: Making Sense of and Using Data

Sub-Strand: Statistical Reasoning and its Application in Real Life

Learning Outcomes:

1. *Design a data collection instrument and justify its appropriateness to collecting everyday life data to address a contextual issue.*
2. *Construct and interpret a variety of data presentation methods including cumulative frequency curves (Ogive), waffle diagrams, etc. and describe the relationship between the measures of dispersion in data displays to solve and/or pose problems.*

Content Standards:

1. *Demonstrate conceptual understanding of data handling in relation to designing and validating a variety of data collection methods.*
2. *Demonstrate an understanding data presentations and analysis for grouped and ungrouped data and describe the relationship between the measures of dispersion in data displays.*

INTRODUCTION AND SECTION SUMMARY

Understanding data is crucial in today's information-driven world. Data collection instruments, such as surveys, interviews, and observations, are used to gather various types of data, including numerical, categorical and ordinal. Once collected, data must be organised and represented visually through graphs, charts and tables to facilitate understanding and interpretation. Data organisation and representation are essential for analysing patterns and trends. Measures of dispersion, like range, standard deviation and variance, help explain the "spread" of the data. These measures are useful for making comparisons and drawing conclusions. Analysis and interpretation of data enable students to make informed decisions and solve real-world problems. Engaging in real-life projects involving data collection, analysis and presentation provides hands-on experience and demonstrates practical applications of data concepts across various subjects. Proficiency in these areas prepares students for further studies in fields such as statistics, economics and data science. The weeks covered by the section are:

Week 9: Data collection instruments

1. *Designing a questionnaire*
2. *Designing an interview guide*
3. *Building an observation guide*
4. *Applying digital technology in data collection*

Week 10: Data organization and representation

1. *Pictorial representation of data (ogive, waffle, box and whisker plots, etc)*
2. *Model and solve real life problems involving data*

Week 11: Analysis and interpretations of data

1. *Analyse and interpret data using measures of dispersion*
2. *Testing suitability of the measures for a given data*

SUMMARY OF PEDAGOGICAL EXEMPLARS

This section emphasises the importance of hands-on activities, visual representations, real-life examples, and data analysis techniques to enhance understanding and application of concepts related to data collection, pictorial representation, real-life problem solving, and data analysis using measures of dispersion.

1. **Hands-on Data Collection:** Engage students in collecting real data using various instruments such as surveys, interviews and observations. This hands-on approach helps them understand the practical aspects of data collection and the types of data they might encounter.
2. **Pictorial Representation:** Utilise various types of graphs and charts, including ogives, waffle charts, box and whisker plots, etc., to visually represent data. This helps students see patterns and trends in data more easily and understand the data distribution.
3. **Real-life Problem Solving:** Present students with real-world problems that require them to collect, analyse and interpret data.

ASSESSMENT SUMMARY

The concepts covered in these topics require learners to demonstrate a comprehensive understanding of different types of data, data collection methods, data organisation, presentation, measures of dispersion, and real-life applications. Assessments should largely target levels 2, 3 & 4 of the Depth of Knowledge (DOK) framework to ensure learners grasp the concepts deeply and can apply them effectively. A variety of assessment strategies should be used, including:

1. Quizzes and Tests: Assess understanding of types of data, data collection methods and measures of dispersion.
2. Practical Assessments: Involve real-life projects on data collection, analysis and presentation, allowing learners to apply their knowledge in authentic scenarios.
3. Homework Assignments: Reinforce learning and assess understanding of data organisation, presentation techniques, and measures of dispersion.
4. Group Activities: Engage learners in collaborative tasks where they work together to analyse and interpret data, fostering collaboration and critical thinking skills.
5. Project Works & Presentations: Require learners to communicate their findings from data analysis, demonstrating their ability to present data effectively.

WEEK 9: DATA COLLECTION INSTRUMENTS

Learning Indicators

1. *Design a data collection instrument (questionnaire, interview guide, checklist, observation guide, etc.) by employing feasible digital technology (where available) and use it to collect real-life data.*
2. *Evaluate a given set of data and/or its instrument by identifying potential problems related to bias, use of language, gender, ethics, cost, time, privacy, cultural sensitivity, etc.*

FOCAL AREA: DATA COLLECTION INSTRUMENTS

Data collection instruments are tools used to collect data from various sources, ensuring that the information obtained is valid and useful for analysis. These instruments are essential for conducting effective research projects, experiments and surveys.

In SHS one, we learnt that data collection instruments can be broadly categorized into two types: quantitative and qualitative. Quantitative instruments are used to collect numerical data, which can be measured and analysed statistically. Examples include surveys and questionnaires. On the other hand, qualitative instruments are used to gather non-numerical data, providing deeper insights into people's thoughts, behaviours, and experiences. Examples include interviews, focus groups and observations.

In this session, we will learn about how to develop these data collections tools in order to collect good data from people.

Designing and validating a questionnaire

Steps to Develop a Simple Questionnaire and validate it to Collect Data

1. Define the Purpose

- Identify the Objective: Clearly state the purpose of the questionnaire. What information do you want to gather? For example, you might want to find out about students' favourite subjects, extracurricular activities, or opinions on school facilities.
- Target Audience: Decide who will be answering the questionnaire. In this case, it would be the students at your school.

2. Develop the Questions

- Types of Questions: Decide on the types of questions to include. There are two main types:
    - o Closed-Ended Questions: These provide specific options for answers, such as multiple choice, yes/no, or rating scales. They are easy to analyse.

Example: “What is your favourite subject? (a) Math (b) Science (c) English (d) History”

  - o Open-Ended Questions: These allow respondents to answer in their own words, providing more detailed information. Example: “What do you like most about your favourite subject?”

- Keep it Simple: Use clear and straightforward language. Avoid complex or ambiguous questions.

3. Organize the Questionnaire

- Introduction: Write a brief introduction explaining the purpose of the questionnaire and assuring respondents that their answers will be confidential.
- Question Order: Start with easy and engaging questions to encourage participation. Group similar questions together and place more sensitive or difficult questions towards the end.
- Length: Keep the questionnaire short and to the point. Aim for around 10-15 questions to ensure it is manageable and doesn’t take too much time to complete.

4. Test the Questionnaire

- Pilot Test: Conduct a pilot test with a small group of students to see if they understand the questions and can complete the questionnaire without difficulty.
- Feedback: Ask for feedback on the clarity and relevance of the questions. Make any necessary adjustments based on this feedback.

5. Distribute the Questionnaire

- Method: Decide how you will distribute the questionnaire. You can do this in person, via email, or through social media platforms.
- Instructions: Provide clear instructions on how to complete and return the questionnaire.

A sample Questionnaire

Introduction:

We are conducting a survey to find out more about our classmates’ favourite subjects and activities. Your answers will help us understand what students enjoy and how we can improve our school experience. Please answer the following questions honestly. Thank you!”

Questions: Please tick [√]

1. What grade are you in?

(a) 6

(b) 7

(c) 8

2. What is your favourite subject?
   (a) Math
   (b) Science
   (c) English
   (d) History
   (e) Literature
   (f) Social Studies
   (g) Government
   (h) French
   (i) Agriculture
   (j) Business
   (k) General Arts
   (l) Physics
   (l) Chemistry
   (m) Biology
   (n) Other (please specify): ________________

3. How much do you enjoy your favourite subject?
   (a) A lot
   (b) Somewhat

4. What extracurricular activities do you participate in? (Check all that apply)
   (a) Sports
   (b) Music
   (c) Art
   (d) Drama
   (e) Other (please specify): ________________

5. What do you like most about your favourite subject?

Designing an interview guide

Steps to Develop a Simple Interview Guide

1. Define the Purpose

- Identify the Objective: Clearly state the purpose of the interview. What specific information do you want to gather? For instance, you might want to learn about students' opinions on school events, their study habits, or their thoughts on new school rules.
- Target Audience: Determine who will be interviewed. In this case, it would be the students at your school.

2. Develop the Interview Questions

- Types of Questions: Create a mix of question types to gather comprehensive information.
  - o Open-Ended Questions: Encourage detailed responses and provide deeper insights. Example: "Can you describe your favourite school event and why you like it?"
  - o Probing Questions: Follow up on initial responses to gain more detail. Example: "Can you explain more about what makes that event special for you?"
- Clarity and Simplicity: Use clear and simple language. Avoid jargon and ensure questions are easy to understand.

3. Organize the Interview Guide

- Introduction: Write a brief introduction for the interview, explaining its purpose and reassuring the interviewee about the confidentiality of their responses.
- Question Order: Start with easy and general questions to make the interviewee comfortable, then move to more specific or sensitive topics.
- Length: Keep the interview concise to maintain the interviewee's engagement. Aim for around 8-12 questions.

4. Prepare for the Interview

- Practice: Conduct a mock interview with a classmate or teacher to practice your questions and improve your interviewing skills.
- Adjust: Make any necessary adjustments to the questions based on feedback from the practice session.

Example of a Simple Interview Guide

Introduction:

Thank you for agreeing to participate in this interview. We are trying to understand students' opinions on various aspects of school life to help improve our school experience. Your responses will be kept confidential. Let's start with some basic questions."

Questions:

1. Icebreaker: Can you tell me a little bit about yourself? (e.g., grade, favourite subject)
2. General Opinion: What do you enjoy most about school?
3. Specific Event: Can you describe your favourite school event and explain why you like it?
4. Study Habits: How do you usually prepare for exams or complete your homework?
5. School Policies: What are your thoughts on the new homework policy implemented this year?
6. Improvement: Is there anything you would like to change about our school? Why?
7. Follow-Up: You mentioned [specific detail from their answer]; can you tell me more about that?

Designing an observation guide

Steps to Develop a Simple Observation Guide

1. Define the Purpose
    - Identify the Objective: Clearly state the purpose of the observation. What specific behaviours or events are you interested in observing? For instance, you might want to observe classroom participation, playground interactions, or the use of school facilities.
    - Target Setting: Determine where the observation will take place. For example, in a classroom, during a sports activity, or in the school canteen.
2. Develop the Observation Criteria
    - Key Behaviours or Events: List the specific behaviours or events you want to observe. These should be clearly defined and relevant to your objective.
        - o Example: If observing classroom participation, look for behaviours such as raising hands, group discussions, or attentiveness.
    - Categories: Group similar behaviours or events into categories to make the observation more organized.
        - o Example: Categories for classroom participation might include "Active Participation," "Passive Participation," and "Distractions."

3. Create the Observation Guide

- Observation Checklist: Develop a checklist or rating scale to record the occurrence and frequency of the behaviours or events you are observing.
  - o Example Checklist:
    - Active Participation:
      - Raises hand
      - Engages in group discussion
    - Passive Participation:
      - Listens attentively
      - Takes notes
    - Distractions:
      - Talks to classmates
      - Uses phone
- Observation Schedule: Decide on the times and duration of your observations. This ensures consistency and helps in comparing data from different sessions.

Example of a Simple Observation Guide

Objective: To observe classroom participation among students during a mathematics lesson.

Observation Criteria:

1. Active Participation:
- o Raises hand to answer questions
- o Engages in group discussions
- o Volunteers for classroom activities

2. Passive Participation:
- o Listens attentively to the teacher
- o Takes notes during the lesson

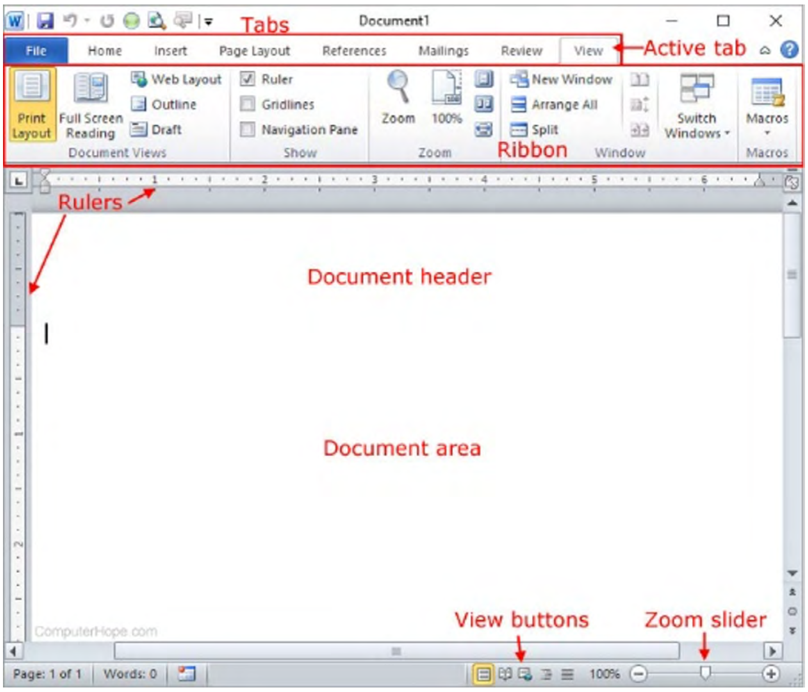
3. Distractions:
- o Talks to classmates during the lesson
- o Uses phone or other electronic devices

Observation Checklist:

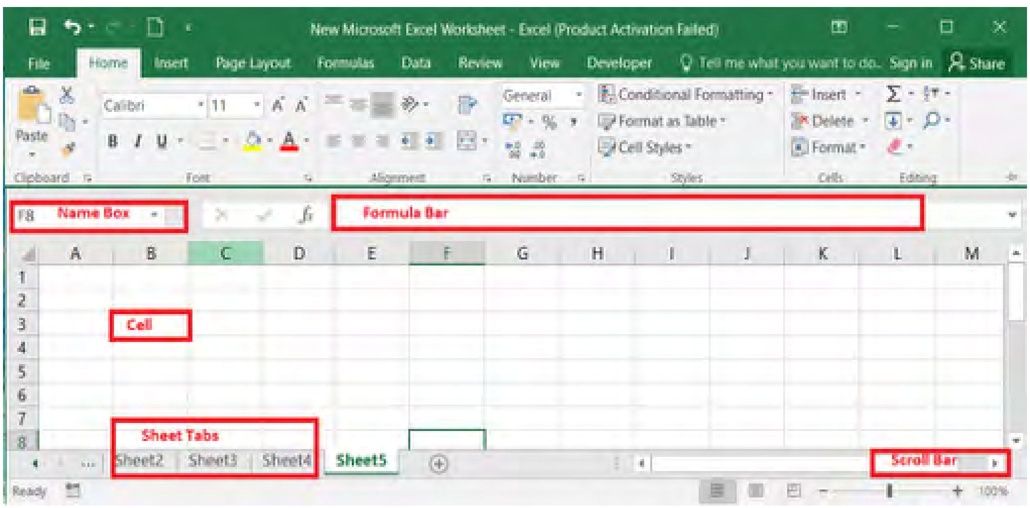
Student Name	Raises Hand	Group Discussion	Volunteers	Listens Attentively	Takes Notes	Talks to Classmates	Uses Phone
John Efa	Yes	No	Yes	Yes	Yes	No	No
Jane Acquaah	No	Yes	No	Yes	Yes	Yes	No

Applying digital technology in data collection

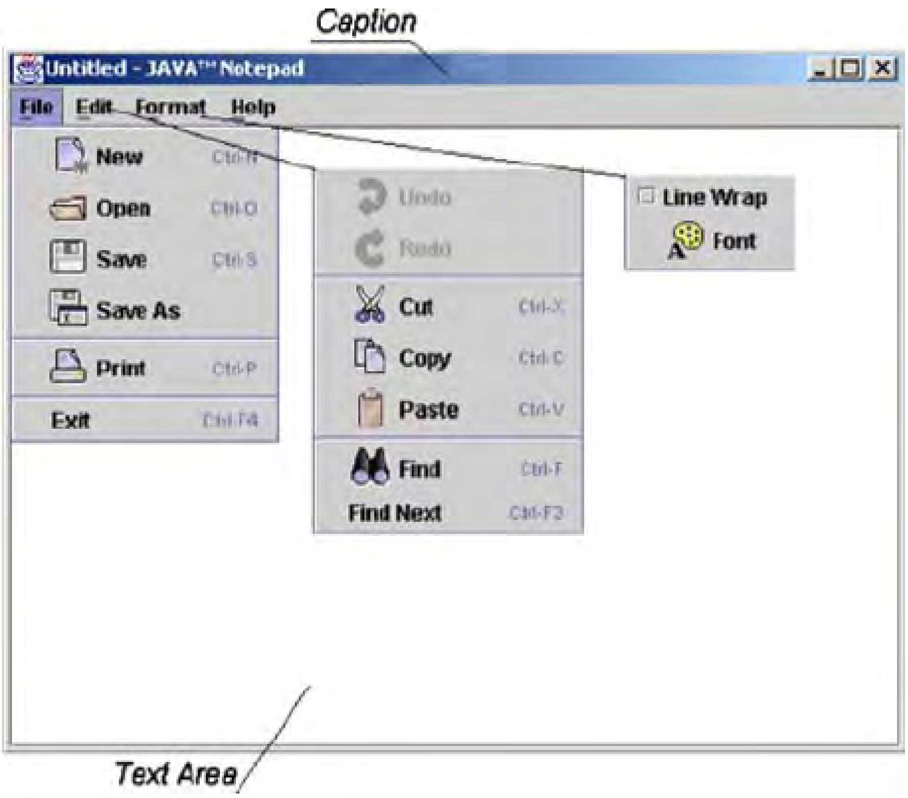
In designing these tools, computer application software such as Microsoft Word, Excel, Notepad, etc. These tools help to make the creating of these tools easier and faster.



Microsoft Word



Microsoft Excel



Notepad Application

Note

Please link up with the ICT tutor to require support from her/him for the learners.

Topic Area that Learners Can Design Data Collection Tools On

Here are some engaging and relevant research topics for students:

Science and Technology

1. Climate Change: The impact of human activities on global warming and climate change.
2. Renewable Energy: Exploring the effectiveness and feasibility of renewable energy sources.
3. Artificial Intelligence: The role of AI in modern society and its potential future applications.
4. Space Exploration: The benefits and challenges of space missions and colonization.
5. Plastic Pollution: Effects of plastic waste on marine life and solutions to reduce it.

Health and Medicine

1. Mental Health: The impact of social media on teenage mental health.
2. Nutrition: The effects of diet on overall health and academic performance.
3. Vaccines: The role of vaccines in preventing diseases and the public perception of vaccination.
4. Addiction: Causes and effects of addiction to substances or behaviors (e.g., technology addiction).
5. Pandemics: Lessons learned from historical pandemics and their influence on current healthcare practices.

Social Sciences

1. Education Systems: Comparing different educational systems around the world and their effectiveness.
2. Bullying: Causes, effects, and prevention strategies for bullying in schools.
3. Cultural Diversity: The importance of cultural diversity in fostering inclusive communities.
4. Gender Equality: Progress and challenges in achieving gender equality in various fields.
5. Immigration: The social and economic impacts of immigration on host countries.

Environmental Studies

1. Sustainable Agriculture: Practices that promote sustainable farming and reduce environmental impact.
2. Deforestation: Causes, consequences, and solutions to deforestation.
3. Water Conservation: The importance of conserving water resources and effective methods to do so.
4. Wildlife Conservation: Strategies to protect endangered species and their habitats.
5. Urbanization: The environmental and social impacts of urbanization.

Economics and Business

1. Globalization: The effects of globalization on local economies and cultures.
2. Entrepreneurship: The challenges and opportunities of starting a small business.
3. Consumer Behavior: Factors influencing consumer choices and trends.
4. Labour Market: The impact of automation and AI on the future of work.

History and Politics

1. World War II: Key events and their lasting impact on global politics and society.

2. Democracy vs. Authoritarianism: Comparing different forms of government and their effectiveness.
3. Colonialism: The effects of colonialism on modern-day countries.
4. Human Rights: Current human rights issues and efforts to address them.

Arts and Literature

1. Literary Analysis: In-depth analysis of a classic novel or play and its relevance today.
2. Art Movements: Exploring the influence of a particular art movement on contemporary art.
3. Film Studies: The impact of cinema on culture and society.
4. Music Therapy: The benefits of music therapy for mental health and well-being.
5. Cultural Heritage: The importance of preserving cultural heritage and traditions.

Technology and Society

1. Social Media: The influence of social media on communication and relationships.
2. Cybersecurity: The importance of cybersecurity in protecting personal and organizational data.
3. E-commerce: The rise of online shopping and its impact on traditional retail.
4. Digital Divide: The gap between those who have access to technology and those who do not.
5. Smart Cities: The development and benefits of smart cities for urban living.

Learners Tasks

Learners should design simple questionnaires, interview guides and observation guides using free hand or available computer applications.

PEDAGOGICAL EXEMPLARS

1. Experiential learning: In collaborative and mixed-gender/ability groups, engage learners to explore and design various data collection instruments around contemporary issues that interest them.
2. Project/Collaborative learning: Engage learners to use the designed instrument to collect the appropriate data and present the report to the class including the procedure adopted in collecting the data.

Note

Learners should be encouraged to use feasible technology to collect the data and talk about the appropriate ways of using the IT tools.

KEY ASSESSMENT

1. Explain briefly how each of the following data collection tools can be designed;

a. Questionnaire

b. Interview guide

c. Observation guide

2. Design a simple questionnaire/interview/observation guide on a topic of interest and pilot it using four respondents.

3. Review the tool and state any three of the things you did in the review.

WEEK 10: DATA ORGANISATION AND REPRESENTATION

Learning Indicator: *Organise and present data (grouped/ungrouped) by means of ogive, waffle diagrams, box and whisker plots, etc., including generating 3D graphs and solve and/or pose problems.*

FOCAL AREA: DATA ORGANIZATION AND REPRESENTATION

Data organization and representation involves the systematic arrangement and visualization of data to make it easier to understand, analyse, and interpret. Effective data organization ensures that information is structured logically and coherently, while data representation uses various methods, such as charts, graphs, tables and diagrams, to present this information visually.

Common methods of data representation include bar graphs, pie charts, histograms, line graphs, and scatter plots. Each of these methods has its unique advantages and is suitable for different types of data and purposes. For example, bar graphs are ideal for comparing discrete categories, while line graphs are useful for showing trends over time.

Pictorial representation of data (ogive, waffle, box and whisker plots, etc.)

Cumulative frequency curve [Ogive]

A cumulative frequency curve, also known as an ogive, is a graphical representation used in statistics to show the cumulative frequency distribution of a dataset. It provides a visual way to see how the cumulative totals of a dataset accumulate over time or intervals, making it easier to understand the overall distribution and identify key statistical measures such as the median, quartiles, and percentiles.

To construct an ogive, we begin with a frequency distribution table that includes class intervals and their corresponding frequencies. The cumulative frequency is calculated by successively adding each class's frequency to the sum of the previous frequencies. This cumulative frequency is then plotted against the upper class boundary of each interval. By connecting these points with a smooth curve, we obtain the ogive.

To create an ogive:

1. Calculate the cumulative frequency for each class interval.
2. Plot the cumulative frequency against the upper boundary of each class interval.
3. Connect the points to form a smooth curve.

Example 4.1

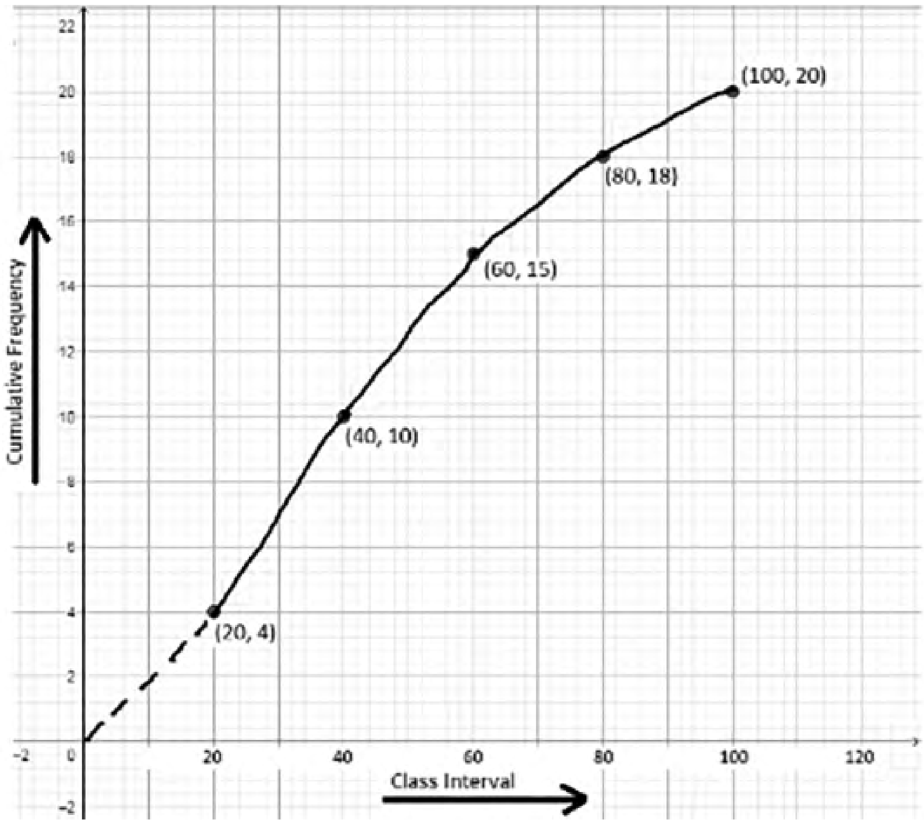
Draw an ogive for the following distribution.

Class Interval	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
Frequency	4	6	5	3	2

Solution

Class Interval	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
Frequency	4	6	5	3	2
Cumulative Frequency	4	10 (4 +6)	15 (4 + 6 + 5)	18 (4 + 6 + 5 + 3)	20 (4 + 6 + 5 + 3 + 2)

Now, plot the points (20, 4), (40, 10), (60, 15), (80, 18) and (100, 20) following steps 3 and 4, and join the points following steps 5 and 6. We get the following ogive.



Scale: On the x-axis, 2 cm : 20 units.

On the y-axis, 1 cm : 2 units

Example 4.2

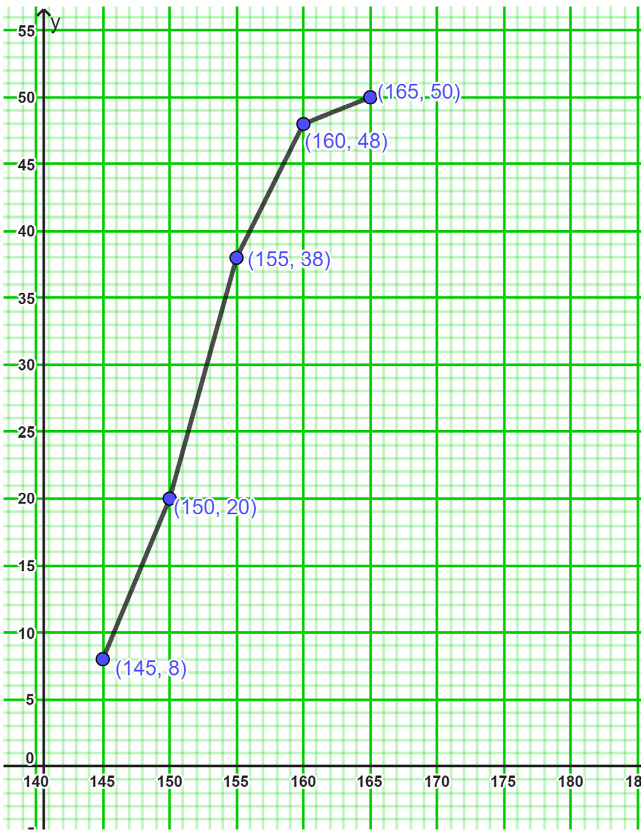
Draw a cumulative frequency curve for the data on the heights of 50 students:

Height (cm)	Frequency
140 - 145	8
145 - 150	12
150 - 155	18
155 - 160	10
160 - 165	2

Solution

1. Calculate the cumulative frequency for each class interval.

Height (cm)	Frequency	Cumulative freq.
140 - 145	8	8
145 - 150	12	20
150 - 155	18	38
155 - 160	10	48
160 - 165	2	50



Types of Ogives

There are two main types of ogives, known as the “less than” ogive and the “greater than” ogive. These two types of ogives provide different perspectives on the cumulative frequency distribution of a dataset.

Less Than Ogive

The “less than” ogive is a cumulative frequency curve that shows the cumulative frequency of data points that are less than or equal to a certain value. It is constructed by plotting the cumulative frequency against the upper class boundary of each class interval.

Steps to Construct a Less Than Ogive:

1. Calculate Cumulative Frequency: Determine the cumulative frequency for each class interval by successively adding the frequencies.

2. Identify Upper Class Boundaries: Use the upper boundary of each class interval for plotting.

3. Plot Points: Plot the cumulative frequency values against the upper class boundaries on a graph.

4. Draw Curve: Connect the plotted points with a smooth curve.

Greater Than Ogive

The “greater than” ogive, on the other hand, shows the cumulative frequency of data points that are greater than or equal to a certain value. It is constructed by plotting the cumulative frequency against the lower class boundary of each class interval.

Steps to Construct a Greater Than Ogive:

1. Calculate Cumulative Frequency: Determine the cumulative frequency for each class interval by successively adding the frequencies, starting from the highest class interval.

2. Identify Lower Class Boundaries: Use the lower boundary of each class interval for plotting.

3. Plot Points: Plot the cumulative frequency values against the lower class boundaries on a graph.

4. Draw Curve: Connect the plotted points with a smooth curve.

Example 4.3

Graph the two ogives for the following frequency distribution of the weekly wages of the given number of workers at Serene Hotel. Hence, find the median.

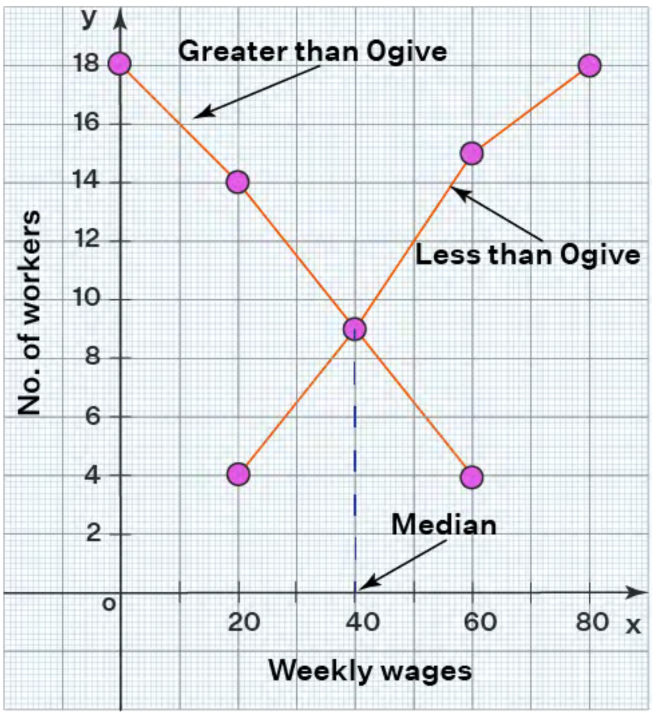
Weekly wages	No. of workers
0-20	4
20-40	5
40-60	6
60-80	3

Solution

Weekly wages	No. of workers	C.F. (Less than)	C.F. (More than)
0-20	4	4	18 (total)
20-40	5	9 (4 + 5)	14 (18 - 4)
40-60	6	15 (9 + 6)	9 (14 - 5)
60-80	3	18 (15 + 3)	3 (9 - 6)

For plotting less than type curve, points (20,4), (40,9), (60,15), and (80,18) are plotted on the graph, and these are joined by freehand to obtain the less than ogive. For plotting greater than type curve, points (0,18), (20,14), (40,9), and (60,3) are plotted on the graph, and these are joined by freehand to obtain the greater than type ogive.

The less than and greater than ogives shown in the graph below.



The median: A perpendicular line on the x-axis is drawn from the point of intersection of these curves. This perpendicular line meets the x-axis at a certain point. This determines the median. Here, the median is 40.

Learning tasks

Learners plot cumulative frequency graphs including using computer application tools to plot the graphs.

Waffle Charts

Waffle charts are a visually appealing and effective way to represent data proportions and percentages. They consist of a grid of small squares or "waffles," each representing a specific portion of the whole, typically 1% or another fixed unit of measure. This makes waffle charts particularly useful for comparing parts of a whole in a clear and concise manner.

Waffle charts are an excellent alternative to pie charts and bar graphs when you want to emphasize the composition of different categories within a dataset. They are easy to read and understand, making them a popular choice for presentations and reports where visual clarity is crucial.

Key Features of Waffle Charts:

1. Grid Structure: A typical waffle chart is composed of a 10×10 grid, totaling 100 squares. Each square represents 1% of the total value, making it easy to visualize proportions at a glance.
2. Color Coding: Different colors are used to represent different categories or segments within the data. This color coding helps in quickly distinguishing between various parts of the dataset.
3. Simplicity and Clarity: The straightforward grid layout makes waffle charts easy to interpret, even for those who may not be familiar with more complex data visualization techniques.

Steps to Create a Waffle Chart:

1. Data Collection: Gather the data you want to represent.
2. Determine Proportions: Convert the data into percentages if not already done.
3. Create Grid: Design a 10×10 grid for 100 squares, or adjust the grid size based on your data.
4. Color Squares: Fill the squares according to the data proportions using distinct colors for each category.
5. Label and Interpret: Add labels and a legend to help interpret the chart.

Why Use Waffle Charts?

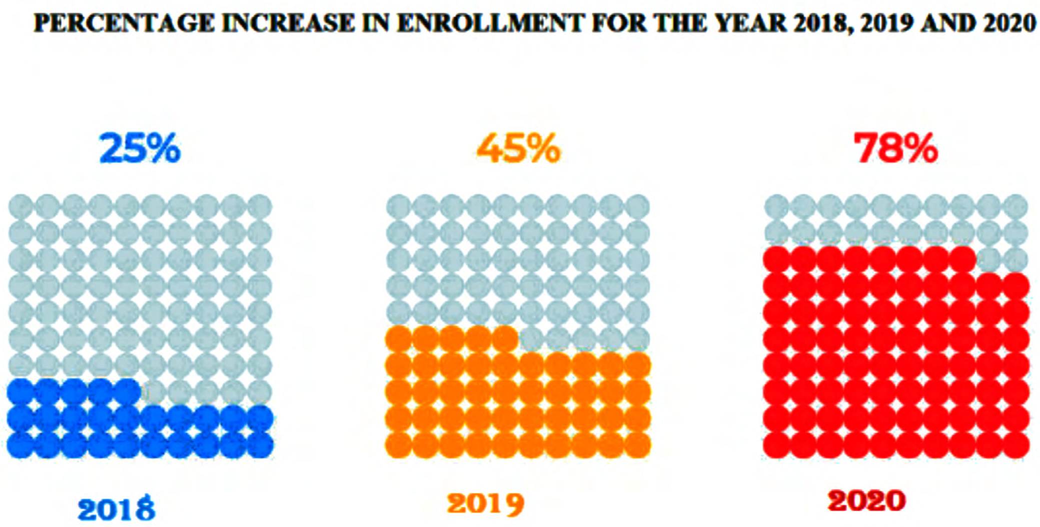
- Effective Comparison: They allow for easy comparison of different parts of a whole.
- Visual Appeal: The grid structure and color coding make them visually engaging.
- Simplifies Complex Data: Breaks down complex data into simple visual chunks.

Example 4.4

The data below shows the percentage increase in enrollment in a Senior High School in Salaga over a period of three years. Represent the data using a waffle graph.

Year	Enrollment Increase(%)
2018	**25**
2019	**45**
2020	**78**

Solution

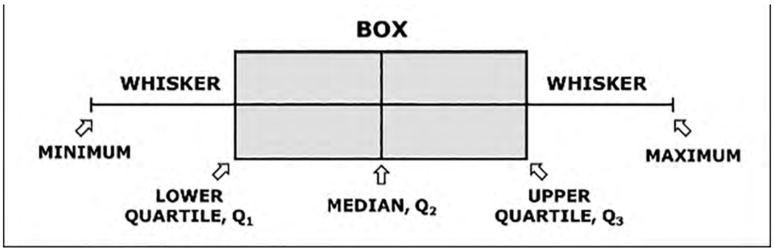


Learning tasks

Learners discuss and make a presentation on waffle charts. Task learners to solve problems on waffle charts.

Box Plots

A box and whisker plot (or box plot) is a graph that displays the data distribution by using five numbers. Those five numbers are the minimum, first (lower) quartile, median, third (upper) quartile and maximum.



In a box and whisker plot:

- The left and right sides of the box are the lower and upper quartiles. The box covers the interquartile interval, where 50% of the data is found.
- The vertical line that splits the box in two is the median. Sometimes, the mean is also indicated by a dot or a cross on the box plot.
- The whiskers are the two lines outside the box, which go from the minimum to the lower quartile (the start of the box) and then from the upper quartile (the end of the box) to the maximum.

Example 4.5

Dziifa threw the dice 20 times and got these results: 6 3 3 6 3 5 6 1 4 6 3 5 5 2 2 2 2 3 2 3

Draw a box plot to represent the data.

Solution

The first thing we need to do is to order the data from smallest to largest: 1 2 2 2 2 2 3 3 3 3 3 3 4 5 5 5 6 6 6 6

Furthermore, we need to calculate the median. Since the number of data points is even, we have.

$$Me = \frac{x_{10+x_{11}}}{2} = 3\frac{+3}{2} = 3$$

After that, we have to calculate the quartiles, thus the lower and upper quartiles.

Find the lower quartile (Q1):

- The lower half of the data set (first 10 values): 1 2 2 2 2 2 3 3 3 3
- The median of this half is the average of the 5th and 6th values: $Q_1 = \frac{2+2}{2} = 2$,

Find the upper quartile (Q3):

- The upper half of the data set (last 10 values): 3 3 4 5 5 5 6 6 6 6

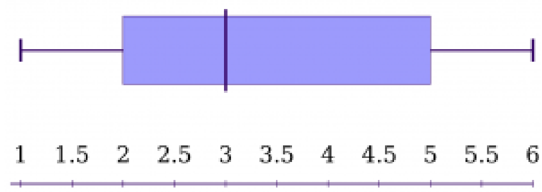
The median of this half is the average of the 5th and 6th values: $Q_1 = \frac{5+5}{2} = 5$.

Now, from the data, the minimum value is 1, and the maximum is 6.

The next step is to scale an appropriate axis for the obtained 5 numbers.

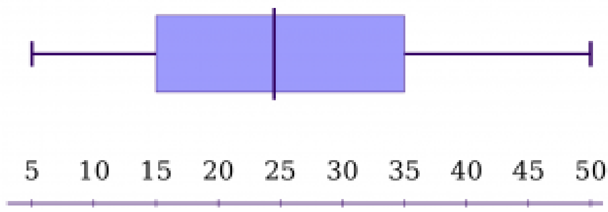
Then, we need to draw a box from the lower quartile value 2 to the value 3, which is the median, and put a vertical line through the median. Then, draw the box from the median to the upper quartile. Furthermore, we have to draw "whiskers". Those are the lines that extend parallel with the scale from the box. In other words, the whisker goes from the lower quartile to the minimum value 1 and from the upper quartile to the maximum value 6.

Finally, our box plot is:



Example 4.6 (Interpreting box and whisker plots)

Find the range, the interquartile range and the median of the data in the box plot below.



Solution

Since the minimum value of the given data is 5 and the maximum is 50, the range is R = 50 – 5 = 45.

The lower quartile is 15, and the upper quartile is 35. Therefore, the interquartile range is = 35 – 15 = 20. Actually, the interquartile range represents the length of the box and the central 50% of the data.

The median is clearly 25.

Learning tasks

Learners discuss and make a presentation on the features box plot. Task learners to solve problems on box plots.

PEDAGOGICAL EXEMPLARS

1. Collaborative learning: In small groups, organise given data using a frequency curve (ogive) and interpret the graph. Use appropriate technology tools such as Microsoft Excel if available. Encourage learners to be fair and impartial towards other learners and help them to acknowledge that there is reward in being truthful and honest citizenship.
2. Experiential Learning: In small groups, organise given data using a waffle diagram. Encourage learners to use the appropriate IT tools in designing waffle charts.
3. Experiential Learning: In small groups, organise given data using a box and whisker plot and interpret a given box plot. Encourage learners to use the appropriate IT tools in making a presentation on interpreting a box plot.

Model and solve real-life problems involving data

We can collect real-life data about numerous issues in our communities [school, home, markets, etc.] that are of interest to us. The data collected can then be organised and presented using one or a combination of the data presentation tools that we have learnt. Let's us take a look at a sample mini-project that students can embark on to realistically apply the knowledge and skills acquired in data collection, organisation and presentations.

Mini-project: Real-life Project on Data Collection, Organisation and Presentation

Questionnaire on Gender Equality and Social Inclusion in Ghanaian Senior High Schools

Instructions for Respondents:

- Please take your time to answer the questions honestly and to the best of your knowledge.
- Your responses will be kept confidential and used only for the purpose of this study.
- If you have any questions or need clarification, feel free to ask the person administering this questionnaire.

Section A: Demographic Information

1. Age:
   - o Under 15 o 15-17
   - o 18-20 o Over 20
2. Gender:
   - o Male o Female
3. Grade Level:
   - o SHS 1 o SHS 2 o SHS 3

Section B: Awareness and Perception

4. How aware are you of gender equality and social inclusion policies in your school?

- o Very aware
- o Somewhat aware
- o Not aware

5. How important do you think gender equality and social inclusion are in the school environment?

- o Very important
- o Important
- o Not important

Section C: Experiences and Observations

6. Have you ever witnessed or experienced gender discrimination in your school?

- o Yes
- o No

7. If yes and you feel able to, please describe the incident (Optional):

____

____

____

____

8. Do you believe that all students, regardless of gender, race, ethnicity, language spoken, background have equal opportunities in academic and extracurricular activities in your school?

- o Strongly agree
- o Agree
- o Neutral
- o Disagree
- o Strongly disagree

9. How inclusive do you think your school's policies and practices are towards students with disabilities?

- o Very inclusive
- o Somewhat inclusive
- o Not inclusive

Section D: Challenges and Suggestions

10. What do you think are the main challenges to achieving gender equality in your school?

____

____

____

____

11. What measures do you think can be taken to improve gender equality and social inclusion in your school?

____

____

____

____

12. Would you like to see more initiatives and programs focused on promoting gender equality and social inclusion in your school?

- **o** Yes
- **o** No

Section E: Additional Comments

13. Please share any additional comments or suggestions regarding gender equality and social inclusion in your school (Optional):

____

____

____

____

Note

This is just a sample; you may modify it to suit your class. Alternatively let students use the ones they developed earlier to collect the data.

Process

Distribute the Questionnaire

- Method: Decide how you will distribute the questionnaire. You can do this in person, via email, or through a social media platform.
- Instructions: Provide clear instructions on how to complete and return the questionnaire.

Collect and Analyse the Data

- Collect Responses: Gather all the completed questionnaires.

- Analyse Data: Summarise the responses. For closed-ended questions, tally the answers and create charts or graphs. For open-ended questions, identify common themes or patterns and write them down.

Report the Findings

- Summary: Write a summary of the findings from the questionnaire.
- Presentation: Present the results to your classmates or teachers, using visual aids like charts and graphs [Waffle charts, box plots, ogive, etc.] to make the data easy to understand.

KEY ASSESSMENT

1. Follow through this activity to construct a cumulative frequency curve.

Suppose we have the following data on the heights of 50 students:

Height (cm)	Frequency
140 - 145	8
145 - 150	12
150 - 155	18
155 - 160	10
160 - 165	2

Constructing a Less Than Ogive

Height (cm)	Frequency	Cumulative Frequency
140 - 145	8	8
145 - 150	12	20
150 - 155	18	38
155 - 160	10	48
160 - 165	2	50

Plot Points: (145, 8), (150, 20), (155, 38), (160, 48), (165, 50)

i. Draw Curve: Connect these points with a smooth line.

Constructing a Greater Than Ogive

Height (cm)	Frequency	Cumulative Frequency
140 - 145	8	50
145 - 150	12	42
150 - 155	18	30
155 - 160	10	12
160 - 165	2	2

Plot Points: (140, 50), (145, 42), (150, 30), (155, 12), (160, 2)

i. Draw Curve: Connect these points with a smooth line.

2. A high school is analyzing the distribution of students' participation in various extracurricular activities. The data for a total of 100 students is as follows:

a. Sports: 35 students

b. Music: 25 students

c. Art: 20 students

d. Science Club: 10 students

e. Drama: 10 students

Create a waffle chart to visually represent the distribution of students' participation in these extracurricular activities.

3. A statistics teacher collects the final exam scores of 15 students in a class. The scores are as follows: 72, 85, 90, 65, 78, 80, 92, 88, 76, 84, 91, 87, 69, 95, 82.

i. Calculate the minimum, first quartile (Q1), median (Q2), third quartile (Q3), and maximum of the exam scores.

ii. Draw a box plot to represent the distribution of the exam scores.

iii. Identify any potential outliers in the data set.

WEEK 11: ANALYSIS AND INTERPRETATIONS OF DATA

Learning Indicator: *Analyse and interpret data using measures of dispersion and justify which of these measures best suit(s) the data.*

FOCAL AREA 1: ANALYSE AND INTERPRET DATA USING MEASURES OF DISPERSION

Measures of dispersion are statistical tools that describe the extent to which data points in a dataset are spread out or scattered. While measures of central tendency, such as the mean, median and mode, provide information about the central value of a dataset, measures of dispersion give us insights into the distribution and variability of the data.

Measures of dispersion help us answer questions such as:

- How spread out are the data points in a dataset?
- Are the data points closely clustered around the mean, or are they widely scattered?
- What is the range of values in the dataset?

Key Measures of Dispersion

Range

The simplest measure of dispersion, the range, is the difference between the highest and lowest values in a dataset. It provides a quick sense of the spread but does not give any information about the distribution of values between the extremes.

Formula: Range = Maximum value - Minimum value

Example 4.6

Calculate the range for the following data values.

45, 55, 63, 76, 67, 84, 75, 48, 62, 65

Solution

From the data;

Maximum value = 84

Minimum or Least value = 45

Range = Maximum value = Minimum value

= 84 – 45

= 39

Real-Life Application of Range

Scenario:

A school is analyzing the performance of students in a recent mathematics test to identify the spread of scores and understand the overall performance variability.

Question:

The mathematics test scores of 10 students in a class are as follows: 45, 78, 56, 89, 92, 67, 81, 74, 58, and 62.

1. Calculate the Range of the Test Scores:

- o Determine the highest and lowest scores.
- o Compute the range to understand the spread of the test scores.

2. Interpret the Results:

- o Explain what the range tells you about the performance variability of the students.
- o Discuss why knowing the range of test scores might be useful for the teacher.

Solution

Steps to Solve:

1. Identify the highest and lowest scores:

- o Highest score: 92
- o Lowest score: 45

2. Calculate the range:

- o Range = Highest score - Lowest score
- o Range = 92 - 45 = **47**

Interpretation:

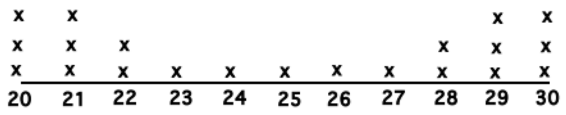
The range of the test scores is 47. This means there is a 47-point difference between the highest and lowest scores in the class.

Discussion:

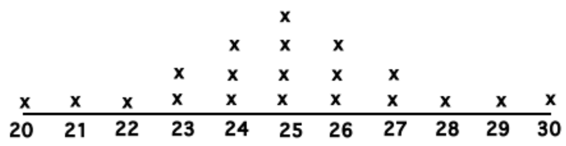
- Performance Variability: The range indicates a considerable spread in the students' performance, suggesting that there is significant difference in their understanding and mastery of the content.
- Usefulness for the Teacher: Understanding the range can help the teacher identify the extent of differences in student performance. A large range might indicate the need for differentiated instruction or additional support for lower-performing

students. It also highlights the need to explore factors contributing to the wide disparity in scores, such as study habits, attendance, or access to resources.

Although the range is easy to compute it is a crude measure of variability. Consider the following two sets of data which have the same mean, 25, and the same range, 10, but obvious differences in the pattern of variability:



Data Set 1



Data Set 2

Data set 1 has most of its values far from the mean and is u-shaped, while data set 2 has most of its values closer to the mean and is mound shaped or bell-shaped. In order to catch the different patterns in variability above, we need a more subtle measure than the range. So, we will later talk about standard deviation and variance.

Quartile Deviation & Interquartile Range

Interquartile Range (IQR): The IQR measures the spread of the middle 50% of the data. It is the difference between the third quartile (Q3) and the first quartile (Q1), effectively ignoring the extremes and focusing on the central portion of the data.

Formula: IQR = Q3 - Q1

Quartile Deviation, also known as the Semi-Interquartile Range, is a measure of statistical dispersion that quantifies the spread of the middle 50% of a dataset. It is particularly useful for understanding the variability in data that is not influenced by outliers or extreme values, making it a robust measure of spread.

Calculation of Quartile Deviation:

Identify the First Quartile (Q1):

- o The first quartile, Q1, is the value below which 25% of the data falls. It marks the end of the first quarter of the dataset.

Identify the Third Quartile (Q3):

- o The third quartile, Q3, is the value below which 75% of the data falls. It marks the end of the third quarter of the dataset.

Compute the Interquartile Range (IQR):

- o IQR = Q3 - Q1

Calculate the Quartile Deviation:

- o Quartile Deviation = IQR / 2

The quartile deviation represents half of the interquartile range, providing a measure of the spread of the middle 50% of the data.

Formula:

Quartile Deviation $= \dfrac{Q3 - Q1}{2}$

Example 4.7

Suppose we have a dataset of test scores: 56, 61, 63, 68, 70, 71, 73, 75, 78, 80, 85

1. **Find Q1 and Q3:**
   - o Q1 (first quartile) = 63
   - o Q3 (third quartile) = 78
2. **Compute IQR:**
   - o IQR = Q3 - Q1 = 78 - 63 = 15
3. **Calculate Quartile Deviation:**
   - o Quartile Deviation $= \dfrac{IQR}{2} = \dfrac{15}{2} = 7.5$

The quartile deviation is 7.5, indicating that the middle 50% of the test scores are spread out by 7.5 points on either side of the median.

Example 4.8

Find the quartile deviation for the following given data.

23, 8, 5, 16, 33, 7, 24, 5, 30, 33, 37, 30, 9, 11, 26, 32

Solution

The given data points are 23, 8, 5, 16, 33, 7, 24, 5, 30, 33, 37, 30, 9, 11, 26, 32

Let us arrange this data in the following ascending order.

5, 5, 7, 8, 9, 11, 16, 23, 24, 26, 30, 30, 32, 33, 33, 37

From the above data we have $Q_1 = \dfrac{(8+9)}{2} = \dfrac{17}{2} = 8.5$, and $Q_3 = \dfrac{(30+32)}{2} = \dfrac{62}{2} = 31$

Quartile Deviation $= \dfrac{Q3 - Q1}{2} = \dfrac{31 - 8.5}{2} = \dfrac{2.5}{2}$ $=11.25$.

Variance

Variance measures the average squared deviation of each data point from the mean. It gives a sense of how much the data points vary from the mean, with larger values indicating greater dispersion.

Formula

	Population	Sample
Ungrouped	$\sigma^2 = \sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N}$	$\sigma^2 = \sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N-1}$
Grouped	$\sigma^2 = \sum_{i=1}^{N} \frac{f(M_i - \bar{X})^2}{N}$	$\sigma^2 = \sum_{i=1}^{N} \frac{f(M_i - \bar{X})^2}{N-1}$

where x_i are the data points, $\bar{x}$ is the mean, and N is the number of data points.

Steps

- Find the mean of the observations.
- Subtract the mean from each observation.
- Square each of these values.
- Add all the values obtained in the previous step.
- Divide the value from step 4 by n (for population variance) or n - 1 (for sample variance).

Example 4.9

i. Suppose we have the data set {3, 5, 8, 1}, and we want to find the population variance.

The mean is given as $\frac{(3 + 5 + 8 + 1)}{4} = 4.25$ and n = 4.

$$\sigma^2 = \frac{\sum[(3 - 4.25)2 + (5 - 4.25)2 + (8 - 4.25)2 + (1 - 4.25)2]}{4} = 6.68.$$

Thus, variance = **6.68**.

Example 4.10

Find the sample variance of the data (3, 4, 7, 12, 14).

Solution

$$\sigma^2 = \frac{\Sigma(x_i - \bar{x})^2}{N - 1}$$

n = 5

$$\text{Mean} = \frac{3 + 4 + 7 + 12 + 14}{5} = 8$$

Sample Variance

$$\frac{\sum[(3-8)2+(4-8)2+(7-8)2+(12-8)2+(14-8)2)}{5-1} = 23.5$$

Answer: Variance = 23.5

Example 4.11

Find the population variance of the data (1.2, 4.5, 6.7, 2.3).

Solution

n = 4

Mean = (1.2 + 4.5 + 6.7 + 2.3) / 4 = 3.675

Population Variance $= \frac{\sum(x_i - \bar{x})^2}{N}$

$$\frac{[(1.2 - 3.675)^2 + (4.5 - 3.675)^2 + (6.7 - 3.675)^2 + (2.3 - 3.675)^2]}{4} = 4.461$$

Answer: Variance = 4.461

Standard Deviation: The standard deviation is the square root of the variance. It provides a measure of dispersion in the same units as the original data, making it more interpretable. A larger standard deviation indicates that the data points are more spread out.

Formula: Standard Deviation $\sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{N}}$

Worked Examples

Earlier we solved some examples of variance. As we have learnt, we can take the square root of the values of the variances to obtain the standard deviations.

Example 4.12

Variance: $\sigma^2 = 6.68$.

So, Standard Deviation: $\sigma = \sqrt{6.68} = 2.58$

Example 4.12

Variance: $\sigma^2 = 23.5$

So, Standard Deviation: $\sigma = \sqrt{23.5} = 4.85$

Real-Life Application Problem on Standard Deviation

Scenario:

A company's human resources department is analyzing the salaries of its employees to understand the salary distribution and ensure fair compensation practices. The HR manager has collected the following annual salaries (in thousands of Ghana cedis) of 10 employees: 45, 52, 47, 49, 50, 60, 55, 53, 58, and 62.

Problem:

1. Calculate the Mean Salary:

- o Find the average salary of the employees.

2. Calculate the Standard Deviation of the Salaries:

- o Determine how much the salaries deviate from the mean salary.

3. Interpret the Results:

- o Explain what the standard deviation tells you about the salary distribution.
- o Discuss why understanding the standard deviation of salaries might be useful for the HR manager.

Steps to Solve

1. Calculate the Mean Salary: Mean (μ) = $\frac{\sum x_i}{N}$

$$\bar{x} = \frac{45 + 52 + 47 + 49 + 50 + 60 + 55 + 53 + 58 + 62}{10}$$

$$\bar{x} = \frac{531}{10}$$

$$\bar{x} = 53.1$$

Calculate the Standard Deviation: Standard Deviation (σ) = $\sum(x_i - \bar{x})^2/N$

$$\sigma = 5.32$$

Interpret the Results

- The mean salary is GH¢53,100. The standard deviation is approximately GH¢5,320.
- This means that the salaries of the employees typically deviate from the mean salary by about GH¢5,320.

Mean Absolute Deviation (MAD): MAD measures the average absolute deviation of each data point from the mean. It provides a straightforward interpretation of the average distance from the mean.

Formula: MAD = $\frac{\sum |x_i - \bar{x}|}{N}$

Worked Examples

Example 4.13

Scenario: A teacher wants to analyse the variability in test scores of 5 students. The scores are as follows: 85, 90, 88, 92, 87.

Solution

1. Calculate the Mean of the Scores:
   $$\text{Mean}(\bar{x}) = \frac{85 + 90 + 88 + 92 + 87}{5} = \frac{442}{5} = 88.4$$

2. Find the Absolute Deviations from the Mean:
   $|85 - 88.4| = 3.4$

   $|90 - 88.4| = 1.6$

   $|88 - 88.4| = 0.4$

   $|92 - 88.4| = 3.6$

   $|87 - 88.4| = 1.4$

Calculate the Mean Absolute Deviation (MAD):

$$\text{MAD} = \frac{3.4 + 1.6 + 0.4 + 3.6 + 1.4}{5} = \frac{10.4}{5} = 2.08$$

Interpretation

The mean absolute deviation of the test scores is 2.08, indicating that, on average, the scores deviate from the mean by 2.08 points. This provides a measure of the variability of the test scores.

Note

In the calculation of the Mean Absolute Deviation (MAD), we focus on the absolute values of the deviations from the mean. This is because the absolute value ensures that all deviations are considered positively, avoiding the issue of negative values canceling out positive values.

That is, when we calculate deviations from the mean, we obtain positive or negative values depending on whether the data points are above or below the mean. To measure the average spread or dispersion of the data, we need a measure that reflects the magnitude of the deviations without being affected by their direction.

In variance we get around this issue in a different way. Here, we square each difference which ensures all the values are positive. We can then square root the variance to find the standard deviation.

PEDAGOGICAL EXEMPLARS

1. In mixed groups, learners collaborate to discuss and solve problems on range and quartile deviations.
2. Using Talk for Learning strategies, learners brainstorm on the meaning of standard deviation and discuss the steps in determining standard deviation and deduce the formula for Standard deviation.

3. Using mixed-ability groupings, learners solve problems, including real-life problems, on standard deviation.
4. Using think-pair-share activities, learners discuss the steps in determining variance and deduce the formula for variance. Learners should be encouraged to use feasible technology to analyse data, determine the variance of the data, and talk about the appropriate ways of using the IT tools.
5. In mixed groups, learners collaborate to discuss and solve problems on mean absolute deviations.

KEY ASSESSMENT

1. Calculate the range for the following data: 60, 65, 70, 12, 52, 40, 48
2. A weather station records the daily high temperatures (in °C) over a week as follows: 28, 30, 27, 29, 31, 32, 30. Calculate mean absolute deviation.
3. A small business owner wants to analyse the monthly sales data for the past year to understand the variability in sales. The monthly sales (in thousands of dollars) for the past 12 months are as follows: 45, 52, 47, 50, 55, 60, 53, 57, 62, 59, 49, 54.
   - **i.** Calculate the mean of the monthly sales.
   - **ii.** Determine the standard deviation of the monthly sales.
   - **iii.** Explain what the standard deviation tells you about the variability of the sales data.
4. A teacher is interested in analysing the study habits of students in a particular class. The number of hours that students studied per week over a period of 10 weeks is recorded as follows: 12, 15, 14, 10, 16, 20, 18, 17, 13, 11.
   - **i.** Calculate the mean number of study hours per week.
   - **ii.** Determine the standard deviation of the study hours.
   - **iii.** Discuss the significance of the standard deviation in understanding the study patterns of the students.
5. A researcher is studying the amount of time (in minutes) spent on physical exercise by a group of 8 individuals in a week. The data collected is as follows: 30, 45, 50, 35, 40, 55, 60, 50.
   - **i.** Calculate the mean amount of time spent on physical exercise.
   - **ii.** Determine the variance of the amount of time spent on physical exercise.
   - **iii.** Explain what the variance indicates about the distribution of exercise times in this group.

Reminder

End of semester comes in Week 12. All scores required to be submitted into the STP should be submitted by the end of Week 11.

Section 4 Review

Week 9: Data collection instruments

Designing a Questionnaire

- Crafting a set of structured questions aimed at gathering specific information from respondents systematically.

Designing an Interview Guide

- Creating a framework of questions and topics to guide a structured or semi-structured interview process for in-depth data collection.

Building an Observation Guide

- Developing a checklist or set of criteria to systematically observe and record behaviors, events, or conditions in a study setting.

Applying Digital Technology in Data Collection

- Utilizing digital tools such as online surveys, mobile apps, and electronic data capture systems to enhance the efficiency and accuracy of data collection.

Week 10: Pictorial Representation of Data (Ogive, Waffle, Box and Whisker Plots, etc.)

- **Ogive:** A cumulative frequency graph that shows the number of observations below a particular value in a dataset, useful for determining percentiles and medians.
- **Waffle Chart:** A grid-based visual representation used to display parts of a whole, often with colored squares to depict proportions.
- **Box and Whisker Plot:** A graphical representation of data that shows the distribution through five-number summaries (minimum, first quartile, median, third quartile, and maximum), highlighting outliers and variability.

Model and Solve Real-Life Problems Involving Data

- Using mathematical models and statistical techniques to interpret data and provide solutions to practical issues such as predicting sales trends, analyzing customer behavior, and optimizing resource allocation based on collected data.

Week 11: Analysis and interpretations of data

Measures of Dispersion

- Range: The difference between the highest and lowest values in a dataset, indicating the spread of the data.
- Variance: The average of the squared differences from the mean, showing how much the data points deviate from the mean.
- Standard Deviation: The square root of the variance, providing a measure of the average distance of each data point from the mean, indicating data spread and consistency.
- Interquartile Range (IQR): The difference between the first quartile (Q1) and the third quartile (Q3), showing the range of the middle 50% of the data.

SECTION 5: RATIOS, RATES AND PROPORTIONS

Strand: Number for Everyday life

Sub-Strand: Proportional Reasoning

Learning Outcomes

1. *Establish the similarities among ratios, rates, and proportions and use these to solve problems.*
2. *Analyse the relevance of ratios, rates and proportions in solving day-to-day problems.*

Content Standards

1. *Demonstrate knowledge and understanding of ratios, rates and proportions and use it to solve real-world problems.*
2. *Demonstrate understanding of proportional reasoning using mathematical connections among ratios, rates and proportions to solve daily problems including compound interest, tax (VAT, E-LEVY), utilities, depreciation, etc.*

INTRODUCTION AND SECTION SUMMARY

In this section, we will explore the fundamental concepts of ratios and rates, their applications, and their relevance to financial mathematics. This section is designed to provide a deep understanding of these concepts and demonstrate their practical use in everyday life and various fields. We will begin by understanding the basic concepts of ratios and rates. We will then apply our understanding of ratios, rates, and proportions to various contexts, by extending the application of ratios to angles, including the use of the Pythagorean theorem in triangles. We shall also investigate everyday applications of ratios, rates and proportions, including percent problems, to see their practical relevance. We will apply ratios, rates, and proportions to various aspects of financial mathematics. These include; health-related calculations, exploring their application in the equitable sharing of properties; investigating how ratios, rates, and proportions are used in sports statistics and performance analysis; learning about their role in currency exchange calculations; applying these concepts to understand and manage utility bills; and explore other practical applications, demonstrating the versatility of ratios, rates, and proportions in financial contexts.

SUMMARY OF PEDAGOGICAL EXEMPLARS

This section highlights the significance of **hands-on activities, visual models, real-life contexts, and interactive problem-solving** to enhance students' understanding and application of **ratios, rates, and proportions** in everyday situations. These strategies help learners develop **conceptual fluency, critical thinking, and problem-solving skills**.

1. **Hands-on Exploration of Ratios and Proportions**

   Engage students in practical activities such as **mixing colors, comparing ingredient quantities in recipes, or scaling models** to help them grasp the concept of **ratio relationships**. This experiential approach strengthens their ability to recognize and apply ratios in different contexts.

2. **Visual Representation**

   Use **double number lines, tape diagrams, ratio tables, and coordinate graphs** to represent ratios and proportional relationships. These tools provide a clear, structured way for students to understand how quantities compare and scale relative to each other.

3. **Real-life Applications and Problem-Solving**

   Introduce authentic real-world problems, such as **calculating speed (distance-time relationships), determining unit prices in shopping, scaling maps, and adjusting recipes for different serving sizes**. These applications help students see the relevance of ratios, rates, and proportions in daily life and decision-making.

4. **Collaborative Learning and Discussion**

   Encourage peer discussions and group projects where students **investigate proportionality** in areas such as **financial literacy (exchange rates, discounts, and interest rates), sports statistics, and population growth trends**. This fosters teamwork, critical thinking, and deeper understanding.

5. **Technology Integration**

   Use digital tools and simulations to explore proportional relationships dynamically. **Interactive graphing software, virtual manipulatives, and online ratio calculators** allow students to experiment with scaling and rate conversions in a more engaging way.

ASSESSMENT SUMMARY

Different forms of assessments should be carried out to ascertain learners' performance on the concepts that will be taught under this section. Teachers are entreated to administer these assessments and record them for onward submission into the Student Transcript

Portal (STP). The following assessment would be conducted and recorded for each learner:

Week 12: *End of Semester Examination*

Week 13: *Class Exercise*

Week 14: *Individual Project Work*

Note

For additional information on how to effectively administer these assessment modes, refer to the Appendices.

WEEK 12: RATIOS, RATES AND PROPORTIONS

Learning Indicators

1. *Explain the concepts of ratio and rate as comparison of quantities.*
2. *Establish the relationships among ratio, rates and proportions.*

FOCAL AREA 1: CONCEPTS OF RATIOS AND RATES

We have experienced the concepts of ratios and rates in Junior High School Mathematics. Here, we shall look at their applications to our daily activities. Let us look at these concepts in the following.

Ratios

The two common types of ratios we'll see are **part-to-part** and **part-to-whole**.

For example, in making lemonade:

1. The ratio of lemon juice to sugar is a **part-to-part** ratio. It compares the amounts of two ingredients.
2. The ratio of lemon juice to lemonade is a **part-to-whole** ratio. It compares the amount of one ingredient to the sum of all ingredients.

Establish **a ratio** by comparing two quantities of the same unit of measure. The ratio can be expressed as $\boldsymbol{a} : \boldsymbol{b}$ **or** $\frac{\boldsymbol{a}}{\boldsymbol{b}}$ and read as '$\boldsymbol{a}$ to $\boldsymbol{b}$'.

Example 5.1

Write a ratio for the number of males to females in your class, school, town, district, region or country. **i.e.** *Number of* **males** : *Number of* **females**

Example 5.2

In a class containing 30 boys and 20 girls. Find the ratio of

a. boys to girls.

b. girls to boys, in the class.

Solution

a. The ratio of boys to girls is given by 30:20 = 3:2

b. The ratio of girls to boys is given by 20:30 = 2:3

Example 5.3

The population of Ghana as of 2024 was 33,000,000. If the female population was 17,000,000. Find the ratio of males to females in Ghana for the 2024 figures.

Solution

Given: total population= 33,000,000. The population of females = 17,000,000 implies the population of males = 33,000,000 – 17,000,000 = 16,000,000

the ratio of males to females 16,000,000 : 17,000,000 = 16:17

Rates

The rate can be seen as a special ratio that compares two quantities measured in different types of units or different dimensions.

Establish the rate by comparing two quantities of different units of measure. i.e. comparing the speed of a vehicle to the distance it covers.

Example 5.4

If a car took 1 *hour* to cover a total distance of 60km, determine its rate.

Solution

The rate of the car is written as $\frac{Total\ distance\ traveled}{time\ taken} = \frac{60km}{1Hr}$ or $60km/Hr$

Example 5.5

If Alberta earns *GH¢*5,000.00 for working 24 hours, what will be her average rate of pay per hour?

Solution

Her average rate of pay per hour= $\frac{Amount\ earned}{time\ used} = \frac{GH¢5,000.00}{24\ hours}$ = GH¢308.33

Learning Tasks

1. In pairs or mixed–ability groups, learners solve problems involving ratios and rates
2. In mixed groupings [gender/ability] learners should model and solve real-world problems involving ratios and rates.

Worked Examples

Example 5.6

In a class of 30 students, the ratio of boys to girls is 2:3. How many boys and girls are in the class?

Solution

Method I: Since the total ratio is 5 (2 + 3), we partition the 30 students into 5 groups of 6 as follows:

We realise that the two groups of boys give a total of 12 boys, while the 3 groups of girls provide a total of 18 girls. Therefore, there are 12 boys and 18 girls in the class.

Method II: Since the ratio of boys to girls is 2:3, make students regroup themselves in 'twos' and 'threes' repeatedly until there are no more students, See the table below:

Step/group	Boys	Girls	Ratio	Total
1*st*	2	3	2:3	5
2*nd*	4	6	4:6	10
3*rd*	6	9	6:9	15
4*th*	8	12	8:12	20
5*th*	10	15	10:15	25
6***th***	12	18	12:18	30

We will see that on the 6^{th} step there are 12 boys and 18 girls making a total of 30 students.

Method III: Let x be the number of parts in the ratio.

Given the total number of students to be 30, we set up the equation: $2x + 3x = 30$

By solving for x: $5x = 30$, we have $x = 6$

Calculating for the number of boys, we have $2x = 2(6) = 12$,

Calculating for the number of girls, we have $3x = 3(6) = 18$

Therefore, there are 12 boys and 18 girls in the class.

Example 5.7

If a vehicle travel 240 kilometers in 4 hours, what is its average speed?

Solution

Using the rate formula: $Speed = \dfrac{Distance\ traveled}{Time\ taken}$

Given: $distance\ travelled = 240\ kilometres$, and $time\ taken = 4\ hours$

Substitute in the values, we have: $Speed = \dfrac{240\ kilometres}{4\ hours}$

Simplifying gives us $speed = 60$ kilometres/$hour$

Example 5.8

If a function reflects in the line $x = y$ determine its ratio.

Solution

Since the function $x = y$ is 1:1, then its ratio is 1:1

FOCAL AREA 2: CONNECTION BETWEEN RATIOS AND RATES

If we know a ratio and want to apply it to a different scenario or population, we can use proportions to set up equivalent ratios and calculate any unknown quantities similarly, if we can use proportions to set up equivalent rates. Proportion can then be thought of as an equation of two equivalent ratios or rates.

Proportion: A proportion is a statement that compares or sets up two equivalent ratios or rates.

Worked Examples

Example 5.9

A dinner for 4 servings calls for 2 cups of rice. How much rice is needed for 10 servings?

Solution

We set up the proportion as $2cups \rightarrow 4servings$ and $xcups \rightarrow 10sevings$

Implies $\dfrac{2\ cups}{4\ servings} = \dfrac{cups}{10\ servings}$

Cross-multiplying we have: $2 \times 10 = 4x$; $20 = 4x$

Solving for x gives $\quad x = \dfrac{20}{4} = 5$

Therefore, 5 cups of rice are needed for 10 servings.

Example 5.10

On a map, 1 inch represents 5 kilometers. How many kilometers are represented by 3.5 inches on the map?

Solution

We set up the proportion as: $\frac{1\ inch}{5\text{ kilometers}} = \frac{3.5\text{ inches}}{x\text{ Kilometers}}$

Cross-multiplying we have: $1x = 5 \times 3.5$; $\quad x = 5 \times 3.5 = 17.5$

Therefore, 3.5 inches on the map represents 17.5 kilometers

Example 5.11

If 3 apples cost GHs10.25, how much would 8 apples cost?

Solution

Step 1: Set up the proportion: $\frac{3}{10.25} = \frac{8}{x}$

Step 2: Cross multiply: $3x = 10.25 \times 8$

Step 3: Solve for x: $x = \frac{10.25 \times 8}{3} = 6$

Therefore, 8 apples would cost GH27.33.

Example 5.12

A recipe calls for a ratio of 2 cups of flour to 3 cups of sugar. If you need 10 cups of flour for a large batch, how much sugar should you use?

Solution

Step 1: Set up the proportion: $\frac{2}{3} = \frac{10}{x}$

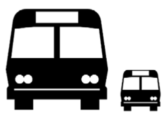
Step 2: Cross multiply: $2x = 30$

Step 3: Solve for x: $x = 15$

Therefore, you need 15 cups of sugar.

Example 5.13

A model car is built to a scale of 1:24. If the real car is 4.8 meters long, how long is the model car?



Solution

Step 1: Set up the ratio: $\frac{1}{24} = \frac{x}{4.8}$

Step 2: Cross multiply: $24x = 4.8$

Step 3: Solve for x: $x = \frac{4.8}{24} = 0.2$ *meters* or 20 *cm*

Example 5.14

The rate of currency exchange between US dollars ($) and Ghana cedis (GH¢) as of July 2024 is GH¢15.38 per US dollar. How much GH¢ is equivalent to 5,275$?

Solution

$$\frac{\text{GH¢}}{\$} = \frac{15.38}{1} = \frac{x}{5{,}275}$$

$$x = 15.38 \times 5{,}275$$

$$= 81{,}129.50$$

Learning Tasks

1. Learners identify and explain the concepts of ratio rates and proportions.
2. Learners, model and discuss possible areas for their applications.

PEDAGOGICAL EXEMPLARS

1. Think-pair share activities: Learners in pairs research and make presentations on the concepts of ratio, rate, and proportion.
2. In mixed-ability groups, learners discuss the concepts and their applications to real life.
3. Collaborative Learning: In mixed-grouping (gender/ability) tasks learners carry out group presentations on the concepts of ratio, rates and proportions.

KEY ASSESSMENT

DoK Level 1 (Recall & Reproduction)

1. If the ratio of cats to dogs in a pet shop is 3:5, and there are 40 animals in total, how many cats are there?
2. If 3 kg of apples cost GH¢7.50, what is the price per kg?
3. In a class of 30 students, the ratio of boys to girls is 2:3. How many girls are in the class?
4. If 4 shirts cost GH¢60, how much would 7 shirts cost at the same rate?
5. A solution is 20% acid. How many milliliters of acid are in 250 mL of the solution?

DoK Level 2 (Skills & Concepts)

6. A car travels 240 miles in 4 hours. What is its average speed in kilometers per hour? (Assume 1 mile = 1.609 km)

7. If 6 workers can complete a job in 10 days, how many days would it take 15 workers to complete the same job (assuming they work at the same rate)?

8. A recipe calls for 2 cups of flour for every 3 eggs. If you want to use 5 eggs, how many cups of flour do you need?

9. A map has a scale of 1:100,000. If two cities are 5 cm apart on the map, what is their actual distance in kilometers?

10. A high school randomly selected 50 students to take a survey about extending their lunch period. Of the students selected:

- 13 were freshmen
- 14 were continuous students

Find the ratio that represents:

a) Part-to-part relationship

b) Part-to-whole relationship

DoK Level 3 (Strategic Thinking)

11. If a 12-metre shadow is cast by a 20-metre-tall building, how tall is a tree that casts an 18-metre shadow at the same time of day?

12. There are students at Du Bois Academy. If the student-to-teacher ratio is 25:1, how many teachers are there in the school if there are 1,000 students?

13. A carpenter needs to scale a furniture design. If the original dimensions of a table are 1.2 m by 0.8 m, and the scale factor is 5:3, what will be the new dimensions of the table?

14. A farmer needs to mix fertilizer in a ratio of 4:5 (fertilizer to water). If he has 20 liters of fertilizer, how much water does he need?

15. A taxi driver covers 150 km using 10 liters of fuel. If fuel consumption is proportional, how many liters of fuelwill be required to cover 375 km?

Hint

- *The Recommended Mode of Assessment for Week 12 is **End of semester Examination.***
- *Refer to Appendix E at the end of **Section 5** for further information on how to go about the end of semester examination.*

WEEK 13: APPLICATIONS OF PROPORTIONS

Learning Indicators

1. *Extend the application of ratios to angles, Pythagoras's theorem and some algebra.*
2. *Application of rates to speed: distance-time graphs.*

FOCAL AREA 1: APPLICATION OF PROPORTIONS

Proportions

Now, we shall consider proportions which compare two ratios or two rates in detail. For instance, by computing the speed of a vehicle we usually compare the distance travelled to the time taken.

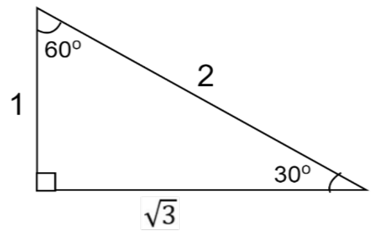
Key points

Equivalent proportions are proportions that are essentially the same although they can look a little different.

How can you tell if proportions are equivalent? Examples of these can be found in plane figures such as similar triangles.

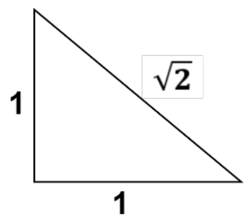
1. Ratios in Angles in Triangles

Application of ratios to angles in triangles.



a) If the interior angles of a triangle are 30° 60° 90° then the ratio of its interior angles is: 30:60:90, or simplified = 1:2:3.

Similarly, the ratio of its side lengths is: $1:\sqrt{3}:2$



b) If the interior angles of a triangle are 45° 45° 90° then the ratio of its interior angles is: 45:45:90, or simplified = 1:1:2.

Similarly, the ratio of its side lengths is: $1:1:\sqrt{2}$

Pythagoras theorem: $a^2 + b^2 = c^2$

Example: In a triangle with sides 3, 4, *and* 5: $3^2 + 4^2 = 5^2$

Trigonometric ratios: sin, cos, tan

Example: In a right-angled triangle,

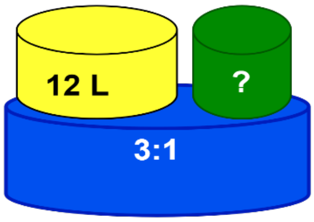
$$sin\theta = \frac{\text{Opposite}}{hypotenuse}$$

$$cos\theta = \frac{\text{Adjacent}}{hypotenuse}$$

$$tan\theta = \text{O}\frac{\text{Opposite}}{\text{Adjacent}}$$

Worked Examples

Example 5.15



Mixing paint colours (2 parts blue: 1 part yellow)

A painter wants to create a soft green colour by mixing yellow and blue paint. If they use 12 litres of yellow paint, how many litres of blue paint should they add to maintain a 3:1 ratio of yellow to blue?

Solution

To solve this, we use the ratio 3 : 1 (*yellow* : *blue*).

If 3 parts = 12 litres of yellow, then

$$\frac{3\ parts}{12\ litres} = \frac{1\ part}{x\ litres}$$

Implies, 1 *part* $= \frac{12}{3} = 4$ litres

Therefore, the painter should add 4 litres of blue paint.

Alternatively

By comparing ratios, we have

Share	Yellow	Blue	Ratio	Total
1*st*	3	1	3:1	5
2*nd*	6	2	6:2	8
3*rd*	9	3	9:3	12
4th	12	4	12:4	16

We noticed from the table above that for every 12 *litres* of yellow there is a corresponding 4 *litres* of blue paint. Thus**,** 3:1 = 12:4

Example 5.16

Godfred is creating a custom purple using a ratio of 2 parts red, 1 part blue, and 1 part white. If he wants to make 20 *litres* of this purple paint in total, how many litres of each color should he use?

Solution

Step-by-step procedure:

- The ratio is 2:1:1, so there are 4 total parts (2 + 1 + 1 = 4).

- Each part represents: $\frac{20\ litres}{4} = 5\ litres$
- Now we can calculate each colour as follows

  Red: 2 parts $= 2 \times 5 = 10\ litres$

  Blue: 1 part $= 1 \times 5 = 5\ litres$

  White: 1 part $= 1 \times 5 = 5\ litres$

FOCAL AREA 2: DISTANCE-TIME GRAPHS

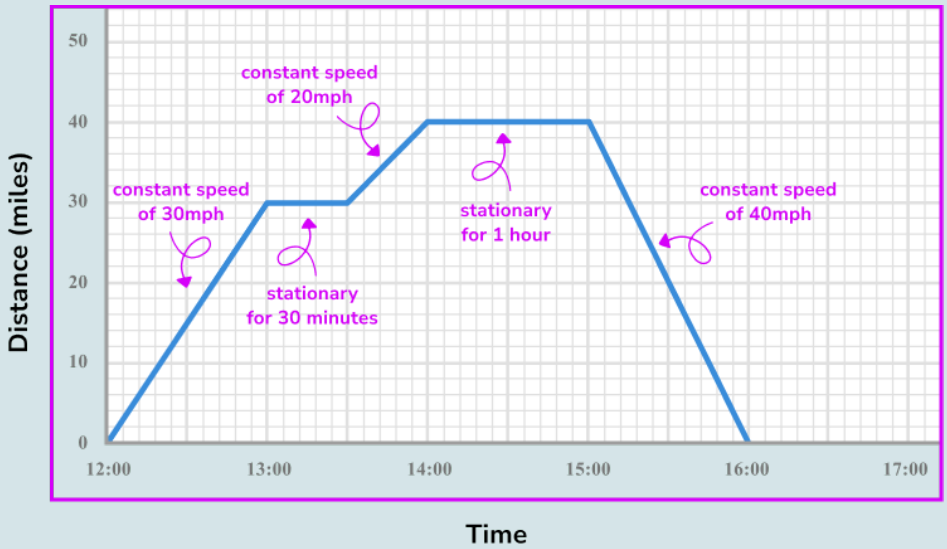
Distance-time graphs are graphs that show the distance an object or person has traveled against time. They can also be referred to as travel graphs.

A distance-time graph will show the distance (in metres, kilometres, miles etc.) on the vertical axis (y-axis) and the time (in seconds, minutes, hours, etc.) on the horizontal axis (x-axis).

Learning Tasks

Learners are guided to establish the following:

1. *Gradient of graphs:*
2. *Calculate speed:*
3. *Calculate acceleration:*
4. *Interpreting graphs:*
5. *Area under the graph:*
6. *Real-world applications:*



Examples

1. Gradient:

a. In a distance-time graph, the gradient (or slope) of the line represents the speed or velocity of the object.

b. It shows how quickly distance is changing with respect to time.

c. Mathematically, $gradient = \frac{rise}{run} = \frac{change\ in\ distance}{change\ in\ time}$.

d. A steeper gradient indicates that more distance is covered in less time, hence higher speed.

e. A gentler gradient means less distance covered in more time, indicating lower speed.

f. A flat or horizontal line (zero gradient) means no change in distance over time, indicating the object is stationary.

Worked examples

Example 5.17

A car travels 120 *km* in 2 *hours*. What is its average speed, and how would this appear on a distance-time graph?

Solution

Average speed $= \frac{120\ km}{2\ hours} = 60\ km/h$. This would appear as a straight line with a gradient of 60 km/h on the graph.

Example 5.18

On a distance-time graph, a line goes from (0 *hours*, 0 *km*) to (3 *hours*, 180 *km*). What is the speed of the object?

Solution

$$Speed = \frac{Change\ in\ distance}{Change\ in\ time} = \frac{180\ km - 0\ km}{3\ hours - 0\ hours} = 60\ km/h$$

Example 5.19

A runner's distance-time graph shows they covered 8 km in 40 minutes. What was their speed in m/s?

Solution

$$time = 40 minutes = \frac{40}{60} hours,\ Speed = \frac{km}{40/60} hours = 12\ km/h$$

$$12\ km/h = 12000\ m\ /\ 3600\ s = 3.33\ m/s$$

PEDAGOGICAL EXEMPLARS

1. Think-pair share activities: Learners in pairs research and make presentations on ***Distance-time graphs***.
2. In mixed-ability groups, learners discuss the concepts and their applications to real life.
3. Collaborative Learning: In mixed-grouping (gender/ability) tasks learners carry out group presentations on the concepts of ***Distance-time graphs***.

KEY ASSESSMENT

DoK Level 1 (Recall & Reproduction)

1. What shape would a distance-time graph show for an object moving at a constant speed?
2. In a distance-time graph, what does a curved line that bends upwards indicate?
3. If two lines intersect on a distance-time graph, what does this mean about the two objects?
4. How is deceleration represented in a distance-time graph?
5. What does the area under the line in a distance-time graph represent?

DoK Level 2 (Skills & Concepts)

6. If an object moves away from its starting point, turns around, and returns to the start, what would its distance-time graph look like?
7. How would you represent an object that moves at a constant speed, then stops for a while, and then moves again at a different constant speed?
8. A car accelerates from rest to 20 m/s over 10 seconds. What is its acceleration, and how would this appear on a distance-time graph?
9. A distance-time graph shows a line from (0s, 0m) to (5s, 50m), then a horizontal line to (10s, 50m). What was the object's average speed over the entire 10 seconds?
10. On a distance-time graph, a line goes from (0s, 0m) to (5s, 45m). What was the object's speed in km/h?

DoK Level 3 (Strategic Thinking)

11. A car travels 30 km north in 30 minutes, then 20 km south in 20 minutes.
    - **a.** Calculate the total distance traveled and the displacement.
    - **b.** How would this journey appear on a distance-time graph?
12. A distance-time graph shows a curved line. At t = 2s, the gradient is 5 m/s, and at t = 4s, the gradient is 9 m/s. What is the average acceleration between these times?

13. A cyclist travels 4 km east in 10 minutes, rests for 5 minutes, then travels 3 km west in 15 minutes.

a. Calculate the average speed for the entire journey.

b. Sketch the distance-time graph for this motion.

14. A car decelerates from 20 m/s to 5 m/s over 5 seconds.

a. Calculate the deceleration.

b. Describe how this would appear on a distance-time graph.

Hint

- *The Recommended Mode of Assessment for Week 13 is **Class Exercise**.*
- *Please ensure to organise it and record the scores for onward submission to the STP.*

Reminder

- *Remind learners about their Portfolio. Engage them to find out where they have reached and offer support where required.*

WEEK 14: FINANCIAL MATHEMATICS

Learning Indicators

1. *Apply the concept of ratios, rates and proportions to solve problems in financial mathematics, health, sports, etc.*
2. *Establish the relevance of ratios, rates and proportions in their day-to-day activities, make generalizations and apply them to solve real-world problems.*

FOCAL AREA 1: APPLICATION OF RATIOS, RATES AND PROPORTIONS TO FINANCIAL MATHEMATICS

In weeks 12 and 13 we encountered ratios, rates, and proportions as fundamental concepts in mathematics. We shall explore these concepts further because they have widespread applications in financial mathematics. Ratios, rates, and proportions can be experienced in the following real-life applications. Consider ratios, rates, and proportions in the field of health:

Here we shall look at some applications of ratio, rate, and proportions involving the concepts stated in focal point 1:

Everyday Applications

Example 5.20

Mass of food: A woman recommends a ratio of 40% carbohydrates, 30% protein, and 30% fat in her mini budget. If she is considering $GHs2000$ worth of calories per day, how much worth of calories should come from carbohydrates?

Solution

Calories from Carb carbohydrates

$= (GH¢) = Total\ Calories\ (GH¢) \times Ratio\ (\%)$

Calories from carbohydrates $(GH¢) = GH¢\ 2000 \times \frac{40}{100} = GH¢800$

Example 5.21

Heart rate and blood pressure ratios: A healthy resting heart rate is typically between 60 − 100 *beats per minute* (*bpm*). If someone's resting heart rate is 80 *bpm*, what is a desirable range for their diastolic blood pressure (lower number)? Generally, a good rule of thumb is that diastolic pressure should be around two-thirds of heart rate plus 2.

Solution

Desired Diastolic Range $= \frac{2}{3} \times \textit{Heart Rate} + 2$

Desired Diastolic Range $= \frac{2}{3} \times 80$ bpm$+ 2 \approx 53.33mmHg$ (millimetres of mercury)

Example 5.22

Body Mass Index (BMI) calculations: BMI is a measure of body fat based on height and weight. The formula is $BMI = \frac{weight\ (kg)}{height\ (m)^2}$. Calculate the BMI of a person who weighs 70 kg and is 1.7 meters tall.

Solution

BMI $= \frac{(70\ kg)}{(1.7\ m)^2} \approx 24.22$

Example 5.23

Division of assets among heirs: A ***will*** states that a collection of antique stools valued at $GH¢100{,}000$ should be divided equally among 3 children. How much will each child receive?

Solution

Share per Child (GH¢) $= \frac{\text{Total Value (GH¢)}}{\text{Number of Heirs}}$

Share per Child (GH¢) $= \frac{\text{GH¢}100{,}000}{3}$

$= \text{GH¢}33{,}333.33$

Example 5.24

Partnership profit sharing: Two partners agree to share profits in a 2:3 ratio. If the total profit for the year is GH¢50,000, how much will each partner receive?

Solution

Partner 1 Share (GH¢) $= \textit{Total Profit}\ (GH¢) \times \textit{Ratio}$

$= \text{GH¢}50{,}000 \times \frac{2}{5}$

$= \text{GH¢}20{,}000$

Partner 2 Share (GH¢) $= \textit{Total Profit}\ (GH¢) \times \textit{Ratio}$

$= GH¢50{,}000 \times \frac{3}{5}$

$= GH¢30{,}000$

Example 5.25

Land area distribution: A farmer has 120 hectares of land and wants to distribute it among his 2 sons in a 3:2 ratio. How much land will each son get?

Solution

Son 1 Land (ha) $= Total\ Land\ (ha) \times Ratio$

$$= 120\ ha \times \frac{3}{5} = 72\ ha$$

Son 2 Land (ha) $= Total\ Land\ (ha) \times Ratio$

$$= 120\ ha \times \frac{2}{5}$$

$$= 48\ ha$$

Example 5.26

Investment portfolio allocation: An investor wants to allocate their 100,000 GH¢ portfolio with 60% in stocks and 40% in bonds. How much should be invested in each asset class?

Solution

Stock Investment (GH¢) $= Total\ Portfolio\ (GH¢) \times Stock\ Ratio$

$$= GH¢\ 100{,}000 \times \frac{60}{100}$$

$$= GH¢60{,}000.$$

Bond Investment (GH¢) $= Total\ Portfolio\ (GH¢) \times Bond\ Ratio$

$$= GH¢100{,}000 \times \frac{60}{100}$$

$$= GH¢\ 40{,}000.$$

Example 5.27

Shooting percentages in basketball: A basketball player attempts 15 shots during a game and makes 10 of them. What is their shooting percentage for the game?

Solution

Win-Loss Ratio: $\frac{Wins\ (made\ shots)}{Total\ Attempts} = Shooting\ Percentage$

Shooting Percentage $= \frac{10\ made\ shots}{15\ attempts}$

$$= 66.67\%$$

Example 5.28

Batting averages in baseball: A baseball player gets 3 hits in 10 at-bats over the first week of the season. What is their batting average?

Solution

Win-Loss Ratio: $\dfrac{Hits\ (Wins)}{At-Bats\ (Total\ Attempts)} = Batting\ Average$

$$\text{Batting Average} = \frac{3\ hits}{10\ at-bats}$$
$$= 0.3\ \left(often\ expressed\ as\ a\ decimal\right)$$

Worked Examples with Currency Exchange

Example 5.29

Exchange rates between currencies: The exchange rate between the Ghanaian cedi (GH¢) and the US dollar (USD) is 1 USD = 10.20 GH¢. You want to convert $50 USD to cedis.

Exchange Rate (Win-Loss Ratio): $\dfrac{\text{Amount in New Currency}}{\text{Amount in Old Currency}}$

- o New Currency (GH¢) = $Exchange\ Rate \times Old\ Currency\ (USD)$
- o New Currency (GH¢) = $\dfrac{10.20\ GH¢}{USD \times \$50\ USD} = 510\ GH¢$

Example 5.30

Commission rates for transactions: When exchanging currency, there might be a commission fee. If you're exchanging $100 (USD) and the commission rate is 2%, what is the total cost in USD?

Win-Loss Ratio (Commission): $\dfrac{Commission\ Fee\ (USD)}{Amount\ Exchanged\ (USD)}$

- o Commission Fee (USD) = $Commission\ Rate\ (\%) \times Amount\ Exchanged\ (USD)$
- o Commission Fee (USD) = $2\% \times \$100\ USD = \$2\ USD$
- o Total Cost (USD) = Amount Exchanged (USD) + Commission Fee (USD)
- o Total Cost (USD) = $100 USD + $2 USD = $102 USD

Example 5.31

Purchasing power comparisons: The purchasing power of a currency refers to how much you can buy with it in a specific location. Even with similar exchange rates, the cost of living can vary between countries.

For instance, GH¢10 might buy you a loaf of bread in Ghana, while you might need $2 USD (equivalent to GH¢ 20.40 based on the exchange rate) to buy the same loaf of bread in the US. Here, the exchange rate favours the cedi, but the higher cost of living in the US means you'd effectively "win" more with \$2 USD in Ghana.

Example 5.32

Currency pair ratios in forex trading: Forex traders speculate on the future movement of currency exchange rates. A currency pair ratio might be used to compare the relative value of two currencies.

For instance: A trader might be looking at the EUR/USD (Euro vs US Dollar) currency pair. If the ratio is currently 1.20, it means it takes 1.20 USD to buy 1 EUR. The trader might analyse various factors to predict if the ratio will increase (EUR strengthens against USD) or decrease (USD strengthens against EUR) in the future, impacting their win-loss scenario.

Worked Examples with Utility Bills

1. Drug Dosage Calculations

Key formulas

- $Dosage = \frac{(Prescribed\ dose \times Patient\ weight)}{Standard\ dose}$

  $-\ Drip\ rate\ (mL/hr) = \frac{(Volume \times Drip\ factor)}{Time}$

Note

This is a real-world application where;

a. Nurses use these calculations daily

b. Pharmacists verify dosages

c. Doctors prescribe based on these ratios

2. Body Mass Index (BMI)

Definition: A measure of body fat based on height and weight

Formula,

$$BMI = \frac{eight\ (kg)}{(height\ (m)^2)}$$

After calculating the BMI, we compare it to the BMI Categories

- $Underweight : < 18.5$
- $Normal\ weight : 18.5 - 24.9$
- $Overweight : 25 - 29.9$
- $Obese : \geq 30$

Worked Examples

Example 5.33

A patient weighing 70 kg is prescribed amoxicillin at a dose of 25 mg/kg/day, to be given in 3 divided doses. How many milligrams of amoxicillin should be given per dose?

Solution

Calculate the total daily dose:

$25 \text{ mg/kg/day} \times 70 \text{ kg} = 1750 \text{ mg/day}$

2. Divide the total daily dose by 3 (as it's given in 3 divided doses):

$1750 \text{ mg} \div 3 = 583.33 \text{ mg}$

Therefore, each dose should be 583.33 mg, which in practice would likely be rounded to 580 mg or 585 mg per dose.

Example 5.34

A woman is 165 cm tall and weighs 68 kg. Calculate her BMI and determine which BMI category she falls into.

Solution

The BMI formula is: $\text{BMI} = \dfrac{eight\ (kg)}{(height\ (m))^2}$

Convert height to meters: 165 cm = 1.65 m

Calculate BMI:

$$BMI = \frac{68}{(1.65)^2} = \frac{68}{2.7225} = 24.98 \text{ kg/m}^2$$

Therefore. the woman's BMI is 24.98 kg/m².

Comparing with BMI Categories:

- Underweight: <18.5
- Normal weight: 18.5-24.9
- Overweight: 25-29.9
- Obese: ≥30

With a BMI of 24.98, this woman falls into the "Normal weight" category, though she is very close to the "Overweight" category threshold.

Example 5.35

The weight of a 173*cm* tall teacher is 82kg. Find the body mass index of the teacher and hence, determine the category of the teacher.

Solution

Convert height to meters: 173 cm = 1.73 m

Calculate BMI:

$$\text{BMI} = \frac{82\text{kg}}{1.73^2\,\text{m}} = 27.398$$

Since the BMI falls within this category - Overweight: 25-29.9, he/she is overweight

KEY ASSESSMENT

DoK Level 1 (Recall & Reproduction)

1. A soccer player takes 5 penalty kicks during a game and successfully scores 3 goals. What is their win-loss ratio (success rate) expressed as a decimal for penalty kicks during this game?
2. A baseball team wins 60 games and loses 40 games in a season. What is their win-loss ratio (winning percentage) expressed as a decimal?
3. Player A has a quarterback rating of 95, while Player B has a rating of 88. Based on this rating system, which player is considered statistically more successful?
4. The exchange rate between the US dollar (USD) and the Japanese Yen (JPY) is 1 USD = 132.5 JPY. You have $200 USD and want to convert it to Yen. How many Yen will you receive?
5. You're exchanging 50 Euros (EUR) for Ghanaian Cedis (GH¢). The exchange rate is 1 EUR = 10.50 GH¢, but there's a 1.5% commission fee. What is the total cost of the transaction in Euros?

DoK Level 2 (Skills & Concepts)

6. Imagine bread costs 5 Euros in France and 300 Mexican Pesos (MXN) in Mexico. The exchange rate is 1 EUR = 20 MXN. In which country would you get more bread for your money (considering exchange rates)? Briefly explain your reasoning.
7. Your water bill shows that you consumed an average of 15 cubic metres (m^3) of water per day for the past month. How many cubic metres did you consume in total for the entire month (assuming 30 days)?
8. Your electricity bill uses a tiered pricing structure:
   - **a.** The first 200 kWh of electricity costs 0.10 GH¢ per kWh.
   - **b.** Any consumption exceeding 200 kWh costs 0.15 GH¢ per kWh.

c. If you used a total of 250 kWh last month, what was your total electricity bill in GH¢?

9. Over the past year, you've monitored your monthly electricity consumption and noticed a significant increase in usage during the summer months. Explain how understanding the concept of consumption rate can help you reduce utility bills.

DoK Level 3 (Strategic Thinking)

10. Identify whether the following relationship is a direct variation, inverse variation, or neither: The distance traveled by a car at a constant speed and the time it takes to travel that distance.

11. A recipe requires 2 cups of flour for every 3 eggs. If you want to bake a recipe that uses 6 eggs, how many cups of flour will you need?

12. The force (f) exerted on an object varies inversely with the square of the distance (d) from the source of the force. If the force is 16 N when the distance is 2 meters, find the force when the distance is increased to 4 meters.

Hint

- *The Recommended Mode of Assessment for Week 14 is **Individual Project Work.***
- *Refer to Appendix F at the end of Section 5 for further information on how to go about the project work.*

Section 5 Review

Over the past three weeks, we have covered essential mathematical concepts related to ratios, rates, and proportions, and their applications in various fields, including financial mathematics.

Week 12: Ratios, Rates, and Proportions

- **Concepts:** We established the basic concepts of ratios, rates, and proportions, learning how to compare quantities and understand their relationships.
- **Connections:** We explored the interconnections between these concepts and solved simple problems to reinforce our understanding.

Week 13: Applications of Proportions

- **Angles and Triangles:** We extended the application of ratios to angles and triangles, including the Pythagorean theorem.
- **Everyday Applications:** We investigated the practical use of ratios, rates, and proportions in everyday scenarios, including percent problems.

- **Distance-Time Graphs:** We used distance-time graphs to model and solve problems, providing a visual understanding of these concepts.

Week 14: Financial Mathematics

- **Health and Property Sharing**: We explored the application of ratios, rates, and proportions in health calculations and the sharing of properties.
- **Sports and Currency Exchange:** We investigated their use in sports statistics and currency exchange.
- **Utility Bills and Other Applications:** We applied these concepts to understand and manage utility bills and other financial scenarios.

These weeks have equipped you with a thorough understanding of ratios, rates, and proportions, and their importance in both theoretical and practical contexts. Continue to apply these concepts in your studies and daily life to enhance your mathematical proficiency and problem-solving skills.

APPENDIX E: END OF FIRST SEMESTER EXAMINATION

1. Sample of principles to consider when designing effective multiple-choice questions (MCQ's)

a. Write questions that are clear and straightforward so learners can easily understand them.

b. Make sure each question matches the learning goals and the topics that have been taught in the course.

c. Add incorrect answer choices (distractors) that seem believable and are closely related to the correct answer.

d. Ensure each question has only one clear and correct answer. Avoid using options like "all of the above" or "none of the above," which can sometimes confuse learners.

e. Create questions with different levels of difficulty to distinguish between learners with varying levels of understanding.

f. Make sure questions are free from cultural, language, or gender bias. Use language and examples that are fair and accessible to all learners, regardless of their background.

2. Sample of principles to consider in developing essay-type questions

a. Write questions that are clear and straightforward so learners can easily understand them.

b. Make sure each question matches the learning goals and the topics that have been taught in the course.

c. Add incorrect answer choices (distractors) that seem believable and are closely related to the correct answer.

d. Ensure each question has only one clear and correct answer. Avoid using options like "all of the above" or "none of the above," which can sometimes confuse learners.

e. Create questions with different levels of difficulty to distinguish between learners with varying levels of understanding.

f. Make sure questions are free from cultural, language, or gender bias. Use language and examples that are fair and accessible to all learners, regardless of their background.

3. Sample Table of Test Specification

Weeks	Learning indicators	Type of Questions	DoK Levels				Total
			1	2	3	4	
1	a. Carry out basic operations on surds b. Rationalize monomial denominators	Multiple Choice	1	2	1	-	4
		Essay type	1	-	-	-	1
2	a. Explain the concepts of indices and logarithms with examples. b. Compose and decompose logarithm laws and properties with exponents and apply the concepts to solve real-life problems.	Multiple Choice	1	2	1	-	4
		Essay type	1		-	-	1
3	a. Investigate real-life problems using laws and properties of indices and logarithms. b. Use mathematical connections to explore the relevance of indices and logarithms and their applications to scientific concepts.	Multiple Choice	1	2	1	-	1
		Essay type	-	1	-	-	1
4	a. Undertake a brief review on sorting and divisibility rules of integers and determine their connections to modular arithmetic using models such as number arrays. b. Model simple situations involving modular arithmetic concepts, connect the ideas to real-world problems, and solve them using appropriate models and technology.	Multiple Choice	-	2	1	-	2
		Essay type	-	1	-	-	1
5	a. Analyse two linear equations in two variables and solve them using the elimination and substitution methods.	Multiple Choice	-	1	-	-	2
		Essay type	-	1	-	-	1

Weeks	Learning indicators	Type of Questions	DoK Levels				Total
			1	2	3	4	
6	a. Solve simultaneous linear equations involving two variables using the graphical method and interpret them. b. Analyse, model, and solve word problems of simultaneous linear equations involving numbers and age, etc.	Multiple Choice	-	2	-	-	2
		Essay type	-	-	1	-	1
7	a. Identify and translate an object or point by a translating vector and describe the image. b. Identify and explain the reflection of an object in a mirror line and describe the image points of shapes in a reflection.	Multiple Choice	1	2	1	-	4
		Essay type	1	-	-	-	1
8	a. Identify shapes with rotational symmetry and show the image of an object (or point) after a rotation about the origin (or point). b. Carry out an enlargement of a plane shape given a scale factor.	Multiple Choice	-	-	-	-	-
		Essay type	-	1	-	-	1

Weeks	Learning indicators	Type of Questions	DoK Levels				Total
			1	2	3	4	
9	a. Design a data collection instrument (questionnaire, interview guide, checklist, observation guide, etc.) by employing feasible digital technology (where available) and use it to collect real-life data. b. Evaluate a given set of data and/or its instrument by identifying potential problems related to bias, use of language, gender, ethics, cost, time, privacy, cultural sensitivity, etc.	Multiple Choice	1	-	-	-	1
		Essay type	1	1	1	-	3
10	a. Organise and present data (grouped/ungrouped) by means of ogive, waffle diagrams, box and whisker plots, etc., including generating 3D graphs and solve and/or pose problems.	Multiple Choice	1	1	1	-	3
		Essay type	-	1	2	-	3
11	a. Analyse and interpret data using measures of dispersion and justify which of these measures best suit(s) the data.	Multiple Choice	2	3	3		8
		Essay type	1	2	1		4
12	a. Explain the concepts of ratio and rate as comparison of quantities. b. Establish the relationships among ratio, rates and proportions.	Multiple Choice	2	3	2		7
		Essay type	1	2	1		4
Total							
Multiple Choice			12 (30%)	16 (40%)	12 (30%)	-	40 (100%)
Essay type			6 (30%)	10 (40%)	6 (30%)		22 (100%)

APPENDIX F: SAMPLE RUBRIC FOR SCORING INDIVIDUAL PROJECT WORK

Criteria	Performance Level	Demands	Examples
1. Data Collection and Organisation	**Excellent (3)**	1. Collects complete WASSCE results for all five years without missing data.	Collected data for 2018–2022 for all programs and subjects.
		2. Organises data by year, program, course, and subject.	Created a table with categories for Science, Arts, Business, and their subjects.
	Good (2)	1. Collects WASSCE results for most of the five years, with minor gaps.	Collected data for 2018–2021 but missed 2022.
		2. Organises data, but some programs or subjects are missing or categorisation is inconsistent.	Organised by Science and Arts but omitted some subjects like Core Maths or Elective French.
	Average (1)	1. Collects incomplete data (e.g., fewer than three years or missing major subjects/programs).	Only collected data for 2020–2022, leaving large gaps.
		2. Data is not categorized or lacks adequate breakdown (e.g., only by year).	Only presented total marks by year, with no details by program or subject.

Criteria	Performance Level	Demands	Examples
2. Graphical Representation	Excellent (3)	1. Creates accurate and well-labeled charts (bar, pie, and line) tailored to the data type.	Made a bar chart comparing yearly program performance and a pie chart for subject percentages.
		2. Uses multiple chart types to showcase various trends effectively.	Combined a line chart for trends over years and a pie chart for percentage distribution.
	Good (2)	1. Charts are appropriate but contain minor errors (e.g., inconsistent labeling or incomplete data).	Made a bar chart for yearly performance but left out one program.
		2. Uses only one type of chart to present data trends.	Created a bar chart but no pie chart or line chart for comparison.
	Average (1)	1. Charts are poorly created or do not match the data type.	Used a pie chart for yearly performance when a bar or line chart would be better.
		2. Contains significant errors, missing labels, or irrelevant graphs.	Made a line chart with mislabeled axes and incomplete data sets.

Criteria	Performance Level	Demands	Examples
3. Analysis and Interpretation	**Excellent (3)**	1. Provides detailed and insightful analysis of trends over time and by subject/program.	Found a steady improvement in Science scores and highlighted consistent performance in Arts.
		2. Gives meaningful interpretations of the data, relating them to specific school factors.	"Improved facilities in 2019 led to a rise in Science and Business program performance."
	Good (2)	1. Provides a basic analysis of trends but lacks depth or skips some details.	"Overall performance improved, but specific subjects were not analysed."
		2. Gives general interpretations without clear connections to school factors.	"Scores went up in 2021 without explaining why."
	Average (1)	1. Analysis is shallow, with inaccurate or incomplete trends.	Claimed overall scores improved without providing supporting data.
		2. Conclusions are disconnected from the data or based on assumptions.	Made claims about performance without using data insights.

Criteria	Performance Level	Demands	Examples
4. Statistical Measures	Excellent (3)	1. Applies accurate frequencies and percentages for all data categories.	Found Science scored 70% of total distinctions and Arts 20%.
		2. Presents statistical results clearly in tables or summaries.	Created a table with year-wise averages and percentage increases.
	Good (2)	1. Uses frequencies and percentages, but results contain minor inaccuracies.	Found subject-wise percentages but missed one program's data.
		2. Statistical summaries are present but not clearly presented.	Created an untidy table with no proper headings or labels.
	Average (1)	1. Fails to calculate percentages accurately or uses incomplete data.	"Claimed 90% of distinctions without considering total number of distinctions."
		2. Statistical summaries are missing or poorly formatted.	No tables, summaries, or clear statistical representation.

SECTION 6: PATTERNS AND RELATIONS INVOLVING SEQUENCE AND SERIES

Strand: Algebraic Reasoning

Sub-Strand: Patterns and Relationships

Learning Outcomes:

1. *Explore patterns of a sequence using plane figures and find the nth term and the sum of n terms of an arithmetic and geometric progression.*
2. *Analyse, model, and solve real-life problems involving financial mathematics and exponential growth.*

Content Standard: *Demonstrate understanding of patterns and relations involving sequence and series and generate strategies for algebraic formulas and use them in solving real-life problems.*

INTRODUCTION AND SECTION SUMMARY

Understanding and recognizing arithmetic and geometric progressions (APs and GPs) is a crucial aspect of algebra with numerous practical applications. Finding the nth term of an arithmetic progression involves using a formula that allows for determining any term in the sequence based on its position and the common difference between terms. Similarly, calculating the sum of n terms finds the total of the first n terms, which is particularly useful in various real-life scenarios. Geometric progressions follow a similar approach but involve a common ratio instead of a common difference. Modeling real-life examples with these concepts can include calculating interest rates over time in financial planning, determining population growth, or analyzing patterns in data sets. Mastery of these techniques enhances problem-solving skills and provides valuable tools for interpreting and predicting patterns in diverse fields such as finance, biology, and engineering.

The weeks covered by the section are:

Week 15: Patterns and relations involving sequences and series 1

- *Explore patterns of a sequence using figures and shapes*
- *Explore the differences and similarities between a sequence and a series*
- *Explore patterns of a sequence to establish the nth term ($U_n = ar^{n-1}$) of an arithmetic*

- *Explore and establish the sum ($S_n = \frac{n}{2}\left(2a + (n - 1)d\right)$ of an arithmetic progression.*

Week 16: Patterns and relations involving sequences and series 2

- *Explore patterns of a sequence to establish the nth ($U_n = a r^{n-1}$) and the sum ($S_n = \frac{a(r^n - 1)}{r - 1}$ where $r > 1$ and $S_n = \frac{a(1 - r^n)}{1 - r}$ where $r < 1$) of the nth term of a geometric expression (GP)*

Week 17: Modelling real life problems involving sequence and series

- *Model and solve real-life problems involving linear and exponential growths.*

SUMMARY OF PEDAGOGICAL EXEMPLARS

Learners will benefit from a variety of teaching and learning strategies aimed at building a solid foundation in recognizing and finding the nth term and the sum of the nth term of arithmetic and geometric progressions (AP) or linear sequences, as well as modeling real-life examples of these concepts.

1. **Graphical Representation:** Use graphing tools to visually represent arithmetic and geometric sequences. Encourage students to plot the terms of sequences on graphs to observe patterns and relationships between terms.
2. **Real-life Applications:** Integrate real-life examples to illustrate the relevance and practical applications of sequences and series. For instance, demonstrate how arithmetic sequences can model situations such as saving money over time or planning installment payments, and how geometric sequences can model population growth or radioactive decay.
3. **Collaborative Learning:** Assign group projects that require students to apply their knowledge of sequences and series to solve real-world problems. This promotes teamwork and enhances their analytical and problem-solving skills. Examples include designing a savings plan or analyzing the growth of investments.
4. **Hands-on Activities:** Engage students in hands-on activities such as creating physical models of sequences and series using objects like blocks or beads. This tactile approach helps solidify their understanding of abstract concepts.

ASSESSMENT SUMMARY

Different types of assessments should be conducted to evaluate learners' understanding of the concepts covered in this section. Teachers are encouraged to administer these assessments and document the results for submission to the Student Transcript Portal (STP). The following assessments will be carried out and recorded for each learner:

Week 15: **Group presentation**

Note

For additional information on how to effectively administer the group presentation, please, refer to the Appendices at the end of Section 6.

WEEK 15: PATTERNS AND RELATIONS INVOLVING SEQUENCE AND SERIES 1

Learning Indicators

1. *Explore patterns of a sequence using plane figures and continue with more terms.*
2. *Recognize and find the nth and the sum of the nth term of an arithmetic and geometric progression (AP) or Linear Sequence.*

FOCAL AREA 1: PATTERNS AND RELATIONS INVOLVING SEQUENCE AND SERIES 1

Patterns and relations involve recognising and describing regularities and relationships within sets of numbers or objects. One of the key concepts in this area are sequences and series, which are essential for developing analytical and problem-solving skills.

Sequences are ordered lists of numbers that follow a specific pattern or rule. Each number in a sequence is called a term. Sequences can be finite or infinite, depending on whether they have an end. Common types of sequences include arithmetic sequences, where the difference between consecutive terms is constant, and geometric sequences, where each term is a fixed multiple of the previous term.

Series refer to the sum of the terms of a sequence. For instance, in an arithmetic series, you add up all the terms of an arithmetic sequence. Similarly, in a geometric series, you sum the terms of a geometric sequence. Understanding series helps in solving problems related to summing up long sequences of numbers efficiently.

Real-Life Applications

- **Finance:** Calculating compound interest involves geometric sequences.
- **Physics:** Understanding wave patterns and oscillations often involves arithmetic and geometric series.
- **Computer Science:** Algorithms and data structures use sequences and series for efficient data processing and storage.

Patterns in real-life



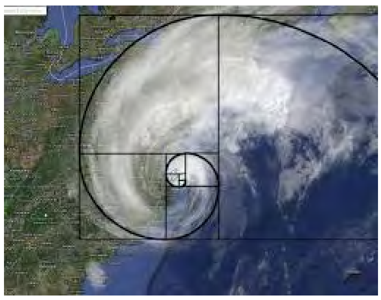
Have you ever stopped to look around and notice all the amazing shapes and patterns we see in the world around us? Mathematics forms the building blocks of the natural world and can be seen in stunning ways. Here are a few of my favourite examples of maths in nature, but there are many other examples as well.

The Fibonacci Sequence

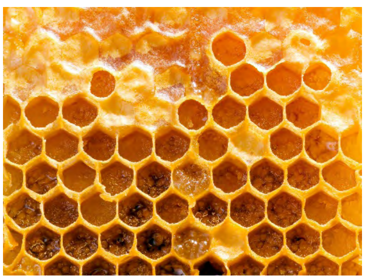
Named after the famous mathematician, Leonardo Fibonacci, this number sequence is a simple, yet profound pattern. Based on Fibonacci's 'rabbit problem,' this sequence begins with the numbers 1 and 1, and then each subsequent number is found by adding the two previous numbers. Therefore, after 1 and 1, the next number is 2 (1 + 1). The next number is 3 (1 + 2) and then 5 (2 + 3) and so on.

What's remarkable is that the numbers in the sequence are often seen in nature. A few examples include the number of spirals in a pine cone, pineapple or seeds in a sunflower, or the number of petals on a flower.





Another of nature's geometric wonders is the hexagon. A regular hexagon has 6 sides of equal length, and this shape is seen again and again in the world around us. The most common example of nature using hexagons is in a bee hive. Bees build their hive using a tessellation of hexagons.

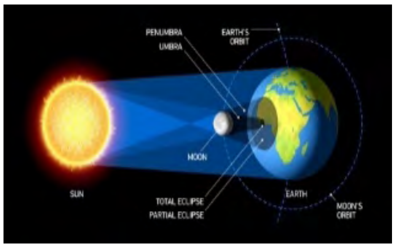


Another common shape in nature is a set of concentric circles, where circles share the same center but have different radii. This means the circles are of varying sizes, one inside the other. A typical example is the ripples in a pond when something hits the water surface. We also see concentric circles in the layers of an onion and the rings of trees that form as they grow and age.

Beyond Earth, we observe similar mathematical features in outer space. For example, our galaxy's shape resembles a Fibonacci spiral. The planets orbit the Sun in concentric paths, and we also see concentric circles in Saturn's rings.

Saturn

We observe a remarkable symmetry in outer space, specifically the alignment between the Earth, Moon, and Sun that enables solar eclipses. Approximately every two years, the Moon aligns between the Sun and Earth in a way that it seems to completely obscure the Sun. This phenomenon is intriguing, given the Moon's significantly smaller size compared to the Sun.



Learning Tasks

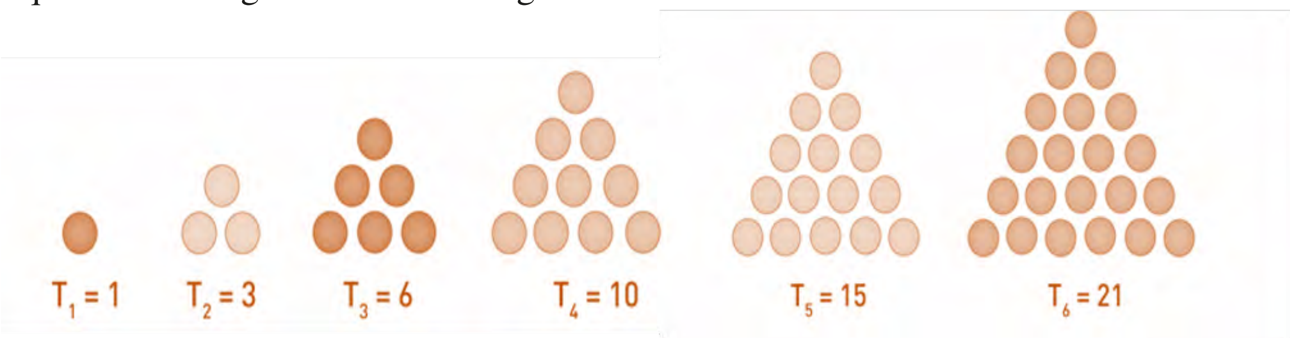
Learners should research and present on various real-life scenarios where patterns can be found.

Explore patterns of a sequence using figures and shapes

We can create a lot of sequences with shapes. When creating these sequences, you need to remember that it should be ordered lists of numbers that follow a specific pattern or rule. Let's take a look at examples of sequences that we can create with shapes.

Triangular Number Sequence

A triangular number sequence is a sequence that is obtained from a pattern forming equilateral triangles. Look at the figure below.



The sequence 1, 3, 6, 10, and so on is a triangular number sequence.

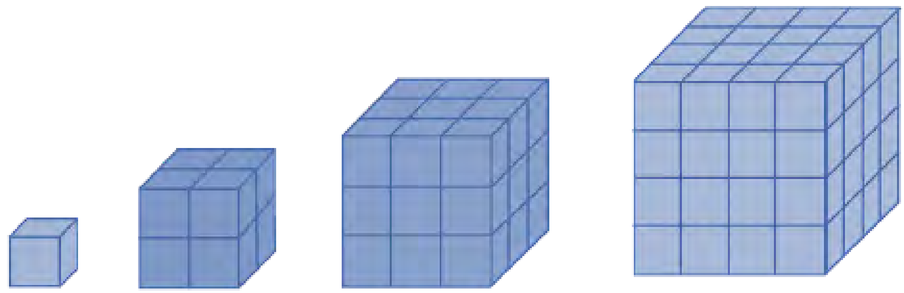
Square Number Sequence

A square number sequence is a sequence that is obtained from a pattern forming squares. Look at the figure below.

The sequence 1, 4, 9, 16, and so on is a square number sequence.

Cube Number Sequence

A cube number sequence is a sequence that is obtained from a pattern forming cubes. Look at the figure below.



The sequence 1, 8, 27, 64, and so on is a cube number sequence.

Learning Tasks

Learners create patterns using shapes and discuss the rule for the pattern.

Explore patterns of a sequence to establish the nth term ($a_1 + (n-1)d$) of an arithmetic

An arithmetic sequence is a sequence of numbers in which each successive term is a sum of its preceding term and a fixed number. This fixed number is called a **common difference.** The terms of the arithmetic sequence are of the form a, a + d, a + 2d, ...

All multiplication tables are arithmetic number patterns. For instance, in the sequences 9, 18, 27, 36, 45, 54 … the common difference is 9. We get the following number by continuously adding 9 to the last number.

General Form of an Arithmetic Sequence

Let the first term of the sequence be a_1. The second term will be a_1+d, the third term will be a_1+2d, and so on. Thus, the nth term can be expressed in terms of a_1 and d.

Express the nth Term

To find a general formula for the nth term, let's observe the pattern:

- 1st term: a_1
- 2nd term: a_1+d
- 3rd term: a_1+2d
- 4th term: a_1+3d
- …
- nth term: $\mathbf{a_1 + (n-1)d}$

Derive the Formula for the nth Term

From the pattern, we can derive that the nth term (a_n) is given by: $a_n = a_1 + (n - 1)d$

Understand the Components of the Formula

- a_n: The nth term of the arithmetic sequence.
- a_1: The first term of the sequence.
- d: The common difference between consecutive terms.
- n: The position of the term in the sequence.

Let's verify this formula with an example. Suppose we have an arithmetic sequence where the first term a_1 is 3, and the common difference d is 5. Find the 4th term (a_4).

Using the formula:

$a4 = a1 + (4 - 1)d$

$a4 = 3 + 3 \times 5$

$a4 = 3 + 15$

$a4 = 18$

Thus, the 4th term of the sequence is 18, confirming that our formula works.

Worked Examples

Example 6.1

Find the formula for the nth term of the arithmetic sequence 2, 5, 8, ... and find the 6^{th} term.

Solution

$a = 2$ and $d = 3$

So

$U_n = 2 + 3(n - 1)$

$\therefore U_n = 3n - 1$

$n = 6$

$\therefore U_n = 3(6) - 1 = 17$

$\therefore$ Therefore, the 6th term of the above sequence is 17.

Example 6.2

Find the 15th term of the arithmetic sequence $-3, -1, 1, 3, ...$ using the arithmetic sequence explicit formula.

Solution

The first term of the given sequence is, $a = -3$

The common difference is, $d = -1 - (-3)$ (or) $1 - (-1)$ (or) $3 - 1 = 2$.

The 15th term of the given sequence is calculated using,

$= a + (n - 1)\, d$

$= -3 + (15 - 1)\, 2 = -3 + 14(2) = -3 + 28 = 25.$

$\therefore$ The 15th term of the given sequence $= 25$.

Example 6.3

Find the common difference of the arithmetic sequence whose first term is $\frac{1}{2}$ and whose 10th term is 9.

Solution

The first term is, a = ½ .

Its 10th term is = 9.

Using the arithmetic sequence formula, $a + (n - 1)\, d$.

Substituting $n = 10$, we get

$9 = (\ 1\ \_\ 2\ ) + (10 - 1)\, d$

$9 = (\ 1\ \_\ 2\ ) + 9d$

Subtracting $\frac{1}{2}$ from both sides,

$\frac{17}{2} = 9d$

Dividing both sides by 9,

$d = \frac{17}{18}$

$\therefore$ The common difference is $\frac{17}{18}$.

Example 6.4

Find the general term (or) n^{th} term of the arithmetic sequence $-\frac{1}{2}, 2, \frac{9}{2}, \ldots$

Solution

The first term of the given sequence is, $a = -\frac{1}{2}$.

The common difference is $d = 2 - \left(-\frac{1}{2}\right) \left(or\right) \frac{9}{2} - 2 = \frac{5}{2}$.

We can find the general term (or) n^{th} term of an arithmetic sequence using the arithmetic sequence explicit formula.

$= a + (n-1)\, d$

$= -\frac{1}{2} + (n-1)\left(\frac{5}{2}\right)$

$= -\frac{1}{2} + \frac{5}{2}n - \frac{5}{2}$

$= \left(\frac{5}{2}\right) n - 3$

$\therefore$ The n^{th} term of the given sequence is $\left(\frac{5}{2}\right) n - 3$.

Sum of Arithmetic Sequence (series)

Express the Sum of the First n Terms

The sum of the first n terms of the arithmetic sequence, denoted by Sn, is the sum of these terms:

$\boldsymbol{Sn = a + (a + d) + (a + 2d) + \cdots + [a + (n-1)d]}$

Write the Sum in Reverse Order

Now, write the sum Sn in reverse order:

$Sn = [a + (n-1)d] + [a + (n-2)d] + \cdots + (a + d) + a$

Add the Two Equations

Add the two expressions for Sn:

$Sn = a + (a + d) + (a + 2d) + \cdots + [a + (n-1)d]$

$Sn = [a + (n-1)d] + [a + (n-2)d] + \cdots + (a + d) + a$

$2Sn = [a + (a + (n - 1)d)] + [(a + d) + (a + (n - 2)d)] + \cdots + [(a + (n - 1)d) + a]$

$2Sn = [2a + (n - 1)d] + [2a + (n - 1)d] + \cdots + [2a + (n - 1)d]$

Simplify the Expression

Notice that there are n terms, each equal to $2a + (n - 1)d$:

$2Sn = n[2a + (n - 1)d]$

Solve for Sn

Divide both sides by 2 to solve for Sn: $Sn = \frac{n}{2}[2a + (n - 1)d]$

Alternative Form (if needed)

Alternatively, the formula can also be written using the first term a and the nth term a_n:

Since $an = a + (n - 1)d$,

$S_n = \frac{n}{2}(a + a_n)$

Let's verify this with an example

Suppose the first term $a = 3$, common difference $d = 5$, and $n = 4$.

Find the sum of the first 4 terms.

Using the formula: $Sn = \frac{n}{2}[2a + (n - 1)d]$

$S4 = \frac{4}{2}[2(3) + (4 - 1)5]$

$S4 = 2[6 + 15]$

$S4 = 2 \times 21$

$S4 = 42$

So, the sum of the first 4 terms is 42.

Worked Examples

Example 6.5

Find the sum of the first 50 terms of the sequence 1, 3, 5, 7, 9, ...

Solution

This is an arithmetic progression, and we can write down

$a = 1$, $d = 2$, $n = 50$

We now use the formula, so that

$Sn = \frac{n}{2}[2a + (n - 1)d]$

$S_{50} = \frac{1}{2} \times 50 \times \left(2 \times 1 + \left(50 - 1\right) \times 2\right]$

$= 25 \times (2 + 49 \times 2)$

$= 25 \times (2 + 98)$

$= 2500$

Example 6.6

Find the sum of the series

$1 + 3{\cdot}5 + 6 + 8{\cdot}5 + \ldots + 101.$

Solution

This is an arithmetic series, because the common difference between the terms is a constant value, 2·5.

We also know that the first term is 1, and the last term is 101. But we do not know how many terms are in the series. So we will need to use the formula for the last term of an arithmetic progression,

$\ell = a + (n - 1)d$

to give us

$101 = 1 + (n - 1) \times 2{\cdot}5.$

Now this is just an equation for n, the number of terms in the series, and we can solve it. If we subtract 1 from each side we get

$100 = (n - 1) \times 2{\cdot}5$

and then dividing both sides by 2·5 gives us

$40 = n - 1$

so that n = 41. Now we can use the formula for the sum of an arithmetic progression, in the version using ℓ, to give us

$S_n = \frac{1}{2} n(a + l)$

$S_{41} = \frac{1}{2} \times 41 \times (1 + 101)$

$= \frac{1}{2} \times 41 \times 102$

$= 41 \times 51$

$= 2091$

Example 6.7

An arithmetic progression has 3 as its first term. Also, the sum of the first 8 terms is twice the sum of the first 5 terms. Find the common difference.

Solution

We are given that $a = 3$. We are also given some information about the sums S_8 and S_5, and we want to find the common difference. So we shall use the formula

$$S_n = \frac{1}{2}n[2a + (n - 1)d]$$

for the sum of the first n terms. This tells us that

$$S_8 = \frac{1}{2} \times 8 \times \left(6 + 7d\right)$$

and that

$$S_5 = \frac{1}{2} \times 5 \times \left(6 + 4d\right)$$

So, using the given fact that $S_8 = 2S_5$, we see that

$$\frac{1}{2} \times 8 \times \left(6 + 7d\right) = 2 \times \frac{1}{2} \times 5 \times \left(6 + 4d\right)$$

$$4 \times (6 + 7d) = 5 \times (6 + 4d)24 + 28d = 30 + 20d$$

$$8d = 6$$

$$d = \frac{3}{4}$$

Example 6.8

Find the sum of the first 10 terms of the arithmetic sequence 3, 7, 11, ...

Solution

$$S_n = \frac{n}{2}\left(2a + (n - 1)d\right.$$

$n = 10$, $a = 3$ *and* $d = 4$

$$S_{10} = \frac{10}{2}\left(2 \times 3 + (10 - 1) \times 4\right.$$

$$S_{10} = 210$$

Learning Tasks

Learners solve problems on arithmetic sequence and sum of arithmetic sequence.

PEDAGOGICAL EXEMPLARS

Using Talk for Learning approaches, engage learners in a whole class to discuss the types of sequences as arithmetic (linear) and geometric (exponential) sequences. Encourage learners to volunteer to lead group discussions, leading to the development of leadership qualities for national development.

Using Collaborative Learning, create convenient groups and task learners to investigate and discuss the meaning of arithmetic progression as a sequence and establish the general rule for the nth term by creating different numerical sequences and investigating the patterns.

KEY ASSESSMENT

1. Write down the first five terms of the AP with first term 8 and common difference 7.

2. Write down the first five terms of the AP with first term 2 and common difference –5.

3. What is the common difference of the AP 11, –1, –13, –25, . . . ?

4. Find the 17th term of the arithmetic progression with first term 5 and common difference 2.

5. Write down the 10th and 19th terms of the APs

a. 8, 11, 14, . . .,

b. 8, 5, 2 . . ..

6. Find the sum of the first 23 terms of the AP 4, –3, –10, . . .

7. An arithmetic series has first term 4 and common difference $\frac{1}{2}$. Find:

a. the sum of the first 20 terms,

b. the sum of the first 100 terms.

8. Find the sum of the arithmetic series with first term 1, common difference 3, and last term 100.

9. The sum of the first 20 terms of an arithmetic series is identical to the sum of the first 22 terms. If the common difference is –2, find the first term.

Hint

- *The Recommended Mode of Assessment for Week 15 is* ***Group presentation.***
- *Refer to Appendix G at the end of Section 6 for further information on how to go about the group presentation.*

WEEK 16: PATTERNS AND RELATIONS INVOLVING SEQUENCE AND SERIES 2

Learning Indicator: *Identify geometric progressions or exponential sequence and find the algebraic expression for the general term.*

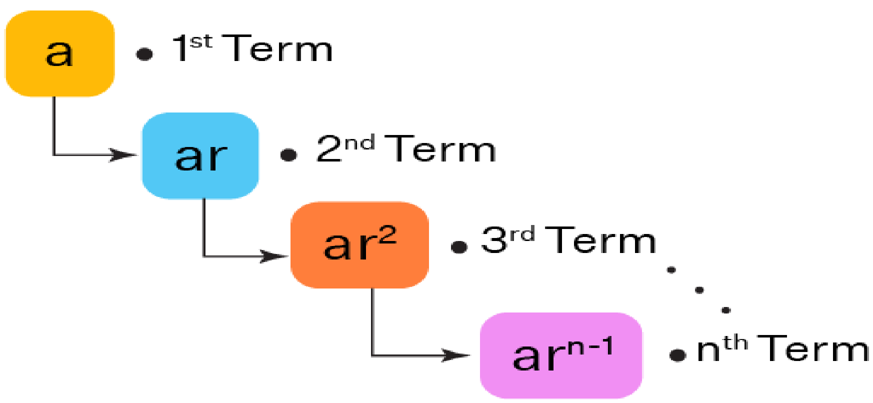
What is a Geometric Sequence/Progression (GP)?

A **geometric sequence/progression** *is a special type of sequence where the* ratio *of every two successive terms is a constant. This ratio is known as a common ratio of the geometric sequence. In other words, in a geometric sequence, every term is multiplied by a constant which results in its next term. So,* a geometric sequence *is in form a*, *ar*, *a* r^2... where '*a*' is the first term and '*r*' is the common ratio of the sequence. The common ratio can be either a positive or a negative number.

Key Characteristics of Geometric Sequences/Progression:

1. Initial Term (a): The first term in the sequence.
2. Common Ratio (r): The factor by which each term is multiplied to get the next term.

The general form of a geometric sequence can be written as:



Examples of Geometric Sequences

1. **Doubling Sequence:** 1, 2, 4, 8, 16, ... (common ratio $r = 2$)
2. **Halving Sequence:** 100, 50, 25, 12.5, ... (common ratio $r = 0.5$)
3. **Negative Ratio Sequence:** 5, -10, 20, -40, ... (common ratio $r = -2$)

Geometric Series

A geometric series is the sum of the terms of a geometric sequence. For example, the series corresponding to the geometric sequence 2, 6, 18, 54, ... is 2 + 6 + 18 + 54 + ...

Sum of a Finite Geometric Series:

The sum of the first n terms of a geometric series can be calculated using the formula:

$$S_n = a\,\frac{1-r^n}{1-r}$$

where S_n is the sum of the first n terms, a is the first term, r is the common ratio, and n is the number of terms.

Sum of an Infinite Geometric Series:

For an infinite geometric series where $|\,r\,| < 1$, the sum can be calculated using the formula: $S = \frac{a}{1-r}$

where S is the sum of the series, a is the first term, and r is the common ratio.

Real-Life Applications of Geometric Sequences and Series

Geometric sequences and series are widely used in various fields such as finance, physics, biology, and computer science. For instance:

- **Finance:** Calculating compound interest where the amount of money grows geometrically.
- **Biology:** Modeling population growth under ideal conditions.
- **Physics:** Analyzing radioactive decay or wave frequencies.
- **Computer Science:** Designing algorithms that perform exponentially growing tasks.

Worked Examples on Geometric sequence

Example 6.9

Find the 8th term of the exponential sequence 2, 6, 18, 54…

Solution

$U_n = ar^{n-1}$ $a = 2$, $r = 3$, $n = 8$

$U_8 = 2 \times 3^{8-1}$

$U_8 = 4372$

Example 6.10

The second and the fourth terms of an exponential sequence are 9 and 4, respectively. Find the sequence.

Solution

$U_2 = ar = 9$ …… .(1)

$U_4 = ar^3 = 4$…….(2)

Solving the equation simultaneously $r = \frac{2}{3}$ and $a = 13.5$

Using a, ar, ar^2, ar^3… the sequence is 13.5, 9, 6, …

Example 6.12

Find the 7th term in the following sequence: 6, 18, 54, 162, …

Solution

Finding the common ratio can be harder than finding the common difference. One way to find it is the

divide each term by the term before it.

$18 \div 6 = 3$, $54 \div 18 = 3$,

$162 \div 54 = 3$ So the common ratio is 3

$a_7 = 6 \times 3^6 = 6 \times 729 = 4{,}374$

So the 7th term of the sequence is 4, 374.

Example 6.13

Find the 8th term in the following sequence: 96, 48, 24, 12, 6, …

Solution

To find the common ratio, divide each term by the one before it.

∴ The common ratio is ½

$a_8 = 96 \times (1 \_ 2)^{8-1} = 96 \times 1\frac{=}{28} = 0.75$

The 8th term of the sequence is 0.75.

Sum of geometric sequence

Example 6.14

Find the sum of the first 7 terms of the G P $\frac{1}{2}$, 1, 2, 4, …

Solution

$S_n = \frac{a(r^n - 1)}{r - 1}$ since $r > 1$ $r = 2$, $a = \frac{1}{2}$, $n = 7$

$S_7 = \frac{\frac{1}{2}(2^7 - 1)}{2 - 1}$

$S_7 = \frac{1}{2}(256 - 1)$

$S_7 = 127.5$

Example 6.15

Find the sum of the geometric series

$2 + 6 + 18 + 54 + \ldots$

where there are 6 terms in the series.

Solution

For this series, we have $a = 2$, $r = 3$ and $n = 6$. So

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_6 = \frac{2(1 - 3^6)}{1 - 3}$$

$$= \frac{2(1 - 729)}{-2}$$

$$= -(-728)$$

$$= 728.$$

Example 6.16

How many terms are there in the sequence 2, 4, 8, . . ., 128?

Solution

In this sequence $a = 2$ and $r = 2$. We also know that the nth term is 128.

But the formula for the nth term is ar^{n-1}. So

$128 = 2 \times 2^{n-1}$

$64 = 2^{n-1}$

$2^6 = 2^{n-1}$

$6 = n - 1$

$n = 7.$

So, there are 7 terms in this geometric progression.

Example 6.17

Find the sum of the infinite geometric series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{6} + \ldots$

Step-by-Step Solution

1. **Identify the first term a and the common ratio r**:
   - The first term a $= \frac{1}{2}$
   - The common ratio r $= \frac{1}{4} \div \frac{1}{2} = \frac{1}{2}$.

Check if the series converges

- For an infinite geometric series to converge, the absolute value of the common ratio must be less than 1 $(i.e., |r| < 1)$.

In this case, $|r| = \left|\frac{1}{2}\right| = \frac{1}{2} < 1$. So the series converges.

Use the formula for the sum of an infinite geometric series

- The sum S of an infinite geometric series with first term a and common ratio r is given by:

  $S = \frac{a}{1 - r}$

Substitute the values of a and r into the formula:

- Here, a $= \frac{1}{2}$ and r $= \frac{1}{2}$. Therefore,

  $S = \frac{1\_2}{1 - \frac{1}{2}}$

Simplify the expression:

$$S = \frac{1\_2}{\frac{1}{2}} = 1$$

The sum of the infinite geometric series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + 1\_\_6 + \ldots is\ 1$.

So, the infinite geometric progression converges to the sum of 1.

Learning Tasks

Learners solve problems on geometric sequence and sum of geometric sequence.

PEDAGOGICAL EXEMPLARS

Using Collaborative Learning, create convenient groups and task learners to investigate the general rule for the nth term and solve problems on geometric sequence.

KEY ASSESSMENT

1. Write down the first five terms of the geometric progression which has first term 1 and common ratio ½.
2. Find the 10th and 20th terms of the GP with first term 3 and common ratio 2.
3. Find the 7th term of the GP 2, −6, 18, . . .,
4. Find the sum of the first five terms of the GP with first term 3 and common ratio 2.
5. Find the sum of the first 20 terms of the GP with first term 3 and common ratio 1.5.
6. How many terms in the GP 4, 3.6, 3.24, . . . are needed so that the sum exceeds 35?
7. Determine if the sequence is arithmetic or geometric, and then find the given term.
   a. 11^{th} term: 5, 3, 1, –1, … _____________
   b. 23^{rd} term: 0.1, 0.15, 0.2, 0.25, … ______________
   c. 6^{th} term: 25, 75, 225, 675, … _______________
   d. 22^{nd} term: –2, –5, –8, –11, –14, … _____________

REMINDER

- *Learners' score on individual class exercise should be ready for submission into the STP.*

WEEK 17: REAL LIFE PROBLEMS INVOLVING SEQUENCE AND SERIES

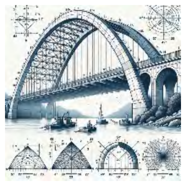
Learning Indicators

1. *Analyse, model, and solve real-life problems involving financial mathematics.*
2. *Analyse, model, and solve real-life problems involving exponential growth.*

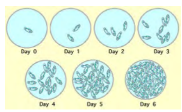
FOCAL AREA: MODELLING REAL LIFE PROBLEMS INVOLVING SEQUENCES AND SERIES

Sequences and series are not just abstract mathematical concepts; they are powerful tools that help us understand and solve real-life problems across various fields. A sequence is an ordered list of numbers, while a series is the sum of the terms of a sequence. These mathematical constructs can model a wide range of phenomena in nature, economics, engineering, and everyday life.

Finance Example: Saving for Retirement: If you save a fixed amount of money each month, the total savings can be modeled using an arithmetic series.



Engineering Example: Bridge Design: Calculating the total length of materials needed for a bridge with regularly spaced supports uses sequences and series.



Biology Example: Population Growth: Modeling the growth of a bacteria colony that doubles every hour involves a geometric sequence.

Environmental Science Example: Carbon Emissions: Analyzing the cumulative impact of annual carbon emissions uses series to understand long-term effects.

Learning Task for Practice

Learners investigate and make presentations on real life situations that involve sequence and series.

Model and solve real-life problems involving linear and exponential growths.

Example 6.18

A man saves GH¢16500 in ten years. In each year after year, he saved GH¢100 more than he did in the previous year. How much did he save in the first year?

Solution

Sequence: $a, a + 100, a + 200, a + 300, \ldots$

$d = (a + 100) - a = 100.$

$n = 10, S_{10} = 16500$

$S_n = \frac{n}{2}(2a + n - 1)d$

$16500 = \frac{n}{2}(2a + 10 - 1100$

$a = 1200$

Hence, he saved GH¢1200 in the first year.

Example 6.19

A job pays a salary of GH¢25,000 for the first year. For the next 19 years, the salary increases by 5% each year. What is the total amount of money earned over the twenty years?

Solution

$r = \frac{100 + 5}{100} = 1.05, a = 25000\ n = 20$

$S_{20} = 25000 + 25000(1.05) + 25000(1.05)^2 + \ldots + 25000(1.05)^{20-1}.$

$S_{20} = \frac{a(r^n - 1)}{r - 1}$

$S_{20} = \frac{25000(1.05^{20} - 1)}{1.05 - 1} = \text{GH¢ } 826{,}648.85$

Example 6.20

An island has a population of 1500 and is growing at a rate of 3% per year. What will the population be after 7 years?

Solution

$start \rightarrow U_1.$

$Year\ 1 \rightarrow U_2.$

$Year\ 2 \rightarrow U_3.$

$\downarrow.$

$Year\ 7 \rightarrow U_8$

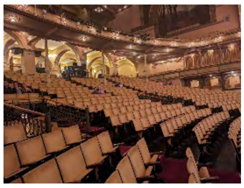
$U_n = ar^{n-1}\ r = 1.03, n = 7\ a = 1500.$

$U_8 = 1500(1.03)^7$

$U_8 = 1844.81$

Therefore, the population of the island will be approximately 1844.

Example 6.21



A local theatre has a large auditorium with 22 rows of seats. There are 18 seats on Row 1 and each row after Row 1 has two more seats than the previous row. How many seats are in Row 22?

Solution

Identify the first term (a) and the common difference (d):

- The first term (a) is the number of seats in Row 1, which is 18.
- The common difference (d) is the increase in the number of seats per row, which is 2.

Using $a_n = a_1 + (n - 1)d$

Where:

- a_n is the n^{th} term,
- a is the first term,
- n is the term number,
- d is the common difference.

$a_{22} = 18 + (22 - 1) \times 2$

$= 18 + 42$

$= 60$

There are 60 seats in Row 22 of the theatre.

Example 6.22

In the theatre example we just solved, there were 22 rows of seating. There were 18 seats on Row 1 and each subsequent row had two more seats than the previous row. What is the seating capacity of the auditorium?

Solution

$S_n = \frac{n}{2}(2a + n - 1)d$

Where:

- S_n is the sum of the first n terms,
- a is the first term,
- n is the number of terms,
- d is the common difference.

$S_{22} = 22 \_ 2\ (2 \times 18 + (22 - 1) \times 2$

$S_{22} = 11(36 + 21 \times 2)$

$S_{22} = 11(36 + 42)$

$S_{22} = 11 \times 78$

$S_{22} = 858$

$\therefore$ The seating capacity of the auditorium is 858 seats.

Example 6.23

As part of your history class, you decided to save for your trip to a historical site. You opened a susu account with GH¢1.00 and on each subsequent day, you deposited a cedi more than on the previous day. How much have you contributed by the end of one year?

Solution

We can write the sequence like this $1 + 2 + 3 + 4\ldots +365$

Using the formula; $S_n = \frac{1}{2}\, n(a + l)$

$= \frac{365}{2}(1 + 365)$

$= \frac{365}{2} \times 366 = 365 \times 183 = 66{,}795.$

Therefore, you would have contributed GH¢66,795.00.

Learning Tasks

Learners solve real life problems on arithmetic and geometric sequence and series.

PEDAGOGICAL EXEMPLARS

Put learners into convenient groups and engage them in pairs to analyse and model real-life problems of sequence involving growth and discuss your findings.

Encourage learners to give fair criticisms to their colleagues' presentations. And in doing so, creating a sustainable discourse for learners to question norms, practices, and opinions; and to reflect on one's own values, perceptions and actions for decision-making as they engage in group and whole class activities on exponential growth.

KEY ASSESSMENT

1. After knee surgery, your trainer tells you to return to your jogging program slowly. He suggests jogging for 12 minutes each day for the first week. Each week thereafter, he suggests that you increase that time by 6 minutes per day. How many weeks will it be before you are up to jogging 60 minutes per day?

2. A factory produces ceramic bowls that are designed to be stacked. The height of each bowl in a stack increases by a constant increment due to the shape of the bowls. The height of the first bowl is 5 cm, and each subsequent bowl adds 0.5 cm to the total height of the stack.

   a. What is the height of a stack of 10 bowls?
   b. If the maximum height that a stack can safely reach is 40 cm, how many bowls can be stacked together without exceeding this height?
   c. What is the total height of a stack of 20 bowls?

3. A restaurant is setting up for a special event and wants to arrange tables in a line to accommodate more guests. Each table can seat 4 people, but when two tables are placed together, they share one side, and thus the total seating capacity increases by 3 additional seats for each additional table added to the line.

   a. If the restaurant starts with 1 table, how many people can be seated?
   b. How many people can be seated if the restaurant sets up 10 tables in a line?
   c. What is the total seating capacity if the restaurant arranges 15 tables in a line?

4. A gardener is building a rectangular fence around a garden. The fence is initially 10 meters long and 6 meters wide. The gardener adds additional fence panels to each side of the rectangular fence to expand the garden. Each fence panel is 2 meters long.

   a. Initial Perimeter Calculation:

      Calculate the initial perimeter of the rectangular fence.

   b. Adding One Panel to Each Side:

      Determine the new perimeter after adding one fence panel to each side of the rectangle.

   c. Adding Two Panels to Each Side:

      Determine the new perimeter after adding two fence panels to each side of the rectangle.

   d. General Pattern:

      Discuss the pattern and formula for the perimeter if n fence panels are added to each side of the rectangle.

5. A badminton tournament begins with 128 teams. After the first round, 64 teams remain. After the second round, 32 teams remain. How many teams remain after the third, fourth, and fifth rounds?

6. You start a chain email and send it to six friends. The next day, each of your friends forwards the email to six people. The process continues for a few days. After how many days will 1296 people have received the email?

7. A soup kitchen makes 16 gallons of soup. Each day, a quarter of the soup is served and the rest is saved for the next day.

a. Write the first five terms of the sequence of the number of fluid ounces of soup left each day.

b. Write an equation that represents the nth term of the sequence.

c. When is all the soup gone? Explain.

Section 6 Review

The weeks covered by the section are:

Week 15: Patterns and relations involving sequence and series 1

Sequence:

- *A sequence is an ordered list of numbers, where each number is called a term.*
- *Terms are arranged in a specific order, following a particular rule or pattern.*
- *Example: 2, 4, 6, 8, 10 (an arithmetic sequence with a common difference of 2).*

Series:

- *A series is the sum of the terms of a sequence.*
- *It represents the cumulative addition of sequence terms.*
- *Example: 2 + 4 + 6 + 8 + 10 (the arithmetic series corresponding to the sequence above).*

Similarities:

- *Both sequences and series are lists of numbers that follow a specific rule or pattern.*
- *Sequences and series can be arithmetic, geometric, or follow other types of mathematical progressions.*

Differences:

- *A sequence is a list of numbers, whereas a series is the sum of a sequence of numbers.*
- *Sequences focus on individual terms, while series focus on the cumulative sum of terms.*

In the lesson we;

- *established the nth term ($U_n = a\,r^{n-1}$) of an arithmetic.*
- *established the sum ($S_n = \frac{n}{2}\big(2a + (n - 1)d$) of an arithmetic progression.*

Week 16: Patterns and relations involving sequence and series 2

In the lesson we;

- *established the nth ($U_n = a\,r^{n-1}$) and the sum ($S_n = \dfrac{a(r^n - 1)}{r - 1}$ where $r > 1$ and $S_n = \dfrac{a(1 - r^n)}{1 - r}$ where $r < 1$) of the nth term of a geometric expression (GP)*

Week 17: Modelling real life problems involving sequence and series

In the lesson we;

- *Modelled and solved real-life problems involving linear and exponential growths.*

RESOURCES

[Math in Nature: 5 Stunning Ways We See Math in the World (mathgeekmama.com)]

APPENDIX G: SAMPLE RUBRIC FOR SCORING GROUP PRESENTATION

Criteria	Marks	Examples
Communication Skills	**3 marks:** Clear, confident voice; steady eye contact; engages audience effectively. **2 marks:** Moderately clear voice; occasional eye contact; limited engagement. **1 mark:** Weak voice; minimal eye contact; no engagement.	**3 marks Example:** "The speed is distance ÷ time. Here, it's 180 km ÷ 3 hours = 60 km/h." **2 marks Example:** "Speed is calculated as distance over time, which is 180 ÷ 3." **1 mark Example:** "The answer is 60."
Design & Creativity	**3 marks:** Creative, appealing visuals; balanced text and images. **2 marks:** Basic layout; visuals support topic but lack impact; slight imbalance in text/images. **1 mark:** Simple layout; unclear visuals; poor integration.	**3 marks Example:** A well-designed graph with a labeled slope and detailed explanation of the speed. **2 marks Example:** A basic graph with correct labels but minimal explanation. **1 mark Example:** A graph with no labels or explanations.
Analytical Skills	**3 marks:** Systematically breaks down the problem; accurately calculates and interprets. **2 marks:** Correct calculation but misses details. **1 mark:** Misinterprets graph; errors in calculations.	**3 marks Example:** Identifies slope as speed and explains how $180 \div 3 = 60$ km/h. **2 marks Example:** Correctly calculates 60 but does not connect it to the graph. **1 mark Example:** Miscalculates or misinterprets the graph.

Criteria	Marks	Examples
Collaboration Skills	**3 marks:** Equal participation; coordinates efforts; credits all members. **2 marks:** Participates but lacks coordination or inclusivity. **1 mark:** Minimal collaboration; work dominated by a few members.	**3 marks Example:** Each group member explains part of the problem and presentation. **2 marks Example:** Only one or two members present while others remain passive. **1 mark Example:** No collaboration or teamwork.
Total score: 12 marks		

SECTION 7: SURFACE AREAS & VOLUMES

Strand: Geometry Around Us

Sub-Strand: Measurement

Learning Outcome: *Determine the volume and capacity of solid shapes and solve problems that involve SI and imperial units in surface area, volume and capacity measurements.*

Content Standard: *Demonstrate conceptual understanding of the measurement of surface area, volume and capacity of solid shapes.*

INTRODUCTION AND SECTION SUMMARY

Solving problems involving SI and imperial units in surface area and volume measurements is essential in many fields, including engineering, construction and everyday life. Surface area calculations in both systems require an understanding of the formulas for different shapes and the ability to convert between units, such as square metres to square feet. Verification of solutions ensures accuracy and consistency in measurements. Similarly, volume measurements in SI units like cubic metres or litres and imperial units like cubic feet or gallons are crucial for tasks such as determining the capacity of containers or calculating the amount of material needed for a project. Real-life problems involving volume measurements can include calculating the amount of water needed to fill a swimming pool, the concrete required for construction or the fuel capacity of a tank. Mastery of these skills equips learners with practical tools for solving real-world problems and enhances their ability to work effectively in various professional and everyday contexts.

Week 18: Real world problems involving SI and imperial units

1. *Review imperial and SI units in surface areas*
2. *Solve problems involving SI and imperial units in surface area*

Week 19: Problems that involve SI and imperial units in volume and capacity measurements.

1. *Review imperial and SI units in volumes*
2. *Solve problems involving SI and imperial units in volume and capacity.*

Week 20: Real world problems that involve the volume/capacity of a 3-D object.

1. *Model and solve real world problems that involves the volume/capacity of a 3-D object.*

SUMMARY OF PEDAGOGICAL EXEMPLARS

Learners will benefit from a variety of teaching and learning strategies aimed at building a solid foundation in solving problems that involve SI and imperial units in surface area and volume measurements, as well as verifying solutions and applying these concepts to real-life scenarios.

1. **Practical Measurements:** Conduct hands-on activities where learners measure the dimensions of real objects (e.g., boxes, cans and rooms) using both SI and imperial units. This helps them understand the practical applications of surface area and volume calculations.
2. **Unit Conversions:** Incorporate exercises that require learners to convert between SI and imperial units. Provide real-world problems that necessitate these conversions, such as calculating the paint needed for a wall (surface area) in square metres and square feet or the volume of a pool in litres and gallons.
3. **Real-life Applications:** Integrate real-life examples to illustrate the relevance of surface area and volume measurements. For instance, demonstrate how surface area calculations are essential for packaging design, while volume measurements are crucial for determining the capacity of containers.
4. **Technology Integration:** Utilise software tools and online calculators that help learners visualise and verify their surface area and volume calculations. These tools can provide immediate feedback and assist in understanding the step-by-step processes involved.
5. **Problem-based Learning:** Present learners with challenging, real-life problems that require surface area and volume calculations in both SI and imperial units. Examples include determining the amount of material needed to cover a roof or the volume of soil required for a garden bed.
6. **Hands-on Projects:** Engage learners in hands-on projects, such as building models of geometric shapes and calculating their surface area and volume using both SI and imperial units. This tactile approach helps solidify their understanding of abstract concepts.

ASSESSMENT SUMMARY

Different types of assessments should be conducted to evaluate learners' understanding of the concepts covered in this section. Teachers are encouraged to administer these assessments and document the results for submission to the Student Transcript Portal (STP). The following assessments will be carried out and recorded for each learner:

Week 18: **Mid-semester Examination**

Note

For additional information on how to effectively administer the mid-semester examination, please, refer to the Appendix at the end of Section 7.

WEEK 18: SURFACE AREA

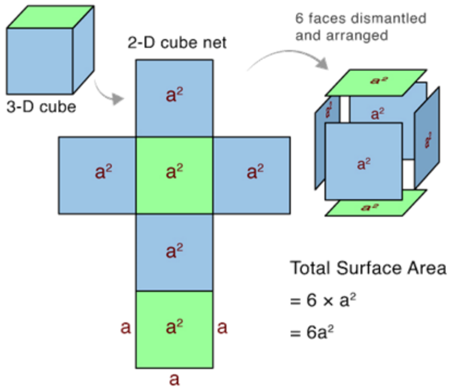
Learning Indicator: *Solve problems that involve SI and imperial units in surface area measurements and verify the solutions.*

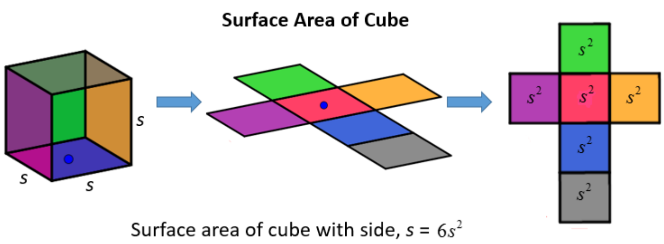
FOCAL AREA: MEASUREMENT OF SURFACE AREAS INVOLVING IMPERIAL AND SI UNITS

Definition of Surface Area

Surface area is the total area that the surface of a three-dimensional object occupies. It is the sum of the areas of all the faces or curved surfaces of the object. For example, the surface area of a cube is the sum of the areas of its six square faces.

Study the images carefully. You realise that when you take any box similar to the ones in the picture and dismantle it, you obtain the net. Now, the surface area of the box will be the sum of the area of the six faces.

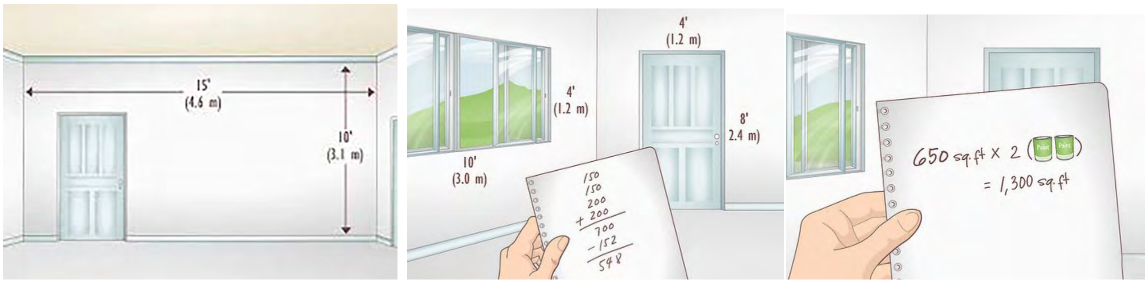




Importance of calculating surface area in real-life scenarios

Calculating surface area is crucial in various real-life situations, for example:

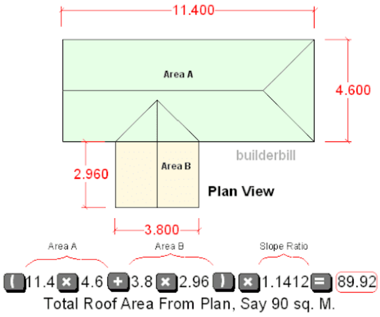
Construction: Determining the amount of paint needed for walls, ceilings and other surfaces.

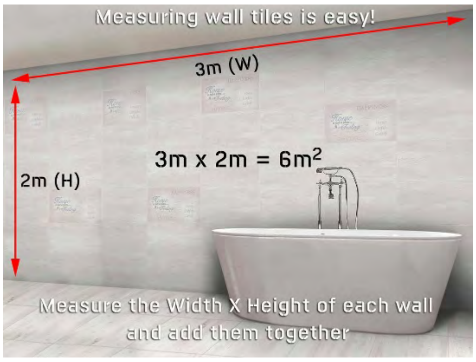


Packaging: Calculating the quantity of material required to wrap an object.



Manufacturing/Construction: Designing and creating objects with specific surface area requirements for functionality and aesthetics.





Units of Measurement

SI (metric) units (square metres, square centimetres). The International System of Units (SI) is the modern form of the metric system and is the most widely used system of measurement.

1. **Square Metre (m^2):** The base unit for measuring surface area in the metric system. One square metre is the area of a square with sides that are one metre in length. It is used for larger surface areas such as rooms, fields and large objects.
2. **Square Centimetre (cm^2):** A smaller unit for measuring surface area. One square centimetre is the area of a square with sides that are one centimetre in length. There are 10 000 square centimetres in a square metre. It is used for measuring smaller surfaces like pages of a book, small objects or parts of larger surfaces.

Imperial units (square feet, square inches). The Imperial system of units is commonly used in the United States and a few other countries.

1. **Square Foot (ft^2):** The base unit for measuring surface area in the Imperial system. One square foot is the area of a square with sides that are one foot in length. It is commonly used in real estate, construction and interior design.
2. **Square Inch (in^2):** A smaller unit for measuring surface area. One square inch is the area of a square with sides that are one inch in length. There are 144 square

inches in a square foot. It is used for measuring smaller surfaces like parts of furniture, books or small household items.

Conversion between square metres and square centimetres

There are 10 000 square centimetres in one square metre. To convert square metres to square centimetres, multiply by 10 000.

Example 7.1

$2\,m^2 = 2 \times 10\,000 = 20\,000\,cm^2$

To convert square centimetres to square metres, divide by 10 000.

Example 7.2

$15\,000\,cm^2 = \frac{15\,000}{10\,000} = 1.5\,m^2$

Conversion between square feet and square inches

There are 144 square inches in one square foot. To convert square feet to square inches, multiply by 144.

Example 7.3

$3\,ft^2 = 3 \times 144 = 432\,in^2$

To convert square inches to square feet, divide by 144.

Example 7.4

$288\,in^2 = \frac{288}{144} = 2\,ft^2$

Conversion Factors

1. **Square Metres to Square Feet**

   1 square metre (m^2) = 10.7639 square feet (ft^2)

2. **Square Feet to Square Metres**

   1 square foot (ft^2) = 0.092903 square metres (m^2)

3. **Square Centimetres to Square Inches**

   1 square centimetre (cm^2) = 0.155 square inches (in^2)

4. **Square Inches to Square Centimetres**

   1 square inch (in^2) = 6.4516 square centimetres (cm^2)

How to Use Conversion Factors in Calculations

1. ***Identify the Unit to Convert from and to:***
   a. Determine the original unit and the desired unit for conversion.

2. ***Find the Appropriate Conversion Factor***
   b. Use the relevant conversion factor for the units you are converting between.

3. ***Multiply or Divide Using the Conversion Factor***
   Examples

   i. Convert 5 square metres to square feet.
   Multiply the original measurement by the conversion factor.
   $5\,\text{m}^2 \times 10.7639 = 53.8195\,\text{ft}^2$

   ii. Convert 200 square centimetres to square Inches.
   Divide the original measurement by the conversion factor.
   $200\,\text{cm}^2 \div 6.4516 = 31.0\,\text{in}^2$

4. ***Double-Check the Calculation***
   Ensure that the units cancel out appropriately, leaving you with the desired unit.

Worked Examples

Example 7.5

Converting SI to Imperial

Convert 8 square metres to square feet.

$8\,\text{m}^2 \times 10.7639 = 86.1112\,\text{ft}^2$

Example 7.6

Converting Imperial to SI

Convert 50 square feet to square metres.

$50\,\text{ft}^2 \div 10.7639 = 4.64515\,\text{m}^2$

Example 7.7

Converting SI to Imperial

Convert 150 square centimetres to square inches.

$150\,\text{cm}^2 \div 6.4516 = 23.25\,\text{in}^2$

Example 7.8

Converting Imperial to SI

Convert 300 square inches to square centimetres.

$300\,\text{in}^2 \times 6.4516 = 1\,935.48\,\text{cm}^2$

Learning Tasks

Learners are to discuss the importance of calculating surface area in real-life scenarios.

1. Learners are to convert Imperial to SI and vice versa.

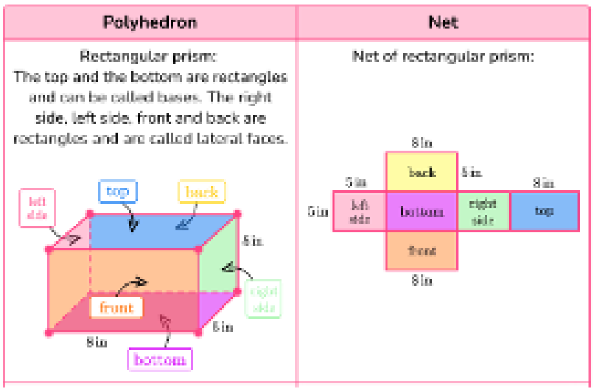
PEDAGOGICAL EXAMPLES

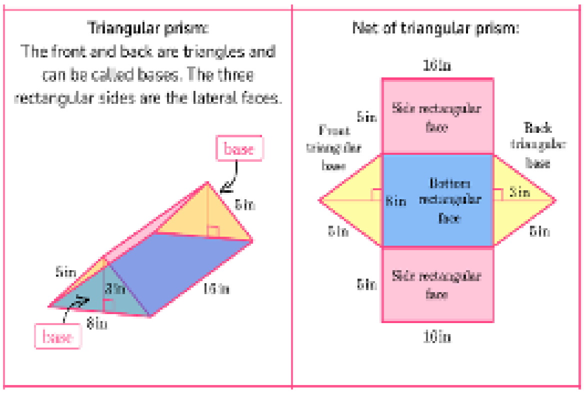
In mixed ability groups, learners discuss the importance of calculating surface area in real-life scenarios.

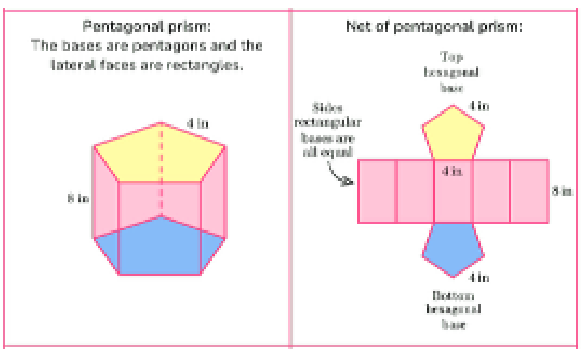
Using think-pair-squares and pairs, learners are to perform conversions from imperial to SI and vice versa.

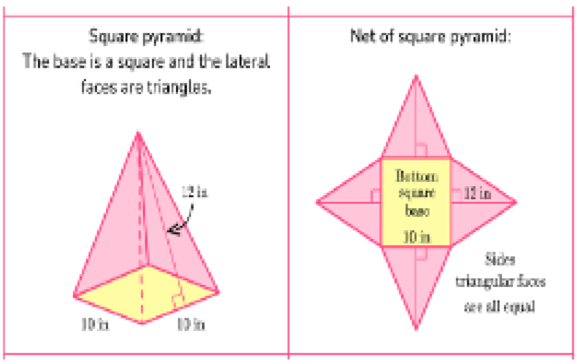
Experiential Learning: In convenient groups, explain, showing practical examples, the difference between volume and surface area. Reward honesty as a strong moral principle as learners discuss their challenges solving problems on surface area measurements.

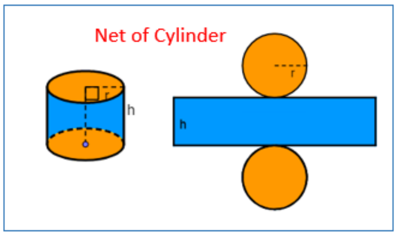
Nets of 3D objects (prisms, cones, pyramids, spheres, etc.)

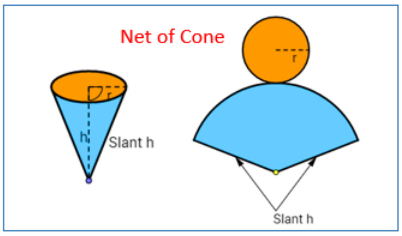




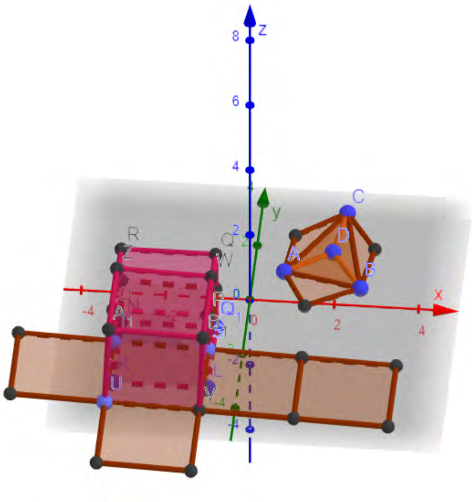


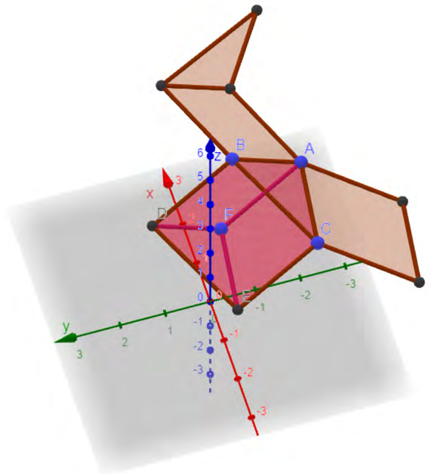






Use Computer Applications such as Geogebra to explore nets

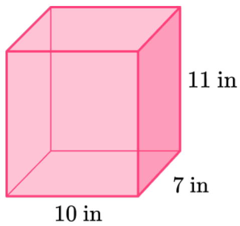




Draw Nets

Example 7.9

Draw the net representing this rectangular prism.



Solution

a. Identify all the faces of the 3D solid.

There are 6 faces on this rectangular prism.

The bottom face (base) is a rectangle with dimensions of 10 inches ×7 inches.

The top face (base) is equal to the bottom.

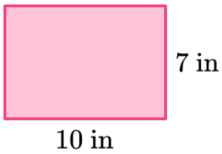
The right face is a rectangle that is 7 inches by 11 inches.

The left face is equal to the right face.

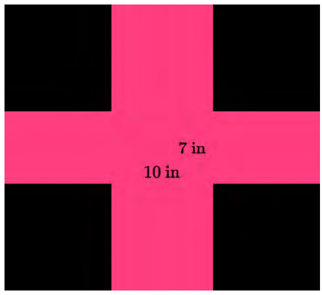
The front face is a rectangle with dimensions of 10 inches ×11 inches.

The back face is equal to the front face.

b. Draw the base

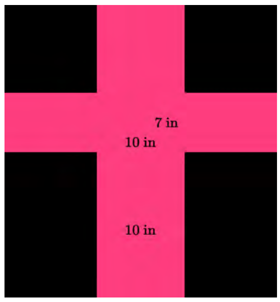


c. Draw the lateral faces and have them connect to the base.



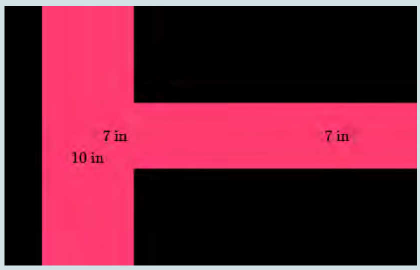
d. If the 3D figure is a prism, draw the second base so that it is connected to one of the lateral sides.

This is a rectangular prism so it has a second base that has dimensions 10 $in \times 7$ in.



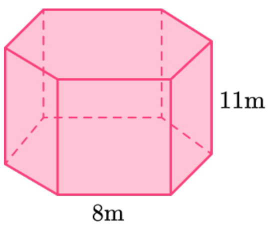
Note

The rectangular prism can be unfolded different ways. For example,



Example 7.10

Draw the net representing this hexagonal prism.



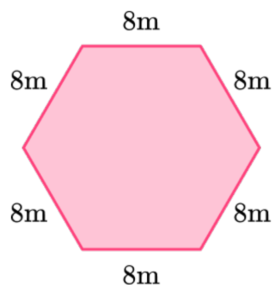
Solution

1. Identify all the faces of the 3D solid.

   The base faces are regular hexagons which means the hexagons have equal length sides and angles.

   There are six lateral faces that are rectangles. Since the base is a regular hexagon, the dimensions of each of the rectangular lateral faces is 8 $m \times 11$ m.

2. Draw the base.



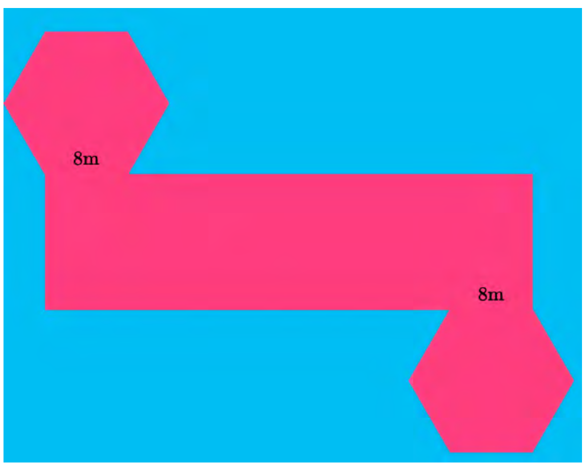
3. Draw the lateral faces and have them connect to the base.

   The side of the hexagon base matches up with the shorter side of the rectangular face so they can be connected. You can connect the hexagon base to any of the rectangles.

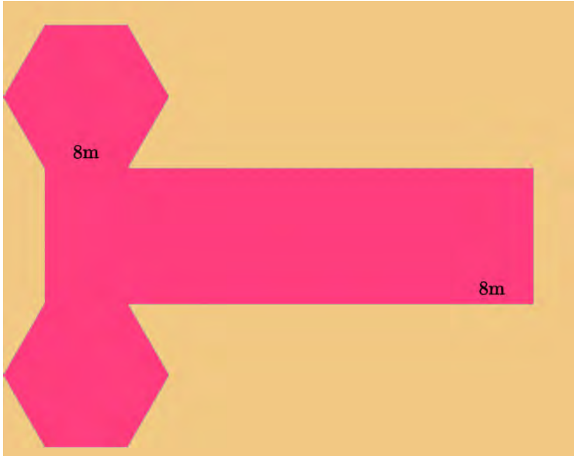
4. If the 3D figure is a prism, draw the second base so that it is connected to one of the lateral sides.

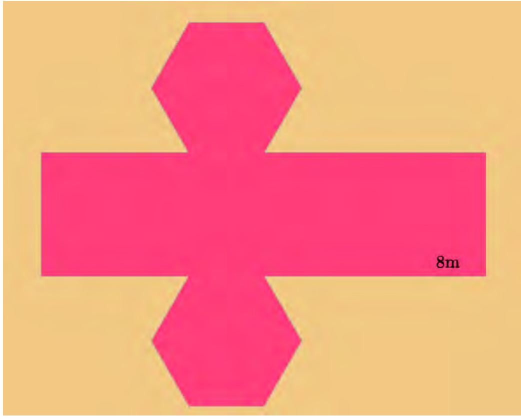
   This is a hexagonal prism so there are two hexagon bases that are equal.

   The second hexagon base can be connected to the rectangular as long as it's on the other side. It can be drawn like this:



You can also draw it like this:





OR

Learning Tasks

Learners draw nets of 3D objects (prisms, cones, pyramids, spheres, etc.)

Determine the surface area of 3D shapes

To calculate the surface area, you may refer to the following formulas.

3D Shape	Total Surface Area (TSA)	Lateral Surface Area (LSA) or Curved Surface Area
Cube	$6a^2$	$4a^2$, where a is the length of each side
Cuboid	$2(lw + wh + lh)$	$2h(l + w)$, where l, w and h are the length, width and height of the cuboid
Cone	$\pi r^2 + \pi rl = \pi r(r + l)$	πrl, where r is the radius and l is the slant height of the cone
Cylinder	$2\pi r^2 + 2\pi rh = 2\pi r(r + h)$	$2\pi rh$, where r is the radius and h is the height of the cylinder
Sphere	$4\pi r^2$, where r is the radius of the sphere	Not applicable

Shape	Base	Surface Area of Prism = (2 × Base Area) + (Base perimeter × height)
Triangular Prism	**Triangle**	Surface area of triangular prism $= bh + (s_1 + s_2 + b)H$ Where: **b**: The length of the base of the triangular face of the prism. **h**: The height of the triangular face (the perpendicular distance from the base b to the opposite vertex of the triangle). **s_1**: The length of one of the other two sides of the triangular face (not the base). **s_2**: The length of the other side of the triangular face (not the base). **H**: The height of the prism (the distance between the two triangular bases, also known as the length of the prism).
Square Prism	**Square**	Surface area of square prism $= 2a^2 + 4ah$ Where: **a**: The length of the side of the square face of the prism. **h**: The height of the prism, which is the distance between the two square faces (also called the length of the prism).
Rectangular Prism	**Rectangle**	Surface area of rectangular prism $= 2(lw + wh + lh)$ Where: **l**: The length of the rectangular prism. **w**: The width of the rectangular prism. **h**: The height of the rectangular prism.

Shape	Base	Surface Area of Prism = (2 × Base Area) + (Base perimeter × height)
Trapezoidal Prism	**Trapezoid**	Surface area of trapezoidal prism = h (b + d) + l (a + b + c + d) Where: **h**: The height of the trapezoid (the perpendicular distance between the two parallel sides b and d). **b**: The length of one of the parallel sides of the trapezoid. **d**: The length of the other parallel side of the trapezoid. **L**: The length of the prism (the distance between the two trapezoidal faces). **a**: The length of one of the non-parallel sides of the trapezoid. **c**: The length of the other non-parallel side of the trapezoid.
Pentagonal Prism	**Pentagon**	Surface area of regular pentagonal prism = 5ab + 5bh Where: **a**: The length of one side of the pentagonal base. **b**: The apothem of the pentagonal base (the perpendicular distance from the center of the pentagon to the midpoint of one of its sides). **h**: The height of the prism (the distance between the two pentagonal bases).

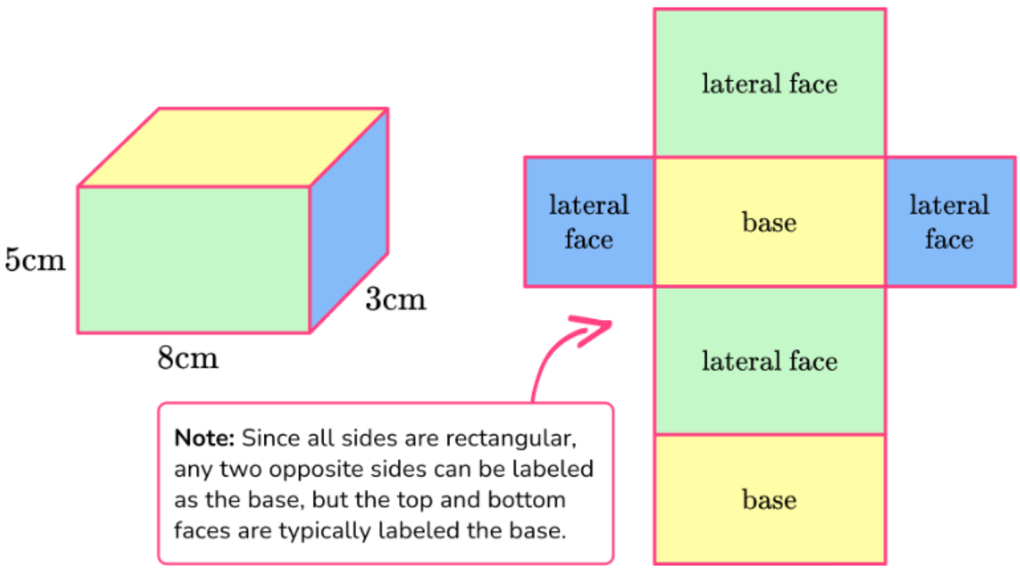
Shape	Base	Surface Area of Prism = (2 × Base Area) + (Base perimeter × height)
Hexagonal Prism	**Hexagon**	Surface area of regular hexagonal prism $= 6ah + 3\sqrt{3a^2}$ Where: **a**: The length of one side of the hexagonal base. **h**: The height of the prism (the distance between the two hexagonal bases).
Octagonal Prism	**Octagon**	Surface area of regular octagonal prism $= 4a^2 (1 + \sqrt{2}) + 8aH$ Where: **a**: The length of one side of the octagonal base. **H**: The height of the prism (the distance between the two octagonal bases).

Worked Examples

Nets can be used to calculate the surface area of three-dimensional shapes.

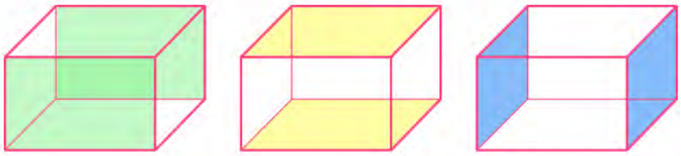
Example 7.11

Here is a rectangular prism and its net.

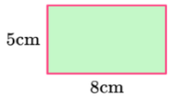
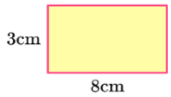
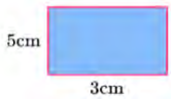


Solution

It has 3 pairs of congruent faces since the opposite faces are the same.



To calculate the surface area of the rectangular prism, calculate the area of each face and then add them together.

Face	Area
	$A = 5 \times 8 = 40 \text{ cm}^2$
	$A = 3 \times 8 = 24 \text{ cm}^2$
	$A= 5 \times 3 = 15 \text{ cm}^2$

The surface area of the prism is the sum of the areas. You can multiply each area by 2 and then add them together or you can add each area twice since each rectangle appears twice in the prism:

$2 \times 40 + 2 \times 24 + 2 \times 15 = 80 + 48 + 30 = 158 \text{ cm}^2$

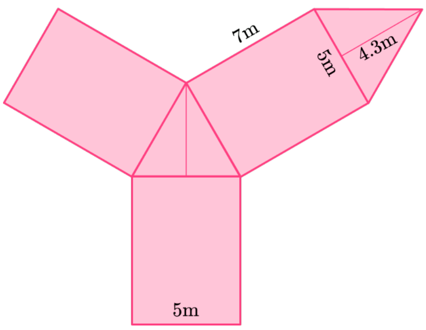
OR

$40 + 40 + 24 + 24 + 15 + 15 = 158 \text{ cm}^2$

The surface area of the rectangular prism is equal to 158 cm^2.

Example 7.12

The net of a triangular prism is given below. Find the surface area (the triangular bases are equilateral triangles).



Solution

There are two triangular bases that are equal. The base length is 5 m and the height is 4.3 m.

There are three rectangular lateral faces that are equal. The base of the rectangles is 5 m and the height of the rectangle is 7 m.

Since the bases are triangles that are equal. Find the area of one and multiply it by two.

Area of triangle $=\frac{1}{2}\times \boldsymbol{b}\times \boldsymbol{h}=\frac{1}{2}\times 5\times 4.3=10.75$

Area of 2 triangles $= 2\times 10.75 = 21.5$

Since all three lateral faces are equal, find the area of one and then multiply it by 3.

Area of rectangle $= b\times h = 5\times 7 =5$

Area of 3 rectangles $= 3\times 35 = 105$

Total surface area $= 21.5 + 105 = 126.5$

The surface area of this triangular prism is 126.5 m^2.

Example 7.13

Find the total surface area of a cylinder if its radius is 3.5 units and height is 6 units.

Solution

We know that the formula to find the total surface area of a cylinder $= 2\pi r(r + \mathrm{h})$

$= 2\times\frac{22}{7}\times 3.5\times(3.5+6) = 2\times\frac{22}{7}\times 3.5\times(9.5) = 209$ unit^2

$\therefore$ The total surface area of the cylinder is 209 unit^2, to the nearest whole unit^2.

Example 7.14

If the radius and slant height of an ice cream cone are 4 inches and 7 inches, respectively. What is its surface area?

Solution

Given: radius = 4 inches and slant height = 7 inches.

The surface area of cone $= \pi r(r + l) = \pi\times 4(4+7) = 3.14\times 4\times 11 = 138.16$ inches^2

$\therefore$ The surface area of the cone is 138 inches^2, to the nearest whole inch^2.

Example 7.15

The total surface area of a cube is 96 square units. Calculate the length of one side of the cube.

Solution

The formula for the surface area of a cube is 6 x area of face = $6 \times \text{side}^2$

Given that the surface area is 96 square units, we can set up the equation:

$96 = 6 \times \text{side}^2$

Divide both sides by 6:

$\text{side}^2 = \frac{96}{6} = 16$

Take the square root of both sides:

$\text{Side} = \sqrt{16} = 4$

$\therefore$ The length of one side of the cube is 4 units.

Example 7.16

The total surface area of a sphere is 154 square units. Calculate the radius of the sphere.

Solution

The formula for the surface area of a sphere is $4\pi r^2$

Given that the surface area is 154 square units, we can set up the equation:

$154 = 4\pi r^2$

Divide both sides by 4π

$r^2 = \frac{154}{4\pi} = \frac{154}{4 \times 3.14} = \frac{154}{12.56} = 12.26$

Take the square root of both sides:

$r = 3.5$

$\therefore$ The radius of the sphere is approximately 3.5 units.

Learning Tasks

Learners calculate the surface area of given 3D shapes.

PEDAGOGICAL EXEMPLARS

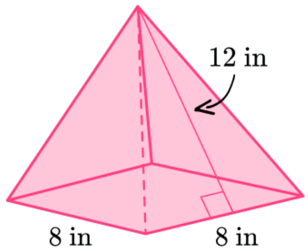
Experiential Learning: In pairs, learners draw nets of 3D objects (prisms, cones, pyramids, spheres, etc.), including the use of computer programmes like GeoGebra.

Collaborative Learning: In pairs, learners calculate the total surface area of 3D objects (prisms, cones, pyramids, spheres, etc.), including the use of computer programmes like GeoGebra.

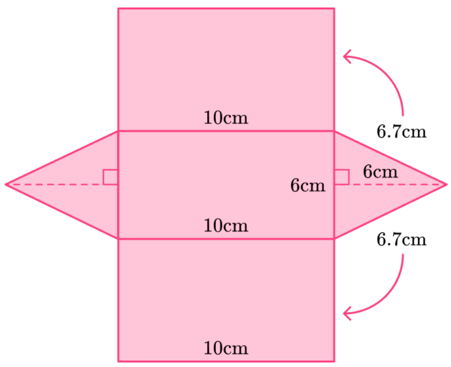
KEY ASSESSMENT

1. Perform the following conversions
   - **i.** Convert 12 square metres to square feet.
   - **ii.** Convert 80 square feet to square metres.
   - **iii.** Convert 250 square centimetres to square inches.
   - **iv.** Convert 450 square inches to square centimetres.

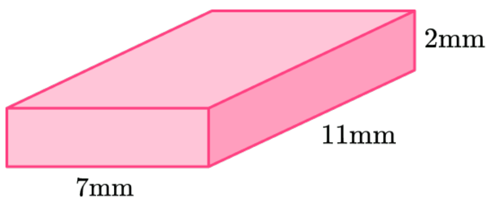
2. Draw a net representing the pyramid below.



3. Calculate the surface area of this prism, using the net.



4. Calculate the surface area of this prism.



5. Find the total surface area of a cube with side length 4 units.
6. Calculate the surface area of a cuboid with dimensions 5 units, 3 units and 2 units.
7. Determine the total surface area of a cone with a base radius of 4 units and a slant height of 7 units.
8. Find the total surface area of a cylinder if its radius is 5 units and height is 10 units.
9. Calculate the surface area of a sphere with a radius of 6 units.

10. Determine the surface area of a triangular prism with a base area of 12 square units, a height of 5 units and a side length of 4 units.
11. Find the total surface area of a square prism with a base side length of 3 units and height of 8 units.
12. Calculate the surface area of a rectangular prism with dimensions 7 units, 4 units and 3 units.
13. The total surface area of a cube is 150 square units. Calculate the length of one side of the cube.
14. The total surface area of a rectangular prism is 94 square units. The dimensions of the base are 3 units by 4 units. Calculate the height of the rectangular prism.
15. The total surface area of a cylinder is 282.6 square units. The radius of the base is 3 units. Calculate the height of the cylinder.

Hint

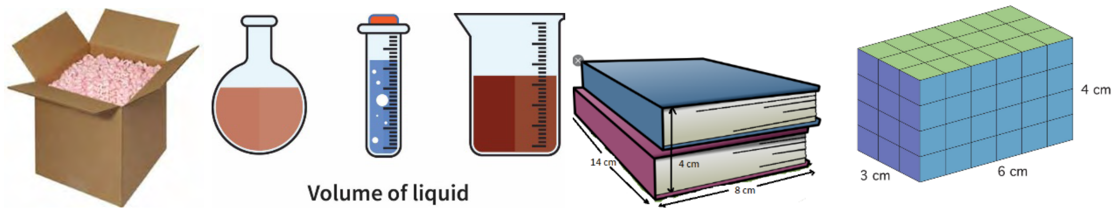
- *The Recommended Mode of Assessment for Week 18 is **Mid-semester Examination.***
- *Refer to Appendix H at the end of Section 7 for further information on how to go about the Mid-semester exam.*

WEEK 19: VOLUME AND CAPACITY

Learning Indicator: *Solve problems that involve SI and imperial units in volume/capacity measurements.*

FOCAL AREA: MEASUREMENTS OF VOLUME AND CAPACITY

What is volume? Volume is the amount of space that a substance or object occupies. It is measured in cubic units, indicating how much three-dimensional space an object takes up. For example, the volume of a cube can be calculated by multiplying the length of one side by itself three times (length × width × height).



What is capacity? Capacity refers to the maximum amount that something can contain, typically used in the context of containers and liquids. It is often measured in litres, millilitres, gallons and other similar units. For instance, the capacity of a water bottle tells us how much water it can hold when full.

Difference between volume and capacity

The total amount of any substance which is contained in a particular space is called the volume. The total potential amount of any substance that can be contained in a space is called the capacity of that space. Thus, the amount of space that a substance takes up is known as volume. The maximum amount of a substance that an object can contain is known as its capacity.

The container here holds some amount of liquid. The total amount of liquid in the container is the volume of the liquid, but the capacity of the container is the potential amount of liquid that can be contained in the entire space of the container.

Volume	Capacity
Volume indicates the total amount of space covered by an object in three-dimensional space.	Capacity refers to the ability of something to hold, absorb or receive an object, like a solid substance, gas or liquid.
Common units (units of measurement) – cm^3, m^3	Common units (units of measurement)- litres, gallons, pounds
Both solid and hollow objects have volume.	Only hollow objects have capacity.

Real-Life relevance of volume and capacity in various fields:

- **Cooking:** Understanding volume and capacity is crucial in cooking for measuring ingredients accurately. Recipes often specify amounts in units of volume (cups, teaspoons, litres), ensuring that the right proportions are used to achieve the desired taste and consistency.
- **Engineering:** Engineers must calculate the volume and capacity of materials and spaces to design structures, machines and systems effectively. For example, determining the volume of a fuel tank or the capacity of a water reservoir is essential for ensuring functionality and safety.
- **Construction:** In construction, volume calculations are necessary for determining the amount of materials required, such as concrete, soil or paint. Capacity measurements help in planning storage and transport of materials to the construction site.
- **Daily Life:** In everyday activities, understanding volume and capacity helps with tasks like filling a swimming pool, selecting the right size container for leftovers or determining the amount of fuel needed for a car trip. It also plays a role in packaging, where knowing the capacity of boxes and bottles helps in efficient storage and transportation.

Common metric units for volume

- **Cubic Metres (m^3):** The cubic metre is the standard unit for measuring large volumes in the metric system. It is used in contexts such as measuring the volume of rooms, storage spaces or large containers.
- **Cubic Centimetres (cm^3):** The cubic centimetre is a smaller unit often used in scientific contexts, particularly in chemistry and medicine, to measure volumes of liquids and solids in laboratory experiments.

- **Litres (*l*):** The litre is a commonly used unit for measuring liquids, especially in everyday contexts such as beverages, fuel and household products. For example, milk, water bottles and soft drinks are often measured in litres.
- **Millilitres (ml):** The millilitre is a smaller unit than the litre and is frequently used in cooking, medicine and scientific experiments. It is often used for measuring small volumes of liquids, such as ingredients in a recipe or doses of medication.

Common imperial units for volume

- **Cubic Feet (ft^3):** The cubic foot is used for measuring larger volumes in contexts such as storage, shipping and construction. It is commonly used to describe the volume of appliances like refrigerators and freezers.
- **Cubic Inches (in^3):** The cubic inch is a smaller unit often used in engineering and automotive contexts to measure engine displacement and small components.
- **Gallons:** Gallons are widely used in the United States for measuring larger quantities of liquids, such as fuel, milk and water. The U.S. gallon is different from the imperial gallon used in the UK.
- **Quarts:** A quart is a quarter of a gallon and is used for smaller liquid measurements. It is commonly used for measuring liquid ingredients in recipes and for products like motor oil.
- **Pints:** A pint is half a quart and is often used for measuring smaller liquid quantities, such as beverages (beer, milk) and ice cream.
- **Fluid Ounces:** Fluid ounces are the smallest common imperial unit for liquid volume, used for very small quantities like drink servings, cooking ingredients and medicine dosages.

SI to Imperial Units

1 Cubic Metre (m^3) = 35.3147 Cubic Feet (ft^3)

Example 7.17

If you have a storage container with a volume of 2 cubic metres, you can convert this to cubic feet by multiplying by 35.3147.

Solution

$2\,m^3 \times 35.3147\,ft^3/m^3 = 70.6294\,ft^3$

Example 7.18

A room has a volume of 3.5 cubic metres.

Solution

To find the volume in cubic feet, you multiply 3.5 by 35.3147.

$3.5\,m^3 \times 35.3147\,ft^3/m^3 = 123.60145\,ft^3$

1 Litre (l) = 0.264172 US Gallons (gal)

Example 7.19

You have a 10-litre container of water. To convert this to gallons, you multiply by 0.264172.

Solution

$10\,l \times 0.264172\,gal/l = 2.64172\,gal$

Example 7.20

A car's fuel tank holds 50 litres of gasoline. To convert this to gallons, you multiply 50 by 0.264172.

$50\,l \times 0.264172\,gal/l = 13.2086\,gal$

Imperial to SI Units

1 Cubic Foot (ft^3) = 0.0283168 Cubic Metres (m^3)

Example 7.21

If you have a refrigerator with a volume of 10 cubic feet, you can convert this to cubic metres by multiplying by 0.0283168.

Solution

$10\,ft^3 \times 0.0283168\,m^3/ft^3 = 0.283168\,m^3$

Example 7.22

A storage unit has a volume of 25 cubic feet. To find the volume in cubic metres, you multiply 25 by 0.0283168.

Solution

$25\,ft^3 \times 0.0283168\,m^3/ft^3 = 0.70792\,m^3$

1 US Gallon (gal) = 3.78541 Litres (L)

Example 7.23

You have a 5-gallon jug of milk. To convert this to litres, you multiply by 3.78541.

Solution

5 gal × 3.78541 *l*/gal = 18.92705 *l*

Example 7.24

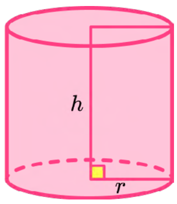
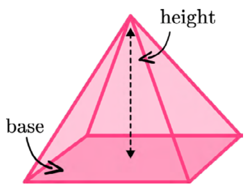
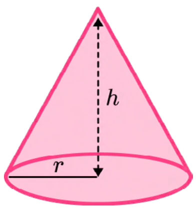
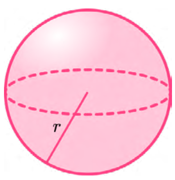
A fuel tank holds 20 gallons of diesel. To convert this to litres, you multiply 20 by 3.78541.

Solution

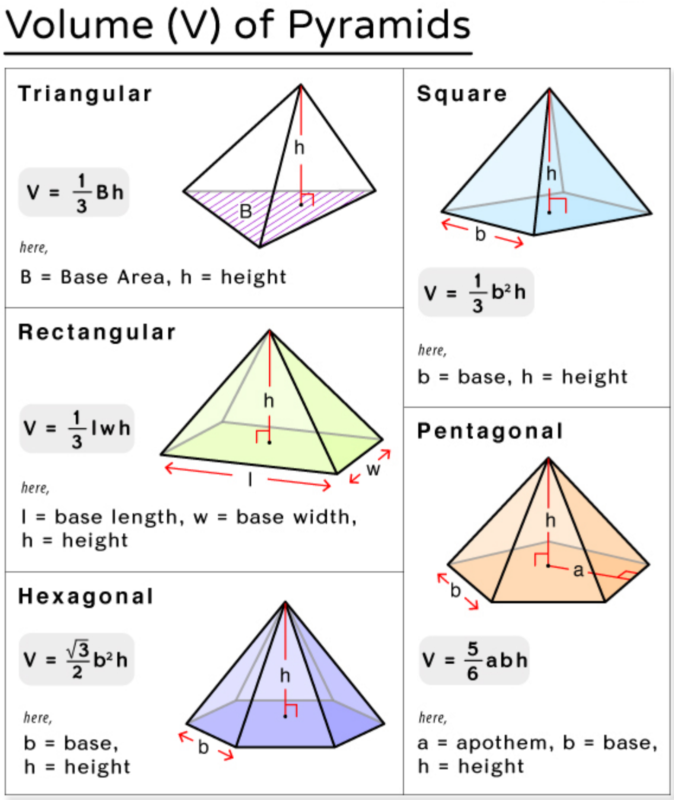
20 gal × 3.78541*l*/gal = 75.7082 *l*

Calculating Volume of 3D shapes

In Year One, we calculated the volume of prisms. We will apply these ideas in calculating volumes of other 3D shapes including composite shapes.

Cylinder		Volume=$\pi r^2 h$
Pyramid		Volume = $\frac{1}{3}$ × Area of base × Height
Cone		Volume=$\frac{1}{3}\pi r^2 h$ (Volume=$\frac{1}{3}$ × Area of base × Height)
Sphere		Volume=$\frac{4}{3}\pi r^3$

For the different Pyramids, note the following



From the table above we realise that the volume of different pyramids can vary based on their base shape and dimensions. Here are key differences to note:

1. **Base Shape**
   a. Pyramids can have different base shapes such as square, rectangle, triangle, pentagon, etc.
   b. The formula for calculating the volume (V) involves the area of the base (B) and the height (h) of the pyramid.

2. **Volume Calculation**
   a. For pyramids with different base shapes, the formula to calculate volume remains consistent: $V = \frac{1}{3} \times B \times h$
   b. The key variation lies in how B (the area of the base) is determined based on the specific shape of the base.

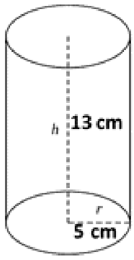
3. **Height and Base Area**
   a. The height (h) of the pyramid is crucial as it represents the perpendicular distance from the base to the apex.
   b. The method to calculate B depends on the shape:
      - **Square or Rectangle:** B = length $\times$ width
      - **Triangle:** B $= \frac{1}{2} \times$ base $\times$ height

- **Polygon:** B is calculated based on the specific geometric properties of the shape.

Worked Examples

Example 7.25

Calculate the volume of the cylinder below, leaving your answer to 2 decimal places. [**Take** $\pi = \frac{22}{7}$]



Solution

Volume = $\pi r^2 h$

From the figure; r = 5 cm and h = 13 cm

Volume = $\frac{22}{7} \times 5^2 \times 13$

Volume = $\frac{22}{7} \times 25 \times 13$

Volume = **1021.43 cm**^{3}

Example 7.26

Calculate the volume of a sphere with diameter 9 *mm*.

Solution

The 3D shape is a sphere. The formula you need to use is:

Volume = $\frac{4}{3}\boldsymbol{\pi r}^3$

You need to substitute the value of the radius of the sphere into the formula. You need to divide the diameter by 2 to calculate the radius, r.

$r = 9 \div 2 = 4.5$

Volume = $\frac{4}{3} \times \pi \times 4.5^3$

Volume = $\frac{243}{2} \times \pi = 381.703$

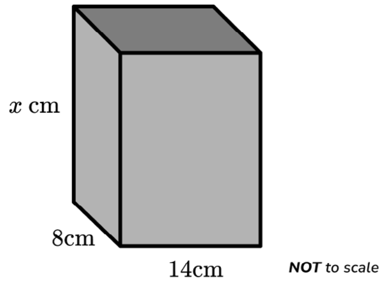
The dimensions of the sphere were given in millimetres, so the volume of the sphere will be in cubic millimetres (mm^3).

V = 382 mm^3 (to 3 significant figures).

Example 7.27

The volume of the cuboid is 2016 cm^3.

Calculate the value of x.



The 3D shape is a cuboid. The formula you need to use is:

Volume $= l \times w \times h$

The values you need to substitute into the formula are:

Volume $= 2016$, $l = 8$ and $w = 14$.

For the height (h) you can use x.

The length (l), the width (w) and the height (h) of the cuboid are interchangeable, so it does not matter which dimension l or w or h.

Volume $= l \times w \times h$

$$2016 = 8 \times 14 \times x$$

$$2016 = 112 \times x$$

$$x = 2016 \div 112$$

$$= 18$$

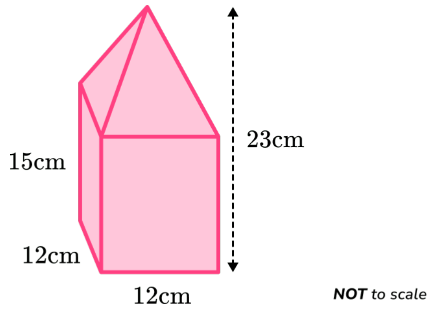
The dimensions of the cuboid were given in centimetres and the volume was given in cubic centimetres (cm^3).

$\therefore x = 18\ cm$

Calculate the volume of composite shapes

Example 7.28

This shape is made from a cuboid and a square based pyramid. Calculate the volume of the 3D shape.



Solution

The 3D shape is made from a cuboid and a pyramid. We can find the volume of each piece and then add the two volumes together. The formulas you need to use are:

Cuboid	Pyramid
Volume $= l \times w \times h$	Volume $= \frac{1}{3} \times$ area of base $\times$ height
$l = 12$, $w = 12$ and $h = 15$	The area of the base is: $12^2 = 12 \times 12 = 144$. The height of the pyramid is: $23 - 15 = 8$.
Volume $= l \times w \times h$ $= 12 \times 12 \times 15 = 2160$	Volume $= \frac{1}{3} \times$ area of base $\times$ height $\frac{1}{3} \times 144 \times 8 = 384$

$\therefore$ The volume of the shape is $2160 + 384 =$ **2544 cm**^{3}.

Learning Tasks

Learners to calculate the volume of given 3D shapes. Learners also create/write questions/problems for their friends to solve.

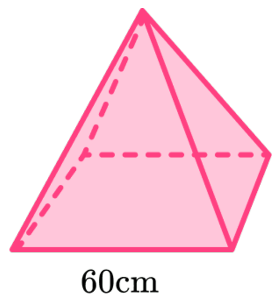
PEDAGOGICAL EXEMPLARS

Collaborative learning: Learners discuss and solve problems that involve the volume of 3-D objects and composite 3-D objects using formulas. Then, write the volume measurement expressed in one SI/imperial unit cubed in another SI/imperial unit cubed.

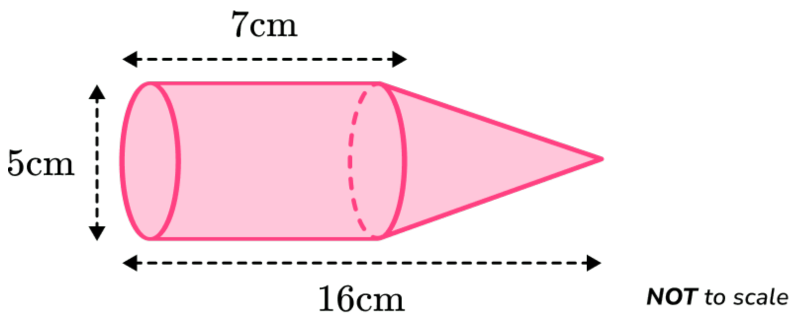
KEY ASSESSMENT

1. The volume of the square pyramid is 66 000 cm^3.

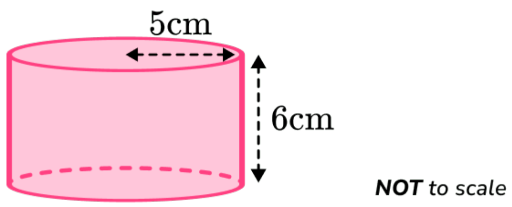
   Calculate the height of the pyramid.



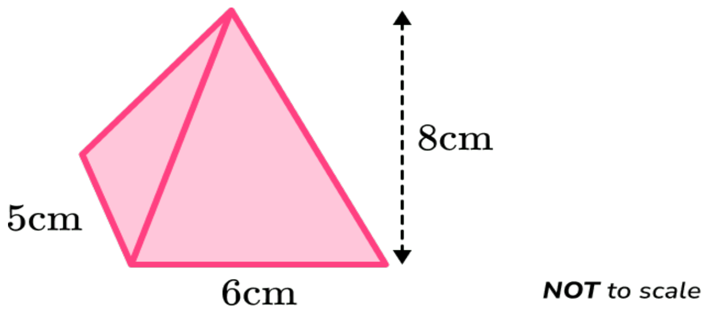
2. This shape is made from a cylinder and a cone. Calculate the volume of the 3D shape.



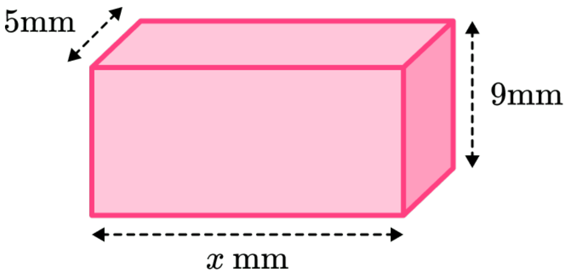
3. Calculate the volume of the cylinder.



4. Calculate the volume of the rectangular based pyramid pyramid.



5. The volume of this cuboid is 720 mm^3. Calculate the length of the missing side.

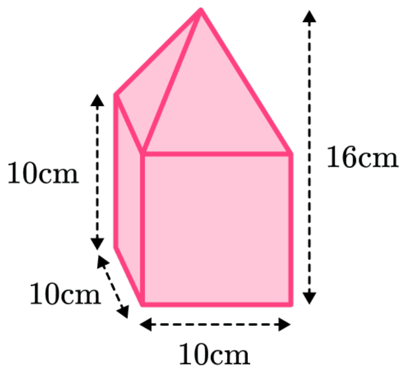


NOT *to scale*

6. The volume of a cylinder is 160π cm^3. The height of the cylinder is 6.4 cm.

Find the radius of the cylinder.

7. Calculate the volume of the shape below. It is made from a cube and a square based pyramid.



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WEEK 20: WORLD PROBLEMS THAT INVOLVES THE VOLUME/CAPACITY OF A 3-D OBJECT

Learning Indicator: *Solve real world problems that involves the volume/capacity of a 3-D object.*

FOCAL AREA: REAL WORLD PROBLEMS THAT INVOLVES THE VOLUME/CAPACITY OF A 3-D OBJECT

Understanding the volume and capacity of three-dimensional objects is crucial for solving many real-world problems. These concepts are widely applicable in various fields such as engineering, architecture, manufacturing and everyday life.

For instance, architects need to calculate the volume of materials required to construct buildings, engineers determine the capacity of tanks and containers and homeowners estimate the amount of paint needed to cover a room. By learning to solve problems involving the volume and capacity of 3-D objects, learners can develop practical skills that are essential for numerous real-life applications.

This topic not only enhances spatial awareness but also integrates mathematical concepts with tangible, everyday scenarios, making learning more relevant and engaging.

General Steps to Solve Real-Life Problems Involving Volume

1. Understand the Problem
   a. Carefully read the problem to understand what is being asked.
   b. Identify the 3-D shape(s) involved (e.g., cube, cylinder, prism).
   c. Determine what information is given (dimensions, capacity, etc.) and what needs to be found.
2. Identify the Relevant Formula
   a. Choose the appropriate volume formula for the given shape
3. Convert Units if Necessary
   a. Ensure all measurements are in the same unit system (e.g., all in metres, centimetres or feet).
   b. Convert units if necessary, using appropriate conversion factors.
4. Plug in the Given Values
   a. Substitute the known values into the volume formula.
5. Perform Calculations
   a. Carry out the necessary arithmetic operations to find the volume.

   b. Use a calculator for complex calculations to ensure accuracy.

6. Interpret the Result

   a. Analyse the result to see if it makes sense in the context of the problem.

   b. If the problem involves cost or additional steps, use the volume to proceed with further calculations.

7. Check Your Work

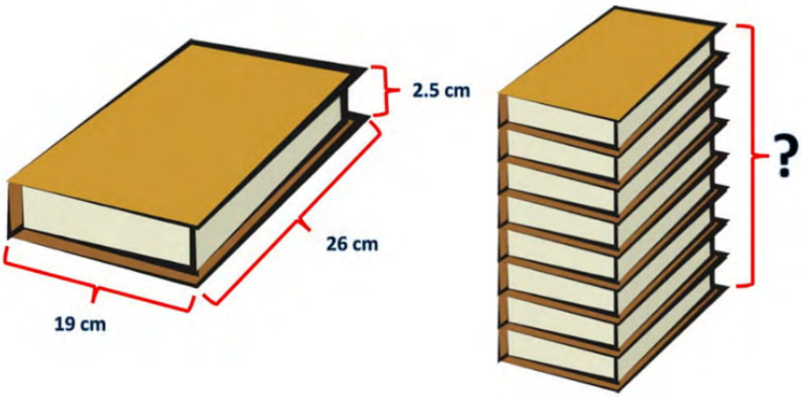
   a. Review each step to ensure there were no mistakes.

   b. Verify that all units are correctly used and converted.

Worked Examples

Example 7.29

A book is 19 cm wide, 26 cm long and 2.5 cm thick. There are 8 similar books placed on the top of each other. What is the volume taken up by them?



Solution

Volume of 8 books = (L × B × H) × 8 books

Volume = (19 × 26 × 2.5) × 8

Volume = 1235 × 8

Volume = **9 880 cm**^{3}

Example 7.30

The pedestal on which a statue is raised is a rectangular concrete solid measuring 9 feet long, 9 feet wide and 6 inches high. How much is the cost of the concrete in the pedestal, if concrete costs GH¢70 per cubic yard?

Solution

To determine the cost of the concrete in the pedestal, we will follow these steps:

Convert the dimensions to the same unit:

The height of the pedestal is given in inches, so we'll convert it to feet to match the other dimensions:

$height\ in\ feet = \dfrac{6\ inches}{12\ inches/foot} = 0.5\ feet$

Calculate the volume in cubic feet:

The volume (V) of a rectangular solid is given by:

V = length × width × height

Substitute the dimensions:

V = 9 feet × 9 feet × 0.5 feet = 40.5 cubic feet

Convert the volume to cubic yards:

There are 27 cubic feet in 1 cubic yard (since 1 yard = 3 feet, thus 1 cubic yard = 3^3 = 27 cubic feet)

$volume\ in\ cubic\ yards = \dfrac{40.5\ cubic\ feet}{27\ cubic\ feet\ /\ cubic\ yard} = 1.5\ cubic\ yards$

Calculate the cost, given the cost of concrete is GH¢70 per cubic yard:

Total cost = 1.5 cubic yards × GH¢70 /cubic yard = GH¢105

∴ The cost of the concrete in the pedestal is GH¢105.

Example 7.31

Ewuradwoa wants to drink milk from a glass that is in the shape of a cylinder. The height of the glass is 15 units and the radius of the base is 3 units. What is the quantity (volume) of milk that she requires to fill the glass completely?

Solution

Given that, the height of the glass = 15 units and the radius of the base = 3 units.

To find the volume of the glass, we need to use the formula for the volume of a cylinder, which is $\pi r^2 h$ cubic units.

The volume of the glass, $V = \pi r^2 h = \pi \times (3^2) \times 15 = \pi \times 135 = 424$ cubic units, to the nearest whole cubic unit.

Therefore, she needs approximately 424 cubic units of milk to fill her glass.

Example 7.32

Ampofo loves playing with building blocks. He has built a structure with 15 cubic blocks. If the edge of each cube is 3in, what the volume of his structure?

Solution

Calculate the volume of one cube:

The volume of cube = Edge × Edge × Edge = 3 in × 3 in × 3 in =27 in^3

There are 15 cubes in his structure.

Volume of the structure = 15 × Volume of one cube = 15 × 27 in^3 = 405 in^3

Therefore, the volume of the structure is 405 in^3.

Example 7.33

Jim wants to know how much his family spends on water for showers. Water costs GH¢**1. 50** for **1, 000** gallons. His family averages **4** showers per day. The average length of a shower is **10** minutes. He places a bucket in his shower and turns on the water. After one minute, the bucket has **2. 5** gallons of water. About how much money does his family spend on water for showers in a **30**-day month?

Solution

Number of gallons of water in one day of showering (four, ten-minute showers):

$$\mathbf{4}(\mathbf{10}\text{ min.})\left(\frac{2.5\ gal.}{1\ min}\right) = 100\ gal.$$

Number of gallons of water in **30** days: $(\mathbf{30}\text{ days})\left(\frac{100\ gal.}{1\ day}\right) = 3\,000\ gal.$

Cost of showering for **30** days: $(\mathbf{3\,000}\text{ gal.})\left(\frac{GH¢1.50}{1000\ gal.}\right) = GH¢\ 4.50$

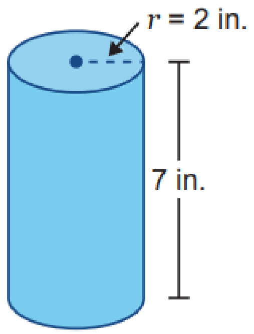
The family spends GH¢**4. 50** in a **30**-day month on water for showers.

Example 7.34

The dimensions of a wax candle are shown below.

What is the volume of the cylinder? Use 3.14 for π.

Solution



V = (π) (r^2) (h)

V = (3.14) (22)(7)

V = **87.92 in**^{3}

Each candle uses about 87.92 cubic inches of wax.

Learning Tasks

Practical Activities for Learners

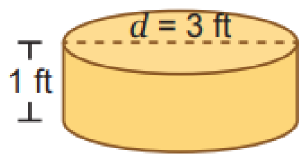
1. How will you help Adzo use a 1*l* milk carton to estimate 750 ml of water?
2. Estimate the number of cubic metres in the classroom. Explain how you determined this estimate. Find the actual measurement. Use this information to help you estimate the volume of another differently sized room, such as the library.
3. I need a box with a volume of 4 000 cubic centimetres to hold a gift I have purchased. Describe what the box might look like. What is an example of a gift that would fit into this box?
4. A container holds 1.5 litres. Is it large enough to make a jug of orange juice if the concentrate is 355 ml and you have to use the concentrate can to add three full cans of water? Explain your thinking.
5. Investigate the capacities of various beverage containers to determine which size container is found most often. Record your findings in a graph or table and present this to the class.

PEDAGOGICAL EXEMPLARS

Collaborative learning: Learners discuss and solve real world problems on volume. While they experience this concept, model tolerance among learners by creating opportunities for Collaborative Learning through mixed-ability grouping.

KEY ASSESSMENT

1. Ananga has a rectangular aquarium that is 12 inches long, 8 inches wide and 8 inches high, providing enough room to safely house 6 guppies. Assuming that the number of guppies that can be safely housed depends upon the size of the aquarium, how many guppies can be safely housed in an aquarium that is 24 inches long, 16 inches wide and 16 inches high?
2. Damien wants to fill a cylindrical sandbox completely with sand. The diagram shows the dimensions of the sandbox. If Damien has 6 cubic feet of sand, does he have enough to fill the sandbox completely? Explain.



3. Adobe is digging a hole for a rectangular swimming pool measuring 38 feet long by 22 feet wide by 8 feet deep. How many gallons of water will the swimming pool hold, assuming that 1 cubic foot = 7.5 gallons
4. A cylindrical can that is four inches tall and has a radius of 1.5 inches can hold 10¢ worth of soda. Assuming that the value of the contents is proportional to the size

(volume) of the can, what would be the value of the soda contained in a can that is 8 inches tall with a radius of 3 inches?

5. Alex made a sketch for a homemade soccer goal he plans to build. The goal will be in the shape of a triangular prism. The legs of the right triangles at the sides of his goal measure 4 ft and 8 ft and the opening along the front is 24 ft. How much space is contained within this goal?

Reminder

- *Learners' score on individual project should be ready for submission into the STP.*
- *Remind learners about their portfolio and ensure that they are poised to get the portfolios ready by Week 23.*

SECTION REVIEW

Here are the key ideas from the section:

1. Real World Problems Involving SI and Imperial Units in Surface Areas

a. Learners were taken through key differences between Imperial units (such as square feet and square inches) and SI units (such as square metres and square centimetres) for measuring surface areas. Understanding these units is essential for solving problems involving different measurement systems.

b. Learners practised converting between SI and Imperial units to solve real-world problems related to surface areas, such as calculating the paint needed for walls or the fabric required for upholstery.

2. Problems Involving SI and Imperial Units in Volume and Capacity Measurements

a. This involves revisiting the key differences between Imperial units (such as cubic feet and gallons) and SI units (such as cubic metres and litres) for measuring volumes. Mastery of these units is important for understanding volume-related problems in different contexts.

b. Learners solved real-world problems by converting between SI and Imperial units, such as determining the amount of liquid a container can hold or the space available in a storage unit.

3. Real World Problems Involving the Volume/Capacity of a 3-D Object

Learners applied their knowledge of volume and capacity to solve practical problems involving 3-D objects, such as calculating the volume of a swimming pool, the capacity of a water tank or the amount of soil needed to fill a garden bed. This involves using both SI and Imperial units where applicable and understanding the context of the problem to select appropriate units and conversion factors.

APPENDIX H: SAMPLE TABLE OF TEST SPECIFICATIONS

Weeks	Learning indicators	Type of Questions	DoK Levels				Total
			1	2	3	4	
13	a. Extend the application of ratios to angles, Pythagoras's theorem and some algebra b. Application of rates to speed: distance-time graphs.	Multiple Choice	3	2	1	-	6
		Essay type	1	-	-	-	1
14	a. Apply the concept of ratios, rates and proportions to solve problems in financial mathematics, health, sports, etc. b. Establish the relevance of ratios, rates and proportions in their day-to-day activities, make generalizations and apply them to solve real-world problems.	Multiple Choice	2	2	2	-	6
		Essay type	-	-	-	-	

Weeks	Learning indicators	Type of Questions	DoK Levels				Total
			1	2	3	4	
15	a. Explore patterns of a sequence using plane figures and continue with more terms. b. Recognise and find the nth and the sum of the nth term of an arithmetic and geometric progression (AP) or Linear Sequence.	Multiple Choice	1	2	2	-	5
		Essay type	-	1	-	-	1
16	Identify geometric progressions or exponential sequence and find the algebraic expression for the general term.	Multiple Choice	1	2	2	-	5
		Essay type	-	1	-	-	1
17	a. Analyse, model, and solve real-life problems involving financial mathematics. b. Analyse, model, and solve real-life problems involving exponential growth.	Multiple Choice	1	2	1	-	4
		Essay type	-	1	-	-	1
18	Solve problems that involve SI and imperial units in surface area measurements and verify the solutions.	Multiple Choice	1	2	1	-	4
		Essay type	-	-	1	-	1
Total							
Multiple Choice			9 (30%)	12 (40%)	9 (30%)	-	30 (100%)
Essay type			1	3	1		5

SECTION 8: WORKING WITH DATA & PROBABILITY EXPERIMENTS

Strand: Making Sense of and Using Data

Sub-Strand 1: Statistical Reasoning and its Application in Real Life

Sub-Strand 2: Probability

Learning Outcomes

1. *Carry out a mini-project involving data handling (data collection, analysis and interpretation) of quantitative and qualitative data beyond the school environment.*
2. *Determine the sample space for simple and compound probability experiments involving dependent events; express the probabilities of given events as fractions, decimals, percentages and/or ratios and solve everyday life problems.*

Content Standards

1. *Demonstrate the ability to carry out a mini-project involving collection, analysis and interpretation of quantitative and qualitative data beyond the school environment.*
2. *Demonstrate conceptual understanding of simple and compound probability experiments involving two dependent events.*

INTRODUCTION AND SECTION SUMMARY

Understanding the process of data collection, analysis and interpretation is essential in solving real-world problems and making informed decisions. Learners will engage in projects that involve gathering data using various methods and tools, such as surveys and experiments. The data collected will be analysed and interpreted using computer applications where applicable, allowing learners to visualise results through graphs, charts and tables. This hands-on approach not only enhances their analytical skills but also demonstrates the practical significance of data in various fields. In addition, learners will explore the concept of sample space in probability, starting with simple experiments, such as flipping a coin or rolling a die, to understand the possible outcomes. They will then progress to compound experiments, where multiple events occur simultaneously or in sequence, requiring a more complex analysis of possible outcomes. The exploration extends to experiments involving two dependent events, where the outcome of one event influences the other. Understanding these probability concepts is crucial for predicting outcomes and making decisions in uncertain situations.

The weeks covered by the section are

Week 21: *Develop and implement a project on data collection, analysis and interpretation [use computer applications for data analysis and interpretations where applicable]*

Week 22

Simple and compound probability experiments:

- *Sample Space from simple experiments*
- *Sample Space from compound experiments*
- *Experiments involving two dependent events*

Applications of Probability to real life

- *Solve everyday life problems involving the probability of two dependent events*

SUMMARY OF PEDAGOGICAL EXEMPLARS

This section focuses on fostering learner engagement through practical activities, critical thinking and the application of probability concepts to real-world scenarios. The teaching approach emphasises interactive learning, the use of technology and contextualised problem-solving to deepen learners' understanding.

1. **Project-Based Learning in Data Collection and Analysis**: Encourage learners to develop and implement projects that involve real-life data collection, analysis and interpretation. By using computer applications where applicable, learners gain hands-on experience with modern tools, which enhances their analytical skills and prepares them for advanced studies. This approach also promotes collaboration and critical thinking as learners navigate through the process of gathering and analysing data.
2. **Exploring Probability Through Experiments**: Introduce learners to the concept of sample space by conducting simple experiments, such as coin tosses or dice rolls. Gradually increase complexity by involving compound experiments and scenarios with dependent events, helping learners understand how probabilities are calculated and applied. These activities not only clarify abstract concepts but also make learning more engaging by connecting it to familiar situations.
3. **Real-World Application of Probability**: Use real-life examples to demonstrate the relevance of probability in everyday decision-making. By modeling scenarios that involve dependent events, learners discover how to predict outcomes and assess risks, which are valuable skills in fields like finance, science and engineering. The emphasis on practical applications ensures that learners see the direct connection between classroom learning and real-world challenges.

ASSESSMENT SUMMARY

The concepts covered in this section require learners to develop a solid understanding of data collection, analysis and the application of probability through both theoretical and practical experiences. Assessments should target levels 2, 3 and 4 of the Depth of Knowledge (DOK) framework, emphasising the application, analysis and synthesis of information. A diverse range of assessment strategies is recommended to ensure comprehensive evaluation of learners' understanding and skills:

1. Quizzes and Tests: Evaluate learners' knowledge of key concepts, including sample space, probability of events and the basics of data collection and analysis. These assessments help ensure that learners can recall and apply fundamental principles.
2. Project-Based Assessments: Involve learners in individual or group projects where they must develop and implement a data collection and analysis plan. These projects should incorporate the use of technology for data analysis, allowing learners to demonstrate their ability to apply theoretical knowledge to real-world situations.
3. Hands-On Experiments: Assess learners' ability to conduct simple and compound probability experiments, including those involving dependent events. These assessments allow learners to demonstrate their understanding of probability in practical scenarios and how it can be applied to predict outcomes.
4. Homework Assignments: Provide opportunities for learners to reinforce their understanding of data analysis techniques and probability concepts. Homework should challenge learners to explore more complex scenarios and deepen their comprehension.
5. Oral Presentations: Require learners to present their findings from projects or experiments, focusing on how they collected, analysed and interpreted data or determined probabilities. This assessment not only evaluates their understanding but also their ability to communicate complex information effectively.
6. Peer Assessments and Group Activities: Encourage collaborative learning by involving learners in group tasks where they analyse data or solve probability problems together. Peer assessments can be used to evaluate both individual contributions and the group's overall performance, fostering teamwork and critical thinking.

WEEK 21: PROJECT ON DATA COLLECTION, ANALYSIS AND INTERPRETATION

Learning Indicators

1. *Develop and implement a project plan for the collection, analysis and interpretation of data with useful conclusions and recommendations (including the use of appropriate computer applications, e.g. Excel) within and beyond the school environment.*
2. *Present a project report including the use of PowerPoint, infographics, etc. and publish it in school magazines, newspapers (Junior Graphic), local radio and TV stations, social media platforms, etc.*

FOCAL AREA 1: DEVELOP AND IMPLEMENT A MINI PROJECT ON DATA COLLECTION

In our various communities, both school and home, we are beset with issues that we can collect data about in order to understand these issues better. Issues on health, social amenities, security and the general well-being of the citizens need good understanding in order to find effective solutions to them. Data collection and analysis are means that can be employed to gain deeper insights into these issues.

For example, data is collected on malaria cases in Ghana every year by the Ghana Statistical Service (GSS) to help the health sector understand the phenomenon better. This enables the government to plan on how to support the ministry of health to combat malaria. Without data on these issues, it may lead to neglect and, eventually, pandemics.

Number of children age 6-59 months measured for malaria through rapid diagnostic test (RDT), by month and region in Ghana in 2019.

	Month			
Region	**September**	**October**	**November**	**Total**
Western	15	164	107	286
Central	53	104	94	251
Greater Accra	0	101	85	186
Volta	9	178	63	250
Eastern	21	122	79	222
Ashanti	12	136	132	280
Brong Ahafo	17	156	88	261

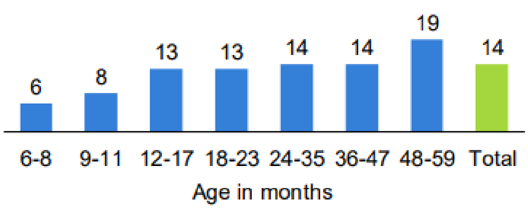
Northern	29	254	226	509
Upper East	22	158	117	297
Upper West	17	173	111	301
Total	**195**	**1 546**	**1 102**	**2 843**

From the table, we realise that the Northern Region recorded the highest number of cases over the three-month period, with Greater Accra recording the least. With this data, the government will be required to provide more resources to the Northern Region to combat malaria. Without this data, one may be tempted to believe that because Greater Accra has the highest population in Ghana (Population and Housing Census, GSS 2021) they would have to be given more resources in terms of health care. But while this may be true, this data shows that in terms of malaria cases, the Northern Region requires more attention and resources for malaria prevention.

Again, the graphs below show the prevalence of malaria in children by age and household wealth.

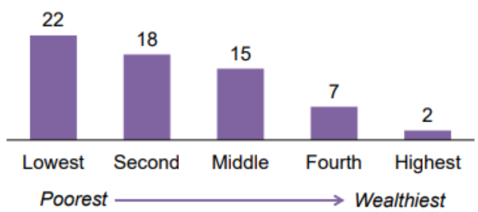
Prevalence of malaria in children by age

Percentage of children age 6-59 months who tested positive for malaria by microscopy



Prevalence of malaria in children by household wealth

Percentage of children age 6-59 months who tested positive for malaria by microscopy



From the graphs, we realise that children between the ages of 48-59 months are most prone to contracting malaria. In addition, it is the poorest people who are more likely to be infected by malaria than wealthier individuals. This data provides us with information on how to allocate resources and which group of individuals should be given more attention.

It is important for learners to embark on mini-projects in their communities on pertinent issues so that they can have a better understanding of these issues and help contribute to dealing with them. For example, assuming one or two learners in your school complained about stomach aches when they ate a particular meal in school. By merely concluding that the stomach aches are as a result of the food may not be enough. However, if you interview more people to gather enough data and realise that they also suffered the same, then your conclusions will likely be true and not just by chance.

Sample Mini-Project Guide

1. Topic/issue: Decide on the topic or issue you will want to collect data on.
2. Decide on which data collection method you want to use.
3. Design the tool to be used to collect the data.
4. Validate the tool by giving it to your teacher, friend or a few people who have similar characteristics as the people you intend to collect the data from.
5. Collect the data.
6. Organise the data
7. Interpret the data
8. Present/publish the results.

Example Mini-Project: Investigating Learners' Study Habits and Academic Performance

1. *Topic/Issue:* Investigating the Relationship Between Learners' Study Habits and Their Academic Performance in Mathematics.
2. *Data Collection Method:* The chosen data collection method is a survey using a questionnaire.
3. *Design the Tool to Collect the Data:* Design a questionnaire with the following sections:
   - **Demographics**:
     - Age
     - Gender
     - Grade level
   - **Study Habits:**
     - Number of hours spent studying per week
     - Preferred study time (morning, afternoon, evening)
     - Study environment (quiet place, library, home, etc.)
     - Use of study aids (tutors, online resources, study groups)
   - **Academic Performance:**
     - Most recent mathematics grade
     - Self-assessment of understanding in mathematics (scale of 1-10)

Sample Questionnaire

Demographics:

1. Age: ______
2. Gender:

   [ ] Male [ ] Female
3. Grade Level: ______

Study Habits:

4. How many hours do you spend studying mathematics per week? ______
5. When do you usually study?

   [ ] Morning

   [ ] Afternoon

   [ ] Evening
6. Where do you prefer to study?

   [ ] Quiet place

   [ ] Library

   [ ] Home

   [ ] Other: ______
7. Do you use any study aids? (Select all that apply)

   [ ] Tutors

   [ ] Online resources

   [ ] Study groups

   [ ] None

Academic Performance:

8. What was your most recent grade in mathematics? ______
9. On a scale of 1-10, how would you rate your understanding of mathematics? ______

4. ***Collect the Data:*** Distribute the questionnaire to 30 students in different grade levels. Collect the completed questionnaires after giving students ample time to respond.
5. ***Organise the Data:*** Compile the responses into a spreadsheet for easy analysis. Create columns for each question and input the responses accordingly.

Sample Data Organisation:

Age	Gender	Grade Level	Hours/ Week	Study Time	Study Environment	Study Aids	Maths Grade	Self-Assessment (1–10)
16	Male	10	5	Evening	Home	Online resources	B	7
15	Female	9	10	Afternoon	Library	Tutors, Study groups	A	9
17	Male	11	3	Morning	Quiet place	None	C	5
16	Female	10	8	Evening	Home	Tutors	B	8
14	Male	9	2	Afternoon	Library	Study groups	C	6
17	Female	12	7	Morning	Home	Online resources	A	8
15	Male	10	6	Evening	Quiet place	Tutors, Study groups	B	7
16	Female	11	9	Afternoon	Library	Online resources	A	9
14	Male	9	4	Morning	Home	None	B	6
17	Female	12	5	Evening	Quiet place	Tutors, Online resources	A	8

6. **Interpret the Data**

Here are some interpretations based on the expanded table, but, note, the more students asked the more the data will be representative of the population as a whole:

i. **Gender Distribution and Study Time**
   - o There are 5 male and 5 female respondents.
   - o The average study time per week for males is lower than for females.
     - Male: (5+3+2+6+4)/5 = 4 hours/week on average
     - Female: (10+8+7+9+5)/5 = 7.8 hours/week on average
     - This means females spend longer time to study than males

ii. **Grade Level and Math Grades**
   - o Students in higher grade levels (11 and 12) tend to have higher maths grades (mostly A's) compared with those in lower grades (9 and 10), where grades vary more.
   - o Grade 12 students both have an A.
   - o Grade 11 students have one A and one C.

iii. **Study Environment and Performance**
   - o Students who study in the library or a quiet place generally have higher maths grades (A's and B's).
   - o Students studying at home show a more mixed performance (ranging from B to C).

iv. **Study Aids and Self-Assessment**
   - o Students using a variety of study aids (tutors, study groups, online resources) tend to rate their self-assessment higher and also have higher maths grades.
   - o Students without any study aids tend to have lower self-assessment scores and varied maths grades.

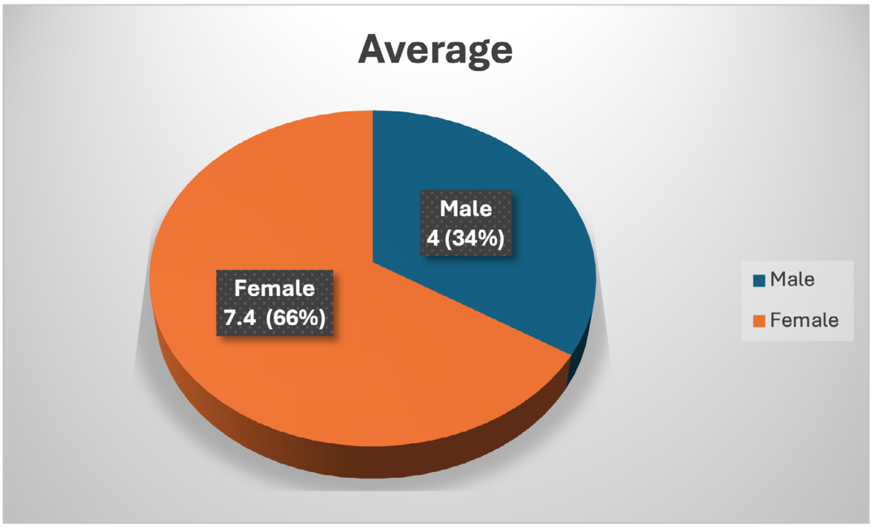
v. **Study Time and Self-Assessment**
   - o There is a positive relationship/correlation between the number of hours studied per week and self-assessment scores.
   - o Students who study more hours per week generally rate themselves higher on the self-assessment scale.

vi. **Time of Study and Grades**
   - o Students who study in the afternoon or evening tend to have higher maths grades and self-assessment scores.

- o Morning study times connect/correlate with lower grades and self-assessment scores, except for one student with an A grade.

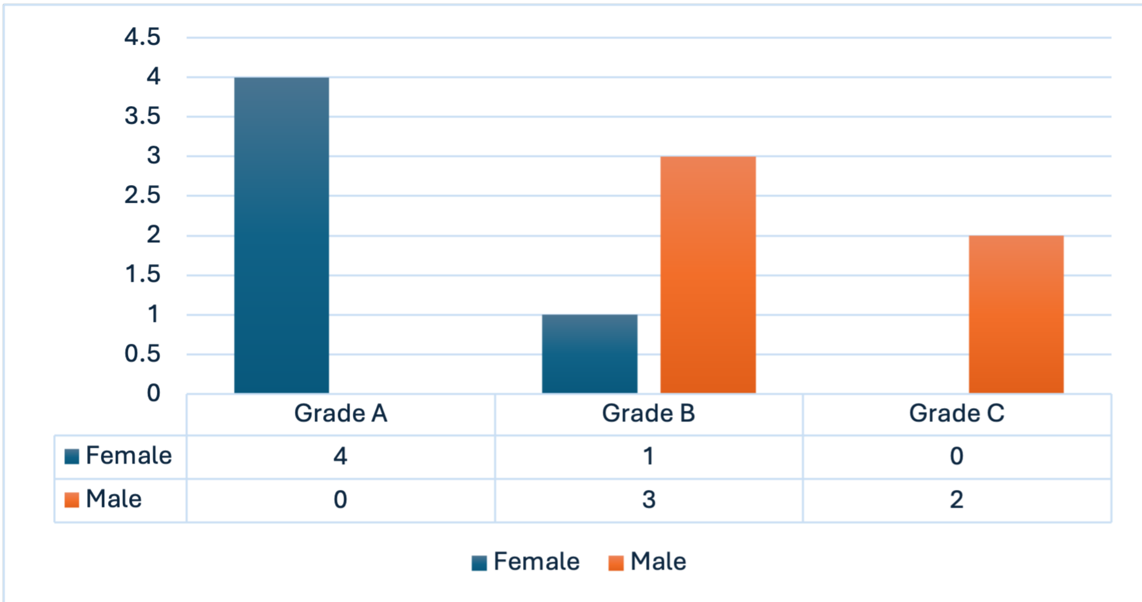
7. ***Present/publish the results***

The pie chart shows the relationship between gender and the average time per week for studying



From the chart, females tend to study for longer hours per week than their male counterparts. That is, when you consider the total number of hours used by both males and females per week females use 66% of the time while their male counterparts use 34% of the time.

The chart below shows the different grades obtained by males and females.



From the graph, four out of the five females had grade A in mathematics with no male obtaining grade A. Three males obtained grade B with one female obtaining grade B. finally, no female obtained grade C with two males obtaining grade C.

It is worth repeating that this is a small sample, so conclusions drawn must be treated with caution.

Learning Tasks

1. Learners select a topic of interest and design a data collection tool (questionnaire, interview guide, observation guide, etc.) on it.
2. Learners collect data from a reasonable sample (e.g. 20 to 50 participants)
3. Learners organise the data and give some interpretations to the data.
4. Learners use various data presentation tools (tables, charts, etc.) to present the data.

PEDAGOGICAL EXEMPLARS

In convenient groups, learners embark on a mini-project by designing a data collection tool to collect data on an issue of interest. Learners are to organise and present the data using appropriate IT tools (where available).

KEY ASSESSMENT

To be carried out as group or individual projects

1. Parents/guardians of a certain community are constantly complaining about the poor performances of their wards in senior high school. As a concerned citizen, embark on a min-project to investigate why students are performing poorly.
2. The residents of a particular urban area have been experiencing an increase in health-related issues, particularly among children. As a concerned community member, embark on a mini-project to investigate the dietary habits and access to nutrititional food to understand why there is a rise in health problems among children.
3. A local neighborhood has been facing severe pollution and waste management issues. The community is concerned about the impact on their health and environment. As an environmental advocate, conduct a mini-project to investigate the community's awareness and practices regarding waste disposal and recycling to determine the root causes of the pollution problem.
4. A rural community has been experiencing high unemployment rates among the young adults. Many young people are unable to find jobs despite completing their education. As a concerned individual, undertake a mini-project to investigate the availability of skill development programs and the alignment of educational curricula with job market requirements to identify why the unemployment rate remains high.

5. In a suburban school district, there is a growing concern among educators about the impact of technology on students' learning and social skills. Teachers and parents have noticed a decline in academic performance and social interaction. As an educational researcher, initiate a mini-project to examine the usage patterns of technology among students and its effects on their academic achievements and social behavior to understand the underlying reasons for these observations.

WEEK 22: SIMPLE AND COMPOUND PROBABILITY EXPERIMENTS

Learning Indicators

1. *List the elements of the sample space from a simple or compound experiment involving two dependent events*
2. *Solve everyday life problems involving the probability of two-independent events*

Revision of concepts

In Year 1, we learnt about some concepts of probability. Let us take a look at these concepts as they will help us understand concepts to be learnt here.

Sample Space

The sample space in probability is the set of all possible outcomes of a particular experiment. It is usually denoted by the symbol S and includes every possible result that can occur in the experiment. For example, the sample space for flipping a coin is S = {heads, tails} and for rolling a six-sided die, the sample space is S = {1,2,3,4,5,6}. The sample space is crucial for determining probabilities, as it provides the basis for calculating the likelihood of specific outcomes.

Probability of Independent Events

An independent probability experiment is a type of experiment where the outcome of one event does not affect the outcome of another event. In other words, the occurrence of one event has no impact on the probability of the other event occurring. Each event is independent of the other.

Worked Examples

Example 8.1

Flipping Two Coins

Flipping a fair coin twice is an independent probability experiment because the result of the first flip (heads or tails) does not affect the result of the second flip.

- Probability of getting heads on the first flip: $\frac{1}{2}$
- Probability of getting heads on the second flip: $\frac{1}{2}$
- Probability of getting heads on both flips: $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Example 8.2: Rolling Two Dice

Rolling two dice is an independent probability experiment because the result of the first roll does not affect the result of the second roll.

- Probability of rolling a 3 on the first die: $\frac{1}{6}$
- Probability of rolling a 4 on the second die: $\frac{1}{6}$
- Probability of rolling a 3 on the first die and a 4 on the second die: $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

Learning Tasks

Learners to list sample space of given probability events and calculate probability for independent events.

FOCAL AREA 1: SIMPLE AND COMPOUND PROBABILITY EXPERIMENTS

Simple Experiment

A simple experiment in probability is an experiment that involves a single action or event, such as flipping a coin, rolling a die or drawing a card from a deck. The outcome of a simple experiment is typically one of a set of mutually exclusive and equally likely outcomes. For example, flipping a coin can result in either "heads" or "tails."

Worked Examples

Example 8.3

What is the probability of getting heads when flipping a fair coin?

Solution

The sample space for flipping a coin is S = {heads, tails}.

There is 1 favourable outcome (heads) out of 2 possible outcomes.

Probability of heads $= \frac{\textit{Number of favorable outcomes}}{\textit{Total number of outcomes}} = \frac{1}{2} = 0.5.$

Example 8.4

What is the probability of rolling a 4 on a fair six-sided die?

Solution

The sample space for rolling a six-sided die is S = {1,2,3,4,5,6}. There is 1 favourable outcome (rolling a 4) out of 6 possible outcomes.

Probability of rolling a 4 $= \frac{1}{6} = 0.167$

Compound Experiment

A compound experiment involves two or more simple experiments conducted together, where the outcome is a combination of the results of the individual experiments. For example, rolling two dice simultaneously or flipping a coin and then rolling a die. The combined results of these simple experiments are analysed to determine probabilities of various combined outcomes.

Worked Examples

Example 8.5

What is the probability of rolling a sum of 7 when rolling two fair six-sided dice?

Solution

The sample space for rolling two dice consists of 36 possible outcomes (6 faces on the first die $\times$ 6 faces on the second die).

The favourable outcomes for a sum of 7 are: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1), which are 6 favourable outcomes.

Probability of a sum of 7 $= \frac{6}{36} = \frac{1}{6} = 0.167$

Example 8.6

What is the probability of getting heads on a coin flip and rolling an even number on a six-sided die?

Solution

The sample space for flipping a coin is {heads, tails} and the sample space for rolling a die is {1,2,3,4,5,6}. The combined sample space is {(heads,1), (heads,2), (heads,3), (heads,4), (heads,5), (heads,6), (tails,1), (tails,2), (tails,3), (tails,4), (tails,5), (tails,6)}.

Thus, a total of 12 possible outcomes.

The favourable outcomes are: (heads,2), (heads,4), (heads,6), which is a total of 3 favourable outcomes.

Probability of heads and an even number $= \frac{3}{12} = \frac{1}{4} = 0.25$

Dependent Events

Dependent events are events where the outcome or occurrence of one event affects the outcome or occurrence of another event. In probability, two events A and B are dependent if the probability of event B occurring is different depending on whether event A has occurred. For example, drawing a card from a deck without replacing it changes the probabilities of subsequent draws because the total number of cards in the deck is reduced.

Example 8.7

If A and B are dependent events, then the probability of A happening AND the probability of B happening, given A, is P(A) × P(B after A).

P(A and B) = P(A) × P(B after A)

P(B after A) can also be written as P(B | A)

then P(A and B) = P(A) × P(B | A)

Probability of Two Dependent Events

Example 8.8

A bowl contains 36 green grapes and 14 purple grapes. You randomly choose a grape, eat it and randomly choose another grape. Find the probability that both events A and B will occur.

Event A: The first grape is green.

Event B: The second grape is green.

Solution

Find P(A) and $P(B|A)$. Then multiply the probabilities.

$P(A) = \frac{36}{50}$ (of the 50 grapes, 36 are green)

$P(B|A) = \frac{35}{49}$ (of the 49 remaining grapes, 35 are green)

$P(A\ and\ B) = \frac{36}{50} \times \frac{35}{49} = \frac{3260}{2450} = 0.514$

The probability that both of the grapes are green is about 51.4%

Example 8.9

A purse contains four GH¢5 notes, five GH¢10 notes and three GH¢20 notes. Two notes are selected without the first selection being replaced. Find P(GH¢5, then GH¢5).

Solution

There are four GH¢5 notes.

There are a total of twelve notes.

P(GH¢5) = $\frac{4}{12}$.

The result of the first draw affected the probability of the second draw.

There are three GH¢5 notes left.

There are a total of eleven notes left.

P(GH¢5 after GH¢5) $= \frac{3}{11}$.

P(GH¢5, then GH¢5) = P(GH¢5) P(GH¢5 after GH¢5) $= \frac{4}{12} \text{ x } \frac{3}{11} = \frac{1}{11}$.

The probability of drawing a GH¢5 note and then another GH¢5 note is $\frac{1}{11}$.

Example 8.10: Probability of Three Dependent Events

A teacher passes around a box with 6 red pens, 9 blue pens and 7 green pens. If Kafui, Ama and Safia are the first to randomly select a pen, what is the probability that all three of them select green pens? P(A and B and C)

Solution

$= P(A) \times P(B|A) \times P(C|A \text{ and } B)$

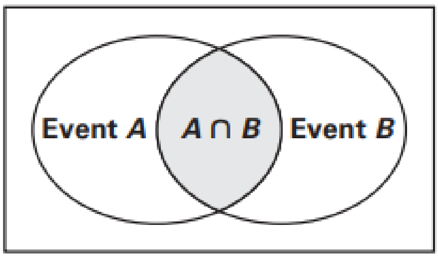
$= \frac{7}{22} \times \frac{6}{21} \times \frac{5}{20} = \frac{1}{11} \times 1 \times \frac{1}{4} = \frac{1}{44} = 0.023.$

Conditional Probability

The formula for dependent events can be rewritten to give a rule for finding conditional probabilities. Dividing both sides of the formula by P(A) gives the following.

$P(B|A) = \frac{P(B \text{ and } A)}{P(A)}$

A Venn diagram shows why this formula for P(B|A) makes sense. Event A is known to have occurred, so it becomes the sample space for P(B|A). The shaded region represents the intersection of A and B, written A ∩ B. This intersection consists of the outcomes in B that are also in A. So, the probability of B given A is the number of outcomes in B that are also in A, divided by the total number of outcomes in A.



P(B|$A) = \frac{\textit{number of outcomes in } A \cap B}{\textit{number of outcomes in } A}$

Dividing both the numerator and denominator by the number of outcomes in the original sample space gives the rule $P(B|A) = \frac{P(B \text{ and } A)}{P(A)}$.

Finding Conditional Probabilities

The table shows the number of males and females with certain hair colours. Find (a) the probability that a listed person has red hair and (b) the probability that a female has red hair.

	Brown hair	Blonde hair	Red hair	Black hair	Other hair
Male	42	11	3	17	27
Female	47	16	13	9	15

Solution

a. P(red hair) $= \frac{\textit{number of people with red hair}}{\textit{total number of people}} = \frac{6}{200} = 0.08$

b. P(red hair|female) $= \frac{\textit{number of red hair females}}{\textit{total number of females}} = \frac{13}{100} = 0.13$

Differences between dependent and independent probability events

Dependent Events	Independent Events
1. The occurrence of one event does affect the probability of another event.	1. The occurrence of one event does not affect the probability of another event.
2. Examples include a power cut in case you don't pay your bill on time, winning the lottery after buying 10 lottery tickets (the more tickets bought, the greater the chance of winning)	2. Examples include riding a bike and watching your favourite movie on a laptop
3. Formula can be written as: $P(A \cap B) = P(A) \times P(B \mid A)$	3. Formula can be written as: $P(A \cap B) = P(A) \times P(B)$

Example: Steps to Check Whether the Probability Belongs to Dependent or Independent Events

- ***Step 1:*** Is it possible for the events to happen in any order? If yes, go to step 2; if no, go to step 3.
- ***Step 2:*** Does one event affect the outcome of the other event? If yes, go to step 4; if no, go to step 3.
- ***Step 3:*** The event is independent. Use the formula for independent events and get the answer.

- ***Step 4:*** The event is dependent. Use the formula for dependent events and get the answer.

Learning tasks

1. Learners discuss the concepts; simple and compound experiments and write their own examples.
2. Learners solve problems on probability of dependent events.
3. Learners discuss the steps in determining whether a probability is dependent or independent.

Calculating probability of dependent events using tree diagrams

Example 8.11

Soccer Game

You are off to soccer and love being the Goalkeeper but the probability of that happening depends on who is the Coach today:

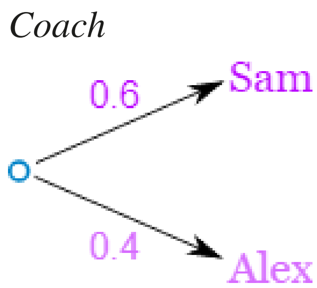
- with Coach Sam, the probability of being the Goalkeeper is **0.5**
- with Coach Alex, the probability of being the Goalkeeper is **0.3**

Sam is Coach more often, 6 out of every 10 games (a probability of **0.6**).

So, what is the probability you will be a Goalkeeper today?

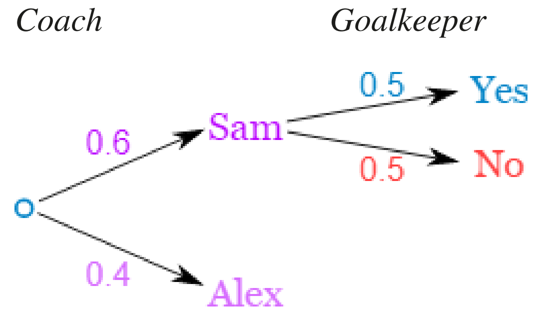
Solution

Let us build the tree diagram. First, we show the two possible coaches: Sam or Alex:

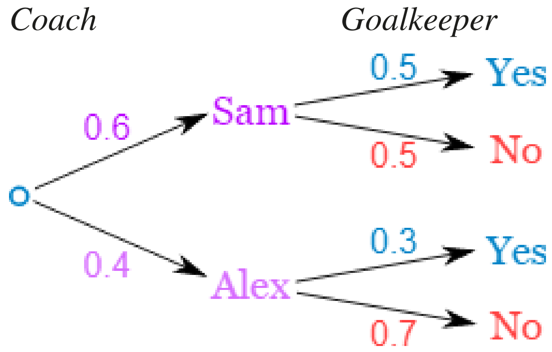


The probability of it being Coach Sam is 0.6, so the probability of Coach Alex must be 0.4 (together the probability is 1)

Now, if you get Sam, there is 0.5 probability of being Goalie (and 0.5 of not being Goalie):

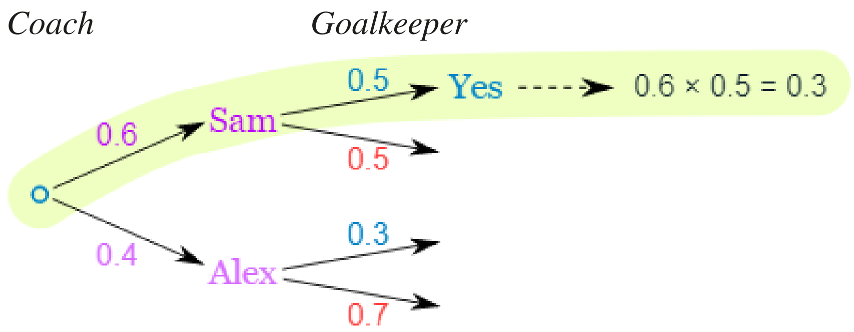


If you get Alex, there is 0.3 probability of being Goalie (and 0.7 not):



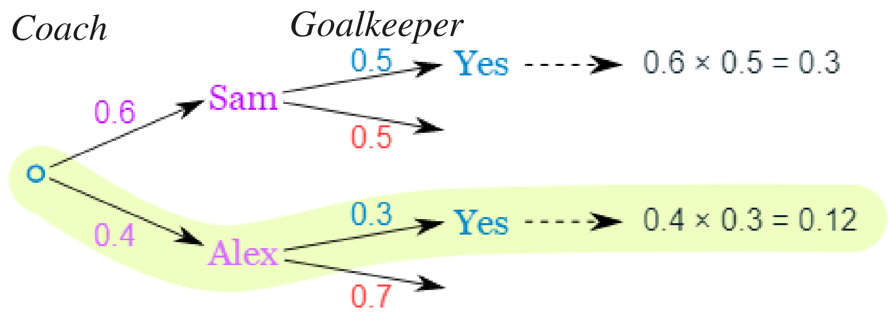
The tree diagram is complete, now let's calculate the overall probabilities. This is done by multiplying each probability along the "branches" of the tree.

Here is how to do it for the "Sam, Yes" branch:



(When we take the 0.6 chance of Sam being the coach and include the 0.5 chance that Sam will let you be Goalkeeper we end up with a 0.3 chance.)

But we are not done yet! We haven't included Alex as Coach:



A 0.4 chance of Alex as Coach, followed by a 0.3 chance gives 0.12.

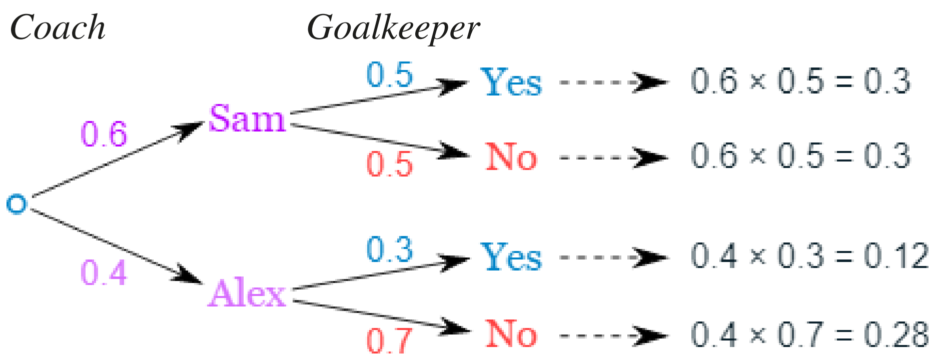
Now we add the column:

0.3 + 0.12 = **0.42 probability** of being a Goalkeeper today

(That is a 42% chance)

Check

One final step: complete the calculations and make sure they add to 1:



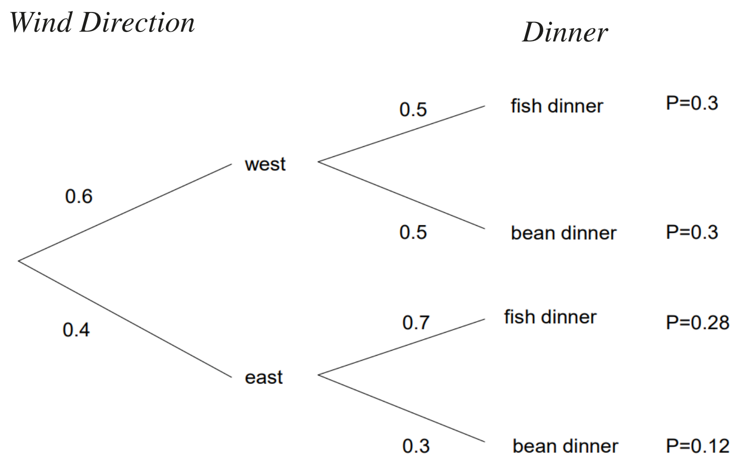
0.3 + 0.3 + 0.12 + 0.28 = 1

Yes, it all adds up.

Example 8.12

A maths teacher is going fishing. He finds that he catches fish 70% of the time when the wind is out of the east. He also finds that he catches fish 50% of the time when the wind is out of the west. If there is a 60% chance of a west wind today, what are his chances of having fish for dinner? Assume that if he doesn't catch fish, he will eat bean dinner.

Solution



P(east, fish) = P(east) x P(fish | east) = 0.4 × 0.7 = 0.28

P(west, fish) = P(west) x P(fish | west) = 0.6 × 0.5 = 0.30

Probability of a fish dinner: 0.28 + 0.3 = 0.58

So, the maths teacher has a 58% chance of having a fish dinner.

Alternatively

- The probability of catching fish is influenced by the wind direction (east or west).

To determine the overall probability of the teacher catching fish, we must consider both possible wind directions and their respective probabilities.

Given Information

1. Probability of catching fish with an east wind: P(Fish | East Wind) = 0.70
2. Probability of catching fish with a west wind: P(Fish | West Wind) = 0.50
3. Probability of east wind: P(East Wind) = 1−0.60=0.40
4. Probability of west wind: P(West Wind) = 0.60

Steps to Find Overall Probability

1. **Calculate the probability of catching fish when the wind is out of the east:**
   P(Fish and East Wind) = P(East Wind) × P(Fish | East Wind)

   = 0.40 × 0.70

   = 0.28

2. **Calculate the probability of catching fish when the wind is out of the west**

   P(Fish and West Wind) = P(West Wind) × P(Fish | West Wind)

   = 0.60 × 0.50

   = 0.30

3. **Add the probabilities from both wind directions to get the total probability of catching fish**

   P(Fish) = P(Fish and East Wind) + P(Fish and West Wind)

   = 0.28 + 0.30

   = 0.58

Conclusion: The probability that the maths teacher will have fish for dinner is 0.58 or 58%.

Learning Tasks

Learners solve problems on probability of dependent events using tree diagrams.

PEDAGOGICAL EXEMPLARS

Using think-pair-share activities, discuss the steps in determining whether a probability is dependent or independent.

In mixed-groupings, learners solve everyday life problems involving the probability of dependent events. Learners pair-share ideas, promote respect for divergent views to ensure inclusivity in the mathematics learning environment.

KEY ASSESSMENT

1. A box contains 5 red marbles and 3 blue marbles. If two marbles are randomly drawn one after the other without replacement, what is the probability that both marbles drawn are red?
2. A deck of 52 playing cards is shuffled and two cards are randomly drawn one after the other without replacement. What is the probability that the first card drawn is a king and the second card drawn is a queen?
3. A jar contains 10 black balls and 6 white balls. If two balls are randomly drawn consecutively without replacement, what is the probability that the first ball is black and the second ball is white?
4. In a bag, there are 7 green apples and 4 red apples. If two apples are randomly picked one after the other without replacement, what is the probability that the first apple picked is green and the second apple picked is red?
5. In a small orchard, there are 5 apple trees and 3 pear trees. If a gardener randomly picks one tree to inspect for pests and then randomly picks another tree without replacement, what is the probability that both trees picked are apple trees?
6. A classroom has 10 girls and 6 boys. If the teacher randomly selects two students one after the other without replacement to participate in a science demonstration, what is the probability that the first student selected is a girl and the second student selected is a boy?
7. A vending machine contains 12 bottles of water and 8 cans of soda. If a customer randomly buys one drink and then another without replacement, what is the probability that the first drink is a bottle of water and the second drink is a can of soda?
8. At a local bakery, there are 7 chocolate donuts and 5 vanilla donuts. If a customer randomly buys two donuts one after the other without replacement, what is the probability that the first donut is chocolate and the second donut is vanilla?

Section Review

The section has covered the following key ideas;

1. **Developing and Implementing Project on Data Collection, Analysis and Interpretation**

   Learners will design a project to collect data on a chosen topic, analyse the data using various methods and interpret the results. Using computer applications for data analysis can enhance accuracy and efficiency, making the interpretation of results more insightful and relevant to real-world scenarios.

2. **Simple and Compound Probability Experiments**

   a. A sample space is the set of all possible outcomes of a simple experiment, such as flipping a coin or rolling a die. Understanding the sample space helps in predicting probabilities and outcomes.

   b. In a compound experiment, the sample space includes all possible outcomes from multiple simple experiments performed together, such as rolling two dice or flipping two coins. This concept helps learners understand more complex probability scenarios.

   c. When two events are dependent, the outcome of one event affects the outcome of the other. Learners explore how to determine the sample space and calculate probabilities when events are dependent, which is crucial in understanding real-life situations like drawing cards from a deck without replacement.

SECTION 9: VECTORS AND TRIGONOMETRY

Strand: Geometry around us

Sub-Strand: Measurement

Learning Outcomes:

1. *Carry out addition, subtraction and scalar multiplication on vectors and investigate with and without technology, some properties (e.g., commutative, associative and distributive properties) of the operations.*
2. *Determine the inverse of trigonometric ratios, calculate angles of elevation and depression in everyday life situations and apply the knowledge to calculate distances and heights.*

Content Standards:

1. *Demonstrate knowledge and understanding of measurement with respect to operations on bearings and vectors.*
2. *Demonstrate an understanding of the inverse of trigonometric ratios, angles of elevation/ depression and apply the knowledge to calculate distances and heights.*

INTRODUCTION AND SECTION SUMMARY

In this section, learners explore two fundamental areas: vector operations and trigonometry. The section investigates addition, subtraction and scalar multiplication of vectors. Through hands-on exploration, learners will discover the properties governing these operations, such as the commutative, associative and distributive laws and application of these concepts to real-world scenarios. The following week begins with a review of basic trigonometric ratios. We will then extend these to inverse trigonometric functions, which allow us to find angles with given ratio values. Finally, we will explore the practical applications of trigonometry in various fields of mathematics in the real world.

The weeks covered by the section are

Week 23: Vectors

Focal Area 1: *Operations and its Properties on Vectors*

a) *Perform addition and subtraction operations on Vectors through investigation*

b) *Perform Scalar Multiplication in Two/Three-space*

Focal Area 2: *Operations and its Properties on Vectors*

a) *Determine the properties (commutative, associative, distributive, etc.) of operations on vectors.*

b) *Model and solve problems involving the operations of vectors.*

Week 24: Trigonometric Ratios

Focal Area 1: *Trigonometric Ratios*

a) *Review trigonometry ratios*

b) *Determine the inverse of trigonometric ratios*

Focal Area 2: *Application of trigonometry*

c) *Calculate angles of elevation and depression in everyday life situations*

d) *Apply the knowledge of trigonometry to calculate distances and heights*

SUMMARY OF PEDAGOGICAL EXEMPLARS

This section focuses on hands-on, **talk-for-learning approach**, initiating with a review of learners' prior knowledge on vectors and their classifications. Learners will engage in **practical measurements, collaborate in teams** and work in **mixed-groupings** to enhance their comprehension of vectors. To foster **collaborative learning** and deepen understanding, learners engage in a series of group discussions exploring the triangular law of vector addition, its application to vector subtraction and the concept of scalar multiplication.

Think-pair-share activities provide opportunities for individual reflection and peer interaction when solving real-world vector problems on properties (commutative, associative, distributive, etc.) of operations on vectors. The concept of trigonometry ratios will be taught through **practical activities**, showcasing its applications in architecture, engineering and navigation. Learners will apply the theorem to identify right triangles and ratios, reinforcing their understanding through practical applications.

ASSESSMENT SUMMARY

Different forms of assessments should be carried out to ascertain learners' performance on the concepts that will be taught under this section. Teachers are entreated to administer these assessments and record them for onward submission into the Student Transcript Portal (STP). The following assessment would be conducted and recorded for each learner:

Week 23: *Portfolio Submission*

Week 24: *End of Semester Examination*

Note

For additional information on how to effectively administer these assessment modes, refer to the Appendices.

WEEK 23: VECTORS

Learning Indicators

1. *Perform addition, subtraction and scalar multiplication on vectors represented as directed line segments in two-space and in Cartesian form in two and three-space.*
2. *Determine the properties (commutative, associative, distributive, etc.) of the operations on vectors through investigation with and without technology.*
3. *Solve problems involving the addition, subtraction and scalar multiplication of vectors, including problems arising from real-world applications.*

FOCAL AREA 1: PROPERTIES AND OPERATIONS OF VECTORS

The concept of vector

Refer the learner to real-life situations where they can experience the concept of vectors. Some referents of vectors are positions and distances of objects from us when we are given specific directions.

Vector as a Visual Representation

Learners imagine a map with straight lines drawn to indicate the position of their school from their homes and discuss their findings in groups.

These lines not only show the distance between the two places (length of the segment) but also the direction they are moving (indicated by a bearing).

Note

A vector has a definite starting point and endpoint to show its sense (indicated by a ray, $\overrightarrow{SH}$).

Definition of terms

In our daily life, we generally come across two types of quantities, namely scalars and vectors.

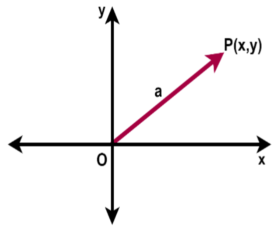
A quantity that has magnitude (length) only is known as a scalar. Examples: mass, length, time, temperature, density, speed, etc., is a scalar.

A quantity that has magnitude as well as direction is called a vector. Examples, force, velocity, acceleration and momentum is a vector.

Also, a directed line segment (with initial point and the terminal point) is called a vector.

Position Vector

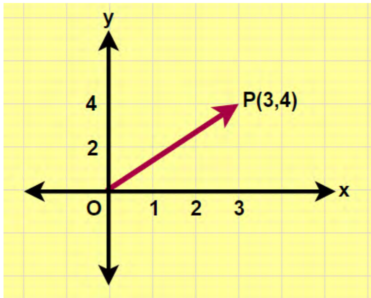
A position vector is a vector that describes the position of a point on the Cartesian plane relative to the origin.



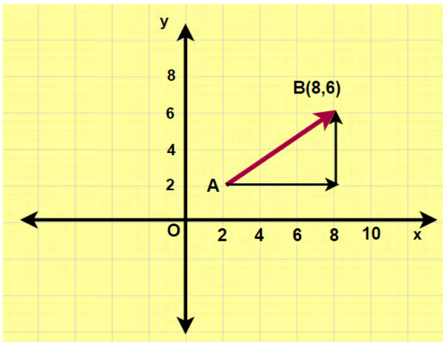
If $P(x, y)$ is any point on the Cartesian plane and O is the origin, then the point P relative to O is a position vector $\overrightarrow{OP}$.

That is $\overrightarrow{OP} = \binom{x}{y}$. For example, the point $P(3, 4)$ on the Cartesian plane will be a position vector $\overrightarrow{OP} = \binom{3}{4}$\

Expressing two given points as a Vector



Given the point $A(x, y)$ and $B(a, b)$ on the Cartesian plane with the origin O, we can express the two points as a vector $\overrightarrow{AB} = -\ \overrightarrow{OA} +\ \overrightarrow{OB}$. That is, $\overrightarrow{AB} = \binom{-x+a}{-y+b}$.



For example, given the point $A(2,2)$ and $B(8,6)$ on the Cartesian plane with the origin O, we can express the two points as $\overrightarrow{AB} = \binom{8}{6} - \binom{2}{2} = \binom{6}{4}$.

When combining the effects of two or more vectors, like forces acting on an object or velocities of moving bodies, we are essentially finding their combined impact. This combined effect is called the **resultant vector**. It is like adding the individual effects together to get a net outcome.

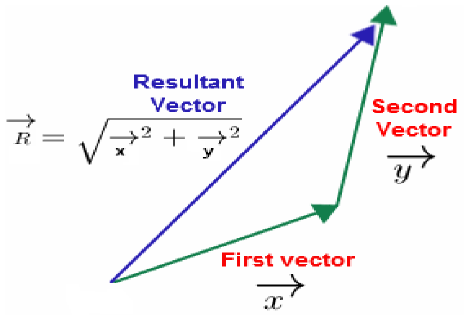
For instance, if you walk 3 kilometres north and then 4 kilometres east, your final position relative to your starting point forms the resultant vector. There are mathematical ways to determine the magnitude and direction of this resultant vector, which we will explore later.

Understanding Resultant Vectors

A resultant vector is the net effect or combination of two or more vectors. It represents the overall direction and magnitude of the combined vectors.

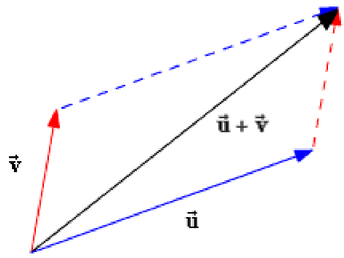
Imagine multiple forces acting on an object; the resultant vector would be the single force that would produce the same effect.

Visualizing Resultant Vectors



1. Head-to-Tail Method

- Draw the first vector.
  - Place the tail of the second vector at the head of the first.
  - The resultant vector is drawn from the tail of the first vector to the head of the second.



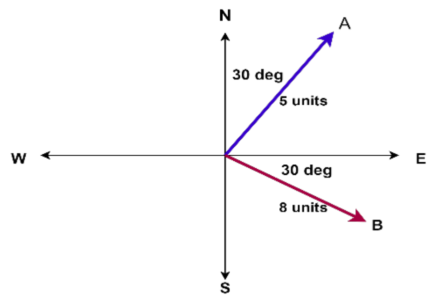
2. Parallelogram Method

- Draw the two vectors starting from the same point.
- Complete the parallelogram using the vectors as adjacent sides.
- The diagonal starting from the same point is the resultant vector.

Calculating Resultant Vectors

If you have the magnitudes and angles of the individual vectors, you can use trigonometry to calculate the magnitude and direction of the resultant vector.

Example 9.1



- Vector A: 5 units at 30 degrees
- Vector B: 8 units at 120 degrees
- To find the resultant vector, you would use the cosine rule and the sine rule to calculate its magnitude.

Solution

$$OA = \begin{pmatrix} 5\ cos\,60^o \\ 5\ sin\,60^o \end{pmatrix} = \begin{pmatrix} 2.5 \\ 2.5\sqrt{3} \end{pmatrix}$$

$$OB = \begin{pmatrix} 8\ cos\,30^o \\ -8\ sin\,30^o \end{pmatrix} = \begin{pmatrix} 4\sqrt{3} \\ -4 \end{pmatrix}$$

$$OA + OB = \begin{pmatrix} 2.5 \\ 2.5\sqrt{3} \end{pmatrix} + \begin{pmatrix} 4\sqrt{3} \\ -4 \end{pmatrix} = \begin{pmatrix} 2.5 + 4\sqrt{3} \\ 2.5\sqrt{3} - 4 \end{pmatrix} = \begin{pmatrix} 9.4 \\ 0.3 \end{pmatrix}$$

Magnitude $= \sqrt{9.4^2 + 0.3^2} = \sqrt{88.45} = 9.4$

Angle with respect to the horizontal $= \tan^{-1}\left(\frac{0.3}{9.4}\right) = 1.83^0$

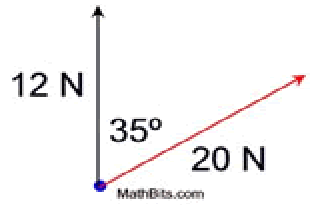
Therefore, the resultant vector has a magnitude of approximately 9.43 units and an angle of approximately 1.8 degrees with respect to the horizontal.

Example 9.2

Given two vectors representing forces acting upon one object, separated by 35º. Find the resultant vector.

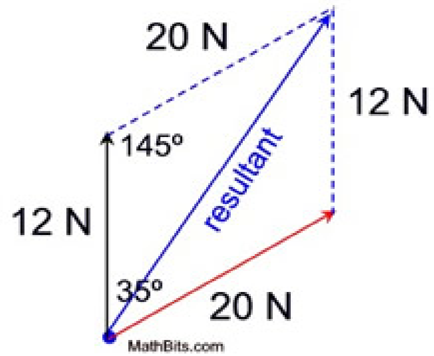
Method 1: (Head to tail method)

In this method, the tail of one vector is "slid up" (translated) to attach to the head of the other vector.



Method 2: (Parallelogram method)

In this method, the original diagram is used to create a parallelogram. The opposite sides of the parallelogram are congruent and the consecutive angles are supplementary. The resultant vector is the diagonal of the parallelogram.



Operations and Properties on Vectors

1. Vector Addition and Subtraction

Understand vector addition and subtraction geometrically and algebraically as follows.

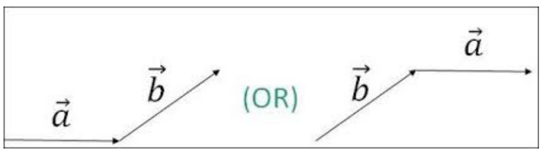
Key Concepts

a) Head-to-tail method for addition

b) Subtraction as addition of the negative vector

c) Parallelogram law for addition

d) Zero vector and its properties

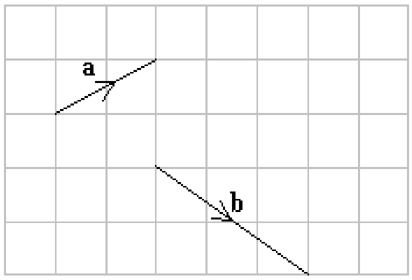


FOCAL AREA I: VECTOR OPERATIONS IN TWO DIMENSIONS

a). Geometric and Algebraic representation of vectors in two–space

Geometric Representation

In two-dimensional space, vectors are visually represented as directed line segments. The key components of this representation are:

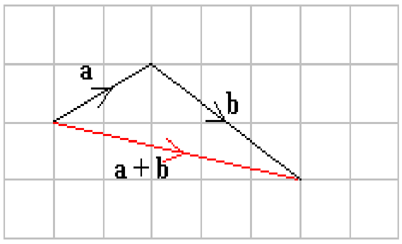


1. Starting point (tail): The origin of the vector.
2. Terminal point (head): The endpoint of the vector, indicating its sense direction.
3. Length: Represents the magnitude of the vector.
4. Sense: Shown by the arrow at the terminal point.

Example 9.3

Find the sum of the two given vectors **a** and **b**.

Solution



Draw the vector $\boldsymbol{a}$.

Draw the 'tail' of vector $\boldsymbol{b}$, joined to the 'nose' of vector $\boldsymbol{a}$.

The vector $\boldsymbol{a} + \boldsymbol{b}$ is from the 'tail' of $\boldsymbol{a}$ to the 'nose' of $\boldsymbol{b}$.

Algebraic Representation

In two-space, vectors are typically represented in Cartesian form as an ordered pair (x, y), where:

- x represents the horizontal component

- y represents the vertical component

Key points to cover:

1. The relationship between geometric and algebraic representations:
2. x is the change in x-coordinate from tail to head
3. y is the change in y-coordinate from tail to head
4. Zero vector: (0, 0)
5. Unit vectors: $\hat{\imath} = (1, 0)$ *and* $\hat{\jmath} = (0, 1)$
6. General form: $\boldsymbol{a} = x\hat{\imath} + y\hat{\jmath}$

Instructional strategies:

1. Use graph paper or digital graphing tools to illustrate the connection between geometric and algebraic representations.
2. Practise converting between the two forms.
3. Introduce real-world examples like displacement or force vectors.

Vector addition and subtraction

Vector Addition

Geometrically:

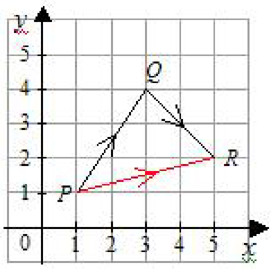
1. Head-to-tail method: Place the tail of the second vector at the head of the first vector. The resultant vector is drawn from the tail of the first to the head of the second.
2. Parallelogram method: Draw a parallelogram using the two vectors as sides. The diagonal of this parallelogram represents the sum.

Algebraically:

For vectors $\mathbf{a} = (x_1, y_1)$ and $\boldsymbol{b} = (x_2, y_2)$:

$\boldsymbol{a} + \boldsymbol{b} = (x_1 + x_2, y_1 + y_2)$

Example 9.4



Given that, $\overrightarrow{PQ} = \binom{2}{3}$ *and* $\overrightarrow{QR} = \binom{2}{-2}$ find the sum of the vectors.

The sum of the vectors $\overrightarrow{PQ}$ *and* $\overrightarrow{QR}$ is the same as the vector $\overrightarrow{PR}$. That is $\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$. Therefore, we add the corresponding components of the vectors as $\overrightarrow{PQ} + \overrightarrow{QR} = \binom{2}{3} + \binom{2}{-2} = \binom{4}{1} = \overrightarrow{PR}$

Vector Subtraction

Geometrically:

To subtract **b** from $\boldsymbol{a}$, add the negative of $\boldsymbol{b}$ to $\boldsymbol{a}$. Visualise this by reversing the direction of $\boldsymbol{b}$ and then using the addition methods.

Algebraically:

$a - b = (x_1 - x_2, y_1 - y_2)$

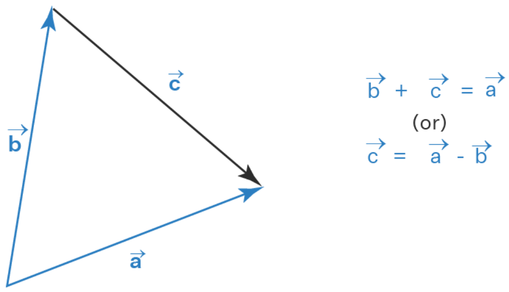
Key concepts to emphasize:

1. Commutative property of addition: $a + b = b + a$
2. Associative property of addition: $(a + b) + c = a + (b + c)$
3. The zero vector is the additive identity
4. The negative of a vector: $-a = (-x, -y)$

First, denote the vector drawn from the endpoint of $\boldsymbol{b}$ to the endpoint of $\boldsymbol{a}$ by $\boldsymbol{c}$.

Note *that* $\boldsymbol{b} + \boldsymbol{c} = \boldsymbol{a}$. *Thus,* $\boldsymbol{c} = \boldsymbol{a} - \boldsymbol{b}$.

In other words, the vector $\boldsymbol{a} - \boldsymbol{b}$ is the vector drawn from the tip of $\boldsymbol{b}$ to the tip of $\boldsymbol{a}$ (if $\boldsymbol{a}$ and $\boldsymbol{b}$ are co-initial).



Scalar multiplication and its effect on vector magnitude and direction

Scalar multiplication involves multiplying a vector by a real number (scalar), affecting its magnitude and potentially its direction.

Unit Vectors

A unit vector has the same direction as the given vector but has a magnitude of one unit. To find a unit vector, divide the vector by its magnitude. Unit vector, $\hat{\boldsymbol{a}}$, $= \frac{\boldsymbol{a}}{|\boldsymbol{a}|}$

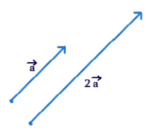
Example 9.5

If $\boldsymbol{a} = (5, 12)$, $|\mathbf{a}| = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$

$\therefore$ Unit vector $\hat{\boldsymbol{a}}$,$= \left(\frac{5}{13}, \frac{12}{13}\right)$

Key points:

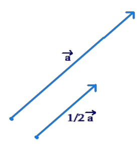
1. **Magnitude effect:** If the magnitude of the scalar is greater than 1, then the new vector is longer than the original vector;



Example 9.6

Consider a vector $\vec{a}$. What happens if you multiply this vector by 2? What will the vector $2\vec{a}$ represent?

- o if it is less than 1, then the new vector is shorter;



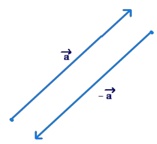
Example 9.7

The vector $\frac{1}{2}\ \vec{a}$ will be a vector in the same direction as $\vec{a}$**,** but with a length equal to half of the length of $\vec{a}$**:**

- **o** if the scalar is equal to 1, the new vector has the same length as the original.

Example: We have seen how to interpret the vector $-\vec{a}$, given the vector $\vec{a}$:

Algebraically:



For a scalar k and vector $\underline{a} = \begin{pmatrix} x \\ y \end{pmatrix}$

$$k\underline{a} = \begin{pmatrix} kx \\ ky \end{pmatrix}$$

Properties:

1. Distributive property: $k(\boldsymbol{a} + \boldsymbol{b}) = k\boldsymbol{a} + k\boldsymbol{b}$
2. Associative property: $(kl)\boldsymbol{a} = k(l\boldsymbol{a})$, where k and l are scalars
3. Vector decomposition: $a = x\boldsymbol{a}\,\hat{\imath} + y\boldsymbol{a}\,\hat{\jmath}$

Property of Vector Addition	Explanation
Existence of identity	For any vector **v**, $\mathbf{v} + \mathbf{0} = \mathbf{v}$ Here, **0** vector is the additive identity.
Existence of inverse	For any vector **v**, $\mathbf{v} + - \mathbf{v} = \mathbf{0}$ and thus an additive inverse exists for every vector.
Commutativity	Addition is commutative; for any two arbitrary vectors **c** and **d**, $\mathbf{c} + \mathbf{d} = \mathbf{d} + \mathbf{c}$
Associativity	Addition is associative; for any three arbitrary vectors **i**, **j** and **k** , $\mathbf{i} + \mathbf{j} + \mathbf{k} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ i.e., the order of addition does not matter.

d) Transition between geometric and algebraic representations

Key transition concepts:

1. From geometric to algebraic:
   - Identify the horizontal and vertical components.
   - Express as an ordered pair (x, y).
2. From algebraic to geometric:
   - Plot the terminal point (x, y) relative to the origin.
   - Draw the arrow from the origin to this point.
3. Magnitude calculation:
   - Geometrically: Measure the length of the arrow.
   - Algebraically: Use the Pythagorean theorem, $|a| = \sqrt{(x^2 + y^2)}$.
4. Direction calculation:
   - Geometrically: Measure the angle with respect to the positive x-axis.
   - Algebraically: Use arctangent, $\theta = tan^{-1}\left(\frac{y}{x}\right)$, considering the appropriate quadrant.
5. Component form to magnitude-direction form:

   $|a| = \sqrt{x^2 + y^2}$

   $\theta = tan^{-1}(y \_ x)$

Real-life uses of vectors

1. Vectors can be used in finding the direction in which the force is applied to move an object.
2. The concept of vectors aids in understanding how gravity uses a force of attraction on an object to work.
3. Vectors can be used in obtaining the motion of a body which is confined to a plane.
4. Vectors help in defining the force applied on a body simultaneously in the three dimensions.
5. In the field of Engineering, for a structure not to collapse, vectors are used where the force is much stronger than the structure will sustain.
6. Vectors are used in various oscillators.

Learning tasks

1. Model real-world scenarios using vector operations; interpret solutions in context; design problems that require vector operations to solve; and propose novel applications of vector operations

2. Explore geometric and algebraic representation of vectors in two-space.
3. Investigate operations of vectors (vector addition and subtraction, Scalar multiplication) and their effects on vector magnitude and direction
4. Transition between geometric and algebraic representations
5. Investigate properties of vectors with respect to commutative, associative and distributive
6. Model Word problem and analyse real-world applications involving vectors.

Worked Examples:

Example 9.8: Vector Addition

Add vectors $\underline{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ *and* $\underline{b} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$

Solution

$$\underline{a} + \underline{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 2-1 \\ 3+4 \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$$

Example 9.9

Given vectors $\underline{u} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\underline{v} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$, find $\underline{u} + \underline{v}$

a. both algebraically and

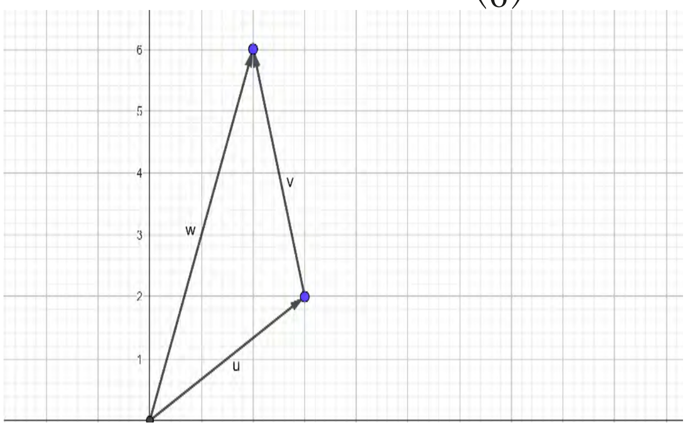
b. geometrically.

Solution

a. Algebraically: $\underline{u} + \underline{v} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 3-1 \\ 2+4 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$

b. Geometrically:

- Draw vector a from the origin to point $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ From the head of $\underline{u}$, draw vector $\underline{v}$ to point $\begin{pmatrix} 2 \\ 6 \end{pmatrix}$
- The resultant $\underline{w}$ vector is from the origin to $\begin{pmatrix} 2 \\ 6 \end{pmatrix}$



Example 9.10

Subtract vector $\underline{p} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ from vector $\underline{\boldsymbol{q}} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

Solution

$$\underline{q} - \underline{p} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 - 5 \\ 3 - (-2) \end{pmatrix} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$$

Example 9.11

Multiply vector $\underline{v} = (2, -3)$ by scalar $k = -2$.

Solution

$$k \cdot \underline{v} = -2 \cdot \begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -2(2) \\ -2(-3) \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$$

Example 9.12

Multiply vector $\underline{v} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$ *by scalar* $k = 4$

Solution

$$4\underline{v} = 4\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 12 \\ -8 \\ 4 \end{pmatrix}$$

Example 9.13

Find the magnitude of vector $v = (3, 4)$

Solution

$$||v|| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

Example 9.14

Find the magnitude and direction of vector $\underline{w} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

Solution

Magnitude $|w| = \sqrt{(4^2 + 3^2)} = \sqrt{25} = 5$

Direction: $\theta = tan^{-1}\left(\frac{y}{x}\right) = tan^{-1}\left(\frac{3}{4}\right) \approx 36.87°$

The vector has a magnitude of 5 *units* and points at an angle of approximately 36.87° above the positive x-axis.

Example 9.15

Decompose vector $\underline{u}$ = (3, 4) into its horizontal and vertical components.

Solution

Horizontal component $\underline{u}x = 3\hat{\imath}$ Vertical component $\underline{u}y = 4\hat{\jmath}$

Verification: $\underline{u} = \underline{u}x + u\_y = 3\hat{\imath} + 4\hat{\jmath} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

PEDAGOGICAL EXEMPLARS

1. Think-pair share activities: In pairs, learners solve real world problems on vectors.
2. Group discussions: In groups, task learners determine the properties (commutative, associative, distributive, etc.) of operations on vectors in component form
3. Group discussions: In groups, task learners to solve some addition, subtraction and scalar multiplication of vectors in component form

KEY ASSESSMENT

1. Define a vector and explain the difference between a scalar and a vector quantity.
2. Explain the parallelogram method for vector addition. Include a diagram in your answer.
3. Given vectors A = $3i + 4j$ and B = $-2i + 5j$, calculate:
   - **a)** A + B
   - **b)** A - B
   - **c)** |A| (magnitude of A).
4. Describe how to find the unit vector in the direction of a given vector.

WEEK 24: TRIGONOMETRIC RATIOS

Learning Indicators

1. *Determine the inverse of trigonometric ratios (graphs excluded) and talk about their applications in the field of astronomy, engineering, physics, geometry and navigation.*
2. *Solve real-life problems involving angles of elevation and depression and identify everyday life situations of these concepts.*

FOCAL AREA 1: TRIGONOMETRIC RATIOS

Reviewing and exploring Trigonometric Ratios

The literal meaning of the word trigonometry is the science of measuring the sides and the angles of triangles.

However, in modern times it is used in a wider context.

> We define trigonometry as the branch of mathematics which deals with the measurements of angles of triangles and the problems related to these angles.

Let's try this Activity

1. Construct right-angled triangles and measure sides and angles to calculate the ratios between them
2. Investigate the triangles to work out the trigonometry ratios and their reciprocals
3. Practise calculating ratios for various angles

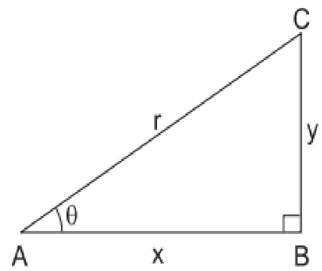
Let $< BAC$ be an acute angle of a right-angled ΔABC.

In right-angled ΔABC, let

base AB x units,

perpendicular BC yunits

and *hypotenuse AC r units*.



We define the following ratios, known as Trigonometric Ratios for θ .

1. $sine\theta = \frac{Perpendicular}{Hypotenuse} = \frac{y}{r} = \frac{Opposite}{Hypotenuse}$, and is written as $sin\theta$.
2. $cosine\theta = \frac{Base}{Hypotenuse} = \frac{x}{r} = \frac{Adjacent}{Hypotenuse}$, and is written as $cos\theta$.
3. $tangent\theta = \frac{Perpendicular}{Base} = \frac{y}{x} = \frac{Opposite}{Ajdacent}$, and is written as $tan\theta$.

4. $cosecant\theta = \frac{Hypotenuse}{Perpendicular} = \frac{r}{y} = \frac{Hypotenuse}{Opposite}$, and is written as $cosec\ \theta$.

5. $secant\theta = \frac{Hypotenuse}{Base} = \frac{r}{x} = \frac{Hypotenuse}{Adjacent}$, and is written as $sec\theta$.

6. $cotangent\theta = \frac{Base}{Perpendicular} = \frac{x}{y} = \frac{Adjacent}{Opposite}$, and is written as $cot\theta$

Determine the inverse of trigonometric ratios

We can see from the ratios above that the reciprocals ratios are:

1. $cosec\theta = \frac{1}{sin\theta}$,
2. $sec\theta = \frac{1}{\cos\theta}$, and
3. $cot\theta = \frac{1}{\tan\theta}$.

N/B: Create mnemonic devices for easy reminder of the ratios (e.g., $SOH-CAH-TOA$)

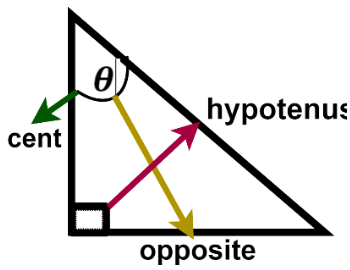
- Sine $= \frac{\text{Opposite}}{\text{Hypotenuse}}$
- Cosine $= \frac{\text{Adjacent}}{\text{Hypotenuse}}$
- Tangent $= \frac{\text{Opposite}}{\text{Adjacent}}$

Learning Tasks

Model and solve problems involving trigonometric ratios.

Try these activities with learners

Learners investigate various triangular shapes, indicate the right angle and one interior angle and establish the ratios ***sine***, ***cosine*** and ***tangent*** of the angle.



A **Right-Angled Triangle,** is a triangle that has one angle that measures 90 degrees (90^o). The side opposite the right angle is called the hypotenuse and the other two sides are called the opposite and adjacent sides relative to a specific angle.

Applications of Trigonometry to real life

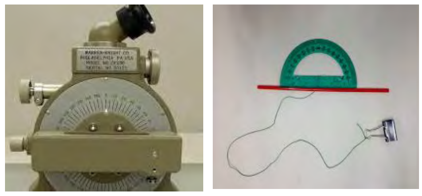
Learners can investigate trigonometric concepts in real-world situations.

Key Applications:

1. *Surveying and navigation*: Calculating distances and heights; GPS and triangulation; etc.
2. *Physics and engineering*: Resolving forces; Oscillations and waves; etc.
3. *Architecture and construction*: Roof angles and support structures; Bridge design; etc.

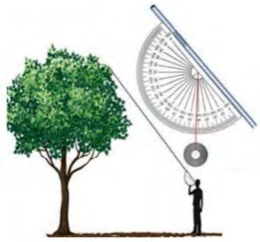
4. *Astronomy*: Calculating distances to celestial bodies; Orbital mechanics; etc.
5. *Medicine*: CT scans and MRI imaging; etc.

Learners can:



clinometers

1. Measure the height of tall objects using clinometers
2. Analyse the angles in various structures (buildings, bridges, etc.)
3. Solve problems related to circular motion
4. Encourage the use of clinometers to determine the height of given high objects such as a tree, using basic trigonometry formulas.

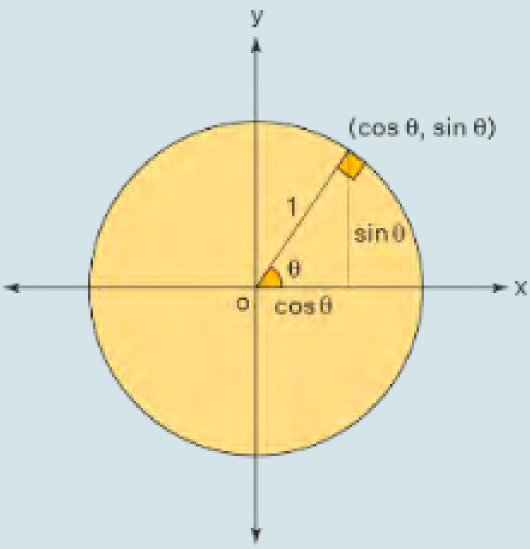


PEDAGOGICAL EXEMPLARS

1. Visual aids: Use diagrams, animations and physical models.
2. Employ interactive software (e.g., GeoGebra) for dynamic visualizations

Learning Tasks

1. Solve problems involving the three basic trigonometric ratios: ***sine*** of an angle, ***cosine*** of an angle and ***tangent*** of an angle.
2. Model and solve problems involving the Reciprocal Trigonometric Ratios.
3. Explore applications of basic trigonometry ratios and their inverses to solve real-world problems.
4. Explore the use of unit circle to calculate the values of basic trigonometric functions- sine, cosine and tangent.



Worked Examples

Example 9.16

In ΔABC, right-angled at B, AB 5 cm and BC 12 cm. Find the values of $sin\ A$, $cos\ A$ *and* $tan\ A$.

Solution

In ΔABC, $< ABC\ 90^o$, $AB = 5cm$ and $BC = 12cm$

By Pythagoras theorem, we have

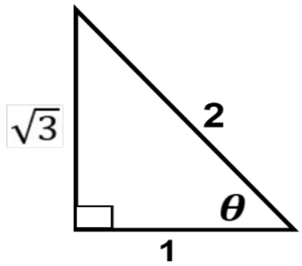
$|AC|^2 = |AB|^2 + |BC|^2 = \{5^2 + 12^2\}\ cm^2 = 169cm^2$

$\therefore AC = \sqrt{169cm^2} = 13cm$

$sinA = \frac{opp.(A)}{hyp.} = \frac{12}{13}$; $cosA = \frac{adj.(A)}{hyp.} = \frac{5}{13}$; and $tanA = \frac{opp.(A)}{adj.(A)} = \frac{12}{5}$

Example 9.17

If $sin\theta = \frac{\sqrt{3}}{2}$ find the values of the Trig-ratios cos and tan



Solution

Given that $sin\theta = \frac{\sqrt{3}}{2}$

By Pythagoras theorem, missing side is $\sqrt{(2^2 - \sqrt{3^2})} = \sqrt{(4-3)} = 1$

Implies $cos\theta = \frac{1}{2}$ *and* $tan\theta = \sqrt{3}$

Example 9.18

If $cos = \frac{3}{5}$, find the values of $cosec$ and cot.

Solution

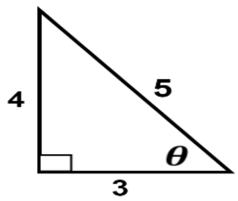
By Pythagoras theorem, missing side is $\sqrt{(5^2 - \sqrt{3^2})} = \sqrt{(25-9)} = 4$

Therefore, $sin\theta$ and $tan\theta$ are as follows:

$sin\theta = \frac{4}{5}$ and $tan\theta = \frac{4}{3}$

$cosec\theta = \frac{1}{sin\theta} = \frac{1}{\left(\frac{4}{5}\right)} = \frac{5}{4}$

$cot\theta = \frac{1}{tan\theta} = \frac{1}{\left(\frac{4}{3}\right)} = \frac{3}{4}$.

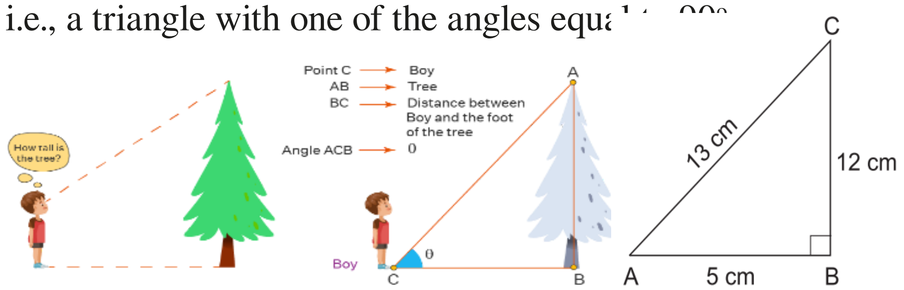


Example 9.19

A boy is standing near a tree. He looks up at the tree and wonders "How tall is the tree?"

Solution

The height of the tree can be found without actually measuring it. What we have here is a right-angled triangle, i.e., a triangle with one of the angles equa



Discuss the appropriate ratios involved. i.e. In a right-angled triangle, ΔABC, *tan of angle* θ is the ratio of the height of the tree to the distance between boy and foot of the tree.

Let us say the angle is θ, then $tan\theta = \dfrac{\text{eight}}{\text{Horizontal distance between boy and tree}} = \dfrac{AB}{CB}$

Learners propose different values for distance and height for this question and discuss the nature of the height.

For instance, if the distance $|CB| = 30m$ and the angle formed $\theta = 45^o$, then

$tan\theta = \dfrac{AB}{CB}$

$\rightarrow tan45° = \dfrac{\text{AB}}{\text{30m}} = \dfrac{AB}{CB}$

$\rightarrow Height = 30tan \times 45°$

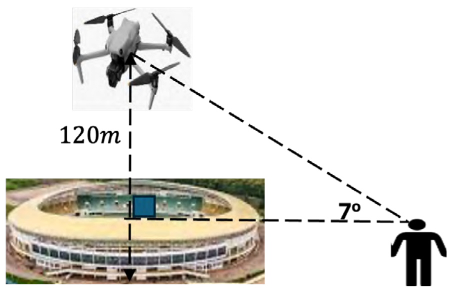
Since, tan 45° = 1

So, Height = 30 m

Example 9.20

A drone was stationed directly 120m above the centre spot of the Aliu Mahama Sports Stadium to monitor events during a match between RTU and Kotoko. A football fan is standing outside the stadium and decided to look up to see the drone at an angle of 7°. How far from the centre spot of the stadium is the football fan standing?

Solution



$tan\,7^o = \dfrac{20m}{distance\ from\ centre\ spot}$

$distance\ from\ centre\ spot = \dfrac{20}{tan\,7^o}$

$= \dfrac{20}{0.1228}$

$\therefore$ *the football fan is 977.32m away from the centre spot*

Special Angles

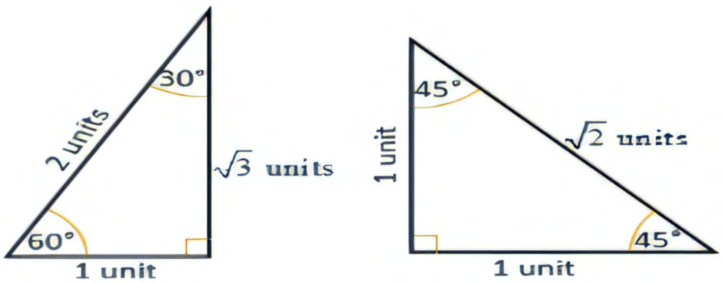
Engage learners to do activities on the following special angles shown below on how to derive the trig ratios of 30°, 45° and 60° from the 30-60-90 and 45-45-90 special triangles

E.g. $Sin60^o = \frac{\sqrt{3}}{2} = cos30^o \; (60^o S + 30^o = 90^o)$

$Sin45^o = \frac{\sqrt{2}}{2} = cos45^o \; (45^o S + 45^o = 90^o)$

$tan45^o = 1$

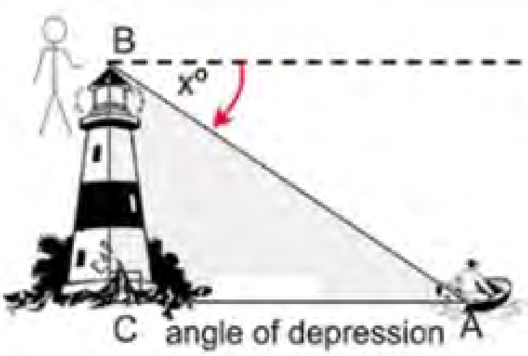
Etc.



	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Examples

Angle of Depression:

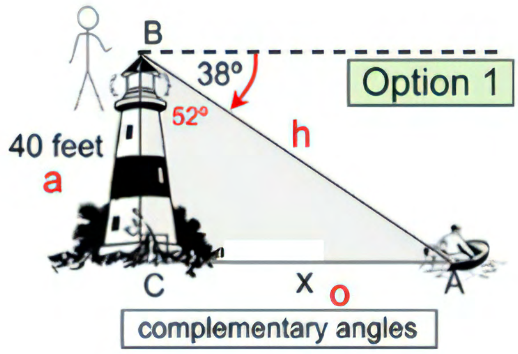


In this diagram, x^o marks the angle of depression of the boat at sea from the top of the lighthouse.

The angle of depression is always **OUTSIDE** the triangle. It is never inside the triangle. It is a downward angle from a horizontal line.

You can think of the angle of depression in relation to the movement of your eyes. You are standing at the top of the lighthouse and you are looking straight ahead. You must lower (*depress*) your eyes to see the boat in the water.

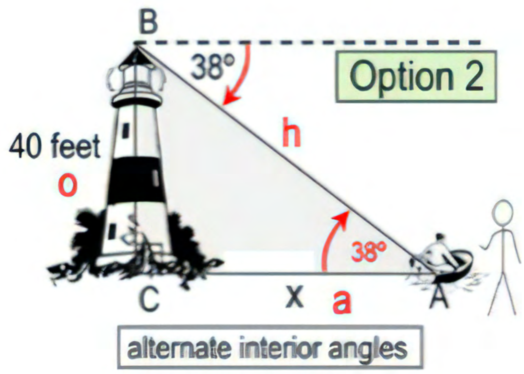
Other examples



When solving a problem with an angle of depression you need to find the measure of an angle **INSIDE** the triangle. There are two options:

Option 1: find the angle **inside** the triangle that is adjacent (next door) to the angle of depression. This adjacent angle will always be the complement of the angle of depression, since the horizontal line and the vertical line are perpendicular (90º). In the diagram to the left, the adjacent angle is 52º.

$$\tan 52 = \frac{opposite}{adjacent} = \frac{x}{40};$$
$$1.2799 = \frac{x}{40};$$
$$x = 51ft.$$



Option 2: utilise the fact that the angle of depression = the angle of elevation and label $\angle BAC$ as 38º **inside** the triangle.

$$\tan 38 = \frac{opposite}{adjacent} = \frac{40}{x}$$
$$0.7811286 = \frac{40}{x};$$
$$x = 51ft.$$

Worked examples

Example 9.21

Find the length of the shadow cast by a 10-foot lamp post when the angle of elevation of the sun is 58º. Find the length to the nearest the *nearest tenth of a foot.*

Solution

- Remember that the "angle of elevation" is from the horizontal ground line upward.
- It is assumed that the lamp post is vertical, making it perpendicular with the ground.
- Shadows are on the ground! If you place the "shadow" on the hypotenuse you have created an apparition (a "ghost"), not a shadow!
- This solution deals with "opposite" and "adjacent" making it a tangent problem.

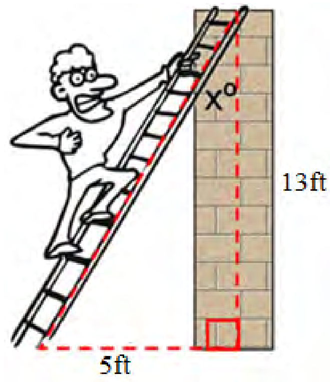
$\tan 58^{o} = \frac{10}{x}$;

$1.6003 = \frac{10}{x}$;

$x = 6.2$ feet

Example 9.22

A ladder leans against a brick wall.
The foot of the ladder is 5 feet from the wall.
The ladder reaches a height of 13 feet on the wall.
Find to the *nearest degree*, the angle the ladder makes with the wall



Solution

- In this problem place x° where the ladder meets the wall. Do not assume that the angle will always be at the ground level.
- It is assumed that the wall is vertical, perpendicular with the ground.
- The foot of the ladder is the bottom of the ladder, where it hits the ground.
- This solution deals with "opposite" and "adjacent" making it a tangent problem.

$\tan x = \frac{5}{13} = 0.3846$; $\tan^{-1}(0.3846) = 21.0^{\circ}$

Example 9.23

A woman, with her eyeline at a height of 1.52 metres, standing 20 metres away from the base of a tree observes its top at an angle of elevation of 30°. How tall is the tree?

Solution

Let x be the height of the tree.

$tan(30°) = \frac{opposite}{adjacent} = \frac{x}{20}$

$x = 20\ tan(30°)$

$x = 20\left(\frac{1}{\sqrt{3}}\right) \approx 11.55\ metres$

But remember, this is the height above the woman's eyes. We need to add her height to get the full tree height:

Total tree height = 11.547 + 1.52 = 13.067 metres

Therefore, the tree is approximately 13.07 *metres* tall.

Example 9.24

From the top of a 75-metre-tall lighthouse, a sailor observes a ship at an angle of depression, n°. The ship was seen to be 280.05 *metres* from the base of the lighthouse. Find the value of n°.

Solution

The distance from the base of the lighthouse to the ship :

$$tan\left(n^\circ\right) = \frac{opposite}{adjacent} = \frac{75}{280.05}$$

$$tan(n^\circ) = \frac{75}{280.05} = 0.2678$$

$$n^\circ = \tan^{-1} 0.2678 \approx 15$$

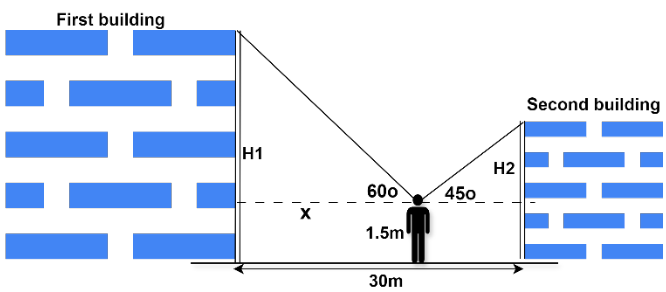
Example 9.25

A 1.5 *metre* tall person stands between two buildings. The angle of elevation to the top of the first building is 60° and to the top of the second building is 45°. If the buildings are 30 metres apart and the height of the first building is twice the second. Find the height of each building.

Solution

Let H_1 and H_2 be the heights of the first and second buildings respectively.

For the first building :



$$\tan 60^\circ = \frac{H_1 - 1.5}{x} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

By clearing the fractions and simplifying, we have

$H_1 - 1.5 = x\, tan(60^\circ)$, Since $tan(60^\circ) = \sqrt{3}$

$$H_1 = x\sqrt{3} + 1.5 \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

For the second building :

$\tan 45° = \frac{H_2 - 1.5}{30 - x}$....................(3)

By clearing the fractions and simplifying, we have

$H_2 - 1.5 = (30 - x) \times tan(45°)$

$H_2 = (30 - x) \times tan(45°) + 1.5$

But $tan(45°) = 1$

Implies $H_2 = 30 - x + 1.5$

$H_2 = 31.5 - x$..........................(4)

We can solve for x using the equations (2) *and* (4) :

Since $H_1 = 2H_2$

$H_1 = x\sqrt{3} + 1.5$................(2) becomes

$H_2 = \frac{x\sqrt{3} + 1.5}{2}$................(5)

Comparing equations (4) and (5)

$31.5 - x = x\frac{\sqrt{3} + 1.5}{2}$

$63 - 2x = x\sqrt{3} + 1.5$

$61.5 = 2x + x\sqrt{3}$

$61.5 = 3.7321x$

$x = 16.4787\ m$

Substitute the value of x into equations 2 and 5

$H_1 = 16.4787\sqrt{3} + 1.5 = 30.0418m$

$H_2 = \frac{H_1}{2} = \frac{30.0418m}{2} = 15.0209\ m$

Example 9.26

An airplane is flying at an altitude of 2000 metres. The pilot observes two towns with angles of depression 30° and 45° in a straight line in front of him. How far apart are the towns?

Solution

Let x be the distance between the two towns.

For the first town: $\tan(30°) = \frac{2000}{d_1}$

$d_1 = \frac{2000}{\tan(30°)}$

$\approx 3464.10\ meters$

For the second town: $\tan(45°) = \frac{2000}{d_2}$

$d_2 = \frac{2000}{tan(45°)} = 2000\ meters$

The distance between the towns is:

$x = d_1 - d_2$

$= 3464.10 - 2000$

$= 1464.10\ meters$

The towns are approximately 1464.10 *meters apart*.

Example 9.27

A person watches a hot air balloon rise vertically at a constant rate. When it is at an angle of elevation of 30°, its height is 100 metres. Two minutes later, the angle of elevation is 60°. What is the balloon's rate of ascent?

Solution

$At\ 30°: tan(30°) = 100 / x$

$x = 100 / tan(30°) \approx 173.21\ metres\ (horizontal\ distance)$

$At\ 60°: tan(60°) = h / 173.21$

$h = 173.21 * tan(60°) \approx 300\ metres$

Height difference $= 300 - 100 = 200\ metres$

Time difference $= 2\ minutes = 120\ seconds$

Rate of ascent $= 200\ metres / 120\ seconds \approx 1.67\ metres/second$

Example 9.28

From a bridge 40 metres above a river, the angle of depression to a boat downstream is 25°. How far is the boat from the point on the river directly below the bridge?

Solution

Let x be the distance from the point below the bridge to the boat.

$tan(25°) = opposite / adjacent = \frac{40}{x}$

$x = \frac{40}{tan(25°)}$

$x \approx 85.78\ meters$

The boat is approximately 85.78 metres from the point on the river directly below the bridge.

Example 9.29

A TV antenna is mounted on top of a 50-metre building. From a point on the ground, the angle of elevation to the base of the antenna is 60° and to the top of the antenna is 65°. Find the height of the antenna.

Solution

Let h be the height of the antenna.

$For\ the\ building : tan(60°) = \frac{50}{x}$

$x = \frac{50}{\tan(60°)} = \frac{50}{1.7321}$

$\approx 28.8667\ m$

$For\ the\ antenna{:}\tan(65°) = \frac{50 + h}{28.87}$

$50 + h = 28.87 * tan(65°)$

$h = 28.8667 * tan(65°) - 50$

$h = 61.9048$

$h \approx 11.9048\ meters$

The antenna is approximately 11.9048 metres tall.

PEDAGOGICAL EXEMPLARS

Experiential learning: In mixed-gender/ability groups, engage learners to explore the immediate school and classroom environment to investigate these angles.

Task learners to free-hand sketch some right-angled triangle and identify the sides relative to the interior angles. Now, present learners with a worksheet containing the basic trigonometry ratios and allow them time to establish the inverses of the basic ratios.

Problem-based learning: In small groups, engage learners to solve simple worked examples on the basic trigonometry ratios and discuss their solutions with colleagues in class.

Group & pair activities: Using think-pair- share, task learners to solve problems on reciprocals/inverses of the trigonometry ratios.

Whole Class discussions and demonstrations: In a whole class, they establish and discuss area in real-life where trigonometry is applied,

Problem-based group learning: Using mixed-ability groups, present learners work task sheets on trigonometry ratios.

Take time to deal with common misconceptions that learners are likely to have.

Individual tasks: Present learners with individual worksheets to complete on real-life applications of trigonometry

KEY ASSESSMENT

1. Sketch a right-angled triangle, label all sides

   Determine the sine, cosine and the tangent of each of the acute angles in the right-angled triangles.

2. In a right-angled triangle, the hypotenuse is 10 *cm* and one angle is 30^o. Find the lengths of the other two sides.
3. Given a right-angled triangle with an angle of 45^o and a hypotenuse of $\sqrt{2}$ units, find the lengths of the other two sides.
4. If $\sin\theta = \frac{3}{5}$ find $\cos\theta$, $\tan\theta$ and the values of θ in degrees.
5. If $\tan A = \sqrt{3}$, find the trigonometric ratios of
   a. $sinA$,
   b. $cosA$
   c. The value of A in degrees
6. If $sec = \frac{25}{7}$ find
   a. the trigonometric ratios of
      i. *sin* and
      ii. *tan*
   b. the value of in degrees
7. If $tan = \frac{15}{8}$, find the values of the Trig-ratios *cos* and *sin*.
8. If $cot = 2$, find the values of the Trig-ratios of *sin*, *cos* and *tan*
9. If $cosec\ \sqrt{10}$, find the values of the Trig-ratios of
   a. sin,
   b. *cos*,
   c. *tan* and
   d. in degrees
10. A person standing on the ground observes a bird on top of a tree at an angle of elevation of 30°.

If the person is 1.5 metres tall and the tree is 5 metres high, how far is the person from the tree?

11. A kite is flying at a height of 75 metres above the ground. The string attached to the kite makes an angle of 60° with the ground.

Find the length of the string.

12. From the top of a lighthouse 60 metres high, the angle of depression of a boat at sea is 35°. How far is the boat from the foot of the lighthouse?

13. A ladder leaning against a wall makes an angle of 65° with the ground.

If the foot of the ladder is 3 metres from the wall, find the height of the point where the ladder touches the wall.

14. Two observers, A and B, are standing on the same horizontal line 50 metres apart. They observe a hot air balloon at the same time. The angles of elevation from A and B to the balloon are 65° and 42° respectively.

Find the height of the balloon above the ground.

15. A man standing on the top of a building observes the top and bottom of a tower at angles of elevation and depression of 45° and 60° respectively.

If the distance between the top of the building and the top of the tower is 50 metres, find the height of the tower.

16. From the top of a hill 200 metres high, the angles of depression of two objects on the same horizontal line as the base of the hill are 30° and 45°.

Find the distance between the two objects.

17. An airplane is flying at a height of 3000 metres above the ground. The angle of depression from the plane to an airport is 25°.

Find the horizontal distance between the airplane and the airport.

18. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building

19. The angles of elevation and depression of the top and the bottom of a tower from the top of a building, 60 m high, are 30° and 60° respectively.

Find the difference between the heights of the building and the tower and the distance between them.

Hint

- *The Recommended Mode of Assessment for Week 24 is* ***End of Semester Examination.***
- *Refer to Appendix I at the end of Section 6 for further information on how to go about the end of semester examination. .*

Section Review

After going through this section, these are the key points to note:

Focal Area 1: Operations and its Properties on Vectors

a. Vectors can be added or subtracted by combining their corresponding components, which can be visually represented through geometric methods like the triangle or parallelogram rules.

b. When a vector is multiplied by a scalar, its magnitude changes proportionally while its direction remains the same, allowing for stretching or shrinking of the vector in two or three dimensions.

Focal Area 2: Operations and its Properties on Vectors

a. The operations on vectors follow specific properties such as commutative (order doesn't matter in addition), associative (grouping doesn't change the result) and distributive (scalar multiplication over addition distributes like in algebra).

b. Vectors can be used to model and solve real-world problems, such as finding the resultant of multiple forces or determining the direction and speed of an object.

Focal Area 1: Trigonometric Ratios

a. Trigonometric ratios (sine, cosine and tangent) relate the angles of a right triangle to the ratios of its sides, forming the basis for many calculations in geometry and real-life applications.

b. Inverse trigonometric functions allow you to determine the angle when the value of a trigonometric ratio is known, essential for solving triangles and other geometric problems.

Focal Area 2: Application of Trigonometry

a. Trigonometry is used to determine angles of elevation or depression, which are angles formed by looking up or down at an object from a specific point, useful in fields such as architecture and navigation.

b. By using trigonometric ratios, you can calculate unknown distances and heights, making it applicable to real-life problems like measuring the height of a building or the distance across a river.

Investigate and Determine Trigonometric Functions of Special Angles

Understanding the exact values of trigonometric functions for special angles (30°, 45°, 60°) enables quick calculations and solutions to problems in both theoretical and applied contexts.

APPENDIX I: SAMPLE TABLE OF TEST SPECIFICATION

Weeks	Learning indicators	Type of Questions	DoK Levels				Total
			1	2	3	4	
13	a. Extend the application of ratios to angles, Pythagoras's theorem and some algebra b. Application of rates to speed: distance-time graphs.	Multiple Choice		2	2	-	4
		Essay type	1	-	-	-	1
14	a. Apply the concept of ratios, rates and proportions to solve problems in financial mathematics, health, sports, etc. b. Establish the relevance of ratios, rates and proportions in their day-to-day activities, make generalizations and apply them to solve real-world problems.	Multiple Choice	2	2		-	4
		Essay type	1		-	-	1
15	a. Explore patterns of a sequence using plane figures and continue with more terms. b. Recognise and find the nth and the sum of the nth term of an arithmetic and geometric progression (AP) or Linear Sequence.	Multiple Choice	1	2	1	-	1
		Essay type	-	1	-	-	1
16	Identify geometric progressions or exponential sequence and find the algebraic expression for the general term.	Multiple Choice	-	2	1	-	2
		Essay type	-	1	-	-	1

Weeks	Learning indicators	Type of Questions	DoK Levels				Total
			1	2	3	4	
17	a. Analyse, model, and solve real-life problems involving financial mathematics. b. Analyse, model, and solve real-life problems involving exponential growth.	Multiple Choice	-	1	-	-	2
		Essay type	-	1	-	-	1
18	Solve problems that involve SI and imperial units in surface area measurements and verify the solutions.	Multiple Choice	-	2	-	-	2
		Essay type	-	-	1	-	1
19	Solve problems that involve SI and imperial units in volume/capacity measurements.	Multiple Choice	1	2	1	-	4
		Essay type	1	-	-	-	1
20	Solve real world problems that involves the volume/ capacity of a 3-D object.	Multiple Choice	-	-	-	-	-
		Essay type	-	1	-	-	1
21	a. Develop and implement a project plan for the collection, analysis and interpretation of data with useful conclusions and recommendations (including the use of appropriate computer applications, e.g. excel) within and beyond the school environment. b. Present a project report including the use of power points, infographics, etc. and publish it in school magazines, newspapers (Junior Graphic), local radio and TV stations, social media platforms, etc.	Multiple Choice	1	-	-	-	1
		Essay type	1	1	1	-	3

Weeks	Learning indicators	Type of Questions	DoK Levels				Total
			1	2	3	4	
22	a. List the elements of the sample space from a simple or compound experiment involving two dependent events b. Solve everyday life problems involving the probability of two-independent events	Multiple Choice	1	1	1	-	3
		Essay type	-	1	2	-	3
23	a. Perform addition, subtraction, and scalar multiplication on vectors represented as directed line segments in two-space, and in Cartesian form in two and three-space. b. Determine the properties (commutative, associative, distributive, etc.) of the operations on vectors through investigation with and without technology. **c.** Solve problems involving the addition, subtraction, and scalar multiplication of vectors, including problems arising from real-world applications.	Multiple Choice	2	3	3		8
		Essay type	1	2	1		4
24	a. Determine the inverse of trigonometric ratios (graphs excluded) and talk about their applications in the field of astronomy, engineering, physics, geometry and navigation. b. Solve real-life problems involving angles of elevation and depression and identify everyday life situations of these concepts.	Multiple Choice	2	3	2		7
		Essay type	1	2	1		4

Weeks	Learning indicators	Type of Questions	DoK Levels				Total
			1	2	3	4	
Total							
Multiple Choice			12 (30%)	16 (40%)	12 (30%)	-	40 (100%)
Essay type			6 (30%)	10 (40%)	6 (30%)		22 ((100%)

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