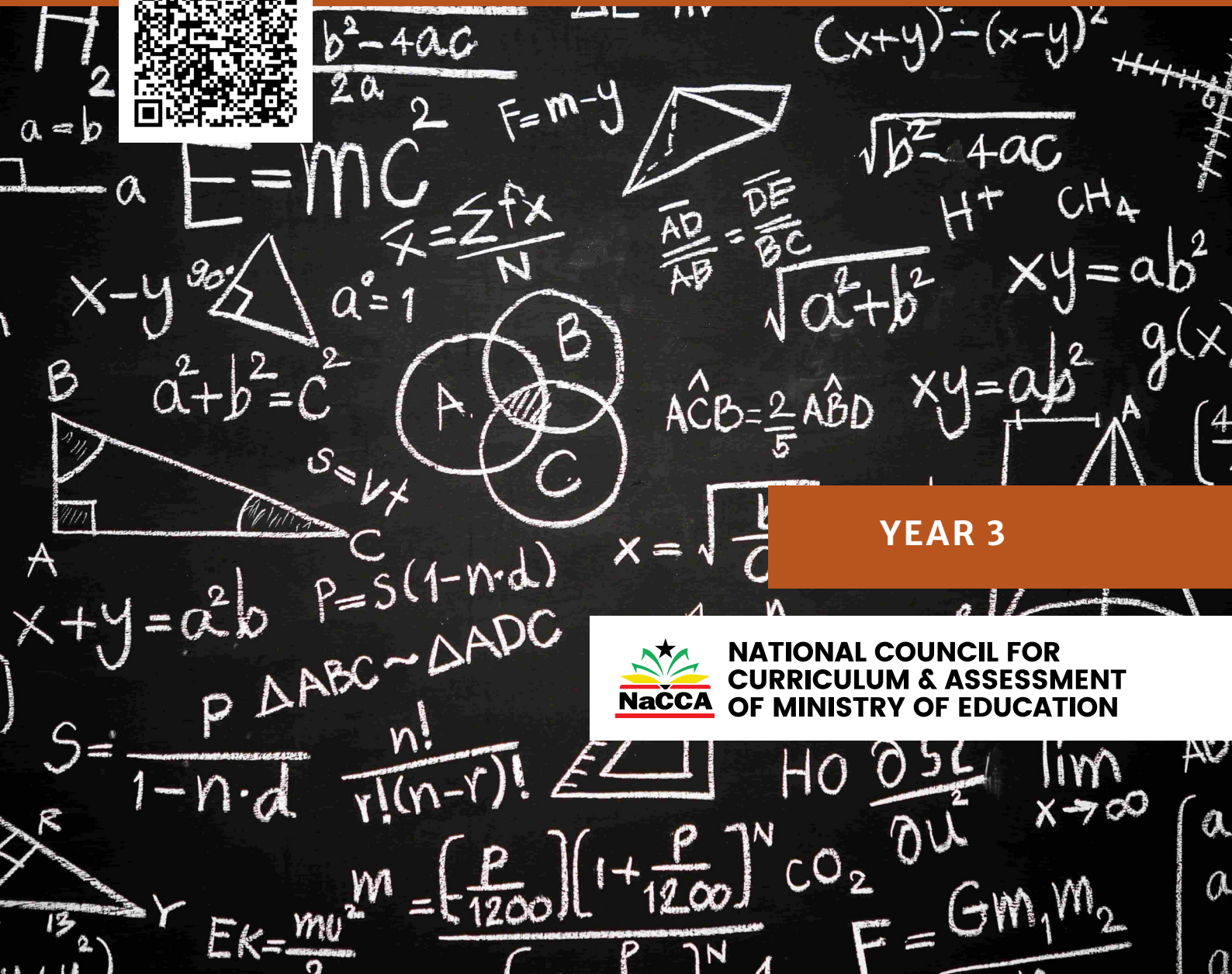
MINISTRY OF EDUCATION

Mathematics

for Senior High Schools

TEACHER MANUAL



YEAR 3

NATIONAL COUNCIL FOR CURRICULUM & ASSESSMENT OF MINISTRY OF EDUCATION

MINISTRY OF EDUCATION

REPUBLIC OF GHANA

Mathematics

For Senior High Schools

Teacher Manual

Year Three

NATIONAL COUNCIL FOR CURRICULUM & ASSESSMENT OF MINISTRY OF EDUCATION

MATHEMATICS TEACHER MANUAL

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Introduction

The National Council for Curriculum and Assessment (NaCCA) has developed the third and final set of curriculum materials to support the implementation of the new Senior High School curriculum.

This Year Three Teacher Manual for covers all aspects of the content, pedagogy, teaching and learning resources and assessment required to effectively support learners in their final year of Senior High School. All the necessary information for the Student Assessment Portal has been included in the Year Three Teacher Manual to simplify the development of weekly learning plans, meaning that teachers have a single reference document.

The Year Three Teacher Manual has two main objectives:

- To support learners to consolidate and deepen their understanding of the content, pedagogy, and assessment across all three years of the new Senior High School curriculum in preparation for the West Africa Senior Secondary Certificate Examination (WASSCE).
- To equip teachers with the tools and resources needed to deliver revision-focused, assessment-led lessons that build on the knowledge learners have acquired from Year One to Year Three.

In this third year, teachers will support learners in preparing for the WASSCE. Each section of the Teacher Manual is therefore structured around the highest Depth of Knowledge (DoK) levels, allowing learners to build progressively on knowledge acquired from Year One to Year Three. For each week, learning tasks and pedagogy are built around the assessment, ensuring that classroom activities directly prepare learners for the demands of the examination. Two sets of sample examination papers are also provided; one administered by Week 12 as a first semester practice examination, and one by Week 20 as a mock examination, to gauge learners' competencies and identify areas requiring remediation.

To further support teachers in delivering Year Three, NaCCA, working with the Ghana Education Service, has developed a subject specific App for each subject.

The Year Three PLC sessions have been revised in response to findings from the 2025 annual evaluation survey. The survey confirmed that teachers have made strong progress in lesson planning, use of varied instructional methods, ICT integration and modes of assessment. To address the identified gap in explaining concepts clearly using examples familiar to learners, each PLC session now includes a dedicated Content Exploration segment. This ten-minute segment brings the subject group together to work through one challenging problem or concept from the week's Teacher Manual content, not to plan how to teach it, but to ensure that every teacher in the group fully understands it.

The PLC sessions in Year Three also place particular emphasis on connecting the week's content to equivalent topics taught in Years One and Two. Teachers are encouraged to note the

recap weeks signposted throughout the Teacher Manual and to use the App to explore how content across all three years links together.

As in previous years, teachers are expected to integrate National Values, Gender Equality and Social Inclusion (GESI), Social and Emotional Learning (SEL), and 21st-century skills into their learning activities, assessment tasks, and rubrics and mark schemes. The subject-specific App will support teachers in doing this throughout the year.

Ghana Education Service acknowledges the dedication of the teachers who have implemented the new curriculum across Years One and Two and recognises that Year Three represents both the culmination of that effort and a critical moment for learners. The Year Three Teacher Manual and its accompanying resources have been developed to ensure that every learner, regardless of background, ability, or school context, is fully prepared to demonstrate what they know and can do in the WASSCE and to progress confidently to further studies, the world of work, and adult life.

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Ama Achiaa - Afriyie	St. Louis SHS, Kumasi	**Food and Nutrition**
Lily-Versta Nyarko	Mancell Girls SHTS, Kumasi	
Osmanu Ibrahim	Mount Mary College of Education, Somanya	**French**
Mawufemor Kwame Agorgli	Akim Asafo SHS	
Comfort Korkor Sam	University for Development Studies, Tamale	**General Science**
Robert Arhin	S.D.A SHS, Akim Sekyere	
Ishmael Yaw Dadson	University of Education, Winneba	**Geography**
Prof Ebenezer Owusu -Sekyere	University for Development Studies, Tamale	
Raymond Nsiah-Asare	Methodist Girls' SHS, Mamfe	
Catherine Ekua Mensah	University of Cape Cape	**Ghanaian Language**
David Sarpei Nunoo	University of Education, Winneba	
Prof Kwasi Adomako	University of Education, Winneba	
Arko Blay Augustine	University of Education, Winneba	**Government**
Josephine Akosua Gbagbo	Ngleshie Amanfro SHS	
Anitha Oforiwah Adu-Boahen	University of Education, Winneba	**History**
Prince Essiaw	Enchi College of Education	
Blessington Dzah	Ziavi SHTS	**Literature-in-English**
Juliana Akomea	Mangoase SHS	
Dorcas Akosua Opoku	Winneba SHS	**Management in Living**
Grace Annagmeng Mwini	Tumu College of Education	
Eric Abban	Mt. Marys College of Education, Somanya	**Mathematics**
Isaac Buabeng	Mamprobi Sempe 2 JHS	

Benjamin Ofori	Cocoa Research Institute of Ghana Primary School	**Music**
Cosmas W. K. Mereku	University of Education, Winneba	
Davies Obiri Danso	New Juaben SHS	
Prof. Joshua Alfred Amuah	University of Ghana, Legon	
Evans Kofi Mati	GES HQ	
Wilson Klebi	GES HQ	
Professor Emmanuel Obed Acquah	University of Education, Winneba	
Chris Ampomah Mensah	Bolgatanga S.H.S	**Performing Arts**
Doreen Deedee Quarcoo	University of Education, Winneba	
Dr. Latipher Osei Appiah-Agyei	University of Education, Winneba	
Albert Sackey	KNUST SHS	**Physical Education and Health (Core and Elective)**
Ebenezer Ewiah-Quarm	Nkroful Agric SHS	
Evans Asare Yeboah	Adisadel College	
John Benedict Nyarko	Adisadel College	
Mary Aku Ogum	University of Cape Coast	
Dr Linus Kweku Labik	KNUST	**Physics**
Sylvester Affram	Kwabeng Anglican SHTS	
Anthony Mensah	Abetifi Presbyterian College of Education	**RME**
Clement Nsorwineh Atigah	Tamale SHS	
Eliel Keelson	Kwame Nkrumah University of Science and Technology	**Robotics**
Isaac Nzoley	Wesley Girls High School	
Jemima Ayensu	Holy Child Girls SHS, Cape Coast	**Social Studies**
Mohammed Adam, Ph.D.	University of Education, Winneba	
Simon Tengan	Wa Senior High Technical School	
Franklina Kabio-Danlebo	University of Ghana	**Spanish**
Mishael Annoh-Achampong	University of Media, Art and Communication	

Augustina Gloria Henyo	West African Examinations Council	**Inclusive Education**
Dr. Gideon Kwesi Obosu	University of Cape Coast	
Dr. Daniel Fobi	University of Education, Winneba	
Dr. Martha-Pearl Okai	University of Cape Coast	
Francis Kweku Essel	Ashesi University	
Richard Doku	Mampong/Akw SHTS For The Deaf	
Yao Mati Abotsi	Ghana Blind Union	
Seth Kwame Ativor	Okuapemman SHS, Akropong	
Bernard Duorinaah	Presbyterian College of Education	
Evans Odei	Achimota Senior High School	

List of Reviewers			
S/N	**Subject**	**Name of Reviewer**	**Institution**
1	Accounting	Ms. Ivy Eklemet	Univ. of Professional Studies, Accra
2	Additional Mathematics	Francis Tornyenor	Pope John SHS & Minor Seminary, Koforidua
3	Agricultural Science	Selina Acheampong	Univ. College of Agriculture and Environmental
4	Agriculture	Mr. Godwin B. K. Doli	Municipal Educ. Office, Amasaman
5	Arabic	Abdullah Musah Abdullah	Univ. of Media Arts & Communication, UniMAC
6	Art and Design Studio	Ernest Kwasi Elelakpodia	Hope College, Gomoa Fetteh
7	Art and Design Foundation	Ernest Kwasi Elelakpodia	Hope College, Gomoa Fetteh
8	Automotive & Metal Technology	Okyere Kojo Alex	Ada College of Education
9	Aviation & Aerospace Engineering	Dr. Augustine Akuoko Kwarteng	Univ. of Mines and Technology
10	Biology	Rev. Joseph Kweku Delanyo	formerly of Hope College, Gomoa Fetteh
11	Biomedical Science	Dr. Irene Adzo Agbo	Univ. of Professional Studies, Accra
12	Building Construction & Woodwork Technology	Mr. Alex A. Appah	Christ Calvary Prep. Sch.
13	Business Management	Mr. Emmanuel Boakye	Ghana Employers Association

14	Chemistry	Joyce Okreyewa Odei	Hope College, Gomoa Fetteh
15	Clothing and Textiles	Rosemary Quarcoo	Department of Clothing and Textiles, UEW
16	Design & Communication Technology	Esme Serwaa Boateng	Ghana National College, Cape Coast
17	Economics	Dr. Kofi Kamasa	Department of Management Studies, UMaT
18	Electrical & Electronics Technology	Dr. Emmanuel Ampoma Affum	Telecom Engineering Department, KNUST
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21	Food and Nutrition	Sylvia Baaba Yankey	
22	French	Gregory Nana Kobina Nketsia Mensah	Mfantsipim School, Cape Coast
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25	Ghanaian Languages	Prof. Regina Caesar	University of Education, Winneba
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28	Information Communication Technology	Felicia Archeresuah	Nkoronza Technical Institute
29	Manufacturing Engineering	Dr. Frank Kwabena Nyarko	Kwame Nkrumah Univ. of Science and Technology
30	Mathematics	Ezekiel Osei Bonsu	Bono East Regional Education Office, Nkoranza
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32	Music	Prof Mark-Millas Fish	University of Education, Winneba
33	Physical Education and Health Core	Francis Maxwell Ayamba	Atebubu College of Education
34	Physical Education and Health Elective	Sulemana Ahmed Tijani	Atebubu College of Education
35	Performing Arts	Dr. Godfred Asare Yeboah	University of Education, Winneba
36	Physics	Peter Bekoe	Our Lady of Mount Carmel Girls SHS, Techiman
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39	Social Studies	Samuel Larbi	St Louis Girls SHS, Kumasi
40	Spanish	Prof. Jonson Asunka	University of Ghana Legon

SECTION 1: LOGICAL REASONING & VARIATIONS

STRAND: NUMBERS FOR EVERYDAY LIFE

Sub-Strand: Proportional Reasoning

Weeks 1, 2 & 3

Learning Outcome: *Establish the validity of logical arguments and use them to make relevant decisions in solving problems.*

Content Standard: *Demonstrate conceptual understanding of logical reasoning to solve real-life problems.*

Weeks 4 & 5

Learning Outcome: *Analyse the impact of variations and conduct simple investigations in solving day-to-day problems.*

Content Standard: *Demonstrate conceptual understanding of proportionality in relation to variation and use it to solve real-life problems.*

INTRODUCTION AND SECTION SUMMARY

In everyday life, we consider issues and make judgments and decisions for our benefit and that of our communities. Logical reasoning in this session examines the process of using logic to evaluate information, make decisions, and reach conclusions. This section will also explore the relationship between two or more characteristics of any subject of interest and how they change or remain stable relative to one another. We refer to this as variation. The concepts of logical reasoning and variation are essential for developing critical thinking, problem-solving, and analytical skills, which will improve learners' academic performance and prepare them for the world of work. Logical reasoning allows learners to assess information, identify patterns, and make informed choices. Both logical reasoning and variation are key components of Home Economics, Science, Engineering, and Economics. The SHS 3 learner is equipped to integrate into society through the concepts learnt in logical reasoning and variation. To maximise these concepts, we will engage learners through experiential learning strategies and regular individual practice.

The weeks, the indicators with their respective focal areas covered by the section are:

Week 1

Investigate to establish differences between variables in situational growth (symbolic logic) and apply it to real-world situations.

- ***Focal Area 1: Concept of Logical Reasoning***
  - *Introduction to logical reasoning*
  - *Making declarative sentences*
  - *Identify true or false statements from declarative statements.*
- ***Focal Area 2: Real-World Applications of logical reasoning***
  - *Situational Growth and Symbolic Logic: Robotics; Game Design; Artificial Intelligence; Computer Science; Philosophy; Law:*
  - *Model and solve real-world issues involving logical reasoning*

Week 2

Make intelligent guesses to establish valid arguments and draw logical conclusions

- ***Focal Area 1: Intelligent guessing***
  - *Establish valid arguments and draw logical conclusions*
  - *Decision-making in solving problems*
- ***Focal Area 2: Using tables and diagrams in decision-making***
  - *Use of Venn diagrams*
  - *Use of Truth tables*
  - *Model and solve real-life issues involving logical reasoning*

Week 3

Carry out mini real-life investigations to solve problems involving logical reasoning, within their local community.

- ***Focal Area: Real-life applications of logical reasoning***
  - *Generate and test the validity of logical arguments from peers.*
  - *Investigations to solve problems involving logical reasoning, within their local community*

Week 4

Use proportional reasoning to investigate the various types of variation (direct and inverse) and extend this to make generalisations.

- ***Focal Area:*** **Variation 1**
  - *Concept, examples and visualisation of proportional reasoning*
  - *Direct proportions*
  - *Inverse proportions*

- *Joint variations*

Week 5

Use proportional reasoning to investigate partial variations, make generalisations and solve real-life issues involving partial variations.

- ***Focal Area:*** **Variation 2**
  - *Partial variations*
  - *Model and solve real-life situations involving proportional reasoning*

SUMMARY OF PEDAGOGICAL EXEMPLARS

This session aims to nurture Senior High School learners who value their own perspectives and those of others, while developing critical thinking skills to address personal and national issues. The teaching of logical reasoning and variations will be guided by strategies that promote independent thought, teamwork, and problem-solving. Learners will engage in activities such as think-pair-share, problem-based learning, peer review, and debates to analyse arguments, draw logical conclusions, and communicate effectively. Group work will support collaborative learning, encouraging learners to explore types of variation—direct, inverse, joint, and partial—and provide solutions to real-life problems. Independent tasks will build skills in evaluating statements, defending positions with logical arguments, and constructing summaries of real-life cases. Class discussions will reinforce foundational knowledge, while presentations will foster communication skills. Throughout, a gender-responsive, learner-centred approach will help learners apply concepts of logic and variation to everyday situations.

ASSESSMENT SUMMARY

This provides a summary of the assessment used against the indicator. It makes provision for information on the assessment strategies and their highest DOK levels, to improve learning outputs.

Table 1: Indicators, DOK levels, assessment modes and strategies

Indicators	DOK Level	Assessment Strategy and/or Mode
Investigate to establish differences between variables in situational growth (symbolic logic) and apply it to real world situations.	Level 3	**Assessment Mode:** Home work. **Assessment Strategy:** Standard questions for individuals.
Make intelligent guessing to establish valid arguments and draw logical conclusions	Level 3	**Assessment Mode:** Debate **Assessment Strategy:** Whole class debate built in groups and built through jigsaw discussion.
Carry out mini real-life investigations to solve problems involving logical reasoning, within their local community.	Level 4	**Assessment Mode:** Group project **Assessment Strategy:** Research on real-life application in learner's environment.

Use proportional reasoning to investigate the various types of variation (direct and inverse) and extend this to make generalisations.	Level 3	**Assessment Mode:** Assignment. **Assessment Strategy**: Individual assignment on essay questions
Use proportional reasoning to investigate partial variations, make generalisations and solve real-life issues involving partial variations.	Level 3	**Assessment Mode:** Problem posing & peer review. **Assessment Strategy:** Generating questions on real-life issues for peers to answer and give feedback

WEEK 1

Learning Indicator: *Investigate to establish differences between variables in situational growth (symbolic logic) and apply it to real world situations*

FOCAL AREA 1: CONCEPT OF LOGICAL REASONING

Understanding Logical Reasoning

Begin by discussing with learners how logical reasoning is used in mathematics and everyday life. Guide learners to recognise that mathematical reasoning involves using established facts, rules and evidence to arrive at valid conclusions.

Logical reasoning helps us to:

a. apply mathematical rules correctly;

b. examine available information;

c. identify patterns and relationships;

d. use evidence to support conclusions; and

e. make sound and justifiable decisions.

Encourage learners to suggest situations in which they have used reasoning to make a decision or solve a problem.

What is Logical Reasoning?

Logical reasoning is the systematic process of using facts, evidence and established rules to analyse information, draw conclusions, make decisions and solve problems.

In Mathematics, logical reasoning helps us determine whether statements and arguments are valid and whether conclusions follow from given information.

Interrogative and Declarative Statements

In everyday communication, we use different kinds of sentences. Some sentences ask questions, while others provide information.

An **interrogative sentence** asks a question and usually ends with a question mark.

Examples

a. "Is 12 an even number?"

b. "Is Accra the capital of Ghana?"

c. "Does a square have four equal sides?"

A **declarative sentence** makes a statement or gives information and usually ends with a full stop.

Examples

a. "12 is an even number."

b. "Accra is the capital of Ghana."

c. "A square has four equal sides."

However, in mathematical logic, not every declarative sentence is regarded as a proposition.

A **proposition**, also called a **logical statement**, is a declarative sentence that has a definite truth value. It must be either true or false, but not both.

For example

a. "12 is an even number." — ***True***

b. "15 is divisible by 2." — ***False***

c. "Every square is a rectangle." — ***True***

Statements based purely on personal opinion, preference or vague information should not normally be used as propositions because their truth values may not be clearly determined.

For example:

"Mathematics is the most enjoyable subject."

This is a declarative sentence, but it expresses an opinion. Different people may have different views about it.

Example 1

Convert each interrogative sentence into a declarative sentence. Then determine whether the resulting sentence is a proposition.

a. Is Accra the capital of Ghana?

b. Is 15 divisible by 3?

c. Does a triangle have four sides?

d. Is mathematics the most enjoyable school subject?

Solution

a. **Interrogative**: Is Accra the capital of Ghana?

Declarative: Accra is the capital of Ghana.

This is a proposition because it has a definite truth value.

b. **Interrogative**: Is 15 divisible by 3?

Declarative: 15 is divisible by 3.

This is a proposition. It is true.

c. **Interrogative**: Does a triangle have four sides?

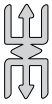
Declarative: A triangle has four sides.

This is a proposition. It is false.

d. **Interrogative**: Is mathematics the most enjoyable school subject?

Declarative: Mathematics is the most enjoyable school subject.

This is a declarative sentence, but it is not suitable as a mathematical proposition because it expresses a personal opinion.



Teacher Note

Emphasise the distinction between a **declarative sentence** and a **proposition**. A proposition must have a definite truth value.

True and False Statements

A proposition may be either **true** or **false**. Consider the following examples.

A: 2 is an even prime number.

This statement is **true** because 2 is both even and prime.

B: 25 is an even number.

This statement is **false** because 25 is odd.

C: The sum of the interior angles of a triangle is 180°.

This statement is **true** in Euclidean geometry.

Negation of a Statement

The **negation** of a statement expresses the opposite of that statement.

If the letter P represents a statement, its negation may be written as:

~P and read as: “not P”.

Consider the following examples.

P: 8 is an even number.

¬P: 8 is not an even number.

Since P is true, ~Pis false.

Q: 15 is greater than 20.

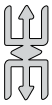
¬Q: 15 is not greater than 20.

Since Q is false, ~Q is true.

For inequalities, care must be taken when forming the negation.

If **A:** , then **¬A:** . Similarly,

if **B:** , then **¬B:** .



Note

The negation of a true proposition is false, while the negation of a false proposition is true.

Simple and Compound Statements

A **simple statement** expresses one complete idea and has a definite truth value.

For example:

P: Ama is a student.

Q: 12 is an even number.

R: A square has four equal sides.

Two or more simple statements may be combined using **logical connectives** to form a **compound statement**.

Common logical connectives include:

not — negation

and — conjunction

or — disjunction

if … then — implication

if and only if — biconditional

Conjunction

A compound statement formed by joining two statements with the word **and** is called a **conjunction**.

The symbol for "and" is ∧

If **P** and **Q** are statements, their conjunction is written as: **P ∧ Q** and read as: "P and Q".

A conjunction is **true only when both component statements are true**.

For example:

P: 8 is an even number.

Q: 8 is greater than 5.

Then:

P ∧ Q: 8 is an even number, and 8 is greater than 5.

Both statements are true, so the conjunction is true.

Consider another example:

A: 6 is even.

B: 6 is a prime number.

Then:

A ∧ B: 6 is even and 6 is a prime number.

The first statement is true, but the second is false. Therefore, the conjunction is false.

Disjunction

A compound statement formed by joining two statements with the word **or** is called a **disjunction**.

The symbol for "or" is: ∨

If P and Q are statements, their disjunction is written as: **P ∨ Q** and read as: "P or Q".

In mathematical logic, **or is inclusive** unless stated otherwise. Therefore, a disjunction is true when at least one of its component statements is true. It is false only when both statements are false.

For example:

P: 9 is an odd number.

Q: 9 is a prime number.

Then:

P ∨ Q: 9 is an odd number, or 9 is a prime number.

Since P is true, the disjunction is true even though Q is false.

Example 2

Consider the statements:

A: 2 is an even number.

B: 2 is an odd number.

Form a conjunction and a disjunction and determine their truth values.

Solution

Conjunction

A ∧ B: "2 is an even number and 2 is an odd number."

A is true, but **B** is false.

Therefore:

A ∧ B is false.

Disjunction

A ∨ B: "2 is an even number or 2 is an odd number."

Since at least one of the statements is true:

A ∨ B is true.

Example 3

Consider the statements:

P: Plane shapes have no volume.

Q: Solid shapes have volume.

Determine the truth values of:

a. P ∧ Q

b. P ∨ Q

Solution

Both P and Q are true. Therefore:

a. **P ∧ Q is true.**

"Plane shapes have no volume and solid shapes have volume."

b. **P ∨ Q is true.**

"Plane shapes have no volume or solid shapes have volume."

Classroom Activity

In pairs, discuss how logical reasoning is useful in mathematics and everyday life. Learners may consider situations such as:

- choosing the most economical means of transport;
- checking whether a mathematical solution is reasonable;
- comparing prices before making a purchase;
- examining evidence before accepting a claim; and
- identifying patterns in numbers or everyday events.

Invite pairs to share their ideas with the class and justify their responses.

Expected Response

Logical reasoning helps us to analyse information, identify relationships, evaluate evidence, solve problems and make informed decisions.

Example 4: Equal and Equivalent Sets

In pairs, consider the statement:

"All equivalent sets are equal sets."

Determine whether the statement is true or false and justify your answer.

Solution

The statement is **false**.

Two sets are **equivalent** if they contain the same number of elements.

Two sets are **equal** if they contain exactly the same elements.

For example:

A = {1, 2, 3}

B = {a, b, c}

A and **B** are equivalent because each contains three elements. However, they are not equal because their elements are different.

Therefore, not all equivalent sets are equal.

Think–Pair–Share Activity

Ask learners to work individually before discussing their answers with a partner.

Convert each interrogative sentence into a declarative sentence.

a. Is 18 divisible by 6?

b. Is a square a quadrilateral?

c. Does 7 + 8 equal 16?

d. Is 21 an odd number?

e. Is every prime number odd?

Solution

a. **Interrogative:** Is 18 divisible by 6?

Declarative: 18 is divisible by 6.

Truth value: True.

b. **Interrogative:** Is a square a quadrilateral?

Declarative: A square is a quadrilateral.

Truth value: True.

c. **Interrogative:** Does 7 + 8 equal 16?

Declarative: 7 + 8 equals 16.

Truth value: False.

d. **Interrogative:** Is 21 an odd number?

Declarative: 21 is an odd number.

Truth value: True.

e. **Interrogative:** Is every prime number odd?

Declarative: Every prime number is odd.

Truth value: False, because 2 is a prime number and is even.

Individual Task

Complete the missing interrogative or declarative form.

Item	Interrogative sentence	Declarative sentence
a.	Is 36 a perfect square?	_____
b.	_____	A rectangle has four sides.
c.	Is 17 an even number?	_____
d.	_____	The sum of 8 and 5 is 13.
e.	Does every square have four equal sides?	_____

Solution

a. **Declarative**: 36 is a perfect square.

b. **Interrogative**: Does a rectangle have four sides?

c. **Declarative**: 17 is an even number.

d. **Interrogative**: Is the sum of 8 and 5 equal to 13?

e. **Declarative**: Every square has four equal sides.

Teachers may extend the task by asking learners to classify each resulting proposition as **true or false.**

FOCAL AREA 2: REAL-WORLD APPLICATIONS OF LOGICAL REASONING

Logical reasoning helps us determine whether a conclusion follows from given information. In mathematics and everyday life, we often encounter statements in which one condition leads to another. Such statements can be expressed using implication.

IMPLICATION

An implication is a compound statement of the form:

If **P**, then **Q**.

It is written symbolically as: $\mathbf{P} \Rightarrow \mathbf{Q}$

and read as: "**P** implies **Q**" or "If **P**, then **Q**."

P is called the ***hypothesis*** or ***antecedent***.

Q is called the ***conclusion*** or ***consequent***.

For example:

P: A number is a multiple of 4.

Q: The number is even.

The implication $P \Rightarrow Q$ means: If a number is a multiple of 4, then the number is even.

This implication is true because every multiple of 4 is even.

For example: 4, 8, 12, 16, 20, ... are all even numbers.

Implication And Set Inclusion

When statements P and Q describe properties of objects or numbers, we may sometimes represent the relationship between them using sets and a Venn diagram.

However, we must first establish that the implication is true before representing it as a subset relationship.

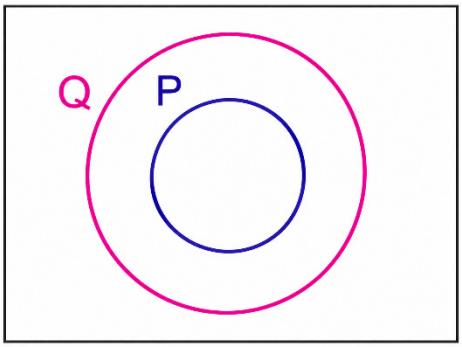
Suppose: **P** = {integers that are multiples of 4}

Q = {even integers}

Every member of P is also a member of Q.

Therefore: $P \subseteq Q$

This can be represented by a Venn diagram in which set P lies completely inside set Q.



The statement P ⇒ Q and the set relationship P ⊆ Q are related in this example because every element having property P also has property Q.

We should not assume that two sets have a subset relationship simply because their descriptions appear in an implication. The relationship must first be established.

Example 1

Consider the following statements:

P: An integer is divisible by 6.

Q: The integer is divisible by 3.

a. Write the implication P ⇒ Q in words.

b. Determine whether the implication is true or false.

c. Represent the relationship using sets.

d. Draw a Venn diagram to illustrate the relationship.

Solution

a. P ⇒ Q means: If an integer is divisible by 6, then it is divisible by 3.

b. Consider some integers divisible by 6:

6, 12, 18, 24, 30, ...

Each of these integers is also divisible by 3.

In general, if an integer n is divisible by 6, then:

n = 6k for some integer k.

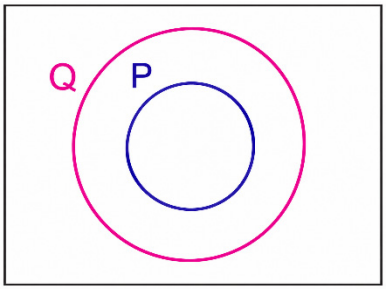
Since: 6k = 3(2k), n is also divisible by 3.

Therefore: P ⇒ Q is true.

c. Let P = {integers divisible by 6} and Q = {integers divisible by 3}

Since every integer divisible by 6 is also divisible by 3: P ⊆ Q

d. Draw set P completely inside set Q.



The Venn diagram is justified because every member of **P** is also a member of **Q**.

THE CONVERSE OF AN IMPLICATION

If $P \Rightarrow Q$ is an implication, then $Q \Rightarrow P$ is called its converse.

The fact that an implication is true does not necessarily mean that its converse is also true.

Consider the previous example:

P: An integer is divisible by 6.

Q: The integer is divisible by 3.

We established that: $P \Rightarrow Q$ is true.

Now consider the converse: $Q \Rightarrow P$

This means: If an integer is divisible by 3, then it is divisible by 6.

This statement is false.

For example, 9 is divisible by 3 but is not divisible by 6.

Therefore, 9 is a **counterexample**.

A **counterexample** is a specific example that shows that a general statement is false.

Only one valid counterexample is needed to show that a general implication is false.

Example 2

Consider the statements:

A: A shape is a square.

B: The shape is a rectangle.

Determine whether each implication is **true** or **false**.

a. $A \Rightarrow B$

b. $B \Rightarrow A$

Solution

a. $A \Rightarrow B$ means:

If a shape is a square, then it is a rectangle.

A square has four right angles and satisfies the defining properties of a rectangle.

Therefore: A ⇒ B is true.

If: A = {squares} and B = {rectangles}, then: A ⊆ B

A Venn diagram may therefore show the set of squares completely inside the set of rectangles.

b. B ⇒ A means: If a shape is a rectangle, then it is a square.

This statement is false because a rectangle does not necessarily have four equal sides.

For example, a rectangle measuring 8 cm by 5 cm is not a square.

Therefore: B ⇒ A is false.

Example 3

Consider the statements:

P: A number is even.

Q: The number is divisible by 4.

A learner claims that: P ⇒ Q

That is: If a number is even, then it is divisible by 4. Is the learner's claim valid?

Solution

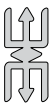
Consider the number 6.

6 is even, but 6 is not divisible by 4.

Therefore, 6 is a counterexample.

Hence: P ⇒ Q is false.

We therefore cannot conclude that: P ⊆ Q.



TEACHER NOTE

Do not draw a subset relationship first and then use the diagram to claim that an implication is true.

First, establish whether the implication is true. A Venn diagram should then be used to represent the relationship that has already been established.

Real-Life Application: School Library Membership

Logical implication and set relationships can also be used to analyse everyday situations.

Suppose a school has the following rule:

Every member of the Reading Club must be a registered member of the school library.

Let: **R** = {learners who belong to the Reading Club}

L = {learners who are registered members of the school library}

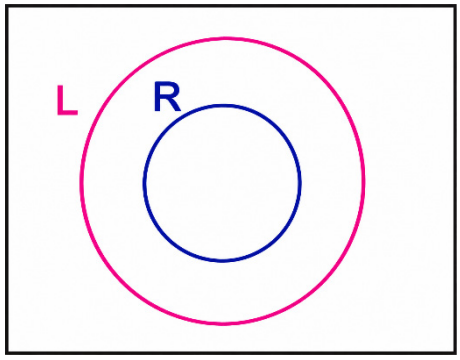
The school rule tells us that every learner in R must also belong to L.

Therefore: R ⊆ L

We can express the relationship as an implication:

If a learner belongs to the Reading Club, then the learner is a registered member of the school library.

Symbolically: R ⇒ L



Example 4

The following learners are registered members of a school library:

Ama, Kofi, Sena, Adwoa, Kwame, Mariama and Yaw.

The members of the Reading Club are: Ama, Sena, Kwame and Mariama.

Let: L = {Ama, Kofi, Sena, Adwoa, Kwame, Mariama, Yaw}

R = {Ama, Sena, Kwame, Mariama}

Answer the following questions.

a. Is R ⊆ L?

b. Is the statement "If a learner belongs to the Reading Club, then the learner is registered with the school library" true?

c. Is the converse true?

d. Give a reason for your answer in (c).

Solution

a. Yes. Every member of R is also a member of L.

Therefore: R ⊆ L

b. The statement:

If a learner belongs to the Reading Club, then the learner is registered with the school library is true.

c. The converse is:

If a learner is registered with the school library, then they belong to the Reading Club.

This statement is false.

d. Kofi, Adwoa and Yaw are registered members of the library but are not members of the Reading Club.

Any one of these learners provides a counterexample to the converse.

Real-Life Investigation: Making Decisions from Evidence

Not every real-life claim can immediately be treated as a mathematical implication. Some claims require evidence before we can decide whether they are reasonable.

Consider the statements:

"If more people use public transport, traffic congestion in the city will decrease."

This is different from the mathematical statement:

"If an integer is divisible by 6, then it is divisible by 3."

The mathematical statement can be established using mathematical reasoning.

The statement about traffic congestion depends on several real-world factors. Evidence must therefore be collected and examined before a conclusion is reached.

Learners should consider questions such as:

1. What evidence supports the claim?
2. What other factors contribute to traffic congestion?
3. Would increase use of public transport necessarily reduce the number of private vehicles on the road?
4. Are there situations in which the conclusion may not follow?
5. Does the available evidence provide sufficient support for the conclusion?

Learning Task: Public Transport and Traffic Congestion

A city authority is considering ways of reducing traffic congestion.

During a class discussion, two groups make the following claims:

Group A:

"If more commuters use reliable public transport instead of private cars, the number of private cars on the road may decrease."

Group B:

"Improving public transport alone may not solve traffic congestion because other factors also contribute to congestion."

In small groups:

1. Explain what each group is claiming.
2. Identify the conclusion made by each group.
3. Suggest at least two pieces of evidence that could be collected to examine each claim.
4. Identify other factors that could affect traffic congestion.
5. Compare the strengths and limitations of the two arguments.
6. Draw a reasoned conclusion based on the available evidence.
7. Explain why a Venn diagram alone cannot prove either claim.

Expected Learning

Learners should recognise that logical reasoning in real-life situations requires them to:

1. identify assumptions;
2. distinguish evidence from opinion;
3. determine whether a conclusion follows from the available information;
4. identify counterexamples or exceptions;
5. consider alternative explanations; and
6. justify conclusions using relevant evidence.

PEDAGOGICAL EXEMPLARS

Think-Pair-Share

Present learners with the statement:

"If an integer is divisible by 10, then it is divisible by 5."

Ask learners to determine individually whether the implication is true.

Learners then discuss their reasoning with a partner before sharing their conclusions with the class.

Encourage learners to use examples, algebraic reasoning and Venn diagrams to justify their conclusions.

Collaborative Learning

In small groups, provide learners with pairs of statements and ask them to:

1. write each implication in the form $P \Rightarrow Q$;
2. determine whether the implication is true or false;
3. provide a reason for their conclusion;

4. give a counterexample where appropriate;
5. determine whether a subset relationship exists; and
6. draw a Venn diagram only when the subset relationship has been established.

Real-Life Investigation

Ask learners to identify a claim from their school or community.

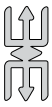
For example:

"Learners who participate regularly in sporting activities are physically fit."

Learners discuss:

1. the meaning of the claim;
2. the evidence required to support the claim;
3. other factors that may affect physical fitness;
4. possible exceptions to the claim; and
5. whether the available evidence is sufficient to support a conclusion.

Emphasise that learners should evaluate the evidence rather than simply declare the statement true or false.



TEACHER NOTE

Help learners distinguish among the following three ideas:

Logical implication: If P holds, then Q follows.

Symbolically: $P \Rightarrow Q$

Set inclusion: Every member of set P is also a member of set Q.

Symbolically: $P \subseteq Q$

Real-life or empirical claim:

A real-life claim may require observation, data and other evidence before its conclusion can be justified.

Where properties P and Q define sets and the implication $P(x) \Rightarrow Q(x)$ holds for every relevant element x, the relationship can be represented by: $P \subseteq Q$

Therefore, establish the logical relationship before using a Venn diagram to represent it.

KEY ASSESSMENT LEVEL 2: HOMEWORK

1. Analyse these statements and label each as True, False, or Debatable.

a. Ghana is a country located in West Africa. ……

b. If n is a number, then $5n$ is greater than n. ……

c. Oti Region in Ghana shares border with the Bono East Region to the South. ……

d. Ghana gained independence from British colonial rule in 1967. ……

e. Mathematics is more important than Robotics. ……

f. A square is a rectangle. ……

g. All prime numbers are odd ……

h. If $x > 5$ then $x > 3$ ……

i. The sum of the interior angles of a triangle is 360° ……

9 marks **(DOK Level 1)**

2. Write a conjunction and a disjunction for each pair of statements and state whether it is "True" or "False".

a. **M: Factors divide a given number.**

b. **N: All factors are composite numbers.** ***4 marks***

c. **S: For $x \geq 9$, x is a negative integer.**

d. **T: For $x \geq 9$, x is a factor of 3.** ***4 marks***

e. **G: Mappings have rules.**

f. **H: Mappings have ordered pairs.** ***4 marks***

12 marks **(DOK Level 2)**

3. Consider the following statements:

P: Teaching staff go on holidays.

Q: No kitchen staff go on holidays.

a. Draw a Venn diagram to illustrate the above statement ***3 marks***

b. Deduce which of the following statements are valid conclusions based on P and Q stating which statement are true and which are false.

i. Mr. Asiedu is a kitchen staff ⇒ Mr. Asiedu does not go on holidays. ***2 marks***

ii. Ms. Freda goes on holidays ⇒ Ms. Freda is a teaching staff. ***2 marks***

iii. Anowa does not go on holidays ⇒ is Anowa is a kitchen staff. ***2 marks***

Total *9 marks* (DOK Level 2)

4. Consider the following two statements:

- **Statement K:** Being sincere is a good virtue.
- **Statement L:** Good virtues are Ghanaian values.

Examine the following three statements. For each one, determine whether the conclusion is definitely true, possibly true, or false based on Statements K and L. Provide a reason for your answer.

a. Auntie Rukia is a Ghanaian.

b. ⇒ Can we conclude that she has good virtues? ***3 marks***

c. Uncle Mensah is very sincere.

d. ⇒ Can we conclude that he is a Ghanaian? ***3 marks***

e. Kojo Baadu is sincere.

f. ⇒ Can we conclude that he has a Ghanaian value? ***3 marks***

Total *9 marks* (DOK Level 2)

PLC Session 1

45 minutes | **Planning & Reviewing the Learning Plan**

1. Begin the session by recapping lessons from the previous year/session/week, focusing on how learning was applied.

2. Use your subject specific app to explore the content before planning. Ask the app to

a. explain the week's concepts in a practical way that will make it easier for learners to understand

b. identify two or three common misconceptions learners may have on the week's content and how to address them using contexts familiar to Ghanaian learners. (NTS 3a, 3g and 3m).

3. Use your subject specific app to plan your lesson. Ask the app to

a. suggest the most effective method for teaching the week's concepts to learners with diverse backgrounds.

b. suggest learning activities, differentiation and essential questions based on the method of teaching (NTS 3a-3q)

c. review your prepared learning plan and address any challenging issues you may have identified in your subject area (NTS 3a-3c).

Note

- Take note of the recap topics linked to Years 1 and 2 while preparing for your lesson.
- Refer to the Values Learning Community Handbook and link the values for the week in your lesson to ensure that learners are applying what they learn.

4. Plan the week's key assessment with your app

a. The key assessment for the week is ***homework***. Chat with your app to plan this assessment, indicating what you and your learners will do before, during and after the homework (NTS 3k-3q).

10 minutes **Content Exploration**

5. As a subject group, select one problem or idea/term in the week's content that is challenging (not a routine procedural question, but one that requires conceptual understanding) and then

a. use the app, ask it to solve the problem step by step and explain the reasoning behind each step (you may be assisted by a teacher who is strong in the content area)

b. all teachers in the group work through a similar problem individually for 3–4 minutes

c. the group then works through the problem together step by step

d. highlight the most common errors made at each step and discuss (NTS 1c, 1e, 2c, 2e, 3e-3j).

Note

- This part is not just about 'How do I teach this?' but how do I teach it for learners to understand?
- If the group still shares the same content gap, use the app: ask it to solve the problem step by step and explain the reasoning behind each step.
- Problems/ideas discussed should be sourced from the Teacher Manual content, not invented separately. This keeps the content exploration directly connected to what teachers will deliver.

30 minutes **Enactment**

6. Enact a teaching activity based on your learning plan (NTS la-1b, 1e-lg, 2c, 2e, 2f, 3c-3j and 3o).

a. Colleagues observe with specific attention to clarity of concept explanation, use of familiar Ghanaian examples and how misconceptions are handled if they arise.

b. After the enactment, ask for structured feedback from the group

i. was the concept explained in a way that a learner with no prior knowledge could follow?

ii. were the examples grounded in contexts familiar to Ghanaian learners?

iii. did any misconceptions surface and if so, how were they addressed?

5 minutes **Reflection**

7. Reflect and share your views on the session. One thing that worked well, what did not work well and what you will do differently. (NTS 1a, 1b).

8. Identify a critical friend to observe and provide feedback on your lesson in relation to the PLC Session conducted (NTS 1f, 1g).

9. Remember to

a. transfer your chats into your learning planner template.

b. read your Teacher Manual and related Learner Material in preparation for the next session. Take note of the recap weeks, which are linked to Years 1 and 2 (NTS 3a).

c. integrate National Values, GESI, SEL, and 21st century skills into your learning activities, assessment tasks, and rubrics/mark schemes.

WEEK 2

Learning Indicator: *Make intelligent guessing to establish valid arguments and draw logical conclusions*

FOCAL AREA 1: INTELLIGENT GUESSING

Observe the pattern of Fibonacci numbers: 1, 1, 2, 3, 5, 8, …

To make an intelligent guess of the next number, a careful study of the trend must be made. Asking useful questions will help. For example: How are successive terms behaving? The next question to ask may be: What is the nature of the change that is taking place?

In this case, the value at a particular term is added to the previous one to obtain the next term.

An intelligent guess can now be made.

Example 1

Observe the pattern and make a guess of the next 3 terms: 1, 3, 6, 10, 15, . . .

Justify why you think your guess is correct and provide a generalisation to support similar works.

Solution

The next 3 numbers will be 21, 28 and 36.

The guess is correct because the successive difference between the terms increases by 1

That is: $1 + 2 = 3$,

$3 + 3 = 6$,

$6 + 4 = 10$,

$10 + 5 = 15$

So, the next number will be

$15 + (5 + 1) = 21$,

$21 + (6 + 1) = 28$ and so

$28 + (7 + 1) = 36$

The rule for the pattern $= \frac{n(n+1)}{2}$

To verify, the 8^{th} term $= \frac{8(8+1)}{2}$

$= \frac{72}{2}$

$= 36$

Example 2

A worker is tasked to fill a 6,400-litre tank with water using a twenty-litre bucket. The worker can fill the tank with 5 buckets a day. However, the tank leaks out 2 litres of water each day. How many days will it take for the labourer to fill the tank to the brim?

Solution

There is a pattern by which the tank is being filled

5 times 20 litres a day = 100 litres of water a day being put in

Reduction of 2 litres a day means there is 98 litres of water remaining in the tank a day.

The number of days to fill the tank = 6400 ÷ 98

= 65 days with 30 litres more to be filled.

Therefore, on the 66th day, the worker would have completed the work since the tank will be full.

Example 3

Mary Akugiri is deciding which course to pursue at the tertiary level. She has identified three conditions, A, B and C, that will influence her decision. These conditions are represented in the Venn diagram below.

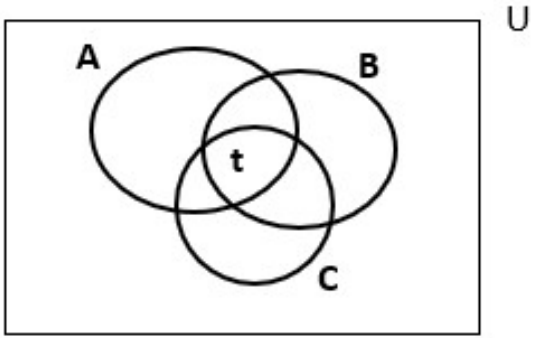


Figure 2.1: *Venn diagram on conditions for choice of a course*

a. Suggest any three possible conditions that Mary may consider when choosing a course to pursue at the tertiary level. Using the conditions you have suggested, explain what region 't' represents.

b. Describe the students represented by the region $C \cap (A \cup B)'$.

Solution

a. Three likely conditions will be the cost involved in offering the course, the prospects of the course and the duration of the course.

If A = cost of the course, B = prospect of the course and C = duration of the course then, region "t": That is the intersection of all three regions, suggests that all three

conditions will have to be considered to make a decision on the choice of course to offer. Course cost, duration and prospects are all concern

b. $C \cap (A \cup B)^I$ can be read as C only. This means that students in that region consider only duration of the course in making decision of a particular course to offer.

Decision Making In Solving Problems

Logical reasoning provides the opportunity to use established patterns to predict the likely occurrence of situations and so be able to decide on how to solve problems. To make a decision, one must consider certain factors. The process of considering such factors implies you are **reasoning logically**.

Example 4: Deciding on which mode of transportation

In Ghana, some of the transportation modes are tro-tro, STC, airplane, private car hiring and okada.

For each of the following scenarios, explain why you will decide on a particular mode of transportation.

a. Travelling from Goaso to Accra, partly along a dusty road, for an interview after school

b. Travelling from Kaneshie with a family of 7 to Madina, both within Accra

c. Travelling from home to buy waakye at a waakye joint through a traffic prone area

Solution

a. Interviews are time bound and one must reach there neat, punctually and less stressed.

STC is more accessible than airplane, less expensive than hiring a private car, more comfortable and can be faster since they don't make multiple stops like tro-tros.

I will make a logical decision to go for STC.

b. From Kaneshie to Madina, both being within Accra.

Tro-tro is less expensive and more intra-city friendly than STC and hiring a private car, and can hold the family size.

Based on these logical reasons, I will decision to go for tro-tro.

c. Traffic congestion requires a swifter transportation mode. It is not cost effective to spend more money on the means of transportation than the cost of the waakye.

Okada is swift within the traffic prone areas and less expensive.

I decide to go for the okada based on these reasons.

Example 5: Deciding where to sell farm produce

In a farming community in Ahafo Region, a farmer has the opportunity to either sell their crops to market-women who come to the village to buy foodstuffs or to send their produce to the market on a market day to sell. Suggest the logical reasons on which the farmer will decide to sell the foodstuff to market-women who come to the village or to send them to the market to sell.

Solution

Selling to the market-women who come to the village:

When the foodstuffs are sold to market-women who come to the village, the farmer will avoid transportation costs.

There is the guarantee of a ready market once the buyers are at the village rather than sending the items to the market.

Stress is reduced as there is no conveying the goods to the market.

The price of the foodstuffs might be higher since the number of farmers selling their goods is fewer than the main market on a market day.

Based on these reasons, the farmer should sell the foodstuffs to the market women who come to the village.

On the other hand, selling their produce to the market:

When the foodstuffs are sent to the market, more buyers will be available than selling to the few women who come to the village. Demand will increase thereby maybe getting a better price.

Going to the market gives the farmer the opportunity to access other items they might need which will not be found in the village.

The farmer will have the opportunity to compare prices from other farmers of the same produce and so may not be cheated by the women who go to the village.

By weighing these reasons logically, the farmer should send the crops to the market on the market day.

Example 6

During a game of Ludo, Odeibea realised that:

X: If she plays a 6, she can capture Yaw's marble.

Y: If she captures Yaw's marble, she will have her final marble in the play zone of Yaw and could be captured by the opponent.

Z: Yaw will take his turn throwing the dice, and Odeibea notices that Yaw's marble is in a vulnerable position.

Using intelligent guessing and logical reasoning, should Odeibea aim at playing a 6 and capture Yaw's marble or not? Support your conclusion with valid arguments.

Solution

Capturing Yaw's marble could prevent Yaw's progress which will be to the advantage of Odeibea.

Odeibea's final marble has the risk of being captured by Yaw or not.

Playing a 6 gives Odeibea another chance to play again ahead of Yaw.

Hence capturing Yaw's marble will delay Yaw's chances of taking a turn and also from completing the game first.

There are 6 numbers on the ludo die and it is not a definite event that Yaw will play a number that will help him to capture Odeibea's marble. Yaw has a slim chance of capturing Odeibea's marble and so Odeibea should target playing a 6 and go ahead to capture Yaw's marble.

Learning Tasks

Your school has received funding to build four new facilities (a science laboratory, a library for e-learning, an athletics field and a 300-capacity dormitory) at four different zones of the campus.

In your group, discuss the logical constraints, and develop and explain your decision-making strategy that satisfy all requirements about the placement of the new facilities at your school.

FOCAL AREA 2: USING TABLES AND DIAGRAMS IN DECISION-MAKING

In the previous focal area, learners used logical reasoning to analyse statements and make decisions. In this focal area, learners will use truth tables and Venn diagrams to examine logical statements and determine whether conclusions follow from given information.

Example 7: Using A Truth Table to Test an Implication

A school has the following rule for membership of its Mathematics Club:

"If a learner completes the registration form and pays the membership fee, then the learner becomes a registered member of the Mathematics Club."

Let **P**: The learner completes the registration form.

Q: The learner pays the membership fee.

R: The learner becomes a registered member of the Mathematics Club.

The rule can be written symbolically as: $(P \wedge Q) \rightarrow R$

a. Construct a truth table for $(P \wedge Q) \rightarrow R$.

b. Identify the condition under which the implication is false.

c. Explain what the truth table tells us about the school rule.

Solution

a. Truth table for $(P \wedge Q) \rightarrow R$

Table 2.1: Truth Table

P	Q	R	$P \wedge Q$	$(P \wedge Q) \rightarrow R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

b. The implication is false only when **P** is ***true***, **Q** is ***true***, but **R** is ***false***.

That is, the implication is false when the learner completes the registration form and pays the membership fee but does not become a registered member of the Mathematics Club.

This corresponds to the second row of the truth table.

c. The truth table helps us determine whether the conditional statement is logically satisfied under each possible combination of truth values.

For example, if the learner completes the registration form and pays the membership fee, then, according to the stated rule, the learner should become a registered member.

If **P** and **Q** are both ***true*** but **R** is ***false***, then the school rule has **not** been satisfied.

IMPORTANT NOTE

A truth table shows the logical relationship among statements. It does not, by itself, establish cause and effect or tell us how likely an event is to occur.

For example, consider the statement:

"If a learner attends extra classes and studies regularly at home, then the learner will pass the examination."

Let: **P**: The learner attends extra classes.

Q: The learner studies regularly at home.

R: The learner passes the examination.

We may write: $(P \wedge Q) \rightarrow R$

A truth table can show the truth values of this compound statement. However, it cannot prove that attending extra classes and studying at home will cause the learner to pass.

Examination performance may also depend on other factors, such as understanding of the subject, quality of preparation, attendance, examination conditions and the nature of the assessment.

Similarly, the truth table cannot tell us the probability that the learner will pass.

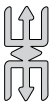
Evidence or appropriate data would be needed to investigate such a real-life claim.

Therefore, teachers should help learners distinguish among:

Logical implication: whether a conclusion follows from stated conditions.

Causation: whether one factor produces or contributes to an outcome.

Probability: how likely an event is to occur.



TEACHER NOTE

Avoid interpreting a truth table as evidence that an action is the "safest strategy" or that it "maximises the chance" of an outcome. Such conclusions require additional information.

The truth table **only** tells us whether the compound proposition is true or false for each combination of truth values.

Example 8: Using A Venn Diagram to Make Logical Conclusions

A school organises three academic clubs:

M: Mathematics Club

S: Science Club

R: Robotics Club

The following information is given:

1. Every member of the Robotics Club is a member of the Science Club.
2. Some members of the Science Club are also members of the Mathematics Club.

Let **M** = {learners who belong to the Mathematics Club}

S = {learners who belong to the Science Club}

R = {learners who belong to the Robotics Club}

a. Draw a Venn diagram to represent the information.

b. Determine whether each of the following conclusions is valid. Give a reason for your answer.

i. Abena is a member of the Robotics Club. Therefore, Abena is a member of the Science Club.

ii. Kojo is a member of the Science Club. Therefore, Kojo is a member of the Robotics Club.

iii. Sena is a member of both the Mathematics Club and the Science Club. Therefore, Sena is a member of the Robotics Club.

Solution

a. From the information given:

Every member of the Robotics Club is a member of the Science Club.

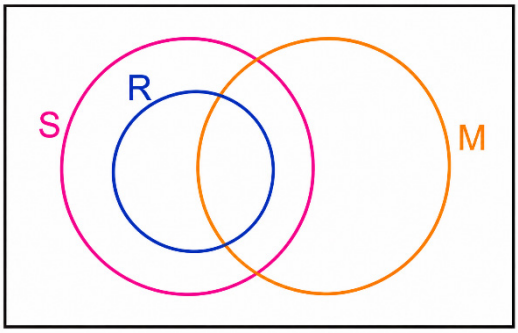
Therefore, $R \subseteq S$

The set R should therefore be drawn completely inside set S.

We are also told that some members of the Science Club belong to the Mathematics Club.

Therefore, $S \cap M \neq \varnothing$

Sets **S** and **M** should overlap.



b.

i. Abena is a member of the Robotics Club. Therefore, Abena is a member of the Science Club.

This conclusion is valid.

Since $R \subseteq S$, every member of R must also be a member of S.

Therefore, if Abena belongs to R, she must also belong to S.

ii. Kojo is a member of the Science Club. Therefore, Kojo is a member of the Robotics Club.

This conclusion is **not** valid.

We know that $R \subseteq S$. However, this does not mean that $S \subseteq R$.

Some learners may belong to the Science Club without belonging to the Robotics Club.

Therefore, being a member of S does not necessarily imply membership in R.

iii. Sena is a member of both the Mathematics Club and the Science Club. Therefore, Sena is a member of the Robotics Club.

This conclusion is **not** valid.

The given information tells us only that Sena belongs to $M \cap S$.

It does not tell us that every learner in $M \cap S$ also belongs to R.

Therefore, there is insufficient information to conclude that Sena is a member of the Robotics Club.

Learning Tasks

The class believes that the development of Ghana's natural resources should be guided by mathematical modelling rather than meeting immediate economic needs.

In your group, select a motion to speak for or against. Prepare to participate in the debate discussing your points through pyramid discussion – here, learners jot down initial thoughts, examples, and evidence. share their ideas with a partner, building on each other's thoughts. Two pairs combine to form groups of four, and the combining of small groups continue until a single group is formed with a single set of thoughts, example and evidence is gathered.

PEDAGOGICAL EXEMPLARS

Collaborative Learning

1. Engage learners in groups, learners discuss to establish and analyse arguments to carry out investigation on logical reasoning including conjunctions.
2. Engage learners in mixed ability groups through problem-based learning approaches to investigate the situation where they logically establish valid arguments and draw logical conclusions in solving problems (in compound statements), ensuring equity at all stages.
3. Engage learners in groups, to discuss, establish and analyse arguments using diagrams (truth tables) to carry out investigations on logical reasoning including conjunctions

Independent Learning

1. Engage individual learners to debate and defend their position on a decision using logical and valid statements.
2. Engage individual learners to write a summary of a real-life case they know and suggest solutions by giving logical reasoning and valid arguments.

KEY ASSESSMENT LEVEL 3: DEBATE

1. Consider the following statements:

- **p**: All farmers are hardworking.
- **q**: No idle person is hardworking.

Using intelligent guessing and logical reasoning state whether or not each of the following is a valid conclusion from p and q:

i. All farmers are not idle people. ***2 marks***

ii. Farmers who are idle are not hardworking. ***2 marks***

iii. Kudjo is hardworking, therefore he is a farmer. ***2 marks***

Total *6 marks* (DOK Level 2)

2. Consider the following statements:

- **p**: Madam Kukua has leadership skills.
- **q**: Madam Kukua heads an SHS.

If p⇒q, which of the following statements are valid?

i. If Madam Kukua does not head an SHS, then she does not have leadership skills. ***2 marks***

ii. If Madam Kukua heads an SHS, then she has leadership skills. ***2 marks***

iii. If Madam Kukua has leadership skills, then she heads an SHS. ***2 marks***

Total *6 marks* (DOK Level 2)

3. Generate a motion to be debated on with your classmates. As a panel member of the debate explain at least 4 tangible points for and 4 tangible points against, that you would expect from the debaters.

Total *12 marks* (DOK Level 3)

4. Adwoa gave these points during a presentation on the growing some flowers.

The flowers grown in the store room had leaves which were very deep green and very healthy.

When the flowers were brought into the open Adwoa noticed that, after one week, that the leaves had started turning yellow and were withering.

On reflection, Danso was of the view that since sunshine supports plant growth, Adwoa's presentation was false.

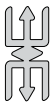
Analyse the validity of Danso's view.

Total *10 marks* (DOK Level 3)

PLC Session 2

45 minutes | **Planning & Reviewing the Learning Plan**

1. Begin the session by recapping lessons from the previous year/session/week, focusing on how learning was applied.
2. Use your subject specific app to explore the content before planning. Ask the app to
   a. explain the week's concepts in a practical way that will make it easier for learners to understand
   b. identify two or three common misconceptions learners may have on the week's content and how to address them using contexts familiar to Ghanaian learners. (NTS 3a, 3g and 3m).
3. Use your subject specific app to plan your lesson. Ask the app to
   a. suggest the most effective method for teaching the week's concepts to learners with diverse backgrounds.
   b. suggest learning activities, differentiation and essential questions based on the method of teaching (NTS 3a-3q)
   c. review your prepared learning plan and address any challenging issues you may have identified in your subject area (NTS 3a-3c).



Note

- Take note of the recap topics linked to Years 1 and 2 while preparing for your lesson.
- Refer to the Values Learning Community Handbook and link the values for the week in your lesson to ensure that learners are applying what they learn.

4. Plan the week's key assessment with your app
   a. The key assessment for the week is ***debate***. Chat with your app to plan this assessment, indicating what you and your learners will do before, during and after the debate (NTS 3k-3q).

10 minutes | **Content Exploration**

5. As a subject group, select one problem or idea/term in the week's content that is challenging (not a routine procedural question, but one that requires conceptual understanding) and then
   a. use the app, ask it to solve the problem step by step and explain the reasoning behind each step (you may be assisted by a teacher who is strong in the content area)
   b. all teachers in the group work through a similar problem individually for 3–4 minutes
   c. the group then works through the problem together step by step

d. highlight the most common errors made at each step and discuss (NTS 1c, 1e, 2c, 2e, 3e-3j).

Note

- This part is not just about 'How do I teach this?' but how do I teach it for learners to understand?
- If the group still shares the same content gap, use the app: ask it to solve the problem step by step and explain the reasoning behind each step.
- Problems/ideas discussed should be sourced from the Teacher Manual content, not invented separately. This keeps the content exploration directly connected to what teachers will deliver.

30 minutes Enactment

6. Enact a teaching activity based on your learning plan (NTS la-1b, 1e-lg, 2c, 2e, 2f, 3c-3j and 3o).

a. Colleagues observe with specific attention to clarity of concept explanation, use of familiar Ghanaian examples and how misc`onceptions are handled if they arise.

b. After the enactment, ask for structured feedback from the group

i. was the concept explained in a way that a learner with no prior knowledge could follow?

ii. were the examples grounded in contexts familiar to Ghanaian learners?

iii. did any misconceptions surface and if so, how were they addressed?

5 minutes Reflection

7. Reflect and share your views on the session. One thing that worked well, what did not work well and what you will do differently. (NTS 1a, 1b).

8. Identify a critical friend to observe and provide feedback on your lesson in relation to the PLC Session conducted (NTS 1f, 1g).

9. Remember to

a. transfer your chats into your learning planner template.

b. read your Teacher Manual and related Learner Material in preparation for the next session. Take note of the recap weeks, which are linked to Years 1 and 2 (NTS 3a).

c. integrate National Values, GESI, SEL, and 21st century skills into your learning activities, assessment tasks, and rubrics/mark schemes.

WEEK 3

Learning Indicator: *Carry out mini real-life investigations to solve problems involving logical reasoning, within their local community*

FOCAL AREA: REAL-LIFE APPLICATIONS OF LOGICAL REASONING

Students make arguments among peers and, occasionally, those who can shout louder, win the argument (debate) of the day. This might not be a fair situation since some winners might not necessarily have raised the most valid points.

Here we will look at how we can support learners to generate and test the validity of logical arguments from peers and investigate to solve problems involving logical reasoning, within their local community.

Examples of situations learners argue on include:

1. Can Ghana ever win a world cup?
2. Which school will win the next National Science and Mathematics Quiz?
3. Who will become the next school prefect?
4. Will the mathematics teacher come to class today?

Some of the responses may seem obvious but that is as a result of a set of logical occurrences over a long period of time. Other issues are not easy to make judgement and conclusions on them. It is easy to make claims on some statements as valid or not.

Example 1

1. The class prefect is the first to come to class ⇒ The class prefect was the first to come to class today. Verify the statement as valid or not.

Solution

Does the class prefect really come to class first?

Did the class prefect come to class before the teacher?

If one statement is false then we do not have a valid statement.

2. A student's subjects that require improvement are Mathematics and Science ⇒ A student failed in mathematics in the previous examination.

Solution

Is the student's subjects that require improvement Mathematics and Science?

Are my weakest subjects that require improvement Mathematics and Science?

If Mathematics and Science are my weakest subjects, did I fail the previous examination?

What logical conclusions do we make on this statement – valid or not?

Investigating Arguments

To solve a problem, we may consciously ,take some steps or perform certain activities. These steps may not be answers in themselves, but are leading us towards answers. In that case, we say we are **investigating**. These steps to approaching a problem can be:

1. Identify the objective for the investigation
2. Gather data on the subject using interview, survey, etc.
3. Analyse data to identify patterns and findings
4. Conclude on the argument

Similarly, Jerome Brunner is of the view that one needs to understand a problem, design a plan, implement the plan and finally look back (Okafor, 2019).

Some challenging situations that might require a solution

1. Your food-box is empty.
2. The end of semester exams is coming up but the expected scope of work has not been covered.
3. The landlord is ejecting you from a room for no fault of yours.

Note: *Guide learners to pose a problem for their peers to generate their own steps in solving a problem.*

Example 2

SHS 3 students are expected to be 17 years. We are in SHS 3 so every member is 17 years.

Solution

We wish to verify the argument of the age of SHS 3 students.

To do that each student should state their age (E.g. 17, 20, 18, 16, 16, 17, 17, 18, 16, 16, 17, 18).

Is everybody 17 years of age?

The expected age is the mean age. Let's find the mean age of the class?

$\sum x = 206$

$= 17.17$

The mean age $= \frac{206}{12} = 17$ years.

What conclusion can we draw?

Example 3

The school has received a technology grant of GH¢ 20,000.00 that must be spent on either upgrading the computer lab or providing tablets for the mathematics department.

In your groups, suggest logical reasons for which of the options will be most appropriate for the school.

Solution

Upgrading the computer lab and providing tablets for the Mathematics Department are both good. Upgrading the computer lab will secure the existing lab from deterioration and have a facelift.

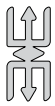
However, the number of computers in the school are few and have a series of challenges so the tablets will rather increase access to ICT tools.

Upgrading the computer lab is capital intensive and the amount may not cater for the lab as well as bringing the outmoded computers to current standard and capacity.

The tablets are handy and can be taken along for field trips and where research will be carried on. The desktops at the computer lab are too bulky to be carried about.

The challenge with power outages is makes it unreliable when using the laptop. In case of power outage, the tablets can be used for some time without going off.

These factors among others are cogent enough to consider providing tablets and upgrading the computer lab for the Mathematics Department.



Note

Engage learners in groups to gather information on the most logical way to optimise the use of water and electricity in your school. Let them carry out their plan of gathering information and present their findings appreciating the findings of other groups.

You may look out for:

Learning Tasks

Learners to undertake research on real-life application of logical reasoning and present their findings to the class:

Learners plan in groups, how they can gather information on the most logical way to optimise the use of water and electricity in their school. They carry out their plans in gathering information and present their findings while appreciating the findings of other groups.

Learners to demonstrate knowledge and skill in:

1. Well-developed appropriate tool for gathering information
2. Collaborative implementation of data collection process

3. Well-organised data to be presented on paper or orally or power point
4. Valid conclusion drawn on the means to optimise the use of water and electricity in your school

PEDAGOGICAL EXEMPLARS

1. In collaborative and gender responsive groups, learners generate and test the validity of logical arguments from peers, then investigations to solve problems involving logical reasoning, within their local community
2. Using Problem-Based Learning approaches learners, in mixed ability groups, carry out mini projects by visiting interesting areas in the community to generate and analyse situational growth and intelligent guessing and present their report to the whole class for intelligent discourse.
3. Individuals to share their appreciation of their involvement in the projects.

KEY ASSESSMENT LEVEL 4: GROUP PROJECT

1. If the computer lab is upgraded, all students will have access, but each student will have limited time. Explain 3 reasons why you think the motion is or is not valid.

Total *8 marks* (DOK Level 4)

2. In a school, the head of the Mathematics Department is requesting tablets. The head of the Agricultural Science department thinks that if tablets are purchased for the mathematics department, only mathematics students will benefit and they will have continuous access during mathematics lessons at the expense of Agricultural science department.

   Give 3 reason why you think the Agricultural Science Department head's view is or is not valid?

Total *8 marks* (DOK Level 4)

3. There is erosion in every part of your home or school community. In convenient groups of where members stay (boarding or day), verify from the stakeholders of your community which of these two: growing of grass and use of stones, to solve the erosion problem will be reasonable.

   Present your findings for discourse in class.

Total *15 marks* (DOK Level 4)

Hint

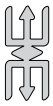


Assign the group project to learners and give them a reasonable time to complete it. Use the SS App to generate the project and the rubric. Learners' projects should be marked immediately, and scores entered into the SAP.

PLC Session 3

45 minutes | **Planning & Reviewing the Learning Plan**

1. Begin the session by recapping lessons from the previous session, focusing on how learning was applied.
2. Use your subject specific app to explore the content before planning. Ask the app to
   a. explain the week's concepts in a practical way that will make it easier for learners to understand.
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3. Use your subject specific app to plan your lesson. Ask the app to
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   c. review your prepared learning plan and address any challenging issues you may have identified in your subject area (NTS 3a-3c).



Note

- Take note of the recap topics linked to Years 1 and 2 while preparing for your lesson.
- Refer to the Values Learning Community Handbook and link the values for the week in your lesson to ensure that learners are applying what they learn.

4. Use your app to plan and administer the mode of assessment for the Student Assessment Portal (SAP).
   a. The key assessment for the week is ***group project***. Chat with your app to plan this assessment by reviewing or developing the assessment items/tasks and marking schemes/rubrics for the project (NTS 3k-3q).

10 minutes | **Content Exploration**

5. As a subject group, select one problem or topic/idea/term in the week's content that is challenging (not a routine procedural question, but one that requires conceptual understanding) and then
   a. use the app, ask it to solve the problem step by step and explain the reasoning behind each step (you may be assisted by a teacher who is strong in the content area)
   b. all teachers in the group work through the problem individually for 3–4 minutes

c. the group then works through it together the problem, step by step

d. highlight the most common errors made at each step (NTS 1c, 1e, 2c, 2e, 3e-3j).

Note

- This part is not 'How do I teach this?' but how do I teach it for learners to understand?
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9. Remember to

a. transfer your chats into your learning planner template.

b. read your Teacher Manual and related Learner Material in preparation for the next session. Take note of the recap weeks, which are linked to Years 1 and 2 (NTS 3a).

c. integrate National Values, GESI, SEL, and 21st century skills into your learning activities, assessment tasks, and rubrics/mark schemes.

WEEK 4

Learning Indicator: *Use proportional reasoning to investigate the various types of variation (direct and inverse) and extend this to make generalisations*

FOCAL AREA: VARIATION 1: INTRODUCTION TO VARIATION

In many real-life situations, a change in one quantity is associated with a predictable change in another. When quantities are related according to a specified mathematical rule, the relationship may be described using variation. For example, the more time you spend studying, the better your understanding tends to become. In mathematics, such relationships between variables are studied under the concept of **variation**.

In this section, we are going to look at four types of variation:

1. Direct Variation
2. Inverse Variation
3. Joint Variation
4. Partial Variation

1. Direct Variation

A quantity is said to vary **directly** as another when an increase (or decrease) in one corresponds to a proportional increase (or decrease) in the other. This is also known as **direct proportionality**.

Mathematical Representation:

If y varies directly as x, then:

$y \propto x$ or $y = kx$

where k is the constant of proportionality.

Example 1

a. The cost of bananas is directly proportional to the weight purchased.

i. If 1 kg costs GH¢5.00, then 2 kg costs GH¢ 10.00, 3 kg costs GH¢ 15.00, and so on.

ii. This forms the relationship: $C = 5w$, where C is the cost and w is the weight in kg.

b. The distance travelled by a car moving at constant speed is directly proportional to the time taken:

$d = st$

where s is the speed and t is time.

2. Inverse Variation

Two quantities vary inversely when their product remains constant; as one increases by a factor, the other decreases by the same factor.

Mathematical Representation:

If y varies inversely as x, then:

$y \alpha \frac{1}{x}$ or $y = \frac{k}{x}$

where k is the constant of proportionality.

Example 2

a. The time taken to complete a job is inversely proportional to the number of people working on it.

i. More people = less time;

ii. Fewer people = more time.

b. The speed of a vehicle is inversely proportional to the time taken for a fixed distance: $s = \frac{d}{t}$. Rearranged as $t = \frac{d}{s}$, which shows inverse variation between t and s.

Using Proportional Reasoning

Proportional reasoning involves comparing ratios and using them to solve problems involving variation.

a. In **direct variation**, the ratio $\frac{y}{x}$ is constant.

b. In **inverse variation**, the product xy is constant.

Table 4.1: Comparison:

Type of Variation	Relationship	General Form	Constant Quantity
Direct Variation	$y \propto x$	$y = kx$	$\frac{y}{x} = k$
Inverse Variation	$y \alpha \frac{1}{x}$	$y = \frac{k}{x}$	$xy = k$

Generalisations from Variation

By investigating different scenarios, students can generalise:

a. In direct variation, doubling one variable doubles the other.

b. In inverse variation, if one variable is doubled, the other is halved (assuming a constant product).

c. The graphs of direct variation are straight lines through the origin.

d. The graphs of inverse variation are rectangular hyperbolas, or a 'reciprocal graph'.

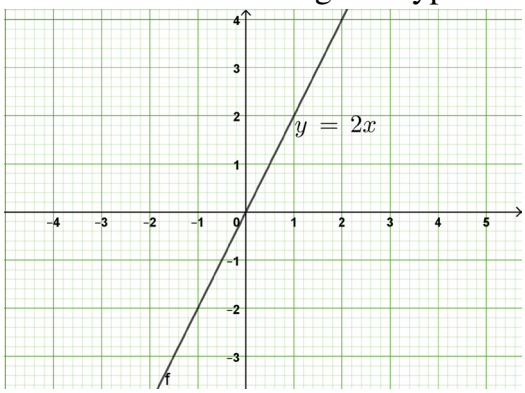


Figure 4.1: *Graph of direct variation*

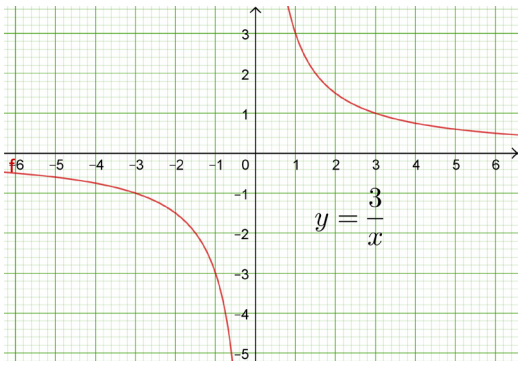


Figure 4.2: *Graph of inverse variation*

Example 3

1. A bag of rice costs GH¢ 120.00. The cost (C) varies directly with the number of bags (n).

a. Express the cost (C) in terms of n.

b. If a customer pays GH¢ 600.00, how many bags did they buy?

Solution

a. $C \propto n$. this implies $C = kn$

$120 = k\ (1)$

$k = 120$

$C = 120n$

b. $C = 120n$

Given, $C = 600$

$600 = 120n$

$n = \frac{600}{120}$

$n = 5$

The customer paid 600 for 5 bags.

2. The time (T) taken to travel from Accra to Kumasi is inversely proportional to the speed (S) of a vehicle: At 60 km/h, the trip takes 4 hours.

a. Derive the equation and find k (the constant of proportionality).

b. Calculate the time if speed increases to 80 km/h.

c. **Debate**: "*Why might this inverse model fail during the festive seasons*?"

Solution

a. Deriving the equation

$$T \alpha \frac{1}{S}$$

$$\Rightarrow T = \frac{k}{S}$$

Given, T = 4 hours and S = 60km/h

$$\Rightarrow 4 = \frac{k}{60}$$

$$k = 240$$

$$T = \frac{240}{S}$$

b. Calculate time when speed is 80 km/h

$$T = \frac{240}{S}$$

$$T = \frac{240}{80}$$

T = 3 hours

The time when speed is 80 km/h is 3 hours

c. **Debate**: Why might this inverse model fail during the festive seasons?

During the festive season, factors such as increased traffic congestion, delays, roadblocks, or accidents may occur. These factors break the assumption of such a high constant speed and free flow because traffic will cause stationary positions and therefore there will be no constant speed.

It can be concluded that the model is limited during unforeseen circumstances.

3. A farming research team is developing a mathematical model to understand how environmental factors affect crop yield. They discover that crop yield (Y) varies directly with rainfall (R) and inversely with temperature (T). In a specific trial: When R = 100 mm and T = 30°C, the yield Y = 50 kg.

a. Using the relationship described, derive the model that expresses Y in terms of R and T, and explain what the constant of proportionality represents in real-life context.

b. Use the model to predict the yield when R = 150 mm and T = 25°C, and justify whether this prediction is reasonable for a real-world setting.

c. Explain how the model might change if fertiliser use is introduced as an additional factor that enhances yield regardless of rainfall and temperature. Support your model with reasoning or hypothetical data.

Solution

a. Derive the model and interpret constant

$$Y \alpha \frac{R}{T} \Rightarrow Y = \frac{kR}{T}$$

$$50 = \frac{k100}{30}$$

$$50 \times 30 = 100k$$

$$k = 15$$

The model that expresses Y in terms of R and T is given by $Y = \frac{15R}{T}$.

Interpretation $k = 15$: Each mm of rain per °C contributes 15 kg of yield

b. Predict and justify reasonableness

$$Y = \frac{15R}{T}$$

$$Y = \frac{15(150)}{25}$$

$$Y = 90$$

With other factors that influence crop yield being constant, as rainfall increased from 100mm (with temperature 30°C) to 150mm (with a lower temperature of 25°C), crop yield increased from 50kg to 90kg. This means that, more rain and lower temp should lead to higher yield of 90 kg.

c. Predict and justify reasonableness

$Y = \frac{kR}{T} + f$, where f is the fertiliser impact

Plants will grow with or without fertiliser. However, fertiliser acts independently to increase yield, especially in poor conditions.

So where R = 100 mm and T = 30°C with K = 15, Y = 50 kg

If f = 20kg/ha then

Y = 70kg

It is obvious that fertiliser has impact and as such the model might change.

3. Joint variation

Joint variation is a variation where two or more quantities relate to each other. For example:

The volume **V** of a cylinder varies jointly with the square of the radius **r** and the height **h**, i.e.,

$$V = kr2h$$

Example 4

1. If y varies jointly as x and z, and $y = 24$ when $x = 2$ and $z = 3$, find the equation connecting y, x and z.

Solution

$y \propto xz$

$y = kxz$

$24 = k \times 2 \times 3$

$24 = 6k$

$k = 4$

Therefore, the equation connecting the y, x and z.

$y = 4xz$

2. The volume of a cone, V, varies jointly as its height, h, and the square of its radius, r.

A cone with a radius measuring 6 feet and a height measuring 10 feet has a volume of 120 cubic feet. Find the volume of a cone having a radius of 12 feet and a height of 2 feet.

Solution

$V \propto hr^2$

$V = khr^2$

$120\pi = k(10 \times 6^2)$

$120\pi = 360k$

$k = \frac{120\pi}{360}$

$k = \frac{\pi}{3}$

$V = \frac{\pi}{3} \times 2 \times 12^2$

$V = \frac{\pi}{3} \times 2 \times 144$

$V = 96\pi$ cubic feet

Learning Tasks

Learners to use direct or inverse variation in modelling a real-world situation and make predictions. They pose problems (questions) on real-life application of direct or inverse variation issues and swap them with their peers to answer and give feedback.

PEDAGOGICAL EXEMPLARS

1. Using small group, engage learners to discuss inverse and joint *variations, model and solve real-life situations involving proportional reasoning.*

2. Relate the variation to scientific concepts and use them to solve real life problems.

KEY ASSESSMENT LEVEL 2: ASSIGNMENT

MCQs (DOK Level 1)

1. Which equation best represents a situation where y varies directly with the square of x?

a. $y = kx$

b. $y = \frac{k}{x^2}$

c. $y = kx^2$

d. $y = k\sqrt{x}$

2. If y varies directly as x, and $y = 12$ when $x = 4$, what is the value of y when $x = 10$?

a. 30

b. 48

c. 33

d. 22

3. A mechanic observes that the time, t, taken to complete a job varies inversely with the number of workers w assigned. If 4 workers take 6 hours to finish the task, how many workers would be required to complete the same job in 2 hours?

a. 8

b. 10

c. 12

d. 14

4. The area A of a triangle varies jointly as the base b and height h. Given that $A = 30$ cm² when $b = 5$ cm and $h = 12$ cm,

a. Find the constant of variation k. ***3 marks***

b. Determine the area when $b = 10$ cm and $h = 15$ cm. ***2 marks***

c. Discuss how doubling both the base and height affects the area. ***3 marks***

Total *8 marks* (DOK Level 2)

5. The number of minutes needed to solve an exercise set of variation problems varies directly as the number of problems and inversely as the number of people working to solve the problems. It takes 4 people 36 minutes to solve 18 problems.

How many minutes will it take 8 people to solve 24 problems?

Total *5 marks* (DOK Level 2)

6. The pressure P of a gas varies jointly as the temperature T and the number of moles n and inversely as the volume V. If $P = 100$ when $n = 2$, $T = 300$ and $V = 10$,

a. Find k. ***3 marks***

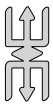
b. Find P when $n = 3$. $T = 250$ and $V = 12$. ***2 marks***

Total *5 marks* (DOK Level 2)

PLC Session 4

45 minutes **Planning & Reviewing the Learning Plan**

1. Begin the session by recapping lessons from the previous year/session/week, focusing on how learning was applied.
2. Use your subject specific app to explore the content before planning. Ask the app to
   - **a.** explain the week's concepts in a practical way that will make it easier for learners to understand
   - **b.** identify two or three common misconceptions learners may have on the week's content and how to address them using contexts familiar to Ghanaian learners. (NTS 3a, 3g and 3m).
3. Use your subject specific app to plan your lesson. Ask the app to
   - **a.** suggest the most effective method for teaching the week's concepts to learners with diverse backgrounds.
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Note

- Take note of the recap topics linked to Years 1 and 2 while preparing for your lesson.
- Refer to the Values Learning Community Handbook and link the values for the week in your lesson to ensure that learners are applying what they learn.

4. Plan the week's key assessment with your app
   - **a.** The key assessment for the week is ***assignment***. Chat with your app to plan this assessment, indicating what you and your learners will do before, during and after the assignment (NTS 3k-3q).

10 minutes **Content Exploration**

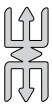
5. As a subject group, select one problem or idea/term in the week's content that is challenging (not a routine procedural question, but one that requires conceptual understanding) and then

a. use the app, ask it to solve the problem step by step and explain the reasoning behind each step (you may be assisted by a teacher who is strong in the content area)

b. all teachers in the group work through a similar problem individually for 3–4 minutes

c. the group then works through the problem together step by step

d. highlight the most common errors made at each step and discuss (NTS 1c, 1e, 2c, 2e, 3e-3j).



Note

- This part is not just about 'How do I teach this?' but how do I teach it for learners to understand?
- If the group still shares the same content gap, use the app: ask it to solve the problem step by step and explain the reasoning behind each step.
- Problems/ideas discussed should be sourced from the Teacher Manual content, not invented separately. This keeps the content exploration directly connected to what teachers will deliver.

30 minutes **Enactment**

6. Enact a teaching activity based on your learning plan (NTS la-1b, 1e-lg, 2c, 2e, 2f, 3c-3j and 3o).

a. Colleagues observe with specific attention to clarity of concept explanation, use of familiar Ghanaian examples and how misconceptions are handled if they arise.

b. After the enactment, ask for structured feedback from the group

i. was the concept explained in a way that a learner with no prior knowledge could follow?

ii. were the examples grounded in contexts familiar to Ghanaian learners?

iii. did any misconceptions surface and if so, how were they addressed?

5 minutes **Reflection**

7. Reflect and share your views on the session. One thing that worked well, what did not work well and what you will do differently. (NTS 1a, 1b).

8. Identify a critical friend to observe and provide feedback on your lesson in relation to the PLC Session conducted (NTS 1f, 1g).

9. Remember to

a. transfer your chats into your learning planner template.

b. read your Teacher Manual and related Learner Material in preparation for the next session. Take note of the recap weeks, which are linked to Years 1 and 2 (NTS 3a).

c. integrate National Values, GESI, SEL, and 21st century skills into your learning activities, assessment tasks, and rubrics/mark schemes.

WEEK 5

Learning Indicator: *Use proportional reasoning to investigate partial variations, make generalisations and solve real-life issues involving partial variations*

FOCAL AREA: VARIATION 2

Introduction

In everyday life, we often encounter situations where a cost or value does not start from zero but includes a fixed component and a variable component. This type of relationship is known as **partial variation**. Understanding partial variation is essential in budgeting, planning services, and interpreting data. It builds on direct variation but adds a constant starting value.

Understanding Partial Variation

Partial variation occurs when one quantity is made up of:

- A fixed value (also called the constant or starting value), and
- A variable part that changes in direct proportion to another quantity.

This can be modelled mathematically as:

$y = kx + c$

Where:

- y is the dependent variable,
- x is the independent variable,
- k is the rate of change,
- c is the fixed component (non-zero starting value).

This equation represents a linear relationship where the graph is a straight line that does not pass through the origin.

Example 1

A taxi charges a flat rate of GH¢ 8.00, plus GH¢ 2.00 per kilometre travelled.

$$\text{Total fare (F)} = 2x + 8$$

Where x is the number of kilometres travelled. The graph of this relationship is a straight line with slope 2 and a y intercept of 8.

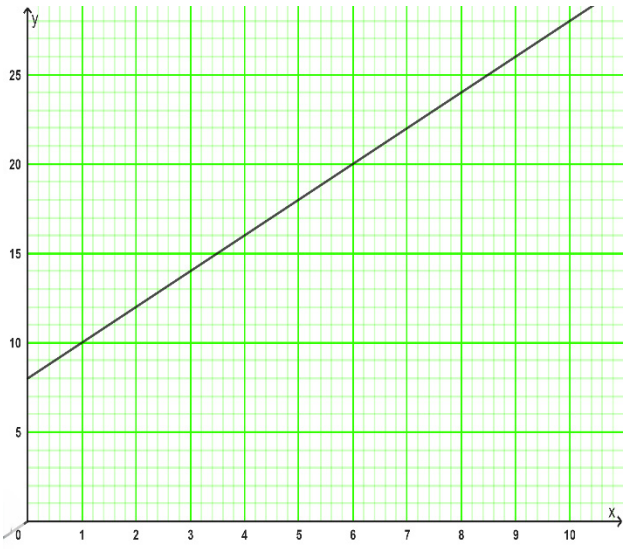


Figure 5.1: *Graph of $y = 2x + 8$*

Example 2

A mobile phone company charges a fixed monthly fee of GH¢ 30.00 and an additional GH¢ 0.10 per minute of call time. Ama's monthly phone bill was GH¢ 65.00.

a. Form an equation that models this situation.

b. How many minutes did Ama use?

c. If her call time increases by 50 minutes next month, estimate her next bill and explain your reasoning.

Solution

a. $T = 30 + 0.10x$, *where x represents the number of minutes Ama was making calls and T represents the total bill charge.*

b. $65 = 30 + 0.10x$

$x = 350$ minutes

c. For her next bill: $350 + 50 = 400$

New bill: $30 + 0.10 \times 400 = 70$

Ama has a fixed cost of GH¢ 30.00 to pay, plus her calls. The increase in her calls will increase her bill from GH¢ 65.00 to GH¢ 70.00.

Example 3

A student investigated the cost of printing at a shop. The cost to print 10 pages was GH¢ 14.00, and to print 30 pages was GH¢ 26.00.

a. Use proportional reasoning to determine the fixed charge and the cost per page.

b. Write the partial variation equation for the cost C and number of pages p.

c. Plot the graph and describe its slope and intercept in context.

d. Predict the cost for 50 pages and justify your approach.

Solution

a. Let C be the cost, k the cost per page and b the fixed charge.

$C = kp + b$

We have two data points:

(10, 14) and (30, 26)

$14 = 10k + b$ ….....1

$26 = 30k + b$ ……..2

Solve equations 1 and 2 simultaneously:

k = 0.6 and b = 8

Therefore, the fixed charge is GH¢ 8

Cost per page = GH¢ 0.60

b. Write the partial variation equation for the cost C and number of pages p:

$C = 0.6p + 8$

c. Plot the graph and describe its slope and intercept in context.

- Plot points: (10, 14), (30, 26)

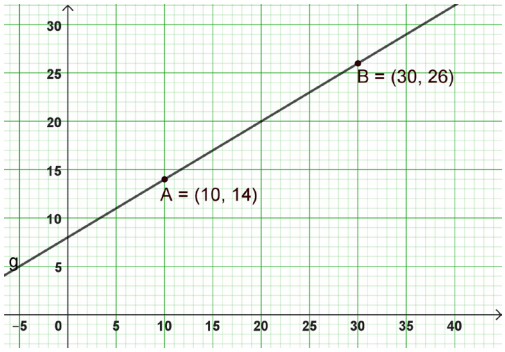


Figure 5.2: *Graph of the line passing through (10, 14) and (30, 26)*

- The slope = 0.6 means, for every extra page, cost increases by GH¢ 0.60
- The intercept on the y-axis = 8 means, even if no page is printed, a setup or service fee of GH¢ 8 is charged

d. Predict the cost for 50 pages and justify your approach.

Using the equation

$C = 0.6p + 8$

$C = 0.6(50) + 8$

$C = 38$

Therefore, the cost is GH¢ 38, assuming the linear relationship holds.

Justification

We used the linear model based on fixed and variable costs, which fits the trend observed from earlier data.

Learning Task

Learners to use partial variations, make generalisations and solve real-life issues involving partial variations. They pose problems (questions) on real-life application of partial variation issues and swap them with their peers to answer and give feedback.

PEDAGOGICAL EXEMPLARS

1. Using group/collaborative strategies, engage learners to revise the concepts of variation (direct, inverse, and joint) and establish the concept of partial.
2. Relate variation to scientific concepts and use them to solve real life problems
3. Individuals to solve problems and model problem involving variation.

KEY ASSESSMENT LEVEL 3: PROBLEM POSING & PEER REVIEW

1. A plumber charges GH¢ 40.00 for a service call and GH¢ 25.00 for each hour of work.

a. Write an equation for the total charge T for h hours of work. ***1 mark***

b. Determine how much a customer would pay for 6.5 hours of work. ***2 marks***

c. Explain how the relationship differs from direct variation. ***2 marks***

d. Modify the equation if the hourly rate increases by GH¢ 5.00 and explain how this affects the graph and total cost. ***3 marks***

Total *8 marks* (DOK Level 3)

2. A car rental agency charges GH¢ 120.00 per day plus GH¢ 1.50 for each kilometer driven.

a. Write a formula for the total cost C in terms of the number of days d and kilometers k. ***1 mark***

b. A customer rents a car for 3 days and drives 200 km. What is the total cost?

2 marks

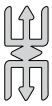
c. If a customer has a budget of GH¢ 500.00, what is the maximum number of kilometers they can drive in 2 days? ***3 marks***

Total *6 marks* (DOK Level 3)

PLC Session 5

45 minutes | **Planning & Reviewing the Learning Plan**

1. Begin the session by recapping lessons from the previous year/session/week, focusing on how learning was applied.
2. Use your subject specific app to explore the content before planning. Ask the app to
   a. explain the week's concepts in a practical way that will make it easier for learners to understand
   b. identify two or three common misconceptions learners may have on the week's content and how to address them using contexts familiar to Ghanaian learners. (NTS 3a, 3g and 3m).
3. Use your subject specific app to plan your lesson. Ask the app to
   a. suggest the most effective method for teaching the week's concepts to learners with diverse backgrounds.
   b. suggest learning activities, differentiation and essential questions based on the method of teaching (NTS 3a-3q)
   c. review your prepared learning plan and address any challenging issues you may have identified in your subject area (NTS 3a-3c).



Note

- Take note of the recap topics linked to Years 1 and 2 while preparing for your lesson.
- Refer to the Values Learning Community Handbook and link the values for the week in your lesson to ensure that learners are applying what they learn.

4. Plan the week's key assessment with your app
   a. The key assessment for the week is ***problem posing & peer review***. Chat with your app to plan this assessment, indicating what you and your learners will do before, during and after the peer review (NTS 3k-3q).

10 minutes | **Content Exploration**

5. As a subject group, select one problem or idea/term in the week's content that is challenging (not a routine procedural question, but one that requires conceptual understanding) and then
   a. use the app, ask it to solve the problem step by step and explain the reasoning behind each step (you may be assisted by a teacher who is strong in the content area)
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Section Review

The essential learning outcomes for Weeks 1 to 5 have focused on developing learners' understanding of logical reasoning and variation and applying these concepts to solve mathematical and day-to-day problems.

During the lessons on logical reasoning, learners explored statements, truth values, logical connectives, implications and the use of tables and Venn diagrams to represent and analyse logical relationships. They also applied logical reasoning to make informed decisions, evaluate arguments and justify conclusions in mathematical and real-life situations.

The lessons on variation introduced learners to relationships between quantities that change in relation to one another. Learners investigated and distinguished among direct variation, inverse variation, joint variation and partial variation. They represented these relationships mathematically, determined constants of variation and used the resulting equations to solve problems.

Learners also applied variation to real-life situations, investigated how changes in one quantity affect another, interpreted relationships between variables and used mathematical reasoning to make predictions and solve practical problems.

Through individual, pair and group activities, learners developed problem-solving, critical-thinking, communication and collaborative skills. They were encouraged to justify their conclusions, examine alternative approaches and connect mathematical ideas to real-life situations.

This section therefore provides a foundation for learners to confidently use logical reasoning and proportional relationships in subsequent mathematical topics and when solving day-to-day problems.

MARKING SCHEMES FOR THE KEY ASSESSMENTS

WEEK 1

Marking Scheme

1. ***1 mark for each correct answer***

i. True

ii. Debatable

iii. False

iv. False

v. Debatable

vi. True

vii. False

viii. True

ix. False

Total = 9 marks

2.

a. **M ∧ N:** Factors divide a given number and all factors are composite numbers. (***1 mark***) **False (*1 mark*)**

i. **M ∨ N:** Factors divide a given number or all factors are composite numbers. (***1 mark***)

ii. **True *(1 mark)***

b. **S ∧ T:** For x ≥ 9, x is a negative integer and x is a factor of 3. (1 mark)
False (1 mark)

i. **S ∨ T:** For x ≥ 9, x could be a multiple of 3 and never a factor of 3. (1 mark)

ii. **False *(1 mark)***

c. **G ∧ H:** Mappings have rules and mappings have ordered pairs. (1 mark)

i. **True (*1 mark*)**

ii. **G ∨ T: Mappings have rules or mappings have ordered pairs. *(1mark)***

iii. **True (*1 mark*)**

3. 3 acres of maize will yield 3 × 2 tons = 6 tons — ***M 1***

7 tons of cassava will yield 7 × 3 tons = 21 tons — ***M 1 A 1*** (all correct answers)

Total = 27 tons — ***A 1***

Since cassava yields greater tons per acre, the higher concentration should be on more acres for cassava and lesser for maize. ***B 1***

Total = 5 marks

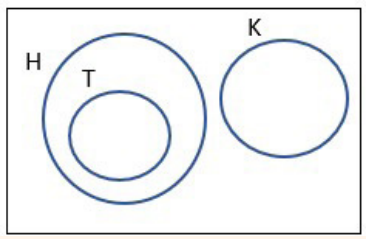
4.

a. Let T = teaching staff

K = kitchen staff

H = people who go on holidays

U = Workforce



B3 *(-1 each error – wrong arrangement of rings, omission of regions, omission of labels of regions)*

Figure 1.4: *Venn diagram for key assessment 4*

b. Valid or invalid conclusions

i. Valid statement – because all kitchen staff do not go for holidays.

$K \cap H^1 = K$ ***B1***

True Statement ***B1***

ii. Invalid statement – because Freda belongs to H and T is a subset of H. Freda can equally belong to A^1.

False statement ***B1, B1***

iii. Invalid statement – because kitchen staff is a subset of H1 which includes other workforce.

False statement ***B1, B1***

Total = *9 marks*

5.

a. Auntie Rukia is a Ghanaian ⇒ Auntie Rukia has good virtues.

- This is reversing the original logic.
- We know that good virtues are Ghanaian values, but we don't know if all Ghanaians possess good virtues.
- So being Ghanaian does not guarantee good virtues. **(*Explanation. 2 marks*)**

Conclusion: **False** – We cannot conclude that all Ghanaians have good virtues.

(*Correct conclusion 1 mark*)

b. Uncle Mensah is very sincere ⇒ Uncle Mensah is a Ghanaian.

- Sincerity is a good virtue, and good virtues are Ghanaian values.
- But having a Ghanaian value does not necessarily mean the person is Ghanaian.
- A non-Ghanaian can also show Ghanaian values. **(*Explanation. 2 marks*)**

***Conclusion*: False** – Sincerity alone does not prove Uncle Mensah is Ghanaian.

(*Correct conclusion 1 mark*)

c. Kojo Baadu is sincere ⇒ Kojo Baadu has a Ghanaian value.

- From Statement K: Sincerity is a good virtue.
- From Statement L: Good virtues are Ghanaian values.

Therefore, **sincerity is a Ghanaian value**. ***(Explanation. 2 marks)***

Conclusion: **True** – If Kojo Baadu is sincere, he possesses a Ghanaian value.

(Correct conclusion 1 mark)

WEEK 2

Marking Scheme

1.

i. All farmers are not idle people.

All hardworking people are not idle. Since farmers are hardworking, then all farmers are not idle. ***B1 - logical reason***

The statement is valid. ***B1 - statement as valid***

ii. Farmers who are idle are not hardworking.

Since all farmers are hardworking and no idle person is hard working, then no farmer is idle in the first place. ***B1 - logical reason***

The statement is therefore invalid as no farmers are idle. ***B1 - statement as invalid***

iii. Kudjo is hardworking, therefore he is a farmer.

Farmers form part of hardworking people but there are other non-farmers who are hard working. ***B1 - logical reason***

This means that Kudjo can be hardworking but may not necessarily be a farmer. Therefore, the statement is invalid. ***B1 - statement as invalid***

2.

i. If Madam Kukua does not head an SHS, then she does not have leadership skills.

Madam Kukua does not head an SHS because she has leadership skills.

B1 - logical reason

The statement is invalid. ***B1 - statement as invalid***

ii. If Madam Kukua heads an SHS, then she has leadership skills.

Since p⇒q is not equal to q⇒p ***B1 - logical reason***

then the statement is invalid. ***B1 - statement as invalid***

iii) If Madam Kukua has leadership skills, then she heads an SHS.

Given the condition p⇒q, the statement is valid. ***B1 - logical reason,***

B1 - statement as valid

3.

Table 2.2: Rubrics for key assessment 3

Indicators (Areas to consider)	Score
Motion	
A relatable motion statement, without ambiguity, clearly identifies two opposing viewpoints as "for" or "against",	4
A relatable motion statement, without ambiguity, opposing viewpoints as "for" or "against" cannot be clearly identified.	3
A relatable motion statement, is ambiguous, opposing viewpoints, ie. "for" or "against" cannot be clearly identified.	2
Motion statement is not relatable, ambiguous, opposing viewpoints, ie. "for" or "against" cannot be clearly identified.	1
Expected points for the motion	
4 or more points that support the motion	4
3 appropriate points that support the motion	3
2 appropriate points that support the motion	2
1 appropriate point that supports the motion	1
Expected points against the motion	
4 or more points that is against the motion	4
3 appropriate points that is against the motion	3
2 appropriate points that is against the motion	2
1 appropriate point that is against the motion	1
Total score	**12**

4.

Table 2.3: Rubrics for key assessment 4

Identifying and justifying Danso's accession.	**2 marks**
Identifying at 3 likely justifications in points raised in Adwoa's presentation as opposing Danso's accession (e.g., There could be light in Adwoa's room)	**2 for each point = 6 marks**
Drawing conclusion on the validity of Danso's accession	**2 marks**
Total	**10 marks**

WEEK 3

Mark Scheme

1.

Table 3.1: Sample rubric for key assessment 1

Description of response	Score
Stating of position as whether the motion is or is not valid	**1 mark**
Reasons for their position as whether the motion is or is not valid. Stating the importance of choosing a computer lab over time of access by learners or vice versa	**1 mark** for each point stated (1 × 3 = **3 marks**)
Explaining the point raised on the importance of choosing a computer lab over time of access by learners or vice versa	1 mark for each point explained (1 × 3 = **3 marks**)
Drawing conclusion on position taken based on the reasons raised	**1 mark**
Total	**8 marks**

2.

Table 3.2: Sample rubric for key assessment 2

Description of response	Score
Stating of position as whether the motion is or is not valid	**1 mark**
Reasons for their position as whether the motion is or is not valid. Statement in support of or against the head of the Agricultural Science Department's view.	**1 mark** for each point stated (1 x 3 = **3 marks**)
Explaining the statement made in support of or against the head of the Agricultural Science Department's view.	**1mark** for each point explained (1 x 3 = **3 marks**)
Drawing conclusion on position taken based on the reasons raised	**1 mark**
Total	**8 marks**

3.

Table 3.3: Sample rubric for key assessment 3

Areas to consider	Details of expected response	Marks
Structure of presentation document	Information includes headings on introduction, process of gathering information, findings from stakeholders, plan to integrate findings in solving erosion problem and conclusion	**1 mark** for each of any 2 headings clearly stated (1× 2 = **2marks**)

Content of presentation document	Statement of introduction	**1mark** for clearly stating what the report is about point explained
Content of presentation document	Explanation of process of gathering information should include mentioning of a tool for collecting data, the process used and the individuals contacted	**1 mark** for each point explained (1× 3=**3marks**)
Content of presentation document	Any 3 findings from stakeholders	**1mark** for each point explained (1× 3=**3marks**)
Content of presentation document	Any 2 statements on plan to integrate findings in solving erosion problem	**1mark** for each point explained (1× 2=**2marks**)
Content of presentation document	Statement of conclusion	**1mark** for a statement of summary
Portfolio	Include availability of tool used, pictures, related research reference or documents on erosion, write-up, audio, slides, pictures, concept maps, etc.	**1mark** for each neat and appropriate material. submitted (1× 3=**3marks**)
Total score		15 marks

WEEK 4

Marking Scheme

1. C
2. A
3. C

Table 4.2: Mark scheme for key assessment 4

Section	Task	Marks
(a)	Correct formula: A = kbh	**1 mark**
	Substitution and solving for k correctly 30 = k×5×12 30 = 60k k = 0.5	**1 mark**
	Correct final answer: k = 0.5	**1 mark**
(b)	Correct substitution using k = 0.5, b = 10, h = 15 A = 0.5 × 10 × 15	**1 mark**
	Correct multiplication and result: A = 75 cm^2	**1 mark**

(c)	Correct explanation using algebra or logical reasoning Let the original base be b and height be h. Original area: A = 0.5 × b × h **New base** = 2b, new height = 2h **New area:** A′ = 0.5 × 2b × 2h = 0.5 ×4bh = 4(0.5bh = 4A	**2 marks**
	Conclusion: Doubling both the base and height causes the area to become four times larger.	**1 mark**
Total		**marks**

5.

Table 4.3: Marking scheme for key assessment 5

Task	Marks
Correctly identifies the variation: $T = \frac{kP}{N}$, where P = the number of problems N = number of people solving	**1 mark**
Substitutes given values $36 = \frac{k \times 18}{4}$	**1 mark**
Solving for k correctly: $36 \times 4 = k \times 18$ $k = \frac{36 \times 4}{18}$ $k = 8$	**1 mark**
Substitutes k = 8, P = 24, N = 8 into the equation $T = \frac{8 \times 24}{8}$	**1 mark**
Correct computation and final answer: T = ***24 minutes***	**1 mark**
Total	**marks**

6.

Table 4.4: Marking scheme for key assessment 6

Task	Marks
Writes correct formula: $P = \frac{knT}{v}$	**1 mark**
Correct substitution with given values $100 = \frac{k \times 2 \times 300}{10}$ $100 = \frac{600k}{10}$ $100 = 60k$	**1 mark**
Solving for k = $\frac{5}{3}$	**1 mark**
Substituting the k = $\frac{5}{3}$, n = 3, T = 250, V = 12 into P = $\frac{\frac{5}{3} \times 3 \times 250}{12}$	**1 mark**
Solving for P $P = \frac{5 \times 250}{12}$ $P = \frac{1250}{12}$ $P = 104.17$	**1 mark**
Total	**5 marks**

WEEK 5

Marking Scheme

1. ***Table 5.1:*** Marking scheme for Key assessment 1

Part	Criteria	Marks
a.	Correctly writing the equation in terms of **T** and **h** T = 40 + 25h	**1 mark**
b.	Correct substitution of h = 6.5 and solving for T. T = 40 + 25×6.5 T = 202.50	**2 marks**
c.	In a direct variation, the relationship is of the form T = kh, where the graph passes through the origin (0,0). Here, the equation is T = 40 + 25h, which includes a fixed starting amount (GH¢40), so the graph does not pass through the origin.	**2 marks**

d.	T = 40 + 30h **Explanation:** The slope of the line increases, meaning the graph becomes steeper. For every hour worked, the total cost increases more rapidly than before.	**3 marks**
Total		8 marks

2. ***Table 5.2:*** Marking scheme for Key Assessment 2

Part	Criteria	Marks
a.	$C = 120d + 1.50k$	**1 mark**
b.	$\text{Cost} = 120 \times 3 + 1.50 \times 200$ $=360 + 300$ $=GH¢660$	**2 marks**
c.	$500 = 120 \times 2 + 1.50k$ $\Rightarrow 240 + 1.50k = 500$ $\Rightarrow 1.50k = 260$ $k = \frac{260}{1.50}$ $k = 173.33$	**2 marks**
	Maximum whole number of kilometres: ***173 km***	**1 mark**
Total		6 marks

SECTION 2: ALGEBRAIC REASONING

STRAND: ALGEBRAIC REASONING

Sub-Strand: Patterns and Relationships

Weeks 6,7,8, and 9

Learning Outcome: *Solve problems on quadratic functions and equations, including real-life problems.*

Content Standard: *Demonstrate understanding of the concept of quadratic functions and equations, and solve real-life problems with them.*

INTRODUCTION AND SECTION SUMMARY

Imagine modelling the height of an object projected into the air, designing a parabolic arch, or modelling the profit of a business. Maybe you're even an economist trying to model the profit of a business based on price. Surprisingly, all these scenarios can be described and analysed using quadratic functions and equations. The quadratic function describes a particular type of relationship between variables that results in a parabolic curve, while quadratic equations help us find specific values of the input variable that make the function equal to zero. They are fundamental tools for modelling and solving problems involving curved paths, optimisation, and various other real-world scenarios.

In this section, we will learn how to solve quadratic equations using the method of grouping, the quadratic formula and graphs. We will also learn how to graph the quadratic function, determine the axis of symmetry, maximum or minimum value, domain and range, where appropriate. We will also apply these concepts to answer real-life problems.

As we delve deeper into this topic, you will see just how versatile and essential quadratic functions and equations are for understanding and solving problems in the world around us.

The weeks, the indicators with their respective focal areas covered by the section are:

Week 6

Identify and solve quadratic equations

***Focal Area*: Identifying and solving quadratic equations**

- *Factorisation of quadratic equations*
- *Model and solve quadratic functions and equations involving real-life applications*

Week 7

Solve quadratic equations graphically and find the maximum and minimum points of quadratic graphs

Focal Area: **Solving quadratic equations using graphs**

- *Graphing and solving quadratic equations*
- *Maximum and minimum*
- *Model and solve real-life situations involving quadratic functions*

Week 8

Identify and explain the axis of symmetry, write its equation and solve linear and quadratic equations simultaneously using graphs.

Focal Area: **Finding equations of axes of symmetry and solving linear and quadratic equations simultaneously using graphs**

- *Finding axis of symmetry from quadratic graphs*
- *Finding axis of symmetry from quadratic equations*
- *Solve linear and quadratic equations simultaneously using graphs.*
- *Solve related real-life problems involving quadratic equations*

Week 9

Use quadratic graphs to solve related equations and solve real-life problems to find the range of values of (x) for which the other value (y) is increasing or decreasing and the range of values (x) for which the other value (y) is positive or negative.

Focal Area: **Real-life problems involving linear and quadratic equations**

- *Find the range of values of (x) for which the other value (y) is increasing or decreasing.*
- *Find the range of values (x) for which the other value (y) is positive or negative.*
- *Model and solve real-life problems involving linear and quadratic equations.*

SUMMARY OF PEDAGOGICAL EXEMPLARS

Learners should engage in hands-on, collaborative and experiential activities that develop their understanding of quadratic equations and functions. Mixed-ability grouping may be used to encourage participation, peer support and mathematical communication.

Provide all learners with opportunities to explore quadratic relationships, solve problems, explain their reasoning, use appropriate formulae and, where available, use technological tools to investigate quadratic functions.

Differentiation should be achieved by varying the level of scaffolding, prompts, representations, and task complexity rather than limiting particular mathematical experiences to specific groups of learners. Learners who require additional support may be provided with worked examples, guiding questions, visual representations, partially completed tasks or peer support. Learners

who demonstrate greater proficiency may be given extension tasks that require deeper reasoning, multiple solution methods, mathematical modelling, justification and investigation using appropriate digital tools.

All learners should be encouraged to communicate their mathematical thinking through oral explanations, written work, diagrams, symbolic representations and presentations, as appropriate.

ASSESSMENT SUMMARY

Table 1 contains the key assessment for each indicator in the section. The highest level of DOK for the indicator is denoted. The mode of assessment to be carried out and strategies that will aid in achieving the indicators are documented.

Table 2: Indicators, DOK levels, assessment modes and strategies

Indicators	DOK Level	Assessment Strategy and/or Mode
Identify and solve quadratic equations.	3	**Strategy:** closed and open-ended standard questions **Mode:** Classwork
Solve quadratic equations graphically and find the maximum and minimum points of quadratic graphs.	3	**Strategy:** Reflective exercise **Mode:** Classwork
Identify and explain the axes of symmetry, write its equation and solve linear and quadratic equations simultaneously using graphs.	3	**Strategy:** Groupwork **Mode:** Presentation
Use quadratic graphs to solve related equations and solve real-life problems to find the range of values of (x) for which the other value (y) is increasing or decreasing and the range of values (x) for which the other value (y) is positive or negative.	3	**Strategy:** Independent assessment **Mode:** Classwork

WEEK 6

Learning Indicator: *Identify and solve quadratic equations*

FOCAL AREA 1: FACTORISATION OF QUADRATIC EQUATIONS

What is a Quadratic Equation?

It is a polynomial equation of degree 2. This means the highest exponent, or power, of the variable is 2.

A quadratic equation in a variable, x , is of the form $ax^2+bx+c=0$, where a, b and c are constants and $a \neq 0$

Similarly, a quadratic equation in a variable, , is of the form , where and are constants and

Example 1

Which of the following is a quadratic equation?

a. $x^2+3x=0$

b. $3x^2-5x+7=0$

c. $x^2+\frac{1}{x}-4=0$

d. $7x+9=0$

e. $x^2-1=0$

f. $(1-x)^2=9$

g. $8x^2=0$

h. $2x+3=\frac{1}{x}$

Solution

a. Is a quadratic equation with $a=1$, $b=3$, and $c=0$

b. Is a quadratic equation with $a=3$, $b=-5$, and $c=7$

c. NOT a quadratic equation.

d. Remove the denominator by multiplying both sides of the equation by x ,

e. $x \times x^2+x \times \frac{1}{x}-4x=x \times 0$

$x^3+1-4x=0$. This is a cubic equation because the highest power of x is 3.

f. NOT a quadratic equation. This is a linear equation because the highest power of x is 1

g. Is a quadratic equation with $a = 1$, $b = 0$, and $c = -1$

h. Is a quadratic equation

Multiply out the bracket and bring everything to the same side:

$1 - 2x + x^2 = 9$

$x^2 - 2x - 8 = 0$

Therefore, $a = 1$, $b = -2$, and $c = -8$

i. IS a quadratic equation with $a = 8$, $b = 0$, and $c = 0$

j. IS a quadratic equation.

k. Remove the denominator by multiplying both sides of the equation by ,

l. $x \times 2x + 3x = x \times \frac{1}{x}$

$2x^2 + 3x = 1$

$2x^2 + 3x - 1 = 0$

Therefore, $a = 2$, $b = 3$, and $c = -1$

Factorising And Finding The Roots Of A Quadratic Equation Using The Method Of Grouping

The roots of the quadratic equation, $ax^2 + bx + c = 0$, are the values of x that satisfy

$ax^2 + bx + c = 0$

Steps for Solving Quadratic Equations ($ax^2 + bx + c = 0$) **by Grouping:**

1. Identify a, b, and c
2. Calculate the product of a and c, i.e., $a \times c = ac$
3. Find two numbers whose product is equal to ***ac*** and whose sum is equal to ***b***.
4. Rewrite the middle term by replacing the term ***bx*** with the two numbers you found in step 3.
5. Group the first two terms and the last two terms, then factor out common factors from each group.
6. Factor out the common binomial factor to obtain the factored form of the quadratic equation.
7. Set each factor equal to zero and solve for ***x*** to find the solutions of the quadratic equation.

Let us use the steps to answer a few questions:

Example 1

Solve $2x^2 + 3x - 5 = 0$

Solution

Step 1

Comparing $2x^2 + 3x - 5 = 0$ to $ax^2 + bx + c = 0$, $a = 2$, $b = 5$, and $c = -5$

Step 2

$a \times c = 2 \times -5 = -10$

Step 3

This means we need two numbers whose product is -10. The negative value tells us that one of the numbers must be negative and the other must be positive. We can consider the factors of -10 to help with this:

- The positive factors of are 1, 2, 5, 10
- The negative factors of are -1, -2, -5, -10
- Therefore, the possible combinations are (1, -10), (2, -5), (5, -2) and (10, -1)
- We also need our combination to have a sum equal to b, in this case $b = 3$.
- So, which of these combinations has a sum of 3?

  (1, -10) $\rightarrow$ $1 + (-10) = -9 \neq 3$

  (2, -5) $\rightarrow$ $2 + (-5) = -3 \neq 3$

  (5, -2) $\rightarrow$ $5 + (-2) = 3 = 3$

  Therefore, the desired values are and since their product will give us and their sum is 3.

Step 4

$3x = 5x - 2x$ or $3x = -2x + 5x$

Rewrite $2x^2 + 3x - 5 = 0$ as either $2x^2 + 5x - 2x - 5 = 0$ or $2x^2 - 2x + 5x - 5 = 0$

Step 5

$2x^2 + 5x - 2x - 5 = 0$ or $2x^2 - 2x + 5x - 5 = 0$

$(2x^2 + 5x) - (2x + 5x) = 0$ or $(2x^2 - 2x) + (5x - 5) = 0$

Step 6

$(2x^2 + 5x) - (2x + 5) = 0$ or $(2x^2 + 5x) - (2x + 5) = 0$

$x(2x + 5) - 1(2x + 5) = 0$ or $2x(x - 1) + 5(x - 1) = 0$

$(2x + 5)(x - 1) = 0$ or $(x - 1)(2x + 5) = 0$

Step 7

$+ 5 = 0 \Rightarrow x = -\frac{5}{2} = -2\frac{1}{2}$ or $x - 1 = 0 \Rightarrow x = 1$

Example 2

Find the root of $4x^2 - 9 = 0$

Note, the root is the solution of the equation. The roots or solutions of a quadratic equation are the values of that satisfy the equation. Graphically, they correspond to the x-coordinates of the points where the graph intersects or touches the x-axis.

Solution

$4x^2 - 9 = 0$ can be rewritten as $4x^2 + 0x - 9 = 0$

This means that $a = 4,\ b = 0$ and $c = -9$

So, $ac = 4 \times -9 = -36$

We have $6 \times -6 = -36$ and $6 + -6 = 0$

$4x^2 + 6x - 6x - 9 = 0$

$2x(2x + 3) - 3(2x + 3) = 0$

$(2x - 3)(2x + 3) = 0$

$2x - 3 = 0 \Rightarrow x = \frac{3}{2} = 1\frac{1}{2}$

or $2x + 3 = 0 \Rightarrow x = -\frac{3}{2} = -1\frac{1}{2}$

Example 3

Solve $3x^2 + 8x = 0$

Solution

$3x^2 + 8x = 0$ can be rewritten as $3x^2 + 8x + 0 = 0$

This means $a = 3$, $b = 8$, and $c = 0$

Since there are only two terms, we can factorise by taking the 'x out'.

$3x^2 + 8x = 0$

$x(3x + 8) = 0$

so, either $x = 0$ or $3x + 8 = 0 \Rightarrow x = -\frac{8}{3} = -2\frac{2}{3}$

Solving a quadratic equation using the quadratic formula

We can also use the quadratic formula to solve quadratic equations.

This is a very powerful formula to help us solve quadratic equations. The derivation of the formula is not required to be learnt, but it is good to see where it comes from. Learners are expected to know the quadratic formula.

The general quadratic equation is

$ax^2 + bx + c = 0$

$ax^2 + bx = -c$

Divide by a

$$\frac{a}{a}x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Divide the coefficient of x by 2

$$\frac{b}{a} \div 2 = \frac{b}{2a}$$

Note that $\frac{b}{a} = \frac{b}{2a} + \frac{b}{2a}$

Square $\frac{b}{2a}$ and add it to either side of $x^2 + \frac{b}{a}x = -\frac{c}{a}$

$$\left(\frac{b}{2a}\right)^2 = \frac{b}{2a} \times \frac{b}{2a} = \frac{b^2}{4a^2}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$

Recall that $\frac{b}{a} = \frac{b}{2a} + \frac{b}{2a}$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$x^2 + \frac{b}{2a}x + \frac{b}{2a}x + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}$$

Complete the square

$$x\left(x + \frac{b}{2a}\right) + \frac{b}{2a}\left(x + \frac{b}{2a}\right) = \frac{b^2 - 4ac}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)\left(x + \frac{b}{2a}\right) = \frac{b^2 - 4ac}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\sqrt{\left(x+\frac{b}{2a}\right)^2} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x+\frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{\sqrt{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$x = \frac{-b+\sqrt{b^2-4ac}}{2a}$ or $x = \frac{-b-\sqrt{b^2-4ac}}{2a}$

The roots/solutions are $\frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\frac{-b-\sqrt{b^2-4ac}}{2a}$

Let us use the formula to solve a few quadratic equations

Example 1

Solve for x in the following equation.

$$2x^2 - x - 1 = 0$$

Solution

The quadratic formula is

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Comparing $2x^2 - x - 1 = 0$ to the general equation, $ax^2 + bx + c = 0$

We have $a = 2, b = -1$ and $c = -1$

Substitute 2 for ***a***, -1 for ***b***, and -1 for **c** into the quadratic formula and simplify.

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{1 \pm \sqrt{9}}{4}$$

$$x = \frac{1 \pm 3}{4}$$

$$x = \frac{1+3}{4} = 1$$

$$x = \frac{1-3}{4} = -\frac{1}{2}$$

Example 2

Solve for x in the following equation.

$$x^2 + 4x - 21 = 0$$

Solution

$a = 1, b = 4, c = -21$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-21)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 + 84}}{2}$$

$$x = \frac{-4 \pm \sqrt{100}}{2} = \frac{-4 \pm 10}{2}$$

Either $x = \frac{-4-10}{2} = -\frac{14}{2} = -7$ or $x = \frac{-4+10}{2} = \frac{6}{2} = 3$

FOCAL AREA 2: REAL-LIFE APPLICATIONS OF QUADRATIC EQUATIONS

The quadratic equation has many useful real-world applications. The following are just a few of the many applications of quadratic equations.

Example 1

A rectangular plate has an area of $6m^2$ and a perimeter of 11 metres.

Find the dimensions of the rectangular plate.

Solution

Let L and W respectively represent the length and width of the rectangle.

Area $= L \times W \quad \Rightarrow \quad L \times W = 6$ ……..……(1)

Perimeter $= 2L + 2W \quad \Rightarrow \quad 2L + 2W = 11$ ……...(2)

From (1), $W = \frac{6}{L}$

Substitute, $W = \frac{6}{L}$ into (2)

$2L + 2\left(\frac{6}{L}\right) = 11$

$2L + 2\left(\frac{6}{L}\right) = 11$

$2L + \frac{12}{L} = 11$

Multiply through by L

$2L^2 + 12 = 11L$

$2L^2 - 11L + 12 = 0$

Solve the quadratic equation, either by factorising or using the quadratic formula:

$2L^2 - 3L - 8L + 12 = 0$

$L(2L - 3) - 4(2L - 3) = 0$

$(2L - 3)(L - 4) = 0$

$2L - 3 = 0 \Longrightarrow L = \frac{3}{2} = 1\frac{1}{2}m$

Or $L - 4 = 0 \Longrightarrow L = 4m$

If $L = \frac{3}{2}$, $W = \frac{6}{3/2} = 6 \times \frac{2}{3} = 4m$

If $L = 4$, $W = \frac{6}{4} = \frac{3}{2}m$

Therefore, the rectangle plate is $1\frac{1}{2}m$ by $4m$

Example 2

The product of two consecutive odd numbers is 575. Find the numbers.

Solution

Let the first odd number be x

Therefore, the second odd number is $x + 2$

The product is 575, which implies that

$$x(x + 2) = 575$$

$$x^2 + 2x = 575$$

$$x^2 + 2x = 575$$

$$x^2 + 2x - 575 = 0$$

$$x^2 + 25x - 23x - 575 = 0$$

$$x(x + 25) - 23(x + 25) = 0$$

$$(x + 25)(x - 23) = 0$$

$x + 25 = 0 \Longrightarrow x = -25$ and or

$x - 23 = 0 \Longrightarrow x = 23$

If $x = -25, x + 2 = -25 + 2 = -23$

If $x = 23, x + 2 = 23 + 2 = 25$

So, the numbers are either –23 and –25 or 23 and 25

Example 3

Ama took part in a 40km walk. She walked for 16km at an average speed of km/h. She walked the rest of the race at an average speed of 2km/h, less than the average speed of her first 16km. Given that her total time for the walk was 6 hours, determine the value of x.

Solution

We know that speed $= \frac{dis\tan ce}{time}$

This implies that time $= \frac{dis\tan ce}{speed}$

Let T_1 = Time for the first part of the journey and T_2 = Time for the second part of the journey.

In the first part of the journey, the distance = 16 and the speed is x

This means, $T_1 = \frac{16}{x}$

In the second part of the journey, the distance $40 - 16 = 24$, and speed is $x - 2$, as this speed is 2 less than that of the first journey.

This means, $T_2 = \frac{24}{x-2}$

The total time taken to cover the 40-kilometre journey is 6 hours.

So, $T_1 + T_2 = 6$

This means that, $\frac{16}{x} + \frac{24}{x-2}$

Multiply through by $x(x - 2)$

$$x(x-2)\frac{16}{x} + x(x-2)\frac{24}{x-2} = 6x(x-2)$$

$$\cancel{x}(x-2)\frac{16}{\cancel{x}} + x\cancel{(x-2)}\frac{24}{\cancel{x-2}} = 6x(x-2)$$

$$(x-2)\times 16 + 24x = 6x^2 - 12x$$

$$16x - 32 + 24x = 6x^2 - 12x$$

Write the equation in the general form

$$0 = 6x^2 - 12x - 16x - 24x + 32$$

$$0 = 6x^2 - 52x + 32$$

$$6x^2 - 52x + 32 = 0$$

We can divide by 2 here to make our factorisation easier:

$$3x^2 - 26x + 16 = 0$$

Now solve it, either by factorisation or using the quadratic formula:

$$3x^2 - 24x - 2x + 16 = 0$$

$$3x(x-8) - 2(x-8) = 0$$

$$(3x-2)(x-8) = 0$$

$3x - 2 = 0 \Longrightarrow x = \frac{2}{3}$ or $x - 8 = 0 \Longrightarrow x = 8$

When $x = 8,\ T_1 = \frac{16}{8} = 2$ and $T_2 = 4$

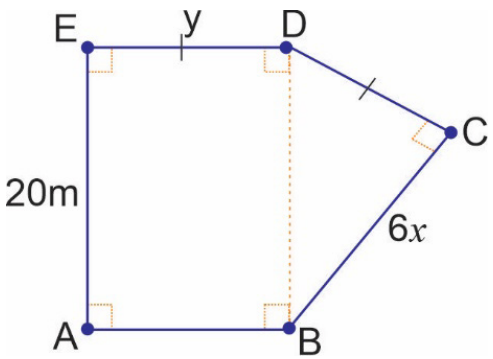
When $x = \frac{2}{3},\ T_1 = \frac{16}{2/3} = 16 \div \frac{2}{3} = 16 \times \frac{3}{2} = 8 \times 3 = 24$ and $T_2 = 6 - 24 = -18$

Since time cannot be negative, we reject $x = \frac{2}{3}$ and conclude that $x = 8$

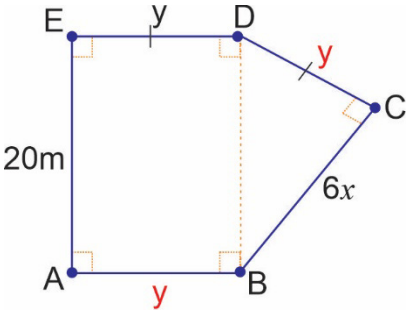
Example 4

The figure below shows a floor space in the form of a pentagon ABCDE whose measurements, in metres(m), are given in terms of x and y. |DE| = |CD| = y, |BC| = 6*x*, and |AE| = 20m. The total area and perimeter of the floor space are 336 and 72m respectively.

Find the value of and .



Solution



Perimeter of ABCDE = $20 + y + y + 6x + y = 20 + 3y + 6x$

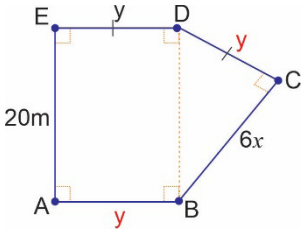
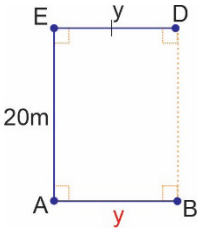
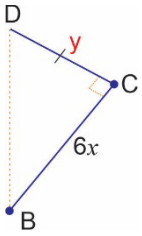
From the question, perimeter of ABCDE = 72

This implies that = $20 + 3y + 6x = 72$

Make y the subject:

$$3y = 52 - 6x$$

$$y = \frac{52 - 6x}{3}$$

Area of  = Area of  + Area of 

Area of ABDE = $20 \times y = 20y$

But $y = \frac{52-6x}{3}$, area of ABDE = $20 \times \left(\frac{52-6x}{3}\right) = \frac{1040-120x}{3}$

Area of BCD = $\frac{1}{2} \times 6x \times y = 3xy$

Therefore, area of BCD = $3x\left(\frac{52-6x}{3}\right) = 52x - 6x^2$

Therefore, area of ABCDE = $20y + 3xy$

Area of ABCDE = $\frac{1040-120x}{3} + 52x - 6x^2$

From the question, area of ABCDE = 336

$$\frac{1040 - 120x}{3} + 52x - 6x^2 = 336$$

Multiply through by 3 to remove it from the denominator:

$$3 \times \frac{(1040 - 120x)}{3} + 3 \times 52x - 3 \times 6x^2 = 3 \times 336$$

$$1040 - 120x + 156x - 18x^2 = 1008$$

$$-18x^2 + 36x + 32 = 0$$

Divide through by –2 to simplify the equation and then solve:

$9x^2 - 18x - 16 = 0$

$9x^2 + 6x - 24x - 16 = 0$

$3x(3x + 2) - 8(3x + 2) = 0$

$(3x - 8)(3x + 2) = 0$

This means that $3x - 8 = 0 \Longrightarrow x = \frac{8}{3}$ or $3x + 2 = 0 \Longrightarrow x = -\frac{2}{3}$

Since the side of the floor cannot be negative, we ignore $x = -\frac{2}{3}$ and conclude that:

$$x = \frac{8}{3} = 2\frac{2}{3}m$$

Substitute this value of x, into our equation to find y:

$$y = \frac{52 - 6 \times \frac{8}{3}}{3} = \frac{52 - 16}{3} = \frac{36}{3} = 12$$

Therefore, $x = 2\frac{2}{3}m$ and $y = 12m$

Learning Tasks

Guide learners to solve tasks related to understanding quadratic equations, quadratic graphs and functions. Provide support systems for learners who may encounter difficulties.

Learners are to:

1. Identify and provide reasonable justification to show whether a given equation is quadratic.
2. Factorise and find the roots of a quadratic equation
3. Apply the quadratic equation to answer real-life problems.

PEDAGOGICAL EXEMPLARS

The aim of the lessons for the week is for all learners to be able solve problems involving quadratic equations. The following pedagogical approach is suggested for facilitators to take learners through.

Group Work/Collaborative Learning

Using this strategy:

1. Review learners' knowledge on quadratic expressions, deal with their misconceptions about such concepts, and extend the ideas to quadratic equations and functions using GeoGebra.
2. Review learners' knowledge on how to solve quadratic expressions using the factorisation method.
3. Engage learners to explore why a≠0 in a quadratic equation.

KEY ASSESSMENT: MID-SEMESTER EXAMINATION

Hint



*Conduct **Mid-semester Examination**. It should cover contents taught from Week 1 to 6. Refer to the **Table of Specification and Structure of the Paper**. Learners' papers should be marked immediately and scores entered into the SAP.*

Structure

Section A: *25 Multiple-choice questions*

Section B: *2 Essay Questions*

Table of specification for item construction for section A and B

Week	Learning Indicator	Types of Questions	DoK Levels				Total
			1	2	3	4	
1	Investigate to establish differences between variables in situational growth (symbolic logic) and apply it to real world situations	MCQs	2	2	–		4
		ESSAY	–	–	–		–
2	Make intelligent guesses to establish valid arguments and draw logical conclusions	MCQs	2	2	2		6
		ESSAY	–	–	1		1
3	Carry out mini real-life investigations to solve problems involving logical reasoning, within their local community	MCQs	–	2	1		3
		ESSAY	–	–	–		
4	Use proportional reasoning to investigate the various types of variation (direct and inverse) and extend this to make generalisations	MCQs	3	2	1		6
		ESSAY					
5	Use proportional reasoning to investigate partial variations, make generalisations and solve real-life issues involving partial variations	MCQs	1	3	2		6
		ESSAY			1		1
		Total	8	11	8		27

Sample Mid-Semester Questions

Part 1 – Objective Test

Each question is followed by four options, lettered A to D. Choose the correct option.

1. Which of the following statements is valid?

a. All frogs are amphibians. All frogs have gills. Therefore, all amphibians have gills.

b. All racers live dangerously. Mills is a racer. Therefore, Mills lives dangerously.

c. If I am literate, then I can read and write. I can read, but I can't write. Thus, I am not literate.

d. If an animal is cute, then it isn't a squid. This animal isn't a squid. Therefore, this animal is cute.

2. Which of the following statements is NOT valid?

a. If I eat peanuts, then I get rashes. I didn't get rashes. Therefore, I didn't eat peanuts.

b. If I am literate, then I can read and write. I can read, but I can't write. Thus, I am not literate.

c. All reptiles are poisonous. All snakes are reptiles. Therefore, all snakes are poisonous.

d. All boys like football. Dela likes football. Hence, Dela is a boy.

3. Which of the following sentences is NOT a statement?

a. It is time for the elective mathematics lesson.

b. She is an intelligent student.

c. Tidy up the classroom.

d. The elective mathematics lesson has ended.

4. Consider the following statements.

P: Kofi is rich

Q: Ama is intelligent

Write a statement to represent PQ

a. Kofi is not rich, and Ama is not intelligent.

b. Kofi is not rich or Ama is intelligent.

c. Kofi is not rich and Ama is intelligent.

d. Kofi is not rich, or Ama is not intelligent.

5. Consider the following statements

X: All quadrilaterals are polygons

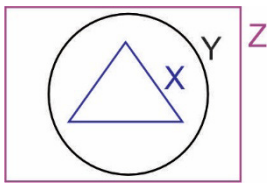
Y: All polygons are 2-D shapes

Z: All quadrilaterals are 2-D shapes.

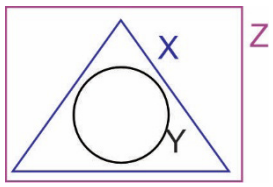
Assuming that X and Y are true, and Z is a valid conclusion from X and Y.

Which of the following Venn Diagrams best describes the three statements?

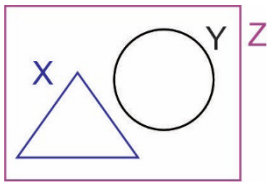
A



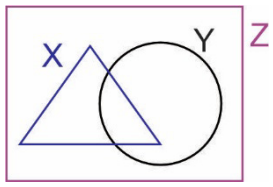
B



C



D



6. Consider the following statements

M: Barbara is hardworking

N: Barbara is rich

If MN, which of the following is a valid conclusion from the statements?

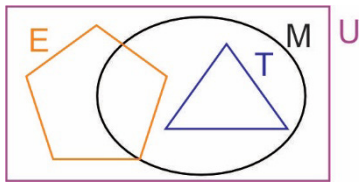
a. If Barbara is rich, then she is hardworking.

b. If Barbara is not hardworking, then she will not be rich.

c. If Barbara is not rich, then she is not hardworking.

d. If Barbara is not hardworking, then she will be rich.

7.



In the Venn diagram, U = workers at a construction company. Ttilers. M = masons and E = electricians.

Which of the following statements is false?

a. No tiler is an electrician

b. All masons are tilers

c. Some electricians are masons

d. There is no tiler who is not a mason

8. If $-\frac{1}{2}$ and 1 are the roots of the quadratic equation , find

9.

a. -0.5

b. 0

c. 0.5

d. 2

10. Consider the following statements.

X: Soldiers are strong

Y: Soldiers are law-abiding

Write in symbolic form the statement: "Soldiers are strong but NOT law-abiding"

a. $X \wedge \sim Y$

b. $X \vee \sim Y$

c. $X \wedge Y$

d. $X \vee Y$

11. Consider the following statements.

P: All teachers are intelligent.

Q: Most teachers are graduates.

Which of the following statements is a valid deduction from P and Q?

a. Kofi is intelligent Kofi is a teacher.

b. Kofi is not intelligent Kofi is not a teacher.

c. Kofi is not a graduate Kofi is not a teacher.

d. Kofi is a graduate Kofi is intelligent.

Part 2

1. Ama took part in a 30km walk. She walked for 10km at an average speed of km/h. She walked the rest of the race at an average speed of 1km/h less than the average speed of her first 10km. Given that her total time for the walk was 7 hours, determine the value of x.

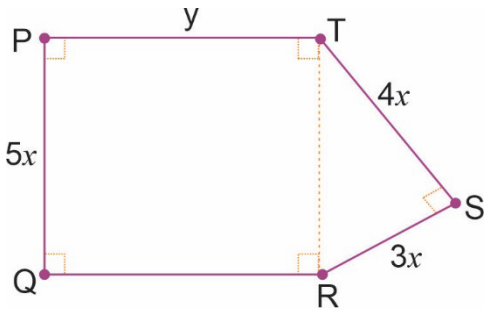
10 marks

2. The figure below shows a floor space in the form of a pentagon PQRST whose measurements, in metres (m), are given in terms of x and y. |PQ|, |TS| = $4x$, |SR| = $3x$,

3. TSR and |PT| = y.

The total area and perimeter of the floor space are 4 and 8m.

Find the values of x and y.

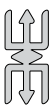


10 marks

PLC Session 6

45 minutes | **Planning & Reviewing the Learning Plan**

1. Begin the session by recapping lessons from the previous session, focusing on how learning was applied.
2. Use your subject specific app to explore the content before planning. Ask the app to
   a. explain the week's concepts in a practical way that will make it easier for learners to understand.
   b. identify three common misconceptions learners may have on the week's content and how to address them using contexts familiar to Ghanaian learners (NTS 3a, 3g and 3m).
3. Use your subject specific app to plan your lesson. Ask the app to
   a. suggest the most effective method for teaching the week's concepts to learners with diverse backgrounds.
   b. suggest learning activities, differentiation and essential questions based on the method of teaching (NTS 3a-3q).
   c. review your prepared learning plan and address any challenging issues you may have identified in your subject area (NTS 3a-3c).



Note

- Take note of the recap topics linked to Years 1 and 2 while preparing for your lesson.
- Refer to the Values Learning Community Handbook and link the values for the week in your lesson to ensure that learners are applying what they learn.

4. Use your app to plan and administer the mode of assessment for the Student Assessment Portal (SAP).
   a. The key assessment for the week is ***mid-semester examination***. Chat with your app to plan this assessment by reviewing or developing the assessment items/tasks and marking schemes/rubrics for the mid-semester examination (NTS 3k-3q).

10 minutes | **Content Exploration**

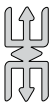
5. As a subject group, select one problem or topic/idea/term in the week's content that is challenging (not a routine procedural question, but one that requires conceptual understanding) and then

a. use the app, ask it to solve the problem step by step and explain the reasoning behind each step (you may be assisted by a teacher who is strong in the content area)

b. all teachers in the group work through the problem individually for 3–4 minutes

c. the group then works through it together the problem, step by step

d. highlight the most common errors made at each step (NTS 1c, 1e, 2c, 2e, 3e-3j).



Note

- This part is not 'How do I teach this?' but how do I teach it for learners to understand?
- If the group still shares the same content gap, use the app: ask it to solve the problem step by step and explain the reasoning behind each step.
- Problems/ideas discussed should be sourced from the Teacher Manual content, not invented separately. This keeps the content exploration directly connected to what teachers will deliver.

30 minutes Enactment

6. Enact a teaching activity based on your learning plan (NTS la-1b, 1e-lg, 2c, 2e, 2f, 3c-3j and 3o).

a. Colleagues observe with specific attention to clarity of concept explanation, use of familiar Ghanaian examples and how misconceptions are handled if they arise.

b. After the enactment, ask for structured feedback from the group

i. was the concept explained in a way that a learner with no prior knowledge could follow?

ii. were the examples grounded in contexts familiar to Ghanaian learners?

iii. did any misconceptions surface and if so, how were they addressed?

5 minutes Reflection

7. Reflect and share your views on the session. One thing that worked well, what did not work well and what you will do differently. (NTS 1a, 1b).

8. Identify a critical friend to observe and provide feedback on your lesson in relation to the PLC Session conducted (NTS 1f, 1g).

9. Remember to

a. transfer your chats into your learning planner template.

b. read your Teacher Manual and related Learner Material in preparation for the next session. Take note of the recap weeks, which are linked to Years 1 and 2 (NTS 3a).

c. integrate National Values, GESI, SEL, and 21st century skills into your learning activities, assessment tasks, and rubrics/mark schemes.

WEEK 7

Learning Indicator: *Solve quadratic equations graphically and find the maximum and minimum points of quadratic graphs*

FOCAL AREA 1: SOLVING QUADRATIC EQUATIONS USING GRAPHS

1. *Graphing and solving quadratic equations*
2. *Maximum and minimum*
3. *Model and solve real-life situations involving quadratic functions*

Solving A Quadratic Equation Using The Graph

We will learn how to graph quadratic functions for a given range of independent values. We will use the graph to find the roots (the x-coordinates of the points where the graph intersects or touches the x-axis), maximum or minimum values and points.

Finally, we will look at how to determine the solutions of a quadratic equation when provided with a pre-drawn graph. Using graphs to solve quadratic equations offers a valuable approach for obtaining approximate solutions or roots.

How to draw the quadratic graph given the domain

Steps

1. **Calculate corresponding y-values:** Substitute each domain value (usually x-values) into the equation to find the corresponding y-value.
2. **Plot the points:** Plot all points obtained in step 1
3. **Sketch the curve:** Draw a smooth, U-shaped (or inverted U-shaped) curve through the plotted points within the specified domain. The curve should be symmetrical about the axis of symmetry. The curves must not be drawn with a ruler, but your axes should be!

Example 1

a. Copy and complete the table of values below for the relation $y = x^2 - 4x - 21$

b. for $-4 \leq x \leq 8$

x	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8
y	11	0			-21	-24			-21	-16	-9		

Using a scale of 2 cm to 2 units on the *x-axis* and 2 cm to 5 units on the y-axis. Draw the graph of the relation.

Solution

We complete the table

When $x = -2$, $y = (-2)^2 - 4(-2) - 21 \Rightarrow y = -9$

When $x = -1$, $y = (-1)^2 - 4(-1) - 21 \Rightarrow y = -16$

When $x = 2$, $y = (2)^2 - 4(2) - 21 \Rightarrow y = -25$

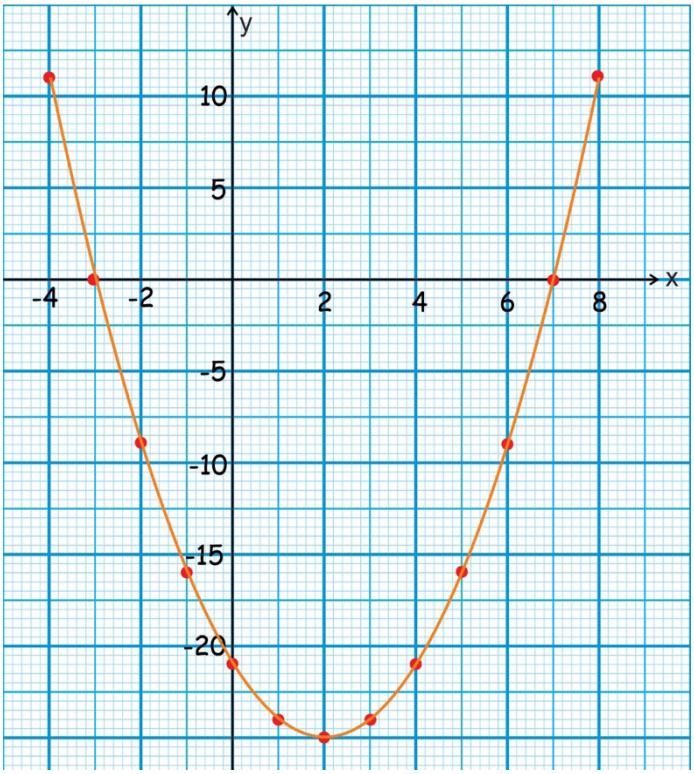
When $x = 3$, $y = (3)^2 - 4(3) - 21 \Rightarrow y = -24$

When $x = 7$, $y = (7)^2 - 4(7) - 21 \Rightarrow y = 0$

When $x = 8$, $y = (8)^2 - 4(8) - 21 \Rightarrow y = 11$

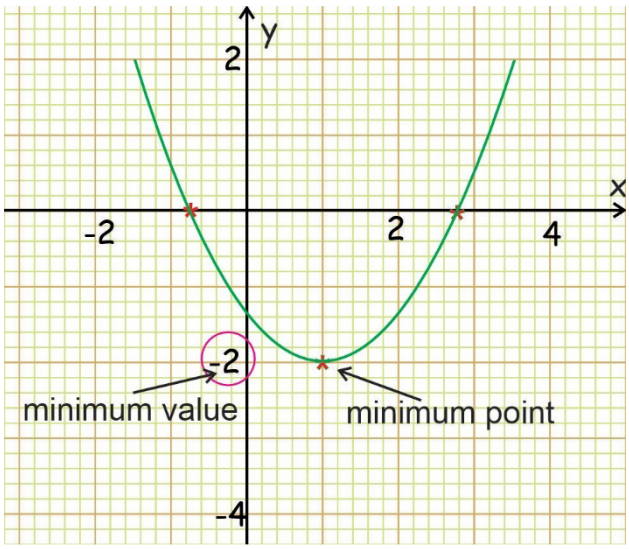
x	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8
y	11	0	-9	-16	-21	-24	-25	-24	-21	-16	-9	0	11

c.



FOCAL AREA 2: FINDING THE MINIMUM AND MAXIMUM VALUE OF A QUADRATIC GRAPH

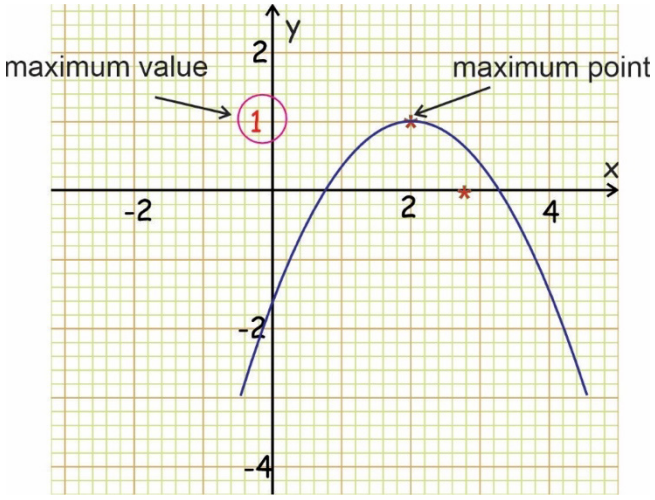
The quadratic function is given by $y = ax^2 + bx + c$, where a, b, and c are constants with $a \neq 0$. The point where the graph 'turns' is called the **vertex**. If the graph opens upwards, the y-coordinate of the vertex is a minimum (concave up) as demonstrated on the graph below:



When the graph has a minimum value then the coefficient of 'x^2' in the quadratic function $y = ax^2 + bx + c$, is positive (i.e. a > 0).

For example, the function $y = 7x^2 + 6x - 5$ has a minimum value because the coefficient of is positive 7.

On the other hand, if the quadratic graph open downwards, the vertex is a maximum (concave down), as illustrated on the graph below:



When the graph has a maximum value, then the coefficient of 'x^2' in the quadratic function $y = ax^2 + bx + c,$ is negative (i.e., a < 0). For example, the function $y = 6 + 3x - 2x^2$ has a maximum value because the coefficient of x^2 is -2.

Solving or the roots using the graph

The roots of the quadratic equation are the values of x at which the graph intersects/crosses the x-axis. If the graph does not intersect the x-axis, it means the roots are imaginary/they are not real numbers.

Example 2

a. Copy and complete the following table for the relation y = $x^2 - 3x - 4$ for $-2 \leq x \leq 5$.

x	-2	-1	0	1	2	3	4	5
y				-6			0	

b. Using a scale of 2 cm to 1 unit on the x axis and 2 cm to 2 units on the y axis, draw the graph of $y = x^2 - 3x - 4$ for $-2 \leq x \leq 5$.

c. From the graph, find;

i. The minimum value of $y = x^2 - 3x - 4$.

ii. The minimum point of $y = x^2 - 3x - 4$

iii. Values of x for which $y = 0$

Solution

a. We complete the above table

When $x = -2$, $y = (-2)^2 - 3(-2) - 4 \Rightarrow y = 6$

When $x = -1$, $y = (-1)^2 - 3(-1) - 4 \Rightarrow y = 0$

When $x = 0$, $y = (0)^2 - 3(0) - 4 \Rightarrow y = -4$

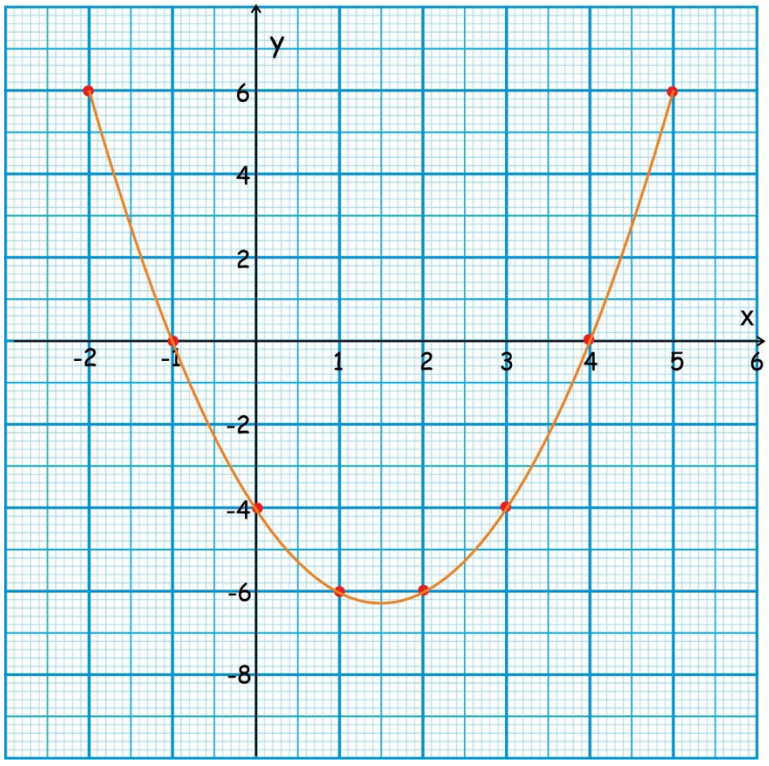
When $x = 2$, $y = (2)^2 - 3(2) - 4 \Rightarrow y = -6$

When $x = 3$, $y = (3)^2 - 3(3) - 4 \Rightarrow y = -4$

When $x = 5$, $y = (5)^2 - 3(5) - 4 \Rightarrow y = 6$

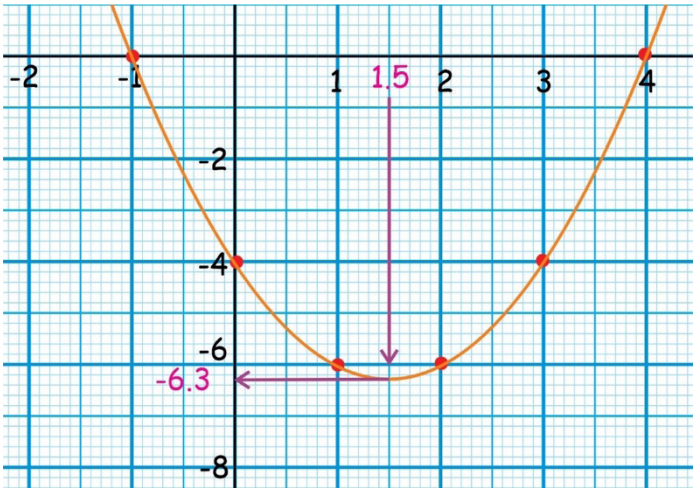
x	-2	-1	0	1	2	3	4	5
y	6.0	0.0	-4.0	-6.0	-6.0	-4.0	0.0	6.0

b.



Note, it is important to see that we do not draw a 'flat bottom' on our graph, but know that the graph goes down in a smooth curve to a minimum value halfway between $x = 1$ and $x = 2$. We know it is half way between because of the line of symmetry, but we will look at that in week 8.

c.



i. From the graph, the minimum point of y is (1.5, −6.3)

Although the accuracy of this depends on how well drawn the graph is. Because of the line of symmetry, we know the x value is definitely 1.5, but to confirm the y value we could input −1.5 into the equation and get the value −6.25.

ii. From the graph the minimum value of y = $x^2 - 3x - 4$ is y = −6.3.

iii. The graph intersects the x-axis at x = −1 and x = 4. Therefore, when y = 0, x is −1 or 4.

iv. These are known as the roots.

Example 5

a. Copy and complete the following table of values for the relation $y = 2 + x - x^2$, for $-2 \leq x \leq 3$

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5	3
y			0	1.25	2	2.25	2	1.25			

b. Draw the graph of the relation, using a scale of 2 cm to 1 unit on both axes.

c. From your graph find:

i. the greatest value of $y = 2 + x - x^2$ and the value of x at which this occurs.

ii. the roots $y = 2 + x - x^2$

Solution

a. We complete the table

When $x = -2$, $y = 2 + (-2) - (-2)^2 \Rightarrow y = -4$

When $x = 1.5$, $y = 2 + (-1.5) - (-1.5)^2 \Rightarrow y = -1.75$

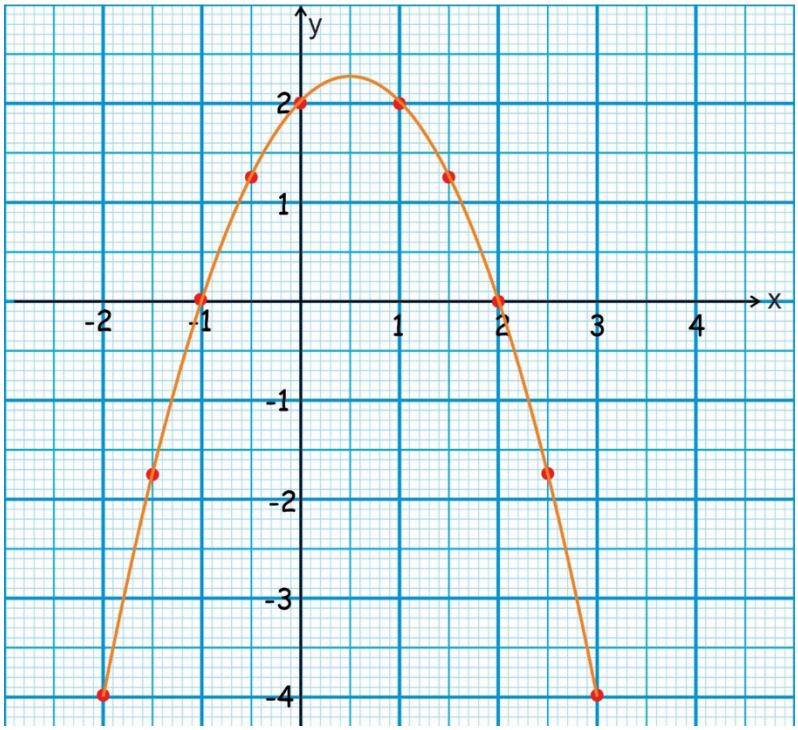
When $x = 2$, $y = 2 + (2) - (2)^2 \Rightarrow y = 0$

When $x = 2.5$, $y = 2 + (2.5) - (2.5)^2 \Rightarrow y = -1.75$

When $x = 3$, $y = 2 + (3) - (3)^2 \Rightarrow y = -4$

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5	3
y	**-4**	**-1.75**	0	1.25	2	2.25	2	1.25	**0**	**-1.75**	**-4**

b.



c.

i. The greatest value of $y = 2 + x - x^2$ is 2.25 and it occurs when $x = 0.5$

ii. The roots of $y = 2 + x - x^2$ are and 2

Example 6

The following is an incomplete table for the relation $y = (x + 1)(3 - x)$ for $-2 \leq x \leq 4$

x	-2	-1	0	1	2	3	4
y	-5	0	3				

a. Copy and complete the table

b. Using 2 cm as 1 unit on the x-axis and 2cm to 2 units on the y-axis, draw the graph of the relation for the given interval.

c. Use your graph to find:

i. the maximum value

ii. the coordinates of the maximum point

iii. the value of x for which $y = 0$

Solution

a. We complete the table

When $x = 1$, $y = (1 + 1)(3 - 1) \Rightarrow y = 4$

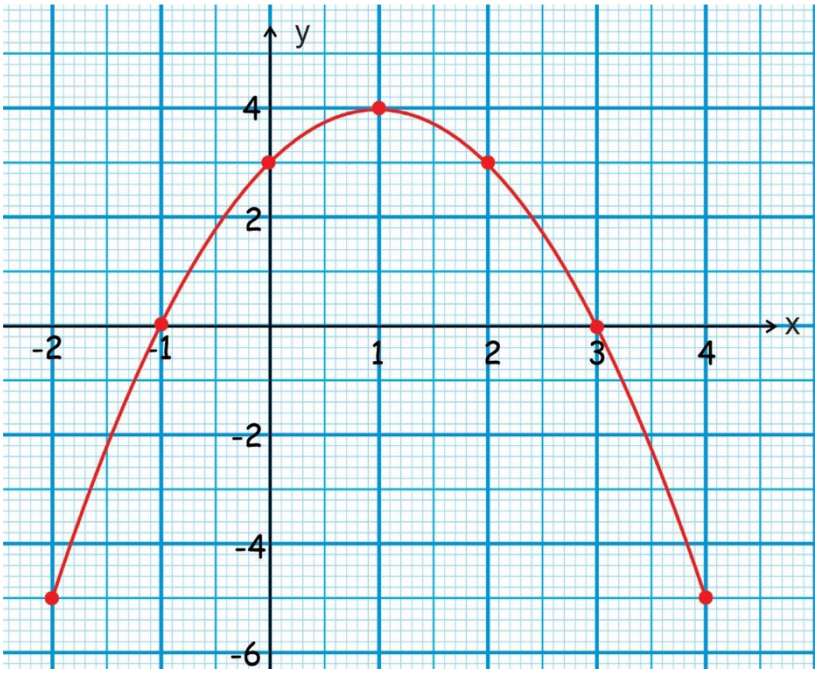
When $x = 2$, $y = (2 + 1)(3 - 2) \Rightarrow y = 3$

When $x = 3$, $y = (3 + 1)(3 - 3) \Rightarrow y = 0$

When $x = 4$, $y = (4 + 1)(3 - 4) \Rightarrow y = -5$

x	-2	-1	0	1	2	3	4
y	-5	0	3	**4**	**3**	**0**	**-5**

b.



c.

i. The maximum value is 4

ii. The coordinate of the maximum point is (1, 4)

iii. This is the point where the graph intersects the x-axis. When y is 0, $x = -1$ or $x = 3$

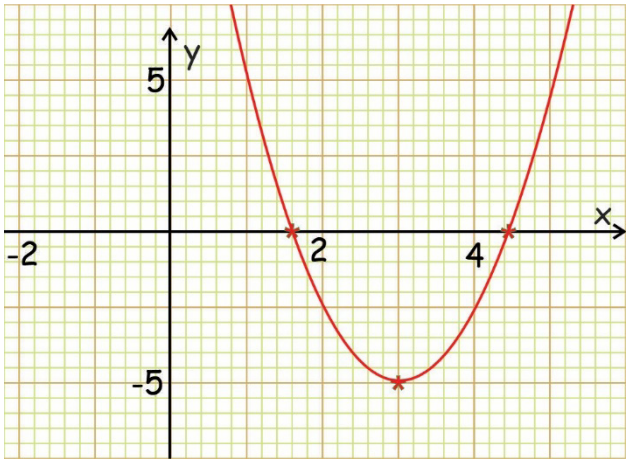
Example 7

The diagram on the next page shows the graph of a quadratic function.

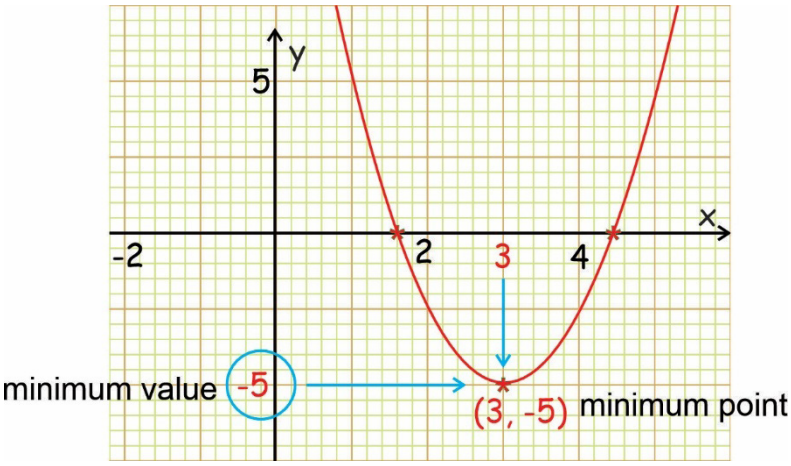
State:

a. minimum value

b. minimum point



Solution



a. The minimum point is (3, –5)

b. The minimum value is the y-coordinate of the minimum point.

c. ∴ minimum value is –5

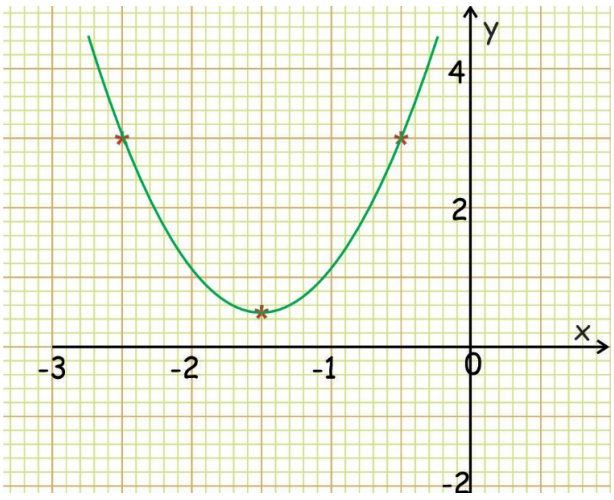
Example 8

The diagram below shows the graph of a quadratic function.

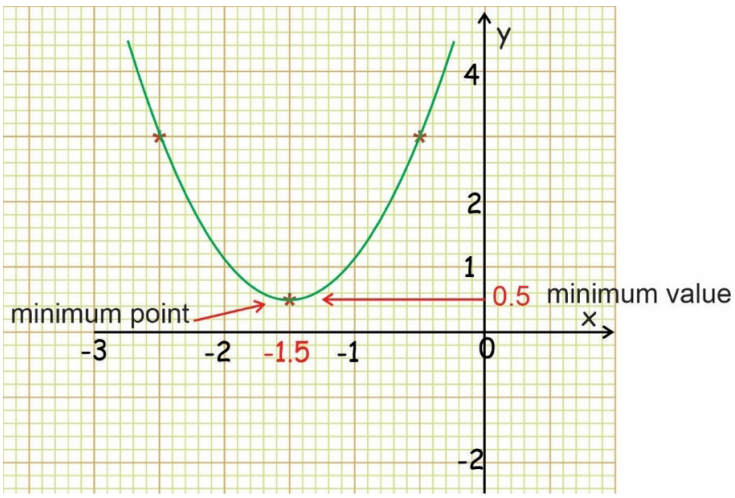
State:

a. minimum value

b. minimum point



Solution



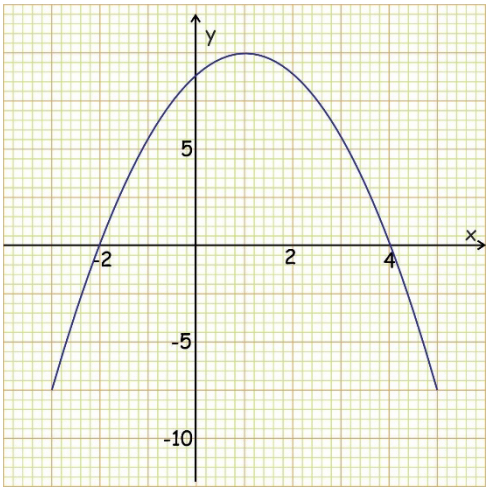
a. The minimum point is (-1.5, 0.5)

b. The minimum value is 0.5

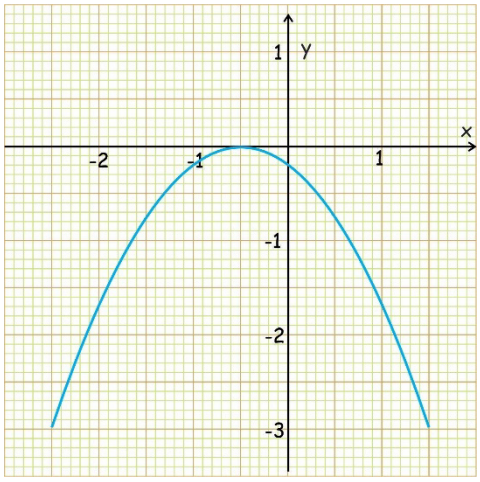
Example 9

Find the maximum values and points of the following graphs.

a.



b.



Solution

a. The maximum value is 10, and the maximum point is (1, 10)

b. The maximum value is 0, and the maximum point is (-0.5, 0)

Real-Life Application of a Quadratic Graph

Example 1

A model rocket is launched from the roof of a building. Its flight path is modelled by Where H = 10t + 30t −5t is the height of the rocket above the ground in metres and t is the time after the launch in seconds.

a. Use the relation to copy and complete the table for a few values of t

t	0	1	2	3	4	5	6	7
H		35				35		

b. Using a scale of 2cm to 1 second on the x-axis and 2cm to 10m on the y-axis, draw a graph for the relation for the interval $0 \leq t \leq 7$

c. Use your graph to find

i. The rocket's maximum height?

ii. The time it took the rocket to reach its maximum height?

iii. The height of the rocket 2.5 seconds after it was launched.

The time it took the rocket to hit the ground.

Solution

a. When t = 0, $H = 10 + 30(0) - 5(0)^2 = 10$

When t = 2, $H = 10 + 30(2) - 5(2)^2 = 50$

When t = 3, $H = 10 + 30(3) - 5(3)^2 = 55$

When t = 4, $H = 10 + 30(4) - 5(4)^2 = 50$

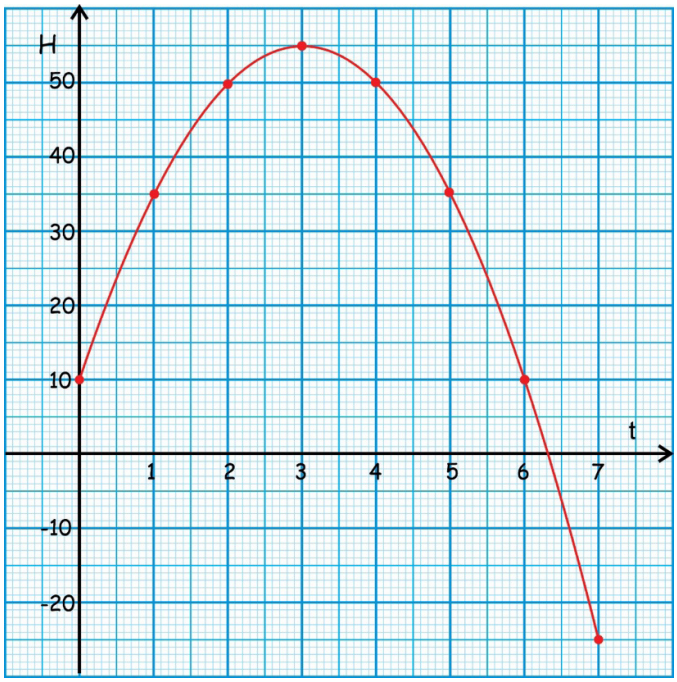
When t = 6, $H = 10 + 30(6) - 5(6)^2 = 10$

When t = 7, $H = 10 + 30(7) - 5(7)^2 = -25$

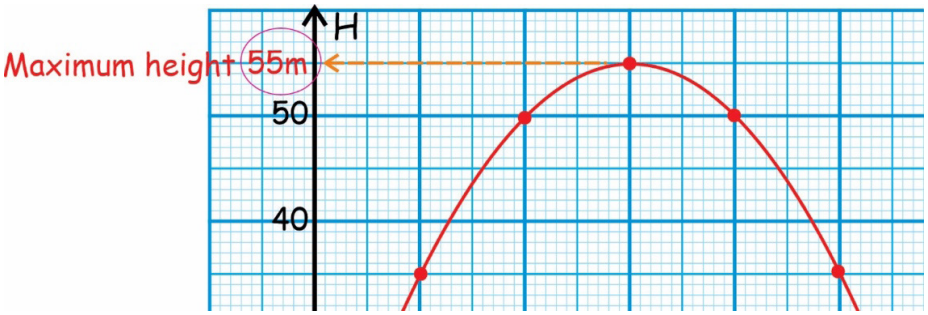
The completed table is shown below

Time (t seconds)	0	1	2	3	4	5	6	7
Height (H metres)	10	35	50	55	50	35	10	-25

b. The figure on the next page shows the graph of the relation $H = 10 + 30t - 5t^2$

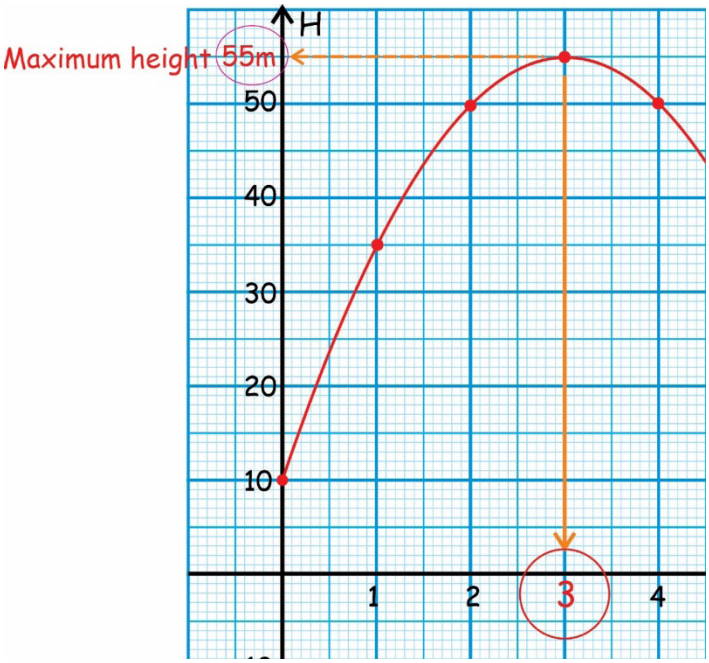


The mathematical model may continue beyond impact, but the physical model is valid only until the rocket reaches ground leve. From the graph, the maximum height will be equal to the value of H at the turning point.



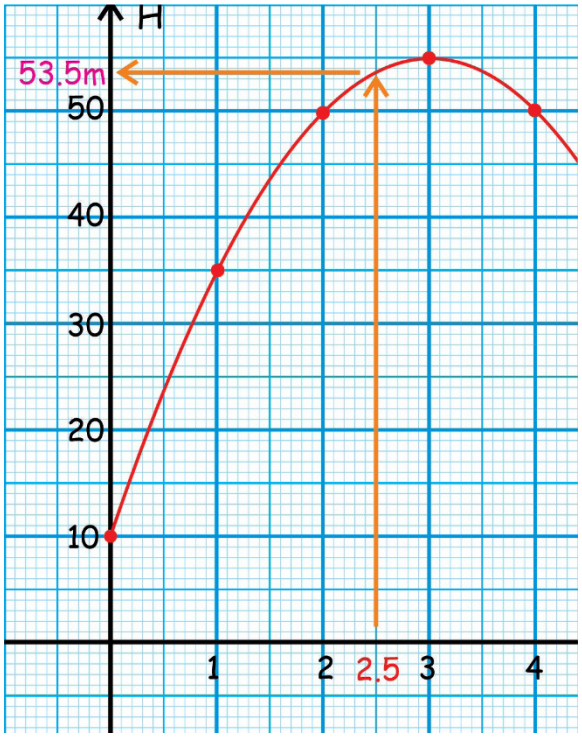
i. From the graph, the maximum height = 55m

ii. This is the time the rocket reached a height of 55m



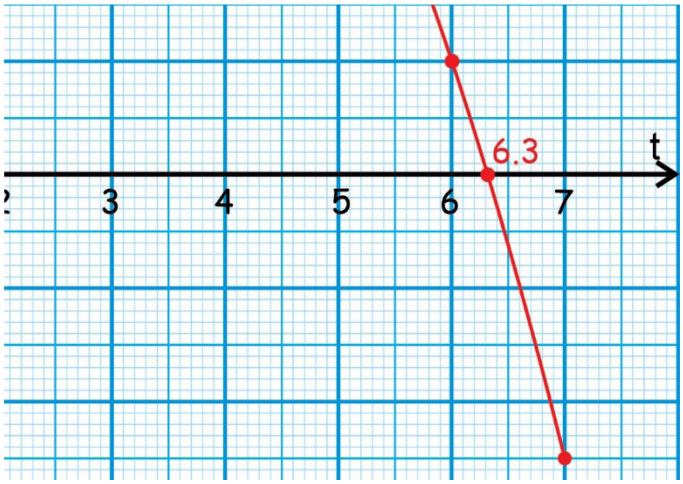
It took the rocket 3 seconds to reach the maximum height.

iii.



The height of the rocket 2.5 seconds after it was launched is about 53.5m

iv. The time it took the rocket to hit the ground. This is the same as calculating the time the rocket reached a height of 0m, as this is ground level, which is the same as the time the graph intersects the x-axis, as this is H = 0.



It took the rocket about 6.3 seconds to hit the ground, after this point, the graph is inaccurate, as the rocket will either stop, or skid along the floor, or burrow into the ground, but at a slower rate because of the increased friction. Also, the model is no longer physically applicable after the rocket reaches the ground.

Learning Tasks

Guide learners to solve tasks related to understanding quadratic equations, quadratic graphs and functions. Provide support systems for learners who may encounter difficulties.

Learners are to:

1. Draw quadratic graphs
2. Use the graphs to find the roots, maximum or minimum values and maximum or minimum points of a quadratic graph.
3. Apply the quadratic graph to answer real-life problems.

PEDAGOGICAL EXEMPLARS

The aim of the lessons for the week is for all learners to be able to solve problems involving quadratic graphs. The following pedagogical approaches are suggested to take learners through.

1. **Talk for learning**: Review, through a whole-class discussion, learners' knowledge on how to draw graphs of a linear equation, deal with their misconceptions about such concepts, and extend the ideas to draw the graph of quadratic functions.
2. **Experiential learning:** In convenient groups, engage learners to draw and identify the properties of a quadratic graph.
3. **Think-pair share:** Using pair activities, solve quadratic problems using the graphical method.

KEY ASSESSMENT LEVEL 3: REFLECTIVE EXERCISE

1.

a. Copy and complete the table of $y = 2x^2 - 4x - 5$ for the range $-2 \leq x \leq 4$.

x	-2	-1	0	1	2	3	4
y	11		-5				11

2 marks

b. Using a scale of 2cm to 1 unit on the x – axis and 2 cm to 2 units on the y – axis, draw a graph of $y = 2x^2 - 4x - 5$.

3 marks

c. Use your graph to find the

i. the minimum value

ii. minimum point

iii. the roots of $2x^2 - 4x - 5$

4 marks

Total *9 marks*

2. A model rocket is launched from a platform. Its flight path is modelled by $H = 12 + 24t - 4t^2$ where H is the height of the rocket above the ground in meters and t is the time after the launch in seconds.

a. Use the relation to copy and complete the table for a few values of

t	0	1	2	3	4	5	6	6.5
H					44			-1

3 marks

b. Using a scale of 2cm to 1 second on the x-axis and 1cm to 5m on the y-axis, draw a graph for the relation for the interval $0 \leq t \leq 6.5$.

3 marks

c. Use your graph to find

i. The rocket's maximum height?

ii. The height of the rocket 5.5 seconds after it was launched.

iii. The time it took the rocket to hit the ground.

4 marks

Total *10 marks*

PLC Session 7

45 minutes **Planning & Reviewing the Learning Plan**

1. Begin the session by recapping lessons from the previous year/session/week, focusing on how learning was applied.

2. Use your subject specific app to explore the content before planning. Ask the app to

a. explain the week's concepts in a practical way that will make it easier for learners to understand

b. identify two or three common misconceptions learners may have on the week's content and how to address them using contexts familiar to Ghanaian learners. (NTS 3a, 3g and 3m).

3. Use your subject-specific app to plan your lesson. Ask the app to

a. suggest the most effective method for teaching the week's concepts to learners with diverse backgrounds.

b. suggest learning activities, differentiation and essential questions based on the method of teaching (NTS 3a-3q)

c. review your prepared learning plan and address any challenging issues you may have identified in your subject area (NTS 3a-3c).

Note

- Take note of the recap topics linked to Years 1 and 2 while preparing for your lesson.
- Refer to the Values Learning Community Handbook and link the values for the week in your lesson to ensure that learners are applying what they learn.

4. Plan the week's key assessment with your app

a. The key assessment for the week is a ***reflective exercise***. Chat with your app to plan this assessment, indicating what you and your learners will do before, during and after the exercise (NTS 3k-3q).

10 minutes **Content Exploration**

5. As a subject group, select one problem or idea/term in the week's content that is challenging (not a routine procedural question, but one that requires conceptual understanding) and then

a. use the app, ask it to solve the problem step by step and explain the reasoning behind each step (you may be assisted by a teacher who is strong in the content area)

b. all teachers in the group work through a similar problem individually for 3–4 minutes

c. the group then works through the problem together step by step

d. highlight the most common errors made at each step and discuss (NTS 1c, 1e, 2c, 2e, 3e-3j).

Note

- This part is not just about 'How do I teach this?' but how do I teach it for learners to understand?
- If the group still shares the same content gap, use the app: ask it to solve the problem step by step and explain the reasoning behind each step.
- Problems/ideas discussed should be sourced from the Teacher Manual content, not invented separately. This keeps the content exploration directly connected to what teachers will deliver.

30 minutes **Enactment**

6. Enact a teaching activity based on your learning plan (NTS la-1b, 1e-lg, 2c, 2e, 2f, 3c-3j and 3o).

a. Colleagues observe with specific attention to clarity of concept explanation, use of familiar Ghanaian examples, and how misconceptions are handled if they arise.

b. After the enactment, ask for structured feedback from the group

i. was the concept explained in a way that a learner with no prior knowledge could follow?

ii. were the examples grounded in contexts familiar to Ghanaian learners?

iii. did any misconceptions surface and if so, how were they addressed?

5 minutes **Reflection**

7. Reflect and share your views on the session. One thing that worked well, what did not work well and what you will do differently. (NTS 1a, 1b).

8. Identify a critical friend to observe and provide feedback on your lesson in relation to the PLC Session conducted (NTS 1f, 1g).

9. Remember to

a. transfer your chats into your learning planner template.

b. read your Teacher Manual and related Learner Material in preparation for the next session. Take note of the recap weeks, which are linked to Years 1 and 2 (NTS 3a).

c. integrate National Values, GESI, SEL, and 21st century skills into your learning activities, assessment tasks, and rubrics/mark schemes.

WEEK 8

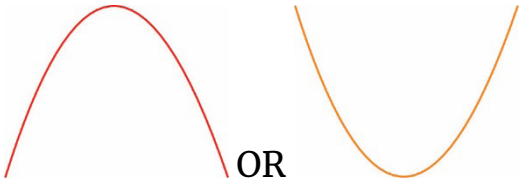
Learning Indicator: *Identify and explain the axes of symmetry, write their equations and solve linear and quadratic equations simultaneously using graphs*

FOCAL AREA 1: PARABOLIC OR QUADRATIC FUNCTIONS

The general quadratic function is of the form $f(x) = y = ax^2 + bx + c$, where a, b and c are constants and $a \neq 0$ .Also, x and y are variables.

Graphs of Parabolic Functions

The graph of a parabolic function of the form $f(x) = y = ax^2 + bx + c$ takes the shape of either of the diagrams below. These types of graphs are symmetrical, with the line of symmetry occurring at $x = -\frac{b}{2a}$


OR

The orientation of the graph of is dependent on the value of a

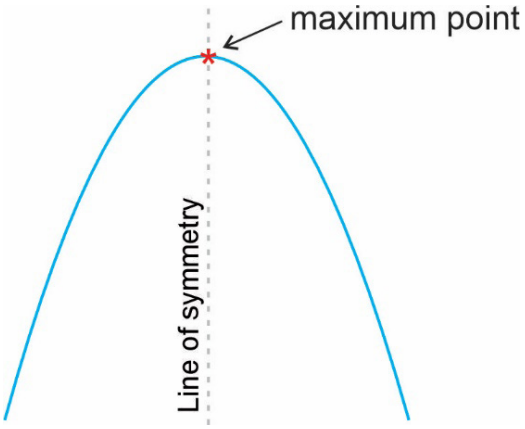
If $\boldsymbol{a < 0}$, the parabola opens downwards and has a maximum point.

The coordinates of the turning point are $\left(-\frac{b}{2a}, f(-\frac{b}{2a})\right)$ and the maximum value is $f\left(-\frac{b}{2a}\right)$

The line of symmetry is $x = -\frac{b}{2a}$

The graph is increasing when $x < -\frac{b}{2a}$

The graph is decreasing when $x > -\frac{b}{2a}$

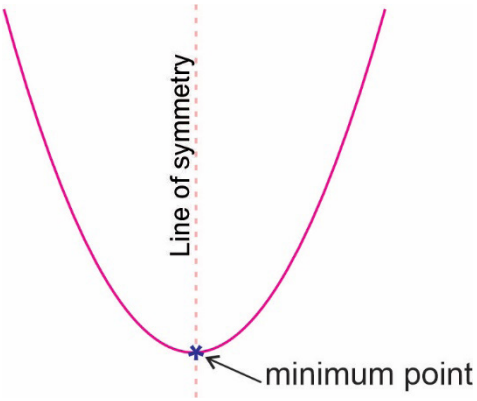


If $\boldsymbol{a > 0}$, the parabola opens upwards and has a minimum point.

The coordinates of the turning point is $\left(-\frac{b}{2a}, f(-\frac{b}{2a})\right)$ and the minimum value is $f\left(-\frac{b}{2a}\right)$

The line of symmetry is $x = -\frac{b}{2a}$

The graph is increasing when $x > -\frac{b}{2a}$



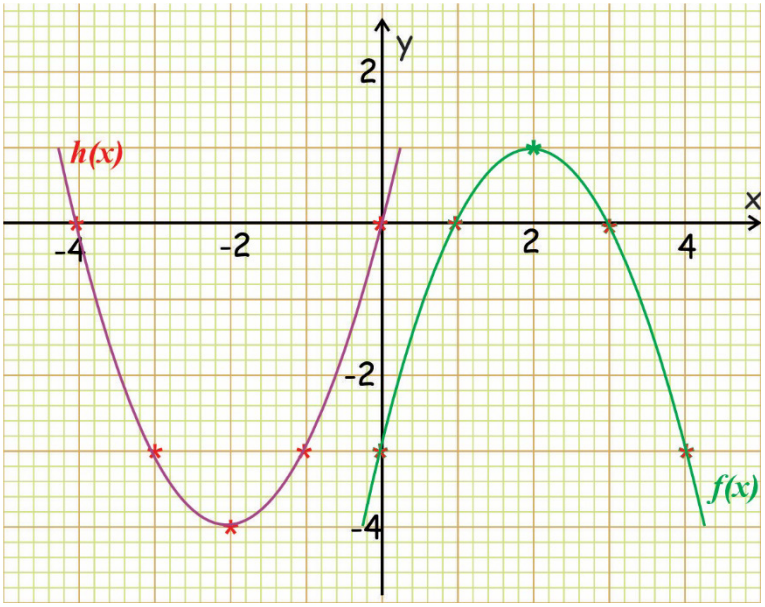
The graph is decreasing when $x < -\frac{b}{2a}$

Therefore, for a quadratic of the form $f(x) = y = ax^2 + bx + c$, the maximum or minimum value is $f\left(-\frac{b}{2a}\right)$.

If the graph is written in the form $f(x) = a(x-h)^2 + k$ the minimum or maximum value is k and the line of symmetry is $x = h$ or $x - h = 0$ or and the turning point is at (h, k).

Example 1

Study the graph below and use it to answer the questions.



a. For $f(x)$, find

- **i.** maximum point
- **ii.** maximum value
- **iii.** equation of the line of symmetry
- **iv.** the coordinates the y-intercept
- **v.** the coordinates of the x-intercepts

b. For $h(x)$, find

- **i.** minimum point
- **ii.** minimum value
- **iii.** equation of the line of symmetry
- **iv.** the coordinates of the y-intercept
- **v.** the coordinates of the x-intercepts

Solution

a. From the f(x) graph,

i. The maximum point or turning point is (2, 1).

ii. The y-coordinate of the maximum point is equal to the maximum value. Therefore, the maximum value is 1.

iii. The line of symmetry is $x = 2$.

iv. The y-intercept is the point where the curve intersects the y-axis. From the graph, the y-intercept is (0, −3).

v. The coordinates of the x-intercepts are the points where the graph intersects the x-axis. From the graph, the x-intercepts are (1, 0) and (3, 0)

b. From the h(x) graph,

i. The minimum point or turning point is (−2, −4).

ii. The y-coordinate of the minimum point is equal to the minimum value. Therefore, the minimum value is −4

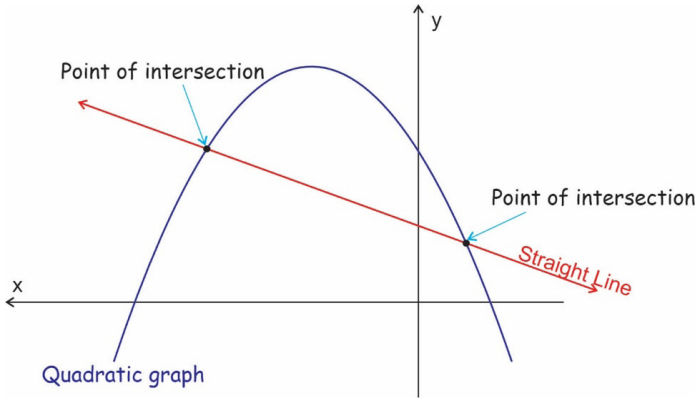
iii. The line of symmetry is $x = -2$

iv. The y-intercept is the point where the curve intersects the y-axis. From the graph, the y-intercept is (0, 0)

v. The coordinates of the x-intercept are the points where the graph intersects the x-axis. From the graph, the x-intercepts are (0, 0) and (−4, 0)

FOCAL AREA 2: SOLVING LINEAR AND QUADRATIC EQUATIONS USING GRAPHS

Two simultaneous equations, one linear and one quadratic, have, at most, two pairs of solutions. Graphically, the equations give a straight line and a curve. The values of x and y at the points of intersection are the solutions of the equations. The graph below shows the point of intersection of the straight line and the quadratic curve. Note: it is possible for the line to only touch the curve, in which case there would only be one solution, or it could miss it altogether, and there would be no solutions.



Example 1

a. Copy and complete the table of values for the relation $y = x^2 - 5x + 5$ for $-1 \leq x \leq 6$.

x	-1	0	1	2	3	4	5	6
y	11		1				5	11

b. Using scales of 2cm to represent 1 unit on the x–axis and 2 cm to represent 2 units on the y–axis, draw the graph of for.

c. Use the graph to find the solution set of

i. $x^2 - 5x + 5 = 0$

ii. $x^2 + 2x + 5 = 7x + 2$

Solution

a. Substitute the values into the equation find the corresponding values of y.

When $x = -1$, y = $(-1)^2 - 5(-1) + 5$, $\therefore y = 11$

When $x = 0$, y = $(0)^2 - 5(0) + 5$, $\therefore y = 5$

When $x = 2$, y = $(2)^2 - 5(2) + 5$, $\therefore y = -1$

When $x = 3$, y = $(3)^2 - 5(3) + 5$, $\therefore y = -1$

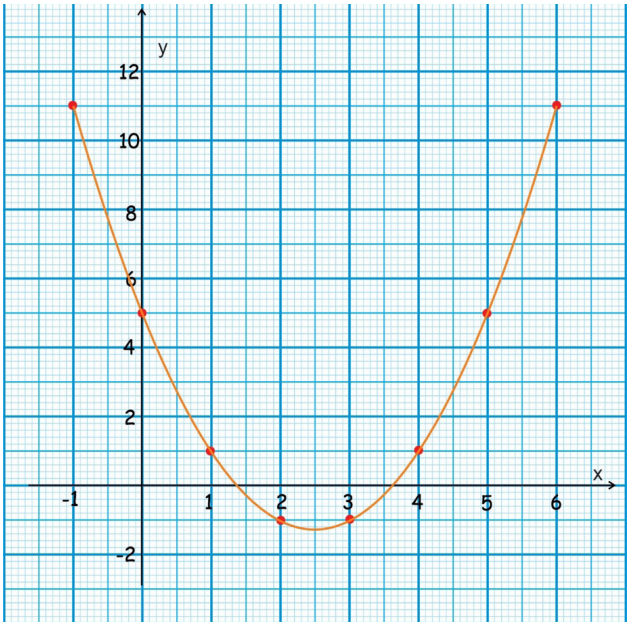
When $x = 4$, y = $(4)^2 - 5(4) + 5$, $\therefore y = 1$

When $x = 6$, y = $(6)^2 - 5(6) + 5$, $\therefore y = 11$

Now we complete the above table with the calculated values of y:

x	-1	0	1	2	3	4	5	6
y	11	**5**	1	**-1**	**-1**	**1**	5	11

b. We now plot the points and draw the quadratic curve as shown below.

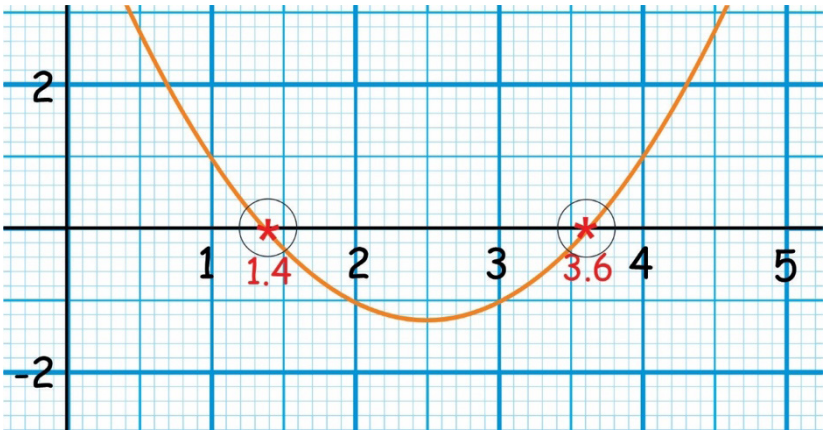


c.

i. $x^2 - 5x + 5 = 0$.

The drawn graph is $y = x^2 - 5x + 5$

This tells us that the solutions are when $y = 0$. This is the point the graph intersects the x-axis



From the graph, the x-axis intersects the quadratic curve at $x = 1.4$ and $x = 3.6$.

Hence the solution of $x^2 - 5x + 5 = 0$ is $x = 1.4$ and $x = 3.6$.

ii. To find the solution set of $x^2 + 2x + 5 = 7x + 2$, we rearrange the equation by grouping like terms, thus $x^2 + 2x - 7x + 5 - 2 = 0$, this gives $x^2 - 5x + 3 = 0$.

We now compare the equation $y = x^2 - 5x + 5$ to $x^2 - 5x + 3 = 0$, to obtain the value

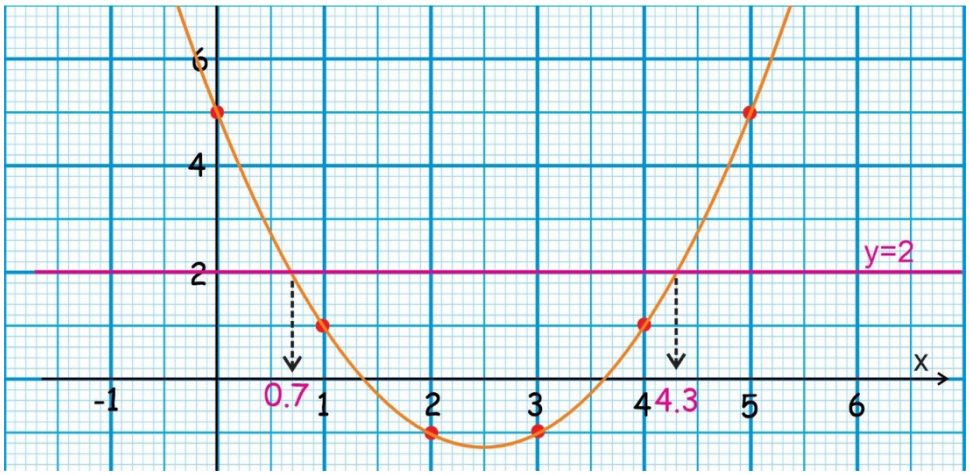
We can subtract $x^2 - 5x + 3 = 0$ from $y = x^2 - 5x + 5$.

$$x^2 - 5x + 5 = y$$

$$- \quad \underline{x^2 - 5x + 3 = 0}$$

$$y = 2$$

Therefore, $y = 2$, so we draw a horizontal line to intersect the graph at y = 2.



The line meets the quadratic curve at $x = 0.7$ and $x = 4.3$.

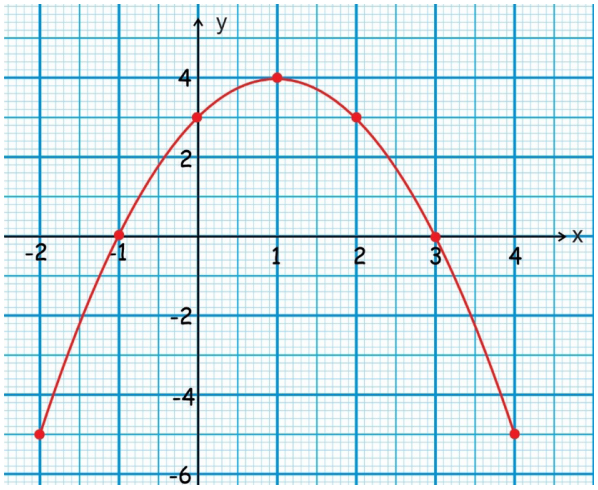
Hence, the solution of $x^2 + 2x + 5 = 7x + 2$ is $x = 0.7$ and $x = 4.3$.

Example 2

The diagram below shows the graph of $y = 3 + 2x - x^2$. Use the graph to solve

a. $6 + 2x - x^2 = 0$

b. $2 + x - x^2 = 0$



Solution

a. $6 + 2x - x^2 = 0$.

The drawn graph is $y = 3 + 2x - x^2$. So, we have to modify the equation, $6 + 2x - x^2 = 0$, to have a component of the graph, $3 + 2x - x^2$

Since the coefficients of the x and the x^2 terms are the same for both functions, we will just subtract 3 to both sides of the equation

$6 - 3 + 2x - x^2 = 0 - 3$

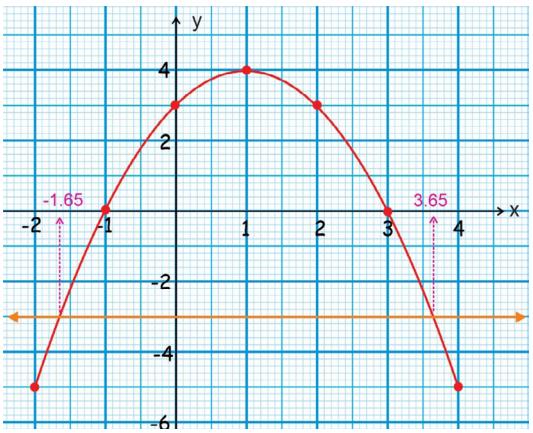
This gives $3 + 2x - x^2 = -3$

We know that $y = 3 + 2x - x^2$, therefore, our equation becomes $y = -3$

An **alternative** method is to find the difference between the graph and the equation.

$3 + 2x - x^2 - (6 + 2x - x^2) = -3$

This means that drawing or following the line $y = -3$ on the graph will lead us to the solution.



From the graph, the solution of $6 + 2x - x^2 = 0$ is either $x = -1.65$ or $x = 3.65$

b. $2 + x - x^2 = 0$

Add x to both sides of the equation

$2 + x + x - x^2 = 0 + x$

$2 + 2x - x^2 = x$

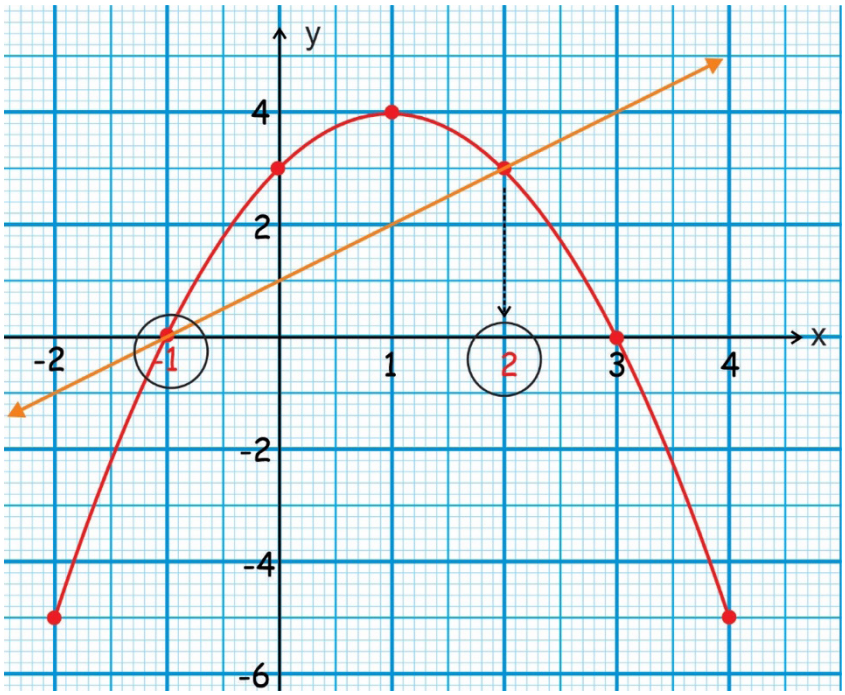
Add 1 to both sides of the equation

$2 + 1 + 2x - x^2 = x + 1$

$3 + 2x - x^2 = x + 1$

$y = x + 1$

Draw the line $y = x + 1$ on the graph and write the values of x at which it intersects the curve as the solution.



From the graph, the curve and line intersect at the solution, so the solution is $x = -1$ and $x = 2$

Learning Tasks

Guide learners to solve tasks related to understanding quadratic equations, quadratic graphs and functions. Provide support systems for learners who may encounter difficulties.

Learners are to:

1. Identify and write the equation of line of symmetry for quadratic graphs.
2. solve linear and quadratic simultaneous equations.

PEDAGOGICAL EXEMPLARS

The aim of the lessons for the week is for all learners to be able solve problems involving quadratic graphs with specific emphasis on using the graph to answer linear and quadratic functions simultaneously. The following pedagogical approach is suggested for facilitators to take learners through.

1. **Collaborative and gender responsive groups**. Use an integrated learning approach to establish, analyse and interpret quadratic graphs using diagrams to carry out investigations on quadratic equations concurrently through problem solving approaches.
2. **Problem-based learning**. Task learners to determine the solution set of equations in a whole class discussion and identify and brainstorm the meaning of axes of symmetry.
3. **Small gender responsive grouping**. Identify and brainstorm the connection between straight line and quadratic equations and find their points of intersections.

KEY ASSESSMENT LEVEL 3: GROUP PRESENTATION

1.

a. Copy and complete the table of values for the relation $y = 3x^2 - 5x + 4$ for $-2 \leq x \leq 4$

x	-2	-1	0	1	2	3	4
y	26			2			32

2 marks

b. Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 5 units on the y-axis, draw on the graph of $= 3x^2 - 5x + 4$.

3 marks

c. Use your graph to solve $3x^2 - 5x + 4 = 10 + 3x$

4 marks

Total *10 marks*

PLC Session 8

45 minutes | **Planning & Reviewing the Learning Plan**

1. Begin the session by recapping lessons from the previous year/session/week, focusing on how learning was applied.
2. Use your subject specific app to explore the content before planning. Ask the app to
   a. explain the week's concepts in a practical way that will make it easier for learners to understand

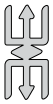
b. identify two or three common misconceptions learners may have on the week's content and how to address them using contexts familiar to Ghanaian learners. (NTS 3a, 3g and 3m).

3. Use your subject specific app to plan your lesson. Ask the app to

a. suggest the most effective method for teaching the week's concepts to learners with diverse backgrounds.

b. suggest learning activities, differentiation and essential questions based on the method of teaching (NTS 3a-3q)

c. review your prepared learning plan and address any challenging issues you may have identified in your subject area (NTS 3a-3c).



Note

- Take note of the recap topics linked to Years 1 and 2 while preparing for your lesson.
- Refer to the Values Learning Community Handbook and link the values for the week in your lesson to ensure that learners are applying what they learn.

4. Plan the week's key assessment with your app

a. The key assessment for the week is ***group presentation***. Chat with your app to plan this assessment, indicating what you and your learners will do before, during and after the presentations (NTS 3k-3q).

10 minutes **Content Exploration**

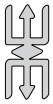
5. As a subject group, select one problem or idea/term in the week's content that is challenging (not a routine procedural question, but one that requires conceptual understanding) and then

a. use the app, ask it to solve the problem step by step and explain the reasoning behind each step (you may be assisted by a teacher who is strong in the content area)

b. all teachers in the group work through a similar problem individually for 3–4 minutes

c. the group then works through the problem together step by step

d. highlight the most common errors made at each step and discuss (NTS 1c, 1e, 2c, 2e, 3e-3j).



Note

- This part is not just about 'How do I teach this?' but how do I teach it for learners to understand?
- If the group still shares the same content gap, use the app: ask it to solve the problem step by step and explain the reasoning behind each step.

- Problems/ideas discussed should be sourced from the Teacher Manual content, not invented separately. This keeps the content exploration directly connected to what teachers will deliver.

30 minutes Enactment

6. Enact a teaching activity based on your learning plan (NTS la-1b, 1e-lg, 2c, 2e, 2f, 3c-3j and 3o).

a. Colleagues observe with specific attention to clarity of concept explanation, use of familiar Ghanaian examples and how misconceptions are handled if they arise.

b. After the enactment, ask for structured feedback from the group

i. was the concept explained in a way that a learner with no prior knowledge could follow?

ii. were the examples grounded in contexts familiar to Ghanaian learners?

iii. did any misconceptions surface and if so, how were they addressed?

5 minutes Reflection

7. Reflect and share your views on the session. One thing that worked well, what did not work well and what you will do differently. (NTS 1a, 1b).

8. Identify a critical friend to observe and provide feedback on your lesson in relation to the PLC Session conducted (NTS 1f, 1g).

9. Remember to

a. transfer your chats into your learning planner template.

b. read your Teacher Manual and related Learner Material in preparation for the next session. Take note of the recap weeks, which are linked to Years 1 and 2 (NTS 3a).

c. integrate National Values, GESI, SEL, and 21st century skills into your learning activities, assessment tasks, and rubrics/mark schemes.

WEEK 9

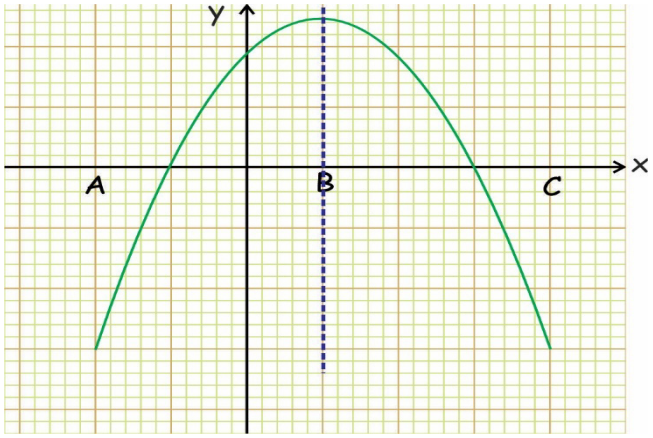
Learning Indicator: *Use quadratic graphs to solve related equations and solve real-life problems to find the range of values of (x) for which the other value (y) is increasing or decreasing and the range of values (x) for which the other value (y) is positive or negative*

FOCAL AREA 1: REAL-LIFE PROBLEMS INVOLVING LINEAR AND QUADRATIC EQUATIONS

1. *Find the range of values of (x) for which the other value (y) is increasing or decreasing.*
2. *Find the range of values (x) for which the other value (y) is positive or negative.*
3. *Model and solve real-life problems involving linear and quadratic equations.*

Finding the Range of Values of for which is Increasing or Decreasing

1. A maximum graph



Reading the curve from left to right, we can see that it increases steadily (i.e., the gradient is positive) from A and peaks at B. The range of values of x for which the y value increases is given by $x < B$

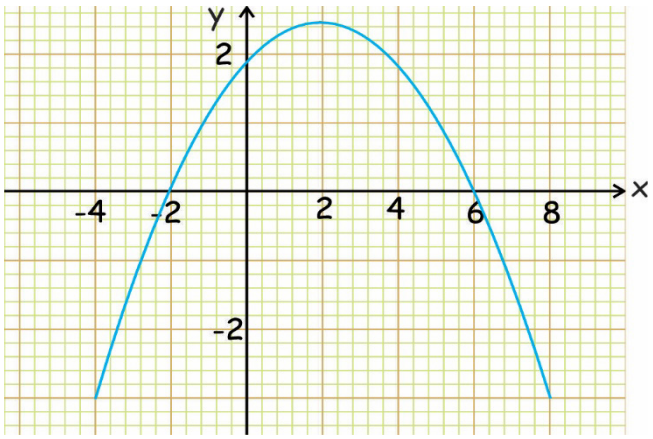
The quadratic curve also decreases (i.e., the gradient is negative) from the peak, B, towards C. The range of values of x for which the y value decreases is given by $x > B$.

Example 1

Use the graph below to find the range of values when the graph is:

a. increasing

b. decreasing



Solution

a. The graph is increasing when $x < 2$.

b. The graph is decreasing when $x > 2$.

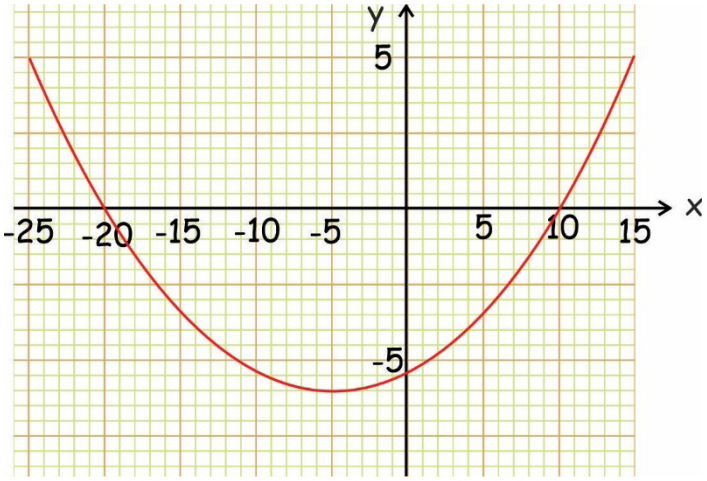
2. Minimum graph

Example 2

Use the graph below to find the range of values of x for which;

a. y increases as x increases

b. y decreases as x increases.



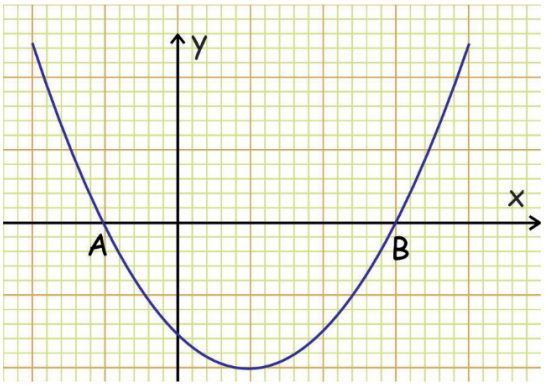
a. The range of values of x for which y increases as x increases is given by $x > -5$.

b. The range of values of x for which y decreases as x increases is given by $x < -5$.

Finding the Range of Values of *x* for which is *y* Positive or Negative

The value of y is negative (i.e., y < 0) if the curve is below the x–axis and positive (i.e. y > 0) if the curve is above the x–axis.

1. Minimum graph



From the above graph, the range of values of x for which $y \leq 0$ is given by $A \leq x \leq B$ and the values of x for which $y > 0$ is $x <$ A or $x >$ B.

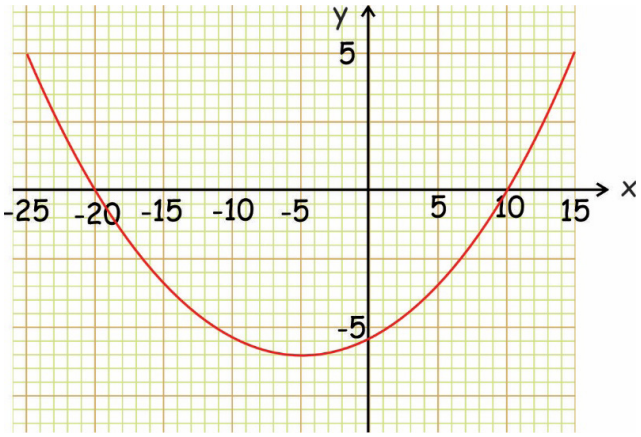
Example 1

Use the curve below to answer the following questions.

Find:

a. the range of values of x for which $y \leq 0$

b. the range of values of x for which $y > 0$.

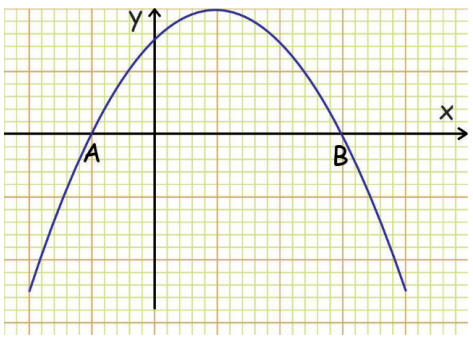


Solution

a. From the graph the range of values for which $y \leq 0$ is $-20 \leq x \leq 10$.

b. From the graph the range of values of x for which y > 0 is x < –20 and x > 10

2. The maximum graph



The range of values of x for which $y \geq 0$ is $A \leq x \leq B$ and the range of values of x for which $y < 0$ is $x <$A or $x >$ B.

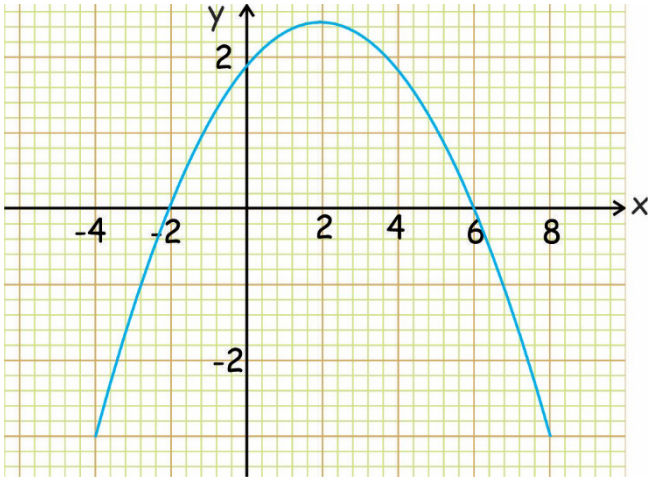
Example 2

Use the quadratic curve to answer the following questions.

Find:

a. the range of values of x for which y is negative.

b. the range of values of x for which y is positive.



Solution

a. From the graph, the range of values of x for which y is negative is $x < -2$ and $x > 6$.

b. From the graph, the range of values of x for which y is positive is $-2 < x \leq 6$

Example 3

a. Copy and complete the following table for the relation $y = 4 + 3x - x^2$ for the interval $-2 \leq x \leq 5$

x	-2	-1	0	1	2	3	4	5
y		0	4			4	0	

b. Using a scale of 2 cm to 1 unit on the x–axis and 2 cm to 2 units on the y–axis, draw the relation.

c. From the graph, find

i. The range of values of x for which y decreases as x increases.

ii. The range of values of x for which y increases as x increases.

iii. The range of values of x for which y is negative.

iv. The range of values of x for which y is positive.

Solution

a. We find the values of y and complete the above table:

When $x = -2$, $y = 4 + 3(-2) - (-2)^2$ $\Rightarrow$ y $= -6$

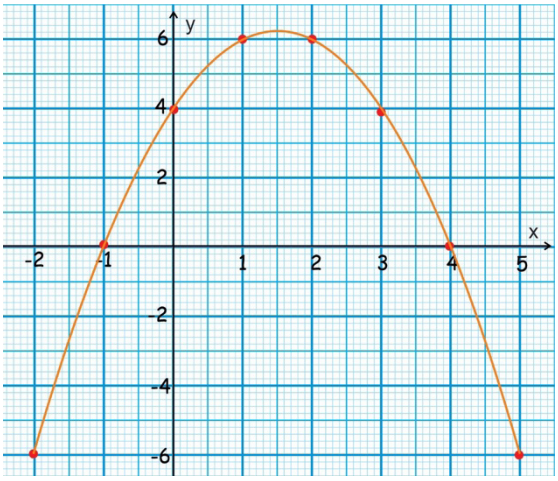
When $x = 1$, $y = 4 + 3(1) - (1)^2$ $\Rightarrow$ y $= 6$

When $x = 2$, $y = 4 + 3(2) - (2)^2$ $\Rightarrow$ y $= 6$

When $x = 5$, $y = 4 + 3(5) - (5)^2$ $\Rightarrow$ y $= -6$

x	-2	-1	0	1	2	3	4	5
y	**-6**	0	4	**6**	**6**	4	0	**-6**

b.



c.

i. The range of values of x for which y decreases as x increases is given by $x > 1.5$

ii. The range of values of x for which y increases as x increases is given by $x < 1.5$

iii. The range of values of x for which y is negative is given by $x < -1$ and $x > 4$

iv. The range of values of x for which y is positive is given by $-1 < x < 4$

Example 4

a. Copy and complete the table of values for the relation $y = x^2 + x - 2$ for $-4 \leq x \leq 3$

x	-4	-3	-2	-1	0	1	2	3
y	10			-2	-2			10

b. Using a scale of 2 cm to 1 unit on the x–axis and 2 cm to 2 units on the y–axis, draw the graph of the relation $y = x^2 + x - 2$ for $-4 \leq x \leq 3$

c. From the graph, find

i. The range of values of x for which y decreases as x increases.

ii. The range of values of x for which y increases as x increases.

iii. The range of values of x for which y is positive

iv. The range of values of x for which y is negative.

Solution

a. We find the values of y and complete the above table;

When $x = -3,\ y = (-3)^2 + (-3) - 2 \Rightarrow y = 4$

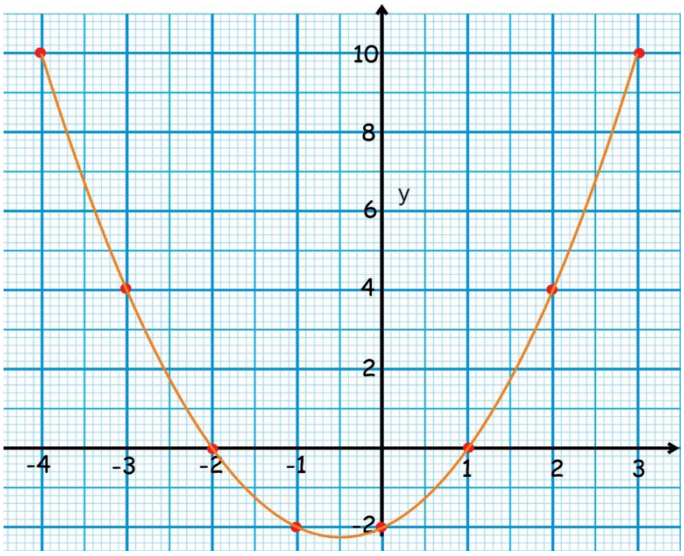
When $x = -2,\ y = (-2)^2 + (-2) - 2 \Rightarrow y = 0$

When $x = 1,\ y = (1)^2 + (1) - 2 \Rightarrow y = 0$

When $x = 2,\ y = (2)^2 + (2) - 2 \Rightarrow y = 4$

x	-4	-3	-2	-1	0	1	2	3
y	10	**4**	**0**	-2	-2	**0**	**4**	10

b.



c.

i. The range of values of x for which y decreases as x increases is given by $x < -0.5$

ii. The range of values of x for which y increases as x increases is given by $x > -0.5$

iii. The range of values of x for which y is positive is given by $x < -2$ and $x > 1$.

iv. The range of values of x for which y is negative is given by $-2 < x < 1$

Example 5

The height of an object launched vertically into the air from the ground at an initial velocity of 80 m/sec can be calculated using $H = 80t - 16t^2$, where H is the height in feet and t is the time in seconds.

a. Prepare a table to show the height of the object for t = 0, t = 0.5, t = 1, t = 1.5, t = 2, t = 3, t = 4 and t = 5s.

b. Using a suitable scale, draw a scaled graph for the relation $H = 80t - 16t^2$.

c. Use your graph to find the time interval during which the height of the object was:

i. less than 64m

ii. greater than 36m

iii. increasing

iv. decreasing

d. Use your graph to find the maximum height attained by the object.

How long did it take the object to reach this height?

e. How long after the object is launched will it hit the ground?

Solution

a. when $t = 0$, $H = 80(0) - 16(0)^2 = 0$

when $t = 0.5$, $H = 80(0.5) - 16(0.5)^2 = 36$

when $t = 1$, $H = 80(1) - 16(1)^2 = 64$

when $t = 1.5$, $H = 80(1.5) - 16(1.5)^2 = 84$

when $t = 2$, $H = 80(2) - 16(2)^2 = 96$

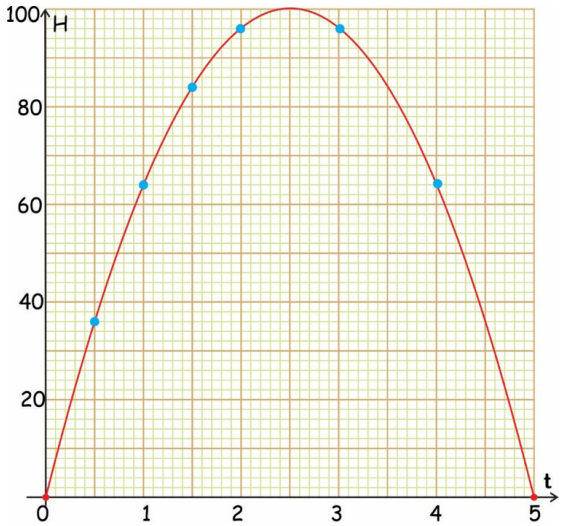
when $t = 3$, $H = 80(3) - 16(3)^2 = 84$

when $t = 4$, $H = 80(4) - 16(4)^2 = 64$

when $t = 5$, $H = 80(5) - 16(5)^2 = 0$

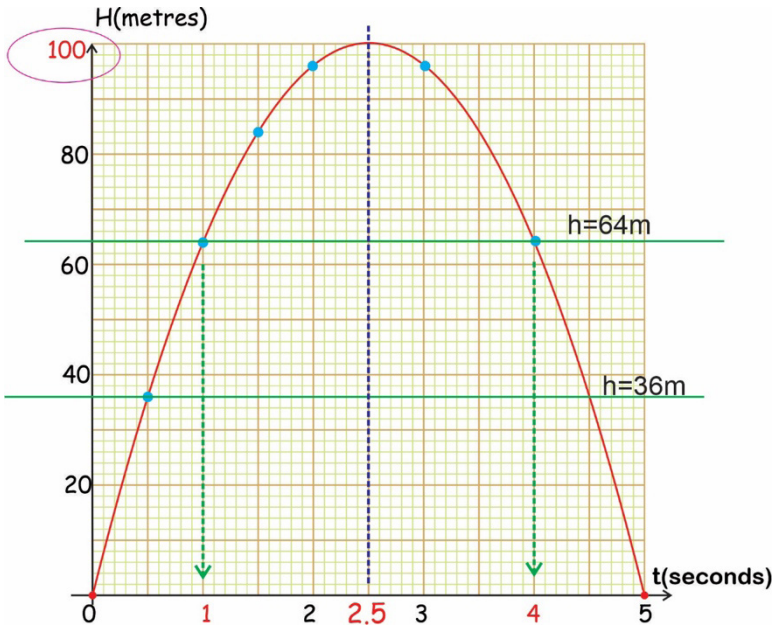
t	0	0.5	1	1.5	2	3	4	5
$H = 80t - 16t^2$	0	36	64	84	96	84	64	0

b.



c. Use your graph to find the time interval during which the height of the object was

d.



i. less than 64m

when $0 \leq t < 1$ and $4 < t \leq 5$

ii. greater than 36m

when $0.5 < t < 4.5$

iii. increasing

when $0 \leq t < 2.5$

iv. decreasing

when $2.5 < t \leq 5$

e. The maximum height attained by the object is 100m and this is reached when $t = 2.5$s.

The object hits the ground when $H = 0$. This happens when $t = 5$ seconds. Therefore, it takes 5 seconds from launch to land.

Learning Tasks

Guide learners to solve tasks related to understanding quadratic equations, quadratic graphs and functions. Provide support systems for learners who may encounter difficulties.

Learners are to:

1. Find the range of values of x for which a quadratic graph is increasing or decreasing
2. Obtain the range of values of x for which a graph is less than or greater than 0
3. Apply the quadratic graphs to answer real-life questions.

PEDAGOGICAL EXEMPLARS

The aim of the lessons for the week is for all learners to be able to solve problems involving quadratic graphs. The following pedagogical approach is suggested for facilitators to take learners through.

1. Using talk for learning in a GESI aware class discussion, engage learners to investigate the range of values of x for which y increases or decreases.
2. Using talk for learning in a GESI aware class discussion, engage learners to investigate the range of values of x for which y is positive or negative.
3. Using talk for learning in a GESI aware class discussion, model and solve real life problems involving linear and quadratic equations.

KEY ASSESSMENT LEVEL 3: CLASSWORK

1. The height of an object launched from the ground with an initial velocity of 60 ft/sec can be calculated using $H = 60t - 12t^2$, where H is the height in feet and t is the time in seconds.

a. Prepare a distribution table to show the object's height for t = 0, t = 0.5s, t = 1s, t = 1.5s,

3 marks

b. Using a suitable scale, draw a scaled graph for the relation $H = 60t - 12t^2$.

3 marks

c. Use your graph to find the time interval during which the height of the object was

i. less than 63m

ii. greater than 48m

iii. increasing

iv. decreasing

6 marks

d. Use your graph to find the maximum height attained by the object. How long did it take the object to reach this height? ***3 marks***

e. How long after the object is launched will it hit the ground? ***1 mark***

Total *16 marks*

2. The entrance to an athletic field is in the shape of a parabolic archway. The archway is modelled by the equation $H = 12x - x^2$, where represents the distance, in feet, that the arch is above the ground for any x value.

a. Use the relation to copy and complete the table for the values of x.

x	0	1	2	3	4	5	6	7	8	9	10
H	0	11			32	35		35			

3 marks

b. Using a scale of 2cm to 2 feet on the x-axis and 2cm to 5 feet on the y-axis, draw a graph for the relation $H = 12x - x^2$. ***3 marks***

c. Use your graph to find the

i. values of x for which the arch is 30 feet above the ground.

ii. range of values of x for which the height of the arc is greater than 25 feet.

iii. the maximum height of the arch above the ground.

5 marks

Total *11 marks*

PLC Session 9

45 minutes **Planning & Reviewing the Learning Plan**

1. Begin the session by recapping lessons from the previous year/session/week, focusing on how learning was applied.

2. Use your subject specific app to explore the content before planning. Ask the app to

a. explain the week's concepts in a practical way that will make it easier for learners to understand

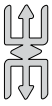
b. identify two or three common misconceptions learners may have on the week's content and how to address them using contexts familiar to Ghanaian learners. (NTS 3a, 3g and 3m).

3. Use your subject specific app to plan your lesson. Ask the app to

a. suggest the most effective method for teaching the week's concepts to learners with diverse backgrounds.

b. suggest learning activities, differentiation and essential questions based on the method of teaching (NTS 3a-3q)

c. review your prepared learning plan and address any challenging issues you may have identified in your subject area (NTS 3a-3c).



Note

- Take note of the recap topics linked to Years 1 and 2 while preparing for your lesson.
- Refer to the Values Learning Community Handbook and link the values for the week in your lesson to ensure that learners are applying what they learn.

4. Plan the week's key assessment with your app

a. The key assessment for the week is ***classwork***. Chat with your app to plan this assessment, indicating what you and your learners will do before, during and after the classwork (NTS 3k-3q).

10 minutes **Content Exploration**

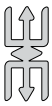
5. As a subject group, select one problem or idea/term in the week's content that is challenging (not a routine procedural question, but one that requires conceptual understanding) and then

a. use the app, ask it to solve the problem step by step and explain the reasoning behind each step (you may be assisted by a teacher who is strong in the content area)

b. all teachers in the group work through a similar problem individually for 3–4 minutes

c. the group then works through the problem together step by step

d. highlight the most common errors made at each step and discuss (NTS 1c, 1e, 2c, 2e, 3e-3j).



Note

- This part is not just about 'How do I teach this?' but how do I teach it for learners to understand?
- If the group still shares the same content gap, use the app: ask it to solve the problem step by step and explain the reasoning behind each step.
- Problems/ideas discussed should be sourced from the Teacher Manual content, not invented separately. This keeps the content exploration directly connected to what teachers will deliver.

30 minutes **Enactment**

6. Enact a teaching activity based on your learning plan (NTS la-1b, 1e-lg, 2c, 2e, 2f, 3c-3j and 3o).

a. Colleagues observe with specific attention to clarity of concept explanation, use of familiar Ghanaian examples and how misconceptions are handled if they arise.

b. After the enactment, ask for structured feedback from the group

i. was the concept explained in a way that a learner with no prior knowledge could follow?

ii. were the examples grounded in contexts familiar to Ghanaian learners?

iii. did any misconceptions surface and if so, how were they addressed?

5 minutes **Reflection**

7. Reflect and share your views on the session. One thing that worked well, what did not work well and what you will do differently. (NTS 1a, 1b).
8. Identify a critical friend to observe and provide feedback on your lesson in relation to the PLC Session conducted (NTS 1f, 1g).
9. Remember to
   a. transfer your chats into your learning planner template.
   b. read your Teacher Manual and related Learner Material in preparation for the next session. Take note of the recap weeks, which are linked to Years 1 and 2 (NTS 3a).
   c. integrate National Values, GESI, SEL, and 21st century skills into your learning activities, assessment tasks, and rubrics/mark schemes.

Section 2 Review

The section has developed learners' understanding of quadratic equations and quadratic functions. Learners solved quadratic equations using factorisation by grouping, the quadratic formula and graphical methods, and applied these techniques to mathematical and real-life problems.

Learners also investigated the graphs of quadratic functions, identified roots, maximum or minimum points and values, and determined the equation of the axis of symmetry. They used graphs to solve linear and quadratic equations simultaneously by identifying the intersection points of the corresponding graphs.

In addition, learners determined the intervals of x for which quadratic functions are increasing or decreasing and the values of x for which the functions are positive, negative, non-positive or non-negative. Real-life applications helped learners interpret quadratic models, identify relevant domains, and draw meaningful conclusions from graphs.

Through collaborative, investigative and problem-solving activities, learners developed their ability to interpret graphical information, justify mathematical conclusions and apply quadratic functions and equations to real-life situations.

MARKING SCHEMES FOR THE KEY ASSESSMENTS

WEEK 6

MCQs

1.	B	**2.**	D
3.	C	**4.**	C
5.	B	**6.**	C
7.	B	**8.**	B
9.	A	**10.**	B

ESSAY

Question 1

The time taken for the first part was $T_1 = \frac{10}{x}$ hours.

The remaining distance was $(30 - 10)km = 20km$.

The average speed for the rest of the race was $(x - 1)$ km/h.

The time taken for the rest of the race was $T_2 = \frac{20}{x-1}$

The total time for the walk was 7 hours.

So, $T_1 + T_2 = 7$.

$$\frac{10}{x} + \frac{20}{x-1} = 7$$

A correct method to solve the equation, for example:

Multiply through by $x(x - 1)$ and simplify to get the quadratic equation

$$7x^2 - 37x + 10 = 0$$

Factorise

$$7x^2 - 37x + 10 = 0$$
$$7x^2 - 2x - 35x + 10 = 0$$
$$x(7x - 2) - 5(7x - 2) = 0$$
$$(x - 5)(7x - 2) = 0$$
$$x = 5, x = \frac{2}{7}$$

Check the validity of the x

If $x = 5$, Speed for the first 10km = 5km/h.

Speed for the remaining 20km = 5 − 1 = 4km/h.

Time for the first 10km = $\frac{10}{5} = 2$ hours.

Time for the remaining 20km = $\frac{20}{4} = 5$ hours.

B1 for correctly expressing the time for the first part

$T_1 = \frac{10}{x}$ or equivalent

B1 Correctly identifying the speed for the second part ((x−1) km/h)

B1 for correctly expressing the time for the second part

$T_2 = \frac{20}{x-1}$ or equivalent

M1 for forming equation (at least one term correct)

A1 for correct equation

$\frac{10}{x} + \frac{20}{x-1}$, or equivalent

Sample mark scheme, but allow any correct method equivalent.

M1 for simplifying equation

A1 for correct quadratic equation.

$$7x^2 - 37x + 10 = 0$$

M1 for factorising the quadratic equation

A1 for 5 and $\frac{2}{7}$ (−0.5 each error)

Total time = 2 + 5 = 7 hours. This solution is valid.

If $x = \frac{2}{7}$ Speed for the first 10km = $\frac{2}{7}$ km/h.

Speed for the remaining 20km = $\frac{2}{7} - 1 = -\frac{5}{7}$ km/h.

A negative speed is not physically possible. Therefore, this solution is not valid.

Therefore, we conclude that the value x of is 5.

B1 for validating the solution and stating the correct value of

Total = 10 marks

Question 2

Perimeter of the shape = 8m

$\Rightarrow 5x + y + 4x + 3x + y = 8$

$12x + 2y = 8$

$6x + y = 4$ ..... (P)

Area of the shape $= 4m^2$

$\Rightarrow 5xy + \frac{1}{2} \times 4x \times 3x = 4$

$5xy + 6x^2 = 4$ .....(A)

From (1), $y = 4 - 6x$

substitute $y = 4 - 6x$ *into equation* (2)

$5x(4 - 6x) + 6x^2 = 4$

$20x - 30x^2 + 6x^2 = 4$

A correct method for solving the quadratic, for example:

$-24x^2 + 20x - 4 = 0$

$6x^2 - 5x + 1 = 0$

$6x^2 - 3x - 2x + 1 = 0$

$3x(2x - 1) - 1(2x - 1) = 0$

$(3x - 1)(2x - 1) = 0$

$x = \frac{1}{3}$ or $x = \frac{1}{2}$

If $x = \frac{1}{3}$, $y = 4 - 6 \times \frac{1}{3} = 4 - 2 = 2$

If $x = \frac{1}{2}$, $y = 4 - 6 \times \frac{1}{2} = 4 - 3 = 1$

In conclusion, either $x = \frac{1}{3}$ and $y = 2$

Or $x = \frac{1}{2}$ and $y = 1$

M1 for correctly attempting to form equation (P)

A1 for the correct equation or its equivalence

M1 for forming equation (A)

A1 for the correct equation or its equivalence

M1 for forming quadratic equation by substitution

A1 for correction equation

($6x^2 - 5x + 1 = 0$ or its equivalence)

M1 correct method attempted for solving for x

A1 for $x = \frac{1}{2}$ and $x = \frac{1}{3}$ (−0.5 each error)

M1 for attempting to input their x to solve for y

A1 for y = 1 and y = 2 (−0.5 each error)

Total =10 marks

WEEK 7

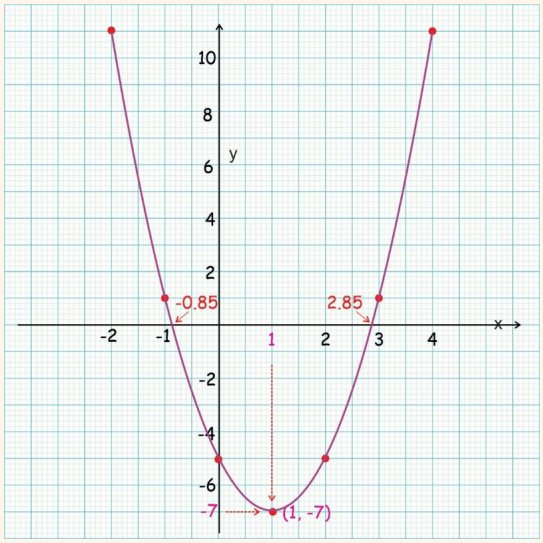
Question One

1.

x	-2	-1	0	1	2	3	4
y	11	**1**	-5	**-7**	**-5**	**1**	11

B2 (0.5 for each correct entry)

b.



B3 for the graph

For example:

B1 for a correct shape.

B1 for going through the correct points

B1 for a smooth curve drawn

c. From the graph

i. the minimum value is -7

ii. minimum point is $(1, -7)$

iii. the roots of $-2x^2 - 4x - 5 = 0$ are -0.85 and 2.85

A1 for the minimum value

A1 for the minimum point

M1 for reading roots

A1 for the roots (0.5 for each)

Allow follow through if their graph is incorrectly drawn but their answers to part c are correct for their graph.

Total 9 marks

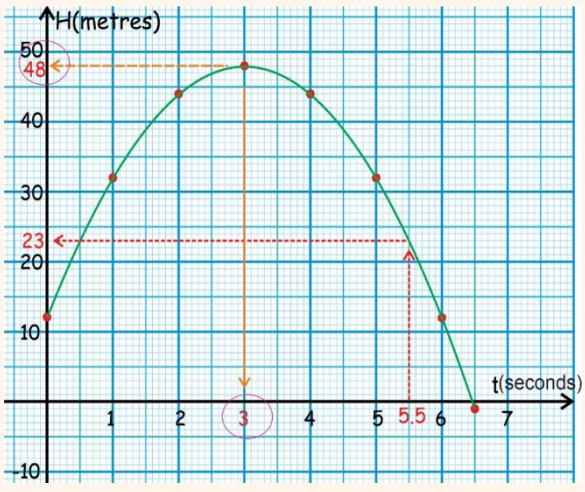
Question 2

a.

t	0	1	2	3	4	5	6	6.5
H	12	32	44	48	44	32	12	-1

B3 for table (0.5 for each correct entry)

b.



B3 for graph

For example:

B1 for a correct shape

B1 for going through the correct points

B1 for a smooth curve drawn)

c.

i. The rockets maximum height is 48 metres

ii. In 5.5 seconds, the rocket was 23 metres above the ground.

iii. It took the rocket about 6.45 seconds to hit the ground.

A1 for maximum height (48 metres)

M1 for reading from graph, or substituting t = 5.5 into the relation

A1 for 23 metres, or correct reading from their graph.

A1 for time the rocket hit the ground

Allow follow through if their graph is incorrectly drawn but their answers to part c are correct for their graph.

Total 10 marks

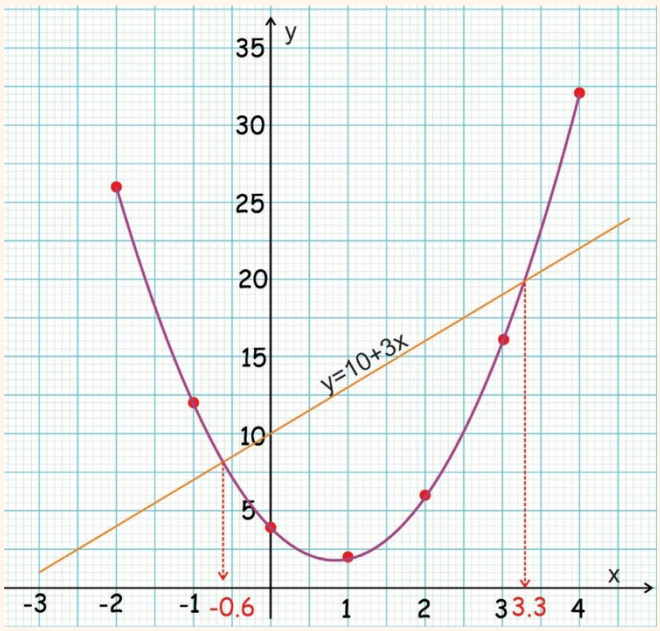
WEEK 8

a.

x	-2	-1	0	1	2	3	4
y	26	**12**	**4**	2	**6**	**16**	32

B2 for table (0.5 for each correct entry)

b.



B3 for graph

For example

B1 for a correct shape

B1 for going through the correct points

B1 for a smooth curve drawn

From the graph the solution for

$3x^2 - 5x + 4 = 10 + 3x$ is

$x = -0.6$ and $x = 3.3$

B2 for linear graph correctly drawn on the graph in part b

M1 for reading their intersection values from their graph, even if graphs incorrect

A1 for -0.6 and 3.3 (0.5 for each)

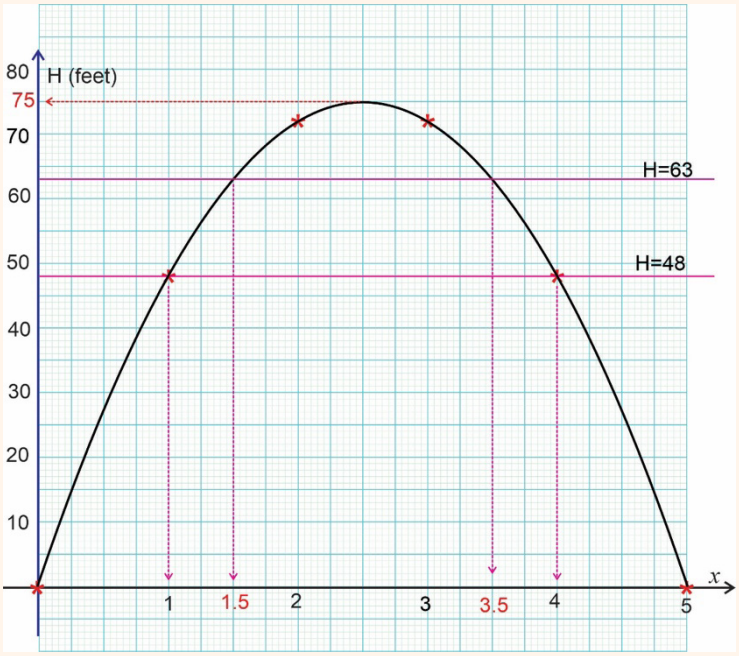
WEEK 9

Question 1

a.

t	0	1	2	3	4	5
$H = 60t - 12t^2$	0	48	72	72	48	0

B3 (0.5 for each correct entry)



B3 for the graph

For example:

B1 for a correct shape

B1 for going through the correct points

B1 for a smooth curve drawn)

b.

i. when H is less than 63m, $t < 1.5$ or $t > 3.5$

ii. $1 < t < 4$

ii. $t < 2.5$

iii. $t > 2.5$

M1 for reading correctly from their graph

A1 for $t < 1.5$ and $t > 3.5$ (0.5 for each), allow follow through from their graph

M1 for reading correctly from their graph, or the table of values

A1 for $1 < t < 4$, allow follow through

A1 for $t < 2.5$

A1 for $t > 2.5$

c. The maximum height is 75m. It took 2.5 seconds to reach the maximum height

M1 for reading their maximum height from the graph

A1 for 75m

A1 for 2.5seconds

d. It took 5 seconds for the object to hit the ground.

A1 for the time it took to hit the ground

Total 16 marks

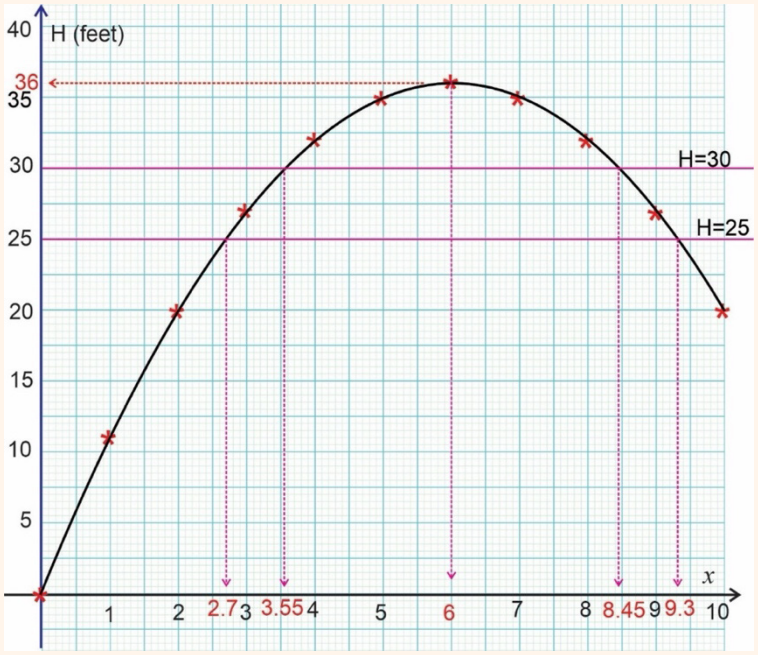
Question 2

a.

x	0	1	2	3	4	5	6	7	8	9	10
H	0	11	**20**	**27**	32	35	**36**	35	**32**	**27**	**20**

B3 (0.5 for each correct entry)

b.



B3 for the graph

(***For example:***

B1 for a correct shape

B1 for going through the correct points

B1 for a smooth curve drawn)

c.

i. from the graph $x = 3.55$ and $x = 8.45$

ii. $2.7 < x < 9.3$

iii. maximum height feet = 36 feet

M1 for reading from their graph

A1 for $x = 3.55$ and x = 8.45 (0.5 for each), allow follow through from their graph

M1 for reading from their graph

A1 for $2.7 < x < 9.3$, allow follow through from their graph

A1 for max height = 36 feet

Total 11 marks

SECTION 3: CIRCLE THEOREMS & GEOMETRIC CONSTRUCTION

STRAND: GEOMETRY AROUND US

Sub-Strand: Spatial Sense

Weeks 10 & 11

Learning Outcome: *Draw circles for given radii and use the circle theorems; identify the tangent as perpendicular to the radius at the point of contact and verify that tangents drawn from an external point to the same circle are equal when measured from their point of contact.*

Content Standard: *Demonstrate a conceptual understanding of spatial sense with respect to circles and their theorems, and apply their properties to solve everyday life problems.*

Weeks 12 & 13

Learning Outcome: *Perform geometric construction of quadrilaterals and given loci.*

Content Standard: *Demonstrate knowledge and understanding of geometrical construction, use the knowledge to construct plane shapes, and apply these in the world around them.*

INTRODUCTION AND SECTION SUMMARY

Circle theorems and geometric constructions are fundamental pillars of Euclidean geometry, offering powerful tools for understanding the properties of circles and for creating precise geometric figures using only a compass and straightedge. These topics have been central to mathematical inquiry for centuries, laying the groundwork for more advanced concepts in geometry, trigonometry, and even architecture and engineering.

Under the circle theorem, we will delve into a collection of proven statements that describe the relationships between various parts of a circle, such as its chords, tangents, angles, and arcs. Understanding these theorems allows for the deduction of unknown angles and lengths within circular figures, providing a logical framework for solving a wide range of geometric problems.

Under geometric constructions, we will focus on the art and science of accurately drawing geometric shapes and figures using only two primary tools: a straightedge (for drawing straight lines) and a compass (for drawing circles and arcs). While seemingly simple, these tools enable

the construction of remarkably complex figures and are crucial for visualising and proving geometric principles. Geometric constructions emphasise precision, logical thinking, and the foundational axioms of geometry. They provide a hands-on approach to understanding geometric relationships and are an essential skill for anyone delving deeper into mathematical concepts or practical design.

The weeks, the indicators with their respective focal areas covered by the section are:

Week 10

a. Identify parts of a circle and draw circles for given radii and through points.

b. Discuss the circle theorems by identifying the statements, proofs, examples, and applications.

Focal Area 1: Circle and Circle Theorem

- *Identify the parts of a circle*
- *Draw circles for given radii and through points.*

Focal Area 2: Applications of Circle Theorems

- Identify the circle theorem statements.
- Model and prove with examples the circle theorems.

Week 11

a. Identify the tangent as perpendicular to the radius at the point of contact and verify the Alternate Segment Theorem.

b. Verify that tangents drawn from an external point to the same circle are equal when measured from their point of contact.

Focal Area 1: The tangent of a circle 1

- *Identify and prove that the tangent as perpendicular to the radius at the point of contact.*
- *Verify the Alternate Segment Theorem.*

Focal Area 2: The tangent of a circle 2

- Prove with examples that tangents drawn from an external point to the same circle are equal when measured from their point of touch.

Week 12

Recall the construction of various angles such as 750, 1050, 1350, and 1500.

Focal Area: Understanding Geometric Constructions

- *Review of angles from Junior High School*
- *Constructing angles of 75˚,105˚,135˚ and 150˚*

Week 13

a. Construct a triangle or a quadrilateral under given conditions.

b. Construct a particular locus for a given condition.

Focal Area 1: Construction of quadrilaterals Concept, examples and visualisation of surds

- *Construct a triangle.*
- *Construct a quadrilateral.*

Focal Area 2: Construction of locus

- *Theorems and conditions of locus*
- *Construction of locus*

SUMMARY OF PEDAGOGICAL EXEMPLARS

This section requires hands-on activities where learners engage with the construction of plane shapes. Learners should be given the opportunity to work in groups to develop their real-life questions and find answers. Therefore, experiential learning activities and mixed-ability groupings should dominate the lessons on these concepts. All learners, regardless of their learning abilities, should be supported to participate fully in investigations and presentations of findings. However, considerations and accommodations must be made for the different groups. Specifically, offer approaching-proficiency learners the chance to make oral presentations. Then, extend activities for above-average or highly proficient learners to use computer applications to solve problems.

ASSESSMENT SUMMARY

Table 1 contains the key assessment for each indicator in the section. The highest of DOK for the indicator is denoted. The mode of assessment to be carried out and strategies that will aid in achieving the indicators are documented.

Table 1: Indicators, DOK levels, assessment modes and strategies

Indicators	DOK Level	Assessment Strategy and/or Mode
1. Identify parts of a circle and draw circles for given radii and through points. 2. Discuss the circle theorems by identifying the statements, proofs, examples, and applications.	3	**Strategy:** The questions will contain ten (10) compulsory questions (Q1 to Q10) from all sub-strands of the curriculum. There will be eight (8) questions (Q11 to Q18) with two (2) questions from each strand. Candidates must select six (6) questions to answer, with at least one question from each strand. **Mode:** Mock Exams

1. Identify the tangent as perpendicular to the radius at the point of contact and verify the Alternate Segment Theorem. 2. Verify that tangents drawn from an external point to the same circle are equal when measured from their point of contact.	3	**Strategy:** Learners through reflective questions (or KWL) and justify their solutions to questions to the mock exams. **Mode:** Class Exercises
Recall the construction of various angles such as $75^0, 105^0, 135^0,$ and 150^0.	2	**Strategy:** Standard level 2 essay questions **Mode:** Home Work
Construct a triangle or a quadrilateral under given conditions. Construct a particular locus for a given condition.	2	**Strategy:** Standard level 2 essay questions **Mode:** Group work and presentation

WEEK 10

Learning Indicators

1. *Identify parts of a circle and draw circles for given radii and through points*
2. *Discuss the circle theorems by identifying the statements, proofs, examples and applications*

FOCAL AREA 1: CIRCLE AND CIRCLE THEOREMS

A Circle

A circle is the set of all points in a plane that are a fixed distance from a fixed point called the centre.

Parts of a Circle

Figure 10.1 is an example of a circle. A, B, C, D and E are points on the circumference of the circle. Point O is the centre of the circle.

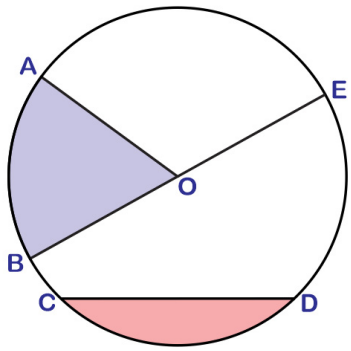


Figure 10.1: *A diagram illustrating parts of a circle.*

1. **Centre**

   The centre of the circle is the fixed point from which all points on the boundary of the circle are equidistant. In ***Figure 10.1***, the centre is point O.

2. **Circumference**

   The perimeter of a circle is called the **circumference**. It is the distance around the circle as shown in ***Figure 10.2***.

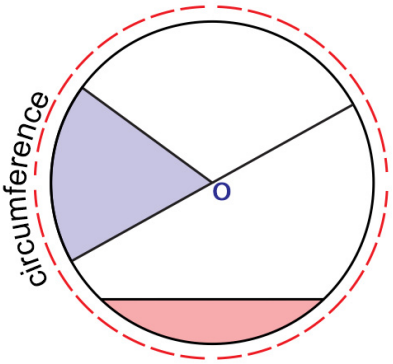


Figure 10.2: *Circumference of a circle*

3. Arc

A part of the circumference is called an **arc**, as shown in figures 10.3 and 10.4.

Major arc: A major arc is greater than half the circumference.

Minor arc: A minor arc is less than half the circumference.

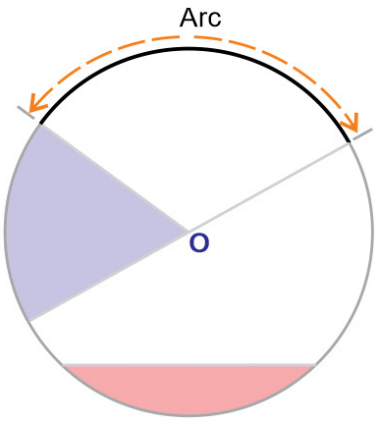


Figure 10.3: *Illustration of an arc of a circle*

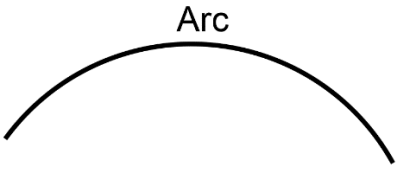


Figure 10.4: *Arc of a circle*

4. Radius

A line segment joining the centre of a circle to any point on its circumference. In ***Figure 10.1***, $|OB| = |OA| = |OC| = |OD| = |OE| =$ Radius(r).

5. Diameter

The diameter of a circle is any straight-line segment that passes through the centre of the circle and whose endpoints lie on the circumference of the circle.

The diameter is the longest chord of the circle.

In ***Figure 10.1***, |BE| = diameter = 2 × radius = $2r$.

6. Sector

Figure 10.5 is an example of a **sector**. The region is bounded by two radii and an arc. Point O is the centre of the circle from which the sector was obtained, and lines OA and OB are radii.

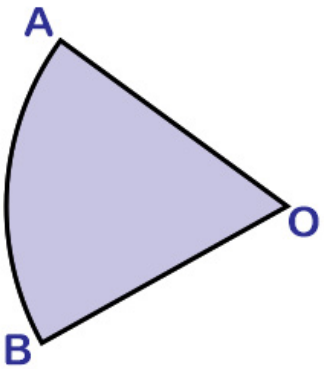


Figure 10.5: *Sector of a circle*

Major sector: A major sector has a central angle which is more than 180°.

Minor sector: A minor sector has a central angle which is less than 180°.

7. **Semicircle:** Half a circle. The central angle of a semicircle is 180 degrees. Note that a minor arc is less than a semicircle, a major arc is greater than a semicircle, and an arc whose endpoints are the endpoints of a diameter is a semicircle.
8. **Quadrant:** A quarter of a circle created by two radii perpendicular to each other. Also, defined as a sector whose central angle is 90°.
9. **Chord:** Line **CD** is called a **chord**. A straight line connecting two points on the circumference of a circle. Also, defined as a line segment whose endpoints lie on the circumference of a circle.



Figure 10.7: *Line CD*

***Note** that the diameter is also a chord. It is the longest chord.*

10. **Segment:** The space shaded pink is called a **segment.** This is the region bounded by a chord and an arc

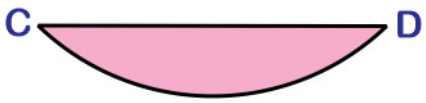


Figure 10.6: *A segment of a circle*

Major segment: a segment where the arc is greater than half the circumference.

Minor segment: a segment where the arc is less than half the circumference.

11. **Geometric Secant:** A secant to a circle is a line that crosses the circle at exactly two distinct points.
12. **Tangent:** A tangent to a circle is a straight line that intersects the circle at exactly one point, i.e., a tangent just touches the circumference of the circle.

How To Construct A Circle Of A Given Radius

We need a ruler and a pair of compasses to construct a circle with a given radius.

Example 1

Construct a circle with radius 5cm

Solution

The steps below will help you with the construction

Step 1: With the aid of a ruler, open your compass to a distance of 5cm.

Step 2: Mark a point O on a piece of paper. This point will be the centre of the circle you are about to construct.

Step 3: Place the pointer of the compass at point O.

Step 4: Keeping the compass point fixed at O, rotate the compass through a complete turn to draw the circle. ***Figure 10.8*** shows an example of the expected result.

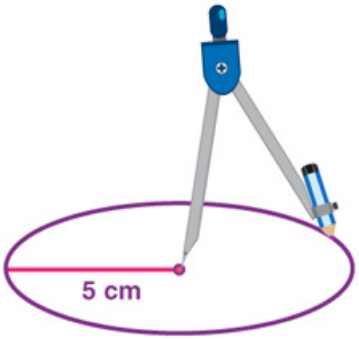


Figure 10.8: *Construction of a circle with radius 5cm.*

Note that an endless number of circles can have the same centre

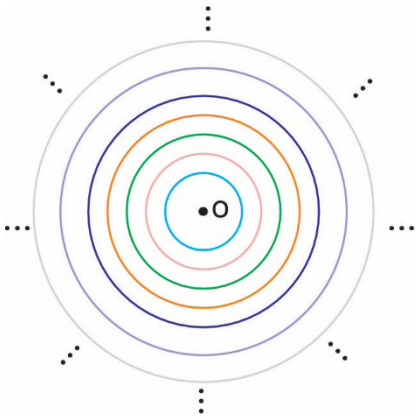


Figure 10.9: *Many circles with the same centre, O.*

Construct a circle through three points

An endless number of circles can travel through a single point, as illustrated in ***Figure 10.10.***

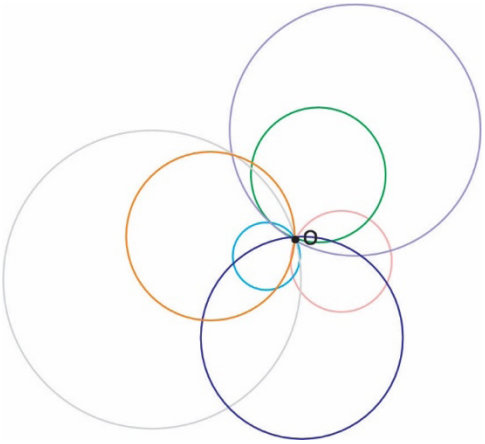


Figure 10.10: *Some circles passing through point O*

Likewise, there are infinitely many circles that can pass through two points. ***Figure 10.11*** shows some circles passing through two points, X and Y.

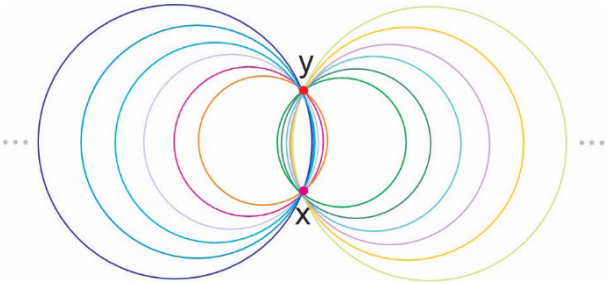


Figure 10.11: *Some circles passing through points X and Y*

However, **only ONE** circle can go through three points which are NOT on a straight line (non-collinear). This is the same as saying that exactly one circle passes through any three non-collinear points.

When such a circle is created through the vertices of a triangle, it is called a **circumscribed circle**. The centre of the circle is called a **circumcentre**. The circumcentre is equidistant from all three vertices.

How do we create such a special circle?

Let us use the example to learn how to do this.

Example 2

Figure 10.12 shows the position of three points X, Y and Z.

Using a ruler and a pair of compasses only, construct a circle passing through the three points.

Label the centre of the circle, O.

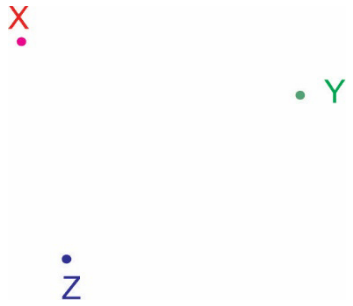


Figure 10.12: *Three points, X , Y and Z.*

Solution

All three points X, Y and Z must be equidistant from the circle's centre, O.

This means that we seek to find a point, O, such that |OX|=|OY|=|OZ|

The steps below will help you with the construction

1. **Step 1**: Join points X and Y with a straight line and draw a perpendicular bisector, l_1 , of |XY|. This is the same as constructing the locus of all points equidistant from points X and Y. Any point on l_1 will be equidistant from X and Y.

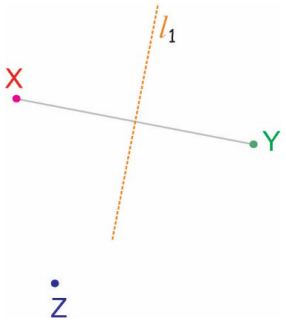


Figure 10.13: *Three points X, Y and Z and a perpendicular bisector of line XY*

2. **Step 2:** Join points X and Z with a straight line and draw a perpendicular bisector, l_2 , of |XZ|. This is the same as constructing the locus of all points equidistant from points X and Z. Any point on l_2 will be equidistant from X and Z.

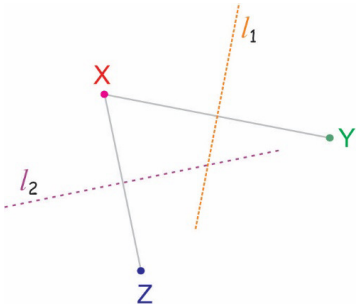


Figure 10.14: *Points X, Y and Z, perpendicular bisectors of XY and XZ*

3. **Step 3**: Indicate the point of intersection of l_1 and l_2 as O. Point O is equidistant from X, Y, and Z. This means that |OX|=|OY|=|OZ|

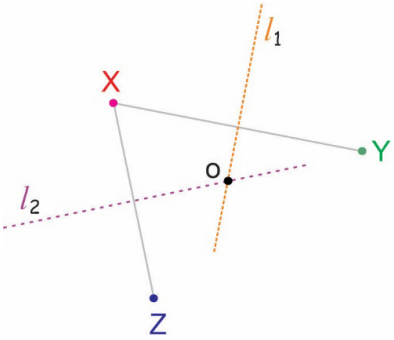


Figure 10.15: *Locating a point equidistant from X, Y and Z.*

4. With the aid of a pair of compasses, draw a circle with centre O and radius |OX| or |OY| or |OZ|.

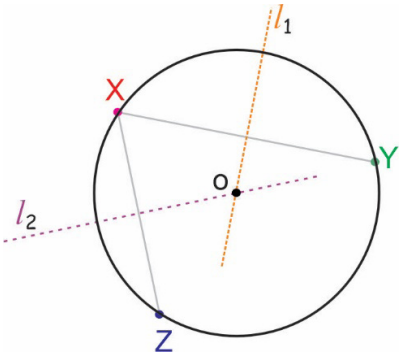


Figure 10.16: *A circle intersecting three points*

Example 3

Make a copy of the triangle below and construct a circumscribed circle (i.e., a circle that passes through all three vertices of the triangle).

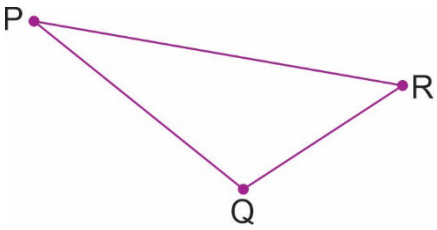


Figure 10.17: *Triangle PQR*

Solution

The steps below will help you with the construction.

Step 1: Construct a copy of the triangle. (the orientation of your copy of PQR is not important)

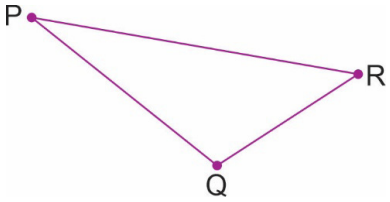


Figure 10.18: *Triangle PQR*

Step 2: Construct a perpendicular bisector of any two sides of the triangle. Label the point of intersection as O.

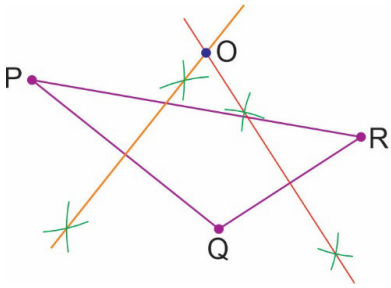


Figure 10.19: *Perpendicular bisectors of sides of a triangle.*

Step 3: With the pin of the compass at O and using |OP| or |OQ| or |OR| as the radius, construct a circle.

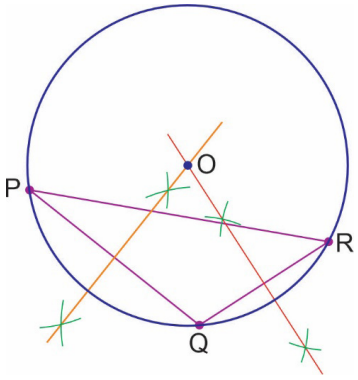


Figure 10.20: *A cyclic triangle PQR with centre O.*

Example 4

a. Using a ruler and pair of compasses only, construct a right-angled isosceles triangle DEF with its hypotenuse = 113*mm*

b. Construct a circle to intersect all three vertices of the triangle.

c. Measure the radius of the circle

d. Calculate the perimeter of triangle DEF

Solution

a.

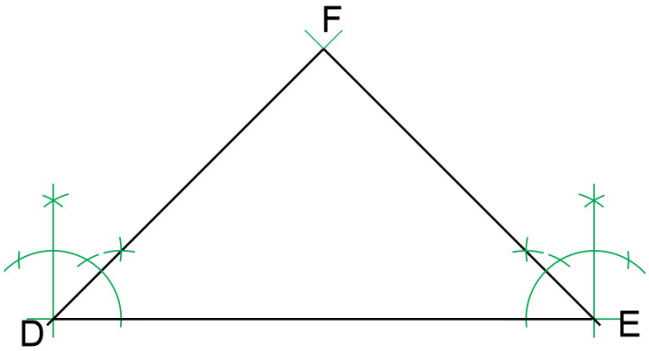


Figure 10.21: *Right-angle isosceles triangle DEF with hypotenuse 113mmnot drawn to scale*

b. To find the centre of the circle, we will construct a perpendicular bisector of lines DF and EF.

Using |OD| as the radius and O as the centre, construct a circle.

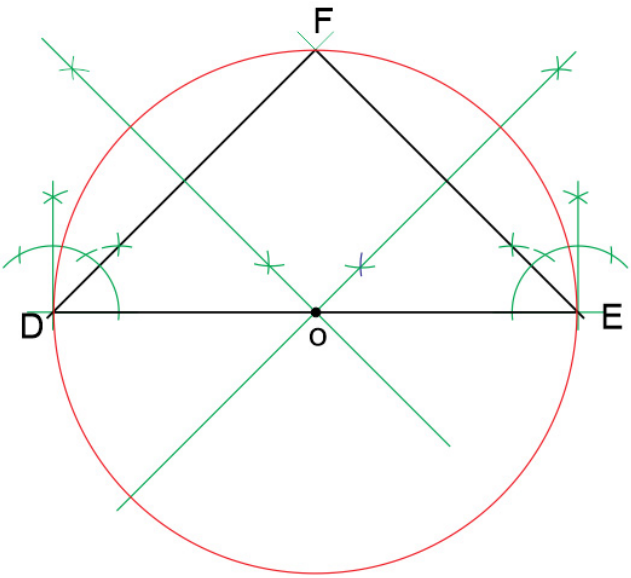


Figure 10.22: *Cyclic triangle DEF*

c. Use your ruler to measure the radius of the circle.

Radius of the circle = 56.5*mm*

Note, that in this case, the hypotenuse of the right-angled triangle is the diameter of the circle.

Hence, the radius is half of the hypotenuse $= \frac{113}{2} = 56.5mm$. We will go on to learn that any right-angled cyclic triangle will always have its hypotenuse as the diameter of its circumcircle.

d. Perimeter of triangle DEF $= 80mm + 80mm + 113mm = 273mm$

Circle Theorems

Circle theorems are a set of rules or properties related to angles, lines, and points on or in a circle. These theorems are used in geometry to answer questions relating to circles.

Circle Theorem Statements

Statement One

The angle subtended by an arc at the centre of a circle is twice the angle subtended by the same arc at any point on the remaining circumference.

In ***Figure 10.24***, A, B and C are points on the circle. O is the centre of the circle. The angle subtended by the chord, |AB|, at the centre is and at the circumference is .

The circle theorem statement one means that:

$$x = 2y \text{ or } y = \frac{1}{2}x$$

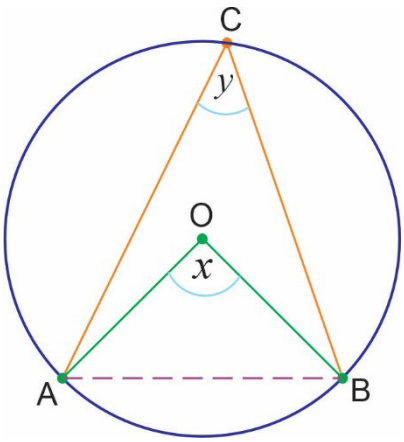


Figure 10.23: *Cyclic triangle ABC with centre O*

Statement Two

The angle subtended by the diameter at the circumference is a right angle.

In ***Figure 10.25***, O, A, and B are points on the circumference of the circle with centre V. Line AVB represents the diameter of the circle.

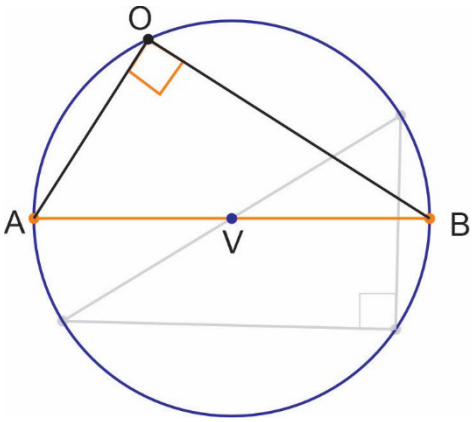


Figure 10.24: *Cyclic triangle OAB with centre V*

Angle $AOB = 90^{\circ}$

Statement Three

The angles subtended at the circumference by the same chord or arc in the same segment are equal.

In ***Figure 10.26***, O, A, B and C are points on the circumference of the circle.

Angle OCA = Angle OBA since they are subtended at the circumference by the same chord |OA|

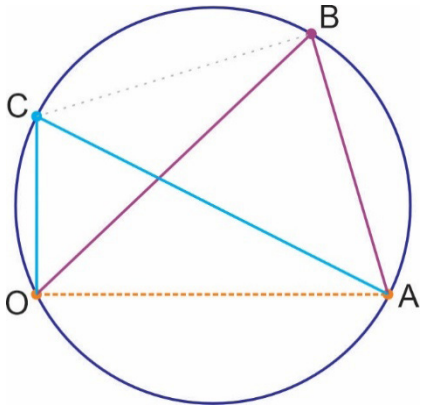


Figure 10.25: *A circle with points O, A, B and C on its circumference*

Statement Four

Two equal chords subtend equal angles at the centre of the circle.

Statement Five

If the angles subtended by two chords at the centre are equal, then the two chords are equal. Statement Five is similar to Statement Four. One implies the other. Statement Five is the converse of Statement Four.

In ***Figure 10.27***, point O is the centre of the circle and points X, Y, A and B are points on the circumference of the circle. Chord |XY|=|AB|.

From *Statement Four,* if |XY| is equal to |AB|, then angle XOY = angle AOB.

From *Statement Five*, if angle XOY = angle AOB, then |XY|=|AB|

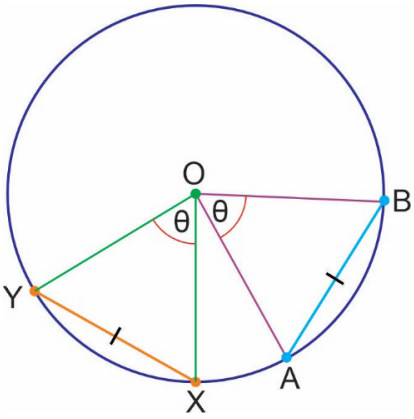


Figure 10.26: *Circle with points Y, X, A and B on its circumference. It has centre O.*

Statement Six

The opposite angles in a cyclic quadrilateral are supplementary (add up to 180˚)

Figure 10.28 shows a cyclic quadrilateral with internal angles *a, b. c* and *d*. Angles and are opposite to each other. Likewise, *b* and *d*.

This means that: ***a + c = 180˚ and b + d = 180˚***

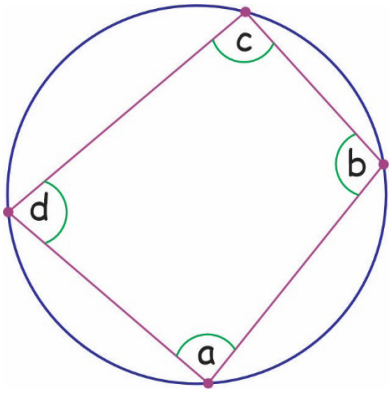


Figure 10.27: *A cyclic quadrilateral with internal angles a, b, c and d.*

Statement Seven

The angle between a tangent and a chord at a point of contact is equal to the angle in the alternate segment.

In ***Figure 10.29***, ABC is a inscribed triangle. The circle is tangent to Line DAE at point A. This means that angle BAE is equal to angle ACB. This is the same as saying $x = y$.

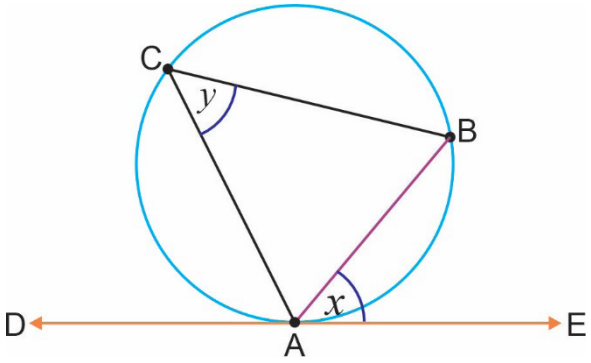


Figure 10.28: *Line DAE tangent to a circle at point A*

Proof of Circle Theorem Statements

Let us prove some of these statements.

Poof of Statement One

Statement: The angle subtended by a chord at the centre is twice the angle subtended by it at the circumference.

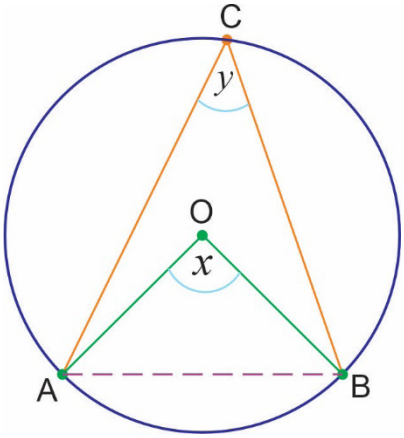


Figure 10.29: *Cyclic triangle ABC with centre O.*

Proof

In ***Figure 10.30***, we seek to prove that $x = 2y$

Draw a straight line from point C, through point O, to intersect the circle at point D.

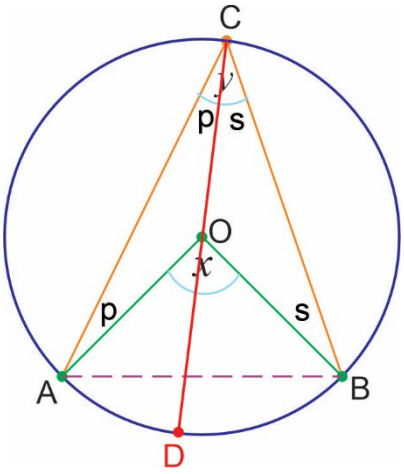


Figure 10.30: *Cyclic triangle ABC with centre O and CD as a diameter.*

This will generate two triangles. They are triangle AOC and triangle BOC.

Both triangles are isosceles since two of their sides are radii. For triangle AOC, angle OAC = angle OCA = P.

For triangle BOC, angle OBC = angle OCB = s

Remember that the *exterior angle of a triangle theorem* states that, for any triangle, the measure of the exterior angle of a triangle is equal to the sum of the measures of the two non-adjacent interior angles. Therefore, from this, we have AOD = 2p and BOD = 2s

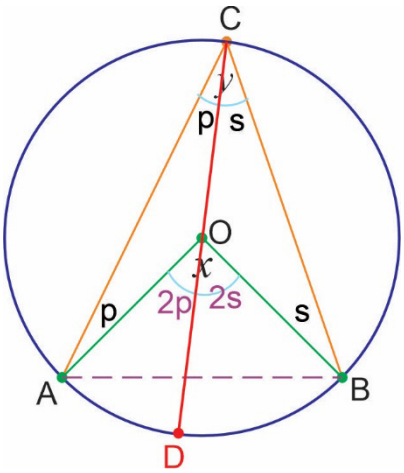


Figure 10.31: *Cyclic triangle ABC with centre O and CD as a diameter.*

This means that the angle at the centre

$$x = 2p + 2s$$

$$x = 2(p + s) \text{ .....(1)}$$

but $y = p + s$

substitute $y = p + s$ into equation (1)

This gives $x = 2y$ which proves the theorem.

Proof of Statement Two

Statement: The angle subtended by the diameter at the circumference is a right angle.

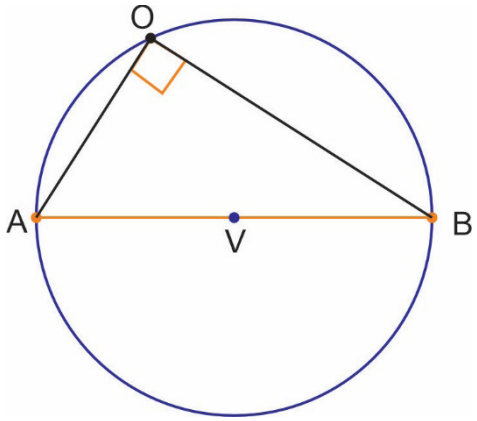


Figure 10.32: *Cyclic triangle OAB with centre V.*

Proof

We need to prove that $\angle AOB = 90°$

From ***Statement 1***, 'The angle subtended by a chord at the centre is twice the angle subtended by it at the circumference.'

So, we have $\angle AVB = 2 \times \angle AOB$.

We know that the angle formed by a straight line is 180°.

This implies that ∠AVB = 180° as AB is a straight line (diameter).

So, we have which implies that 2 × ∠AOB = 90°.

Proof of Statement Three

Statement: The angles subtended at the circumference by the same arc are equal.

Proof: Consider the following figure, which shows an arc AB subtending angles ACB and ADB at two arbitrary points, C and D, on the circumference. O is the centre of the circle.

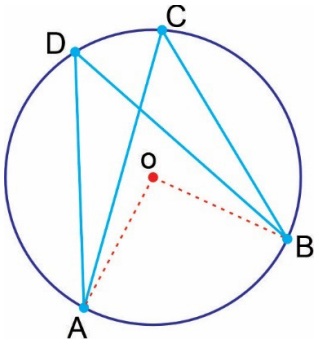


Figure 10.33: *Circle ABCD with centre O.*

We need to prove that ∠ACB = ∠ADB.

Using the circle theorem, 'The angle subtended by a chord at the centre is twice the angle subtended by it at the circumference,'

We know that $\angle ACB = \frac{1}{2} \times \angle AOB$ ……..……(1)

Likewise, $\angle ADB = \frac{1}{2} \times \angle AOB$ ……(2)

From equations (1) and (2), we get ∠ACB = ∠ADB, which proves the theorem.

Proof of Statement Four

Two equal chords subtend equal angles at the centre of the circle.

Proof: Consider the circle given below with centre O and two chords |AB| and |XY|, such that |AB| = |XY|.

We seek to prove $\angle AOB = \angle XOY$

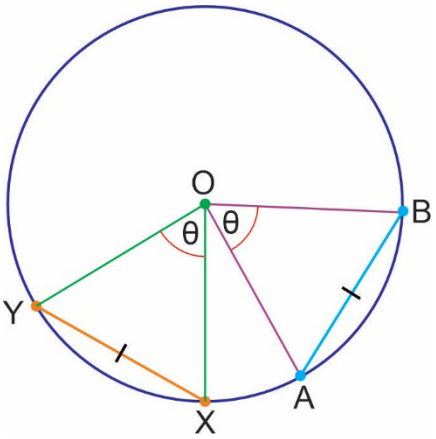


Figure 10.34: *Circle with centre O with points Y, X, A and B on its circumference.*

In triangles XOY and AOB, we have OA = OB (Radii), OX = OY (Radii) and AB = XY (Given)

So, triangles XOY and AOB are congruent by the Side-Side-Side (SSS) congruence rule.

So, we have ∠XOY = ∠AOB since corresponding angles of congruent triangles are equal.

Proof of Statement Five

Statement: If the angles subtended by two chords at the centre are equal, then the two chords are equal.

Proof: Consider ***Figure 10.36***. It is a circle with centre O and two chords, |AB| and |CD|, such that ∠AOB = ∠COD.

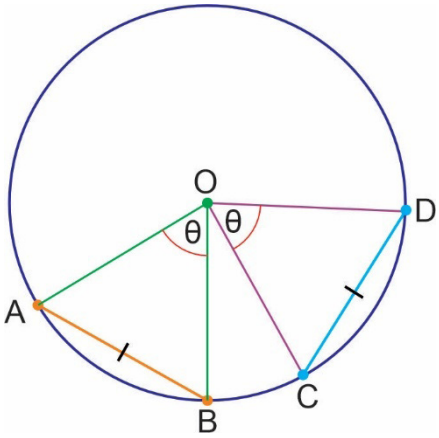


Figure 10.35: *Circle ABCD with centre O*

We need to prove that |AB| = |CD|.

In triangles AOB and COD, we have |OA| = |OC|=|OB| = |OD|= (Radii) and ∠AOB = ∠COD (Given)

So, triangles AOB and COD are congruent by the Side-Angle-Side congruence rule.

So, we have |AB| = |CD| (Corresponding parts of congruent triangles).

The proof of the remaining two statements will be dealt with in our next lesson.

Example 1

In ***Figure 10.37***, A and C are points on the circumference of the circle with centre O. Line AT is tangent to the circle at point A, angle OAC = and angle ATO = 28˚.

Calculate the measure of angle *x*

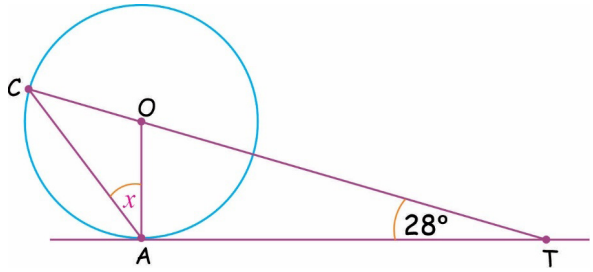


Figure 10.36: *A tangent to a circle at point A.*

Solution

From the theorem: "The angle between the radius and the tangent at the point of contact is 90" We have ∠OAT = 90°.

This implies that

$$90° + 28° + \angle AOT = 180°$$

$$\Rightarrow \angle AOT = 180° - (90° + 28°)$$

$$\text{angle } AOT = 62°$$

Since $OA = OC\ (radii\ of\ the\ circle) \Rightarrow \angle OAC = \angle OCA = x$

Using the exterior angle theorem, we have $\angle OAC + \angle OCA = 62°$

$$\Rightarrow x + x = 62°$$

$$\Rightarrow 2x = 62°$$

$$\Rightarrow x = 31°$$

Example 2

Figure 10.38 shows a cyclic quadrilateral ABCD with centre O, angle BAD = 63°, angle BOD = *y* and angle BCD = *x*.

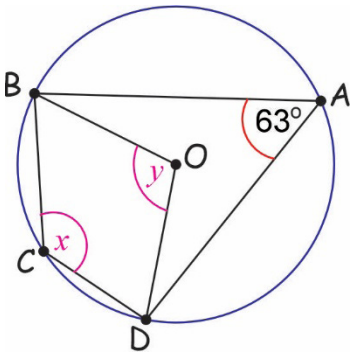


Figure 10.37: *Cyclic quadrilateral ABCD with centre O.*

Calculate the measure of angle:

i. *y*

ii. *x*

Solution

i. Since the angle subtended at the centre is twice the angle subtended at the circumference, we have $y = 2 \times 63 = 126°$.

ii. Since opposite angles in a cyclic quadrilateral add up to 180°, we have $x + 63 = 180°$

$x = 180° - 63° = 117°$

Learning Tasks

This learning module aims to equip learners with an understanding of circle theorems and their applications. Implement differentiated learning support mechanisms to address individual learning challenges.

Upon completion, learners will be able to:

1. Identify and state relevant circle theorems for given geometric situations.
2. Apply circle theorems to calculate unknown angles in circles.
3. Utilise circle theorems to solve geometric problems involving circles.

PEDAGOGICAL EXEMPLARS

When teaching circle theorems, a variety of pedagogical approaches can be employed to make the concepts more accessible, engaging, and memorable for learners.

Two examples of such are:

1. **Inquiry-Based Learning/Discovery Approach**

   With this strategy, instead of directly stating the theorems, guide learners to discover the relationships themselves through hands-on activities. To do this, first provide pre-drawn circles with various points and lines (chords, tangents, angles at the centre). Then, ask learners to measure angles using protractors and observe patterns. For example, have them draw several angles subtended by the same arc at the circumference and compare their measures. Finally, pose guiding questions that lead to the formulation of the theorem. Encourage group discussions and conjectures.

2. **Group work/collaborative learning**

   Using this strategy:

   a. Review learners' knowledge of parts of a circle. Address their misconceptions about such concepts, and in pairs, draw circles with various radii.

   b. In groups task learners to engage in discussions to recall the definition of circles and their properties.

   c. Review learners' knowledge on the various circle theorems and their proofs.

   d. Engage learners to apply the circle theorem statements to answer questions.

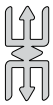
KEY ASSESSMENT LEVEL 3: RESEARCH

1. Provide a comprehensive note with supported diagrams and examples on any five (5) of the circle theorems you have learnt.
2. Research and write a page about the usefulness of these circle theorems in real life.

PLC Session 10

45 minutes | **Planning & Reviewing the Learning Plan**

1. Begin the session by recapping lessons from the previous year/session/week, focusing on how learning was applied.
2. Use your subject specific app to explore the content before planning. Ask the app to
   a. explain the week's concepts in a practical way that will make it easier for learners to understand
   b. identify two or three common misconceptions learners may have on the week's content and how to address them using contexts familiar to Ghanaian learners. (NTS 3a, 3g and 3m).
3. Use your subject specific app to plan your lesson. Ask the app to
   a. suggest the most effective method for teaching the week's concepts to learners with diverse backgrounds.
   b. suggest learning activities, differentiation and essential questions based on the method of teaching (NTS 3a-3q)
   c. review your prepared learning plan and address any challenging issues you may have identified in your subject area (NTS 3a-3c).



Note

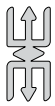
- Take note of the recap topics linked to Years 1 and 2 while preparing for your lesson.
- Refer to the Values Learning Community Handbook and link the values for the week in your lesson to ensure that learners are applying what they learn.

4. Plan the week's key assessment with your app
   a. The key assessment for the week is ***research***. Chat with your app to plan this assessment, indicating what you and your learners will do before, during and after the research (NTS 3k-3q).

10 minutes | **Content Exploration**

5. As a subject group, select one problem or idea/term in the week's content that is challenging (not a routine procedural question, but one that requires conceptual understanding) and then
   a. use the app, ask it to solve the problem step by step and explain the reasoning behind each step (you may be assisted by a teacher who is strong in the content area)
   b. all teachers in the group work through a similar problem individually for 3–4 minutes
   c. the group then works through the problem together step by step

d. highlight the most common errors made at each step and discuss (NTS 1c, 1e, 2c, 2e, 3e-3j).



Note

1. This part is not just about 'How do I teach this?' but how do I teach it for learners to understand?
2. If the group still shares the same content gap, use the app: ask it to solve the problem step by step and explain the reasoning behind each step.
3. Problems/ideas discussed should be sourced from the Teacher Manual content, not invented separately. This keeps the content exploration directly connected to what teachers will deliver.

30 minutes Enactment

6. Enact a teaching activity based on your learning plan (NTS la-1b, 1e-lg, 2c, 2e, 2f, 3c-3j and 3o).

a. Colleagues observe with specific attention to clarity of concept explanation, use of familiar Ghanaian examples and how misconceptions are handled if they arise.

b. After the enactment, ask for structured feedback from the group

i. was the concept explained in a way that a learner with no prior knowledge could follow?

ii. were the examples grounded in contexts familiar to Ghanaian learners?

iii. did any misconceptions surface and if so, how were they addressed?

5 minutes Reflection

7. Reflect and share your views on the session. One thing that worked well, what did not work well and what you will do differently. (NTS 1a, 1b).

8. Identify a critical friend to observe and provide feedback on your lesson in relation to the PLC Session conducted (NTS 1f, 1g).

9. Remember to

a. transfer your chats into your learning planner template.

b. read your Teacher Manual and related Learner Material in preparation for the next session. Take note of the recap weeks, which are linked to Years 1 and 2 (NTS 3a).

c. integrate National Values, GESI, SEL, and 21st century skills into your learning activities, assessment tasks, and rubrics/mark schemes.

WEEK 11

Learning Indicators

1. *Identify the tangent as perpendicular to the radius at the point of contact and verify the Alternate Segment Theorem*
2. *Verify that tangents drawn from an external point to the same circle are equal when measured from their point of contact*

FOCAL AREA 1: THE TANGENT OF A CIRCLE 1

A tangent to a circle is a straight line that touches the circle at exactly one point, called the point of contact. There can be an infinite number of tangents to a circle. These tangents follow certain properties that can be used as identities to perform mathematical computations on circles.

We will learn how to prove two of those properties in detail.

Our focus is to prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

In ***Figure 11.1***, O is the Centre of the circle and line L is tangent to the circle at point A.

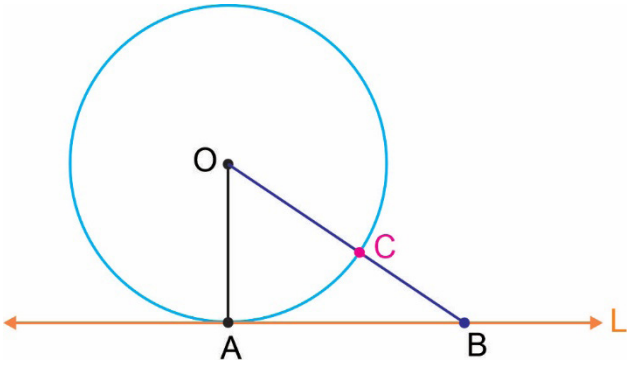


Figure 11.1: *A tangent to a circle at point A*

From prior knowledge, we know that for all line segments drawn from a point (say O) to a line (say L), the shortest line is perpendicular to L.

In ***Figure 11.1***, |OA|=|OC| as they are radii of the circle.

Also, |OB| = |OC| + |CB|. So clearly, |OC| < |OB| and |OA| < |OB|

This logic holds for every point B on the tangent, L, except point A.

So |OA| is shorter than any other line segment joining O to any point on L. **Hence, OA is perpendicular to line L**

Therefore, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Let us use this understanding to answer a question.

Example 1

In the diagram, O is the centre of the circle.

Line AC is tangent to the circle at B and $BOC = 120^{\circ}$. Calculate the value of ABD.

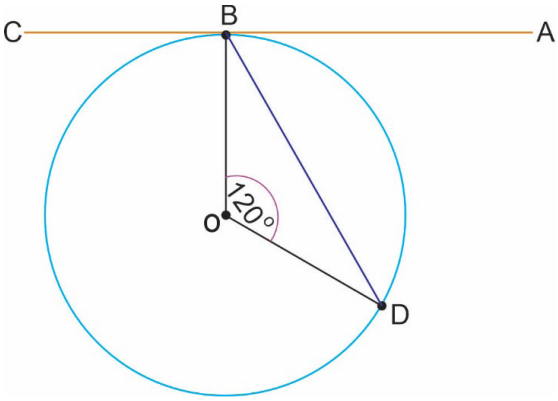


Figure 11.2: *A tangent to a circle at point B*

Solution

The triangle BOD is isosceles, since |OB|=|OD|=radius. This implies angle OBD = angle ODB

Let OBD = ODB = x

$x + x + 120° = 180°$

$2x = 180° - 120°$

$2x = 60°$

$x = 30°$

But $OBA = 90°$

$ABD = 90° - x = 90 - 30 = 60°$

$\therefore\ ABD = 60°$

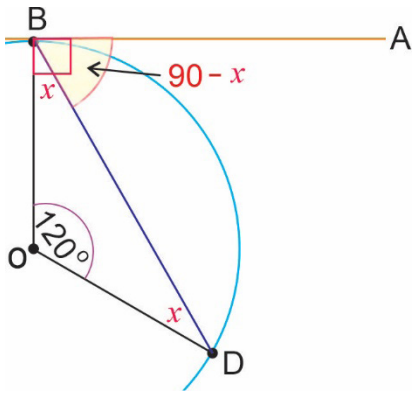


Figure 11.3: *Triangle BOD*

Theorem 2

Alternate Segment Theorem

The Alternate Segment Theorem states that "For any circle, the angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment subtended by that chord."

The alternate segment theorem is also known as the tangent-chord theorem.

Let us use ***Figure 11.4*** to break down the theorem.

In the Figure, ABC is a cyclic triangle with centre O. The circle is tangent to Line DAE at point A. The shaded part is a minor segment. Its alternate segment is the unshaded part of the circle. According to the alternate segment theorem, angle BAE is equal to ACB. i.e. $x = y$

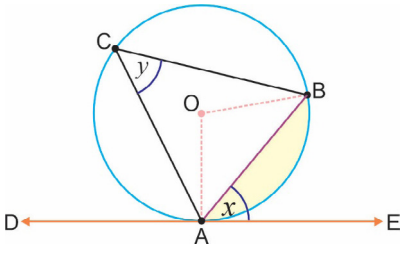


Figure 11.4: *Tangent to a circle at point A*

Proof

From our previous proof, we established that the angle the radius or the diameter makes with the tangent at the point of tangency is 90˚. Therefore, OAE = 90

This implies that OAB + x = 90

solving for OAB, we have OAB = 90 – x

Since triangle OAB is isosceles,

$OAB = OBA = 90 - x$

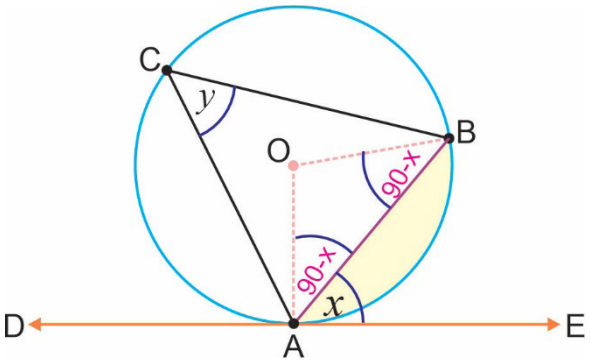


Figure 11.5: *Tangent to a circle at point A*

Also, $AOB + (90 - x) + (90 - x) = 180°$

solving for AOB, we have $AOB = 180 - 90 - 90 + 2x$

This implies $AOB = 2x$……(1)

Recall that the angle at the centre of the circle is twice the angle at the circumference of the circle. This means that, $AOB = 2y$……..(2)

Comparing (1) to (2), we have $2x = 2y$

This means $x = y$, proving the theorem.

Example 2

Find the unknown angles (angle ACP and angle BAC) in the figure, given that the chord BC makes the angle 65° with the tangent line PQ.

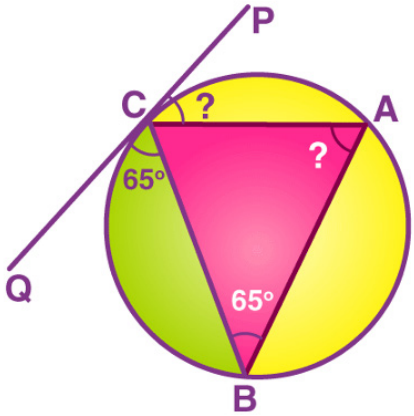


Figure 11.6: *A tangent to a circle at point C*

Solution

Given that, ∠QCB = 65°. By using the alternate segment theorem, we can say that ∠CAB = 65°. Similarly, by using the angles in the alternate segment, ∠PCA = 65° Therefore, ∠CAB = 65° and ∠PCA = 65°.

Example 3

Find the angle ∠QPS in the given figure.

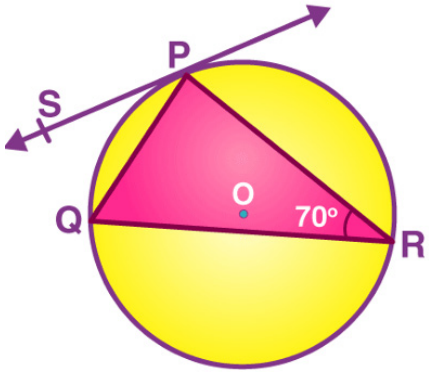


Figure 11.7: *A tangent to a circle at point P*

Solution

Given that, ∠PRQ = 70°.

By using the alternate segment theorem, ∠QPS = ∠PRQ. Hence, ∠QPS = 70°.

Alternate Segment Theorem for Quadrilaterals

In ***Figure 11.8***, line DAE is a tangent to the circle at point A. BCD are points on the circumference of the circle and ABCD is a cyclic quadrilateral.

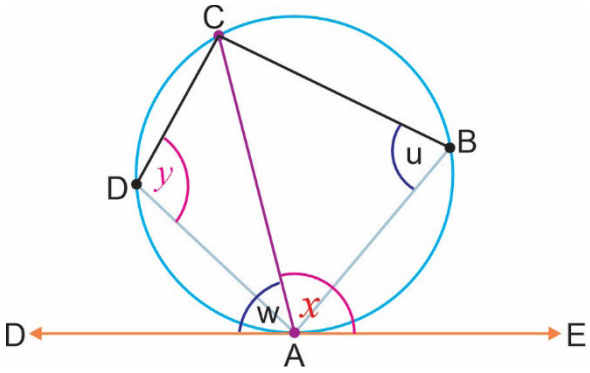


Figure 11.8: *Line DAE tangent to cyclic quadrilateral ABCD at point A*

From the alternate segment theorem, ∠w = ∠u ……..... (1)

We need to prove that ∠ = ∠

We know from our previous theorems that opposite angles of a cyclic quadrilateral are supplementary; their sum is 180°. This means ∠y + ∠u = 180° ……..…(2)

Since ∠w and ∠x are adjacent at point A on a straight line, we can conclude that

∠w + ∠x = 180° ……..…..(3)

From (2) and (3), we have ∠y + ∠u = ∠w+ ∠x

from (1), we have ∠w = ∠u

Therefore, ∠y + ∠w = ∠w+ ∠x

Eliminating ∠w, we have ∠y = ∠x

Theorem 3

Tangents drawn from the same external point to a circle are equal in length.

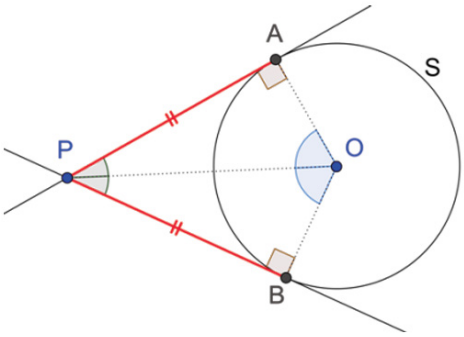


Figure 11.9: *Lines PA and PB tangent to a circle with centre O at points A and B.*

In ***Figure 11.9***, lines AP and BP are tangents to circle, S, at points A and B. point O is the centre of the circle. According to the equal tangent theorem,

|PA| = |PB|. Also, ∠POA = ∠POB and ∠APO = ∠BPO

Proof: All three parts will be proved if we show that ΔPAO is congruent to ΔPBO.

Comparing the two triangles, we see that:

1. OA = OB (radii of the same circle)
2. OP = hypotenuse of both triangles.
3. From our first theorem, ∠OAP = ∠OBP = 90. So, by the RHS congruence (right angle–hypotenuse–side) property of triangles, triangle OAP is congruent or the same as triangle OBP. This follows that the lengths of the tangents BP and AP are equal. Also, line OP bisects BPA and AOB.

   Hence, angle BOP = angle AOP and angle BPO = angle APO.

Example 4

Consider a chord AB of length 9 cm in a circle of radius 5 cm. Tangents at A and B intersect at C, as shown below:

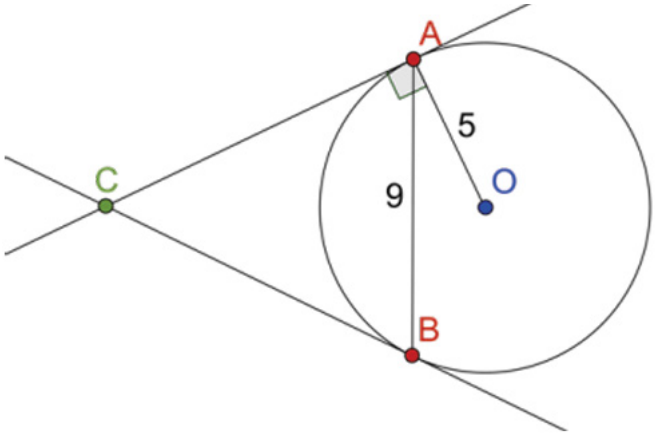


Figure 11.10: *Lines AC and BC are tangent to a circle at points A and B, with the centre at O.*

What are the lengths of the tangents CA and CB?

Solution

Connect point O to C with a straight line to intersect AB at D

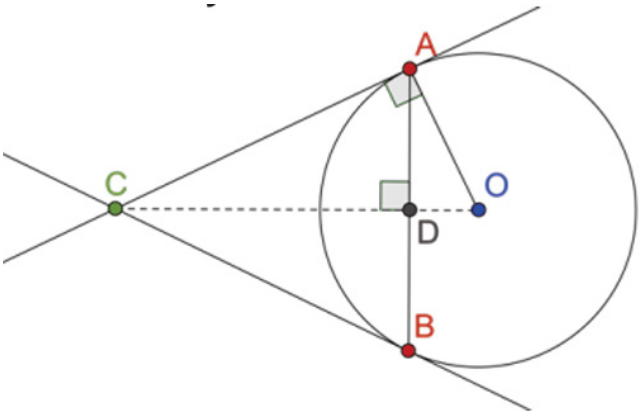


Figure 11.11: *Lines AC and BC tangent to a circle with centre O at points A and B*

Since AB = 9 units, AD = $\frac{1}{2} \times 9 = 4.5$ units.

Sin(AOC)=$\frac{4.5}{5}$

AOC= $\text{Sin}^{-1}(0.9) = 64.16$

Tan 64.16= $\frac{|AC|}{5}$

$|AC| = 5tan64.16 = 10.32$ units.

Therefore, the lengths of the tangents AC and BC is 10.32 units

Learning Tasks

This learning module aims to equip learners with an understanding of circle theorems and their applications. Implement differentiated learning support mechanisms to address individual learning challenges.

Upon completion, learners will be able to:

1. Identify and state relevant circle theorems for given geometric situations.
2. Apply circle theorems to calculate unknown angles in circles.
3. Utilise circle theorems to solve geometric problems involving circles.

PEDAGOGICAL EXEMPLARS

When teaching circle theorems, a variety of pedagogical approaches can be employed to make the concepts more accessible, engaging, and memorable for learners.

Two of such are:

1. **Inquiry-Based Learning/Discovery Approach**

   With this strategy, instead of directly stating the theorems, guide learners to discover the relationships themselves through hands-on activities. To do this, first provide pre-drawn circles with various points and lines (chords, tangents, angles at the centre). Then, ask learners to measure angles using protractors and observe patterns. For example, have them draw several angles subtended by the same arc at the circumference and compare their measures. Finally, pose guiding questions that lead to the formulation of the theorem. Encourage group discussions and conjectures.

2. **Group work/collaborative learning.**

   Using this strategy:

   a. Review learners' knowledge of parts of a circle. Address their misconceptions about such concepts, and in pairs, draw circles with various radii.

   b. In groups task learners to engage in discussions to recall the definition of circles and their properties.

   c. Review learners' knowledge on the various circle theorems and their proofs.

   d. Engage learners to apply the circle theorem statements to answer questions.

KEY ASSESSMENT LEVEL 3: CLASS EXERCISE

1. In ***Figure 11.12***, X, Y and Z are points on the circumference of the circle. V is the centre of the circle. OZ and OX are tangents to the circle, OX = 6 cm and XYZ = 113°.

   Find

   a. XOZ — ***5 marks***

   b. Perimeter of quadrilateral OXVZ — ***4 marks***

   Total *9 marks*

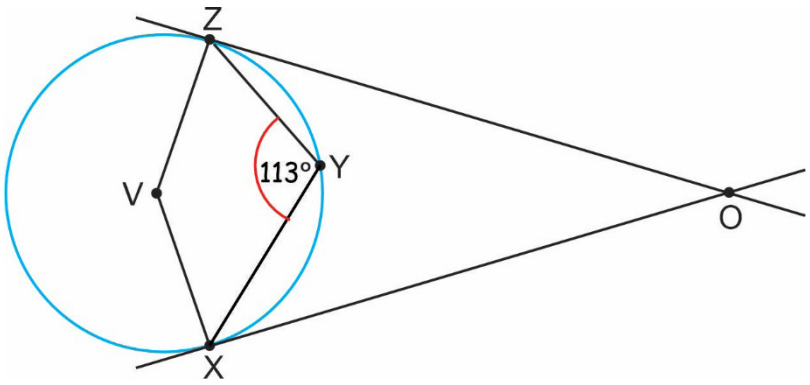


Figure 11.12: *Lines OX and OZ tangent to a circle at points X and Z*

2. In ***Figure 11.13***, OABC is a cyclic quadrilateral. Line DE is a tangent to the circle at point O. |OB| is the diameter of the circle, BOC = 57° and ACB = 50°

a. Calculate

i. ABC *3 marks*

ii. AOE *2 marks*

b. If the diameter of the circle is 10cm, find the perimeter of the quadrilateral OABC *6 marks*

Total *11 marks*

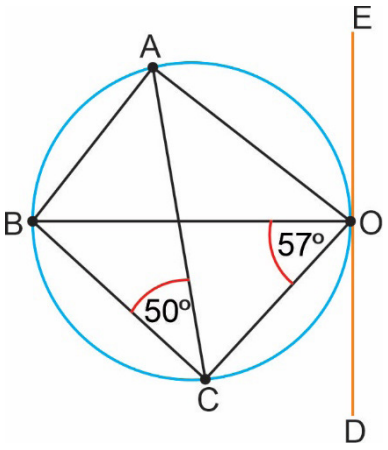


Figure 11.13: *Line DOE tangent to cyclic quadrilateral OABC at point O*

PLC Session 11

45 minutes **Planning & Reviewing the Learning Plan**

1. Begin the session by recapping lessons from the previous year/session/week, focusing on how learning was applied.

2. Use your subject specific app to explore the content before planning. Ask the app to

a. explain the week's concepts in a practical way that will make it easier for learners to understand

b. identify two or three common misconceptions learners may have on the week's content and how to address them using contexts familiar to Ghanaian learners. (NTS 3a, 3g and 3m).

3. Use your subject specific app to plan your lesson. Ask the app to

a. suggest the most effective method for teaching the week's concepts to learners with diverse backgrounds.

b. suggest learning activities, differentiation and essential questions based on the method of teaching (NTS 3a-3q)

c. review your prepared learning plan and address any challenging issues you may have identified in your subject area (NTS 3a-3c).

Note

- Take note of the recap topics linked to Years 1 and 2 while preparing for your lesson.
- Refer to the Values Learning Community Handbook and link the values for the week in your lesson to ensure that learners are applying what they learn.

4. Plan the week's key assessment with your app

a. The key assessment for the week is ***class exercise***. Chat with your app to plan this assessment, indicating what you and your learners will do before, during and after the exercise (NTS 3k-3q).

10 minutes **Content Exploration**

5. As a subject group, select one problem or idea/term in the week's content that is challenging (not a routine procedural question, but one that requires conceptual understanding) and then

a. use the app, ask it to solve the problem step by step and explain the reasoning behind each step (you may be assisted by a teacher who is strong in the content area)

b. all teachers in the group work through a similar problem individually for 3–4 minutes

c. the group then works through the problem together step by step

d. highlight the most common errors made at each step and discuss (NTS 1c, 1e, 2c, 2e, 3e-3j).

Note

- This part is not just about 'How do I teach this?' but how do I teach it for learners to understand?
- If the group still shares the same content gap, use the app: ask it to solve the problem step by step and explain the reasoning behind each step.
- Problems/ideas discussed should be sourced from the Teacher Manual content, not invented separately. This keeps the content exploration directly connected to what teachers will deliver.

30 minutes **Enactment**

6. Enact a teaching activity based on your learning plan (NTS la-1b, 1e-lg, 2c, 2e, 2f, 3c-3j and 3o).

a. Colleagues observe with specific attention to clarity of concept explanation, use of familiar Ghanaian examples and how misconceptions are handled if they arise.

b. After the enactment, ask for structured feedback from the group

i. was the concept explained in a way that a learner with no prior knowledge could follow?

ii. were the examples grounded in contexts familiar to Ghanaian learners?

iii. did any misconceptions surface and if so, how were they addressed?

5 minutes **Reflection**

7. Reflect and share your views on the session. One thing that worked well, what did not work well and what you will do differently. (NTS 1a, 1b).

8. Identify a critical friend to observe and provide feedback on your lesson in relation to the PLC Session conducted (NTS 1f, 1g).

9. Remember to

a. transfer your chats into your learning planner template.

b. read your Teacher Manual and related Learner Material in preparation for the next session. Take note of the recap weeks, which are linked to Years 1 and 2 (NTS 3a).

c. integrate National Values, GESI, SEL, and 21st century skills into your learning activities, assessment tasks, and rubrics/mark schemes.

WEEK 12

Learning Indicator: *Recall the construction of various angles, such as 75°, 105°, 135° and 150°*

FOCAL AREA: UNDERSTANDING GEOMETRIC CONSTRUCTIONS

Construction of 150° angle

Method 1

Step 1: Construct 120°

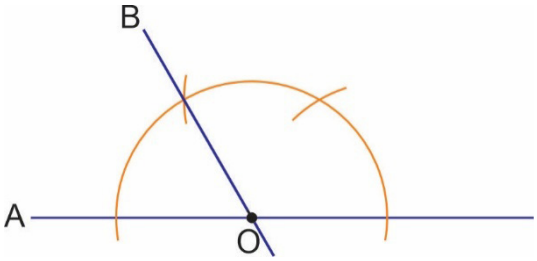


Figure 12.1: *Construction of angle 120°*

Step 2: Bisect angle AOB

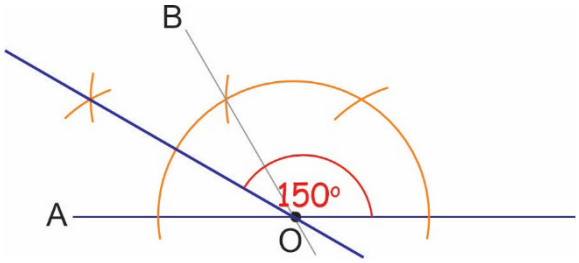


Figure 12.2: *Construction of 150°*

Method 2

Step 1: Construct 90°

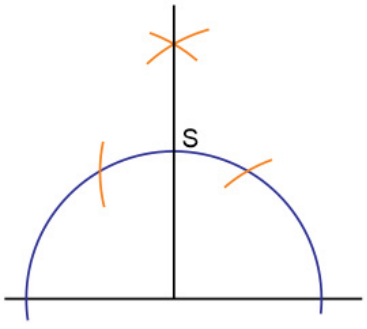


Figure 12.3: *Construction of 90°*

Step 2: With the pin of the compass at S and using radius |PS|, construct an arc to intersect arc TS at O.

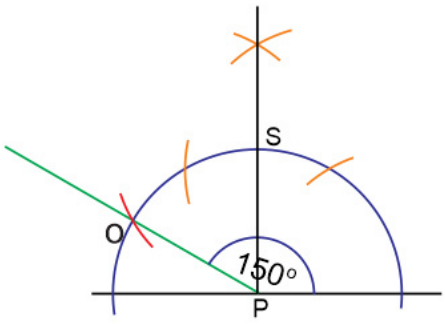


Figure 12.4: *Construction of 150°*

Use your protractor to verify your answer.

Construction of 75°

Method 1

Construct a 60° angle adjacent to a 90° angle and bisect the angle between them.

This gives $60° + \frac{1}{2} \times 30° = 60° + 15° = 75°$

Step 1: Construct 60° adjacent to 90°

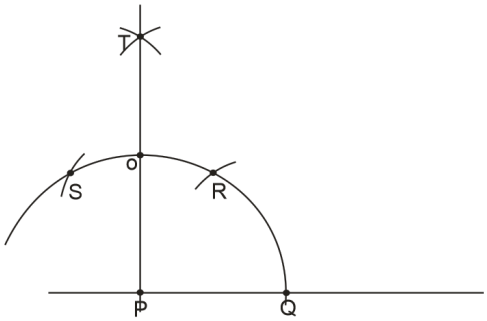


Figure 12.5: *Construction of 90°*

Step 2: Bisect acute angle TPR.

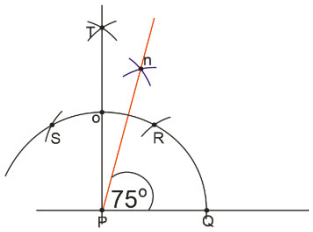


Figure 12.6: *Construction of 75°*

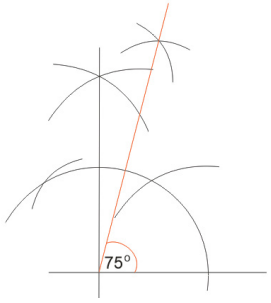


Figure 12.7: *Construction of 75°*

Method 2

First construct 150°, as above, and then bisect it.

Step 1: Construct 150°

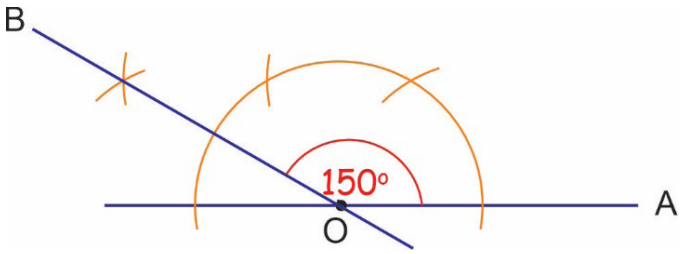


Figure 12.8: *Construction of 150°*

Step 2: Bisect 150°

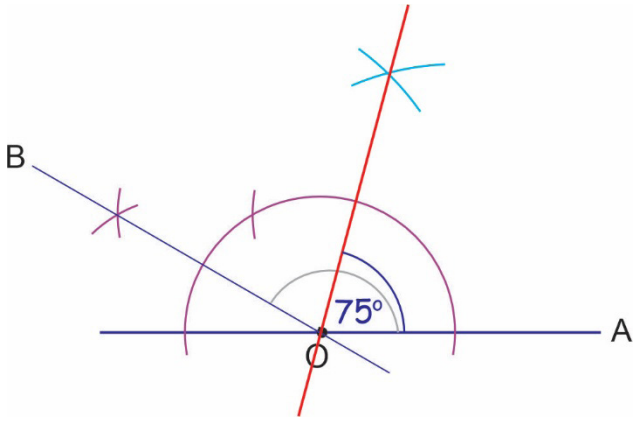


Figure 12.9: *Bisection of 150°*

Construction of 105°

Construct 120° adjacent to 90° and bisect the angle between them.

Step 1: Construct 120° adjacent to 90°.

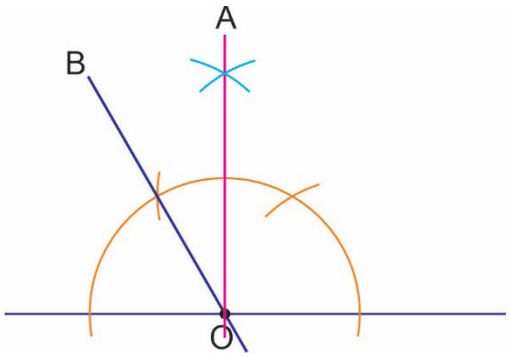


Figure 12.10: *Construction of 90° adjacent to 120°*

Step 2: Bisect AOB

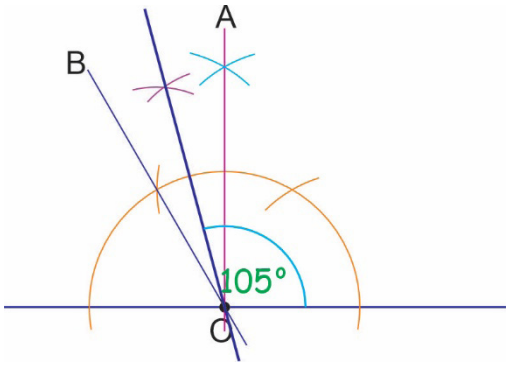


Figure 12.11: *Construction of 105°*

Construction of 135°

Step 1: Construct 90°

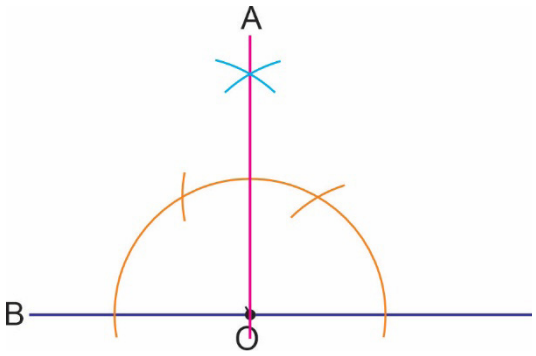


Figure 12.12: *Construction of 90°*

Step 2: Bisect BOA

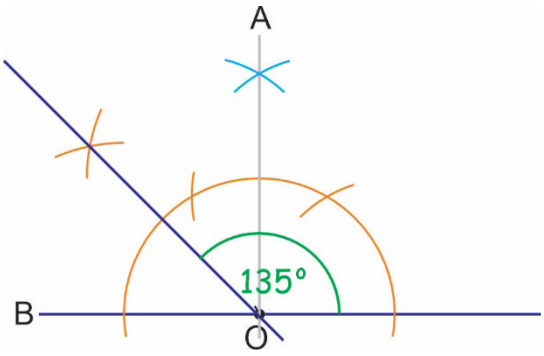


Figure 12.13: *Construction of 135°*

Learning Tasks

Guide learners to solve tasks related to the construction of acute and obtuse angles using appropriate instruments. Provide support systems for learners who may encounter difficulties.

Learners are to use a straight edge (for example, a ruler), a pencil, and a pair of compasses only to construct

1. 75°
2. 105°
3. 135°
4. 150°

PEDAGOGICAL EXEMPLARS

The aim of the lessons for the week is for all learners to be able to use appropriate instruments to construct standard angles. The following pedagogical approach is suggested for teachers to take learners through.

1. Think-pair-share activities*:*
2. In pairs, engage learners to review on the construction of angles from the Junior High School.
3. Group discussions**:**
   - In convenient groups, learners discuss and draw various angles using the appropriate mathematical tools.

KEY ASSESSMENT: PRACTICE PAPER 1

Recommended mode of assessment for week 12 is homework on standard level 2 essay questions.

Home Work

Construction of Angles

Name: ……

Instructions and information

- Use a pencil, ruler, and pair of compasses only to answer the questions.
- Answer all questions.
- Answer the questions in the spaces provided– there may be more space than you need.
- You must show all your working, including any construction marks.
- The marks for each question are shown in brackets.

Advice

- Please read each question carefully before you start to answer it.
- Use your protractor to validate your answers.

1. Construct the bisector of AOB **(*2 marks*)**

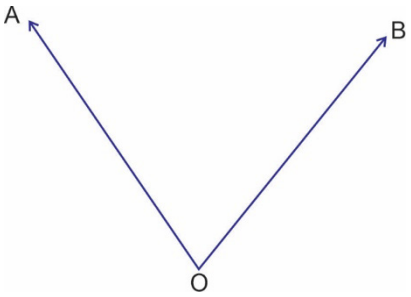


Figure 12.14: *Lines OA and OB intersecting at point O*

2. Construct a line from point O to intersect line L at right angles. **(*2 marks*)**

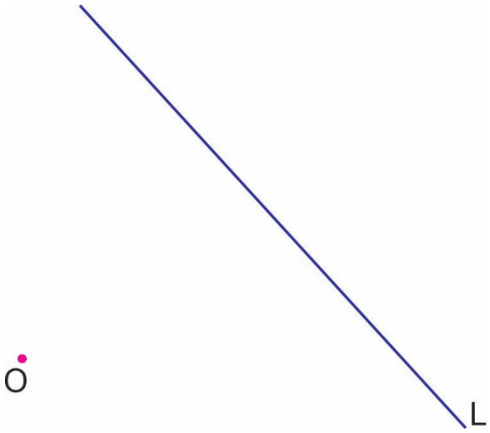


Figure 12.15: *Line L and point O*

3.

a. Construct AOB = 30° **(*2 marks*)**

b. Construct OAB = 105° **(*3 marks*)**



Figure 12.16: *Vertical line OA*

4.

a. Construct a line from point A to intersect arc L at D such that BAD = 135° **(*2 marks*)**

b. Construct a line from point B to intersect arc M at C such that ABC = 150°

(*2 marks*)

c. Join points C and D with a straight line and measure the perimeter of the quadrilateral ABCD. **(*1 mark*)**



Figure 12.17: *Line AB and arcs L and M*

5.

a. Construct a line from point O to intersect line L at B such that AOB = 75° **(*2 marks*)**

b. Measure ABO **(*1 mark*)**

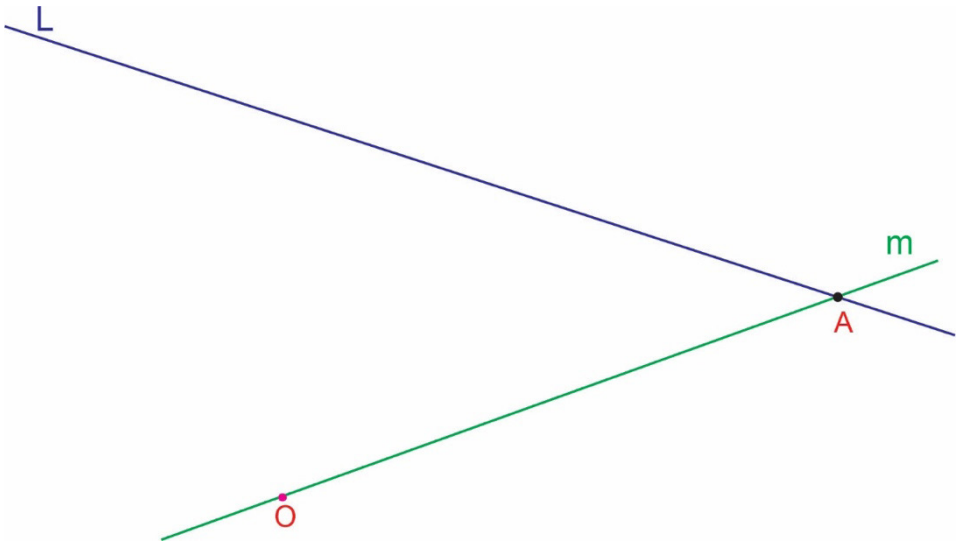


Figure 12.18: *lines m and L intersecting at point A*

Hint

*Conduct **Practice Paper 1**. It should cover contents taught from Year 1 to 3. Refer to the **Table of Specification and Structure of the Paper** at the end of this section. Learners' papers should be marked immediately and scores entered into the SAP.*

PRACTICE MOCK 1

TABLES OF SPECIFICATION

Test Specification (MCQ)

STRAND	SUB-STRAND	DOK LEVELS			TOTAL	
		1	2	3		
Numbers for everyday life	Real Number and Numeration System	1	2		3	5
	Proportional Reasoning	2	0		2	
Algebraic Thinking	Applications of expressions, equations, and inequalities	5	2		7	13
	Patterns and relationships	1	3	2	6	
Geometry Around Us	Spatial sense	2	5	2	9	14
	Measurement	2	3		5	
Making sense of and using data.	Statistical reasoning and its application in real life	3	3		6	8
	Chance		2		2	
TOTAL		16 (40%)	20 (50%)	4 (10%)	40 (100%)	40 (100%)

PAPER II

SECTION A

Section A Test Specification

STRAND	SUB-STRAND	DOK LEVELS			TOTAL	
		1	2	3		
Numbers for everyday life	Real Number and Numeration System		1	1	2	3
	Proportional Reasoning		1		1	
Algebraic Thinking	Applications of expressions, equations and inequalities	1	2		3	3
	Patterns and relationships				0	
Geometry Around Us	Spatial sense			1	1	2
	Measurement		1		1	
Making sense of and using data.	Statistical reasoning and its application in real life		2		2	2
	Chance				0	
TOTAL		1 (10%)	7 (70%)	2 (20%)	10 (100%)	10 (100%)

Part II-Section B

Test Specification

STRAND	SUB-STRAND	DOK LEVELS			TOTAL	
		1	2	3		
Numbers for everyday life	Real Number and Numeration System			1	**1**	**3**
	Proportional Reasoning	1	1		**2**	
Algebraic Thinking	Applications of expressions, equations, and inequalities		1		**1**	**2**
	Patterns and relationships			1	**1**	
Geometry Around Us	Spatial sense		1		**1**	**2**
	Measurement		1		**1**	
Making sense of and using data.	Statistical reasoning and its application in real life	1	1		**2**	**3**
	Chance		1		**1**	
TOTAL		**20**	**40 (80%)**	**20 (20%)**	**10 (100%)**	**10 (100%)**

Test Specification (with Indicators)

MULTIPLE CHOICE QUESTIONS

STRAND	SUB-STRAND	Learning Indicators	DOK LEVELS			TOTAL	
			1	2	3		
Numbers for everyday life	Real Number and Numeration system	**1.1.1.LI.1** Determine the properties of subsets (for two-set problems), their vocabulary and operations, and use them to solve real-life problems.	1			1	5
		1.1.2.LI.1 Establish the concept of fractions and investigate the connections between fractions and decimal numbers.	1			1	
		2.1.1.LI.2 Explain the concepts of indices and logarithms with examples.		1		1	

	Proportional Reasoning	**3.1.2.LI.1** Use proportional reasoning to investigate the various types of variation (direct and inverse) and extend this to make generalisations.	1	1		2	
Algebraic Thinking	Applications of expressions, equations and inequalities	**1.2.1.LI.1** Use numbers, patterns, and variables to formulate mathematical expressions and apply the algebraic order of the four operations to solve problems.					13
		1.2.1.LI.2 Factorise algebraic expressions involving quadratic trinomials.		1		1	
		1.2.1.LI.4 Analyse and apply operations on simple algebraic fractions involving monomial and binomial denominators and determine the conditions under which an algebraic fraction is zero or undefined.	1	1		2	
		1.2.1.LI.1 Construct and interpret formulae for a given task and apply them to problems involving a change of subjects.	1			1	
		1.2.1.LI.2 Solve linear equations and inequalities in one variable for a given problem and relate it to real-life situations.	2	1		3	
		2.2.1.LI.1 Solve simultaneous linear equations involving two variables using the graphical method and interpret them					
		2.2.1.LI.2 Analyse two linear equations in two variables and solve them using the elimination and substitution methods.					

		2.2.1.LI.3 Analyse, model, and solve word problems of simultaneous linear equations involving numbers, age, etc.					
	Patterns and relationships	**1.2.2.AS.1** Extend the knowledge of the coordinates of two points to find the gradient and equation of a straight line.		1	1	2	
		2.2.2.LI.2 Recognise and find the nth term and the sum of the nth term of an arithmetic progression (AP) or linear sequence.			1	1	
		3.2.2.L1.1 Identify and solve quadratic equations.	1			1	
		3.2.2.L1.2 Solve quadratic equations graphically and find the maximum and minimum points of quadratic graphs.		1		1	
		3.2.2.L1.3 Identify and explain the axis of symmetry, write its equation and solve linear and quadratic equations simultaneously using graphs.	1			1	
Geometry Around Us	Spatial sense	**1.3.1.LI.1** Draw and describe angles with various measures, including acute, right, straight, obtuse, and reflex angles, and solve problems on them.		1		1	14
		1.3.1.LI.2 Solve problems that involve parallel lines, perpendicular lines transversal, and pairs of angles formed between them.		1		1	
		1.3.1.LI.3 State and apply the exterior angle theorem of a triangle to solve problems and identify various properties of special triangles.			1	1	

		1.3.1.LI.5 State and use the properties of quadrilaterals and calculate the sums of interior angles and exterior angles of a polygon.	1	1		2	
		3.3.1.LI.1 Identify parts of a circle and draw circles for given radii and through points.		1		1	
		3.3.1.LI.3 Identify the tangent as perpendicular to the radius at the point of contact and verify the Alternate Segment Theorem.	1	1	1	3	
		3.3.1.LI.2 Discuss the circle theorems by identifying the statements, proofs, examples and applications.					
	Measurement	**1.3.2.LI.3** Solve problems using the three primary trigonometric ratios for angles from 0° to 360° in standard position.		1		1	
		1.3.2.LI.3 Solve a contextual problem that involves the perimeter and area of a regular, a composite or an irregular 2-D shape [kites, parallelogram, rhombus and trapezoids].					
		1.3.2.LI.1 Investigate the three basic trigonometric ratios (tangent, sine and cosine) of an acute angle in degrees.					
		1.3.2.LI.3 Solve problems using the three primary trigonometric ratios for angles from 0° to 360° in standard position.		1		1	
		1.3.2.LI.4 Determine the volume of prisms and solve everyday life problems on them.		1		1	

		1.3.2.LI.2 Represent a vector in two-space geometrically as a directed line segment, with directions expressed in different ways (e.g., 320°, N40°W) and algebraically; then recognise vectors with the same magnitude and direction but different positions as equal vectors.		1		1	
		1.3.2.LI.2 Estimate the perimeter and area of a given regular, composite or irregular 2D shape [kites, parallelogram, rhombus and trapezoids].		1		1	
Making sense of and using data.	Statistical reasoning and its application in real life	**1.4.3.LI.2** Analyse (including using appropriate computer applications) and interpret data using descriptive statistics (i.e., measures of central tendency/ location and minimum and maximum values) and justify which of the averages best represents the data.	1			1	
		1.4.1.LI.2 Organise and present data (grouped/ ungrouped) using frequency tables, line graphs, pie charts, multiple bar graphs, infographics, etc., including generating 3D graphs/ charts with appropriate digital technology (where available) and solving problems on them.	1			1	
		2.4.1.LI.2 Analyse and interpret data using measures of dispersion and justify which of these measures best suits the data.	1	1		2	

		2.4.1.LI.2 Evaluate a given set of data and/or its instrument by identifying potential problems related to bias, use of language, gender, ethics, cost, time, privacy, cultural sensitivity, etc.		1		1	
		1.4.1.LI.3 Identify and validate qualitative data collection methods (interviews, observations, focus groups, oral histories, online tracking, social media monitoring, etc.) and use them to collect everyday-life data.		1		1	
	Chance	**1.4.2.LI.1** List the elements of the sample space from a simple or compound experiment involving two independent events.	1			1	
		1.4.2.LI.2 Determine the probabilities of independent events and express the results as fractions, decimals, percentages, and/or ratios.	1			1	8
TOTAL			**16**	**20**	**4**	**40**	**40**
			-40%	**-50%**	**-10%**	**-100%**	**-100%**

PAPER II

SECTION A (WITH INDICATORS)

Strand	Sub-Strand	Learning Indicators	DOK LEVELS			TOTAL	
			1	2	3		
Numbers for everyday life	Real Number and Numeration system	**1.1.1.LI.3** Establish the relationship between and among three sets, including set equations and De Morgan's law					
		2.1.1.LI.2 Explain the concepts of indices and logarithms with examples.					3

		2.1.1.LI.3 Compose and decompose logarithm laws and properties with exponents and apply the concepts to solve real-life problems.			1	1	
		2.2.1.LI.2 Analyse two linear equations in two variables and solve them using the elimination and substitution methods.					
		2.2.1.LI.3 Analyse, model, and solve word problems of simultaneous linear equations involving numbers, age, etc.					
		3.1.2.LI.1 Investigate to establish differences between variables in situational growth (symbolic logic) and apply it to real world situations.					
		3.1.2.LI.2 Make intelligent guessing to establish valid arguments and draw logical conclusions.		1		1	
	Proportional Reasoning	**1.1.2.LI.2** Analyse daily activities/issues/ businesses involving percentage change, including simple problems involving personal or household finance (such as utility bills, exchange rates, project budgeting, school fees, shopping, etc.)					
		2.1.2.LI.1 Explain the concepts ratios and rate as a comparison of quantities.		1		1	

		2.2.2.LI.5 Analyse, model, and solve real-life problems involving exponential growth.					
Algebraic Thinking	Applications of expressions, equations and inequalities	**1.2.2.LI.2** Draw graphs of linear functions and interpret them.		1		1	3
		2.2.1.LI.1 Solve simultaneous linear equations involving two variables using the graphical method and interpret them.		1		1	
		3.2.2.LI.1 Identify and solve quadratic equations.					
		3.2.2.LI.2 Solve quadratic equations graphically and find the maximum and minimum points of quadratic graphs.					
		3.2.2.LI.3 Identify and explain the axis of symmetry, write its equation and solve linear and quadratic equations simultaneously using graphs.					
		3.2.2.LI.4 Use quadratic graphs to solve related equations and solve real-life problems to find the range of values of (x) for which the other value (y) is increasing or decreasing and the range of values (x) for which the other value (y) is positive or negative.					

		1.2.1.LI.2 Solve linear equations and inequalities in one variable for a given problem and relate it to real-life situations.	1			1	
	Patterns and relationships					0	
Geometry Around Us	Spatial sense	**1.3.1.LI.2** Solve problems that involve parallel lines, perpendicular lines, transversal, and pairs of angles formed between them.			1	1	2
		1.3.1.LI.4 Solve problems on the Pythagorean theorem by identifying situations that involve right triangles, verify the formula and apply it.					
		3.3.1.LI.1 Identify parts of a circle and draw circles for given radii and through points.					
	Measurement	**1.3.2.LI.2** Estimate the perimeter and area of a given regular, composite or irregular 2D shape [kites, parallelogram, rhombus, and trapezoids].		1		1	
		1.3.2.LI.3 Solve a contextual problem that involves the perimeter and area of a regular, a composite or an irregular 2-D shape [kites, parallelogram, rhombus and trapezoids].					
		2.3.2.LI.1 Determine the inverse of trigonometric ratios (graphs excluded) and talk about their applications in the fields of astronomy, engineering, physics, geometry and navigation.					

Making sense of and using data.	Statistical reasoning and its application in real life	**1.4.1.LI.2** Organise and present data (grouped/ ungrouped) using frequency tables, line graphs, pie charts, multiple bar graphs, infographics, etc., including generating 3D graphs/charts with appropriate digital technology (where available) and solving problems on them.		1		1	2
		1.4.3.LI.2 Analyse (including using appropriate computer applications) and interpret data using descriptive statistics (i.e., measures of central tendency/location and minimum and maximum values) and justify which of the averages best represents the data.					
		2.1.2.LI.1 Apply the concept of ratios, rates, and proportions to solve problems in financial mathematics, health, sports, etc.					
		3.4.1.LI.1 Establish simple mathematical relationships between two variables in a given observational or experimental context; illustrate using scatter graphs and use them to solve and/or pose problems.					

		3.4.1.LI.2 Collect data from an experimental study in which the interest is based on treatment and non-treatment (control) groups. Illustrate the data using scatter graphs and find the relationship between the variables, if any.		1		1	
	Chance	**3.4.2.LI.2** Solve everyday life problems involving the probability of two dependent and independent events, including addition and multiplication laws.					
TOTAL			**1**	**7**	**2**	**10**	**10**
			(10%)	**-70%**	**-20%**	**-100%**	**-100%**

PAPER II

SECTION B (WITH INDICATORS)

STRAND	SUB-STRAND	LEARNING INDICATORS	DOK LEVELS			TOTAL	
			1	2	3		
Numbers for everyday life	Real Number and Numeration System	**2.1.1.LI.1** Carry out operations on surds and rationalize monomial denominators.					
		2.1.1.LI.3 Compose and decompose logarithm laws and properties with exponents and apply the concepts to solve real-life problems.					
		1.1.1.LI.2 Organise information visually to establish the relationship between and among sets of items (three sets) and apply these to conduct mini-surveys in the school community and beyond.	1			1	3

		2.1.1.LI.1 Investigate real-life problems using laws and properties of indices and logarithms.					
		2.1.2.LI.2 Establish the relationships among ratio, rates and proportions.					
	Proportional Reasoning	**1.1.2.LI.1** Establish the concept of fractions and investigate the connections between fractions and decimal numbers.					
		1.1.2.LI.2 Analyse daily activities/issues/ businesses involving percentage change, including simple problems involving personal or household finance (such as utility bills, exchange rates, project budgeting, school fees, shopping, etc.)					
		2.1.2.LI.1 Explain the concepts of ratios and rates as a comparison of quantities.					
		2.1.2.LI.2 Establish the relationships among ratio, rates and proportions.		1		1	
		2.1.2.LI.1 Apply the concept of ratios, rates, and proportions to solve problems in financial mathematics, health, sports, etc.			1	1	

		3.1.2.LI.1 Use proportional reasoning to investigate the various types of variation (direct and inverse) and extend this to make generalisations.					
Algebraic Thinking	Applications of expressions, equations, and inequalities				1	1	2
		2.2.2.LI.5 Analyse, model, and solve real-life problems involving exponential growth.					
	Patterns and relationships	**3.2.2.LI.2** Solve quadratic equations graphically and find the maximum and minimum points of quadratic graphs.		1		1	
Geometry Around Us	Spatial sense						
	Measurement	**1.3.2.LI.1** Recognise a vector as a quantity with both magnitude and direction, and identify, gather, and interpret information about real-world applications of vectors.					
		1.3.2.LI.2 Represent a vector in two-space geometrically as a directed line segment, with directions expressed in different ways (e.g., 320°, N40°W) and algebraically; then recognise vectors with the same magnitude and direction but different positions as equal vectors.					

		1.3.2.LI.2 Estimate the perimeter and area of a given regular, composite or irregular 2D shape [kites, parallelogram, rhombus and trapezoids].		1		1	
		1.3.2.LI.3 Solve a contextual problem that involves the perimeter and area of a regular, a composite or an irregular 2-D shape [kites, parallelogram, rhombus and trapezoids].		1		1	
		2.3.2.LI.1 Perform addition, subtraction, and scalar multiplication on vectors represented as directed line segments in two-space and in Cartesian form in two and three-space.					2
		2.3.2.LI.3 Solve problems involving the addition, subtraction, and scalar multiplication of vectors,					
		including problems arising from real-world applications.					
		2.3.2.LI.1 Determine the inverse of trigonometric ratios (graphs excluded) and talk about their applications in the fields of astronomy, engineering, physics, geometry and navigation.					

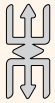
Making sense of and using data.	Statistical reasoning and its application in real life	**1.4.1.LI.2** Organise and present data (grouped/ ungrouped) using frequency tables, line graphs, pie charts, multiple bar graphs, infographics, etc., including generating 3D graphs/charts with appropriate digital technology (where available) and solving problems on them.					3
		1.4.3.LI.2 Analyse (including using appropriate computer applications) and interpret data using descriptive statistics (i.e., measures of central tendency/ location and minimum and maximum values) and justify which of the averages best represents the data.					
		2.4.1.LI.1 Organise and present data (grouped/ungrouped) by means of the ogive, waffle diagrams, box and whisker plots, etc., including generating 3D graphs and solving and/or posing problems.	1			1	
		2.4.1.LI.2 Analyse and interpret data using measures of dispersion and justify which of these measures best suits the data.		1		1	
	Chance	**2.4.2.LI.1** List the elements of the sample space from a simple or compound experiment involving two dependent events.					

		2.4.2.LI.2 Solve everyday life problems involving the probability of two independent events.					
		2.4.2.LI.2 Solve everyday life problems involving the probability of two independent events.		1		1	
TOTAL			**2**	**6**	**2**	**10**	**10**

PLC Session 12

45 minutes | **Planning & Reviewing the Learning Plan**

1. Begin the session by recapping lessons from the previous session, focusing on how learning was applied.
2. Use your subject specific app to explore the content before planning. Ask the app to
   a. explain the week's concepts in a practical way that will make it easier for learners to understand.
   b. identify three common misconceptions learners may have on the week's content and how to address them using contexts familiar to Ghanaian learners (NTS 3a, 3g and 3m).
3. Use your subject specific app to plan your lesson. Ask the app to
   a. suggest the most effective method for teaching the week's concepts to learners with diverse backgrounds.
   b. suggest learning activities, differentiation and essential questions based on the method of teaching (NTS 3a-3q).
   c. review your prepared learning plan and address any challenging issues you may have identified in your subject area (NTS 3a-3c).



Note

- Take note of the recap topics linked to Years 1 and 2 while preparing for your lesson.
- Refer to the Values Learning Community Handbook and link the values for the week in your lesson to ensure that learners are applying what they learn.

4. Use your app to plan and administer the mode of assessment for the Student Assessment Portal (SAP).

a. The key assessment for the week is ***practice paper 2***. Chat with your app to plan this assessment by reviewing or developing the assessment items/tasks and marking schemes/rubrics for the practice paper (NTS 3k-3q).

10 minutes Content Exploration

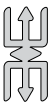
5. As a subject group, select one problem or topic/idea/term in the week's content that is challenging (not a routine procedural question, but one that requires conceptual understanding) and then

a. use the app, ask it to solve the problem step by step and explain the reasoning behind each step (you may be assisted by a teacher who is strong in the content area)

b. all teachers in the group work through the problem individually for 3–4 minutes

c. the group then works through it together the problem, step by step

d. highlight the most common errors made at each step (NTS 1c, 1e, 2c, 2e, 3e-3j).



Note

- This part is not 'How do I teach this?' but how do I teach it for learners to understand?
- If the group still shares the same content gap, use the app: ask it to solve the problem step by step and explain the reasoning behind each step.
- Problems/ideas discussed should be sourced from the Teacher Manual content, not invented separately. This keeps the content exploration directly connected to what teachers will deliver.

30 minutes Enactment

6. Enact a teaching activity based on your learning plan (NTS la-1b, 1e-lg, 2c, 2e, 2f, 3c-3j and 3o).

a. Colleagues observe with specific attention to clarity of concept explanation, use of familiar Ghanaian examples and how misconceptions are handled if they arise.

b. After the enactment, ask for structured feedback from the group

i. was the concept explained in a way that a learner with no prior knowledge could follow?

ii. were the examples grounded in contexts familiar to Ghanaian learners?

iii. did any misconceptions surface and if so, how were they addressed?

5 minutes Reflection

7. Reflect and share your views on the session. One thing that worked well, what did not work well and what you will do differently. (NTS 1a, 1b).

8. Identify a critical friend to observe and provide feedback on your lesson in relation to the PLC Session conducted (NTS 1f, 1g).

9. Remember to

a. transfer your chats into your learning planner template.

b. read your Teacher Manual and related Learner Material in preparation for the next session. Take note of the recap weeks, which are linked to Years 1 and 2 (NTS 3a).

c. integrate National Values, GESI, SEL, and 21st century skills into your learning activities, assessment tasks, and rubrics/mark schemes.

WEEK 13

Learning Indicators

1. Construct a triangle or a quadrilateral under given conditions
2. Construct a particular locus for a given condition

FOCAL AREA 1: CONSTRUCTION OF TRIANGLES AND QUADRILATERALS

Constructing Triangles

A triangle can be constructed using a ruler, pencil, and compass only if the following information is provided

1. All three sides of the triangle are provided
2. Two sides and the included angle are given
3. One side and two angles are given.

Example 1

Using a ruler and a pair of compasses only, construct triangle ABC such that |AB| = 8cm, |AC| = 10cm, and |BC| = 9cm

Solution

We can choose to construct any of the lines first.

Let us construct line |AC| first.

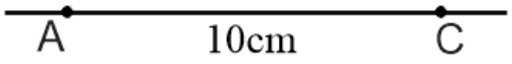


Figure 13.1: *Line AC = 10 cm*

To construct |AB| open your compass to a radius of 8cm.

With the pin of the compass at point **A**, draw an arc

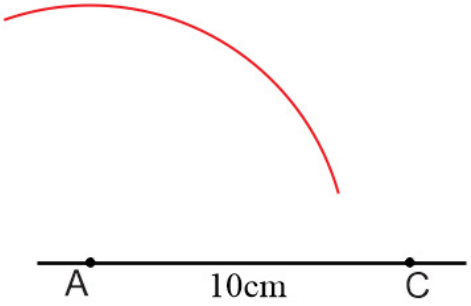


Figure 13.2: *Line AC and an 8cm arc*

Next, open the compass to a length of 9cm. With the pin of the compass at **C**, draw an arc to intersect the arc in the previous step. Indicate the point of intersection of the two arcs as **B**.

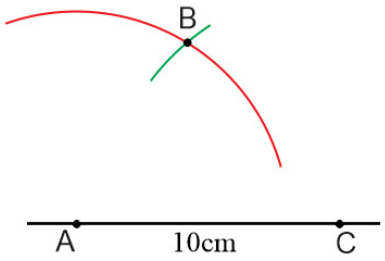


Figure 13.3: *Construction of triangle ABC*

Finally, join A to B and B to C with straight lines

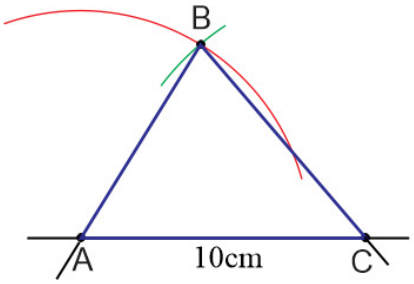


Figure 13.4: *Triangle ABC*

Example 2

Using a ruler and a pair of compasses only, construct triangle XYZ such that |XY| = 12cm, |XZ| = 10cm, and < YXZ = 75°. **Measure |YZ|**

Solution

Construct line |XY|, indicating the points **X** and **Y**, which are 12cm apart.

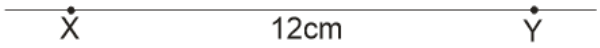


Figure 13.5: *Line XY=12cm*

Using point **X** as the vertex and |XY| as the baseline, construct angle 75° (as taught in Week 12) at **X**.

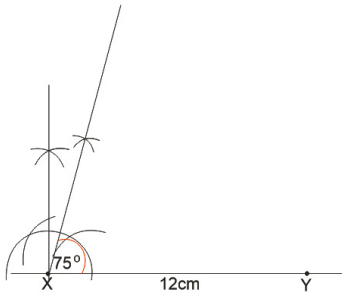


Figure 13.6: *Construction of 75°*

Locate the position of point **Z** on the arm of the angle such that it is 10cm from **X**.

Complete the triangle by joining the points **Z** and **Y** with a straight line.

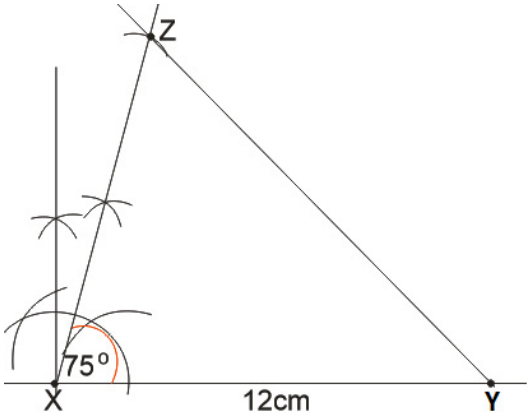


Figure 13.7: *Triangle XYZ*

With a ruler, measure line | YZ | = 13.5 cm

Example 3

Using a ruler and a pair of compasses only, construct triangle **PQR** such that |QR | 8cm, < PQR105, < PRQ 45°. Measure:

a. |PQ|

b. |PR|

Solution

i. First construct line **QR** indicating clearly the points **Q** and **R** which are 8cm apart.



Figure 13.8: *Line QR =8cm*

ii. Construct angle 105° at **Q**

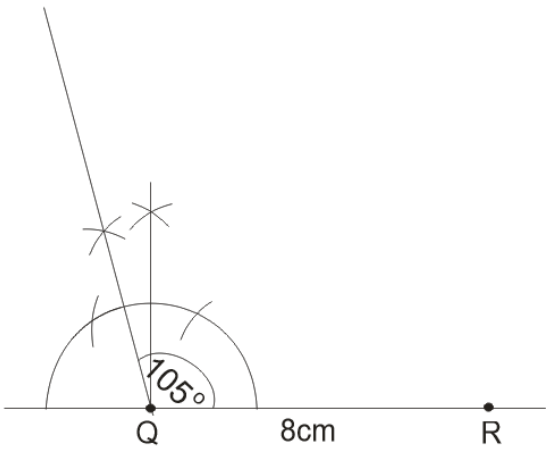


Figure 13.9: *Construction of 105°*

iii. Construct angle 45° at R.

iv. Name the point of intersection as **P** to complete the construction.

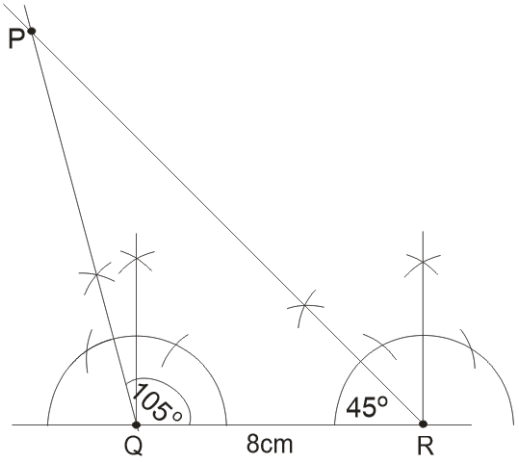


Figure 13.10: *Triangle PQR*

Construction of Quadrilaterals

Example 4

Using a ruler and a pair of compasses only construct a quadrilateral **ABCD**, such that |AB| = |BC| = 11cm, |AD| = 7.5cm, angle ABC = 105° and AD//BC.

Solution

a. First construct line |AB|



Figure 13.11: *Line AB = 11 cm*

b. Next, construct angle 105° at **B**,

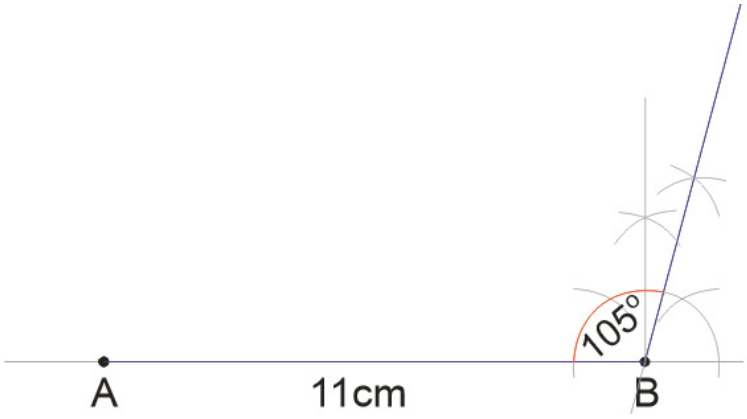


Figure 13.12: *Construction of 105° at point B*

c. Now, since BC//AD (line **BC** is parallel to line **AD**), we construct angle 75° at **A** (from properties of parallel lines, co-interior angles add up to 180°)

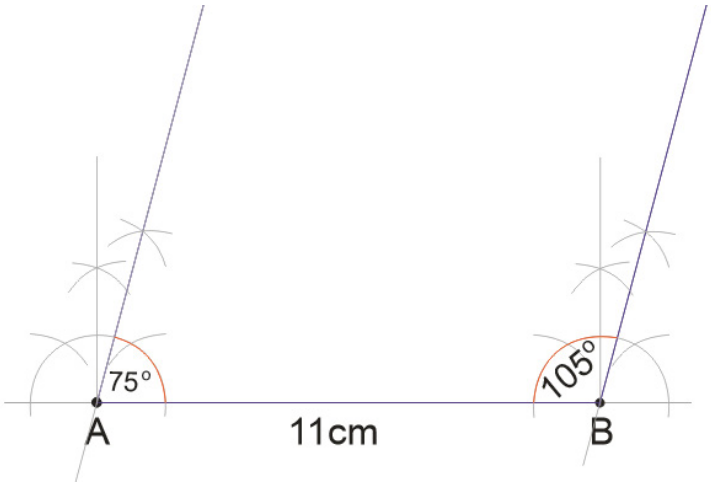


Figure 13.13: *Construction of 75° at point A and 105° at point **B** on line **AB** = 11 cm*

d. Locate **D**, which is 7.5cm from **A**.

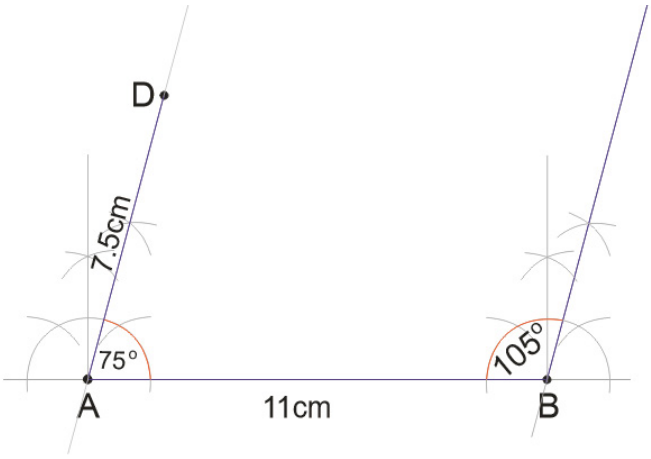


Figure 13.14: *Construction of 75° at point A and 105° at point **B** on line **AB** = 11 cm*

e. Likewise, locate **C**, which is 11cm from **B**, and join the points **D** and **C** with a straight line.

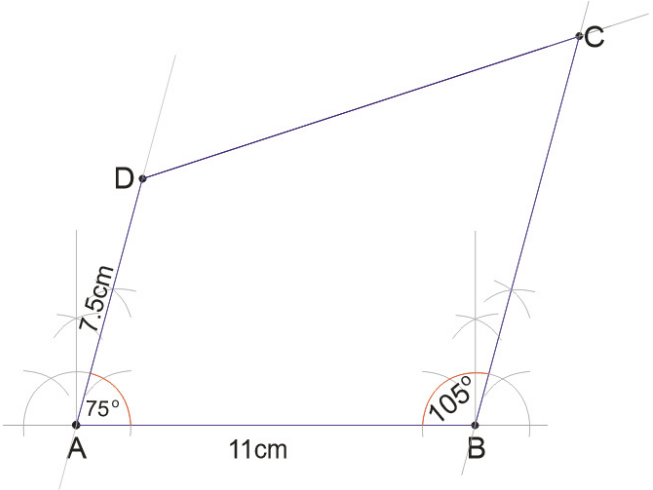


Figure 13.15: *Quadrilateral ABCD*

FOCAL AREA 2: CONSTRUCTION OF LOCUS

- *Theorems and conditions of locus*
- *Construction of locus*

Locus

A locus is a set of all points that satisfy a given condition or set of conditions. Think of it as a path traced by a point moving according to specific rules.

Locus theorem 1

Locus of points equidistant from a single point

Method: Using the point as the centre and the given distance as the radius, construct a circle. The locus is the circle.

Example 5

Figure 13.16 shows triangle **ABC**.

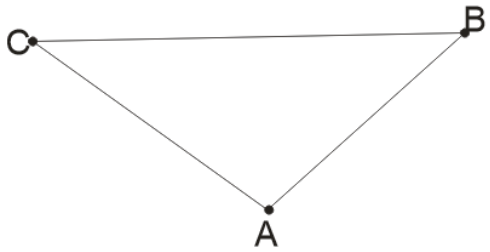


Figure 13.16: *Triangle ABC*

a. With the aid of a ruler and a pair of compasses only, make a copy of triangle **ABC**.

b. Construct the locus p, of points 5cm from **A**.

Solution

Open your compass to a radius of 5cm. With the pin of the compass at **A**, construct a complete circle.

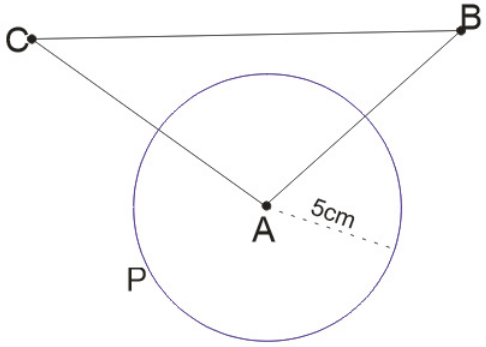


Figure 13.17: *Triangle ABC and circle P with centre A*

Locus theorem 2

Locus of points equidistant from two points

Method: Construct a perpendicular bisector of the line segment determined by the two points. The locus is the perpendicular bisector.

Example 6

Fig 13.18 shows triangle **ABC**.

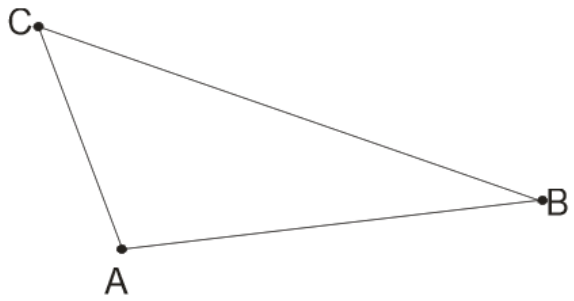


Figure 13.18: *Triangle ABC*

a. Using a ruler and a pair of compasses only, make a copy of triangle **ABC.**

b. Construct the locus, m, of points equidistant from **A** and **B**.

Solution

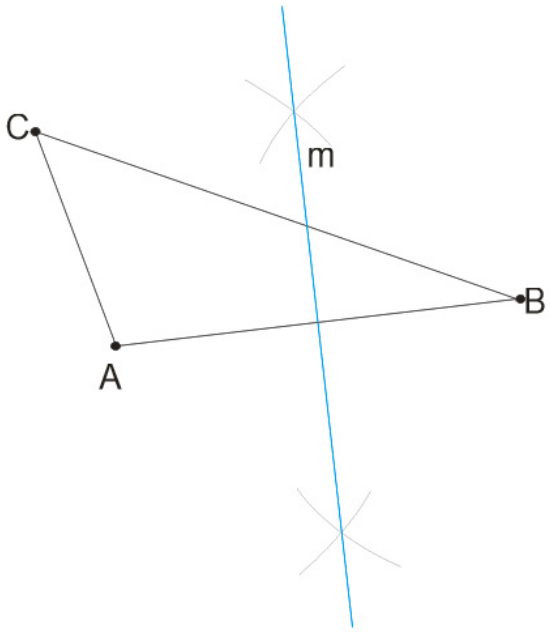


Figure 13.19: *Triangle ABC and line m bisecting line AB*

Line m is the locus

Locus theorem 3

Locus of points equidistant from two intersecting lines.

Method: Bisect the angle of intersection of the two lines. The locus is the **line** bisecting the angle.

Example 7

Figure 13.20a shows the intersection of line **AB** and **CD**.

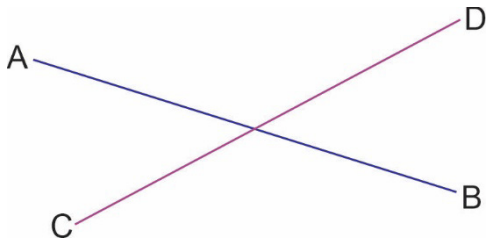


Figure 13.20a: *lines AB and CD*

a. Using a ruler and a pair of compasses only, make a copy of the figure

b. Construct the locus **P**, of points equidistant from **AB** and **CD.**

Solution

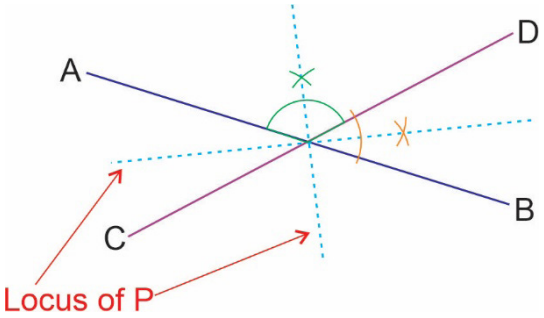


Figure 13.20b: *Construction of loci equidistant from **AB** and **CD***

Locus theorem 4

Locus of points equidistant from two parallel lines

The locus equidistant from two parallel lines, |AB| and |CD| is a line parallel to both **AB** and **CD** and halfway between them.

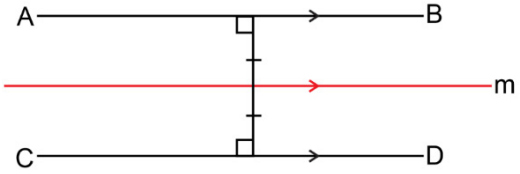


Figure 13.21: *line m equidistant from AB and CD*

In the diagram above, the locus is line m. Line m is parallel to and equidistant from lines **AB** and **CD**. Line m is also parallel to **AB** and **CD**.

Example 8

The diagram below represents a construction of two parallel lines m_1 and m_2.

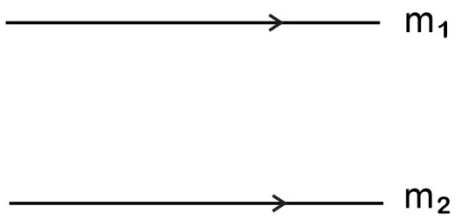


Figure 13.22: *Parallel lines m_1 and m_2*

Construct the locus d, equidistant m_1 from and m_2.

Solution

1. **Step 1:** Construct a perpendicular at any point **O** on line m_1 to intersect line m_2 at point **A**.

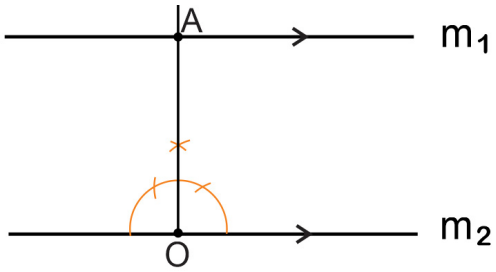


Figure 13.23: *A perpendicular transversal intersecting parallel lines m_1 and m_2 at point O and A*

2. **Step 2:** Construct a perpendicular bisector of line |OA|

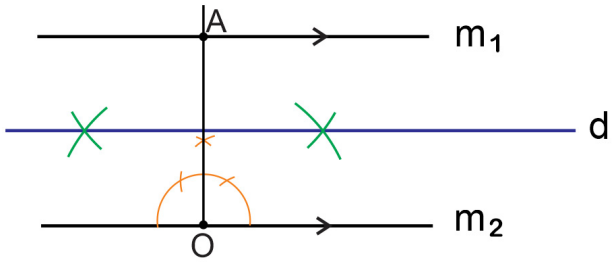


Figure 13.24: *Line* **d** *equidistant from parallel lines m_1 and m_2*

Line **d** is the locus of points equidistant from m_1 and m_2.

Locus theorem 5

1. Locus of points equidistant from a line.
2. This is a pair of parallel lines a distance away from the given line.
3. This theorem is illustrated in ***Figure 13.25***
4. The figure illustrates the locus of points, p, of points m units from line **AB**.
5. The locus, p, represents the dotted lines on opposite sides of line **AB**.

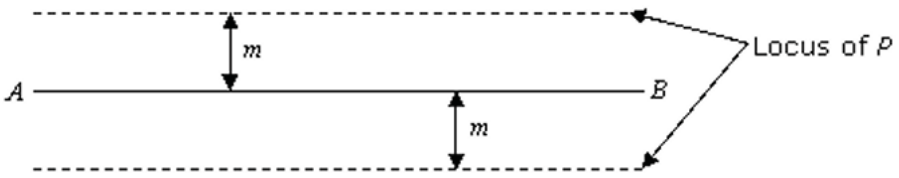


Figure 13.25: *Loci p equidistant from line AB*

Locus theorem 6

Constructing a perpendicular from a point to a line.

Method: Open the compass to a distance more than the perpendicular distance from the point *O*, to the line.

1. **Step 1:** Draw an arc to intersect the line at the two points X and Y.

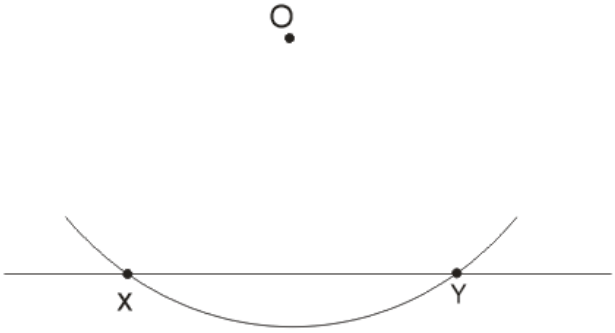


Figure 13.26: *Construction of an arc from point O to line XY*

2. **Step 2:** With the compass point at X and using the radius |OX|, draw an arc on the other side of the line.

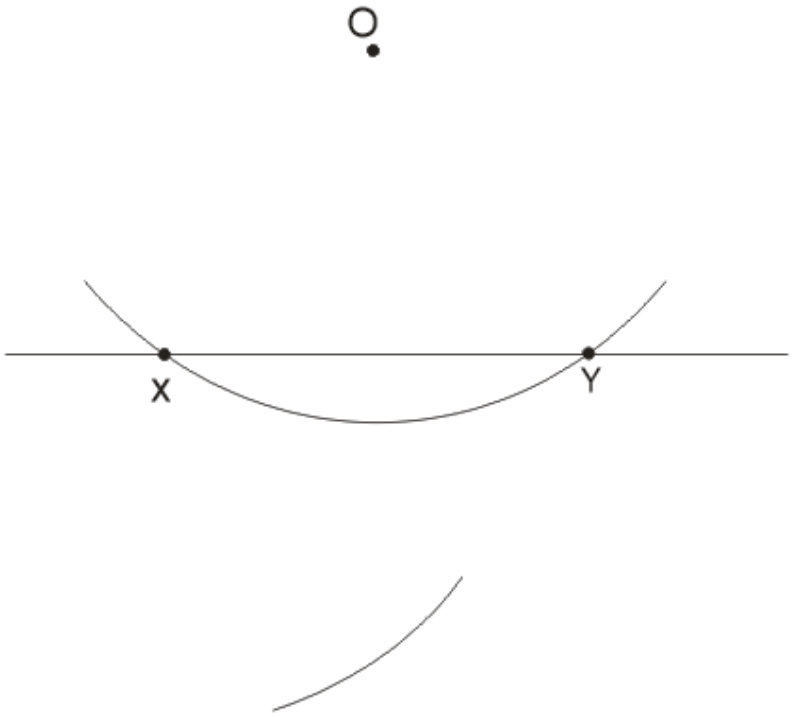


Figure 13.27: *Construction of a perpendicular from O to line XY*

3. **Step 3:** With the compass point at Y and using the same radius as used in 1, draw an arc to intersect the one drawn in 1 at Z.

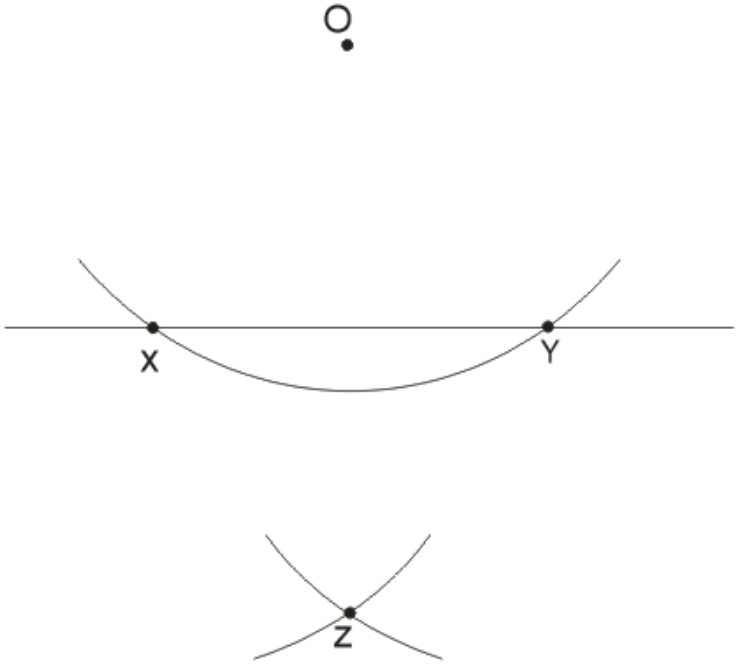


Figure 13.28: *Construction of a perpendicular from point O to line XY*

1. Finally, join the points **O** and **Z** with a straight line.

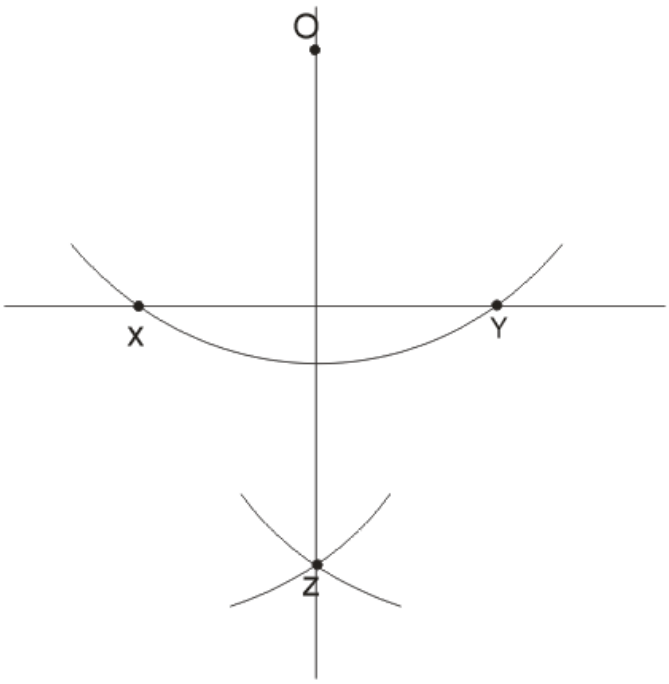


Figure 13.29: *Construction of a perpendicular from point O to line XY*

Locus theorem 7

Locus of points equidistant from three non-collinear fixed points.

Method

1. First, join the three points to make a triangle.

2. Construct the perpendicular bisectors of any two sides of the triangle.
3. The intersection of the two perpendicular bisectors is the locus. This locus is a single point.

NOTE

This locus is a single point called the circumcentre. This point can be used to construct a circumscribed circle.

Example 9

1. Consider the triangle below.

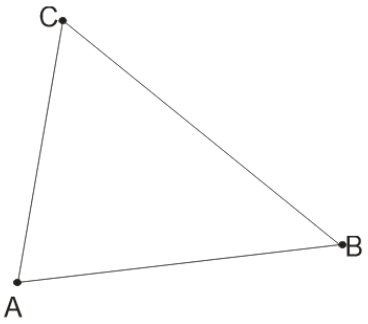


Figure 13.30: *Triangle ABC*

2. Find the point, **O**, equidistant from **A**, **B**, and **C**.

Solution

We first construct the perpendicular bisector of any of the two sides. Let's choose sides **AB** and **AC**.

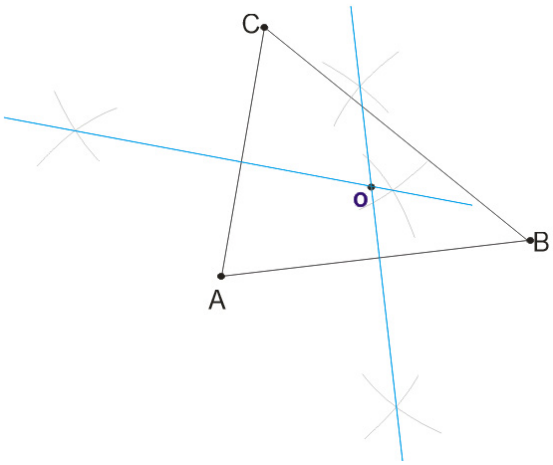


Figure 13.31: *Perpendicular bisectors of sides **AB** and **AC** of triangle **ABC***

The intersection of the two perpendicular bisectors is the locus.

The locus is just a single point which is equidistant from all three points, **A**, **B** and **C**.

Example 10

a. Using a ruler and a pair of compasses only, construct a parallelogram PQRS such that $|PQ| = 8$cm, $|QR| = 6$cm and $= 11.5$cm.

b. Construct the locus u, 5cm from **Q**

c. Construct the locus of points t, equidistant from |PS| and |SR|

d. Indicate the point of intersection of u and t as **A** and **B**

Solution

First construct triangle PQR. This is a triangle with all sides given.

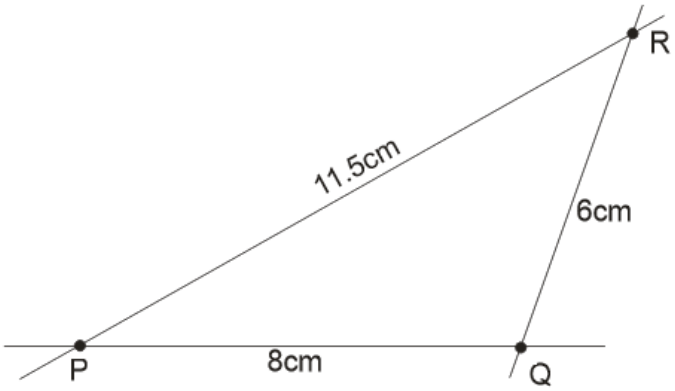


Figure 13.32: *Triangle PQR*

To locate the last vertex of the parallelogram, open your compass to a distance of 8cm. with the pin of the compass at **R**, scribe an arc [arc should be in line with **PQ**]

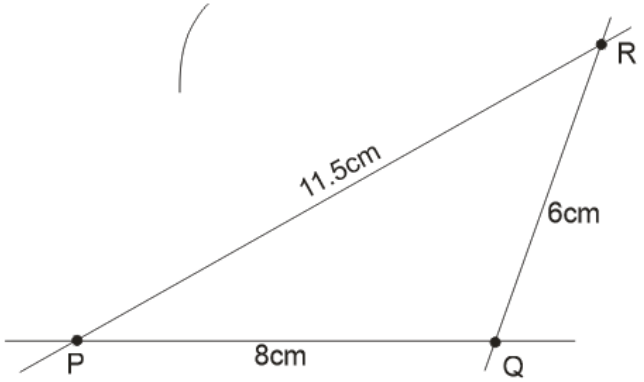


Figure 13.33: *Triangle PQR*

Likewise, open your compass to a distance of 6cm. with the pin of your compass at P scribe an arc to intersect the previous one. Indicate the point of intersection of the two arcs as S.

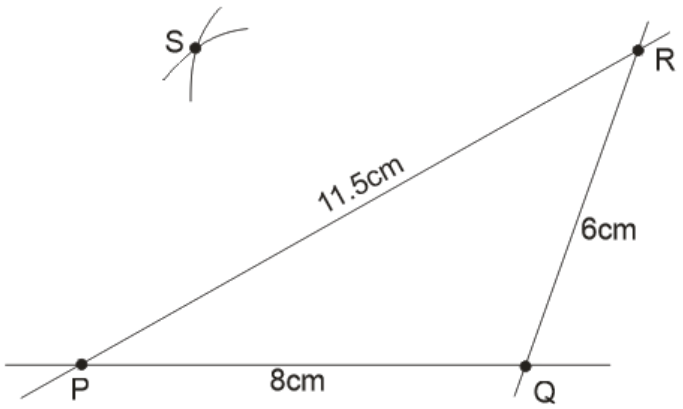


Figure 13.34: *Construction of parallelogram PQRS*

Finally, join **P** and **S**, **R** and **S** with straight lines to complete the (a) part of the question.

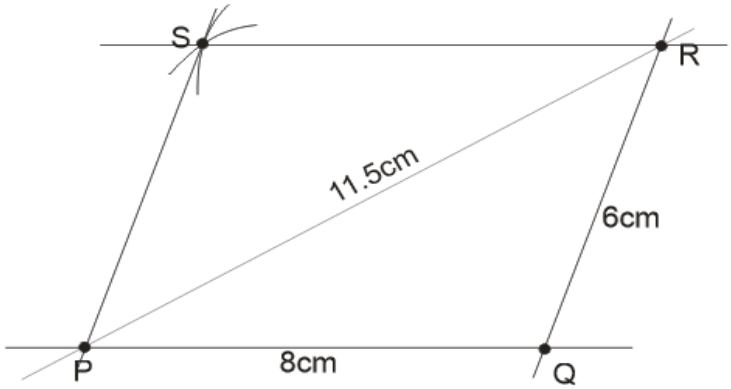


Figure 13.35: *Parallelogram PQRS*

We then construct the locus, 5cm from **Q**. That means using **Q** as our centre, construct a circle with a radius of 5cm.

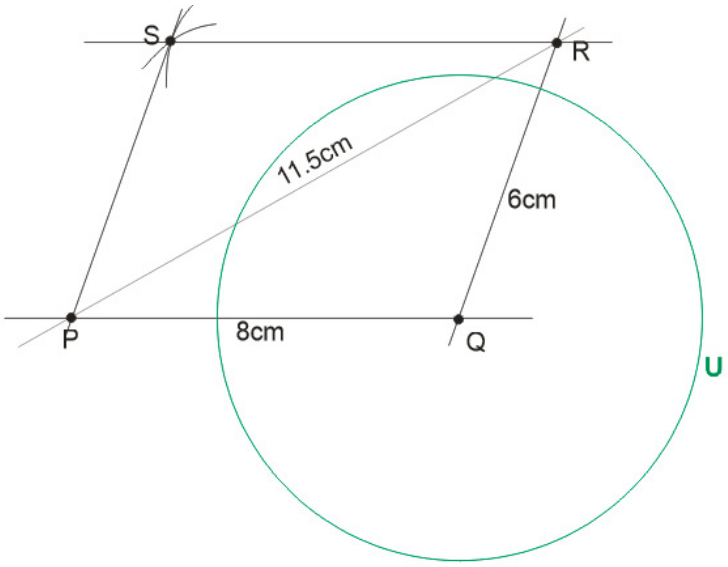


Figure 13.36: *Parallelogram PQRS and circle u with centre Q*

Construct the locus of points equidistant from |PS| and |SR|. This is a bisector of angle PSR.

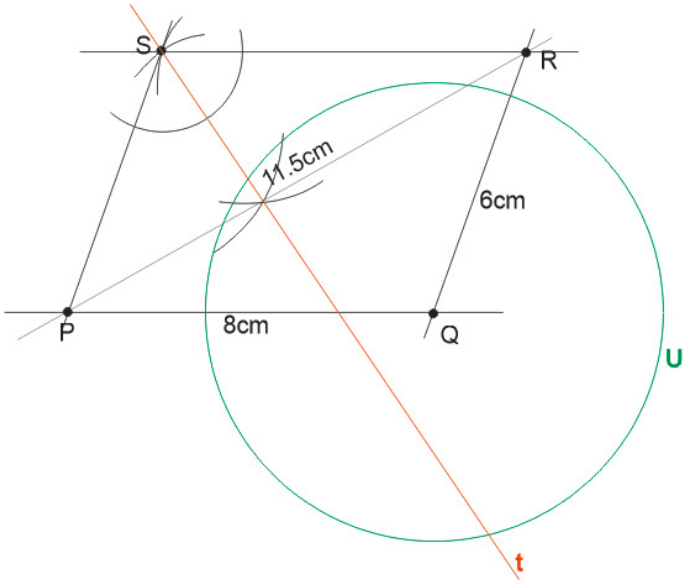


Figure 13.37: *Parallelogram PQRS and circle u with centre Q*

We then indicate the points of intersection of loci *u* and *t* as A and B to complete the construction.

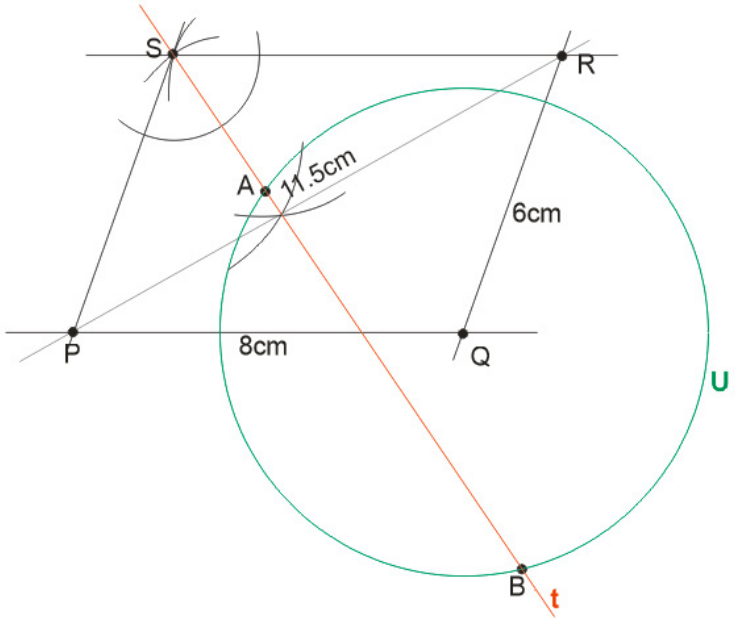


Figure 13.38: *Parallelogram PQRS and circle u with centre Q*

Learning Tasks

Guide learners to apply their knowledge of the construction of angles to construct triangles and quadrilaterals using appropriate instruments. Provide support systems for learners who may encounter difficulties.

PEDAGOGICAL EXEMPLARS

The aim of the lessons for the week is for all learners to be able to use the appropriate instrument to construct triangles, quadrilaterals, and loci. The following pedagogical approach is suggested for facilitators to take learners through.

1. **Group discussions**

   In small groups, learners draw various triangles and quadrilaterals using the appropriate tools. Learners discuss the concept of loci and draw various loci for triangles and quadrilaterals using the appropriate mathematical tools.

2. Using experiential learning, engage learners to construct and verify a triangle or a quadrilateral under given conditions

3. Using talk for learning, guide learners to discuss and work out examples for the various locus theorems.

KEY ASSESSMENT LEVEL 2: GROUP WORK & PRESENTATION

1. The diagram shows the scale plan of a restaurant. |AB| = 6cm, |AD| = 5cm, DAB = 105 and

2. ABC = 90°, with 1cm = 1m in real life

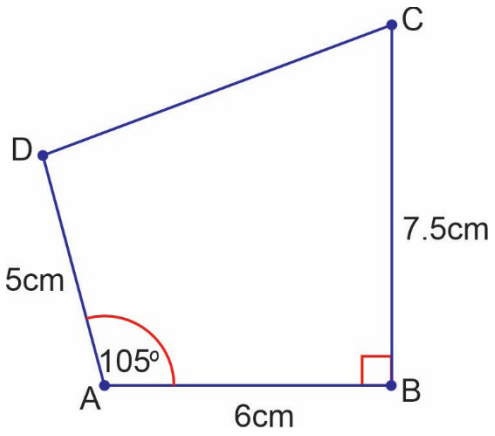


Figure 13.39: *Quadrilateral ABCD*

Scale 1cm:1m

a. Using a ruler and a pair of compasses only, make a scaled construction of ABCD

[*5 marks*]

b. A washing basin in the restaurant is equidistant from BC and CD and exactly 5.5m from A. On the diagram, mark the position of the basin with a dot and label it as O

[*4 marks*]

c. Find the perimeter of the floor space of the restaurant. **[*2 marks*]**

[Total *11 marks*]

3. Town M is 20km from town N and 22km from P while N is 18km from P. A market is to be built to serve the three towns. It is to be located such that traders from N and M will always travel equal distance to access the market while traders from P will travel exactly 10km to reach the market.

a. Using a ruler and a pair of compasses only, find by construction, the possible locations, A and B, for the market. Use a scale of 1cm to 2km. **[*9 marks*]**

b. Find the distance between the two locations? **[*1 mark*]**

c. Measure and record the distances of the locations from town N. **[*2 marks*]**

d. Which of the two possible locations would be more convenient for the three towns? Justify your answer. **[*1 mark*]**

[Total *13 marks*]

PLC Session 13

45 minutes | **Planning & Reviewing the Learning Plan**

1. Begin the session by recapping lessons from the previous year/session/week, focusing on how learning was applied.

2. Use your subject-specific app to explore the content before planning. Ask the app to

a. explain the week's concepts in a practical way that will make it easier for learners to understand

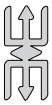
b. identify two or three common misconceptions learners may have on the week's content and how to address them using contexts familiar to Ghanaian learners. (NTS 3a, 3g and 3m).

3. Use your subject-specific app to plan your lesson. Ask the app to

a. suggest the most effective method for teaching the week's concepts to learners with diverse backgrounds.

b. suggest learning activities, differentiation and essential questions based on the method of teaching (NTS 3a-3q)

c. review your prepared learning plan and address any challenging issues you may have identified in your subject area (NTS 3a-3c).



Note

- Take note of the recap topics linked to Years 1 and 2 while preparing for your lesson.
- Refer to the Values Learning Community Handbook and link the values for the week in your lesson to ensure that learners are applying what they learn.

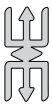
4. Plan the week's key assessment with your app

a. The key assessment for the week is ***group work & presentation***. Chat with your app to plan this assessment, indicating what you and your learners will do before, during and after the presentations (NTS 3k-3q).

10 minutes Content Exploration

5. As a subject group, select one problem or idea/term in the week's content that is challenging (not a routine procedural question, but one that requires conceptual understanding) and then

- **a.** use the app, ask it to solve the problem step by step and explain the reasoning behind each step (you may be assisted by a teacher who is strong in the content area)
- **b.** all teachers in the group work through a similar problem individually for 3–4 minutes
- **c.** the group then works through the problem together step by step
- **d.** highlight the most common errors made at each step and discuss (NTS 1c, 1e, 2c, 2e, 3e-3j).



Note

- This part is not just about 'How do I teach this?' but how do I teach it for learners to understand?
- If the group still shares the same content gap, use the app: ask it to solve the problem step by step and explain the reasoning behind each step.
- Problems/ideas discussed should be sourced from the Teacher Manual content, not invented separately. This keeps the content exploration directly connected to what teachers will deliver.

30 minutes Enactment

6. Enact a teaching activity based on your learning plan (NTS la-1b, 1e-lg, 2c, 2e, 2f, 3c-3j and 3o).

- **a.** Colleagues observe with specific attention to clarity of concept explanation, use of familiar Ghanaian examples and how misconceptions are handled if they arise.
- **b.** After the enactment, ask for structured feedback from the group
  - **i.** was the concept explained in a way that a learner with no prior knowledge could follow?
  - **ii.** were the examples grounded in contexts familiar to Ghanaian learners?
  - **iii.** did any misconceptions surface and if so, how were they addressed?

5 minutes Reflection

7. Reflect and share your views on the session. One thing that worked well, what did not work well and what you will do differently. (NTS 1a, 1b).

8. Identify a critical friend to observe and provide feedback on your lesson in relation to the PLC Session conducted (NTS 1f, 1g).

9. Remember to

a. transfer your chats into your learning planner template.

b. read your Teacher Manual and related Learner Material in preparation for the next session. Take note of the recap weeks, which are linked to Years 1 and 2 (NTS 3a).

c. integrate National Values, GESI, SEL, and 21st century skills into your learning activities, assessment tasks, and rubrics/mark schemes.

Section 3 Review

This section provided a recap of the key concepts covered during weeks 10 to 13. It served as an overview of what learners should have understood during this period. These weeks laid a solid groundwork for further exploration into geometry. We reviewed the various parts of a circle and learned how to construct common acute and obtuse angles. Using our understanding of circle components, we delved into circle theorems, including their statements and proofs.

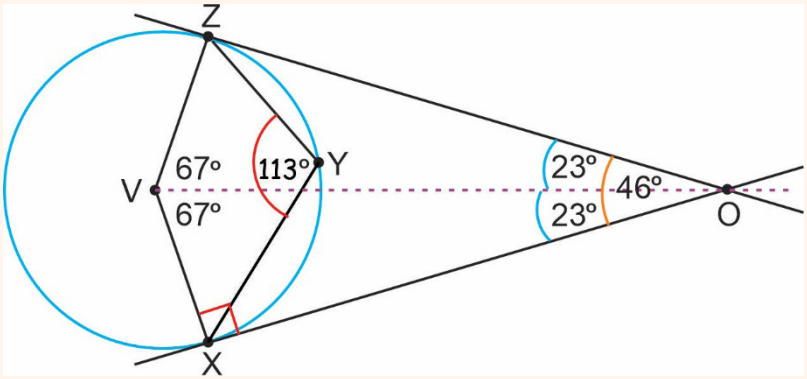
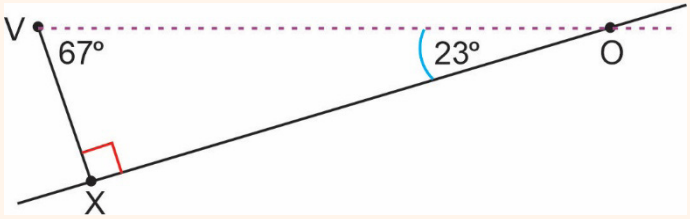
Additionally, we explored the construction of triangles and quadrilaterals. To conclude, we examined the concept of loci and applied it to solve practical, real-world problems.

To enhance teaching and learning, the following resources are recommended:

A complete set of mathematical instruments, a whiteboard, educational videos, mini whiteboards, dry erase markers and erasers, calculators, graph/grid paper, and digital tools such as computers, mobile phones, and YouTube videos.

MARKING SCHEMES FOR THE KEY ASSESSMENTS

WEEK 11

Solution	Marks
Question One	
Since the angle at the centre of a circle is twice the angle at the circumference, we have reflex angle XVZ=2 × 113 = 226°.	M1 for finding XVZ A1 for XVZ = 134
Since the sum of angles at a point is 360, the obtuse angle XVZ = 360– 226 = 134	M1 for 360 – their 226
Since the sum of angles in a quadrilateral (OXVZ) =360, We have, XOZ + 90° + 134° + 90° = 360°, Or **VOX** = 180° – 90°– 67° = 23°, so **XOZ** = 2 x 23 **XOZ** = 46°	M1 for correct method to find XOZ A1 for XOZ=46
Connect points V and O with a straight line. From the equal triangle theorem, triangle OXV is congruent (equal) to triangle OZV. So, angle XOV = $\frac{1}{2} \times 46 = 23^\circ$	M1 for correct method to find the radius of the circle
 **Figure 11.14:** *Lines OZ and OX tangent to a circle with centre V*	A1 for 6tan 23 or 6/ tan67, or 2.5468 seen or equivalent.
From triangle OXV,  **Figure 11.15:** *Triangle OXV*	M1 for correct method for finding the perimeter A1 for answers which round to 17cm
Tan23 = $\frac{xv}{6}$, |XV| = $6 \tan 23$ = 2.568cm Perimeter of OXVZ = 6 + 6 + 2.568 + 2.568 = 17.09cm	

Question Two

a.

i. Since |OB| is the diameter, it implies that

BCO = BAO = 90°

This means ACO + 50°, So ACO = 40°

Since ABO and ACO are subtended at the circumference on chord |AO|, their measures are equal. This implies ABO = 40

From triangle OBC, we have OBC + 90° + 57° = 180°

OBC = 33°

Therefore, ABC = 40° + 33° = 73°

ii. From the alternative segment theorem, AOE = ABO = 90°

i.

B1 for finding AOB,

M1 for applying the opposite angle theorem **A1** for ABC = 73

OR

B1 for ABO,

M1 for finding OBC

A1 for ABC=73

Alternative solution

i. Since ACB and AOB are subtended by chord |AB| at the circumference,

ACB = AOB = 50°

This implies AOC = 57° + 50° = 107°

Also, since opposite angles in a cyclic quadrilateral sum up to 180°, we have ABC + AOC = 180°

ABC = 180° −107° = 43°

ii. Since the radius or the diameter is perpendicular to the tangent,

AOB + AOE = 90°

AOE = 90° − 50° = 40°

ii.

M1 for applying the alternate angle theorem or the tangent theorem.

A1 for AOE=40

b. From OBC,

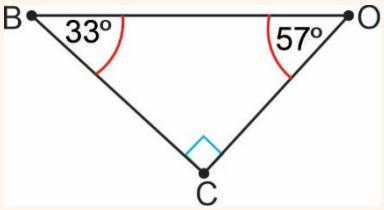


Figure 11.16: *Triangle OBC*

We have Sin 57°= $\frac{|BC|}{10}$, |BC| = $10Sin57^{\circ}$

Also, Sin 33° = $\frac{|OC|}{10}$, |OC| = $10Sin33^{\circ}$

b.

M1 for correct method to find side |OC| or |BC|

A0.5 for |OC|

A0.5 for |BC|

M1 for solving for either |OA| or |AB|

A0.5 for |OA|

A0.5 for |AB|

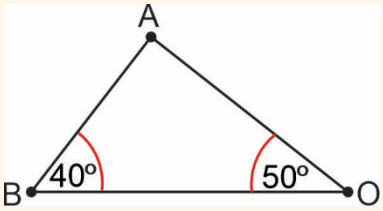


Figure 11.17: *Triangle OAB*

From OAB, we have sin50° = $\frac{|AB|}{10}$, |AB| = $10\sin 50$

Also, sin40° = $\frac{|OA|}{10}$, |OA| = $10\sin 40$

Perimeter of OABC = *10sin57 + 10sin33 + 10sin50 + 10sin40*

= 27.92cm

M1 for correct method to find perimeter

A1 for answers which round to 28

WEEK 12

Question One

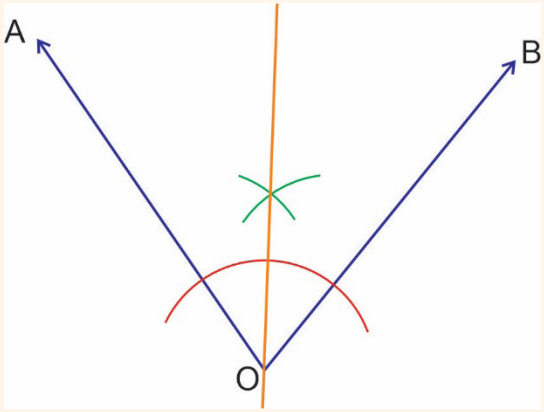


Figure 12.19: *Angle bisector of AOB*

M1 A1 for angle bisector.

Note, for method marks, construction markings on the diagram need to be seen throughout the answers.

For accuracy marks allow a tolerance of +/– 3 degrees.

Question Two

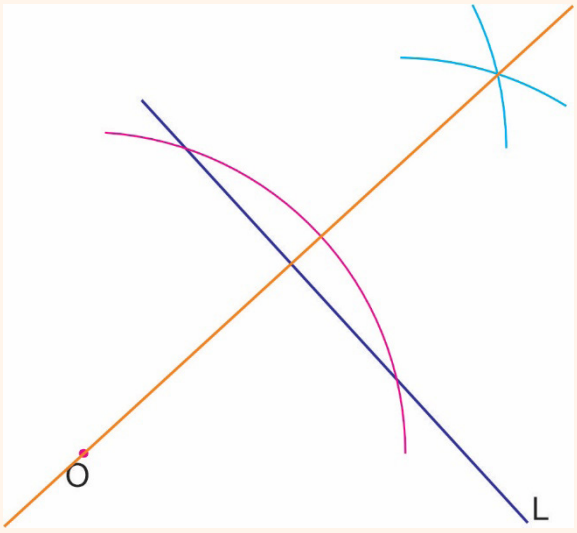


Figure 12.20: *A perpendicular from point O to line L*

M1A1 for correct construction

Question Three

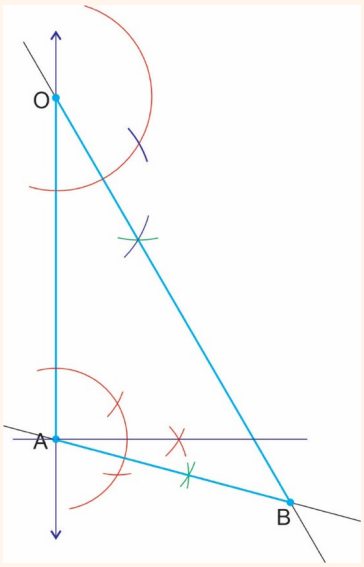


Figure 12.21: *Triangle AOB*

a. M1/A1 for construction AOB 30°

b. M1 for construction of 90-degree angle

M1 for bisecting 30°

A1 for final angle of 105° degrees

Question Four

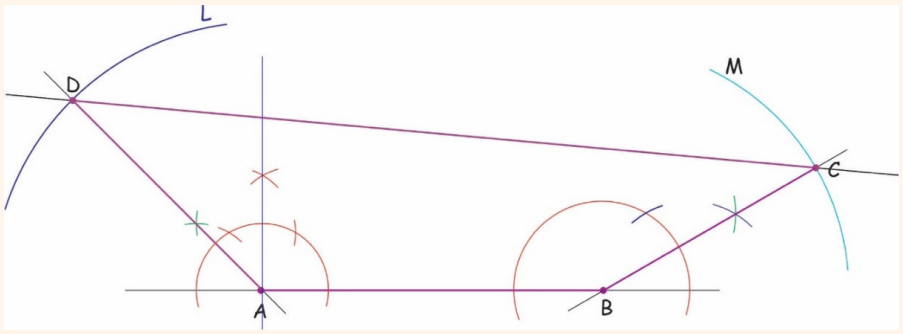


Figure 12.22: *Quadrilateral ABCD*

a. M1/A1 for locating point D

b. M1/A1 for locating point C

c. A1 for correctly adding the lengths of *their* perimeter together.

Question Five

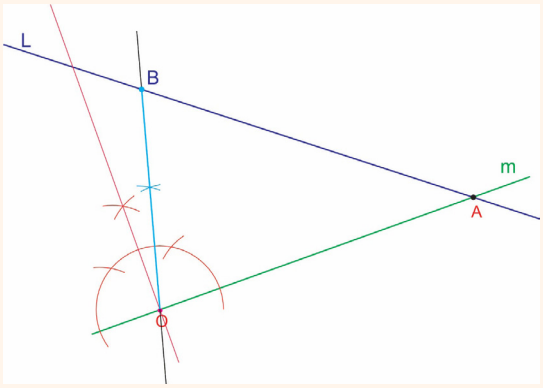


Figure 12.23: *triangle AOB*

a. M1/A1 for angle AOB = 75°

b. A1 for correctly measuring *their* ABO

WEEK 13

Question One

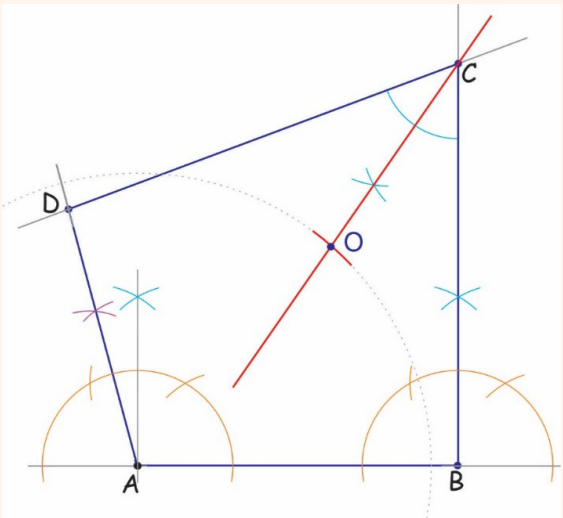


Figure 13.40: *Quadrilateral ABCD*

a. A1 line AB

M1/A1 for angle BAD=105

A1 for line BC

A1 for line CD

b. M1/A1 for the angle bisector of BCD

M1/A1 for locating O

c. A1 for CD = 7.8m +/- 0.5m

A1 for their perimeter.

5+6+7.5+7.8 = 26.3m

Question Two

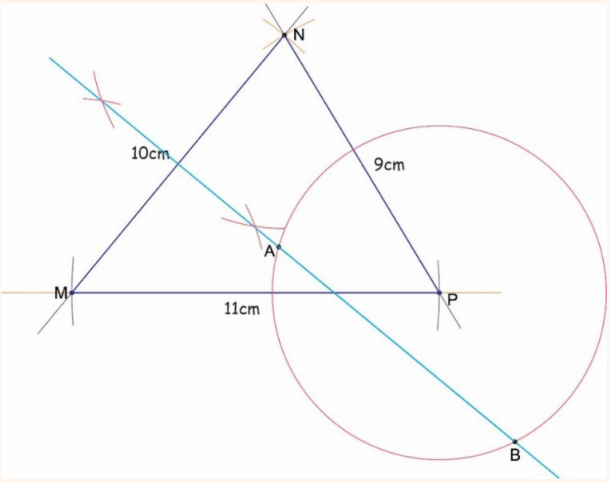


Figure 13.41: *Triangle MNP and a circle with centre P*

b. The distance between A and B = 9.2cm.
This is the same as 18.4km in reality.

c. On paper, |AN|=6.2cm. In reality, AN = 12.4km
On paper, |BN|=13.9cm. In reality, |BN|=27.8km

d. Location A, which is 12.4km from N, rather than 27.8km.

a. A1 for line MP

M1/A1 for MN

M1/A1 for NP

M1/A1 for perpendicular bisector of line MN

M1 for circle radius 5cm from **P**

A1 for locating points A and **B**

b. A1 for either 9.2cm or 18.4km

c. A1 for AN

A1 for BN

d. B1 for choosing location A and justifying it because it is the closer location

SECTION 4: TRIGONOMETRY GRAPHS

STRAND: GEOMETRY AROUND US

Sub-Strand: Measurement

Weeks 14 & 15

Learning Outcome: Draw graphs of given trigonometric functions and use them to determine equations and solve related problems.

Content Standard: *Demonstrate conceptual understanding of trigonometric graphs and use them to solve trigonometric equations.*

INTRODUCTION AND SESSION SUMMARY

The basic trigonometric functions are sine (sin), Cosine (cos), and Tangent (tan). The graphs of these functions give a visual representation of their behaviour over a range of angles. These functions extend into waveforms when graphed on the Cartesian plane.

Each trigonometric function exhibits a **periodic pattern**, meaning the graph repeats itself at regular intervals. These patterns are especially useful for modelling real-world phenomena such as sound waves, light waves, tides, and circular motion.

Here are the key characteristics of the three primary trigonometric graphs:

1. **Sine Graph ($y = \sin x$)**
   - ***Domain***: All real numbers
   - ***Range***: $[-1,1]$
2. **Cosine Graph ($y = \cos x$)**
   - ***Domain***: All real numbers
   - ***Range***: [-1, 1]
3. **Tangent Graph ($y = \tan x$)**
   - ***Domain***: All real numbers except 90°, 270°, 450°, 630° …..
   - ***Range***: All real numbers

Understanding these graphs involves recognising how changes to the equation affect their appearance. Mastery of trigonometry graphs is foundational for more advanced topics in maths, physics, and engineering.

In this section, we will learn how to graph these functions over a given range of angles. This will help us make inferences and use them to solve equations

The weeks, the indicators with their respective focal areas covered by the section are:

Week 14

- Draw graphs of given trigonometric functions and use them to solve related problems

  Focal Area: *Trigonometric graphs*

  - *Draw trigonometric graphs and use them to solve related problems.*

Week 15

- Use trigonometric graphs to determine equations.

  Focal Area: *Determining equations using trigonometric graphs*

  - *Use trigonometric graphs to determine equations and solve related problems.*

SUMMARY OF PEDAGOGICAL EXEMPLARS

This section is designed to be highly interactive, focusing on hands-on activities where learners actively engage with graphing trigonometric functions. To foster collaborative learning, students will work in groups to complete tables of values for provided trigonometric functions and then graph them.

Therefore, the lessons on these concepts should primarily feature **experiential learning activities** and **mixed-ability groupings**. All learners, irrespective of their current proficiency levels, will be fully supported to participate in investigations and present their findings.

To ensure effective differentiation and accommodation for diverse learning needs:

- Approaching-proficiency learners will be given dedicated opportunities to make oral presentations of their work, building confidence and communication skills.
- Above-average or highly proficient learners will have extended activities that involve using computer applications (e.g., graphing software, online calculators) to solve more complex problems related to trigonometric functions and their graphs.

ASSESSMENT SUMMARY

Table 1 contains the key assessment for each indicator in the section. The highest DOK for the indicator is denoted. The mode of assessment to be carried out and strategies that will aid in achieving the indicators are documented.

Table 1: Indicators, DOK levels, assessment modes, and strategies

Indicators	DOK Level	Assessment Strategy and/or Mode
Draw graphs of given trigonometric functions and use them to solve related problems	2	The assessment mode for Week 14 is Group work and presentation **Strategy:** Provide essay questions on trigonometric graphs for learners to answer in groups and present their answers to the class
Use trigonometric graphs to determine equations.	3	The assessment mode for week 15 is multiple representations. **Strategy:** Provide learners with real-life situations for learners to move between multiple representations (verbal description → graph → writing equation)

WEEK 14

Learning Indicator: *Draw graphs of given trigonometric functions and use them to solve related problems*

FOCAL AREA: TRIGONOMETRIC GRAPHS

Drawing trigonometric graphs and using them to solve related problems.

Trigonometry Graphs

The graph of a trigonometric function is a visual representation of the function for a given range of angles. The horizontal axis of a trigonometric graph represents the angle, usually written as θ, and the y-axis represents the function of that angle.

Throughout this lesson, we will examine the graphs of trigonometric functions for angles expressed in both degrees and radians. Before we consider the graphs of trigonometric functions, let us look at the relationship between degrees and radian measurement.

Relationship Between Degree and Radian Measurement

In mathematics, an angle can be measured in degrees (°) or in radians (rad).

A **degree** is a traditional unit of measurement for angles. One full circle is **360°**.

A **radian** is another way to measure angles. It is based on the length of the arc of a circle.

One radian is the angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle.

The Relationship

The circumference of a circle = $2\pi\mathbf{r}$

One full circle = 360°.

In radians, one full circle = $\frac{arc\ length}{radius} = \frac{2\pi r}{r} = 2\pi$

So, $360° = 2\pi radians$

Dividing both sides by 360, we have:

$$\frac{360°}{360} = \frac{2\pi}{360}\ radians$$

$$\frac{360 \times 1°}{360} = \frac{2\pi}{360}\ radians$$

$$1° = \frac{\pi}{180}\ radians$$

And dividing both sides by 2π:

$$\frac{360°}{2\pi} = \frac{2\pi \times 1 radians}{2\pi}$$

$$\frac{180°}{\pi} = 1\ radian$$

To convert **degrees to radians**: multiply the degree by $= \frac{\pi}{180}$

To convert **radians to degrees**: multiply the radian by $= \frac{180}{\pi}$

Angle in degrees	Angle in radians	Angle in degrees	Angle in radians
15°	$\frac{\pi}{12}$	195°	$\frac{13\pi}{12}$
30°	$\frac{\pi}{6}$	210°	$\frac{7\pi}{6}$
45°	$\frac{\pi}{4}$	225°	$\frac{5\pi}{4}$
60°	$\frac{\pi}{3}$	240°	$\frac{4\pi}{3}$
75°	$\frac{5\pi}{12}$	255°	$\frac{17\pi}{12}$
90°	$\frac{\pi}{2}$	270°	$\frac{3\pi}{2}$
105°	$\frac{7\pi}{12}$	285°	$\frac{19\pi}{12}$
120°	$\frac{2\pi}{3}$	300°	$\frac{5\pi}{3}$
135°	$\frac{3\pi}{4}$	315°	$\frac{7\pi}{4}$
150°	$\frac{5\pi}{6}$	330°	$\frac{11\pi}{6}$
165°	$\frac{11\pi}{12}$	345°	$\frac{23\pi}{12}$
180°	π	360°	2π

The graph of $y = sin\,\theta$

Figure 14.1 shows a diagram of the sine graph for $-360° \leq \theta \leq 360°$

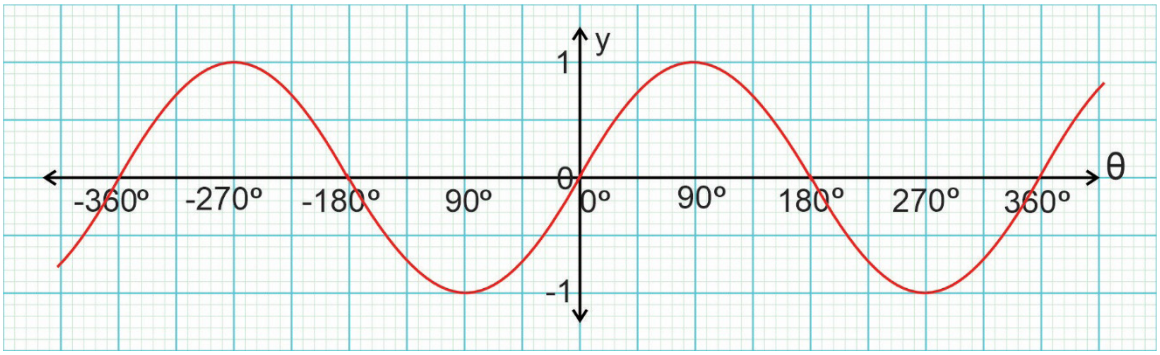


Figure 14.1: *Graph of* $y = sin\,\theta$ *for* $-360° \leq \theta \leq 360°$

Properties of the graph of $y = \sin\theta$

1. The graph passes through the origin.
2. The graph is continuous and repeats every 360°
3. The maximum value is +1
4. The minimum value is
5. The zeros are $\theta = 180n$ where is an integer. The zeros = {…, -360°, −180°, 0°, 180°, 360°, …..}
6. The domain is all real numbers. Domain = $\{\theta : \theta \in R\}$
7. The range is $-1 \leq y \leq 1$

The graph of $y = \cos\theta$

The diagram shows a graph of for

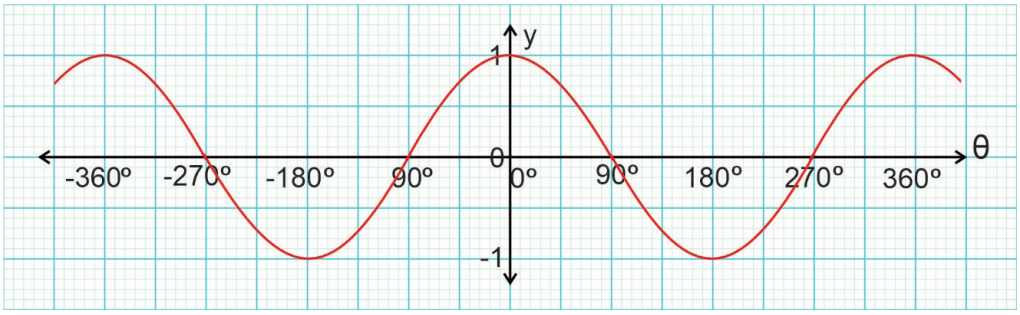


Figure 14.2: *Graph of y = Cos θ for* $-360^\circ \leq \theta \leq 360^\circ$

Properties of the graph of $y = \cos\theta$

1. The graph does NOT pass through the origin, unlike the Sin and the Tan graphs
2. The graph is continuous and repeats every 360
3. The maximum value is +1
4. The minimum value is −1
5. The zeros are $\theta = 90^\circ + 180n$ where is an integer. Zeros = {…,−270°, −90°, 90°, 270°, …..}
6. The domain is all real numbers. Domain = $\{\theta : \theta \in R\}$
7. The range is $-1 \leq y \leq 1$

The graphs of $y = \sin\theta$ and $y - \cos\theta$

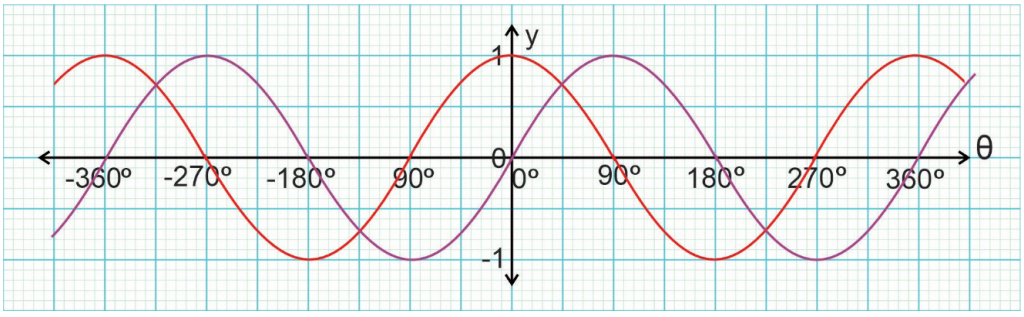


Figure 14.3: *Graph of y = sin θ and y = cos θ for* $-360^\circ \leq \theta \leq 360^\circ$

Relationship between the sine and the cosine graph

1. The graphs have the same shape
2. If you shift any of the graphs horizontally, either to the left or right by $90°$, you will get the other graph. This means $\cos(90-\theta)=\sin\theta$. Likewise, $\sin(90-\theta)=\cos\theta$.

The graph of $y=\tan\theta$

This graph has a unique shape compared with the sine and cosine graphs

Figure 14.4 shows a diagram of the tan graph for $-360° \leq \theta \leq 360°$

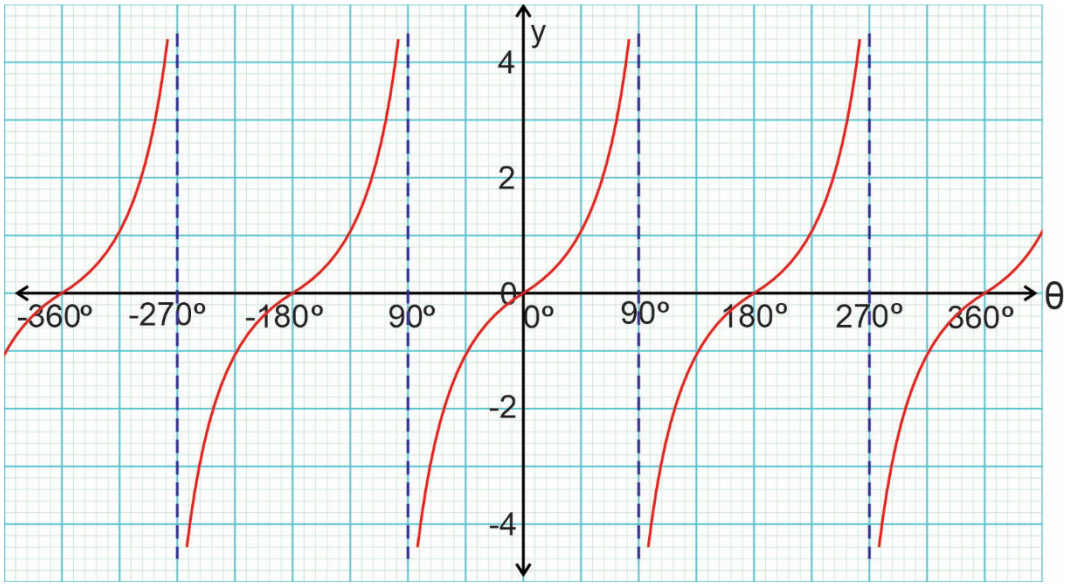


Figure 14.4: *Graph of y = tan θ for* $-360° \leq \theta \leq 360°$

Properties of the graph of $y=\tan\theta$

1. The graph passes through the origin.
2. The graph is NOT continuous and repeats every 180°.
3. It has no maximum value (maximum value approaches ∞)
4. It has no minimum value (minimum value approaches −∞). The tangent function has no maximum or minimum value; it is unbounded above and below.
5. The zeros are $\theta = 180n$ where is an integer. Zeros = {…,-360°, −180°, 0°, 180°, 360°,..}
6. Domain = $\{\theta : \theta \in R$ except $\theta = 90° + 180°n$ where n is an integer}.
7. The range is $-\infty \leq y \leq \infty$ or $y \in R$
8. Vertical asymptotes at $\theta = 90° + 180°n$ where n is an integer.

Drawing Trigonometric Graphs

Drawing a trigonometric graph follows the same procedure as drawing a quadratic graph.

Example 1

a. Copy and complete the ***Table 14.1***

Table 14.1: Partially completed table for the relation

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$y = 2sinx$	0		1.73		1.73						-1.73		

b. Using a scale of 2 cm to 60° on the x-axis and 2cm to 1 unit on the y – axis, draw the graph of $y = 2sinx$, for $0° \leq x \leq 360°$.

c. Use your graph to find the (i) maximum point (ii) minimum point.

Solution

Table 14.2: Completed table for the relation $y = 2sinx$

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$y = 2sinx$	0	1	1.73	2	1.73	1	0	−1	−1.73	−2	-1.73	−1	0

For example

When $x = 180°$

$y = 2\sin 180°$

$y = 0$

When $x = 240°$

$y = 2\sin 240°$

$y = -1.73$

Now we plot the above table values

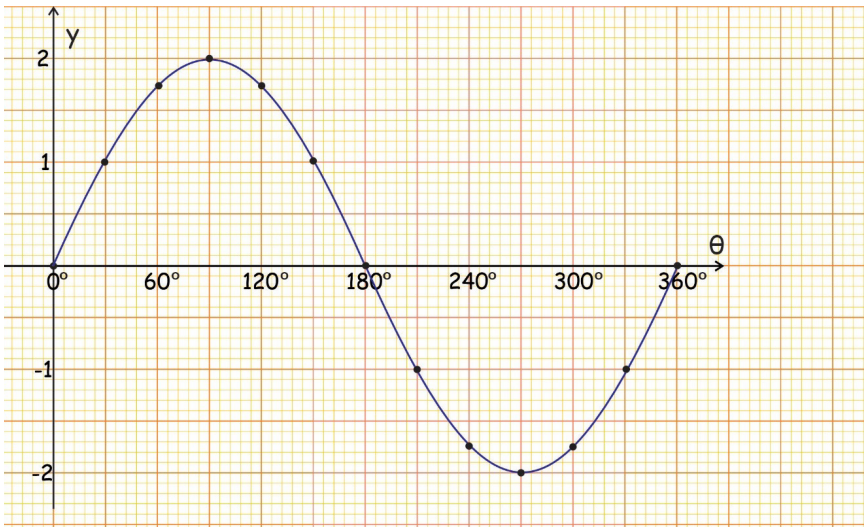


Figure 14.5: *Graph of $y = 2Sinx$ for $0° \leq x \leq 360°$*

c. From the graph, the

i. maximum point is (90°, 2)

ii. minimum point is $(270°, -2)$

Example 2

Copy and complete the table of the function $y = 2\sin x - \cos 2x$ for the interval $0° \leq x \leq 180°$.

Table 14.3: Partially completed table for the relation $y = 2sinx - cos2x$

x	0°	30°	60°	90°	120°	150°	180°
$y = 2\sin x - \cos 2x$		0.5					-1.0

Using a scale of 2 cm to 30° on the x-axis and 2cm to 1 unit on the y–axis draw the graph of $y = 2sinx - cos2x$, for $0° \leq x \leq 180°$

Solution

Table 14.4: Completed table for the relation $y = 2sinx - cos2x$

x	0°	30°	60°	90°	120°	150°	180°
$y = 2\sin x - \cos 2x$	-1	0.5	2.23	3	2.23	0.5	-1.0

When $x = 0°$

$$y = 2\sin(0) - \cos2(0)$$

$$y = -1$$

When $x = 60°$

$$y = 2\sin(60) - \cos2(60)$$

$$y = 2.23$$

When $x = 90°$

$$y = 2\sin(90) - \cos2(90)$$

$$y = 3$$

When $x = 120°$

$$y = 2\sin(120) - \cos2(120)$$

$$y = 2.23$$

When $x = 150°$

$$y = 2\sin(150) - \cos2(150)$$

$$y = 0.5$$

Plotting the points on the graph

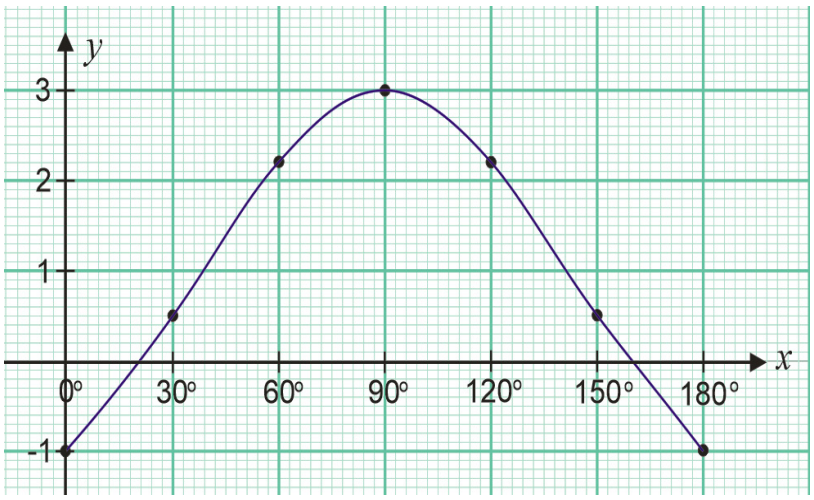


Figure 14.5: *Graph of y = 2sinx − cos2x , for 0° ≤ x ≤ 180°*

Transformation of Trigonometric Graphs

Amplitude, Period, and Phase Shift

The **Amplitude** is the vertical height from the centre line to the peak (trough). We can also measure the height from the highest to the lowest point and divide the result by 2. That is the maximum value minus the minimum value, and divide the result by 2

Examples are illustrated below

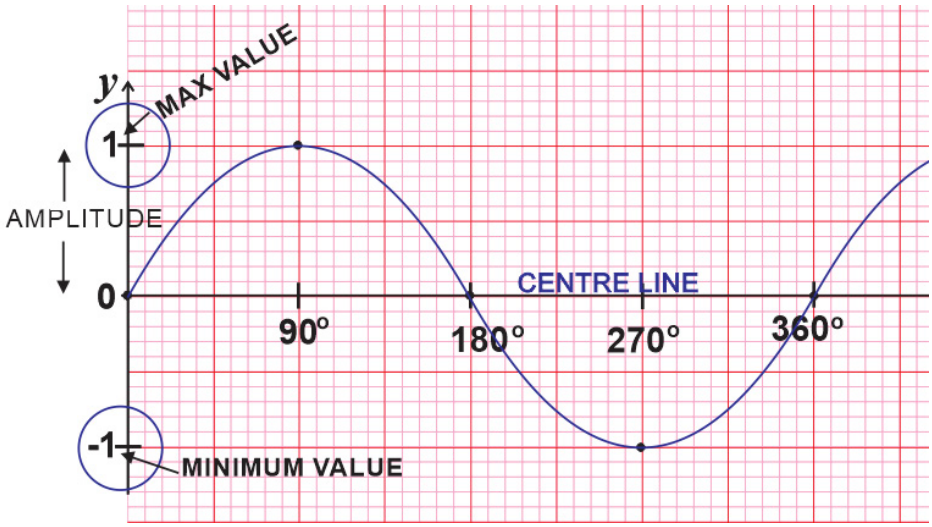


Figure 14.6: *Trigonometric transformation*

The centre line is the line $y = 0$. The amplitude is the vertical distance from the centre line to either the maximum or the minimum value. With the graph illustrated above, the amplitude is 1 or Amplitude $= \frac{(1 - -1)}{2} = 1$

The **Period** is the distance required for the graph to complete a full cycle. The period goes from one peak to the next (or from one point to the next matching point)

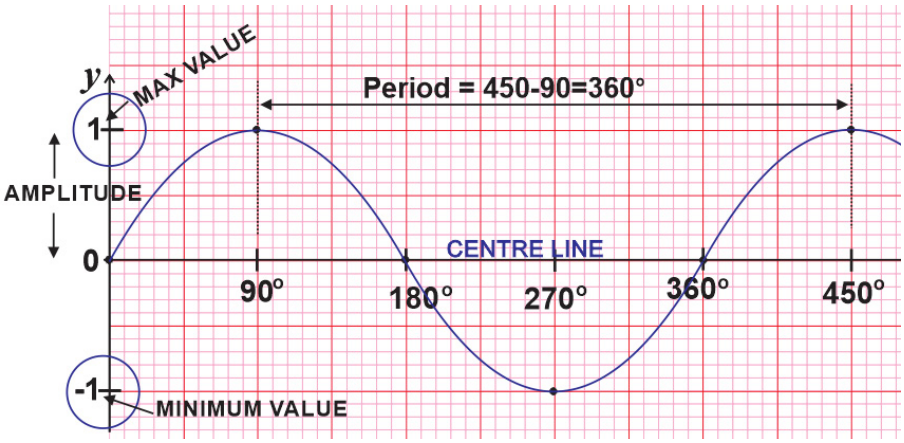


Figure 14.7: *Transformation of Trigonometric graph (Period)*

The period in the trigonometric graph above $= 450° + 90° = 360°$

A **phase shift** is the horizontal displacement of the graph from its basic position; it may be to the left or right.

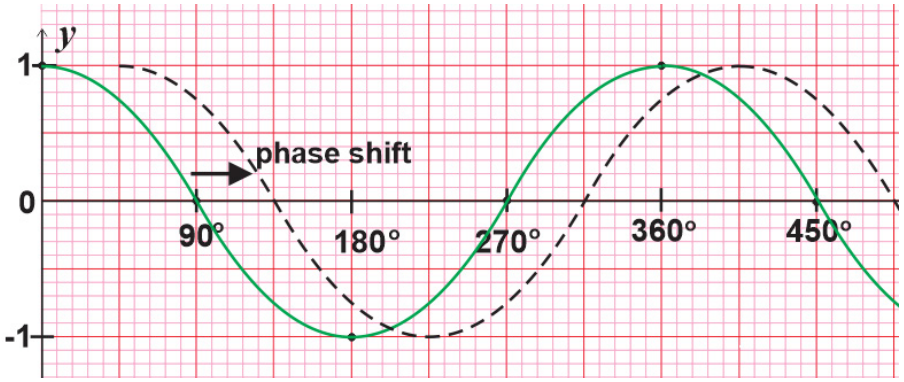


Figure 14.8: *Transformation of Trigonometric graph (phase shift)*

- The **dotted curve is a phase shift of the green curve by $+45°$ (to the right).**
- The **green curve is a phase shift of the dotted curve by $-45°$ (to the left).**
- **A shift to the right is positive, and a shift to the left is negative.**
- A **vertical shift** is the vertical displacement of the graph upward or downward from its basic position.

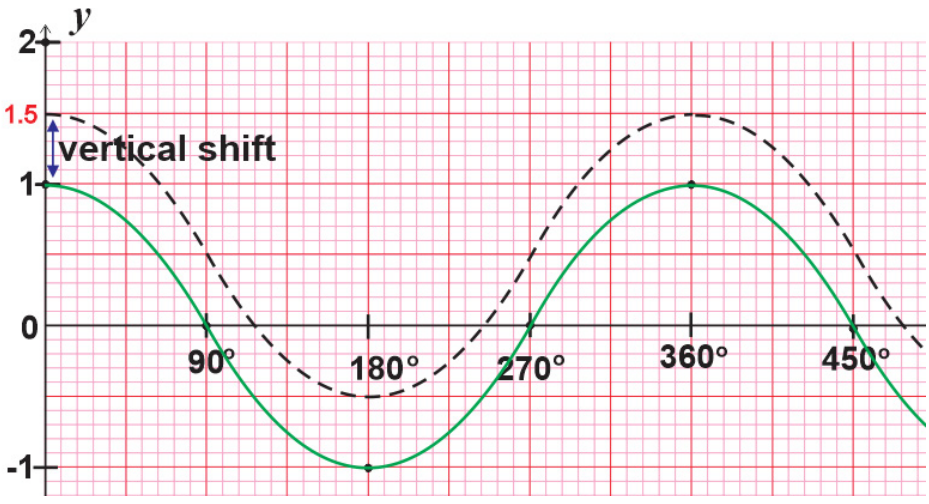


Figure 14.9: *Transformation of Trigonometric graph (Vertical shift)*

The dotted graph is the same as the green graph but has been shifted upwards(vertical) by 0.5

A shift up is positive and a shift down is negative.

In General, $If\ y = ASin(Bx + c) + D$ or $y = ACos(Bx + c) + D$

|A| is amplitude

$\frac{360}{B}$ is the period

$-\frac{c}{B}$ is the phase shift

D is the vertical shift

For example, if $2sin(3x - 90) + 1$

This implies, Amplitude = 2, meaning the graph will be twice as tall as $y = sinx$

Period $= \frac{360}{3} = 120°$, it means the graph will repeat itself every $120°$

Phase shift $= -\frac{-90}{3} = 30°$, it implies the graph will be shifted to the right by $30°$ (positive goes to the right)

Vertical shift = 1, this tells us the centre line will be instead of so it will be shifted upwards by 1 unit.

Note that the Tangent graph has no Amplitude and its range $= \{-\infty \leq x \leq +\infty\}$. It has no maximum or minimum values.

The range of the cosine and sine graphs $= \{-|A| \leq x \leq |A|\}$

The graph of $y = \sin x$ (blue curve) and $y = 2\sin(3x - 90) + 1$ (red curve)

are illustrated below.

Table 14.5: Table for the relation $y = 2\sin(3x - 90) + 1$ and $y = sinx$

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°
$sinx$	0			1			0			-1
$y = 2\sin(3x - 90) + 1$	-1	1	3	1	-1	1	3	1	-1	1

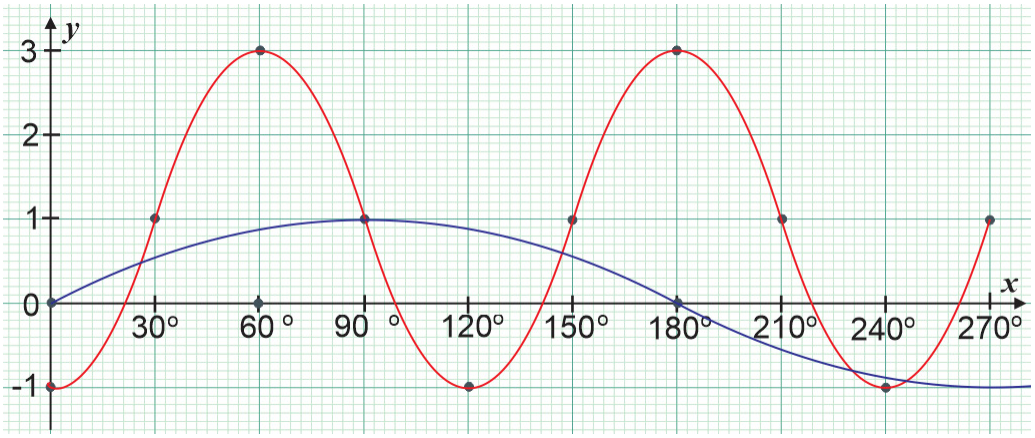


Figure 14.10: *Graphs of* $y = 2\sin(3x - 90) + 1$ *and* $y = sinx$

We can apply this knowledge to write the equation of the Sine and Cosine given a sketch of the graph. *If* $y = ASin(Bx + c) + D$ or $y = ACos(Bx + c) + D$

1. $A = |\frac{(Maximum\ Value - Minimum\ Value)}{2}|$
2. Equate the period to $\frac{360}{B}$ and solve for B
3. $\mathbf{D} = \frac{maximum\ value + minimum\ value}{2}$
4. Equate the phase shift to $-\frac{c}{B}$ and solve for C

To find the **phase shift** for a graph of the form $\boldsymbol{y = aCos(bx + c) + d}$, we identify one basic cycle of the graph that begins at the maximum value, decreases to the minimum value and increases to the maximum value. Find the x-coordinate of the end points of the cycle $[x_0, x_1]$, The value of the phase shift is the lower end point of the interval of the basic cycle: phase shift $= x_0$

To find the **phase shift** for a graph of the form $\boldsymbol{y = aSin(bx + c) + d}$, we identify one basic cycle of the graph that begins at $y = k$ (where k is the centre line). Find the x-coordinates of the end point of the cycle $[x_0, x_1]$. The value of the phase shift is the lower end point of the interval of the basic cycle: phase shift $= x_0$.

Note that the equation of the sine and cosine differ only by a phase shift as illustrated below

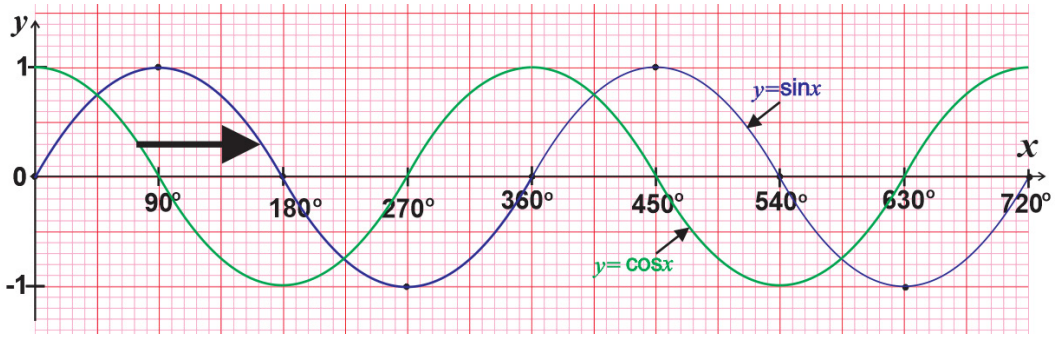


Figure 14.11: *Graphs of y = Sinx and y = Cosx*

Sinx is a phase shift of 90° of *Cosx*

Example 3

The diagram below represents a trigonometric graph.

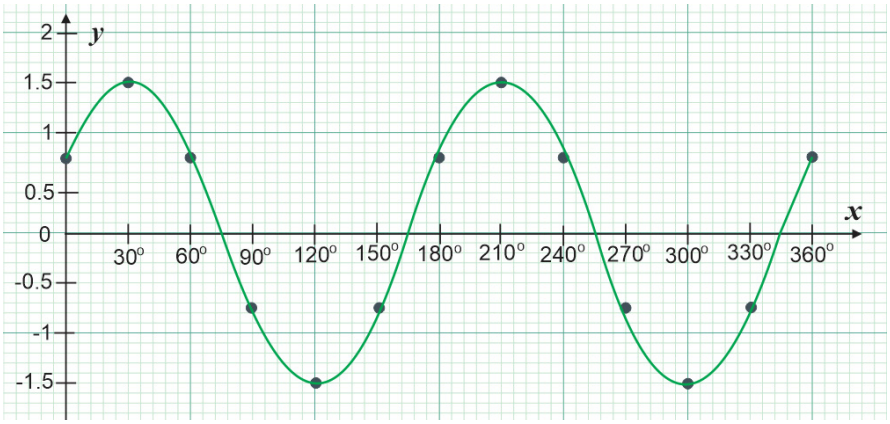


Figure 14.12: *Graph of a trigonometric function*

a. State its

i. Amplitude

ii. Period

iii. Vertical shift

b. Write the equation of the graph in the form $y = aCos(bx + c) + d$, where a, b and c are constants.

c. Write the equation of the graph in the form $y = aSin(bx + c) + d$, where a, b and c are constants.

Solution

a. Amplitude $= \left|\frac{(Maximum\ Value - Minimum\ Value)}{2}\right|$

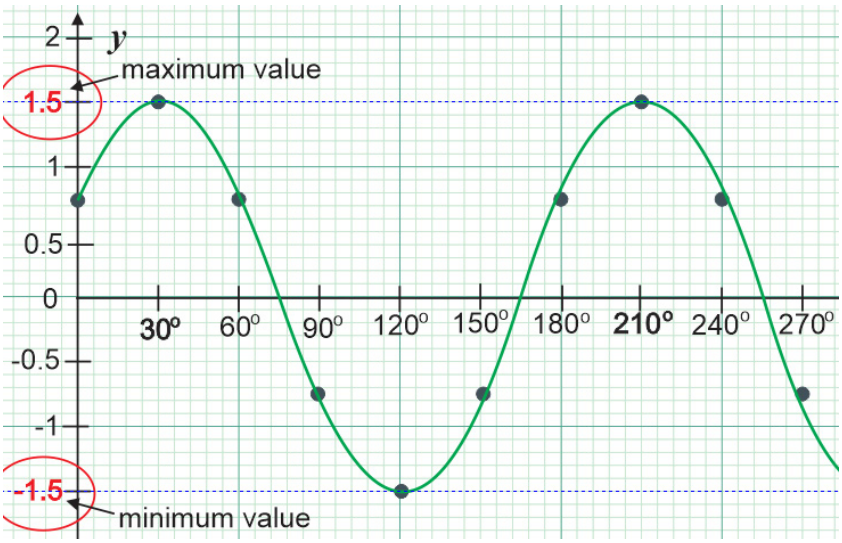


Figure 14.13: *Making inferences with a trigonometric graph (amplitude)*

i. From the graph, the maximum value is 1.5 and the minimum value is −1.5

Therefore, **Amplitude** $= \frac{1.5 - (-1.5)}{2} = 1.5$

ii. Period is the distance between two peaks

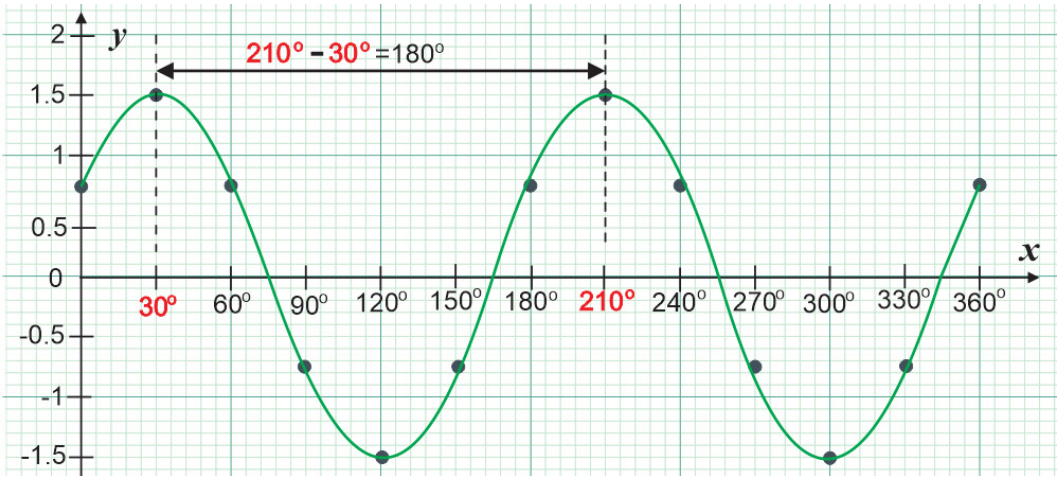


Figure 14.14: *Making inferences with a trigonometric graph (period)*

Hence, from the graph Period $= 210° - 30° = 180°$

iii. Vertical shift $= \frac{maximum\ value + minimum\ value}{2} = \frac{1.5 + (-1.5)}{2} = 0$

b. $y = aCos(bx + c) + d$

c. $a =$ Amplitude $= 1.5$

d. $d =$ Vertical shift $= 0$

To find b, we will equate the period to $\frac{360}{B}$

$$\Rightarrow 180 = \frac{360}{B}$$

$$180b = 360°$$

$$b = \frac{360}{180} = 2$$

To find c,we have to equate $-\frac{c}{B}$ to the phase shift.

To find the phase shift for a graph of the form $y = aCos(bx + c) + d$, we identify one basic cycle of the graph. Find the x-coordinates of the end points of the cycle $[x_0, x_1]$, the value of the phase shift is the lower end point of the interval of the basic cycle: phase shift $= x_0$

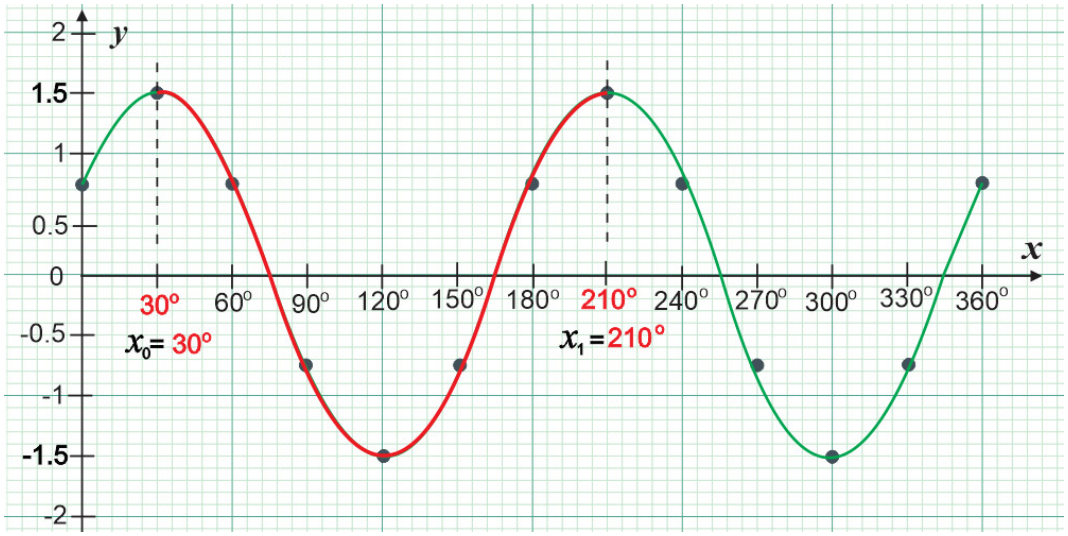


Figure 14.15: *Making inferences with a trigonometric graph (Cosine equation)*

From the graph, we have $x_0 = 30° \Rightarrow Phase\ Shift = 30°$

we then equate the phase shift to $-\frac{c}{B}$ and solve for c

$-\frac{c}{b} = 30$, note that $b = 2$

$$-\frac{c}{2} = 30$$

$$c = -60°$$

Hence, the equation of the graph is $y = 1.5Cos(2x - 60)$ or $y = 1.5Cos2(x - 30)$

e. The process of finding the amplitude and vertical shift is the same for both equations.

To find the phase shift for a graph of the form $y = aSin(bx + c) + d$, we identify one basic cycle of the graph that begins at $y = 0$. Find the x-coordinates of the end points of the cycle $[x_0, x_1]$. The value of the phase shift is the lower end point of the interval of the basic cycle: phase shift $= x_0$.

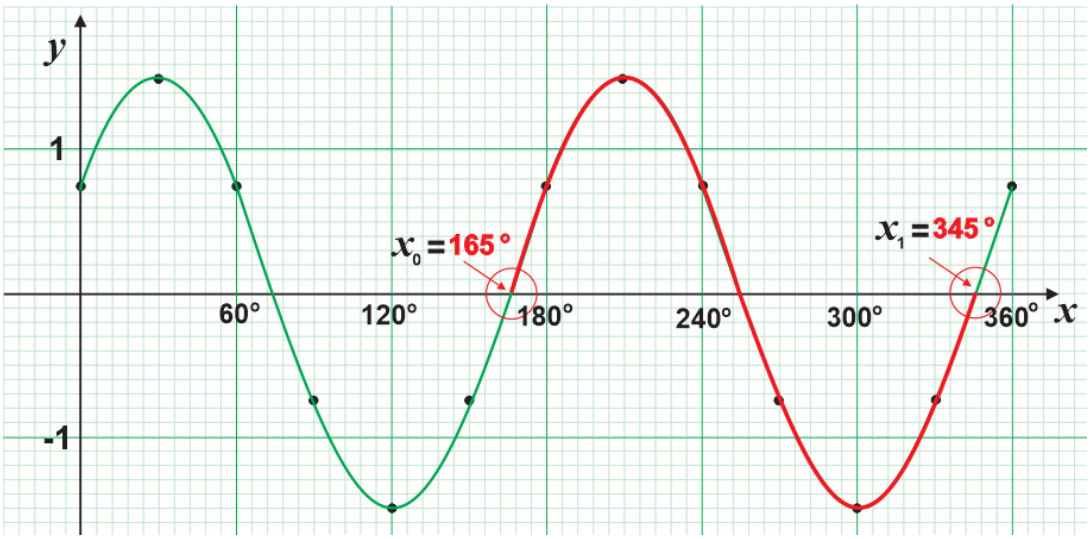


Figure 14.16: *Making inferences with a trigonometric graph (Sine equation)*

From the graph, we have $x_0 = 165° \Rightarrow Phase\ Shift = 165°$

we then equate the phase shift to $-\frac{c}{B}$ and solve for c

$$-\frac{c}{b} = 165 \text{ , note that } b = 2$$

$$-\frac{c}{2} = 165$$

$$c = -330°$$

Hence, the equation of the graph is $y = 1.5Sin(2x - 330°)$ or $y = 1.5Sin2(x - 165°)$

Learning Tasks

Guide learners to construct tables of values for points satisfying a given trigonometric function. Subsequently, guide them to plot these points and draw the resulting graph. Provide opportunities for learners to utilise their drawn graphs to answer related questions.

PEDAGOGICAL EXEMPLARS

The aim of this week's lesson is to enable all learners to solve problems involving trigonometric graphs. The following pedagogical approaches are recommended:

1. **Whole-Class Review and Discussion:** Initiate a discussion with the entire class to review learners' existing knowledge of plotting quadratic equations. Address any misconceptions, then extend these concepts to the graphing of trigonometric functions.

2. **Small Group Collaborative Learning:** Organise learners into small groups to discuss and explore sine, cosine, and tangent graphs, incorporating illustrative examples. Within these groups, learners will also practice drawing sine, cosine, and tangent graphs.

KEY ASSESSMENT LEVEL 2: GROUP WORK

1.

a. Copy and complete the table of values for the relation $y = 1 - 2\cos\theta$. ***3 marks***

Table 14.6: Partially completed table for the relation $y = 1 - 2\cos\theta$

θ	0°	45°	90°	135°	180°	225°	270°	315°	360°
$y = 1 - 2\cos\theta$		-0.41				2.41			-1

b. Using a scale of 2 cm to 45 on the θ-axis and 2cm to 1 unit on the y–axis, draw the graph of $y = 1 - 2\cos\theta$, for $0° \leq x \leq 360°$ ***4 marks***

c. Use your drawn graph to find the

i. the maximum value, ***1 mark***

ii. the coordinate at which the maximum occurs, ***1 mark***

iii. values of θ for $y = 0$, correct your answer to one significant figure. ***2 marks***

iv. Line of symmetry ***1 mark***

Total *12 marks*

2. Copy and complete the table of the function $y = 2\sin 3x$ for the interval $0° \leq x \leq 180°$ ***3 marks***

Table 14.7: Partially completed table for the relation $y = 2\sin 3x$

x	0°	30°	60°	90°	120°	150°	180°
$y = 2\sin 3x$		2					0

a. Using a scale of 2 cm to on the x-axis and 2cm to 1 unit on the y–axis, draw the graph of $y = 2\sin 3x$, for $0° \leq x \leq 180°$ ***4 marks***

c. Use your drawn graph to find the

i. the minimum point ***1 mark***

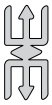
ii. the maximum point ***1 mark***

iii. equation of the line of symmetry ***1 mark***

PLC Session 14

45 minutes **Planning & Reviewing the Learning Plan**

1. Begin the session by recapping lessons from the previous year/session/week, focusing on how learning was applied.
2. Use your subject-specific app to explore the content before planning. Ask the app to
   a. explain the week's concepts in a practical way that will make it easier for learners to understand
   b. identify two or three common misconceptions learners may have on the week's content and how to address them using contexts familiar to Ghanaian learners. (NTS 3a, 3g and 3m).
3. Use your subject-specific app to plan your lesson. Ask the app to
   a. suggest the most effective method for teaching the week's concepts to learners with diverse backgrounds.
   b. suggest learning activities, differentiation and essential questions based on the method of teaching (NTS 3a-3q).
   c. review your prepared learning plan and address any challenging issues you may have identified in your subject area (NTS 3a-3c).



Note

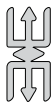
- Take note of the recap topics linked to Years 1 and 2 while preparing for your lesson.
- Refer to the Values Learning Community Handbook and link the values for the week in your lesson to ensure that learners are applying what they learn.

4. Plan the week's key assessment with your app
   a. The key assessment for the week is ***group work***. Chat with your app to plan this assessment, indicating what you and your learners will do before, during and after the group work (NTS 3k-3q).

10 minutes **Content Exploration**

5. As a subject group, select one problem or idea/term in the week's content that is challenging (not a routine procedural question, but one that requires conceptual understanding) and then
   a. use the app, ask it to solve the problem step by step and explain the reasoning behind each step (you may be assisted by a teacher who is strong in the content area)
   b. all teachers in the group work through a similar problem individually for 3–4 minutes
   c. the group then works through the problem together step by step

d. highlight the most common errors made at each step and discuss (NTS 1c, 1e, 2c, 2e, 3e-3j).



Note

- This part is not just about 'How do I teach this?' but how do I teach it for learners to understand?
- If the group still shares the same content gap, use the app: ask it to solve the problem step by step and explain the reasoning behind each step.
- Problems/ideas discussed should be sourced from the Teacher Manual content, not invented separately. This keeps the content exploration directly connected to what teachers will deliver.

30 minutes Enactment

6. Enact a teaching activity based on your learning plan (NTS la-1b, 1e-lg, 2c, 2e, 2f, 3c-3j and 3o).

a. Colleagues observe with specific attention to clarity of concept explanation, use of familiar Ghanaian examples and how misconceptions are handled if they arise.

b. After the enactment, ask for structured feedback from the group

i. was the concept explained in a way that a learner with no prior knowledge could follow?

ii. were the examples grounded in contexts familiar to Ghanaian learners?

iii. did any misconceptions surface and if so, how were they addressed?

5 minutes Reflection

7. Reflect and share your views on the session. One thing that worked well, what did not work well and what you will do differently. (NTS 1a, 1b).

8. Identify a critical friend to observe and provide feedback on your lesson in relation to the PLC Session conducted (NTS 1f, 1g).

9. Remember to

a. transfer your chats into your learning planner template.

b. read your Teacher Manual and related Learner Material in preparation for the next session. Take note of the recap weeks, which are linked to Years 1 and 2 (NTS 3a).

c. integrate National Values, GESI, SEL, and 21st century skills into your learning activities, assessment tasks, and rubrics/mark schemes.

WEEK 15

Learning Indicator: *Use a trigonometry graph to determine equations*

FOCAL AREA: USE TRIGONOMETRIC GRAPHS TO SOLVE EQUATIONS AND RELATED PROBLEMS

Use trigonometric graphs to determine equations and solve related problems.

Finding the solution set using the trigonometry graph

We will use the following examples to learn how to solve trigonometry equations using a graph.

Example 1

a. Copy and complete the table below for the relation $y = \sin x - 3\cos x$ for values x of from 0° to 180° at intervals of 30°.

Table 15.1: Partially completed table for the relation $y = \sin x - 3\cos x$

x	0°	30°	60°	90°	120°	150°	180°
$y = \sin x - 3\cos x$	−3.00	−2.10	−0.63			3.10	

b. Using a scale of 2 cm to 30° on the -axis and 2 cm to 1 unit on the y-axis draw a graph of y.

c. Use your graph to estimate, in the given interval, the truth set of the equations

i. $\sin x - 3\cos x = 0$

ii. $\sin x - 3\cos x = 1$

Solution

a. We complete the above table

Table 15.2: Completed table for the relation $y = \sin x - 3\cos x$

x	0°	30°	60°	90°	120°	150°	180°
$y = \sin x - 3\cos x$	−3.00	−2.10	−0.63	1.00	2.37	3.10	3.00

When $x = 90^\circ$

$y = \sin(90) - 3\cos(90)$

$y = 1$

When $x = 120^\circ$

$y = \sin(120) - 3\cos(120)$

$y = 2.37$

When $x = 180°$

$y = \sin(180) - 3\cos(180)$

$y = 3$

b. Plotting the graph of the function

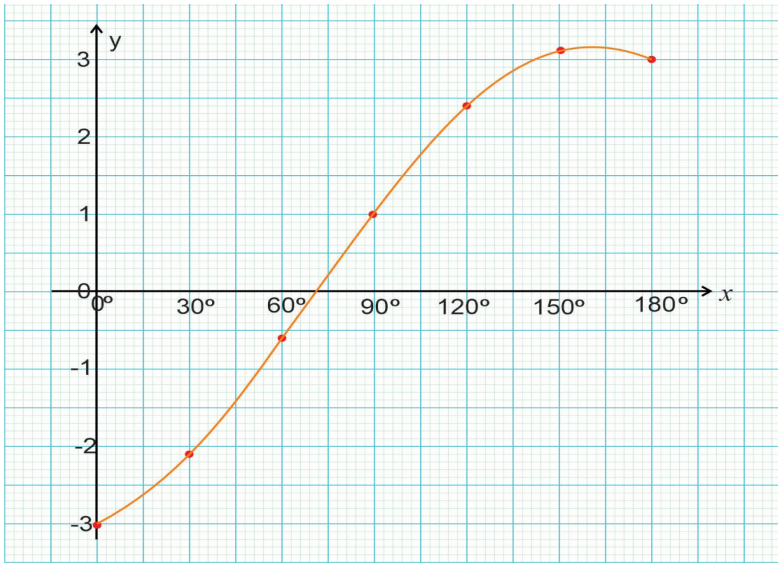


Figure 15.1: *graph of y = sinx − 3cos x*

c.

i. Comparing the two equations $\sin x - 3\cos x = 0$ but

$$y = \sin x - 3\cos x$$

$$y = 0$$

From the graph, the curve intersects the line $y = 0$ at $x \approx 72°$

Accept any value between $70°$ to $72°$.

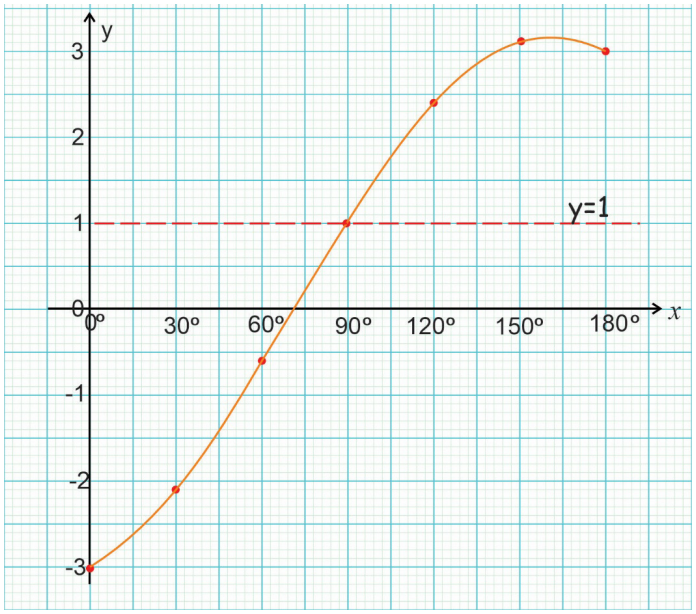


Figure 15.2: *graphs of y = sinx − 3cos x and y = 1*

Hence, the solution set of is given by: $\{x : x \approx 72^\circ\}$

ii. Comparing $\sin x - 3\cos x = 1$ and $y = \sin x - 3\cos x$, $y = 1$.
From the graph, the curve, and the line y = 1 intersect at x = 90°
Hence, the solution set of $\sin x - 3\cos x = 1$ is given by: $\{x : x = 90^\circ\}$

Example 2

a. Copy and complete the table below for the relation $y = 2\cos x$ for values of x from 0° to 360° at intervals of 45°.

Table 15.3: Partially completed table for the relation $y = 2\cos x$

x	0°	45°	90°	135°	180°	225°	270°	315°	360°
$y = 2\cos x$			0		−2		0		

b. Using a scale of 2 cm to 45° on the x-axis and 2 cm to 1 unit on the y-axis, draw a graph of y.

c. Use your graph to estimate, in the given interval, the truth set of the equations

i. $1 + 2\cos x = 0$

ii. the minimum point of $y = 2\cos x$

iii. equation of the line of symmetry

Solution

a. We copy and complete the above table.

Table 15.4: Completed table of the relation $y = 2\cos x$

x	0°	45°	90°	135°	180°	225°	270°	315°	360°
$y = 2\cos x$	2	1.41	0	−1.41	−2	−1.41	0	1.41	2

b. We plot the above points on the graph and join them with a free-hand curve

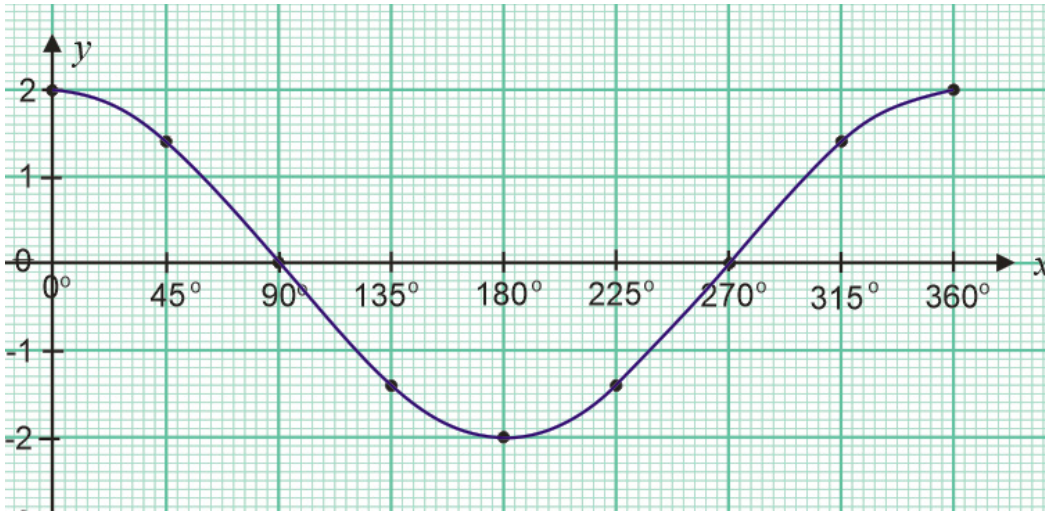


Figure 15.3: *Graph of* $y = 2\cos x$

c.

i. Comparing the two equations $y = 2\cos x$ and $1 + 2\cos x = 0$, we obtained $y = -1$.

Draw the line $y = -1$. on the graph and find its truth set. That is the values of x for which the line and the curve intersect.

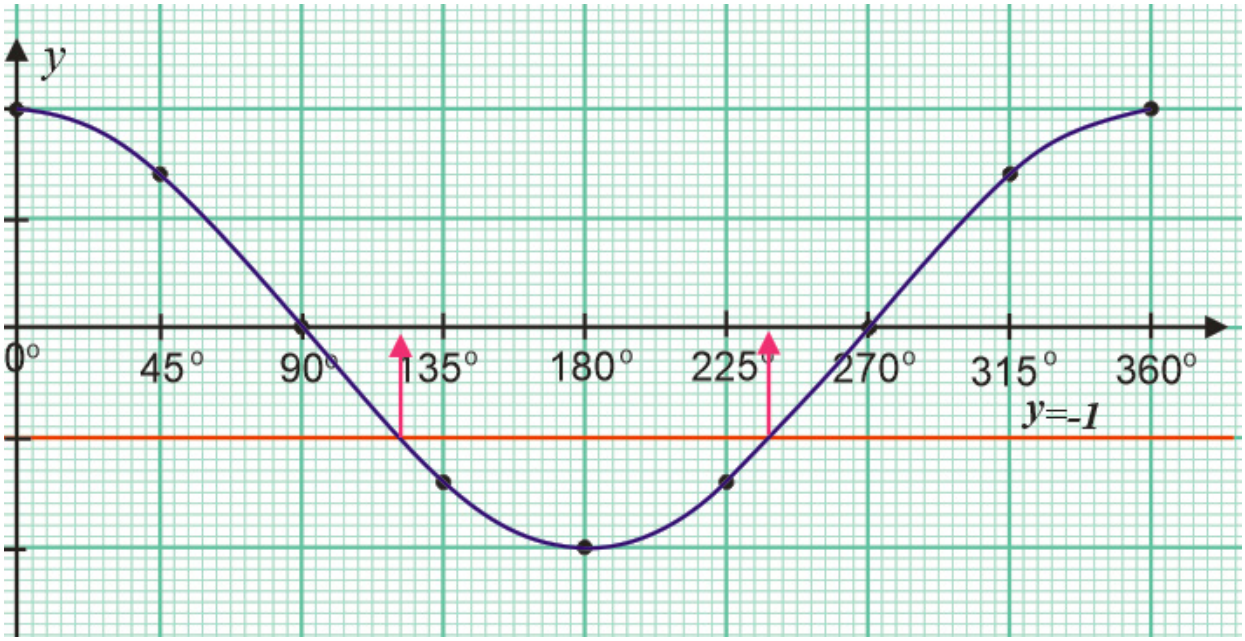


Figure 15.4: *Graph of y = 2cosx and line y = −1*

From the graph, the line $y = -1$ intersect the curve at $x \approx 121.5°$ and $x \approx 238.5°$ as shown in the graph above.

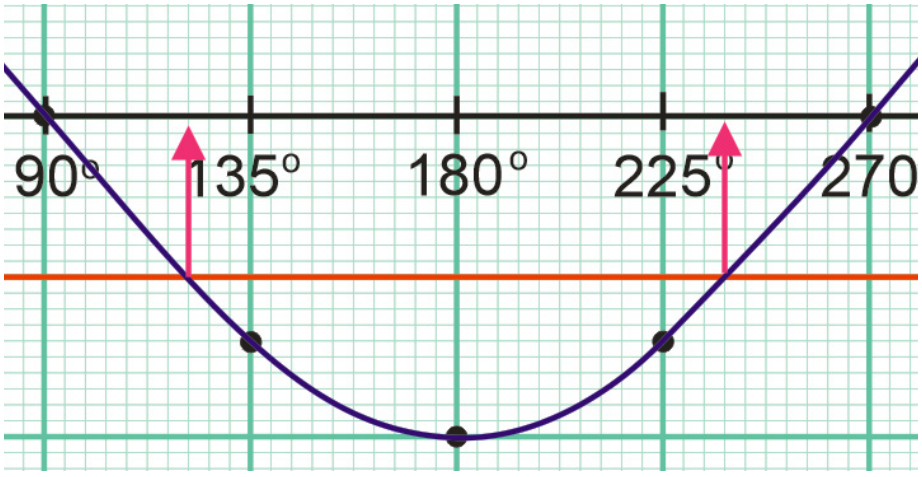


Figure 15.5: *A zoom in of a part of the graph of y = 2cosx and y = −1*

Zooming in clearly shows that the intersection of the line and the curve falls on the 7th sub-line after 90° and also the third sub-line after 225°. Using the given scale, we have

$$x \approx 90 + 7 \times 4.5 \approx 121.5° \text{ and } x \approx 225 + 3 \times 4.5 \approx 238.5°$$

ii. The minimum point is the lowest point of the graph over the stated interval.

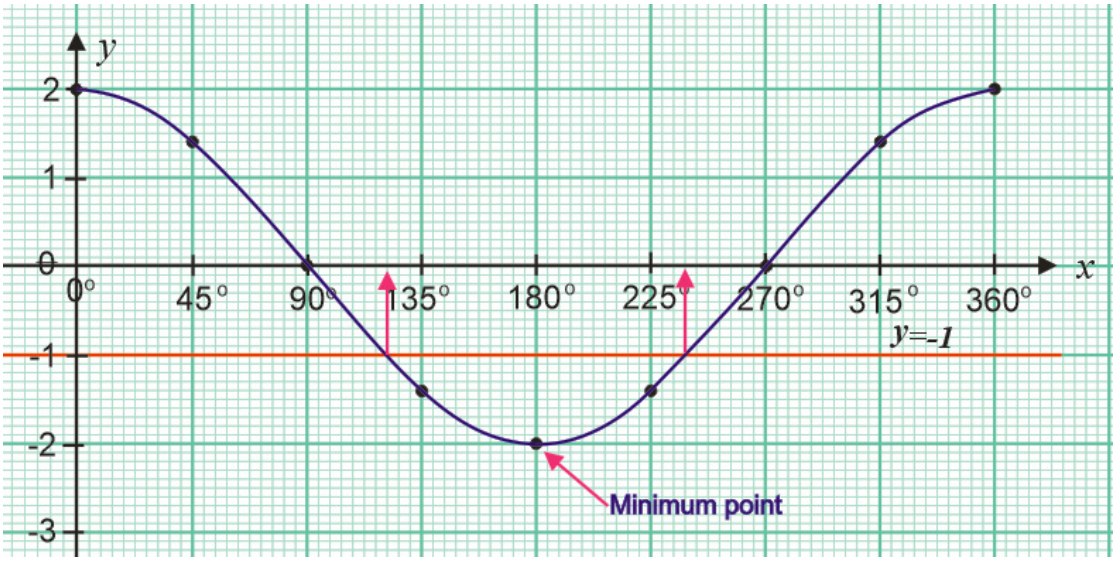


Figure 15.6: *Graph of y = 2cosx showing the minimum point*

From the graph, the minimum point as indicated as (180°, −2). The y-coordinate of this point is called the minimum value, which is −2. The minimum value occurs when $x = 180°$.

iii. The line of symmetry divides the curve within the range given into equal halves.

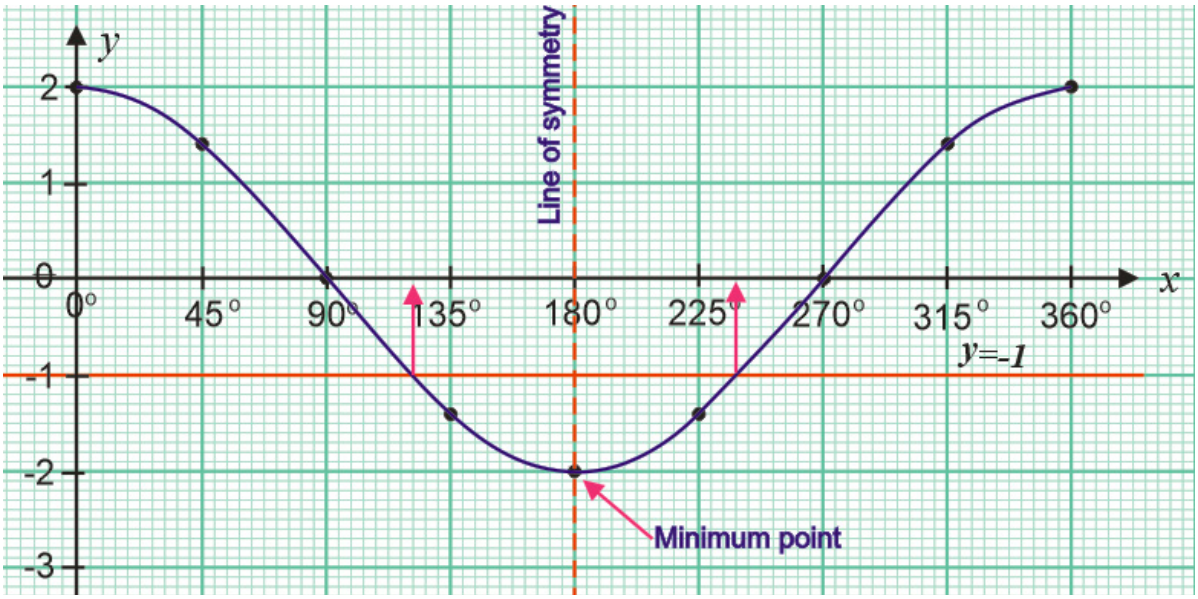


Figure 15.7: *Graph of y = 2cosx showing the equation of the line of symmetry*

From the graph, the line of symmetry is indicated by the broken line. It intersects the x-axis at 180. Hence, the line of symmetry is; $x = 180°$

KEY ASSESSMENT LEVEL 3: COMPUTATIONS & INTERPRETATIONS

1.

a. Copy and complete the table below for the relation $y = 2\sin x + 3$ for values of x from 0° to 180° at intervals of 30°. ***2 marks***

Table 15.5: Partially completed table of the relation $y = 2\sin x + 3$

x	0°	30°	60°	90°	120°	150°	180°
$y = 2\sin x + 3$	3.00					4.00	3.00

b. Using a scale of 2 cm to 30° on the x-axis and 2 cm to 1 unit on the y-axis, draw a graph of $y = 2\sin x + 3$ ***4 marks***

c. Use your graph to estimate, in the given interval,

i. Solve the equation $2\sin x - 1 = 0$ **3 marks**

ii. the maximum value of y and the value of x at which it occurs ***2 marks***

Total *11 marks*

2.

a. Copy and complete the table below for the relation $y = 3\cos x + 2\sin x$ for values of x from 0° to 180° at intervals of 30°. ***2 marks***

Table 15.6: Partially completed table of the relation $y = 3\cos x + 2\sin x$

x	0°	30°	60°	90°	120°	150°	180°
$y = 3\cos x + 2\sin x$	3.00		3.23				−3.00

b. Using a scale of 2 cm to 30° on the x-axis and 2 cm to 1 unit on the y-axis draw a graph of $y = 3\cos x + 2\sin x$. ***4 marks***

c. Use your graph to estimate, in the given interval, the truth set of the equations.

Correct your answer to one decimal place.

i. $6\cos x + 4\sin x = 1$ ***4 marks***

ii. $3\cos x + 2\sin x = -1$ ***4 marks***

Total *14 marks*

PLC Session 15

45 minutes **Planning & Reviewing the Learning Plan**

1. Begin the session by recapping lessons from the previous year/session/week, focusing on how learning was applied.

2. Use your subject specific app to explore the content before planning. Ask the app to

a. explain the week's concepts in a practical way that will make it easier for learners to understand

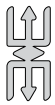
b. identify two or three common misconceptions learners may have on the week's content and how to address them using contexts familiar to Ghanaian learners. (NTS 3a, 3g and 3m).

3. Use your subject specific app to plan your lesson. Ask the app to

   a. suggest the most effective method for teaching the week's concepts to learners with diverse backgrounds.

   b. suggest learning activities, differentiation and essential questions based on the method of teaching (NTS 3a-3q)

   c. review your prepared learning plan and address any challenging issues you may have identified in your subject area (NTS 3a-3c).



Note

- Take note of the recap topics linked to Years 1 and 2 while preparing for your lesson.
- Refer to the Values Learning Community Handbook and link the values for the week in your lesson to ensure that learners are applying what they learn.

4. Plan the week's key assessment with your app

   a. The key assessment for the week is ***computations***. Chat with your app to plan this assessment, indicating what you and your learners will do before, during and after the assessment (NTS 3k-3q).

10 minutes **Content Exploration**

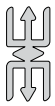
5. As a subject group, select one problem or idea/term in the week's content that is challenging (not a routine procedural question, but one that requires conceptual understanding) and then

   a. use the app, ask it to solve the problem step by step and explain the reasoning behind each step (you may be assisted by a teacher who is strong in the content area)

   b. all teachers in the group work through a similar problem individually for 3–4 minutes

   c. the group then works through the problem together step by step

   d. highlight the most common errors made at each step and discuss (NTS 1c, 1e, 2c, 2e, 3e-3j).



Note

- This part is not just about 'How do I teach this?' but how do I teach it for learners to understand?
- If the group still shares the same content gap, use the app: ask it to solve the problem step by step and explain the reasoning behind each step.
- Problems/ideas discussed should be sourced from the Teacher Manual content, not invented separately. This keeps the content exploration directly connected to what teachers will deliver.

30 minutes **Enactment**

6. Enact a teaching activity based on your learning plan (NTS la-1b, 1e-lg, 2c, 2e, 2f, 3c-3j and 3o).
   a. Colleagues observe with specific attention to clarity of concept explanation, use of familiar Ghanaian examples and how misconceptions are handled if they arise.
   b. After the enactment, ask for structured feedback from the group
      i. was the concept explained in a way that a learner with no prior knowledge could follow?
      ii. were the examples grounded in contexts familiar to Ghanaian learners?
      iii. did any misconceptions surface and if so, how were they addressed?

5 minutes **Reflection**

7. Reflect and share your views on the session. One thing that worked well, what did not work well and what you will do differently. (NTS 1a, 1b).
8. Identify a critical friend to observe and provide feedback on your lesson in relation to the PLC Session conducted (NTS 1f, 1g).
9. Remember to
   a. transfer your chats into your learning planner template.
   b. read your Teacher Manual and related Learner Material in preparation for the next session. Take note of the recap weeks, which are linked to Years 1 and 2 (NTS 3a).
   c. integrate National Values, GESI, SEL, and 21st century skills into your learning activities, assessment tasks, and rubrics/mark schemes.

Section 4 Review

This section summarises the core concepts covered in Weeks 14 and 15, providing an essential overview of the learning outcomes during this period. These weeks are crucial for establishing a foundational understanding of trigonometry, paving the way for more advanced topics. During this period, learners developed proficiency in:

1. **Understanding Basic Trigonometric Functions**
   a. Identifying and describing the fundamental characteristics, shapes, and properties of the sine, cosine, and tangent functions.
2. **Graphing Trigonometric Functions**
   - Constructing tables of values for various trigonometric functions.
   - Utilising these tables to accurately plot points and draw the corresponding graphs.

3. Analysing Trigonometric Graphs

a. Interpreting drawn graphs to answer related questions, including:

- Determining minimum and maximum values and their corresponding points.
- Identifying equations of lines of symmetry.
- Graphically solving trigonometric equations.

Recommended Resources for Enhanced Teaching and Learning

To optimise the teaching and learning experience, the following resources are highly recommended:

- Complete set of mathematical instruments
- Whiteboard
- Educational videos
- Mini whiteboards
- Dry erase markers and erasers
- Calculators
- Graph/grid paper
- Digital tools (e.g., computers, mobile phones, YouTube videos)

MARKING SCHEMES FOR THE KEY ASSESSMENTS

WEEK 14

Question One

a.

Table 14.8: Completed table for the relation $y = 1 - 2\cos\theta$

θ	0°	45°	90°	135°	180°	225°	270°	315°	360°
y	**-1**	-0.41	**1**	**2.41**	**3**	2.41	**1**	**-0.41**	-1

B3 for table (−0.5ee)

B1 for correct axes

B2 for points plotted correctly (−0.5 for each error)

b.

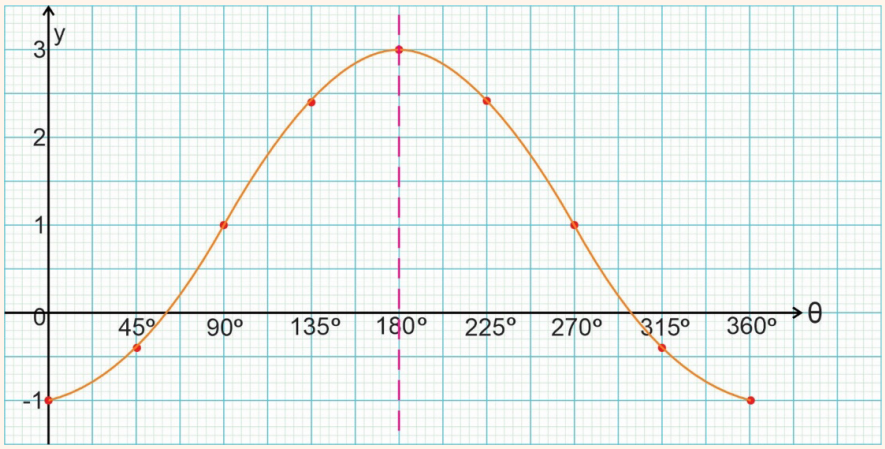


Figure 14.17: *Graph of* $y = 1 - 2\cos\theta$ *for* $0^\circ \leq \theta \leq 360^\circ$

B1 for a smooth curve joining the points

A1 for max value

A1 for maximum point

A1 for $\theta = 60$

A1 for $\theta = 180^\circ$

A1 for line of symmetry

c.

i. maximum value = 3

ii. maximum point = (180°, 3)

iii. when $y = 0$, $\theta = 60, 300^\circ$

iv. line of symmetry is $\theta = 180^\circ$

Question Two

a.

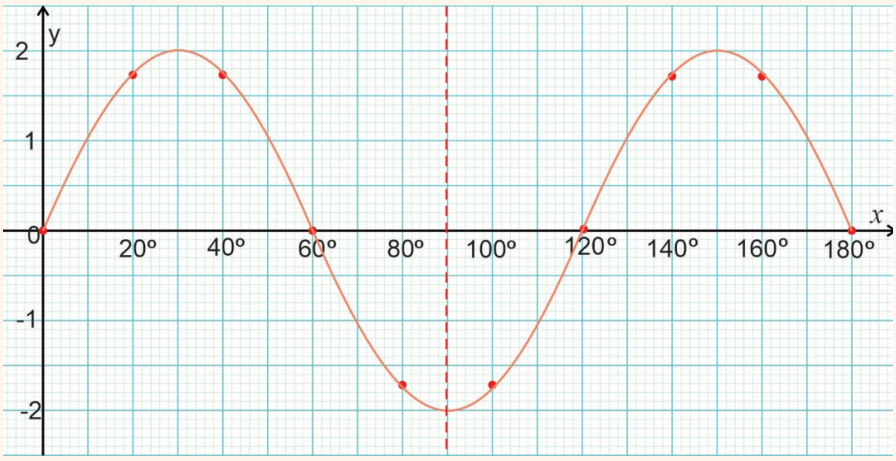
Table 14.9: Completed table for the relation $y = 2\sin 3x$

x	0°	30°	60°	90°	120°	150°	180°
$y = 2\sin 3x$	**0**	2	**0**	**-2**	**0**	**2**	0

B3 for table (−0.5 each error)

B1 for correct axes

B2 for points plotted correctly (−0.5 for each error)



B1 for a smooth curve joining the points

A1 for their minimum point

A1 for their min value

A1 for line of symmetry

Figure 14.18: *Graph of $y = 2\sin 3x$ for $0° \leq x \leq 180°$*

b.

i. minimum point $= (90°, -2)$

ii. maximum value $= 2$

iii. line of symmetry is $x = 90°$

WEEK 15

Question One

a.

Table 15.7: Completed table for the relation $y = 2\sin x + 3$

x	0	30	60	90	120	150	180
y	3.00	4.00	4.73	5.00	4.73	4.00	3.00

b. Plotting the above points, we obtain

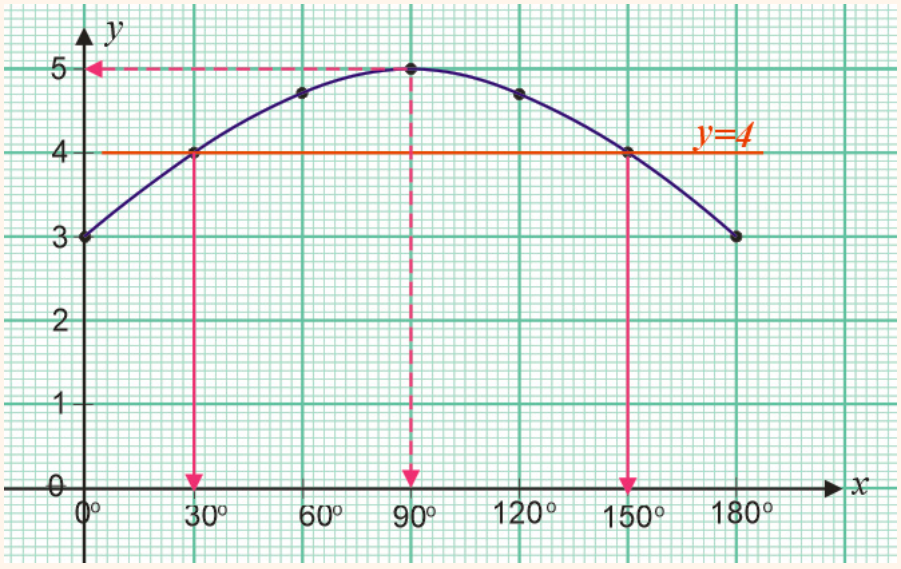


Figure 15.8: *Graph of $y = 2\sin x + 3$ and $y = 4$*

a. **B2** for completing table (−1 each error)

b. **B1** for correct axes

B2 for points plotted correctly (−0.5 for each error)

B1 for a smooth curve joining the points

c.

i. Comparing the two equations $y = 2\sin x + 3$

$$y = 4$$

From the graph, when $y = 4$, $x = 30$, or $x = 150$

ii. maximum value = 5

occurs when $x = 90°$

Question Two

a.

Table 15.8: Completed table of the relation $y = 3\cos x + 2\sin x$

x	0°	30°	60°	90°	120°	150°	180°
y	3.00	3.60	3.20	2.00	0.23	−1.60	−3.00

b.

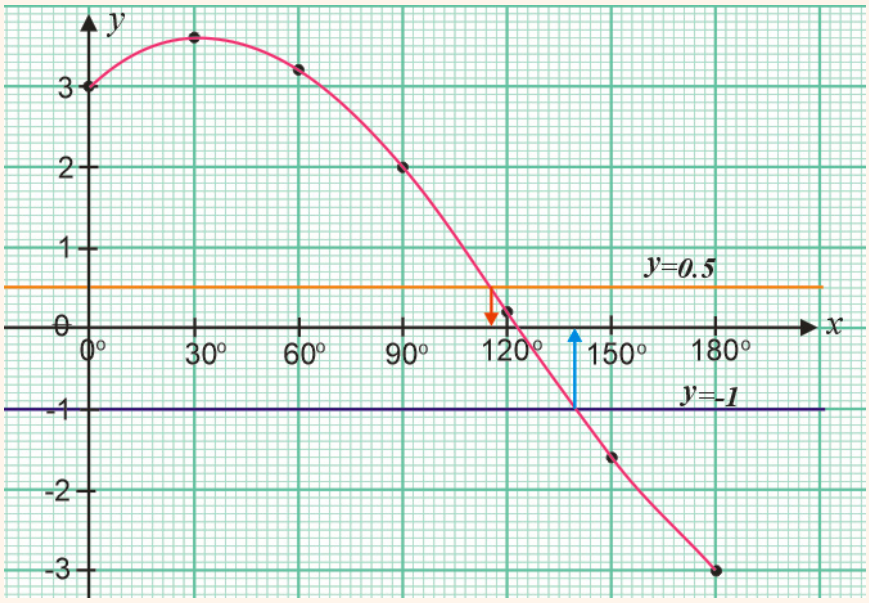


Figure 15.9: *Graph of y = 3cosx + 2sinx and y = 4*

c.

i. $y = 0.5$

$x = 115.5°$

ii. $y = -1$

$x = 139.5°$

c.

i. **B1** for $y = 4$

M1 for drawing line $y = 4$ on the graph

A1 for $x = 30$ and $x = 150$ (−0.5 each error)

ii. **A1** for max value

A1 for $x = 90°$

a.

B2 for completing (−0.5 each error)

b.

B1 for correct axes

B2 for points plotted correctly (−0.5 for each error)

B1 for a smooth curve joining the points

B4 for graph (−0.5 each error)

c.

i. **B1** for y = 0.5

M1 for drawing line $y = 0.5$

M1/A1 for reading value from graph

$x = 115.5° \pm 3°$

ii. **B1** for y = **−1**

M1 for drawing line $y = -1$

M1/A1 for reading value from graph

$x = 139.5° \pm 3°$

SECTION 5: DATA HANDLING & PROBABILITY

STRAND: MAKING SENSE OF AND USING DATA

Sub-Strand 1: Statistical Reasoning And Its Applications In Real Life

Weeks 16&17

Learning Outcome: *Establish simple mathematical relationships between two variables in a given observational or experimental context; illustrate using scatter graphs and use them to solve and/or pose problems.*

Content Standards: *Demonstrate understanding of data handling involving simple mathematical relationships of bivariate data in observational and experimental contexts.*

Week18

Learning Outcome: *Compare different data sets and use mathematical vocabulary and language (concretely, pictorially, symbolically, spoken or written, etc.) to contribute to mathematical and everyday discussions using valid and reliable data to explain and justify the information produced from the data.*

Content Standard: *Demonstrate the ability to compare different data sets and use appropriate vocabulary to contribute to mathematical and everyday discussions and make inferences about information gathered.*

Sub-Strand 2: Probability/Chance

Weeks 19&20

Learning Outcomes: *Explain how a given probability from print and electronic media influences individual decisions; select and analyse real-life data to solve problems involving the probability of two dependent and independent events.*

Content Standard: *Demonstrate an understanding of the role of probability in society and apply probability reasoning to analyse and make useful predictions about everyday life events.*

INTRODUCTION AND SECTION SUMMARY

This section, spanning weeks 16-20, is divided into two sub-strands: statistical reasoning (weeks 16-18) and probability (weeks 19-20). The statistical reasoning component will equip learners to engage with real-world data and media representations by exploring how to reason with data.
Key areas of focus include:

- **Bivariate Data Analysis:** Establishing relationships between two variables, identifying bivariate data sets, differentiating between dependent and independent variables, and plotting/interpreting graphs of bivariate data.
- **Experimental Data:** Collecting data through simple experiments, constructing scatter plots from experimental results, and interpreting the findings.
- **Data Comparison:** Comparing different data sets to determine the most representative one for a given scenario.
- **Media Literacy:** Empowering teachers to guide learners in interpreting data presented in media, making informed inferences, and drawing logical conclusions.

The probability component will examine the application of probability in media and provide support for learners in solving problems involving both dependent and independent probability events.

The weeks and the indicators covered by the section are:

Week 16

Establish simple mathematical relationships between two variables in a given observational or experimental context; illustrate using scatter graphs and use them to solve and/or pose problems.

Focal Area: Establishing the relationship between two variables

- *Identify variables in a given data*
- *Identify whether or not a given table show a bivariate data.*
- *Identify dependent and independent variables.*
- *Plot and interpret graphs of bivariate data.*

Week 17

Collect data from an experimental study in which the interest is based on a treatment and a non-treatment (control) group. Illustrate the data using scatter graphs and find the relationship between the variables, if any.

Focal Area: Data collection on simple experiments

- *Collect data on simple experiments*
- *Draw and interpret scatter plots constructed from experimental data.*

Week 18

1. Compare the quality of different data sets using knowledge and skills acquired from data analysis techniques learned in earlier statistics work.

2. Discuss data information published in local and/or international media platforms (local and international TV stations, newspapers, journals, etc.) by making useful inferences from the data to conclude and/or to explain other phenomena.

Focal Area 1: Comparing different data sets

- *Justify which of the two given data best represents a given situation.*
- *Interpret and make recommendations on a given data.*

Focal Area 2: Interpreting data presented on media platforms

- *Identify and use data from various media platforms*
- *Interpret data information presented in the media*
- *Make useful inferences and conclude from the data presented in the media*

Week 19

Identify examples from print and electronic media, e.g., newspapers, television, the Internet, where probability is used, and explain how the given probability influences individual decisions.

Focal Area: Identification of probability usage in the media

- *Identify examples of the use of probability in real life context including the media*
- *Give explanation and justifications on how these probability events influences individual decisions*

Week 20

Solve everyday life problems involving the probability of two dependent and independent events, including addition and multiplication laws.

Focal Area: Real-life problems involving probability

- *Solve problems involving dependent and independent probability events*
- *Use addition and multiplication laws to solve probability problems.*

SUMMARY OF PEDAGOGICAL EXEMPLARS

This section requires hands-on activities where learners engage in practical activities with quadratic equations and functions. Learners should be given the platform to work in groups to develop their real-life questions and find answers. The following pedagogical approach is suggested for teachers to take learners through.

1. **Reflection:** Learners review previous knowledge on types of data and the collection and organisation of data.
2. **Think-Pair-Share:** Learners pair up to discuss mathematical relationships between two variables and differentiate dependent and independent variables in a bivariate data.
3. **Problem-Based Learning:** In convenient groups, learners collect data of interest within or outside the school community, present them graphically, and predict future occurrences.

4. **Collaborative Exploration:** Learners, in convenient groups, explore the probabilities that are applied in the media outlets and also identify the likely predictions and decisions.
5. **Project-based Learning:** In convenient groups, learners are tasked to collect data of interest within or outside the school community (including using social media platforms or any technological means for collecting data). Learners are assigned project work (individually or in groups) to obtain current data about contemporary issues of interest, then make conclusions, give criticisms, and make useful recommendations.
6. **Experiential Learning:** Learners draw scatter plots for bivariate data. Learners pair to collect data on simple experiments and interpret data presented in graphs (scatter plots), draw conclusions, and give justification for the conclusions. Compare the quality of the given data sets and conclude on the appropriateness of the data
7. Using talk for learning: Learners debate where they support their argument with data and the use of appropriate mathematics terminology on probability.

ASSESSMENT SUMMARY

Table 1 contains the key assessment for each indicator in the section. The highest level of DOK for the indicator is denoted. The mode of assessment to be carried out and strategies that will aid in achieving the indicators are documented.

Table 3: Indicators, DOK levels, assessment modes and strategies

Indicators	DOK Level	Assessment Strategy and/or Mode
Establish simple mathematical relationships between two variables in a given observational or experimental context; illustrate using scatter graphs and use them to solve and/or pose problems.	**Level 3**	**Assessment Mode:** Group project work **Assessment Strategy:** Group work and presentation to find the relationship between two variables.
Collect data from an experimental study in which the interest is based on a treatment and non-treatment (control) groups. Illustrate the data using scatter graphs and find the relationship between the variables, if any.	**Level 3**	**Assessment Mode:** Group project **Assessment Strategy:** Learners collect bivariate data on their own from the school community, represent their data using scatter plots, and make conclusions.
Compare the quality of different data sets using knowledge and skills acquired data analysis techniques you learned in earlier statistics work. Discuss data information published in local and/or international media platforms (local and international TV stations, newspapers, journals, etc.) by making useful inferences from the data to conclude and/or to explain other phenomena.	**Level 3**	**Assessment Mode:** Project Work **Assessment Strategy:** Learners collect bivariate data from documents on electronic media or print media, or textbooks, and present their data using scatter plots and draw conclusions. Compare their data with that of other groups and decide which of the data best represents a given situation.

Identify examples from print and electronic media, e.g., newspapers, television, the Internet, where probability is used, and explain how the given probability influences individual decisions.	**Level 3**	**Assessment Mode:** Individual Research **Assessment Strategy:** Learners to investigate real-life situations from media outlets (internet/radio/print/ interview/etc) to find out the application of probability, and make predictions and provide an explanation on how they influence decisions.
Solve everyday life problems involving the probability of two dependent and independent events, including addition and multiplication laws.	**Level 3**	**Assessment Mode:** Class Exercise **Assessment Strategy:** Provide standard WASSCE-style questions as a class exercise.

WEEK 16

Learning Indicator: *Establish simple mathematical relationships between two variables in a given observational or experimental context; illustrate using scatter graphs and use them to solve and/or pose problems*

FOCAL AREA: ESTABLISHING THE RELATIONSHIP BETWEEN TWO VARIABLES

Variable

In statistics, a variable is a characteristic or quantity that can take different values for different individuals, objects or observations. Variables may describe values like weight in ***kg***, height in metres, favourite food, satisfaction level, age, and gender.

Example 1

Given the table below, identify the variables involved.

Table 16.1: Expenditure versus number of cars sold

Expenditure on advertisement (GHC)	Number of cars sold
25,000	25
30,000	40
35,000	43
40,000	48

Solution

Variable 1: Expenditure on advertisement

Variable 2: Number of cars sold.

Independent Variable

The **independent variable** is the variable used to help explain or predict changes in another variable. It is also referred to as the **Predictor or Explanatory** variable.

Example: **Hours** studied (may be associated with exam score), **Price** of an item (may be associated with the quantity sold), **Temperature** (determines the amount of water consumed).

Usually, the independent variable can be determined when used in a context.

Ask yourself: "What is being changed, used, or controlled to see its effect?" That is the independent variable.

Characteristics of the Independent Variable

1. It is also known as an input, predictor, or explanatory variable.
2. It is the cause or factor being tested.
3. It does not depend on other variables in the analysis.
4. In a graph, it is usually displayed on the x-axis.

Dependent Variable

The **dependent variable** is the variable that changes as a result of changes in the independent variable. It is also referred to as the **Outcome or Response** variable.

Example: **Exam scores** (outcome of hours of study), **Expenditure** (determined by the income), **Amount of water consumed** (determined by the temperature).

Usually, the dependent variable can be determined when used in a context.

Ask yourself: "What changes because of what I did or something that was done?" That's the dependent variable.

Characteristics Of The Dependent Variable

1. It is also known as an output, response, or measured variable.
2. It is **what** the researcher is trying to explain, predict, or observe.
3. It is **NOT** directly controlled. It shows a response to some other change.
4. In a graph, it is usually displayed on the y-axis.

Example 2

State the independent and dependent variables in the following research scenarios:

a. Capacity and sizes of bottles

b. Speed and Distance Travelled

c. Effect of study time and test scores

d. Effect of temperature and sales of ice cream

e. Area of plane shapes and their dimensions.

Solution

a. Capacity and sizes of bottles

Independent Variable = size of the bottles

Dependent Variable = Capacity of the bottles

Reason: The capacity is dependent on the size of the bottles

b. Speed and Distance Travelled

Independent variable = speed

Dependent variable = distance travelled.

Reason: The distance travelled is dependent on the speed of the car.

c. Effect of study time and test scores

Independent variable = Study time

Dependent variable = Test scores

Reason: Your test scores are usually dependent on the amount of time you study.

d. Effect of temperature and sales of ice cream

Independent variable = temperature

Dependent variable = Sales of ice cream

Reason: Sales of ice cream are usually dependent on the temperature of the weather.

e. Area of plane shapes and their dimensions.

Independent variable = dimensions of the plane shape

Dependent variable = Area of the plane shape

Reason: Without the dimensions, you cannot find the area.

Univariate Data

The word "Uni" means "one," and "variate" is another word for "variable." Therefore, "univariate data" refers to data with only one variable or a single characteristic. Examples include the daily temperature in Accra, exam scores for a subject, the weight of babies at birth, and the wages of teachers.

The table below shows an example of univariate data. It represents the outcome of a survey in which a sample of learners at Diaso SHS were asked to give their preferred holiday destination.

Table 16.2: Preferred holiday destination of students

Holiday Destination	Number of students
France	32
South Africa	87
Kenya	43
Egypt	107
Saudi Arabia	126
China	63

The only variable in this data is '**holiday destination'**

You can describe patterns in univariate data using central tendencies (mean, mode, and median) and dispersion (range, interquartile range, standard deviation, variance)

Frequency distribution tables, bar charts, histograms, pie charts, and frequency polygons can be used to present univariate data.

Bivariate Data

"Bi" means "two". So, bivariate data" means data that involves **exactly two** variables. Therefore, bivariate data involves **two variables** that are measured or observed for the same subject or event. The goal is to understand whether and how the two variables are related.

The table shows an example of bivariate data. It gives the income and expenditure of eight cocoa farmers.

Table 16.3: Bivariate data of income versus expenditure

Name	Naa	Nimo	Atsu	Bushra	Yaa	Yakoba	Yaw	Mbo
Income	8,000	11,200	9420	13800	7300	6000	10900	11300
Expenditure	4700	7500	6500	5200	5100	4800	9200	10100

This data contains two variables, income and expenditure.

Example 3

Ask learners to give five examples of bivariate data.

Solution

Examples could include:

a. hours worked and wages earned,

b. years of education and annual income,

c. amount of rainfall and maize yield,

d. alcohol consumption and cholesterol level

e. age of cancer patients and length of survival

Differences between Univariate and Bivariate data

Table 16.4: Distinguishing features of univariate and bivariate data

Univariate data	Bivariate data
Involves one variable	Involves two variables measured for each observation.
Describes the distribution of that variable, such as its typical value and spread.	Examines how the two variables are associated

Does not classify variables as dependent or independent.	The variables may be classified as dependent and independent when the question calls for it, but this is not always necessary.
Example: the heights of learners in a class	***Example***: each learner's height and age.

To **PRESENT** bivariate data, we can use a **scatter plot** (also known as a **scatter diagram, or scatter graph**). It allows us to visualise the relationship between the two variables.

Scatter Plot

A scatter plot shows how two variables are related. Each point on the graph represents a pair of values. Find below an example of a scatter plot. It shows the relationship between daily temperature and the number of people who visit the beach.

Table 16.5: Table showing average daily temperature and the corresponding number of visitors to the beach

Average Daily Temp/^{o}C	24.5	25	25.5	26	26.5	27	27	27.5	28	28.5	29
Number of visitors	60	100	200	100	150	180	250	350	310	330	400

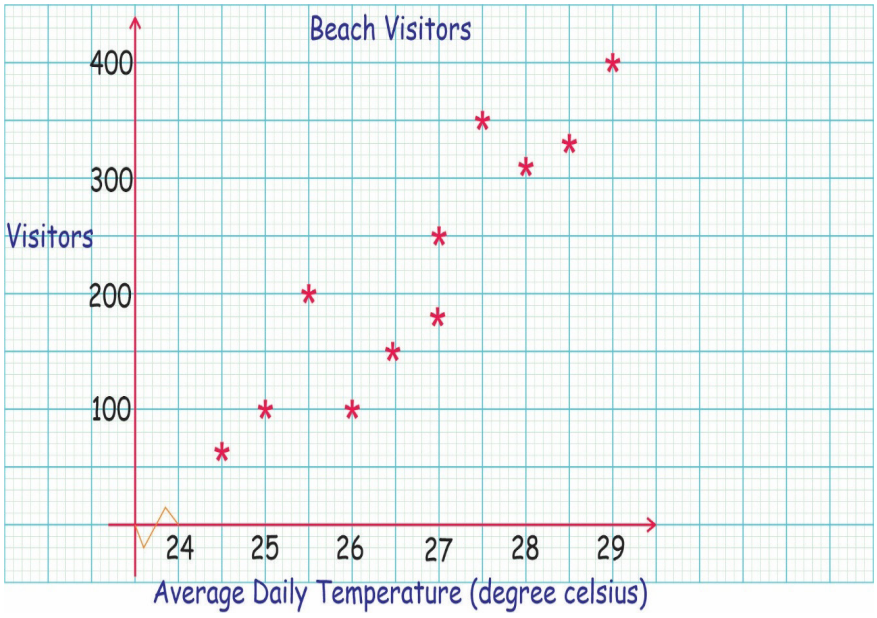


Figure 16.1: *Scatter plot of beach visitors*

Interpreting Graphs of Bivariate Data

When analysing **bivariate data** using graphs of scatter plots, the goal is to understand the **relationship** between the two variables. To do this, we either look at the directions of the relationship, the form of the relationship, or the strength of the relationship. Here, we will concentrate on the direction of the relationship.

Direction of the Relationship

The direction of the relationship could either be

1. **Positive correlation:** As one variable increases, the other also increases
2. (e.g., Hours studied vs. Test score)

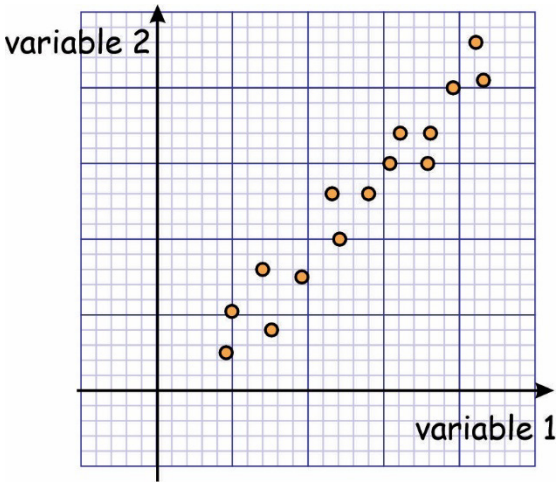


Figure 16.2: *A positive correlation*

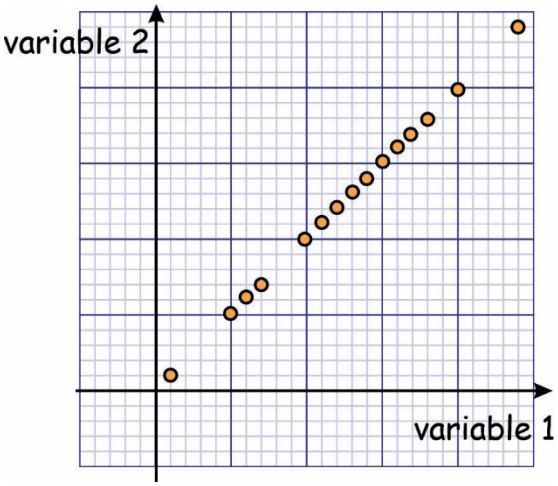


Figure 16.3: *A positive perfect correlation*

3. **Negative correlation:** As one variable increases, the other decreases
4. (e.g., Alcohol consumption vs. Health level)

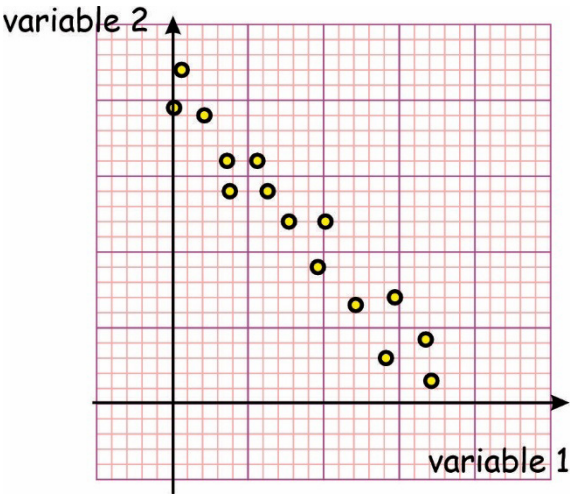


Figure 16.4: *A negative correlation*

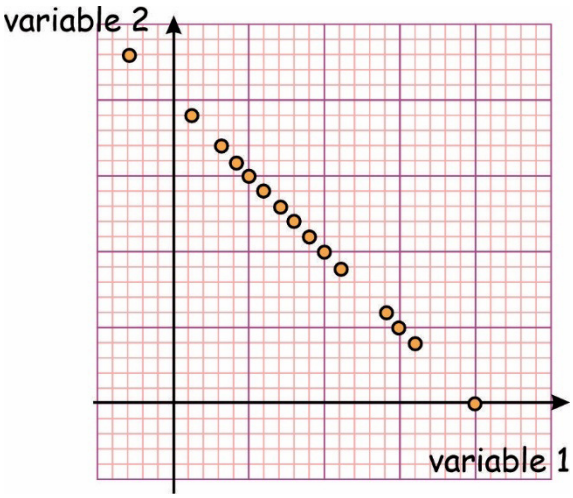


Figure 16.5: *A negative perfect correlation*

5. **No correlation:** No clear pattern or relationship
6. (e.g., Shoe size vs. Intelligence)

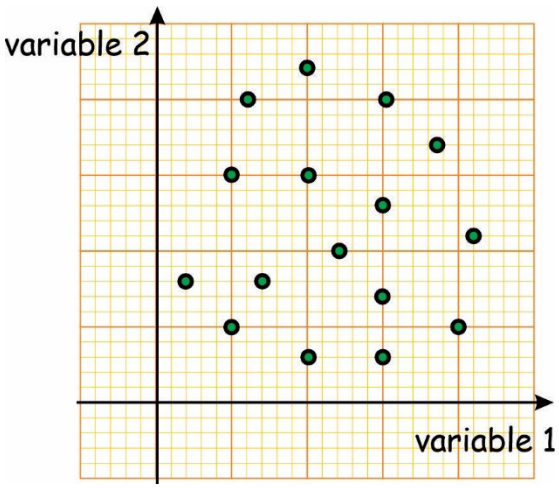


Figure 16.6: *A no correlation scatter plot*

Note that correlation does not imply causation.

Example 4

The tables show the outcomes of the data collected.

(a) ***Table 16.6: Number of sides versus size of interior angle***

Number of sides	5	10	12	13	14	15
Size of the interior angle	108	144	150	152.3	154.3	156

(b) ***Table 16.7:*** Mass of a sample of bags

Mass (kg)	63	64	65	66	67	68
Number of bags	5	8	3	20	12	2

(c) ***Table 16.8:*** Number of workers and the time used to complete a task.

Number of workers	10	20	30	40	50	60
Completion time (days)	58	34	25	19	16	12

1. Which of the tables shows bivariate data?
2. State the independent and dependent variables. Justify your response.

Solution

Table 16.6 shows bivariate data.

- Independent variable = Number of sides
- Dependent variable = Size of interior angles
- The number of sides of a polygon determines the size of the interior angle.

- There is a positive correlation between the number of sides and the size of the interior angle of a regular polygon. The higher the number of sides, the larger the interior angle.

Table 16.7 shows univariate data.

- There is no relationship between the mass of bags and the number of bags.

Table 16.8 shows bivariate data.

- Independent variable = number of workers
- Dependent variable = completion time
- The completion time is dependent on the number of workers.
- There is a negative correlation between the number of workers and the completion time. The higher the number of workers, the lower the completion time.

How to Construct A Scatter Plot Manually

Follow the steps below

Step 1: Identify the variables. Since we are dealing with bivariate data, it will involve two variables, an independent variable and a dependent variable.

Step 2: Label the axes and scale them, putting the independent variable on the x axis and the dependent variable on the y axis.

Step 3: Convert each data point into (x, y) coordinates and plot them on the graph. If two or more points fall on the same point, place them side-by-side.

Example 5

The data below shows the temperature of the day and the number of people wearing jackets.

Table 16.9: Temperature versus number of people wearing jacket.

Temperature (x)	35	6	17	15	21	20	10	7
Number wearing jackets (y)	11	45	22	30	20	25	32	42

a. Draw a scatter plot for the data.

b. Interpret your graph

Solution

1. Steps involved in drawing a scatter diagram

a. Draw two perpendicular lines to form the x-y plane on a sheet of graph paper.

b. Choose an appropriate scale for the axes (x-axis representing temperature, the independent variable, and y-axis representing number wearing jackets, the dependent variable)

c. The ordered pairs (35, 11), (6, 45), (17, 22), (15, 30), (21, 20), (20, 25), (10, 32) and (7, 42) are plotted in the x-y plane.

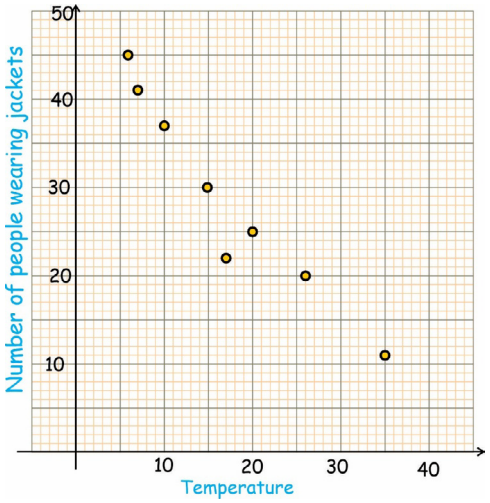


Figure 16.7: *Scatter plot of temperature and number of people wearing jackets*

2. There is a clear negative correlation shown in the graph, with fewer people wearing jackets at higher temperatures and more wearing jackets at lower temperatures.

How to Construct A Scatter Diagram Using Excel

Step 1: Open a new document in the Excel spreadsheet. A section of the interface should look like the picture below

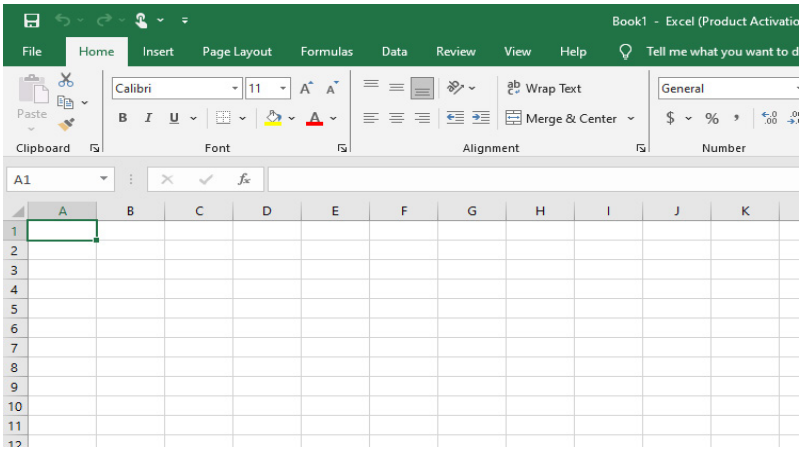


Figure 16.8: *Excel spreadsheet page*

Step 2: Input your data in the first and second columns of the spreadsheet. As an example, let's input the data in ***Table 16.9***

Table 16.10: Temperature versus number of people wearing jacket.

Temperature (x)	35	6	17	15	21	20	10	7
Number wearing jackets (y)	11	45	22	30	20	25	32	42

Your spreadsheet should be similar to the one below

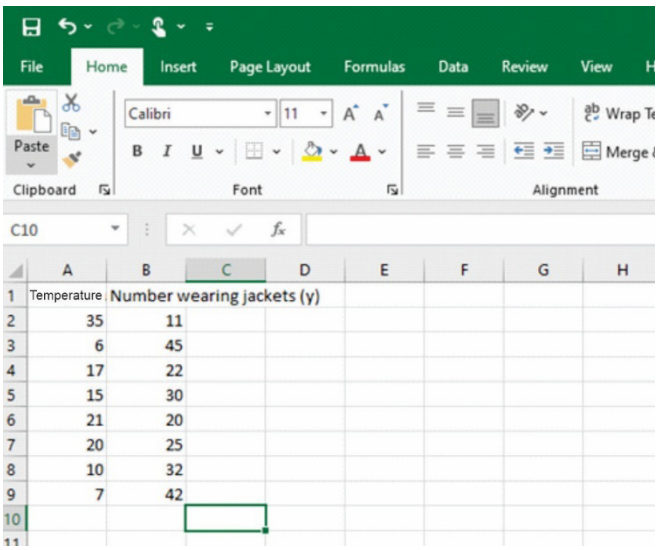


Figure 16.10: *Excel spreadsheet page showing data of temperature and number of people wearing jackets.*

Step 3: Select the data, go to **Insert**, and choose **Scatter** from the drop-down menu.

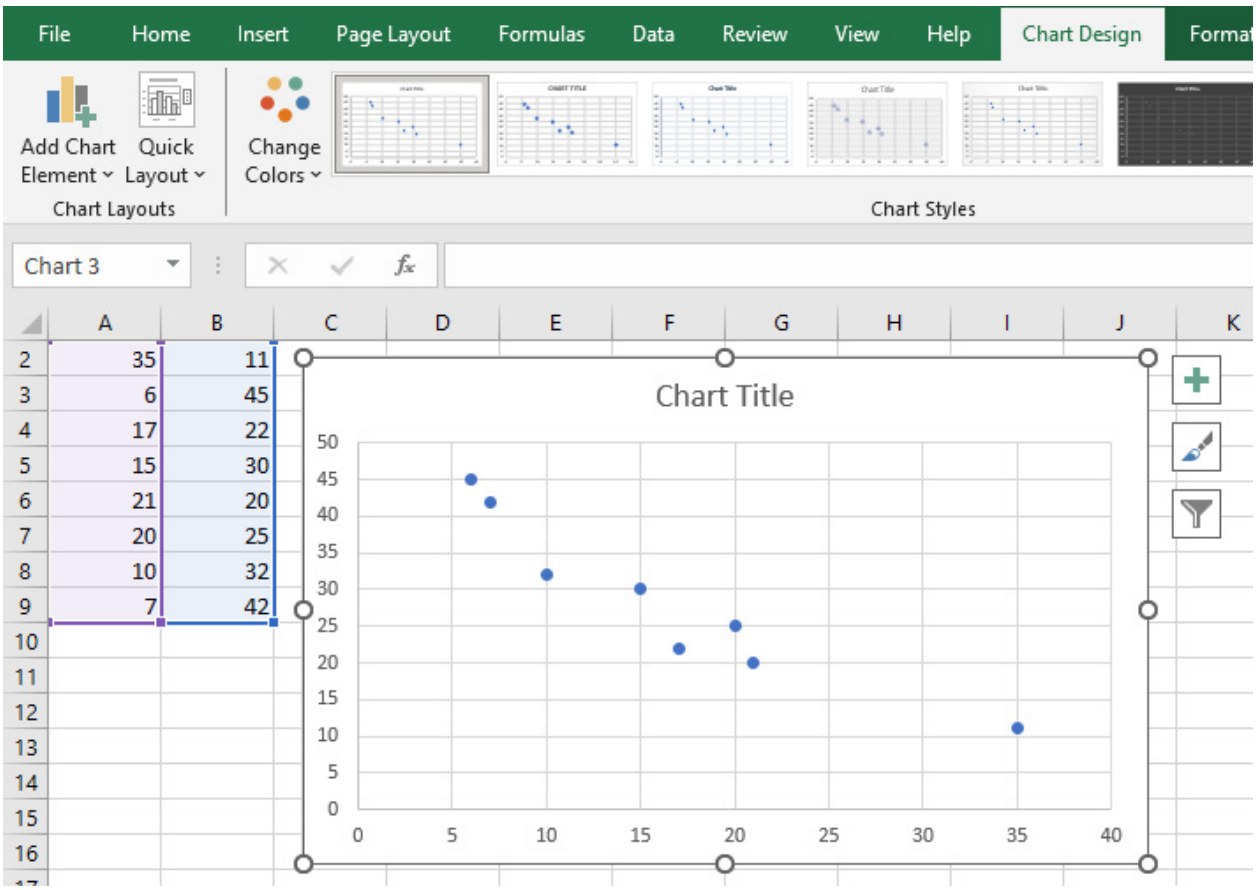


Figure 16.10: *Excel spreadsheet page showing bivariate data and a scatter plot of temperature and number of people wearing jackets.*

Step 4: Insert the title and the description of the x and y axes. To do this, click on the plus sign beside the scatter graph. This will open a dropdown menu titled Chart Elements. Choose **"Axis Titles"** from the dropdown menu.

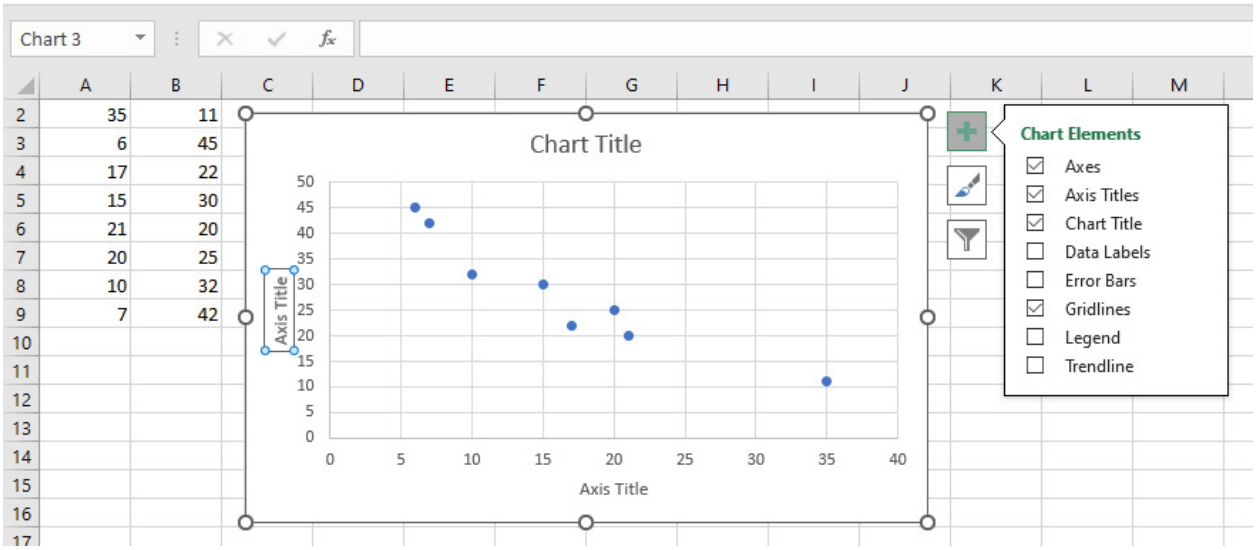


Figure 16.11: *Excel spreadsheet page showing bivariate data and a scatter plot*

Step 5: Your final plot should be similar to the chart below.

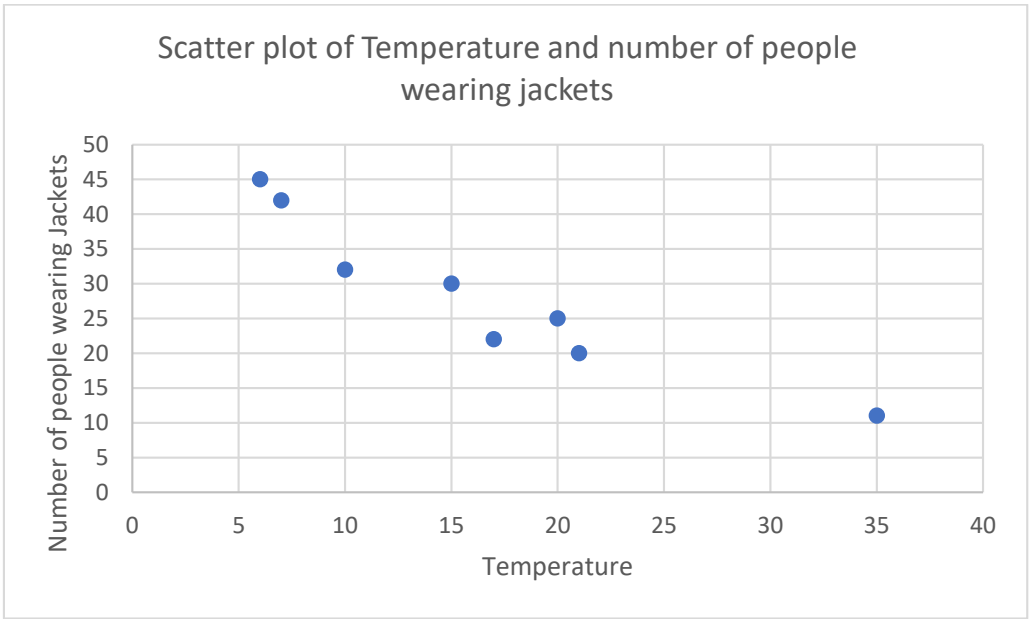


Figure 16.12: *Scatter plot of temperature and number of people wearing jackets.*

Learning Tasks

Guide learners in applying their knowledge of plotting points by creating scatter plots. Help them distinguish between univariate and bivariate data, as well as independent and dependent variables, using relevant real-life examples. After creating the scatter plots, assist them in identifying the relationship between the variables. Ensure those who face difficulties receive the necessary support.

PEDAGOGICAL EXEMPLARS

The aim of this week's lessons is for all learners to establish the relationship between two variables. The following pedagogical approach is suggested for facilitators to guide learners through.

1. **Reflection**: Conduct a review of learners' prior knowledge concerning data types, as well as methods for data collection and organisation.
2. **Collaborative Learning (Think-Pair-Share):** Engage learners in a think-pair-share activity to help them distinguish between dependent and independent variables within a bivariate data set.
3. **Project-Based Learning:** Assign group tasks where learners collect data relevant to their interests, either from the school environment or beyond, making use of social media or other technological platforms for data gathering.
4. **Experiential Learning:** Provide direct guidance to learners as they apply their knowledge to draw scatter plots for their bivariate data.

KEY ASSESSMENT LEVEL 3: GROUP WORK & PRESENTATIONS

Project: Finding the relationship between two variables using a scatter plot.

Project Goal: To create and interpret scatter plots and to analyse the relationship between two quantitative variables, identify different types of correlations, and draw conclusions from data.

Question

Select a real-world topic involving two variables, gather or find data for these related variables, create a scatter plot, and then analyse the correlation (or absence of one) between them. Present your findings and discuss the implications of your observations.

The materials you will need include:

- Graph paper or graphing software (e.g., GeoGebra, Microsoft Excel, Google Sheets)
- Rulers
- Calculators (optional)
- Access to the internet for data collection (optional)
- Presentation tools (e.g., poster board, Google Slides, PowerPoint)

Project Steps

Phase 1: Brainstorming and Data Selection (Day 1)

a. **Introduction to Scatter Plots:** In your groups, review what scatter plots are, their purpose, and the concept of correlation (positive, negative, no correlation).

b. **Brainstorming Variables:** As a small group, brainstorm pairs of quantitative variables that *might* have a relationship and are easily assessable. For example, you may choose one of the examples listed below.

- Hours studied and test scores in a particular subject.
- Daily temperature and water sales.
- Height of a student and his/her Shoe size
- Population of regional capitals in Ghana and the number of JHS.
- Distance from school and Travel time
- Mid-term and end-of-term scores.
- Weight of a student and his/her height.

c. **Topic Selection:** Choose ONE pair of variables you are interested in investigating. Clearly define your independent (x-axis) and dependent (y-axis) variables.

d. **Data Collection Strategy**

- Collect at least 20 data points for your chosen variables.
- **Option A: Self-Collected Data:** Collect your own data (e.g., surveying classmates, measuring physical attributes, etc).
- **Option B: Found Data:** You can research data online from reliable sources (e.g., government statistics websites, sports statistics, scientific studies, reputable news organisations). You must cite your sources.
- **Option C: Teacher-Provided Data:** You may ask a teacher or a parent to provide you a data.

Phase 2: Creating the Scatter Plot (Day 2)

a. **Organising Data:** Organise your collected data in a table with two columns (one for each variable).

b. **Choosing a Method:** Decide whether to create the scatter plot by hand on graph paper or using technology.

- **By Hand:** Provide the scale used for each axis, label the axes, and give the graph a clear title.
- **Using Technology:** Use an Excel Spreadsheet or any appropriate software.

Phase 3: Analysis and Interpretation (Day 3–4)

a. **Observing the Scatter Plot:** Analyse your generated scatter plot.

b. **Addressing Key Questions:** Use your scatter plot to answer the following questions

- What is your independent variable (x-axis)? What are its units?
- What is your dependent variable (y-axis)? What are its units?

- Describe the overall pattern you observe in the scatter plot.
- Does there appear to be a correlation between your two variables? If so, what type (positive, negative, or no correlation)?
- How strong does the correlation appear to be (strong, moderate, weak)? Explain your reasoning.
- What conclusions can you draw about the relationship between your two variables based on your scatter plot?
- Does correlation imply causation? Explain this concept in the context of your data.
- What are some limitations of your data collection or analysis?
- If you were to continue this study, what would you do differently, or what other variables might you consider?

Phase 4: Presentation (Day 5)

a. Prepare Presentation: Prepare a presentation (e.g., poster, slide show, written report) that includes:

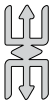
- Project Title
- Names of group members/students
- A brief introduction of the chosen variables and the research question.
- A displayed data table.
- Your scatter plot (large and clear).
- Answers to all analysis questions from Phase 3.
- A concluding summary of your findings.
- State the sources you used for the data and sources of your data, research materials (if any).
- Provide a brief statement acknowledging any individual or institution other than your group members and your teacher who contributed to the successful completion of your project. (Optional)
- Example: *We'd like to express our sincere gratitude to [Name of Individual or institution], whose insights and support were invaluable to the successful completion of this project.*

b. Presentation: You will present your project to your class and answer any questions your classmates ask.

PLC Session 16

45 minutes | **Planning & Reviewing the Learning Plan**

1. Begin the session by recapping lessons from the previous year/session/week, focusing on how learning was applied.
2. Use your subject-specific app to explore the content before planning. Ask the app to
   a. explain the week's concepts in a practical way that will make it easier for learners to understand
   b. identify two or three common misconceptions learners may have on the week's content and how to address them using contexts familiar to Ghanaian learners. (NTS 3a, 3g and 3m).
3. Use your subject-specific app to plan your lesson. Ask the app to
   a. suggest the most effective method for teaching the week's concepts to learners with diverse backgrounds.
   b. suggest learning activities, differentiation and essential questions based on the method of teaching (NTS 3a-3q)
   c. review your prepared learning plan and address any challenging issues you may have identified in your subject area (NTS 3a-3c).



Note

- Take note of the recap topics linked to Years 1 and 2 while preparing for your lesson.
- Refer to the Values Learning Community Handbook and link the values for the week in your lesson to ensure that learners are applying what they learn.

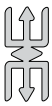
4. Plan the week's key assessment with your app
   a. The key assessment for the week is ***group work & presentation***. Chat with your app to plan this assessment, indicating what you and your learners will do before, during and after the presentation (NTS 3k-3q).

10 minutes | **Content Exploration**

5. As a subject group, select one problem or idea/term in the week's content that is challenging (not a routine procedural question, but one that requires conceptual understanding) and then
   a. use the app, ask it to solve the problem step by step and explain the reasoning behind each step (you may be assisted by a teacher who is strong in the content area)
   b. all teachers in the group work through a similar problem individually for 3–4 minutes

c. the group then works through the problem together step by step

d. highlight the most common errors made at each step and discuss (NTS 1c, 1e, 2c, 2e, 3e-3j).



Note

- This part is not just about 'How do I teach this?' but how do I teach it for learners to understand?
- If the group still shares the same content gap, use the app: ask it to solve the problem step by step and explain the reasoning behind each step.
- Problems/ideas discussed should be sourced from the Teacher Manual content, not invented separately. This keeps the content exploration directly connected to what teachers will deliver.

30 minutes **Enactment**

6. Enact a teaching activity based on your learning plan (NTS la-1b, 1e-lg, 2c, 2e, 2f, 3c-3j and 3o).

a. Colleagues observe with specific attention to clarity of concept explanation, use of familiar Ghanaian examples and how misconceptions are handled if they arise.

b. After the enactment, ask for structured feedback from the group

i. was the concept explained in a way that a learner with no prior knowledge could follow?

ii. were the examples grounded in contexts familiar to Ghanaian learners?

iii. did any misconceptions surface and if so, how were they addressed?

5 minutes **Reflection**

7. Reflect and share your views on the session. One thing that worked well, what did not work well and what you will do differently. (NTS 1a, 1b).

8. Identify a critical friend to observe and provide feedback on your lesson in relation to the PLC Session conducted (NTS 1f, 1g).

9. Remember to

a. transfer your chats into your learning planner template.

b. read your Teacher Manual and related Learner Material in preparation for the next session. Take note of the recap weeks, which are linked to Years 1 and 2 (NTS 3a).

c. integrate National Values, GESI, SEL, and 21st century skills into your learning activities, assessment tasks, and rubrics/mark schemes.

WEEK 17

Learning Indicator: *Collect data from an experimental study in which the interest is based on a treatment and a non-treatment (also known as a control) group. Illustrate the data using scatter graphs and find the relationship between the variables, if any*

FOCAL AREA: DATA COLLECTION ON SIMPLE EXPERIMENTS

What is an Experimental Study?

An experimental study is a method of collecting data by intentionally changing one variable (called the independent variable) to observe how it influences another variable (called the dependent variable). In an experimental study, the researcher controls or manipulates one variable and measures the outcome of another.

Examples

Table 17.1: Examples of experimental study.

Experiment Topic	Independent Variable	Dependent Variable	Variable that a researcher may choose to control
Dropping a ball to find out the number of times it bounces.	Drop Height	Number of bounces	Ball type or surface
Finding out how different amounts of sunlight affect plant growth	Hours of Sunlight	Plant Height	Water, soil, and plant type
How study time affects test scores	Study Time	Test Score	Test type, room conditions
How temperature of water affect dissolving time?	Water Temperature	Dissolving Time	Amount of sugar, volume of water
How the mass applied to a rubber band affects the stretch.	Mass Applied	Length Stretched	Rubber band type, air temperature

Structure of an Experimental Study

The structure of an experimental study includes.

- **Independent Variable:** The variable you change.
- **Dependent Variable:** The variable you measure.
- **Controlled Condition(s):** Factor(s) kept constant for fairness.
- **Repeated Trials:** Performing an activity multiple times for accuracy.

How to Conduct an Experimental Study

Step 1: Identify the Research Question

Ask a clear question about the relationship between two variables.

Step 2: Identify the Variables

Step 3: Plan the Experiment

Decide on the materials and tools you need, the number of trials you will undertake, and the number of times you will repeat each trial for accuracy (repeat each trial at least 3 times and take the average for accuracy). Also, decide what must be kept constant to make the experiment fair.

Step 4: Collect the Data

Experiment as planned and record the data in a table.

Step 5: Present the data using a scatter plot.

Step 6: Interpret the pattern you observe.

Let us use the example below to enhance our understanding of how to conduct a simple experimental study.

Example 1

Research Question

Investigating the Relationship Between Drop Height and Bounce Height of a Ball.

Sample Solution

Let's first define the variables in the research question.

Independent variable = Drop height (Determined by the height from which to drop the ball)

Dependent variable = Bounce height (No control over the highest point the bounce will reach)

So, in this research, we will control the height from which we will drop the ball, the type of floor we want to drop the ball on, the type of ball we want to drop, and maybe the air temperature.

When a ball is dropped on a hard surface, all things being equal, it will bounce more than once. We will be only interested in the first bounce, as this will be the highest bounce.

Let's use the steps to answer the question

Step 1: Identify the Research Question

How does the height from which a ball is dropped affect how high it bounces?

Step 2: Identify the Variables

Independent Variable = drop height

Dependent Variable = bounce height

Step 3: Plan the Experiment

Materials

A rubber ball**,** a metre rule**,** a flat hard surface, and a notebook for recording data

To simplify this example, we will undertake 7 trials. We will drop the ball in intervals of 30cm and repeat each trial three times, and take the average bounce height.

Step 4: Collect the Data

Procedure

1. Drop the ball from different heights (30cm, 60cm, 90cm, 120cm, 150cm, 180cm, 210cm).
2. Measure the bounce height each time using a ruler.
3. Repeat each measurement 3 times and record the average.
4. Record all results in a table.

Table 17.2: Drop height versus average bounce height data.

Drop Height(cm)	Bounce Height (cm)			Average bounce height.
	First trial	Second trial	Third trial	
30	19	17	18	18
60	39	42	39	40
90	58	59	60	59
120	72	72	72	72
150	83	86	83	84
180	103	103	109	105
210	146	144	142	144

Step 5: Present the data using a scatter plot. Don't forget to:

a. Label the **x-axis** as Drop Height (independent variable).

b. Label the **y-axis** as Bounce Height (dependent variable).

c. Plot the data points from your table.

d. Look for a **trend** or pattern.

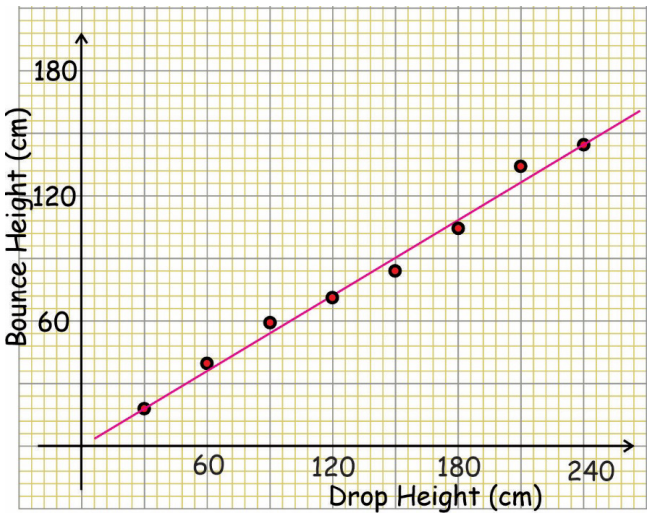


Figure 17.1: *Scatter plot of drop height and average bounce height*

Step 6: Interpreting the Scatter Plot

From the graph, we can observe:

a. As the drop height increases, the bounce height also increases.

b. This shows a positive correlation.

c. A line of best fit can help identify the relationship and predict unknown values. A straight line of best fit may be appropriate when the scatter plot shows an approximately linear relationship. The line should have roughly equal numbers of points on either side of it, and does not need to go through any of the points. Readings can then be taken from this line to make estimates for other values.

Using the Graph to Solve Problems

Problem: If a ball is dropped from 70 cm, what is the expected bounce height?

Solution



Figure 17.2: *Making inferences based on the scatter plot*

From the graph, we can read up from 70cm drop height until we reach the line of best fit and then read across to the bounce height. From this we can see that the expected bounce height is 42cm

Let us now turn our attention to an observational study

Observational study

An observational study is a method of collecting data by **watching and recording** what happens in real-life situations without interfering with or manipulating any variables.

In mathematics, particularly in data analysis, we employ observational studies to investigate relationships between two variables in a natural setting. In an observational study, the researcher does **not** control or manipulate any of the variables; they only observe and record.

Examples of Observational Studies

Table 17.3: Examples of observational study

Observation Context	Variable 1	Variable 2
Observing people at a bus stop	Time of day	Number of people waiting
Monitoring weather and clothing choices	Temperature (°C)	Number of people in jackets
Watching students in the library	Time spent reading (mins)	Number of books borrowed
Observing a football field	Minutes played	Number of goals scored

How to Conduct an Observational Study

Steps

1. Choose two variables you want to study.
2. Decide when and where you will observe (e.g., during break time in the school library).
3. Use a table to collect data at regular intervals.
4. Draw a scatter plot to visualise the relationship.
5. Interpret the pattern you observe.

Example 1

A Math and a Physics test are administered to 10 learners in SHS 1. Each learner takes **both tests during the same week**. The ordered pairs below represent each learner's scores in the two tests. The data is plotted as a scatter plot to explore the relationship between **Math and Physics test scores**.

Learner Scores

Learner	Math Score	Physics Score
A	20	30
B	30	40
C	30	35
D	40	50
E	46	60
F	50	60
G	56	70
H	60	76
I	60	70
J	70	86

Task

- Plot the Math scores on the **x-axis** and Physics scores on the **y-axis**.
- Draw and label the scatter plot.
- Identify the type of correlation.
- Interpret the trend.
- Draw a conclusion.

Solution

Distribution Table for the Graph

Math Scores (x)	Physics Scores (y)
20	30
30	40
30	35
40	50
46	60
50	60
56	70
60	76
60	70
70	86

Graph of the relation

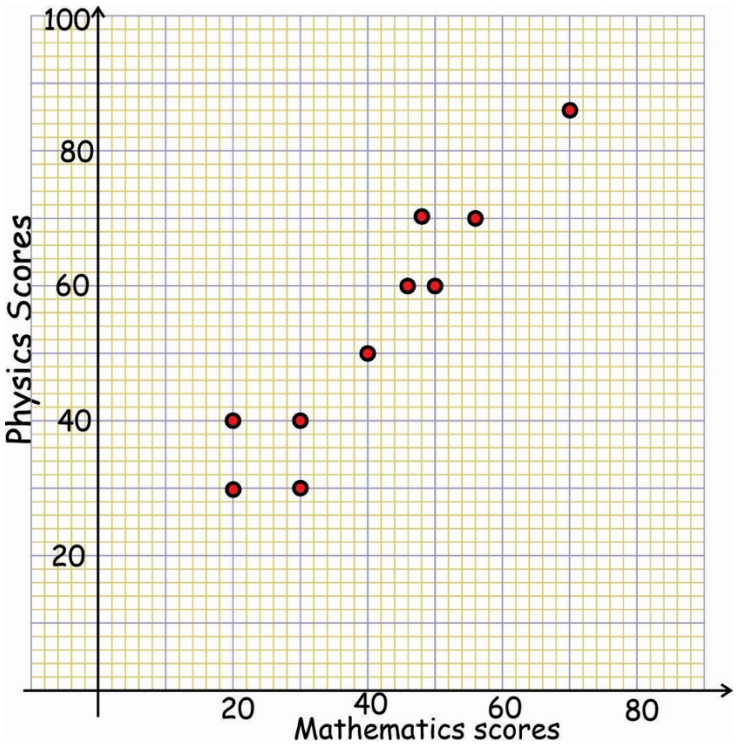


Figure 17.3: *Scatter plot*

Observation

- As Math scores increase, Physics scores also generally increase.
- The points show a pattern trending **upwards**, even if not perfectly aligned.

Interpretation

- Learners who performed better in Math also tended to perform better in Physics.
- This suggests a **positive correlation** between performance in Math and Physics.

Conclusion

There is a **moderate to strong positive correlation** between Math and Physics scores. Learners with higher Math scores generally had higher Physics scores. This supports the idea that strengths in one subject may be associated with strengths in another related subject.

Learning Tasks

Review learners' prior knowledge of data collection methods. Support them in applying these methods to gather data through simple experimental activities. Guide learners step-by-step through the process of conducting an experimental study. Assist them in using scatter plots to represent their collected data and identify simple mathematical relationships between the variables involved. Help learners interpret patterns and correlations in their scatter plots. Encourage them to use the graphs to solve real-life problems and to pose meaningful questions based on their findings.

PEDAGOGICAL EXEMPLARS

The aim of this week's lessons is for all learners to establish the relationship between two variables in an observational or experimental study. The following pedagogical approach is suggested for facilitators to guide learners through.

1. **Reflection:** Begin with a review of learners' prior knowledge on bivariate data and various data collection methods. This sets the foundation for a deeper understanding.
2. **Collaborative Learning (Think-Pair-Share):** Engage learners in a think-pair-share activity to help them differentiate between observational and experimental studies within the context of bivariate data.
3. **Project-Based Learning:** Organise learners into groups to carry out a small project. Each group should collect either observational or experimental data from their surroundings and present their findings using a scatter plot.
4. **Experiential Learning:** Guide learners to apply their understanding of correlation by interpreting their scatter plots and identifying the type of relationship between the two variables in their study.

KEY ASSESSMENT LEVEL 3: GROUP WORK

1. The graph below shows bivariate data (Height versus Age) of a sample of SHS students.

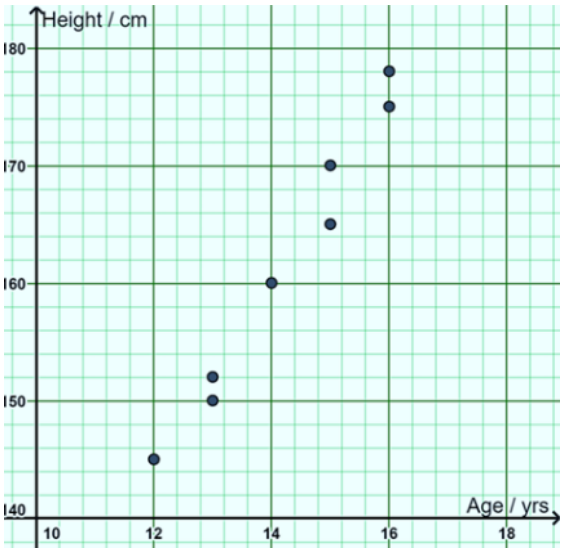


Figure 17.4: *Scatter plot of age and height of SHS students.*

a. Propose an appropriate title for the graph **(*1 mark*)**

b. Analyse the trend on the graph **(*3 marks*)**

c. In what ways can this inform the Parent-Teacher Association of your school in supporting teaching and learning? Give 3 examples of how the information could be used. **(*3 marks*)**

Total *7 marks*

2. ***Table 17.6*** shows the regional percentage distribution of children, 5-17 years, who worked for 15 hours and above per week and school attendance in Ghana.

Table 17.6: Children who worked for at least 15 hours and school attendance.

Region	Percentage of children who work for 15 hours or more	Percentage of learners attending school
Western	61	13
Central	74	12
Greater Accra	35	24
Volta	41	20
Eastern	71	10
Ashanti	67	10
Brong Ahafo	59	18
Northern	33	22
Upper East	39	21
Upper West	28	25

a. Show this information on a scatter plot. **(4 marks)**

b. Describe the trends in the plot. **(*2 marks*)**

c. Propose any 2 suggestions to stakeholders based on the trends you identified. **(*2 marks*)**

Total *8 marks*

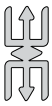
PLC Session 17

45 minutes | **Planning & Reviewing the Learning Plan**

1. Begin the session by recapping lessons from the previous year/session/week, focusing on how learning was applied.
2. Use your subject specific app to explore the content before planning. Ask the app to
   - **a.** explain the week's concepts in a practical way that will make it easier for learners to understand
   - **b.** identify two or three common misconceptions learners may have on the week's content and how to address them using contexts familiar to Ghanaian learners. (NTS 3a, 3g and 3m).
3. Use your subject specific app to plan your lesson. Ask the app to
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b. suggest learning activities, differentiation and essential questions based on the method of teaching (NTS 3a-3q)

c. review your prepared learning plan and address any challenging issues you may have identified in your subject area (NTS 3a-3c).



Note

- Take note of the recap topics linked to Years 1 and 2 while preparing for your lesson.
- Refer to the Values Learning Community Handbook and link the values for the week in your lesson to ensure that learners are applying what they learn.

4. Plan the week's key assessment with your app

a. The key assessment for the week is ***group work***. Chat with your app to plan this assessment, indicating what you and your learners will do before, during and after the group work (NTS 3k-3q).

10 minutes **Content Exploration**

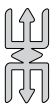
5. As a subject group, select one problem or idea/term in the week's content that is challenging (not a routine procedural question, but one that requires conceptual understanding) and then

a. use the app, ask it to solve the problem step by step and explain the reasoning behind each step (you may be assisted by a teacher who is strong in the content area)

b. all teachers in the group work through a similar problem individually for 3–4 minutes

c. the group then works through the problem together step by step

d. highlight the most common errors made at each step and discuss (NTS 1c, 1e, 2c, 2e, 3e-3j).



Note

- This part is not just about 'How do I teach this?' but how do I teach it for learners to understand?
- If the group still shares the same content gap, use the app: ask it to solve the problem step by step and explain the reasoning behind each step.
- Problems/ideas discussed should be sourced from the Teacher Manual content, not invented separately. This keeps the content exploration directly connected to what teachers will deliver.

30 minutes Enactment

6. Enact a teaching activity based on your learning plan (NTS la-1b, 1e-lg, 2c, 2e, 2f, 3c-3j and 3o).

- **a.** Colleagues observe with specific attention to clarity of concept explanation, use of familiar Ghanaian examples and how misconceptions are handled if they arise.
- **b.** After the enactment, ask for structured feedback from the group
  - **i.** was the concept explained in a way that a learner with no prior knowledge could follow?
  - **ii.** were the examples grounded in contexts familiar to Ghanaian learners?
  - **iii.** did any misconceptions surface and if so, how were they addressed?

5 minutes Reflection

7. Reflect and share your views on the session. One thing that worked well, what did not work well and what you will do differently. (NTS 1a, 1b).

8. Identify a critical friend to observe and provide feedback on your lesson in relation to the PLC Session conducted (NTS 1f, 1g).

9. Remember to

- **a.** transfer your chats into your learning planner template.
- **b.** read your Teacher Manual and related Learner Material in preparation for the next session. Take note of the recap weeks, which are linked to Years 1 and 2 (NTS 3a).
- **c.** integrate National Values, GESI, SEL, and 21st century skills into your learning activities, assessment tasks, and rubrics/mark schemes.

WEEK 18

Learning Indicators

1. *Compare the quality of different data sets using knowledge and skills acquired from data analysis techniques you learned in earlier statistics work*
2. *Discuss data information published in local and/or international media platforms (local and international TV stations, newspapers, journals, etc.) by making useful inferences from the data to draw conclusions and/or to explain other phenomena*

FOCAL AREA 1: COMPARING DIFFERENT DATA SETS

Review of Bivariate Data and Scatter Graphs

Bivariate Data

Bivariate data consists of pairs of values that show how two variables are related. The relationship between the variables can be positive, negative, or non-existent.

Scatter Graphs

Scatter graphs allow us to visualise the relationship between two variables. Each point represents a pair of values.

- If the points show an upward trend → positive correlation
- If the points show a downward trend → negative correlation
- If the points are scattered with no pattern → no correlation

Describing the relationship between two variables

The scatter plot helps us to visualise the relationship between two variables. An alternative way to describe the relationship is to use a linear function (**a straight line**). Note that this is used to approximate the relationship between the variables being compared, as the data points do not form a straight line in most cases. Hence, the approximated line is called a **line of best fit**. It is a line that represents the overall linear trend, with points reasonably distributed around it. Because of this, the line of best fit is usually used to predict data values within the given data. This is known as **interpolation**. When used to predict data values outside a given data, it is called **extrapolation**. Pairs of data values (points) relatively far from the line of best fit are called outliers. In this lesson, we will obtain the line of best fit through **observation**.

Comparing Two Bivariate Data Sets

Example 1

Two sales representatives, Naa and Yaw, are being considered for a promotion. Their weekly performance over 5 weeks is shown below.

Table 18.1: Naa's sales performance

Hours worked	30	35	40	45	50
Sales made	12	13	16	19	20

Table 18.2: Yaw's sales performance

Hours worked	30	35	40	45	50
Sales made	15	16	17	17	18

Analysis and Interpretation

1. **Step 1:** Plot the data for both Naa and Yaw using scatter graphs.

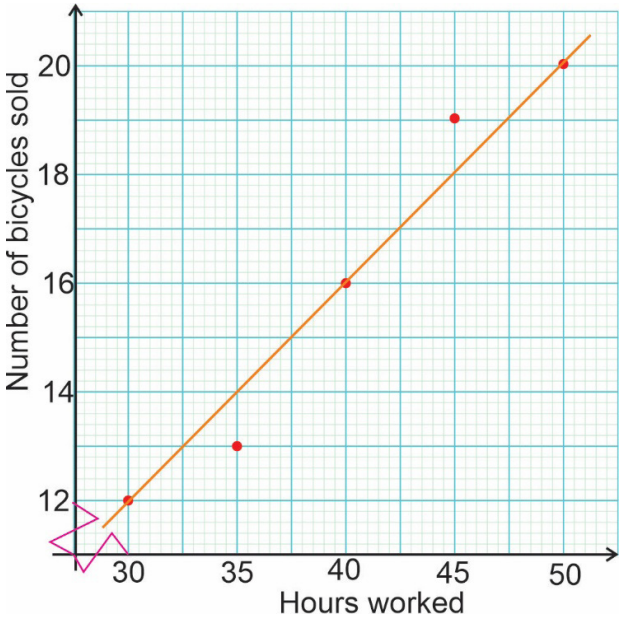


Figure 18.1: *Scatter plot of Naa's sales performance*

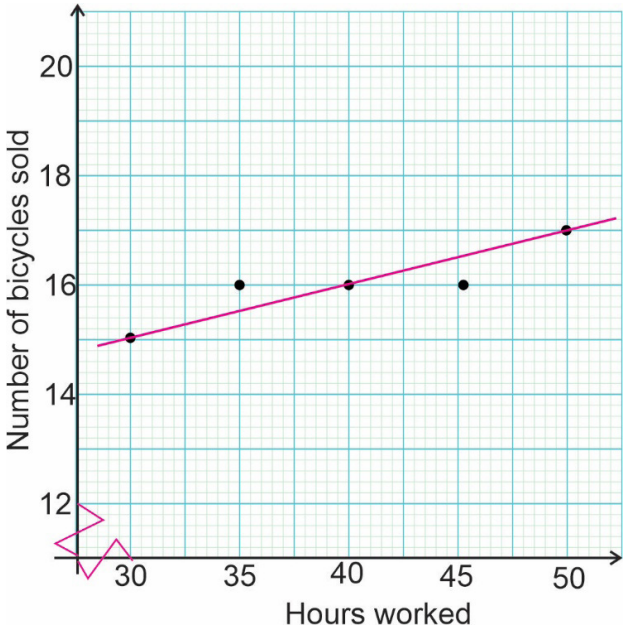


Figure 18.2: *Scatter plot of Yaw's sales performance*

2. **Step 2:** Identify the type of relationship

   Naa: Shows a strong positive correlation – as hours increase, so do sales of bicycles.

   Yaw: Shows a weak or no correlation – sales do not increase that much despite working more hours.

3. **Step 3:** Compare the trends and consistency

   Naa has a steady and consistent increase in sales with more work.

   Yaw starts high, but the increase in sales is minimal, showing reduced efficiency with more hours.

4. **Step 4:** Conclusion

   Naa's data better represents a productive relationship between work and results. The linear growth in sales with effort justifies selecting Naa for a promotion.

Example 2

The tables below present data on tree growth observed after applying two distinct fertiliser brands: **Surge** and **Boom**

Table 18.3: Tree growth using Surge fertiliser

Week	1	2	3	4	5
Height (mm)	75	100	100	145	155

Table 18.4: Tree growth using Boom fertiliser

Week	1	2	3	4	5
Height (mm)	50	80	120	160	200

Which of the fertilisers will you recommend? Justify your choice.

1. **Step 1:** Plot the data for both Surge fertiliser and Boom fertiliser using scatter graphs

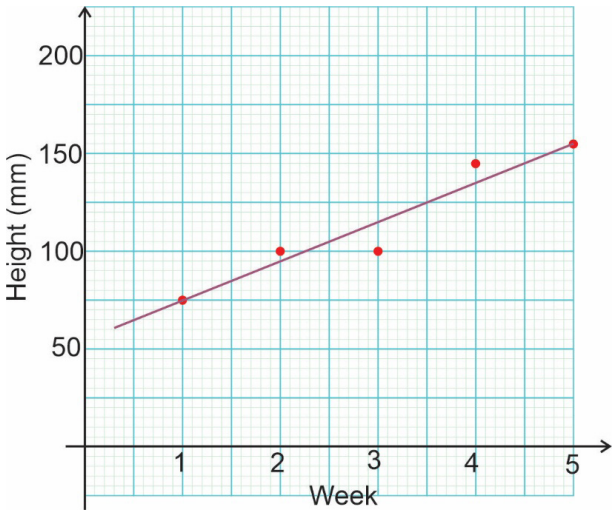


Figure 18.3: *Scatter plot of tree growth using surge fertiliser*



Figure 18.4: *Scatter plot of tree growth using boom fertiliser*

2. **Step 2: Identify the type of relationship**

   Surge: Shows a weak positive correlation – the growth of the trees is promising in the first week, but slows dramatically after that.

   Boom: Shows a strong positive correlation – it shows a consistent growth as the weeks go by.

3. **Step 3: Compare the trends and consistency**

   - Boom fertiliser shows steady growth as it is consistent and effective over time.
   - Surge fertiliser gives quick but limited growth since growth stalls early.

4. **Step 4: Conclusion**

Boom Fertiliser is more effective for long-term plant development.

Example 3

As a speaker in a school debate on the topic: "The rise of HIV/AIDS cases in Ghana, a cause for worry or a mere public scare," gather data on HIV/AIDS cases from the Ghana AIDS Commission or other reputable sources (radio, TV, newspapers, etc.), analyse the data (including using any suitable technological tools), and use charts or tables to support your arguments for or against the motion.

Solution

HIV DATA

Table 18.5: HIV DATA

Metric	Value
Total PLHIV (2024 estimate)	334 721 people
HIV prevalence (ages 15–49)	1.7%
New infections (2023)	17 774 (16,076 adults & 1,698 children)
Daily new infections	50 per day (2025 report)
AIDS-related deaths (2023)	12 614

Source: (pmc.ncbi.nlm.nih.gov, facebook.com)

2019-FACT-SHEET-22-06-2020-revised11.pdf

[Note: PLHIV = People Living with HIV]

Supporting Chart

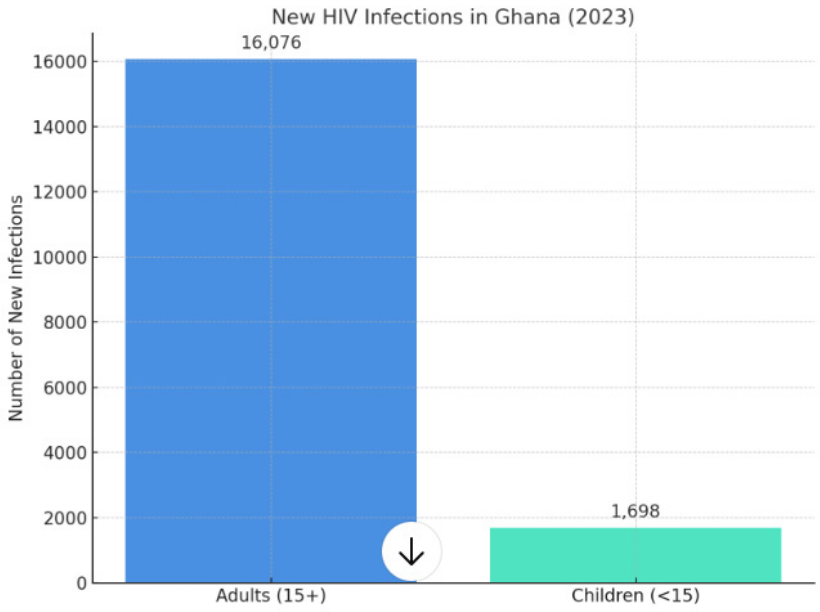


Figure 18.5: *Bar chart showing new HIV infections in Ghana (2023)*

A bar chart showing new HIV infections in Ghana for the year 2023. It shows that the majority of new HIV infections in Ghana in 2023 occurred in adults, but with a notable number still affecting children, emphasising that the rise in cases is a real cause for concern, not just a scare.

Points for Debate: "A Cause for Worry"

1. Substantial HIV Burden Remains
   - Over 334 000 people live with HIV in Ghana as of 2024 (facebook.com).
   - Though prevalence has stabilized (1.6–1.7%), the number of people living with HIV is growing.
2. Disturbingly High Daily New Infections
   - About 50 new cases per day in 2025 (facebook.com).
   - This indicates ongoing transmission and insufficient prevention.
3. Child Infections: A Critical Gap
   - Nearly 1,700 new child infections in 2023 (3news.com).
   - Mother-to-child transmission remains a public health concern.
4. Mortality Still High
   - Over 12 000 AIDS-related deaths in 2023 (3news.com, ghanawebbers.com).
   - Despite Antiretroviral Therapy (ART) advances, HIV/AIDS has not declined.
5. Uneven Regional Burden
   - Urban and southern regions (e.g., Greater Accra, Ashanti) show higher prevalence (pmc.ncbi.nlm.nih.gov).
   - Untargeted interventions may widen disparities.

Conclusion & Debate Standpoint

The data demonstrate a continued and concerning HIV burden in Ghana:

- With 50 new cases daily, significant child infections, and thousands of deaths annually, HIV remains a serious health issue.
- Although interventions have improved care and awareness, transmission remains active, especially in key populations and children.
- Therefore, the rise (or plateau) of HIV/AIDS cases should be considered a valid cause for concern, not just public alarm.

FOCAL AREA 2: INTERPRETING DATA PRESENTED ON MEDIA PLATFORMS

What is Media-Reported Data?

These are data presented on media platforms. The media platforms include news websites, TV stations, Social media platforms (e.g., Facebook, TikTok, WhatsApp, YouTube, etc.) and Online dashboards (e.g., COVID-19 trackers)

Bivariate Data in the Media

Bivariate data consists of two related variables. In the media, these usually appear in the form of tables, scatter plots, bar or line graphs showing two quantities changing together and Infographics.

How to analyse bivariate data from a media platform

After validating the data,

1. Extract the data into a table of ordered pairs (x, y)
2. Plot the data on a scatter graph
3. Analyse the trend. Look out for a correlation between the variables. Do you find any trend? Can we estimate or predict values using the trend? Answering these questions will help you analyse the data.

Example 1

Screen Time and Sleep

An online health article reported data collected from teens aged 15–18 on average daily screen time and hours of sleep.

Table 18.6: Screen time and sleep time

Screen time(hrs)	3	6	8	5	1	2	1	6	4	7
Sleep time (hrs)	7.5	6.5	5.2	6.7	8.2	8	8.5	6.1	7	5.7

a. Present the data using a scatter plot.

b. What kind of correlation exists between screen time and sleep duration?

c. What might be the implication of this trend?

d. Is this data likely observational or experimental? Explain your answer.

Solution

a.

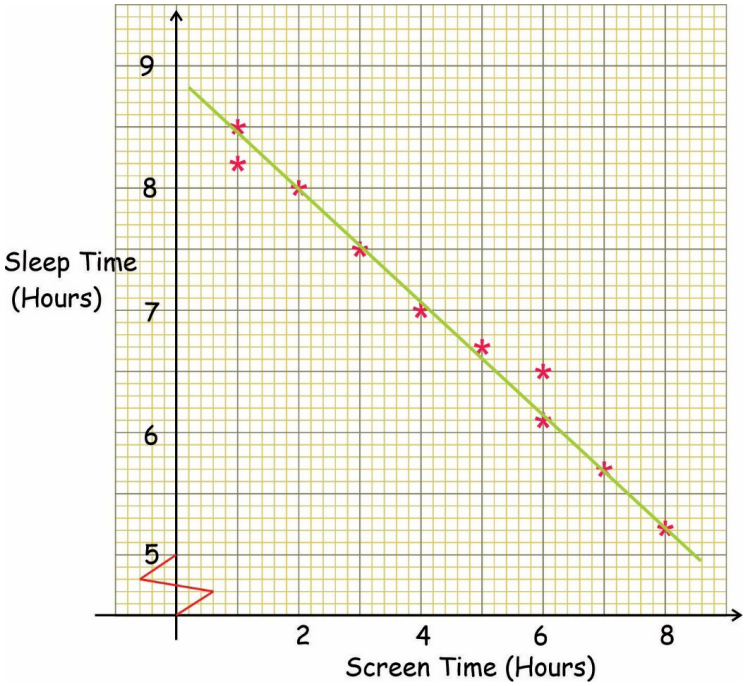


Figure 18.6: *Scatter plot of screen time versus sleep time.*

b. There is a negative correlation between screen time and sleep duration. As screen time increases, sleep decreases.

c. Higher screen time is associated with lower sleep duration in this data set.

d. The data is observational as the researchers observed the variables without deliberately assigning/manipulating screen time.

Example 2

The number of goals scored versus the number of goals conceded by a sample of seven football clubs in the Ghana Premier League for the 2024/2025 season is shown in the bar chart.

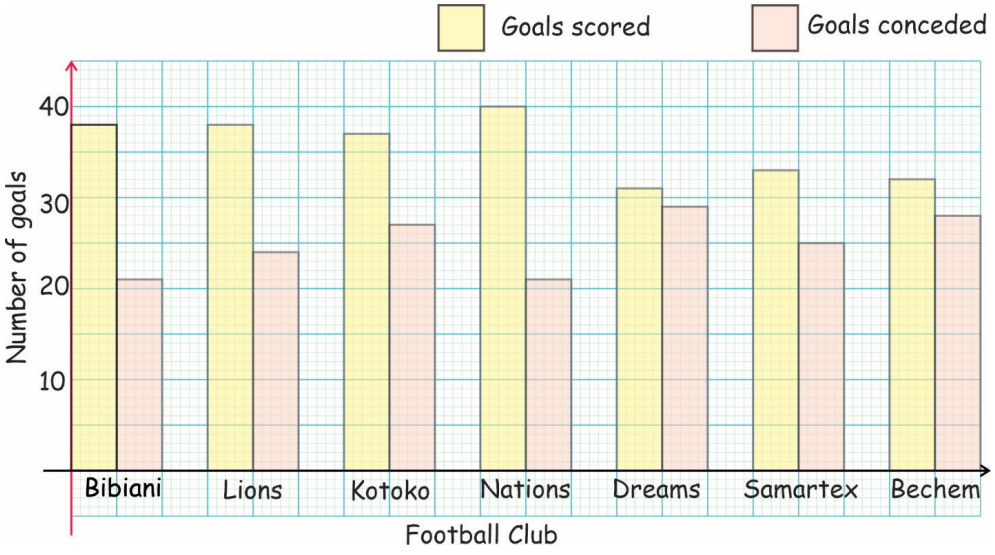


Figure 18.7: *Bar chart showing football clubs and number of goals scored in the 2024/2025 season.*

a. Copy and complete the table

Table 18.7: A partially completed table showing goals scored versus goals conceded

Football Club	Bibiani	Lions	Kotoko	Nations	Dreams	Samartex	Bechem
Goals Scored	38					33	
Goals Conceded						25	

b. Present the data using a scatter plot.

c. What kind of correlation exists between goals scored and goals conceded?

d. What might be the implication of this trend?

Solution

a.

Table 18.8: A completed table showing goals scored versus goals conceded.

Football Club	Bibiani	Lions	Kotoko	Nations	Dreams	Samartex	Bechem
Goals Scored	38	38	37	40	31	33	32
Goals Conceded	21	24	27	21	29	25	28

b. Present the data using a scatter plot.

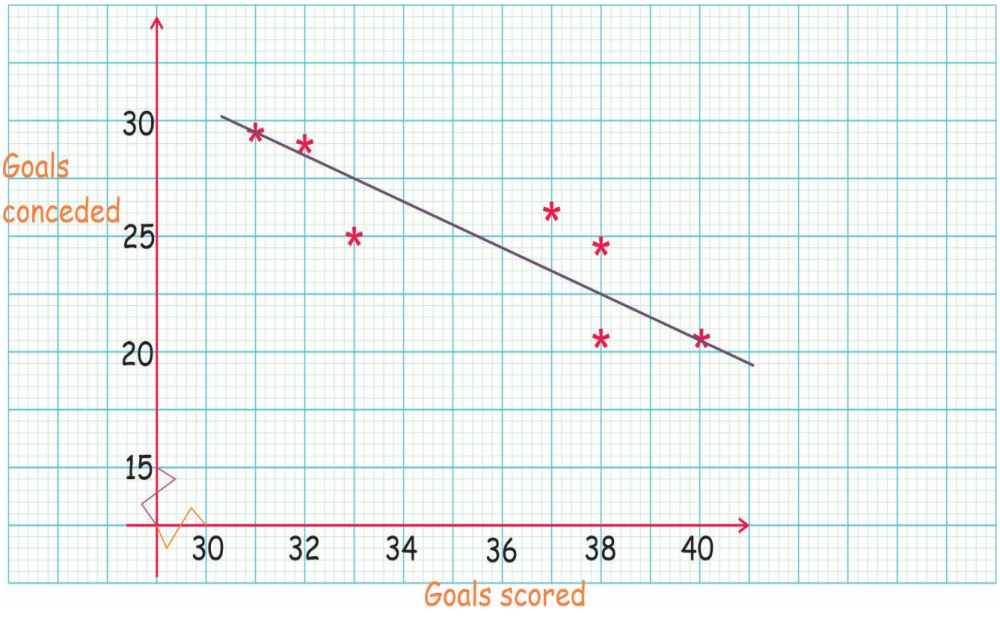


Figure 18.8: *Scatter of goals scored versus goals conceded.*

c. What kind of correlation exists between goals scored and goals conceded?

There is a negative correlation between goals scored and goals conceded.

d. In this sample, clubs that scored more goals tended to concede fewer goals.

Learning Tasks

Guide learners to apply their knowledge of scatter plots and correlation to analyse and solve problems using real-life data. This resource will equip learners to critically evaluate and determine which of two given data sets most accurately reflects a given scenario. Also, students will gain proficiency in identifying, analysing, and interpreting bivariate data collected from media platforms.

PEDAGOGICAL EXEMPLARS

This week's lessons aim to compare the quality of two given data sets and make inferences from published data. The following pedagogical approach is suggested for facilitators to guide learners through.

1. **Experiential Learning:** In convenient groups, task learners to compare the quality of given data sets and draw conclusions on the appropriateness of the data in relation to the purpose of the data and its analysis techniques.
2. Using the talk for learning strategy, engage learners in a class debate to support their arguments with data and the use of appropriate mathematical terminology.
3. **Project-based learning:** Assign project work to learners (individually or in groups) to obtain current data about a contemporary issue of interest, then make conclusions, give criticisms, and make useful recommendations.
4. **Group discussions**: In small groups, learners discuss data published on a local media platform and draw conclusions with justifications.

KEY ASSESSMENT: PRACTICE PAPER 2

Recommended mode of assessment for week 18 is group Project Work and presentation.

Project Work (Work in small groups)

Project Goal: To collect data from a media platform, make inferences about the data, and draw conclusions from the data.

Question

Select a simple project topic that allows you to gather easily data from media sources such as newspapers, television, radio, or online platforms. Use appropriate charts or graphs to present the data. Analyse the information to identify any patterns or relationships, and then share your findings, highlighting the meaning and implications of your observations.

Project Steps

Step 1: Choose a Simple Topic

- Pick a real-world issue or situation involving two related variables (bivariate data).

- Make sure it's something you can find data on easily from media sources (TV, radio, newspapers, websites, etc.).

Examples:

- Fuel prices and transport fares
- Rainfall and crop harvest
- Temperature and electricity usage
- Advertisement spending and product sales

Step 2: Gather Data

- Look for reliable sources such as news websites, government reports, social media pages of institutions, or TV/radio broadcasts.
- Record the data clearly in a table or spreadsheet.
- Make sure both variables are numerical or clearly measurable.

Step 3: Organise the Data

- Create a data table showing both variables side by side.
- Clean up any missing or inconsistent entries.

Step 4: Represent the Data Visually

- Use appropriate charts or graphs to show the relationship between the two variables.

Recommended graphs

- Scatter plot (best for identifying trends or correlations)
- Bar chart (if data is categorical)
- Line graph (if showing changes over time)

Step 5: Analyse the Data

- Look for patterns, trends, or correlations:
  - Is the relationship positive, negative, or no clear trend?
  - Are there any outliers (data points that don't follow the pattern)?
- Use statistical tools if needed (mean, correlation coefficient, etc.).

Step 6: Present Your Findings

- Write a summary of:
  - What the data shows
  - The type of relationship observed

- Key patterns or surprises

Step 7: Discuss the Implications

- Explain why the relationship matters.
- What can people, businesses, or governments learn from this?
- Suggest possible actions or solutions based on your findings.

Hint

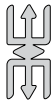


*Conduct **Practice Paper 2**. It should cover contents taught from Year 1 to 3. Refer to the **Table of Specification and Structure of the Paper** at the end of this section. Learners' papers should be marked immediately and scores entered into the SAP.*

PLC Session 18

45 minutes | **Planning & Reviewing the Learning Plan**

1. Begin the session by recapping lessons from the previous session, focusing on how learning was applied.
2. Use your subject-specific app to explore the content before planning. Ask the app to
   a. explain the week's concepts in a practical way that will make it easier for learners to understand.
   b. identify three common misconceptions learners may have on the week's content and how to address them using contexts familiar to Ghanaian learners (NTS 3a, 3g and 3m).
3. Use your subject-specific app to plan your lesson. Ask the app to
   a. suggest the most effective method for teaching the week's concepts to learners with diverse backgrounds.
   b. suggest learning activities, differentiation and essential questions based on the method of teaching (NTS 3a-3q).
   c. review your prepared learning plan and address any challenging issues you may have identified in your subject area (NTS 3a-3c).



Note

- Take note of the recap topics linked to Years 1 and 2 while preparing for your lesson.
- Refer to the Values Learning Community Handbook and link the values for the week in your lesson to ensure that learners are applying what they learn.

4. Use your app to plan and administer the mode of assessment for the Student Assessment Portal (SAP).

   a. The key assessment for the week is ***practice paper 2***. Chat with your app to plan this assessment by reviewing or developing the assessment items/tasks and marking schemes/rubrics for the practice paper (NTS 3k-3q).

10 minutes **Content Exploration**

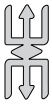
5. As a subject group, select one problem or topic/idea/term in the week's content that is challenging (not a routine procedural question, but one that requires conceptual understanding) and then

   a. use the app, ask it to solve the problem step by step and explain the reasoning behind each step (you may be assisted by a teacher who is strong in the content area)

   b. all teachers in the group work through the problem individually for 3–4 minutes

   c. the group then works through it together the problem, step by step

   d. highlight the most common errors made at each step (NTS 1c, 1e, 2c, 2e, 3e-3j).



Note

- This part is not *'How do I teach this?'* but how do I teach it for learners to understand?
- If the group still shares the same content gap, use the app: ask it to solve the problem step by step and explain the reasoning behind each step.
- Problems/ideas discussed should be sourced from the Teacher Manual content, not invented separately. This keeps the content exploration directly connected to what teachers will deliver.

30 minutes **Enactment**

6. Enact a teaching activity based on your learning plan (NTS la-1b, 1e-lg, 2c, 2e, 2f, 3c-3j and 3o).

   a. Colleagues observe with specific attention to clarity of concept explanation, use of familiar Ghanaian examples and how misconceptions are handled if they arise.

   b. After the enactment, ask for structured feedback from the group

      i. was the concept explained in a way that a learner with no prior knowledge could follow?

      ii. were the examples grounded in contexts familiar to Ghanaian learners?

      iii. did any misconceptions surface and if so, how were they addressed?

5 minutes Reflection

7. Reflect and share your views on the session. One thing that worked well, what did not work well and what you will do differently. (NTS 1a, 1b).

8. Identify a critical friend to observe and provide feedback on your lesson in relation to the PLC Session conducted (NTS 1f, 1g).

9. Remember to

a. transfer your chats into your learning planner template.

b. read your Teacher Manual and related Learner Material in preparation for the next session. Take note of the recap weeks, which are linked to Years 1 and 2 (NTS 3a).

c. integrate National Values, GESI, SEL, and 21st century skills into your learning activities, assessment tasks, and rubrics/mark schemes.

WEEK 19

Learning Indicator: *Identify examples from print and electronic media, e.g., newspapers, television, the Internet, where probability is used, and explain how the given probability influences individual decisions*

FOCAL AREA: IDENTIFICATION OF PROBABILITY USAGE IN THE MEDIA

The Role of Probability in Society

1. **Weather Forecasting in Ghana**

   Role: The Ghana Meteorological Agency uses probability to forecast rainfall and other weather events.

   Example: During the rainy season, forecasts may say there's an 80% chance of rain in Accra or Kumasi.

   Impact: Farmers rely on these forecasts to decide when to plant or harvest crops.

   The public and transport services prepare for possible flooding.

2. **Healthcare and Disease Control**

   Role: Health authorities use probability to assess disease risks and plan interventions.

   Example: The Ghana Health Service estimates the likelihood of malaria outbreaks based on climate and region. Probabilities are also used to predict HIV transmission risks and track progress.

   Impact: Helps with distributing mosquito nets or ARV drugs to high-risk areas.

3. **Insurance in Ghana (e.g., NHIS and Private Insurance)**

   Role: Insurance companies like SIC use probability to assess the likelihood of health issues, accidents, or death.

   Example: NHIS may use probability data to estimate future health service costs.

   Impact: Ensures insurance premiums are affordable and sustainable.

4. **Banking and Microfinance**

   Role: Banks and fintech companies use probability to assess loan default risks.

   Example: A microfinance institution may estimate that there's a 30% chance a new borrower might not repay a loan.

   Impact: Affects decisions on who qualifies for a loan and at what interest rate.

5. **Elections and Public Opinion**

   Role: Probability is used in pre-election polls by institutions like CDD-Ghana or Afrobarometer.

   Example: A poll might say there's a 60% chance the incumbent party will win in a specific region.

   Impact: Guides campaign strategies and helps voters and analysts understand political trends.

6. **Sports and Betting**

   Role: Betting companies in Ghana (e.g., Betway, Odibets) use probability to set odds.

   Example: The odds for a Hearts of Oak win versus Asante Kotoko reflect the calculated probability of that outcome.

   Impact: Influences public bets and betting strategies.

7. **Education (WASSCE/BECE Analysis)**

   Role: Teachers and students use probability to guess which topics are most likely to appear in exams.

   Example: If a question on probability appeared three years in a row in BECE, teachers might estimate it has a lower chance this year.

   Impact: Helps students prioritise their studies, although it's not always accurate and is a high-risk strategy.

8. **Disaster Preparedness**

   Role: Authorities assess the chance of natural disasters like floods or droughts.

   Example: NADMO (National Disaster Management Organisation) uses past data to estimate the probability of flood-prone zones in Accra.

   Impact: Helps in allocating emergency resources and planning evacuation drills.

9. **Agriculture (Cocoa Farming, Food Security)**

   Role: COCOBOD and other bodies use probability to predict crop yields based on rainfall, pests, and market trends.

   Example: A 20% chance of drought in the Ashanti region could mean lower cocoa yields.

   Impact: Influences export decisions and farmer support programmes.

10. **Crime Prevention and Policing**

    Role: Security services analyse crime data to estimate where crimes are more likely to occur.

    Example: Police may deploy more patrols in areas with a high probability of robbery based on past incidents.

Impact: Improves safety and resource allocation.

Probability In The Media

Probability is a measure of how likely an event is to occur, expressed as a number between 0 and 1, or as a percentage. Probability is a powerful tool used in media to influence decisions. Understanding how it's calculated and communicated will help you make **informed decisions**.

Why It Matters

Media outlets frequently use probability to inform or persuade the public, especially in areas such as health, weather, elections, and sports.

Find Below Some Examples Of Media Statements and Their Impact

1. **Media Statement:** "The vaccine reduces the risk of infection by 90%."

   Meaning: In a group of people, those vaccinated are 90% less likely to get infected compared with those who are not vaccinated.

   Impact: Influences individual decisions on whether to get vaccinated. It will also affect public policy and funding for healthcare.

2. **Media Statement:** "Candidate A has a 65% chance of winning the election."

   Impact: May affect voter turnout ("my vote won't matter" effect) or shape campaign strategies and media narratives.

3. **Media Statement:** "Team X has a 60% probability of winning tonight's game."

   Impact: Affects betting behaviour and fan expectations. Used in fantasy sports decisions.

Example 1

The Ghana Meteorological Service reports a 65% probability of rain in Accra tomorrow.

a. What common misunderstanding might people have about this forecast?

b. What does this 65% probability actually mean?

c. Based on this information, what actions might someone choose to take?

Solution

a. Some people may wrongly think that it will rain over 65% of the area or that it will rain for 65% of the day. Others might believe it means it will definitely rain, just not sure when.

b. It means that, based on similar weather conditions in the past, rain occurred 65% of the time. In other words, if these same conditions happened 100 times, it rained on 65 of those cases.

1. Carry an umbrella or wear a raincoat.
2. Postpone or adjust outdoor plans or events.
3. Farmers might prepare their fields or delay irrigation.
4. Drivers may be more cautious, anticipating wet roads.

Example 2

A recent study found that a new flu vaccine is 87% effective.

a. How would you explain this 87% effectiveness to a friend in simple terms?

b. Why is this information important for individuals and the public?

c. What is a possible misunderstanding someone might have about this result?

Solution

a. You can say that if 100 people take the flu vaccine, about 87 of them will be protected from getting the flu, while about 13 might still get ill despite taking the vaccine, but their illness may be milder.

b. It helps people decide whether to get the vaccine. If many people get vaccinated, the flu is less likely to spread, which protects the whole community, especially those who can't get vaccinated.

c. Some might think the vaccine guarantees they won't get the flu at all. Others may assume the 87% means only 87 out of every 100 doses of the vaccine are effective.

Example 3

A sports website predicts that Medeama FC has a 70% chance of winning their next match."

a. What does this 70% chance mean in context?

b. How might fans or gamblers respond to this information?

c. What is the possible misunderstanding someone might have about the prediction?

Solution

a. It means that, based on previous performance, current form, and other factors, Medeama FC is likely to win the match 70 times out of 100 if the same situation happened repeatedly. It shows a high probability, but not a guarantee.

b. (1) Fans may feel more confident and hopeful about their team's chances.

c. (2) Gamblers might be more likely to bet on Medeama FC to win, expecting a strong performance.

Some might even bet on the opponent, hoping for an upset and a bigger payout.

d. Some people might wrongly think that Medeama FC is guaranteed to win or that the result is already certain. Others may misinterpret it as meaning the team will win by 70% of the match duration, which is incorrect.

Example 4

a. Can the media use probability in misleading ways?

b. What are some questions you should ask when you see a probability in the media?

c. Why is it important to understand probability in real life?

Solution

a. Yes. For example, stating a 90% success rate without explaining the sample size or other factors can mislead people into overestimating effectiveness.

b. What data is it based on?

c. Who is providing the information?

d. How recent and reliable is the source?

e. It helps you make informed decisions, avoid being misled by the media, and understand risks better.

Learning Tasks

Guide learners to examine real-life examples of how probability is used in newspapers, television, and online media. Support them to analyse how these probabilities influence public decisions and opinions. Encourage them to calculate simple probabilities from media data and interpret the results.

PEDAGOGICAL EXEMPLARS

This week's lessons are designed to support learners to analyse how probability influences public decisions and opinions. The following pedagogical approach is suggested for facilitators to guide learners through.

1. **Pair work:** Engage learners in pairs to identify examples of the use of probability in real-life contexts, including the media
2. **Group discussions:** In small groups, guide and encourage learners to discuss the role of probability in society by applying probability reasoning to investigate and make useful guesses about everyday-life events and use them to make judgements and suitable decisions in their everyday activities.

KEY ASSESSMENT LEVEL 3: INDIVIDUAL RESEARCH WORK

1. Undertake the following project independently

a. Project Topic

Investigating the Impact and Misconceptions of Media Statements that Use Probability.

b. Project Objective: To investigate how probability is used in media statements (e.g., news, radio, websites) and to assess the public's understanding, misconceptions, and the real-life effects of such statements.

c. Instructions

i. Choose 3 probability media statements from reliable sources (TV, radio, internet, or newspapers).

ii. ***Examples***: "There is a 70% chance of rain tomorrow." "The new malaria vaccine is 80% effective."

iii. For each statement, complete the sections below:

- ***Media Statement:*** *(Write the full statement using probability)*
  - ***Example***: "There is an 80% chance of rain in Accra tomorrow."
- State the source of the statement and the day you assessed the source. *(E.g., Ghana Web, Joy FM, TV3, Daily Graphic, Interview, assessed 23/02/2026)*
- What the statement means: *(Give a clear, simple explanation of the probability in the context)*
- Provide at least **two** common misconceptions people may have about the statement. *(How might people misunderstand or misinterpret the statement?)*
- Give at least 3 Real-Life Impacts of the statement: (*How does this statement influence decisions, behavior, or planning?*)

d. Reflection Questions: *(Answer in full sentences)*

i. What did you learn about how people interpret probability in media?

ii. Why is it important for media outlets to clearly explain probability?

iii. How can schools or public education reduce misunderstandings like these?

Total 30 marks

2. The graph below shows the monthly trends in rainfall and temperature in Ghana, based on a study by Musah et al. (2018)."

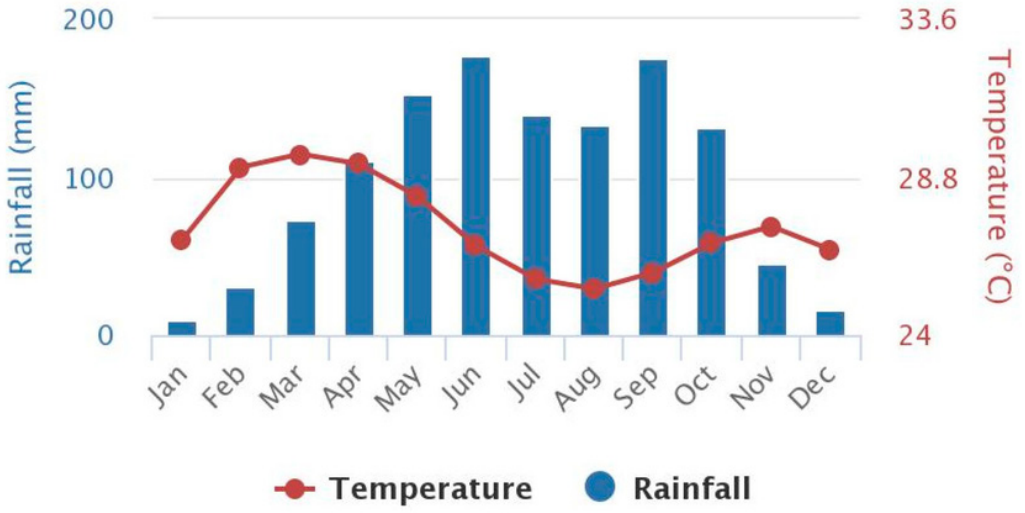


Figure 19.1: *Monthly variations in rainfall and temperature in Ghana*

Assume a temperature (t) is considered high if $t \geq 28.80°C$, and rainfall (f) is considered heavy if $f \geq 100$ mm.

a. Find the probability of having a low temperature ($t < 28.80°C$) within the year. If you are to report this probability figure in the media, what might be the impact and possible misconceptions? ***5 marks***

b. Determine the probability of experiencing heavy rainfall ($f \geq 100$ mm) within the year. If you are to report this probability figure in the media, state one misconception someone might have. ***5 marks***

c. Calculate the probability of experiencing Low temperatures and High rainfall within the year. How will you explain your answer to a layman? ***5 marks***

Total *15 marks*

PLC Session 19

45 minutes | **Planning & Reviewing the Learning Plan**

1. Begin the session by recapping lessons from the previous year/session/week, focusing on how learning was applied.

2. Use your subject-specific app to explore the content before planning. Ask the app to

a. explain the week's concepts in a practical way that will make it easier for learners to understand

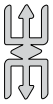
b. identify two or three common misconceptions learners may have on the week's content and how to address them using contexts familiar to Ghanaian learners. (NTS 3a, 3g and 3m).

3. Use your subject-specific app to plan your lesson. Ask the app to

a. suggest the most effective method for teaching the week's concepts to learners with diverse backgrounds.

b. suggest learning activities, differentiation and essential questions based on the method of teaching (NTS 3a-3q)

c. review your prepared learning plan and address any challenging issues you may have identified in your subject area (NTS 3a-3c).



Note

- Take note of the recap topics linked to Years 1 and 2 while preparing for your lesson.
- Refer to the Values Learning Community Handbook and link the values for the week in your lesson to ensure that learners are applying what they learn.

4. Plan the week's key assessment with your app

a. The key assessment for the week is ***individual research***. Chat with your app to plan this assessment, indicating what you and your learners will do before, during and after the research (NTS 3k-3q).

10 minutes **Content Exploration**

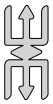
5. As a subject group, select one problem or idea/term in the week's content that is challenging (not a routine procedural question, but one that requires conceptual understanding) and then

a. use the app, ask it to solve the problem step by step and explain the reasoning behind each step (you may be assisted by a teacher who is strong in the content area)

b. all teachers in the group work through a similar problem individually for 3–4 minutes

c. the group then works through the problem together step by step

d. highlight the most common errors made at each step and discuss (NTS 1c, 1e, 2c, 2e, 3e-3j).



Note

- This part is not just about 'How do I teach this?' but how do I teach it for learners to understand?
- If the group still shares the same content gap, use the app: ask it to solve the problem step by step and explain the reasoning behind each step.
- Problems/ideas discussed should be sourced from the Teacher Manual content, not invented separately. This keeps the content exploration directly connected to what teachers will deliver.

30 minutes Enactment

6. Enact a teaching activity based on your learning plan (NTS la-1b, 1e-lg, 2c, 2e, 2f, 3c-3j and 3o).

- **a.** Colleagues observe with specific attention to clarity of concept explanation, use of familiar Ghanaian examples and how misconceptions are handled if they arise.
- **b.** After the enactment, ask for structured feedback from the group
  - **i.** was the concept explained in a way that a learner with no prior knowledge could follow?
  - **ii.** were the examples grounded in contexts familiar to Ghanaian learners?
  - **iii.** did any misconceptions surface and if so, how were they addressed?

5 minutes Reflection

7. Reflect and share your views on the session. One thing that worked well, what did not work well and what you will do differently. (NTS 1a, 1b).

8. Identify a critical friend to observe and provide feedback on your lesson in relation to the PLC Session conducted (NTS 1f, 1g).

9. Remember to

- **a.** transfer your chats into your learning planner template.
- **b.** read your Teacher Manual and related Learner Material in preparation for the next session. Take note of the recap weeks, which are linked to Years 1 and 2 (NTS 3a).
- **c.** integrate National Values, GESI, SEL, and 21st century skills into your learning activities, assessment tasks, and rubrics/mark schemes.

WEEK 20

Learning Indicator: *Solve everyday life problems involving the probability of two dependent and independent events, including addition and multiplication laws*

FOCAL AREA: REAL-LIFE PROBLEMS INVOLVING PROBABILITY

What is Probability?

Probability is a measure of the likelihood that an event will occur. It ranges from 0 (impossible) to 1 (certain). For a finite sample space with equal outcomes,

Theoretical probability of an event $= \frac{number\ of\ elements\ in\ the\ event}{total\ number\ of\ possible\ outcomes} = \frac{n(E)}{n(s)}$

PROBABILITY TERMINOLOGIES

Experiment

A planned activity carried out under controlled conditions. This activity can be performed any number of times and has a well-defined set of possible outcomes.

Examples of experiments include rolling dice, spinning a wheel, drawing balls from a bag at random, and tossing a coin. The result of an experiment is called the **outcome.** The set of all possible outcomes is called the **sample space** (S).

Event (E)

An event is a set of outcomes of an experiment. The event is a subset of the sample space.

P(theoretical probability of an event) $= \frac{number\ of\ elements\ in\ the\ event}{total\ number\ of\ possible\ outcomes} = \frac{n(E)}{n(s)}$

As a number, the probability of an event is between 0 and 1 inclusive. i.e. $0 \leq P(E) \leq 1$

The complement of an event

The complement of an event A is denoted by A' (A prime). From set theory, we know that represents all elements that are not in A but are in the universal set. Under probability, the universal set is equivalent to the Sample Space.

Note that $P(A) + P(A') = 1$. Applying the basic change of subject, we have

$P(A) = 1 - P(A')$ and $P(A') = 1 - P(A)$

Probability of the complement of an event

Let $P(E) = r$ be the probability that event E will occur.

The probability that event E will **not** occur is written as $P(E') = 1 - r$

For example, if the probability that it will rain today $= \frac{2}{7}$, then the probability that it will not rain today $= 1 - \frac{2}{7} = \frac{5}{7}$.

A Trial

Any specific performance of an experiment is called a trial. Suppose you are conducting an experiment involving tossing a single coin multiple times. Each toss of the coin is a trial.

Similarly, if you have a class of 20 students and want to select 4 students, each selection is a trial. To calculate probabilities involving compound events, we typically number the trials.

Conditional Probability

The conditional probability of **A** given **B** is written as P(A|B), meaning "the probability of A, given B."

P(A|B) is the probability that event A occurs assuming that event B has already occurred.

The formula for P(A|B) is $P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{P(A \cap B)}{P(B)}$ where $P(B) > 0$.

Independent Events

Two events are *independent* if the occurrence of one does not affect the occurrence of the other event. For example, flipping a coin does not impact the result from rolling a die.

Generally, A and B are independent if P(A and B) = P(A) × P(B).

Examples of activities that can generate independent events

1. Tossing a coin or a die more than once
2. Tossing a single die twice or two dice once.
3. Choosing a card from a deck and spinning a wheel.
4. Tossing a coin and rolling a die
5. Randomly drawing two or more items with replacement.

Dependent Events

Two events are dependent when the outcome of the first event affects the outcome of the second event. For an event to be considered dependent, one event must influence the other event.

Examples of activities that can generate dependent events

1. Drawing items without replacement.
2. Selecting individuals to form a group or a committee.
3. Selecting individuals for a position where one person cannot occupy multiple positions.

Mutually Exclusive Events

These are events which cannot occur at the same time.

For example, when flipping a coin, you can get either heads or tails, but not both.

Examples of activities that can generate mutually exclusive events

1. Tossing a coin once to observe a head and a tail
2. Selecting one card from a pack of cards to observe an Ace and a six
3. When a single die is rolled, the events "getting a 2" and "getting a 5" are mutually exclusive because both outcomes cannot occur on the same roll.

Two Basic Rules of Probability

These rules help determine if two events are independent, dependent or mutually exclusive.

The Multiplication Rule

If A and B are two events in a sample space, then $P(A \text{ and } B) = P(A \cap B) = P(B) \times P(A|B)$.

If A and B are independent, then $P(A|B) = P(A)$.

In this case, $P(A \cap B) = P(A|B) \times P(B) = P(A) \times P(B)$.

If A and B are mutually exclusive, then $P(A \cap B) = 0$.

The Addition Rule

If A and B are events in a sample space, then

$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If A and B are mutually exclusive, $P(A \cap B) = 0$, so $P(A \cup B) = P(A) + P(B)$.

Drawing objects without replacement

This means that after each item is drawn, it is not put back into the group before the next draw is done. This means that subsequent draws will not have the same sample space. Therefore, each draw will depend on the previous draw(s).

Probability of Two Dependent Events

This is calculated as:

The probability of the **first** event, multiplied by the probability of the **second** event after the first event occurs.

= [Probability of 1^{st} event] × [Probability of 2^{nd} event after the first event occurs]

If the first event is **X** and the second event is **Y**, then the probability of the second event after the first has occurred is written as P(Y|X).

Let us use this understanding to answer questions

Example 1

A social media quiz app offers a 30% chance to win a prize per spin. If you spin twice, what is the probability that you win both times?

Solution

The question is an example of an independent event. The 30% chance of winning does not change regardless of the number of spins.

P(Win twice) $= \frac{30}{100} \times \frac{30}{100} = \frac{900}{10000}\ \frac{9}{100} = 0.09 = 9\%$

Example 2

A box contains 5 red and 3 green beads. Kojo randomly draws two beads from the box without replacement.

Find the probability that:

a. The first bead is red and the second bead is green

b. Both beads are green

Solution

a. This is an example of a dependent event since we are drawing without replacement.

Before selecting the first bead, the contents of the box will look like the one in the picture.

The box will contain 5 red and 3 green beads

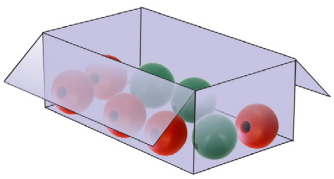


Figure 20.1: *Five red and three green beads in an open box*

Therefore, there are 8 possible individual selections

This means that $n(S_1) = 5 + 3 = 8$

P(the first bead drawn is red) $= \frac{5}{8}$

After drawing the first red bead, the box will look like the picture below

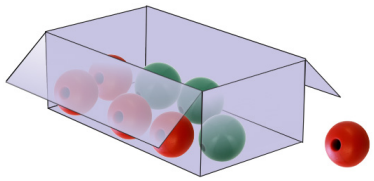


Figure 20.2: *Four red and three green beads in an open box, with one red bead outside the box.*

The total number of beads **in the box** will reduce to 7. We will have 4 red and 3 green beads remaining in the box.

Therefore, the Sample Space (S_2) for the event of selecting the second bead (second trial) is {4 red and 3 green beads}

This means that $n(S_2) = 4 + 3 = 7$

P(the second bead drawn is green) $= \frac{3}{7}$

Therefore, P(the first bead is red and the second bead is green) $= \frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$

a. Both beads are green. This statement means the first bead drawn is green and the second bead drawn is also green.

Before selecting the first bead, the contents of the box will look like the picture below. The box will contain 5 red and 3 green beads

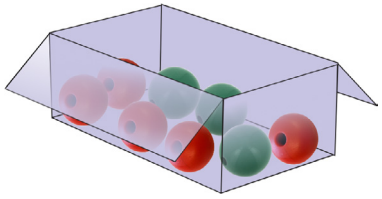


Figure 20.3: *Five red and three green beads in an open box*

Therefore, the Sample Space (S_1) for the event of selecting the first bead is

{5 red and 3 green beads}

This means that $n(S_1) = 5 + 3 = 8$

P(the first bead drawn is green) $= \frac{3}{8}$

After drawing the first green bead, the box will look like the picture below

The total number of beads in the box will reduce to 7. We will have 5 red and 2 green beads remaining in the box.

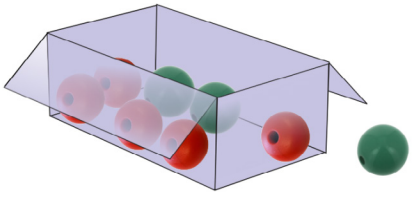


Figure 20.4: *Five red and two green beads in an open box, with one green bead outside the box.*

Therefore, the Sample Space (S_2) for the event of selecting the second bead is

{5 red and 2 green beads}

This means that $n(S_2) = 5 + 2 = 7$

P(the second bead drawn is green) $= \frac{2}{7}$

Therefore, P(both beads drawn are green) $= \frac{3}{8} \times \frac{2}{7} = \frac{6}{56} = \frac{3}{28}$

Example 3

A building contractor submitted bids for two independent contracts, X and Y.

The probability that he will win contract X is 0.5, and the probability that he will not win contract Y is 0.3.

Calculate the probability that the contractor will:

a. Win both contracts

b. Win exactly one of the contracts

c. Win neither of the contracts

Solution

P(Win X) = 0.5

P(Not Win Y) = 0.3 → So, P(Win Y) = 1 – 0.3 = 0.7

Contracts X and Y are independent

a. Probability of winning both contracts:

Since the events are independent:

P(Win X and Win Y) = P(Win X) × P(Win Y)

= 0.5 × 0.7

= 0.35

Therefore, the probability of winning both contracts is 0.35 = 35%

b. Probability of winning exactly one contract:

There are two ways to win exactly one contract:

i. P(Win X and not win Y) = 0.5 × 0.3 = 0.15

ii. P(Not win X and Win Y) = (1 − 0.5) × 0.7 = 0.5 × 0.7 = 0.35

P(Win exactly one contract)

c. Probability of winning neither contract:

P(Not WinX) = 1 − 0.5 = 0.5

P(Not WinY) = 0.3

P(winning neither contract) = P(Not win X) × P(Not win Y)

$$= 0.5 \times 0.3 = 0.15$$

A standard pack of playing cards

The diagram below shows all 52 cards in the pack.

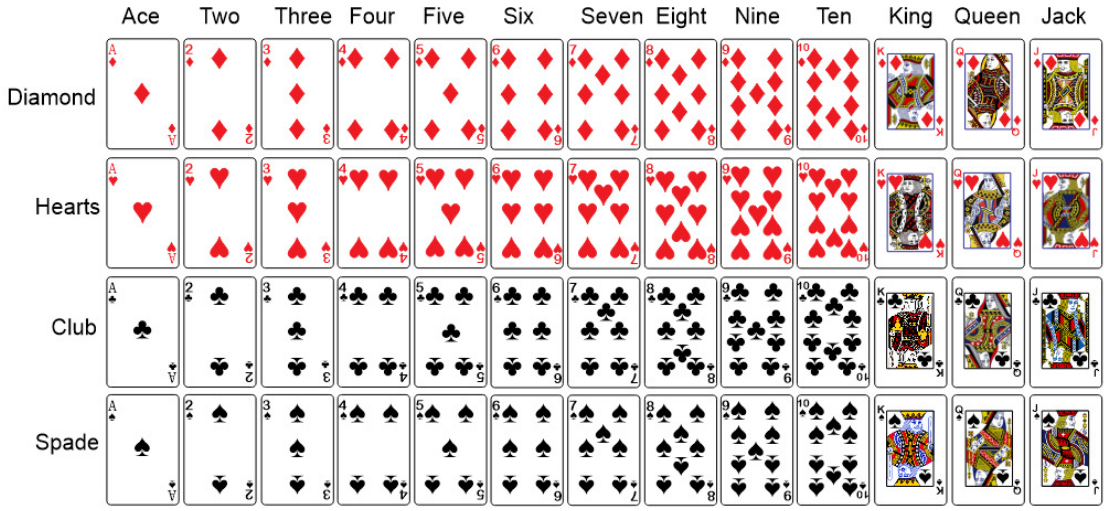


Figure 20.5: *A standard pack of playing cards*

Properties of a standard deck of playing cards

- There are a total of 52 cards in a deck.
- There are four suits: hearts, diamonds, spades, and clubs. Thus, there are 13 hearts, 13 diamonds, 13 spades, and 13 clubs.
- There are 13 ranks of cards. These ranks are Ace (=1), 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. Ace is considered a number card for the purposes of exercises.
- The diamonds and hearts are printed in red. The clubs and spades are printed in black. So, there are 26 red cards and 26 black cards.
- Each rank has four cards in it (one for each of the four suits). This means there are four Aces, four 2s, four 3s, four 4s, and so on.
- The Jacks, Queens, and Kings are all considered picture or face cards. Thus, there are three face cards for each suit, making a total of 12 face cards in the deck.

We are going to use this understanding to answer the next example

Example 4

Two cards are selected at random from a standard deck of playing cards without replacement.

Find the probability of selecting:

a. An ace, followed by a picture card, leave your answer as a fraction.

b. A spade, followed by a heart. Leave your answer as a fraction.

c. Two even-numbered cards.

Solution

a. There are 4 Aces in a pack of cards.

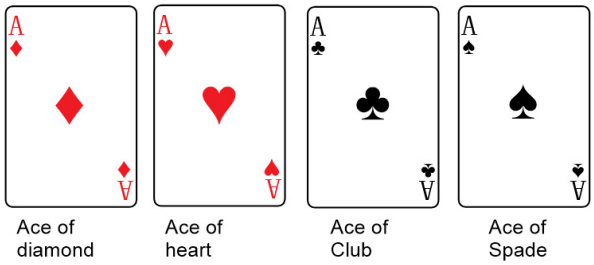


Figure 20.6: *All the aces in a pack of playing cards*

Recall that a standard pack of cards has a total of 52 cards.

So, P(selecting an Ace in the first trial) $= \frac{4}{52} = \frac{1}{13}$

On the second trial, there will be 51 cards remaining. Do not forget we selected one Ace card in the first trial, which we did not replace.

There are 12 picture cards in the deck, and none were picked in the first trial.

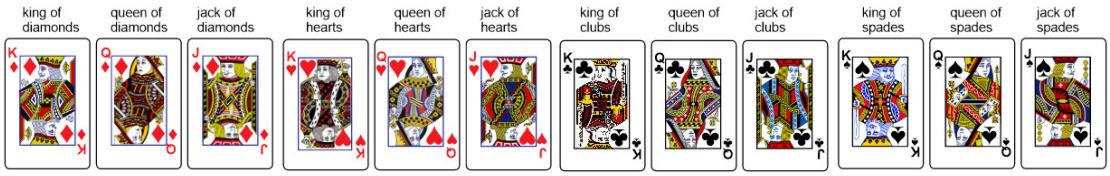


Figure 20.7: *All picture or face cards in a pack of playing cards.*

So, P(selecting a picture card in the second trial) $= \frac{12}{51} = \frac{4}{17}$

Therefore, P(selecting an ace, followed by a picture card) $= \frac{1}{13} \times \frac{4}{17} = \frac{4}{221}$

b. There are 13 spades in a pack of cards.

Recall that a standard pack of cards has a total of 52 cards.

So, P(selecting a spade in the first trial) $= \frac{13}{52} = \frac{1}{4}$

On the second trial, there will be 51 cards remaining. Do not forget we picked one spade in the first trial, which we did not replace.

There are 13 hearts in the deck, and none were picked on the first trial.

So, P(selecting a heart on the second trial) $= \frac{13}{51}$

Therefore, P(selecting a spade, followed by a heart) $= \frac{1}{4} \times \frac{13}{51} = \frac{13}{204}$

c. There are 36 number cards in a pack of cards. Out of these, 20 are even. These are 2, 4, 6, 8, and 10s of each of the four suits.

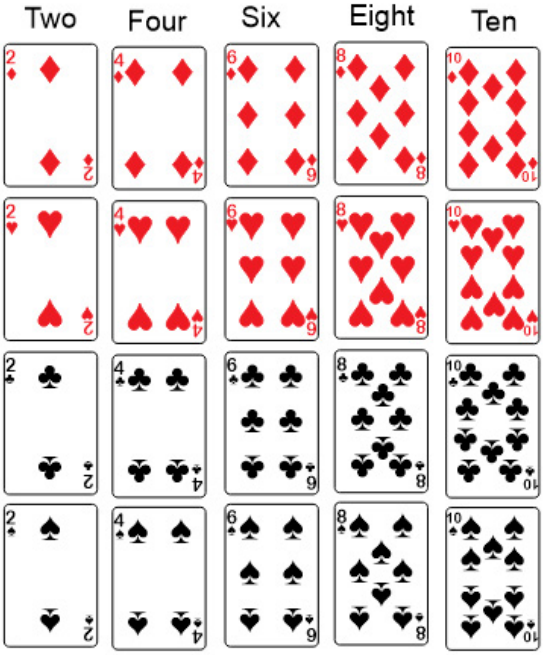


Figure 20.8: *All even-numbered cards in a pack of playing cards.*

Recall that a standard pack of cards has a total of 52 cards.

So, P(selecting an even-numbered card on the first trial) $= \frac{20}{52} = \frac{5}{13}$

On the second trial, there will be 51 cards remaining.

Do not forget we picked one even-numbered card in the first trial, which we did not replace.

This means there will be 19 even-number cards remaining.

So, P(selecting an even number card on the second trial) $= \frac{19}{51}$

Therefore, P(two even number cards) $= \frac{5}{13} \times \frac{19}{51} = \frac{95}{663}$

Learning Tasks

Guide and encourage learners to discuss and solve everyday life problems involving the probability of two dependent and independent events using addition and multiplication laws.

Define and distinguish between independent and dependent events. Calculate probabilities involving independent and dependent events.

PEDAGOGICAL EXEMPLARS

This week's lessons are designed to support learners in answering probability questions involving dependent and independent events. The following pedagogical approach is suggested for facilitators to guide learners through.

1. **Pair work:** Engage learners in pairs to distinguish between independent and dependent events.

2. **Group discussions:** In small groups, guide learners to apply the addition and multiplication rules to answer real-life independent and dependent probability questions.

KEY ASSESSMENT LEVEL 3: INDIVIDUAL CLASSWORK

Question One

Two cards are drawn at random from a standard deck of 52 playing cards without replacement.

Find the probability of selecting:

a. A king, followed by a red card. Leave your answer as a fraction. ***4 marks***

b. A diamond, followed by a black card. Leave your answer as a fraction. ***2 marks***

c. Two odd-numbered cards. Leave your answer as a fraction. ***2 marks***

Total *8 marks*

Question Two

Three boxes labelled A, B, and C contain green, yellow, and orange balls, respectively. All the balls are of the same size.

- The ratio of balls in A to B is 3:5
- The ratio of balls in B to C is 2:3

All the balls are poured into a large container and thoroughly mixed.

a. What is the probability of randomly selecting a green ball? ***4 marks***

b. Two balls are drawn one after the other with replacement. Find the probability of:

i. Picking a yellow ball and an orange ball (in any order) ***3 marks***

ii. Picking a yellow ball first, then a green ball ***2 marks***

Total *9 marks*

PLC Session 20

45 minutes | **Planning & Reviewing the Learning Plan**

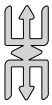
1. Begin the session by recapping lessons from the previous year/session/week, focusing on how learning was applied.
2. Use your subject specific app to explore the content before planning. Ask the app to
   a. explain the week's concepts in a practical way that will make it easier for learners to understand
   b. identify two or three common misconceptions learners may have on the week's content and how to address them using contexts familiar to Ghanaian learners. (NTS 3a, 3g and 3m).

3. Use your subject specific app to plan your lesson. Ask the app to

a. suggest the most effective method for teaching the week's concepts to learners with diverse backgrounds.

b. suggest learning activities, differentiation and essential questions based on the method of teaching (NTS 3a-3q)

c. review your prepared learning plan and address any challenging issues you may have identified in your subject area (NTS 3a-3c).



Note

- Take note of the recap topics linked to Years 1 and 2 while preparing for your lesson.
- Refer to the Values Learning Community Handbook and link the values for the week in your lesson to ensure that learners are applying what they learn.

4. Plan the week's key assessment with your app

a. The key assessment for the week is ***individual classwork***. Chat with your app to plan this assessment, indicating what you and your learners will do before, during and after the classwork (NTS 3k-3q).

10 minutes **Content Exploration**

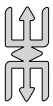
5. As a subject group, select one problem or idea/term in the week's content that is challenging (not a routine procedural question, but one that requires conceptual understanding) and then

a. use the app, ask it to solve the problem step by step and explain the reasoning behind each step (you may be assisted by a teacher who is strong in the content area)

b. all teachers in the group work through a similar problem individually for 3–4 minutes

c. the group then works through the problem together step by step

d. highlight the most common errors made at each step and discuss (NTS 1c, 1e, 2c, 2e, 3e-3j).



Note

- This part is not just about 'How do I teach this?' but how do I teach it for learners to understand?
- If the group still shares the same content gap, use the app: ask it to solve the problem step by step and explain the reasoning behind each step.
- Problems/ideas discussed should be sourced from the Teacher Manual content, not invented separately. This keeps the content exploration directly connected to what teachers will deliver.

30 minutes **Enactment**

6. Enact a teaching activity based on your learning plan (NTS la-1b, 1e-lg, 2c, 2e, 2f, 3c-3j and 3o).
   a. Colleagues observe with specific attention to clarity of concept explanation, use of familiar Ghanaian examples and how misconceptions are handled if they arise.
   b. After the enactment, ask for structured feedback from the group
      i. was the concept explained in a way that a learner with no prior knowledge could follow?
      ii. were the examples grounded in contexts familiar to Ghanaian learners?
      iii. did any misconceptions surface and if so, how were they addressed?

5 minutes **Reflection**

7. Reflect and share your views on the session. One thing that worked well, what did not work well and what you will do differently. (NTS 1a, 1b).
8. Identify a critical friend to observe and provide feedback on your lesson in relation to the PLC Session conducted (NTS 1f, 1g).
9. Remember to
   a. transfer your chats into your learning planner template.
   b. read your Teacher Manual and related Learner Material in preparation for the next session. Take note of the recap weeks, which are linked to Years 1 and 2 (NTS 3a).
   c. integrate National Values, GESI, SEL, and 21st century skills into your learning activities, assessment tasks, and rubrics/mark schemes.

Section 5 Review

This section recaps the key concepts covered over the past five weeks (weeks 16-20), summarising the core statistical and probability knowledge learners should have gained. This period was crucial in building a strong foundation for learners continued learning in these areas.

We started by differentiating between dependent and independent variables and then delved into bivariate data, exploring how to represent it effectively using scatter plots. We also learned how to make inferences from bivariate data and understand the role of probability in media outlets. We wrapped up by examining dependent and independent events and how to solve problems involving them.

To help with teaching and learning these vital concepts, which are applicable in many fields, we recommend the following resources:

- Maths posters
- Whiteboard and graph boards

- Videos (including YouTube videos)
- Mini whiteboards, dry erase markers, and erasers
- Calculators
- Paper grids
- Technological tools like computers and mobile phones

MARKING SCHEMES FOR THE KEY ASSESSMENTS

WEEK 16

ASSESSMENT AND SCORING

You will be assessed on:

- **Accuracy of Scatter Plot:** Correct labelling, scaling, and plotting of points.
- **Data Collection/Sourcing:** Appropriateness of data, sufficient number of points, proper citation (if applicable).
- **Analysis and Interpretation:** Thoroughness and accuracy of answers to analysis questions and understanding of correlation.
- **Presentation:** Clarity, organisation, engagement, ability to explain your findings.

SCATTER PLOT PROJECT RUBRIC

Table 16.11: Scatter plot project rubric

Criteria	Excellent (4 marks)	Good (3 marks)	Satisfactory (2 marks)	Needs Improvement (1 mark)
1. Topic Selection & Relevance	The topic is clearly stated, relevant to real life, and shows originality.	The topic is clear and relevant but lacks originality.	The topic is somewhat clear; relevance to real life is weak.	The topic is unclear or irrelevant to the task.
2. Data Collection	Data is accurate, sufficient, relevant, and collected from reliable sources.	Data is mostly accurate, relevant, and sufficient.	Data is limited in quantity or somewhat relevant.	Data is incomplete, inaccurate, or irrelevant.
3. Scatter Plot	Correctly drawn with appropriate labels, scale, and title; points are accurately plotted.	Mostly correct with minor errors in labels, scale, or plotting.	Contains some errors in plotting or labelling that affect interpretation.	Scatter plot is poorly done or missing.
4. Analysis of Correlation	Clearly identifies correlation type (positive, negative, or none) and justifies conclusion with evidence from the plot.	Identifies correlation type but explanation is brief or lacks detail.	Attempts to identify correlation but explanation is unclear or incorrect.	No attempt to identify correlation or explanation is missing.

5. Presentation of Findings	Findings are well-organised, logically presented, and supported with data; implications are insightful and realistic.	Findings are clear and supported by data; implications are relevant but basic.	Findings are partly supported by data; implications are weak or unclear.	Findings are unclear, unsupported, or missing.
6. Communication Skills	Speaks clearly and confidently, uses correct terms, and engages audience effectively.	Speaks clearly with minor errors; some audience engagement.	Speech has several errors; little audience engagement.	Mumbles or reads without engaging the audience.

Scoring Guide

- **Excellent**: 21–24 marks
- Good: 16–20 marks
- **Satisfactory**: 10–15 marks
- **Needs Improvement**: Below 10 marks

How to Use This Rubric

- **Circulate Beforehand:** Provide this rubric to learners when you introduce the project so they understand the expectations from the beginning.
- **Self-Assessment/Peer Review:** Learners can use this rubric to self-assess their work or peer-review a classmate's project before final submission.
- **Feedback:** Provide specific, actionable feedback to learners on their strengths and areas for improvement.

WEEK 17

Mark Scheme

1.

Age (years)	12	13	13	14	15	15	16	16
Height (cm)	145	150	152	160	165	170	175	178

a. Appropriate Title for the Graph – (***1 mark***)

- Correct, clear, and relevant title that reflects both variables.
    - ***Example:*** *"Relationship Between Age and Height of SHS Students"* (***1 mark***)

b. Analyse the Trend on the Graph – (***3 marks***)

- Identification of positive correlation between age and height (***2 marks***)

- Statement that height increases with age or any other correct trend observed **(*1 mark*)**

c. Ways This Can Inform the PTA – **(*3 marks*)**

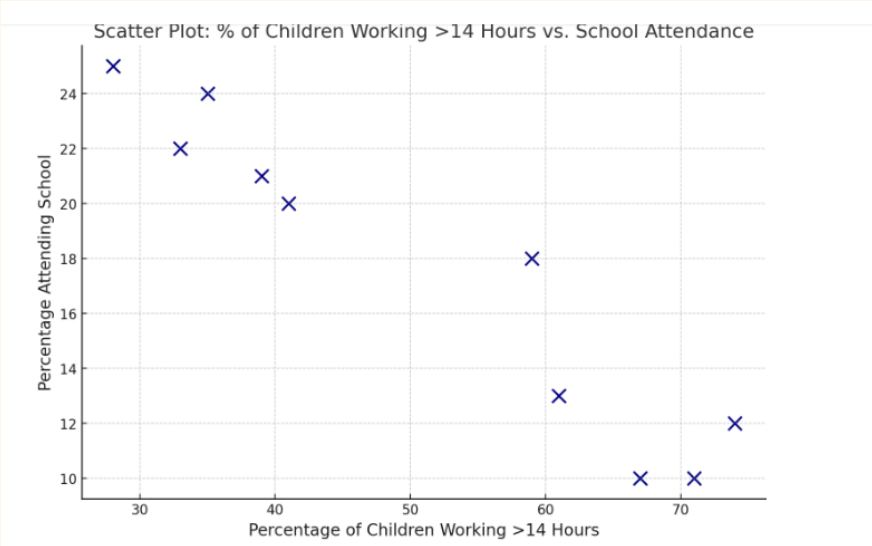
Award **1 mark** for each relevant way stated **(max *3 marks*)**

Examples

- Understanding the growth patterns of students
- Planning sports and physical activities
- Addressing nutritional needs
- Counselling and guidance for parents.

2.

a.



B1 for title

B1 for labelled axes

B1 for correct scale

B1 for correct plotting

Figure 20.9: *Scatter plot of children who worked for at least 15 hours and school attendance.*

b. There is a **negative correlation** between the **percentage of children who work at least 15 hours per week** and the **percentage attending school**. Points are almost linear. This means that, a**s child labour increases**, attendance at school **decreases.**

c. **B1** for identifying a negative correlation between the variables

d. **B1** for explaining the type of correlation. ***Example***: As child labour increases, school attendance decreases, or its equivalent.

e. ***1 mark*** for each reasonable suggestion. ***For example:***

Parents should be educated on the effects of child labour and the government should strengthen child protection policies.

The government could **introduce flexible school programmes where learners can learn from different times of the day and venues.**

WEEK 18

Table 18.9: Rubric for project.

Criteria	Excellent (4 marks)	Very good (3 marks)	Satisfactory (2 marks)	Needs improvement (1 marks)
1. Topic Selection & Relevance	The topic is clear, realistic, and relevant; it allows easy data collection	The topic is appropriate but slightly broad or lacks clarity	The topic is chosen, but not clearly related to real-world variables	The topic is unclear or not suitable for bivariate data
2. Data Collection	Data is accurate, complete, and from credible sources	Data is mostly complete and reliable	Some data is missing, or source reliability is unclear	The data is insufficient or unrelated
3. Data Representation	Charts/graphs are clear, accurate, and appropriate for the data	Visuals are mostly appropriate but may lack clarity or labelling	Visuals are present but unclear, mislabelled, or poorly chosen	Charts/graphs are missing or unrelated to the data
4. Data Analysis	Analysis clearly identifies patterns, trends, and relationships	Some patterns or trends are identified, but with limited explanation	Basic analysis with minimal understanding shown	No meaningful analysis provided
5. Conclusion & Implications	Findings are well explained with thoughtful implications or solutions	Findings are clear, but implications are basic or general	Basic summary given with little insight or relevance	The conclusion is missing or unrelated to the findings
6. Presentation & Organisation	The presentation is well-organised and clear. All required components are present. Student articulates findings clearly.	The presentation is well-organised, but it is missing a component or lacks clarity in certain areas. Students can articulate findings.	The presentation is well-organised, but it is missing some components or lacks clarity in certain areas. Students may struggle to articulate findings.	Presentation is disorganised, unclear, or missing several key components. Student struggles significantly to articulate findings.

GRADING SCALE

- ***20–24 marks:*** Excellent
- ***15–19 marks:*** Good
- ***10–14 marks:*** Fair
- ***Below 10 marks:*** Needs Improvement

How to Use This Rubric

- **Circulate Beforehand:** Provide this rubric to learners when you introduce the project so they understand the expectations from the beginning.
- **Self-Assessment/Peer Review:** Learners can use this rubric to self-assess their work or peer-review a classmate's project before final submission.
- **Feedback:** Use the "Teacher Comments" section to provide specific, actionable feedback to learners on their strengths and areas for improvement.

WEEK 19

Question One

Topic: "Investigating the Impact and Misconceptions of Media Statements That Use Probability"

MARKING SCHEME / RUBRIC (TOTAL: *30 MARKS*)

Table 19.1: Rubric for project

Criteria	Details	Marks	
1. Selection of media statements	At least 3 relevant and clear media statements involving probability identified	Score **1 mark** for each appropriate probability statement made **(up to a maximum of 3 marks)**	
2. Appropriate and diverse sources used (radio, internet, newspaper, interview, etc.)		**1 mark** for each media source mentioned and 1 mark for the date of retrieving statement **(6 marks)**	
3. Explanation of probability concepts used	Appropriate explanation of what each probability statement means in context	**2 marks** for each clear, simple explanation of the probability in the context of the statement **(6 marks)**	**1 mark** for simple but not clear explanation of the probability in the context of the statement

4. Identification of Misconceptions Realistic and clear misconceptions identified for each statement.	**2 marks** each of 2 clearly stated real-life misconceptions (**2 x 2=4 marks**) **2 marks** for additional point(s) explained. **(Total = 4+2=6)**	**1 mark** each if a statement is not clear or not real-life example of misconception
5. Analysis of Real-Life Impact Clearly shows how the probability statements influence decisions, behaviors, or plans in real life	**2 marks** each of 2 clear explanations of how probability statements influence decisions, behaviors, or plans in real life (2 x 2=4). **2 marks** for additional point(s) explained. **(Total = 4+2=6)**	**1 mark** for an explanation which is not clear.
6. Reflection and Critical Thinking Thoughtful answers to reflection questions- Shows understanding of the importance of clear communication and public awareness	**1 mark** for each thoughtful answer to a, b and c to reflection questions **(Total = 1 × 3 = 3)**	

Grading Guide

Table 19.2: Grading guide for project

Score Range	Performance Level	Feedback Suggestion
24–30 marks	Excellent	Shows deep understanding of probability and its real-life effects. Great effort in all sections.
18–23 marks	Very Good	Good analysis with minor gaps or missing details. Strong grasp of key ideas.
14–17 marks	Satisfactory	Basic understanding shown, but lacks depth or has incomplete explanations.
Below 14 marks	Needs Improvement	Little understanding of probability or weak media analysis. Incomplete or unclear responses.

Question Two

Monthly Rainfall and Temperature Data – Ghana

Given Criteria

- **High temperature:** $t \geq 28.80°C$
- **Low temperature:** $t < 28.80°C$
- **Heavy rainfall:** $f \geq 100$ mm

a. Probability of Low Temperature within the Year

From the graph, January, May, June, July, August, September, October, November, and December, a total of 9 months out of 12 months in the year have temperatures less than 28.80°C **B1**

Probability = Number of favorable months ÷ Total months

P(low temperature) $= \frac{9}{12} = 0.75$ or 75% **M1A1**

Impact: For example, People may expect cooler conditions in most months and may adjust clothing, farming, or energy use accordingly. **B1**

Possible misconception: For example, some may think that temperatures are low all year round, or that extreme cold is common, when in reality it's just lower than the "high" threshold, not necessarily cold. **B1**

b. Probability of Experiencing Heavy Rainfall ($f \geq 100$ mm)

From the graph, April, May, June, July, August, September, and October, a total of 7 months, have at least 100mm of rainfall. **B1**

P(heavy rainfall) $= \frac{7}{12} = 0.583$ or $\approx 58.3\%$ **M1A1**

Possible misconception: Gives any two reasonable misconceptions, for example, people may think that it rains heavily all the time, or that rainfall is constant across the country, even though this is based on averages and patterns that vary by region and year. **B2**

c. Probability of Low Temperatures AND Heavy Rainfall

We need months where $t < 28.80$°C and $f \geq 100$ mm

From the graph, May, June, July, August, September, and October satisfy these conditions = 6 months. **B1**

Probability $= \frac{6}{12} = 0.5$ or 50% **M1A1**

Layman's Explanation

"Out of every 12 months in a year, about 6 months are likely to have both cooler temperatures and heavy rains. This usually happens during the rainy season, when the weather is cooler because of the clouds and rainfall." **B2**

Marking Scheme Summary

Table 19.3: Marking scheme

Part	Description	Marks
(a)	Correct count (9), Correct calculation $\frac{9}{12} = 0.75$	B1 for correct count M1 for finding his/her probability A1 for the probability figure.
	Explanation of media impact	B1 for media impact

	Identification of misconception(s)	B1 for misconception
(b)	Correct count (7), correct probability ($\frac{7}{12}$ = 0.583)	B1 for correct count M1 for finding his/her probability A1 for the probability figure
	One misconception identified	B2 for stating the misconception
(c)	Correct count (6), correct probability ($\frac{6}{12}$ = 0.5)	B1 for correct count M1 for finding his/her probability A1 for the probability figure
	Clear explanation in simple terms	B2 for clear explanation
Total		**15 marks**

WEEK 20

Table 20.1: Marking Scheme.

Solution	Marks
Question One a. P(1st King) = $\frac{4}{52}$ If the king selected was red (P= $\frac{2}{52}$), P(2nd Red card) = $\frac{25}{51}$ If the king selected was black, (P= $\frac{2}{52}$), P(2nd Red card) = $\frac{26}{51}$ Therefore, P(a king followed by a red card) = $\frac{2}{52} \times \frac{25}{51} + \frac{2}{52} \times \frac{26}{51}$ = $\frac{25}{1326} + \frac{1}{51} = \frac{1}{26}$ b. P(1st diamond)= $\frac{13}{52} = \frac{1}{4}$ P(2nd black) = $\frac{26}{51}$ P(1st diamond and 2nd black card) = $\frac{1}{4} \times \frac{26}{51} = \frac{13}{102}$ c. P(1st odd) = $\frac{20}{52}$ and P(2nd odd) = $\frac{19}{51}$ P(1st odd and 2nd odd) = $\frac{20}{52} \times \frac{19}{51} = \frac{95}{663}$	A1 for P(1st king) M1 for forming equation 1 M1 for forming equation 2 A1 for $\frac{1}{26}$ M1 for forming equation A1 for $\frac{13}{102}$ M1 for forming equation A1 for $\frac{95}{663}$
Question Two $\frac{A}{B} = \frac{3}{5} \Rightarrow A = \frac{3}{5}B$ Also, $\frac{B}{C} = \frac{2}{3} \Rightarrow C = \frac{3}{2}B$ Therefore, A: B: C = $\frac{3}{5}B : B : \frac{3}{2}B$. This is the same as A: B: C = 6B:10B:15B Total ratio = 31B P(selecting a green ball) = P(A) = $\frac{6B}{31B} = \frac{6}{31}$	B2 for finding his/her ratio or total number of balls M1 for finding the probability of a green ball A1 for $\frac{6}{31}$

b.	M1 for forming equation
i. Probability of a yellow ball and an orange ball (any order): Two possible orders: P(Yellow then Orange) = $\frac{10}{31} \times \frac{15}{31} = \frac{150}{961}$ P(Orange then Yellow = $\frac{15}{31} \times \frac{10}{31} = \frac{150}{961}$	M1 for either multiplying it by 2, or repeating A1 for $\frac{300}{961}$
P(yellow and orange ball in any order) = $\frac{150}{961} + \frac{150}{961}$ $= \frac{300}{961}$	M1 for forming equation A1 for $\frac{60}{961}$
ii. P(Yellow and Green) = $\frac{10}{31} \times \frac{6}{31} = \frac{60}{961}$	

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